

Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.3.1-a+b-sin-^m-c+d-sin-ⁿ-A+B-sin-

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Contents

1	Introduction	3
1.1	Listing of CAS systems tested	3
1.2	Results	3
1.3	Performance	7
1.4	list of integrals that has no closed form antiderivative	8
1.5	list of integrals solved by CAS but has no known antiderivative	8
1.6	list of integrals solved by CAS but failed verification	8
1.7	Timing	9
1.8	Verification	9
1.9	Important notes about some of the results	9
1.9.1	Important note about Maxima results	9
1.9.2	Important note about FriCAS and Giac/XCAS results	10
1.9.3	Important note about finding leaf size of antiderivative	10
1.9.4	Important note about Mupad results	11
1.10	Design of the test system	11
2	detailed summary tables of results	13
2.1	List of integrals sorted by grade for each CAS	13
2.1.1	Rubi	13
2.1.2	Mathematica	13
2.1.3	Maple	14
2.1.4	Maxima	14
2.1.5	FriCAS	15
2.1.6	Sympy	15
2.1.7	Giac	15
2.1.8	Mupad	16
2.2	Detailed conclusion table per each integral for all CAS systems	17
2.3	Detailed conclusion table specific for Rubi results	76
3	Listing of integrals	87
3.1	$\int (d \sin(e + fx))^n (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) dx$	87
3.2	$\int (d \sin(e + fx))^n (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) dx$	91
3.3	$\int (d \sin(e + fx))^n (a + a \sin(e + fx)) (A + B \sin(e + fx)) dx$	94
3.4	$\int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{a+a \sin(e+fx)} dx$	97
3.5	$\int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$	100

3.6	$\int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^3} dx \dots\dots\dots$	103
3.7	$\int (d \sin(e+fx))^n (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx)) dx \dots\dots\dots$	106
3.8	$\int (d \sin(e+fx))^n (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx)) dx \dots\dots\dots$	110
3.9	$\int (d \sin(e+fx))^n \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx)) dx \dots\dots\dots$	114
3.10	$\int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{\sqrt{a+a \sin(e+fx)}} dx \dots\dots\dots$	117
3.11	$\int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^{3/2}} dx \dots\dots\dots$	121
3.12	$\int (d \sin(e+fx))^n (a+a \sin(e+fx))^m (A+B \sin(e+fx)) dx \dots\dots\dots$	126
3.13	$\int (d \sin(e+fx))^n (a-a \sin(e+fx))(a+a \sin(e+fx))^m dx \dots\dots\dots$	129
3.14	$\int \sin^n(c+dx)(a+a \sin(c+dx))^{-2-n} (-1-n-(-2-n) \sin(c+dx)) dx \dots\dots$	132
3.15	$\int \sin^{-2-m}(c+dx)(a+a \sin(c+dx))^m (1+m-m \sin(c+dx)) dx \dots\dots\dots$	134
3.16	$\int \frac{\sin^2(e+fx)(A+B \sin(e+fx))}{(a+b \sin(e+fx))^2} dx \dots\dots\dots$	139
3.17	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^4 dx \dots\dots\dots$	145
3.18	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^3 dx \dots\dots\dots$	149
3.19	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^2 dx \dots\dots\dots$	153
3.20	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx)) dx \dots\dots\dots$	157
3.21	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx \dots\dots\dots$	160
3.22	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx \dots\dots\dots$	164
3.23	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx \dots\dots\dots$	168
3.24	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx \dots\dots\dots$	172
3.25	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx \dots\dots\dots$	177
3.26	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^5 dx \dots\dots\dots$	183
3.27	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^4 dx \dots\dots\dots$	188
3.28	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^3 dx \dots\dots\dots$	193
3.29	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^2 dx \dots\dots\dots$	197
3.30	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx)) dx \dots\dots\dots$	201
3.31	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{c-c \sin(e+fx)} dx \dots\dots\dots$	205
3.32	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx \dots\dots\dots$	210
3.33	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx \dots\dots\dots$	215
3.34	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx \dots\dots\dots$	220
3.35	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx \dots\dots\dots$	225
3.36	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^6} dx \dots\dots\dots$	231
3.37	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^7} dx \dots\dots\dots$	238
3.38	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^6 dx \dots\dots\dots$	244
3.39	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^5 dx \dots\dots\dots$	250
3.40	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^4 dx \dots\dots\dots$	255
3.41	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^3 dx \dots\dots\dots$	260
3.42	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^2 dx \dots\dots\dots$	264
3.43	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx)) dx \dots\dots\dots$	268
3.44	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{c-c \sin(e+fx)} dx \dots\dots\dots$	272
3.45	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx \dots\dots\dots$	278
3.46	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx \dots\dots\dots$	284
3.47	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx \dots\dots\dots$	290

3.48	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx$	296
3.49	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6} dx$	302
3.50	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^7} dx$	307
3.51	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^8} dx$	313
3.52	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{a+a \sin(e+fx)} dx$	320
3.53	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{a+a \sin(e+fx)} dx$	328
3.54	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{a+a \sin(e+fx)} dx$	334
3.55	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{a+a \sin(e+fx)} dx$	339
3.56	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))} dx$	343
3.57	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^2} dx$	346
3.58	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^3} dx$	350
3.59	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^4} dx$	354
3.60	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^5}{(a+a \sin(e+fx))^2} dx$	359
3.61	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{(a+a \sin(e+fx))^2} dx$	369
3.62	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{(a+a \sin(e+fx))^2} dx$	377
3.63	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{(a+a \sin(e+fx))^2} dx$	383
3.64	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$	388
3.65	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))} dx$	392
3.66	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^2} dx$	396
3.67	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^3} dx$	399
3.68	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^4} dx$	404
3.69	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^5} dx$	410
3.70	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^5}{(a+a \sin(e+fx))^3} dx$	417
3.71	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{(a+a \sin(e+fx))^3} dx$	427
3.72	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{(a+a \sin(e+fx))^3} dx$	435
3.73	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{(a+a \sin(e+fx))^3} dx$	441
3.74	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{(a+a \sin(e+fx))^3} dx$	446
3.75	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))} dx$	450
3.76	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^2} dx$	454
3.77	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^3} dx$	459
3.78	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^4} dx$	463
3.79	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^5} dx$	470
3.80	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^6} dx$	478
3.81	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	483
3.82	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	487
3.83	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	491
3.84	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	494

3.85	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	497
3.86	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	501
3.87	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	505
3.88	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	509
3.89	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	513
3.90	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	518
3.91	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	522
3.92	$\int (a+a \sin(e+fx))^2 (A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	526
3.93	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	529
3.94	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	533
3.95	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	537
3.96	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	542
3.97	$\int \frac{(a+a \sin(e+fx))^2 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$	547
3.98	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	552
3.99	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	556
3.100	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	560
3.101	$\int (a+a \sin(e+fx))^3 (A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	564
3.102	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	567
3.103	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	572
3.104	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	577
3.105	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	582
3.106	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$	587
3.107	$\int \frac{(a+a \sin(e+fx))^3 (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$	592
3.108	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{a+a \sin(e+fx)} dx$	597
3.109	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{a+a \sin(e+fx)} dx$	601
3.110	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{a+a \sin(e+fx)} dx$	604
3.111	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{a+a \sin(e+fx)} dx$	607
3.112	$\int \frac{(a+a \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{A+B \sin(e+fx)} dx$	624
3.113	$\int \frac{(a+a \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{A+B \sin(e+fx)} dx$	628
3.114	$\int \frac{(a+a \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{A+B \sin(e+fx)} dx$	632
3.115	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^2} dx$	636
3.116	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^2} dx$	640
3.117	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^2} dx$	644
3.118	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^2} dx$	648
3.119	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^2} dx$	652
3.120	$\int \frac{(a+a \sin(e+fx))^2 \sqrt{c-c \sin(e+fx)}}{A+B \sin(e+fx)} dx$	655
3.121	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2 (c-c \sin(e+fx))^{3/2}} dx$	659

3.122	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^{5/2}} dx$	663
3.123	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^3} dx$	668
3.124	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^3} dx$	672
3.125	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^3} dx$	676
3.126	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^3} dx$	680
3.127	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^3} dx$	684
3.128	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3 \sqrt{c-c \sin(e+fx)}} dx$	687
3.129	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^{3/2}} dx$	691
3.130	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^{5/2}} dx$	696
3.131	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	701
3.132	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	704
3.133	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	707
3.134	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	710
3.135	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	713
3.136	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	718
3.137	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	721
3.138	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	724
3.139	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	727
3.140	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	731
3.141	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	735
3.142	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	738
3.143	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	741
3.144	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	746
3.145	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	750
3.146	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	754
3.147	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$	757
3.148	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$	760
3.149	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2} dx$	763
3.150	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2} dx$	767
3.151	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx$	771
3.152	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)} dx$	775
3.153	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	778
3.154	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	782
3.155	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	786
3.156	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	790
3.157	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$	794
3.158	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$	797
3.159	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{13/2}} dx$	800

3.160	$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2} dx$	803
3.161	$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$	808
3.162	$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$	812
3.163	$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$	816
3.164	$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx$	820
3.165	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	823
3.166	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	827
3.167	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	831
3.168	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$	835
3.169	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$	840
3.170	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$	844
3.171	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{13/2}} dx$	847
3.172	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{15/2}} dx$	851
3.173	$\int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{17/2}} dx$	855
3.174	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{\sqrt{a+a \sin(e+fx)}} dx$	859
3.175	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{\sqrt{a+a \sin(e+fx)}} dx$	863
3.176	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{\sqrt{a+a \sin(e+fx)}} dx$	868
3.177	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c-c \sin(e+fx)}} dx$	873
3.178	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{3/2}} dx$	876
3.179	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{5/2}} dx$	879
3.180	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^{3/2}} dx$	883
3.181	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^{3/2}} dx$	887
3.182	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^{3/2}} dx$	891
3.183	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{3/2}} dx$	895
3.184	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2} \sqrt{c-c \sin(e+fx)}} dx$	898
3.185	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2} (c-c \sin(e+fx))^{3/2}} dx$	901
3.186	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2} (c-c \sin(e+fx))^{5/2}} dx$	904
3.187	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^{5/2}} dx$	908
3.188	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^{5/2}} dx$	913
3.189	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^{5/2}} dx$	917
3.190	$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^{5/2}} dx$	921
3.191	$\int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{5/2}} dx$	925
3.192	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2} \sqrt{c-c \sin(e+fx)}} dx$	928
3.193	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2} (c-c \sin(e+fx))^{3/2}} dx$	932
3.194	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2} (c-c \sin(e+fx))^{5/2}} dx$	936
3.195	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^n dx$	940

3.196	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx$	944
3.197	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx$	947
3.198	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx)) dx$	950
3.199	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx$	953
3.200	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{c-c \sin(e+fx)} dx$	956
3.201	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx$	959
3.202	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx$	962
3.203	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	965
3.204	$\int \frac{(A+B \sin(e+fx))(c+c \sin(e+fx))^m}{\sqrt{a-a \sin(e+fx)}} dx$	968
3.205	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$	971
3.206	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$	975
3.207	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx$	979
3.208	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$	982
3.209	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$	985
3.210	$\int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$	989
3.211	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-4-m} dx$	992
3.212	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-3-m} dx$	995
3.213	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-2-m} dx$	998
3.214	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-1-m} dx$	1001
3.215	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-m} dx$	1005
3.216	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{1-m} dx$	1009
3.217	$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{2-m} dx$	1014
3.218	$\int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^n (B(3 - n) - B(4 + n) \sin(e + fx)) dx$	1017
3.219	$\int (a - a \sin(e + fx))^3 (c + c \sin(e + fx))^n (B(3 - n) + B(4 + n) \sin(e + fx)) dx$	1020
3.220	$\int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^3 (B(-3 + m) - B(4 + m) \sin(e + fx)) dx$	1023
3.221	$\int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^3 (B(-3 + m) + B(4 + m) \sin(e + fx)) dx$	1026
3.222	$\int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n (B(m - n) - B(1 + m + n) \sin(e + fx)) dx$	1029
3.223	$\int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^n (B(m - n) + B(1 + m + n) \sin(e + fx)) dx$	1031
3.224	$\int \sin^3(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1033
3.225	$\int \sin^2(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1037
3.226	$\int \sin(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1041
3.227	$\int (a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1044
3.228	$\int \csc(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1047
3.229	$\int \csc^2(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1050
3.230	$\int \csc^3(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1054
3.231	$\int \csc^4(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1057
3.232	$\int \csc^5(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1060
3.233	$\int \csc^6(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1063
3.234	$\int \csc^7(c + dx)(a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$	1067
3.235	$\int \frac{\sin^4(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1071
3.236	$\int \frac{\sin^3(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1077
3.237	$\int \frac{\sin^2(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1082
3.238	$\int \frac{\sin(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1086
3.239	$\int \frac{A-A \sin(c+dx)}{(a+a \sin(c+dx))^3} dx$	1090
3.240	$\int \frac{\csc(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1093

3.241	$\int \frac{\csc^2(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1097
3.242	$\int \frac{\csc^3(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1101
3.243	$\int \frac{\csc^4(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$	1105
3.244	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1109
3.245	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1114
3.246	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1118
3.247	$\int (a+a \sin(e+fx))(A+B \sin(e+fx)) dx$	1121
3.248	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1123
3.249	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1128
3.250	$\int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1134
3.251	$\int (a+a \sin(e+fx))^2(A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1139
3.252	$\int (a+a \sin(e+fx))^2(A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1145
3.253	$\int (a+a \sin(e+fx))^2(A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1150
3.254	$\int (a+a \sin(e+fx))^2(A+B \sin(e+fx)) dx$	1154
3.255	$\int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1157
3.256	$\int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1164
3.257	$\int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1172
3.258	$\int (a+a \sin(e+fx))^3(A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1180
3.259	$\int (a+a \sin(e+fx))^3(A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1187
3.260	$\int (a+a \sin(e+fx))^3(A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1193
3.261	$\int (a+a \sin(e+fx))^3(A+B \sin(e+fx)) dx$	1198
3.262	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1202
3.263	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1211
3.264	$\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1221
3.265	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{a+a \sin(e+fx)} dx$	1233
3.266	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{a+a \sin(e+fx)} dx$	1238
3.267	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{a+a \sin(e+fx)} dx$	1244
3.268	$\int \frac{A+B \sin(e+fx)}{a+a \sin(e+fx)} dx$	1248
3.269	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))} dx$	1251
3.270	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))^2} dx$	1255
3.271	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))^3} dx$	1260
3.272	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^2} dx$	1267
3.273	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^2} dx$	1272
3.274	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$	1278
3.275	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2} dx$	1282
3.276	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))} dx$	1285
3.277	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))^2} dx$	1289
3.278	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))^3} dx$	1295
3.279	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^3} dx$	1303

3.280	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^3} dx$	1313
3.281	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^3} dx$	1319
3.282	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3} dx$	1323
3.283	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))} dx$	1327
3.284	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))^2} dx$	1332
3.285	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))^3} dx$	1339
3.286	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1349
3.287	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1353
3.288	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1357
3.289	$\int \sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx)) dx$	1360
3.290	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1363
3.291	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1367
3.292	$\int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1371
3.293	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1376
3.294	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1382
3.295	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1387
3.296	$\int (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx)) dx$	1391
3.297	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1394
3.298	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1398
3.299	$\int \frac{(a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1402
3.300	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c+d \sin(e+fx))^3 dx$	1407
3.301	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1414
3.302	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1419
3.303	$\int (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx)) dx$	1423
3.304	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1426
3.305	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1430
3.306	$\int \frac{(a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1435
3.307	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{\sqrt{a+a \sin(e+fx)}} dx$	1440
3.308	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{\sqrt{a+a \sin(e+fx)}} dx$	1446
3.309	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{\sqrt{a+a \sin(e+fx)}} dx$	1451
3.310	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$	1455
3.311	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))} dx$	1458
3.312	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))^2} dx$	1463
3.313	$\int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))^3} dx$	1468
3.314	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^{3/2}} dx$	1476
3.315	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^{3/2}} dx$	1483
3.316	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^{3/2}} dx$	1488
3.317	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$	1492

3.318	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))} dx$	1495
3.319	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))^2} dx$	1501
3.320	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))^3} dx$	1508
3.321	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^{5/2}} dx$	1518
3.322	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^{5/2}} dx$	1524
3.323	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^{5/2}} dx$	1529
3.324	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$	1533
3.325	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))} dx$	1537
3.326	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))^2} dx$	1544
3.327	$\int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))^3} dx$	1554
3.328	$\int (a+a \sin(e+fx))^2(A+B \sin(e+fx))(c+d \sin(e+fx))^n dx$	1564
3.329	$\int (a+a \sin(e+fx))(A+B \sin(e+fx))(c+d \sin(e+fx))^n dx$	1567
3.330	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{a+a \sin(e+fx)} dx$	1570
3.331	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{(a+a \sin(e+fx))^2} dx$	1573
3.332	$\int (a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))(c+d \sin(e+fx))^n dx$	1576
3.333	$\int \sqrt{a+a \sin(e+fx)}(A+B \sin(e+fx))(c+d \sin(e+fx))^n dx$	1580
3.334	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{\sqrt{a+a \sin(e+fx)}} dx$	1583
3.335	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{(a+a \sin(e+fx))^{3/2}} dx$	1587
3.336	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))(c+d \sin(e+fx))^2 dx$	1591
3.337	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))(c+d \sin(e+fx)) dx$	1595
3.338	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx)) dx$	1598
3.339	$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$	1601
3.340	$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1605
3.341	$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$	1609
3.342	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))(c+d \sin(e+fx))^{3/2} dx$	1614
3.343	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))\sqrt{c+d \sin(e+fx)} dx$	1619
3.344	$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{\sqrt{c+d \sin(e+fx)}} dx$	1623
3.345	$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{(c+d \sin(e+fx))^{3/2}} dx$	1627
3.346	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))(c+d \sin(e+fx))^n dx$	1631
3.347	$\int (a+a \sin(e+fx))^m(A+B \sin(e+fx))(c+d \sin(e+fx))^{-1-m} dx$	1635
3.348	$\int (a-a \sin(e+fx))(a+a \sin(e+fx))^m(c+d \sin(e+fx))^n dx$	1639
3.349	$\int (a-a \sin(e+fx))(a+a \sin(e+fx))^m(c+d \sin(e+fx))^{-1-m} dx$	1642
3.350	$\int (a+a \sin(e+fx))^m(c+d \sin(e+fx))^{-2-m}(d-(c-d)m+(c+(c-d)m) \sin(e+fx)) dx$	1645
3.351	$\int (a-a \sin(e+fx))^m(c+d \sin(e+fx))^{-2-m}(d+(c+d)m+(c+(c+d)m) \sin(e+fx)) dx$	1647
3.352	$\int \frac{(a+b \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$	1649
3.353	$\int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^{3/2}}{(a+b \sin(e+fx))^{3/2}} dx$	1659
3.354	$\int \frac{(A+B \sin(e+fx))\sqrt{c+d \sin(e+fx)}}{(a+b \sin(e+fx))^{3/2}} dx$	1664
3.355	$\int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2}\sqrt{c+d \sin(e+fx)}} dx$	1669
3.356	$\int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2}(c+d \sin(e+fx))^{3/2}} dx$	1673

3.357	$\int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2}(c+d \sin(e+fx))^{5/2}} dx$	1678
3.358	$\int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$	1683
4	Listing of Grading functions	1685
4.0.1	Mathematica and Rubi grading function	1685
4.0.2	Maple grading function	1687
4.0.3	Sympy grading function	1690
4.0.4	SageMath grading function	1692

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [358]. This is test number [76].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (358)	% 0.00 (0)
Mathematica	% 97.21 (348)	% 2.79 (10)
Maple	% 81.01 (290)	% 18.99 (68)
Maxima	% 37.15 (133)	% 62.85 (225)
Fricas	% 76.82 (275)	% 23.18 (83)
Sympy	% 25.14 (90)	% 74.86 (268)
Giac	% 36.31 (130)	% 63.69 (228)
Mupad	% 49.72 (178)	% 50.28 (180)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

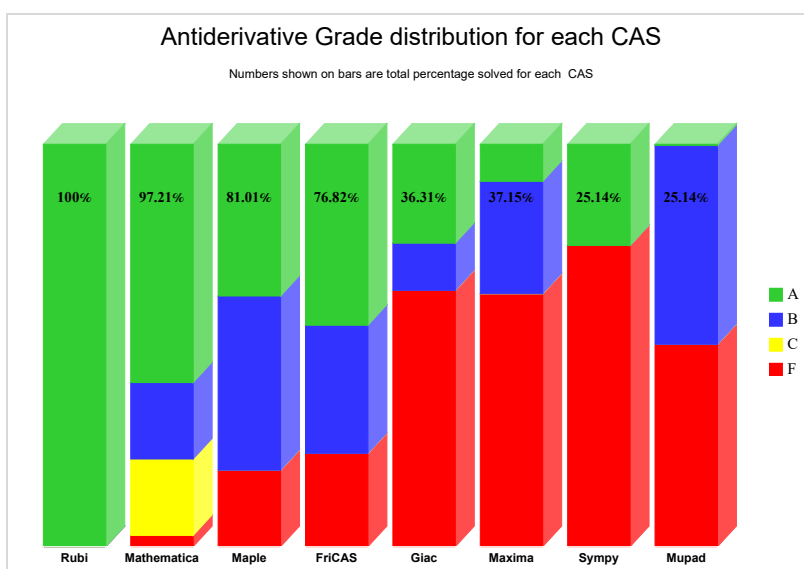
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

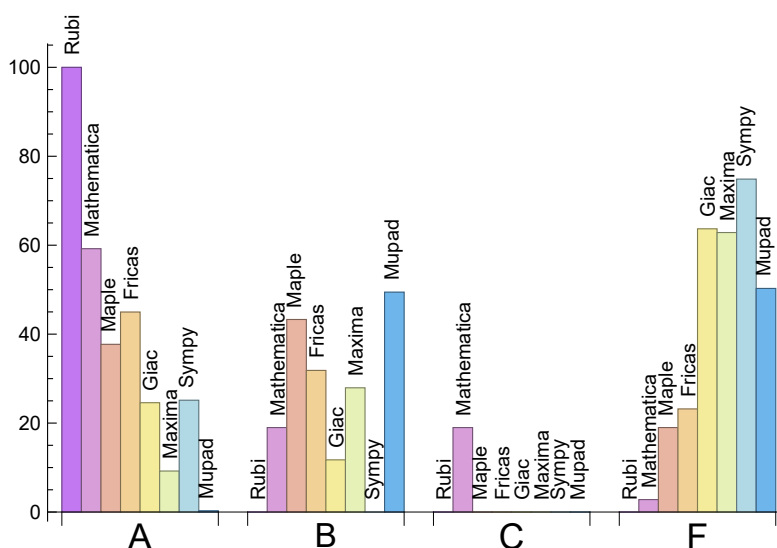
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	59.22	18.99	18.99	2.79
Maple	37.71	43.30	0.00	18.99
Maxima	9.22	27.93	0.00	62.85
Fricas	44.97	31.84	0.00	23.18
Sympy	25.14	0.00	0.00	74.86
Giac	24.58	11.73	0.00	63.69
Mupad	0.28	49.44	0.00	50.28

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	10	80.00 %	20.00 %	0.00 %
Maple	68	100.00 %	0.00 %	0.00 %
Maxima	225	83.11 %	8.00 %	8.89 %
Fricas	83	98.80 %	0.00 %	1.20 %
Sympy	268	30.60 %	69.03 %	0.37 %
Giac	228	33.33 %	27.19 %	39.47 %
Mupad	180	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

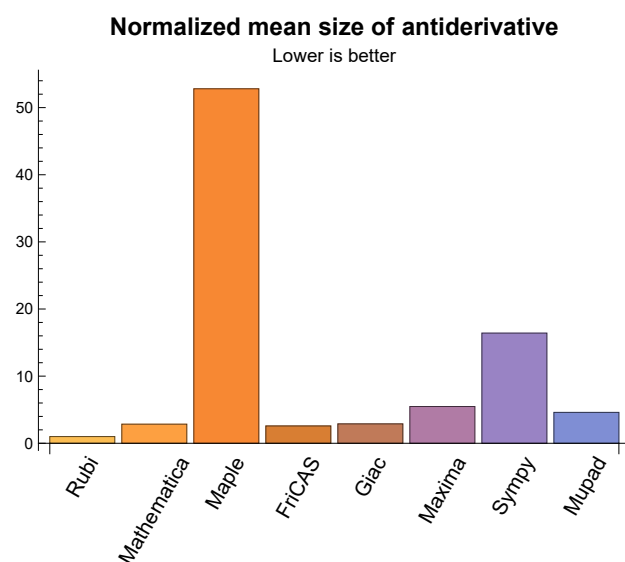
1.3 Performance

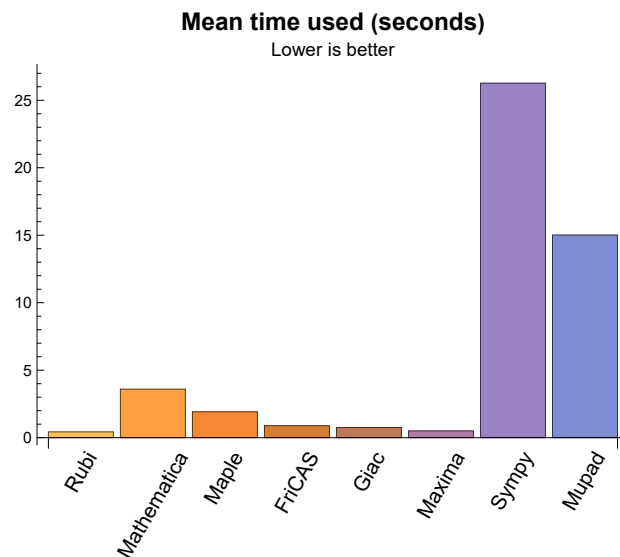
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.43	179.38	1.00	156.00	1.00
Mathematica	3.59	496.05	2.85	223.00	1.47
Maple	1.92	38573.66	52.81	274.00	1.97
Maxima	0.50	782.17	5.48	506.00	4.15
Fricas	0.89	527.67	2.58	234.00	1.77
Sympy	26.27	2372.78	16.41	1443.00	12.12
Giac	0.75	311.58	2.89	215.00	1.50
Mupad	15.01	833.30	4.60	298.50	2.09

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{358}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {7, 8, 10, 11, 12, 88, 195, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 215, 216, 217, 243, 304, 305, 306, 313, 320, 326, 327, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

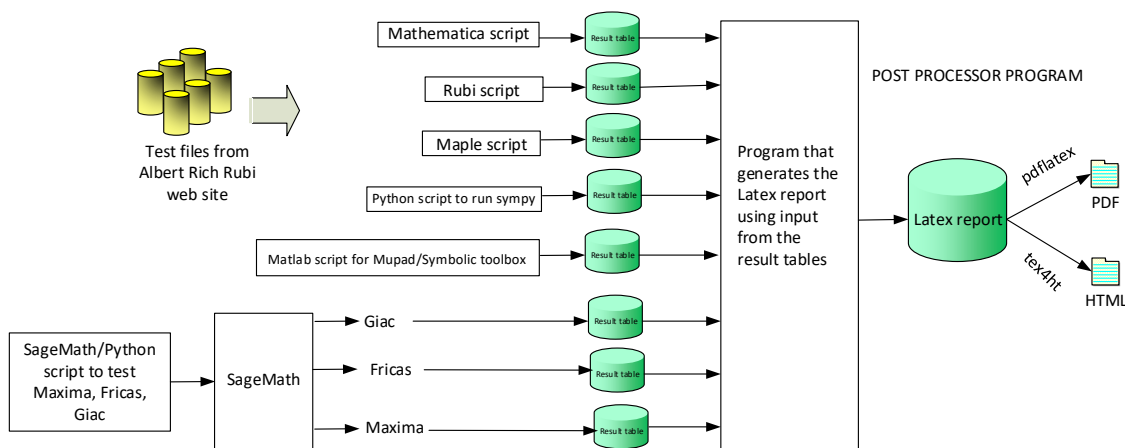
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 4, 5, 6, 7, 10, 15, 16, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 69, 70, 71, 74, 75, 77, 80, 81, 82, 83, 84, 85, 86, 87, 88, 91, 92, 101, 108, 109, 110, 111, 116, 117, 118, 119, 123, 124, 125, 126, 127, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 173, 174, 175, 177, 178, 179, 180, 181, 182, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 203, 204, 206, 207, 208, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 235, 238, 239, 241, 242, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 266, 267, 269, 270, 271, 275, 276, 277, 279, 281, 282, 285, 286, 287, 288, 289, 293, 294, 295, 296, 298, 299, 302, 303, 305, 306, 332, 334, 336, 337, 341, 350, 351, 352, 358 }

B grade: { 8, 11, 12, 14, 21, 22, 32, 33, 34, 35, 46, 47, 48, 49, 50, 55, 63, 64, 67, 68, 72, 73, 76, 78, 79, 89, 90, 98, 99, 100, 115, 170, 171, 172, 232, 233, 234, 236, 237, 240, 243, 264, 265, 268, 272, 273, 274, 278, 280, 283, 284, 297, 301, 304, 335, 339, 340, 342, 343, 344, 345, 346, 347, 353, 354, 355, 356, 357 }

C grade: { 3, 9, 93, 94, 95, 96, 97, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 135, 136, 176, 183, 195, 198, 199, 200, 201, 202, 205, 209, 210, 214, 215, 216, 217, 248, 249, 250, 290, 291, 292, 300, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 338 }

F grade: { 13, 196, 197, 328, 329, 330, 331, 333, 348, 349 }

2.1.3 Maple

A grade: { 18, 20, 21, 23, 24, 25, 32, 35, 36, 37, 43, 50, 51, 55, 56, 57, 58, 59, 63, 65, 68, 69, 74, 75, 80, 81, 82, 83, 84, 85, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 137, 139, 140, 141, 142, 147, 149, 150, 151, 152, 160, 161, 162, 163, 177, 185, 191, 194, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 268, 269, 275, 281, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 310, 311, 358 }

B grade: { 16, 17, 19, 22, 26, 27, 28, 29, 30, 31, 33, 34, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 52, 53, 54, 60, 61, 62, 64, 66, 67, 70, 71, 72, 73, 76, 77, 78, 79, 86, 87, 88, 95, 96, 97, 104, 105, 106, 107, 114, 122, 131, 135, 136, 138, 143, 144, 145, 146, 148, 153, 154, 155, 156, 157, 158, 159, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 190, 192, 193, 235, 248, 249, 250, 255, 256, 257, 260, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352, 353, 354, 355, 356, 357 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351 }

2.1.4 Maxima

A grade: { 17, 18, 20, 30, 43, 56, 66, 77, 135, 176, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 261, 358 }

B grade: { 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79, 80, 108, 109, 110, 111, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 144, 155, 168, 182, 189, 205, 206, 207, 235, 236, 237, 238, 239, 240, 241, 242, 243, 260, 265, 266, 267, 268, 272, 273, 274, 275, 279, 280, 281, 282 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 112, 113, 114, 120, 121, 122, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 248, 249, 250, 255, 256, 257, 262, 263, 264, 269, 270, 271, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357 }

2.1.5 FriCAS

A grade: { 14, 15, 17, 18, 19, 20, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 89, 102, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 137, 138, 139, 140, 141, 142, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 178, 179, 184, 185, 186, 191, 192, 193, 194, 206, 207, 211, 212, 213, 222, 223, 224, 225, 226, 227, 228, 229, 244, 245, 246, 247, 248, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 268, 275, 282, 286, 287, 288, 289, 290, 293, 294, 295, 296, 300, 301, 302, 303, 350, 351, 358 }

B grade: { 16, 21, 22, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51, 55, 63, 64, 71, 72, 73, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 170, 205, 218, 219, 220, 221, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 249, 250, 257, 264, 265, 266, 267, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 283, 284, 285, 291, 292, 297, 298, 299, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 135, 136, 143, 144, 145, 153, 154, 155, 156, 165, 166, 167, 168, 169, 174, 175, 176, 177, 180, 181, 182, 183, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.1.6 Sympy

A grade: { 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 224, 225, 226, 227, 235, 236, 237, 238, 239, 244, 245, 246, 247, 251, 252, 253, 254, 258, 259, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 281, 282 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 37, 49, 50, 51, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 231, 232, 233, 234, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 262, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358 }

2.1.7 Giac

A grade: { 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 52, 53, 54, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 70, 71, 72, 73, 74, 75, 77, 224, 225, 226, 227, 228, 230, 231, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 254, 255, 258, 259, 260, 261, 266, 268, 269, 274, 275, 276, 277, 281, 282, 358 }

B grade: { 15, 16, 21, 34, 35, 36, 37, 48, 49, 50, 51, 55, 61, 67, 68, 69, 76, 78, 79, 80, 229, 232, 234, 250, 256, 257, 262, 263, 264, 265, 267, 270, 271, 272, 273, 278, 279, 280, 283, 284, 285, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357 }

2.1.8 Mupad

A grade: { 358 }

B grade: { 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 111, 118, 119, 125, 126, 127, 131, 132, 133, 134, 138, 139, 140, 141, 142, 147, 148, 149, 150, 151, 152, 158, 159, 160, 161, 162, 163, 164, 171, 172, 173, 191, 205, 206, 207, 211, 212, 213, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 310, 350, 351, 352 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 120, 121, 122, 123, 124, 128, 129, 130, 135, 136, 137, 143, 144, 145, 146, 153, 154, 155, 156, 157, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 208, 209, 210, 214, 215, 216, 217, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 353, 354, 355, 356, 357 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	248	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	2.349	8.248	0.000	0.447	0.000	0.000	0.000
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	204	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.492	1.548	7.543	0.000	0.441	0.000	0.000	0.000
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	392	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	3.846	5.395	0.000	0.427	0.000	0.000	0.000
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	157	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	0.889	1.973	0.000	0.448	0.000	0.000	0.000
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	212	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.327	11.460	0.000	0.446	0.000	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	260	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.846	4.574	11.615	0.000	0.446	0.000	0.000	0.000
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	596	0	0	0	0	0	-1
normalized size	1	1.00	1.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	18.383	1.087	0.000	0.485	0.000	0.000	0.000
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	478	0	0	0	0	0	-1
normalized size	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	15.404	1.051	0.000	0.478	0.000	0.000	0.000
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	409	0	0	0	0	0	-1
normalized size	1	1.00	2.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	67.653	1.055	0.000	0.467	0.000	0.000	0.000
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	250	0	0	0	0	0	-1
normalized size	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.396	4.857	0.895	0.000	0.462	0.000	0.000	0.000
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	523	0	0	0	0	0	-1
normalized size	1	1.00	2.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	21.903	0.893	0.000	0.455	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	5918	0	0	0	0	0	-1
normalized size	1	1.00	26.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.453	22.178	6.039	0.000	0.451	0.000	0.000	0.000
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	11.252	5.908	0.000	0.457	0.000	0.000	0.000
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	107	0	0	41	0	0	61
normalized size	1	1.00	2.89	0.00	0.00	1.11	0.00	0.00	1.65
time (sec)	N/A	0.119	1.635	4.153	0.000	0.448	0.000	0.000	12.982
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	0	41	0	5502	38
normalized size	1	1.00	1.00	0.00	0.00	1.17	0.00	157.20	1.09
time (sec)	N/A	0.094	0.386	1.659	0.000	0.446	0.000	64.011	12.848
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	147	493	0	804	0	371	3718
normalized size	1	1.00	0.96	3.22	0.00	5.25	0.00	2.42	24.30
time (sec)	N/A	0.393	0.879	0.372	0.000	0.522	0.000	0.177	17.182
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	131	342	336	123	853	184	454
normalized size	1	1.00	0.72	1.88	1.85	0.68	4.69	1.01	2.49
time (sec)	N/A	0.295	0.983	0.586	0.602	0.439	7.882	0.172	14.833

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	95	208	200	102	486	145	389
normalized size	1	1.00	0.67	1.46	1.41	0.72	3.42	1.02	2.74
time (sec)	N/A	0.250	0.843	0.485	0.328	0.436	4.093	0.179	13.800
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	105	74	185	179	82	396	114	345
normalized size	1	1.08	0.76	1.91	1.85	0.85	4.08	1.18	3.56
time (sec)	N/A	0.186	0.675	0.391	0.334	0.447	2.126	0.143	13.381
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	74	73	43	138	58	122
normalized size	1	1.00	0.98	1.51	1.49	0.88	2.82	1.18	2.49
time (sec)	N/A	0.083	0.156	0.287	0.383	0.449	0.788	0.145	14.329
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	125	113	265	116	828	124	111
normalized size	1	1.00	2.23	2.02	4.73	2.07	14.79	2.21	1.98
time (sec)	N/A	0.169	0.928	0.407	0.588	0.424	4.034	0.154	12.636
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	160	160	456	162	700	92	132
normalized size	1	1.00	2.22	2.22	6.33	2.25	9.72	1.28	1.83
time (sec)	N/A	0.225	0.625	0.438	0.435	0.440	8.108	0.149	12.506
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	147	115	737	183	1035	139	172
normalized size	1	1.00	1.41	1.11	7.09	1.76	9.95	1.34	1.65
time (sec)	N/A	0.237	0.736	0.481	0.533	0.414	15.781	0.183	12.981

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	174	159	1080	251	1831	187	228
normalized size	1	1.00	1.23	1.12	7.61	1.77	12.89	1.32	1.61
time (sec)	N/A	0.285	0.902	0.467	0.386	0.423	29.472	0.199	13.223
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	200	203	1425	305	3232	267	310
normalized size	1	1.00	1.14	1.15	8.10	1.73	18.36	1.52	1.76
time (sec)	N/A	0.307	0.888	0.513	0.398	0.429	52.384	0.211	13.345
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	219	569	571	158	1586	278	661
normalized size	1	1.00	0.96	2.48	2.49	0.69	6.93	1.21	2.89
time (sec)	N/A	0.368	2.043	0.670	0.543	0.462	25.002	0.225	15.132
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	163	463	460	135	1210	244	553
normalized size	1	1.00	0.86	2.45	2.43	0.71	6.40	1.29	2.93
time (sec)	N/A	0.296	1.587	0.574	0.341	0.449	15.016	0.214	14.888
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	137	365	360	114	910	208	542
normalized size	1	1.00	0.93	2.48	2.45	0.78	6.19	1.41	3.69
time (sec)	N/A	0.216	1.085	0.571	0.468	0.442	9.139	0.183	14.154
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	54	166	164	75	372	118	238
normalized size	1	1.00	0.61	1.87	1.84	0.84	4.18	1.33	2.67
time (sec)	N/A	0.137	0.149	0.480	0.490	0.424	3.697	0.153	14.205

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	67	186	179	77	396	111	339
normalized size	1	1.00	0.68	1.90	1.83	0.79	4.04	1.13	3.46
time (sec)	N/A	0.149	0.796	0.388	0.365	0.433	2.434	0.147	13.705
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	191	299	624	177	2365	163	244
normalized size	1	1.00	1.63	2.56	5.33	1.51	20.21	1.39	2.09
time (sec)	N/A	0.289	1.299	0.431	0.484	0.429	8.092	0.177	14.701
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	238	198	839	237	2474	135	246
normalized size	1	1.00	2.18	1.82	7.70	2.17	22.70	1.24	2.26
time (sec)	N/A	0.284	0.631	0.460	0.441	0.441	16.123	0.160	14.150
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	278	249	1139	277	1647	159	233
normalized size	1	1.00	2.48	2.22	10.17	2.47	14.71	1.42	2.08
time (sec)	N/A	0.277	0.736	0.505	0.628	0.443	26.923	0.181	14.679
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	191	161	1571	263	2008	229	269
normalized size	1	1.00	2.55	2.15	20.95	3.51	26.77	3.05	3.59
time (sec)	N/A	0.229	0.950	0.474	0.612	0.428	45.068	0.193	13.351
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	261	205	2087	335	3262	301	331
normalized size	1	1.00	2.27	1.78	18.15	2.91	28.37	2.62	2.88
time (sec)	N/A	0.286	1.242	0.526	0.422	0.414	78.088	0.231	13.358

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	285	249	2604	407	4816	373	423
normalized size	1	1.00	1.83	1.60	16.69	2.61	30.87	2.39	2.71
time (sec)	N/A	0.374	1.599	0.513	0.455	0.422	125.915	0.250	13.544
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	313	293	3120	475	0	445	500
normalized size	1	1.00	1.59	1.49	15.84	2.41	0.00	2.26	2.54
time (sec)	N/A	0.465	3.646	0.573	0.689	0.444	0.000	0.325	14.053
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	255	651	661	181	1948	347	812
normalized size	1	1.00	0.96	2.46	2.49	0.68	7.35	1.31	3.06
time (sec)	N/A	0.391	4.308	0.853	0.359	0.502	52.762	0.274	14.869
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	232	611	617	158	1753	301	705
normalized size	1	1.00	1.05	2.75	2.78	0.71	7.90	1.36	3.18
time (sec)	N/A	0.322	2.571	0.763	0.347	0.503	37.384	0.254	14.929
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	568	571	137	1579	273	661
normalized size	1	1.00	1.15	3.14	3.15	0.76	8.72	1.51	3.65
time (sec)	N/A	0.234	1.951	0.674	0.356	0.467	23.432	0.225	14.782
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	64	263	264	92	682	162	325
normalized size	1	1.00	0.55	2.25	2.26	0.79	5.83	1.38	2.78
time (sec)	N/A	0.148	0.229	0.666	0.383	0.454	10.351	0.186	14.288

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	133	364	360	106	910	204	536
normalized size	1	1.00	0.96	2.64	2.61	0.77	6.59	1.48	3.88
time (sec)	N/A	0.200	1.086	0.576	0.344	0.439	8.101	0.190	14.166
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	208	200	100	486	145	390
normalized size	1	1.00	0.68	1.49	1.43	0.71	3.47	1.04	2.79
time (sec)	N/A	0.222	0.854	0.481	0.524	0.433	4.211	0.212	13.664
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	223	449	1139	218	4255	234	323
normalized size	1	1.00	1.43	2.88	7.30	1.40	27.28	1.50	2.07
time (sec)	N/A	0.310	1.424	0.443	0.469	0.437	15.993	0.219	14.120
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	280	399	1386	286	4665	233	341
normalized size	1	1.00	1.72	2.45	8.50	1.75	28.62	1.43	2.09
time (sec)	N/A	0.348	0.877	0.470	0.608	0.451	29.894	0.181	14.005
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	316	323	1685	337	4665	226	336
normalized size	1	1.00	2.07	2.11	11.01	2.20	30.49	1.48	2.20
time (sec)	N/A	0.342	1.105	0.509	0.702	0.445	48.977	0.201	14.002
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	356	374	2118	363	2951	213	316
normalized size	1	1.00	2.36	2.48	14.03	2.40	19.54	1.41	2.09
time (sec)	N/A	0.330	1.212	0.485	0.581	0.442	76.726	0.239	15.978

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	283	205	2701	331	3262	301	346
normalized size	1	1.00	3.68	2.66	35.08	4.30	42.36	3.91	4.49
time (sec)	N/A	0.236	2.516	0.527	0.537	0.426	116.395	0.269	13.392
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	313	249	3390	405	0	373	408
normalized size	1	1.00	2.65	2.11	28.73	3.43	0.00	3.16	3.46
time (sec)	N/A	0.289	2.939	0.519	0.529	0.448	0.000	0.312	13.544
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	339	293	4078	475	0	445	500
normalized size	1	1.00	2.17	1.88	26.14	3.04	0.00	2.85	3.21
time (sec)	N/A	0.375	5.285	0.571	0.572	0.452	0.000	0.364	13.794
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	378	337	4765	541	0	517	577
normalized size	1	1.00	1.92	1.71	24.19	2.75	0.00	2.62	2.93
time (sec)	N/A	0.444	6.694	0.576	0.683	0.455	0.000	0.388	13.963
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	274	678	1796	261	6690	343	397
normalized size	1	1.00	1.44	3.57	9.45	1.37	35.21	1.81	2.09
time (sec)	N/A	0.363	2.362	0.429	0.644	0.461	26.866	0.202	14.778
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	220	449	1120	218	4255	235	319
normalized size	1	1.00	1.40	2.86	7.13	1.39	27.10	1.50	2.03
time (sec)	N/A	0.318	1.398	0.410	0.456	0.458	16.310	0.164	14.012

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	188	299	608	179	2365	164	241
normalized size	1	1.00	1.59	2.53	5.15	1.52	20.04	1.39	2.04
time (sec)	N/A	0.278	1.330	0.396	0.428	0.445	8.105	0.157	14.577
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	127	113	256	117	828	122	110
normalized size	1	1.00	2.23	1.98	4.49	2.05	14.53	2.14	1.93
time (sec)	N/A	0.155	0.586	0.379	0.549	0.434	3.945	0.145	12.941
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	57	35	28	83	41	39
normalized size	1	1.00	1.00	1.63	1.00	0.80	2.37	1.17	1.11
time (sec)	N/A	0.136	0.028	0.369	0.321	0.414	2.532	0.176	12.554
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	108	93	266	73	578	102	118
normalized size	1	1.00	1.71	1.48	4.22	1.16	9.17	1.62	1.87
time (sec)	N/A	0.202	0.592	0.448	0.627	0.416	7.815	0.287	12.323
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	157	145	423	107	1236	177	178
normalized size	1	1.00	1.54	1.42	4.15	1.05	12.12	1.74	1.75
time (sec)	N/A	0.257	0.890	0.546	0.638	0.420	15.149	0.260	12.469
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	240	189	619	141	2468	237	239
normalized size	1	1.00	1.69	1.33	4.36	0.99	17.38	1.67	1.68
time (sec)	N/A	0.307	1.141	0.490	0.350	0.425	28.583	0.200	13.233

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	354	778	2982	370	10608	412	500
normalized size	1	1.00	1.48	3.24	12.42	1.54	44.20	1.72	2.08
time (sec)	N/A	0.410	2.066	0.515	0.758	0.464	68.938	0.259	14.825
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	311	549	2094	322	7337	369	414
normalized size	1	1.00	1.73	3.05	11.63	1.79	40.76	2.05	2.30
time (sec)	N/A	0.362	1.319	0.464	0.522	0.436	44.148	0.225	14.952
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	274	399	1378	291	4665	233	336
normalized size	1	1.00	1.69	2.46	8.51	1.80	28.80	1.44	2.07
time (sec)	N/A	0.332	0.883	0.453	0.482	0.447	25.785	0.202	14.341
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	234	198	833	242	2474	136	242
normalized size	1	1.00	2.17	1.83	7.71	2.24	22.91	1.26	2.24
time (sec)	N/A	0.277	0.596	0.426	0.461	0.447	15.057	0.192	14.527
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	156	160	452	166	702	92	133
normalized size	1	1.00	2.17	2.22	6.28	2.31	9.75	1.28	1.85
time (sec)	N/A	0.207	0.584	0.415	0.457	0.436	7.386	0.160	12.826
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	110	97	265	69	578	102	117
normalized size	1	1.00	1.77	1.56	4.27	1.11	9.32	1.65	1.89
time (sec)	N/A	0.197	0.512	0.444	0.349	0.408	7.228	0.176	12.286

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	145	47	41	469	87	82
normalized size	1	1.00	0.85	2.34	0.76	0.66	7.56	1.40	1.32
time (sec)	N/A	0.140	0.125	0.352	0.386	0.420	7.631	0.196	12.383
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	237	183	651	116	2674	235	183
normalized size	1	1.00	2.55	1.97	7.00	1.25	28.75	2.53	1.97
time (sec)	N/A	0.220	1.015	0.498	0.472	0.424	27.731	0.198	12.450
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	285	233	835	151	4228	295	197
normalized size	1	1.00	2.11	1.73	6.19	1.12	31.32	2.19	1.46
time (sec)	N/A	0.270	0.964	0.502	0.368	0.434	49.831	0.230	12.749
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	329	277	998	187	5868	355	337
normalized size	1	1.00	1.88	1.58	5.70	1.07	33.53	2.03	1.93
time (sec)	N/A	0.325	1.155	0.533	0.571	0.434	90.732	0.249	12.883
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	388	649	3282	429	10608	376	501
normalized size	1	1.00	1.60	2.67	13.51	1.77	43.65	1.55	2.06
time (sec)	N/A	0.412	2.502	0.521	0.566	0.461	112.095	0.284	14.754
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	348	474	2394	392	7337	305	419
normalized size	1	1.00	1.73	2.36	11.91	1.95	36.50	1.52	2.08
time (sec)	N/A	0.392	1.606	0.495	0.520	0.456	74.016	0.259	14.711

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	308	323	1679	338	4665	226	333
normalized size	1	1.00	2.01	2.11	10.97	2.21	30.49	1.48	2.18
time (sec)	N/A	0.331	1.092	0.481	0.522	0.441	46.582	0.210	14.365
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	272	249	1134	279	1647	159	230
normalized size	1	1.00	2.47	2.26	10.31	2.54	14.97	1.45	2.09
time (sec)	N/A	0.265	0.698	0.467	0.468	0.422	26.026	0.209	15.081
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	139	115	733	191	1035	138	172
normalized size	1	1.00	1.35	1.12	7.12	1.85	10.05	1.34	1.67
time (sec)	N/A	0.225	0.824	0.453	0.354	0.418	13.789	0.169	13.026
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	156	145	423	106	1236	175	178
normalized size	1	1.00	1.53	1.42	4.15	1.04	12.12	1.72	1.75
time (sec)	N/A	0.249	0.839	0.515	0.420	0.410	14.445	0.178	12.429
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	237	185	650	108	2674	235	183
normalized size	1	1.00	2.63	2.06	7.22	1.20	29.71	2.61	2.03
time (sec)	N/A	0.204	1.051	0.463	0.374	0.424	27.389	0.203	12.469
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	65	227	60	56	1098	134	126
normalized size	1	1.00	0.77	2.70	0.71	0.67	13.07	1.60	1.50
time (sec)	N/A	0.152	0.206	0.421	0.363	0.410	20.934	0.354	14.421

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	325	271	1019	150	6135	355	217
normalized size	1	1.00	2.69	2.24	8.42	1.24	50.70	2.93	1.79
time (sec)	N/A	0.223	1.142	0.511	0.408	0.435	92.701	0.269	13.158
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	373	321	1201	185	8396	415	231
normalized size	1	1.00	2.30	1.98	7.41	1.14	51.83	2.56	1.43
time (sec)	N/A	0.289	1.414	0.562	0.566	0.437	155.471	0.268	13.139
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	401	365	1387	221	0	475	474
normalized size	1	1.00	1.96	1.78	6.77	1.08	0.00	2.32	2.31
time (sec)	N/A	0.345	3.548	0.509	0.577	0.457	0.000	0.304	14.522
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	149	119	0	287	0	0	-1
normalized size	1	1.00	0.75	0.60	0.00	1.45	0.00	0.00	-0.01
time (sec)	N/A	0.487	2.988	1.301	0.000	0.436	0.000	0.000	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	123	103	0	243	0	0	-1
normalized size	1	1.00	0.78	0.66	0.00	1.55	0.00	0.00	-0.01
time (sec)	N/A	0.412	1.518	1.230	0.000	0.432	0.000	0.000	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	81	0	184	0	0	-1
normalized size	1	1.00	0.90	0.70	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.320	0.958	1.109	0.000	0.424	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	87	63	0	130	0	0	-1
normalized size	1	1.00	1.19	0.86	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.240	0.422	1.310	0.000	0.422	0.000	0.000	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	166	159	0	254	0	0	-1
normalized size	1	1.00	1.36	1.30	0.00	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.336	1.264	1.486	0.000	0.442	0.000	0.000	0.000
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	157	227	0	318	0	0	-1
normalized size	1	1.00	1.37	1.97	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.318	1.606	1.239	0.000	0.449	0.000	0.000	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	199	268	0	394	0	0	-1
normalized size	1	1.00	1.58	2.13	0.00	3.13	0.00	0.00	-0.01
time (sec)	N/A	0.335	2.240	1.607	0.000	0.461	0.000	0.000	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	217	352	0	490	0	0	-1
normalized size	1	1.00	1.33	2.16	0.00	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.373	3.451	1.733	0.000	0.455	0.000	0.000	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1355	121	0	358	0	0	-1
normalized size	1	1.00	6.45	0.58	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.554	6.642	1.157	0.000	0.449	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	1173	105	0	313	0	0	-1
normalized size	1	1.00	7.02	0.63	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.453	6.515	1.258	0.000	0.444	0.000	0.000	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	83	0	228	0	0	-1
normalized size	1	1.00	0.88	0.69	0.00	1.90	0.00	0.00	-0.01
time (sec)	N/A	0.387	4.594	1.269	0.000	0.430	0.000	0.000	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	193	0	0	-1
normalized size	1	1.00	1.10	0.80	0.00	2.38	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.582	1.295	0.000	0.440	0.000	0.000	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	175	197	0	310	0	0	-1
normalized size	1	1.00	1.09	1.22	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.442	1.182	1.489	0.000	0.446	0.000	0.000	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	355	282	0	385	0	0	-1
normalized size	1	1.00	2.02	1.60	0.00	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.481	0.896	1.465	0.000	0.451	0.000	0.000	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	344	386	0	449	0	0	-1
normalized size	1	1.00	1.97	2.21	0.00	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.478	1.170	1.599	0.000	0.453	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	342	354	0	521	0	0	-1
normalized size	1	1.00	1.95	2.02	0.00	2.98	0.00	0.00	-0.01
time (sec)	N/A	0.491	1.771	1.690	0.000	0.464	0.000	0.000	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	357	440	0	654	0	0	-1
normalized size	1	1.00	1.61	1.98	0.00	2.95	0.00	0.00	-0.00
time (sec)	N/A	0.511	2.687	1.697	0.000	0.489	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	1569	121	0	405	0	0	-1
normalized size	1	1.00	7.47	0.58	0.00	1.93	0.00	0.00	-0.00
time (sec)	N/A	0.534	6.834	1.369	0.000	0.465	0.000	0.000	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1351	105	0	334	0	0	-1
normalized size	1	1.00	8.39	0.65	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.472	6.714	1.341	0.000	0.440	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1157	83	0	287	0	0	-1
normalized size	1	1.00	9.33	0.67	0.00	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.407	6.505	1.290	0.000	0.455	0.000	0.000	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	89	65	0	232	0	0	-1
normalized size	1	1.00	1.10	0.80	0.00	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.305	1.014	1.240	0.000	0.441	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	193	233	0	353	0	0	-1
normalized size	1	1.00	0.96	1.16	0.00	1.76	0.00	0.00	-0.00
time (sec)	N/A	0.521	1.348	1.556	0.000	0.449	0.000	0.000	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	444	354	0	430	0	0	-1
normalized size	1	1.00	2.04	1.62	0.00	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.546	1.690	1.470	0.000	0.455	0.000	0.000	0.000
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	434	434	0	505	0	0	-1
normalized size	1	1.00	1.93	1.93	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.549	2.209	1.986	0.000	0.461	0.000	0.000	0.000
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	422	524	0	554	0	0	-1
normalized size	1	1.00	1.94	2.41	0.00	2.55	0.00	0.00	-0.00
time (sec)	N/A	0.549	3.093	1.985	0.000	0.471	0.000	0.000	0.000
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	355	432	0	633	0	0	-1
normalized size	1	1.00	1.64	1.99	0.00	2.92	0.00	0.00	-0.00
time (sec)	N/A	0.557	4.377	1.882	0.000	0.485	0.000	0.000	0.000
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	485	526	0	760	0	0	-1
normalized size	1	1.00	1.82	1.98	0.00	2.86	0.00	0.00	-0.00
time (sec)	N/A	0.587	6.833	2.047	0.000	0.489	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	157	111	478	115	0	0	-1
normalized size	1	1.00	0.78	0.56	2.39	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.385	5.590	1.168	0.497	0.440	0.000	0.000	0.000
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	134	95	386	95	0	0	-1
normalized size	1	1.00	0.84	0.60	2.43	0.60	0.00	0.00	-0.01
time (sec)	N/A	0.350	1.758	1.100	0.463	0.432	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	73	294	67	0	0	-1
normalized size	1	1.00	0.96	0.62	2.49	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.317	0.601	1.120	0.463	0.427	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	53	174	44	0	0	128
normalized size	1	1.00	0.60	0.73	2.38	0.60	0.00	0.00	1.75
time (sec)	N/A	0.275	0.203	1.019	0.452	0.426	0.000	0.000	13.150
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	140	130	0	162	0	0	-1
normalized size	1	1.00	1.54	1.43	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.283	0.450	1.294	0.000	0.433	0.000	0.000	0.000
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	284	225	0	231	0	0	-1
normalized size	1	1.00	2.09	1.65	0.00	1.70	0.00	0.00	-0.01
time (sec)	N/A	0.330	0.542	1.280	0.000	0.452	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	404	350	0	282	0	0	-1
normalized size	1	1.00	2.24	1.94	0.00	1.57	0.00	0.00	-0.01
time (sec)	N/A	0.425	0.853	1.783	0.000	0.442	0.000	0.000	0.000
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	953	143	762	153	0	0	-1
normalized size	1	1.00	3.94	0.59	3.15	0.63	0.00	0.00	-0.00
time (sec)	N/A	0.651	6.819	1.360	0.462	0.436	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	159	121	670	133	0	0	-1
normalized size	1	1.00	0.79	0.60	3.33	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.558	2.827	1.507	0.529	0.432	0.000	0.000	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	130	105	577	110	0	0	-1
normalized size	1	1.00	0.84	0.68	3.75	0.71	0.00	0.00	-0.01
time (sec)	N/A	0.479	1.162	1.267	0.461	0.439	0.000	0.000	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	113	81	482	80	0	0	492
normalized size	1	1.00	0.98	0.70	4.19	0.70	0.00	0.00	4.28
time (sec)	N/A	0.408	0.657	1.416	0.581	0.430	0.000	0.000	17.178
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	87	63	343	60	0	0	137
normalized size	1	1.00	1.12	0.81	4.40	0.77	0.00	0.00	1.76
time (sec)	N/A	0.313	0.282	1.308	0.457	0.426	0.000	0.000	17.508

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	176	168	0	217	0	0	-1
normalized size	1	1.00	1.30	1.24	0.00	1.61	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.509	1.780	0.000	0.445	0.000	0.000	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	300	258	0	206	0	0	-1
normalized size	1	1.00	1.71	1.47	0.00	1.18	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.860	1.555	0.000	0.446	0.000	0.000	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	430	426	0	279	0	0	-1
normalized size	1	1.00	1.91	1.89	0.00	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.484	1.411	1.735	0.000	0.462	0.000	0.000	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	176	143	945	166	0	0	-1
normalized size	1	1.00	0.73	0.59	3.90	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.647	4.186	1.471	0.493	0.448	0.000	0.000	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	158	121	854	148	0	0	-1
normalized size	1	1.00	0.76	0.58	4.09	0.71	0.00	0.00	-0.00
time (sec)	N/A	0.567	2.632	1.600	1.255	0.441	0.000	0.000	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	132	105	761	125	0	0	904
normalized size	1	1.00	0.82	0.66	4.76	0.78	0.00	0.00	5.65
time (sec)	N/A	0.480	1.215	1.602	0.483	0.441	0.000	0.000	22.785

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	113	83	663	95	0	0	683
normalized size	1	1.00	0.93	0.69	5.48	0.79	0.00	0.00	5.64
time (sec)	N/A	0.413	0.696	1.464	0.476	0.428	0.000	0.000	19.157
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	89	65	505	77	0	0	479
normalized size	1	1.00	1.05	0.76	5.94	0.91	0.00	0.00	5.64
time (sec)	N/A	0.315	0.303	1.407	0.508	0.445	0.000	0.000	17.535
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	204	200	0	262	0	0	-1
normalized size	1	1.00	1.17	1.15	0.00	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.438	0.772	1.941	0.000	0.442	0.000	0.000	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	357	308	0	277	0	0	-1
normalized size	1	1.00	1.59	1.38	0.00	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.480	1.362	1.689	0.000	0.457	0.000	0.000	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	479	410	0	238	0	0	-1
normalized size	1	1.00	1.86	1.59	0.00	0.92	0.00	0.00	-0.00
time (sec)	N/A	0.555	2.286	2.058	0.000	0.486	0.000	0.000	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	118	174	0	140	0	0	173
normalized size	1	1.00	1.26	1.85	0.00	1.49	0.00	0.00	1.84
time (sec)	N/A	0.339	1.002	0.873	0.000	0.455	0.000	0.000	16.166

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	102	129	0	121	0	0	149
normalized size	1	1.00	1.09	1.37	0.00	1.29	0.00	0.00	1.59
time (sec)	N/A	0.337	0.821	0.853	0.000	0.438	0.000	0.000	14.959
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	91	0	92	0	0	122
normalized size	1	1.00	0.89	0.97	0.00	0.98	0.00	0.00	1.30
time (sec)	N/A	0.333	0.556	0.825	0.000	0.445	0.000	0.000	1.787
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	57	0	61	0	0	75
normalized size	1	1.00	0.68	0.62	0.00	0.66	0.00	0.00	0.82
time (sec)	N/A	0.309	0.175	0.803	0.000	0.436	0.000	0.000	0.938
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	120	395	175	0	0	0	-1
normalized size	1	1.00	1.20	3.95	1.75	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.337	1.133	0.728	0.551	0.834	0.000	0.000	0.000
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	147	403	0	0	0	0	-1
normalized size	1	1.00	1.48	4.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.358	1.157	0.689	0.000	1.174	0.000	0.000	0.000
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	101	137	0	87	0	0	-1
normalized size	1	1.00	1.10	1.49	0.00	0.95	0.00	0.00	-0.01
time (sec)	N/A	0.345	0.539	0.710	0.000	0.428	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	205	0	106	0	0	153
normalized size	1	1.00	1.10	2.18	0.00	1.13	0.00	0.00	1.63
time (sec)	N/A	0.337	0.596	0.723	0.000	0.438	0.000	0.000	17.626
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	205	185	0	148	0	0	323
normalized size	1	1.00	1.40	1.27	0.00	1.01	0.00	0.00	2.21
time (sec)	N/A	0.358	1.486	0.842	0.000	0.450	0.000	0.000	17.532
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	172	147	0	126	0	0	174
normalized size	1	1.00	1.18	1.01	0.00	0.86	0.00	0.00	1.19
time (sec)	N/A	0.361	1.612	0.790	0.000	0.454	0.000	0.000	16.567
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	96	86	0	83	0	0	103
normalized size	1	1.00	0.72	0.64	0.00	0.62	0.00	0.00	0.77
time (sec)	N/A	0.348	0.748	0.748	0.000	0.443	0.000	0.000	1.842
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	81	91	0	87	0	0	122
normalized size	1	1.00	0.84	0.95	0.00	0.91	0.00	0.00	1.27
time (sec)	N/A	0.325	0.556	0.798	0.000	0.447	0.000	0.000	14.292
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	136	494	0	0	0	0	-1
normalized size	1	1.00	0.94	3.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	0.685	0.699	0.000	2.818	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	210	748	366	0	0	0	-1
normalized size	1	1.00	1.33	4.73	2.32	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.861	0.672	0.571	6.007	0.000	0.000	0.000
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	198	594	0	0	0	0	-1
normalized size	1	1.00	1.33	3.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.387	0.945	0.662	0.000	2.949	0.000	0.000	0.000
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	125	223	0	125	0	0	-1
normalized size	1	1.00	1.30	2.32	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.275	0.989	0.659	0.000	0.447	0.000	0.000	0.000
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	123	217	0	134	0	0	245
normalized size	1	1.00	0.84	1.49	0.00	0.92	0.00	0.00	1.68
time (sec)	N/A	0.376	1.352	0.678	0.000	0.469	0.000	0.000	18.994
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	126	339	0	155	0	0	279
normalized size	1	1.00	0.82	2.20	0.00	1.01	0.00	0.00	1.81
time (sec)	N/A	0.373	1.954	0.689	0.000	0.458	0.000	0.000	20.227
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	223	203	0	160	0	0	383
normalized size	1	1.00	1.13	1.03	0.00	0.81	0.00	0.00	1.93
time (sec)	N/A	0.476	2.649	0.901	0.000	0.477	0.000	0.000	18.173

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	113	114	0	117	0	0	131
normalized size	1	1.00	0.63	0.63	0.00	0.65	0.00	0.00	0.73
time (sec)	N/A	0.466	0.799	0.769	0.000	0.460	0.000	0.000	16.003
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	165	147	0	119	0	0	174
normalized size	1	1.00	1.16	1.04	0.00	0.84	0.00	0.00	1.23
time (sec)	N/A	0.362	1.802	0.794	0.000	0.458	0.000	0.000	16.324
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	102	129	0	117	0	0	149
normalized size	1	1.00	1.06	1.34	0.00	1.22	0.00	0.00	1.55
time (sec)	N/A	0.318	0.832	0.804	0.000	0.445	0.000	0.000	2.701
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	177	590	0	0	0	0	-1
normalized size	1	1.00	0.92	3.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	1.498	0.724	0.000	4.601	0.000	0.000	0.000
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	231	846	0	0	0	0	-1
normalized size	1	1.00	1.10	4.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.485	1.645	0.638	0.000	10.487	0.000	0.000	0.000
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	207	1092	506	0	0	0	-1
normalized size	1	1.00	0.98	5.15	2.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.490	1.165	0.648	0.480	1.107	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	204	832	0	0	0	0	-1
normalized size	1	1.00	1.04	4.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.488	1.191	0.688	0.000	6.120	0.000	0.000	0.000
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	145	309	0	165	0	0	-1
normalized size	1	1.00	1.51	3.22	0.00	1.72	0.00	0.00	-0.01
time (sec)	N/A	0.276	2.964	0.678	0.000	0.452	0.000	0.000	0.000
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	368	0	182	0	0	341
normalized size	1	1.00	1.00	2.52	0.00	1.25	0.00	0.00	2.34
time (sec)	N/A	0.376	4.080	0.697	0.000	0.458	0.000	0.000	20.756
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	144	423	0	196	0	0	357
normalized size	1	1.00	0.73	2.16	0.00	1.00	0.00	0.00	1.82
time (sec)	N/A	0.483	5.616	0.718	0.000	0.472	0.000	0.000	20.715
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	269	259	0	183	0	0	482
normalized size	1	1.00	1.08	1.04	0.00	0.73	0.00	0.00	1.93
time (sec)	N/A	0.567	6.850	0.826	0.000	0.528	0.000	0.000	20.031
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	135	142	0	134	0	0	384
normalized size	1	1.00	0.60	0.63	0.00	0.59	0.00	0.00	1.70
time (sec)	N/A	0.559	1.583	0.839	0.000	0.484	0.000	0.000	17.653

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	232	203	0	150	0	0	383
normalized size	1	1.00	1.21	1.06	0.00	0.78	0.00	0.00	1.99
time (sec)	N/A	0.460	2.073	0.850	0.000	0.484	0.000	0.000	18.428
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	212	185	0	142	0	0	321
normalized size	1	1.00	1.49	1.30	0.00	1.00	0.00	0.00	2.26
time (sec)	N/A	0.359	1.852	0.803	0.000	0.460	0.000	0.000	18.169
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	121	174	0	139	0	0	173
normalized size	1	1.00	1.26	1.81	0.00	1.45	0.00	0.00	1.80
time (sec)	N/A	0.325	0.965	0.824	0.000	0.453	0.000	0.000	16.679
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	183	671	0	0	0	0	-1
normalized size	1	1.00	0.77	2.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.569	2.836	0.688	0.000	0.961	0.000	0.000	0.000
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	292	927	0	0	0	0	-1
normalized size	1	1.00	1.08	3.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.593	3.496	0.646	0.000	1.168	0.000	0.000	0.000
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	251	1191	0	0	0	0	-1
normalized size	1	1.00	0.95	4.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	2.573	0.646	0.000	25.359	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	244	1455	749	0	0	0	-1
normalized size	1	1.00	0.92	5.51	2.84	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	2.900	0.661	1.185	44.324	0.000	0.000	0.000
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	238	1019	0	0	0	0	-1
normalized size	1	1.00	0.96	4.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	2.685	0.717	0.000	0.967	0.000	0.000	0.000
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	434	389	0	199	0	0	-1
normalized size	1	1.00	4.52	4.05	0.00	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.272	6.945	0.701	0.000	0.484	0.000	0.000	0.000
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	442	393	0	214	0	0	406
normalized size	1	1.00	3.03	2.69	0.00	1.47	0.00	0.00	2.78
time (sec)	N/A	0.379	6.989	0.730	0.000	0.499	0.000	0.000	22.848
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	442	505	0	234	0	0	827
normalized size	1	1.00	2.19	2.50	0.00	1.16	0.00	0.00	4.09
time (sec)	N/A	0.491	7.106	0.765	0.000	0.537	0.000	0.000	25.256
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	436	560	0	243	0	0	841
normalized size	1	1.00	1.77	2.28	0.00	0.99	0.00	0.00	3.42
time (sec)	N/A	0.591	7.143	0.810	0.000	0.541	0.000	0.000	28.344

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	185	595	0	0	0	0	-1
normalized size	1	1.00	0.94	3.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	1.306	0.717	0.000	4.083	0.000	0.000	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	146	501	0	0	0	0	-1
normalized size	1	1.00	1.00	3.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.365	0.664	0.676	0.000	2.823	0.000	0.000	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	119	399	176	0	0	0	-1
normalized size	1	1.00	1.24	4.16	1.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.322	1.130	0.680	0.795	1.362	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	97	165	0	0	0	0	-1
normalized size	1	1.00	0.86	1.46	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.364	0.337	0.592	0.000	1.412	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	191	302	0	337	0	0	-1
normalized size	1	1.00	1.85	2.93	0.00	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.543	0.595	0.000	0.518	0.000	0.000	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	222	465	0	424	0	0	-1
normalized size	1	1.00	1.45	3.04	0.00	2.77	0.00	0.00	-0.01
time (sec)	N/A	0.355	0.631	0.618	0.000	0.536	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	271	939	0	0	0	0	-1
normalized size	1	1.00	1.00	3.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	3.475	0.657	0.000	14.903	0.000	0.000	0.000
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	212	853	0	0	0	0	-1
normalized size	1	1.00	1.01	4.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	1.604	0.627	0.000	8.938	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	190	759	367	0	0	0	-1
normalized size	1	1.00	1.19	4.77	2.31	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.386	0.864	0.642	0.548	0.945	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	143	408	0	0	0	0	-1
normalized size	1	1.00	1.43	4.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.346	1.176	0.651	0.000	1.068	0.000	0.000	0.000
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	186	303	0	329	0	0	-1
normalized size	1	1.00	1.81	2.94	0.00	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.256	0.562	0.604	0.000	0.515	0.000	0.000	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	178	130	0	272	0	0	-1
normalized size	1	1.00	1.19	0.87	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.373	0.671	0.563	0.000	0.506	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	306	429	0	437	0	0	-1
normalized size	1	1.00	1.41	1.98	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.947	0.553	0.000	0.533	0.000	0.000	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	573	1287	0	0	0	0	-1
normalized size	1	1.00	1.77	3.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.713	7.012	0.675	0.000	45.496	0.000	0.000	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	243	1206	0	0	0	0	-1
normalized size	1	1.00	0.92	4.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.607	2.540	0.640	0.000	1.412	0.000	0.000	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	199	1106	504	0	0	0	-1
normalized size	1	1.00	0.94	5.24	2.39	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.494	1.155	0.631	0.493	1.125	0.000	0.000	0.000
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	179	606	0	0	0	0	-1
normalized size	1	1.00	1.20	4.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.979	0.644	0.000	2.705	0.000	0.000	0.000
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	99	135	0	85	0	0	156
normalized size	1	1.00	1.05	1.44	0.00	0.90	0.00	0.00	1.66
time (sec)	N/A	0.333	0.509	0.661	0.000	0.431	0.000	0.000	14.861

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	214	466	0	408	0	0	-1
normalized size	1	1.00	1.42	3.09	0.00	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.673	0.626	0.000	0.527	0.000	0.000	0.000
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	305	431	0	419	0	0	-1
normalized size	1	1.00	1.47	2.07	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.480	0.961	0.569	0.000	0.546	0.000	0.000	0.000
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	246	151	0	306	0	0	-1
normalized size	1	1.00	1.00	0.62	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.570	0.937	0.602	0.000	0.560	0.000	0.000	0.000
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	2903	0	0	0	0	0	-1
normalized size	1	1.00	16.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.315	14.238	5.774	0.000	0.483	0.000	0.000	0.000
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	180.061	8.099	0.000	0.482	0.000	0.000	0.000
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	180.081	8.309	0.000	0.477	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	462	0	0	0	0	0	-1
normalized size	1	1.00	3.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.289	4.220	4.976	0.000	0.458	0.000	0.000	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	1.832	2.533	0.000	0.435	0.000	0.000	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	7409	0	0	0	0	0	-1
normalized size	1	1.00	60.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.303	25.568	1.087	0.000	0.452	0.000	0.000	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	9240	0	0	0	0	0	-1
normalized size	1	1.00	62.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.330	6.934	8.469	0.000	0.454	0.000	0.000	0.000
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	12580	0	0	0	0	0	-1
normalized size	1	1.00	85.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.332	7.178	9.297	0.000	0.490	0.000	0.000	0.000
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.294	4.870	0.983	0.000	0.458	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	2.383	1.115	0.000	0.462	0.000	0.000	0.000
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	667	0	725	562	0	0	749
normalized size	1	1.00	2.43	0.00	2.64	2.04	0.00	0.00	2.72
time (sec)	N/A	0.503	6.819	1.342	0.546	0.487	0.000	0.000	21.060
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	192	174	0	498	313	0	0	480
normalized size	1	1.16	1.05	0.00	3.00	1.89	0.00	0.00	2.89
time (sec)	N/A	0.354	1.766	1.282	0.533	0.499	0.000	0.000	19.203
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	116	0	323	165	0	0	105
normalized size	1	1.00	1.12	0.00	3.11	1.59	0.00	0.00	1.01
time (sec)	N/A	0.282	0.442	1.280	0.536	0.468	0.000	0.000	1.442
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	200	0	0	0	0	0	-1
normalized size	1	1.00	1.69	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.282	2.229	0.016	0.000	0.467	0.000	0.000	0.000
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	3178	0	0	0	0	0	-1
normalized size	1	1.00	23.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	6.760	1.006	0.000	0.498	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	8147	0	0	0	0	0	-1
normalized size	1	1.00	60.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.332	6.872	1.038	0.000	0.492	0.000	0.000	0.000
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	353	0	0	205	0	0	368
normalized size	1	1.00	1.32	0.00	0.00	0.77	0.00	0.00	1.38
time (sec)	N/A	0.427	14.131	5.935	0.000	0.488	0.000	0.000	22.233
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	269	0	0	137	0	0	239
normalized size	1	1.00	1.41	0.00	0.00	0.72	0.00	0.00	1.25
time (sec)	N/A	0.310	10.734	6.181	0.000	0.485	0.000	0.000	15.484
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	211	0	0	89	0	0	134
normalized size	1	1.00	1.85	0.00	0.00	0.78	0.00	0.00	1.18
time (sec)	N/A	0.222	8.790	5.410	0.000	0.480	0.000	0.000	14.158
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	675	0	0	0	0	0	-1
normalized size	1	1.00	4.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.309	11.616	2.870	0.000	0.448	0.000	0.000	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	2552	0	0	0	0	0	-1
normalized size	1	1.00	16.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	17.204	4.367	0.000	0.479	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	3601	0	0	0	0	0	-1
normalized size	1	1.00	21.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.329	92.471	3.809	0.000	0.478	0.000	0.000	0.000
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	5163	0	0	0	0	0	-1
normalized size	1	1.00	29.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.336	49.215	6.070	0.000	0.471	0.000	0.000	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	63	0	0	78	0	0	64
normalized size	1	1.00	1.85	0.00	0.00	2.29	0.00	0.00	1.88
time (sec)	N/A	0.274	0.540	8.038	0.000	0.480	0.000	0.000	14.447
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	67	0	0	77	0	0	61
normalized size	1	1.00	1.97	0.00	0.00	2.26	0.00	0.00	1.79
time (sec)	N/A	0.238	1.142	8.063	0.000	0.469	0.000	0.000	14.457
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	78	0	0	61
normalized size	1	1.00	2.00	0.00	0.00	2.36	0.00	0.00	1.85
time (sec)	N/A	0.237	1.109	7.862	0.000	0.486	0.000	0.000	14.378
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	0	0	77	0	0	64
normalized size	1	1.00	1.74	0.00	0.00	2.20	0.00	0.00	1.83
time (sec)	N/A	0.238	0.540	8.121	0.000	0.479	0.000	0.000	14.363

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	36	0	0	36
normalized size	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.132	0.487	7.658	0.000	0.487	0.000	0.000	13.542
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	37	0	0	37
normalized size	1	1.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.123	0.475	8.398	0.000	0.496	0.000	0.000	13.497
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	87	158	157	105	440	131	300
normalized size	1	1.00	0.62	1.13	1.12	0.75	3.14	0.94	2.14
time (sec)	N/A	0.186	0.140	0.597	0.429	0.462	8.486	0.171	15.304
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	77	136	138	91	359	113	256
normalized size	1	1.00	0.64	1.12	1.14	0.75	2.97	0.93	2.12
time (sec)	N/A	0.168	0.102	0.497	0.404	0.458	5.693	0.175	15.286
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	55	117	112	77	267	77	292
normalized size	1	1.00	0.57	1.22	1.17	0.80	2.78	0.80	3.04
time (sec)	N/A	0.116	0.476	0.401	0.372	0.442	3.647	0.162	15.030
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	89	86	63	196	77	250
normalized size	1	1.00	0.66	1.09	1.05	0.77	2.39	0.94	3.05
time (sec)	N/A	0.106	0.347	0.406	0.389	0.449	1.373	0.168	15.188

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	74	99	85	92	0	107	212
normalized size	1	1.00	0.97	1.30	1.12	1.21	0.00	1.41	2.79
time (sec)	N/A	0.104	0.147	0.433	0.391	0.481	0.000	0.161	13.227
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	95	83	111	0	153	226
normalized size	1	1.00	0.97	1.20	1.05	1.41	0.00	1.94	2.86
time (sec)	N/A	0.179	0.190	0.361	0.548	0.493	0.000	0.187	13.194
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	142	94	90	152	0	137	220
normalized size	1	1.00	1.82	1.21	1.15	1.95	0.00	1.76	2.82
time (sec)	N/A	0.121	0.038	0.555	0.405	0.481	0.000	0.195	13.500
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	141	103	117	175	0	150	245
normalized size	1	1.00	1.81	1.32	1.50	2.24	0.00	1.92	3.14
time (sec)	N/A	0.131	0.490	0.526	0.447	0.462	0.000	0.180	13.316
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	210	109	145	166	0	174	244
normalized size	1	1.00	2.44	1.27	1.69	1.93	0.00	2.02	2.84
time (sec)	N/A	0.148	0.069	0.621	0.426	0.447	0.000	0.196	13.116
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	268	132	175	201	0	174	244
normalized size	1	1.00	2.55	1.26	1.67	1.91	0.00	1.66	2.32
time (sec)	N/A	0.234	0.074	0.715	0.440	0.428	0.000	0.185	13.155

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	306	155	207	240	0	242	340
normalized size	1	1.00	2.35	1.19	1.59	1.85	0.00	1.86	2.62
time (sec)	N/A	0.197	0.080	0.732	0.598	0.465	0.000	0.205	13.365
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	254	257	715	248	3614	156	326
normalized size	1	1.00	1.97	1.99	5.54	1.92	28.02	1.21	2.53
time (sec)	N/A	0.208	0.945	0.446	0.589	0.458	78.444	0.214	17.076
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	228	155	543	225	2290	113	261
normalized size	1	1.00	2.21	1.50	5.27	2.18	22.23	1.10	2.53
time (sec)	N/A	0.188	0.801	0.433	0.561	0.449	45.407	0.182	16.929
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	189	131	392	204	1268	93	178
normalized size	1	1.00	2.12	1.47	4.40	2.29	14.25	1.04	2.00
time (sec)	N/A	0.173	0.756	0.428	0.639	0.445	25.572	0.192	14.924
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	107	71	348	156	461	63	110
normalized size	1	1.00	1.30	0.87	4.24	1.90	5.62	0.77	1.34
time (sec)	N/A	0.138	0.462	0.415	0.624	0.425	14.110	0.188	13.448
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	92	86	387	154	573	79	134
normalized size	1	1.00	1.59	1.48	6.67	2.66	9.88	1.36	2.31
time (sec)	N/A	0.114	0.239	0.414	0.532	0.407	9.114	0.171	13.309

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	313	130	433	310	0	99	199
normalized size	1	1.00	3.19	1.33	4.42	3.16	0.00	1.01	2.03
time (sec)	N/A	0.164	1.006	0.729	0.586	0.440	0.000	0.194	14.771
Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	167	169	519	406	0	146	210
normalized size	1	1.00	1.48	1.50	4.59	3.59	0.00	1.29	1.86
time (sec)	N/A	0.398	2.974	0.732	0.686	0.451	0.000	0.221	15.774
Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	245	209	622	498	0	180	288
normalized size	1	1.00	1.78	1.51	4.51	3.61	0.00	1.30	2.09
time (sec)	N/A	0.225	3.888	0.870	0.410	0.449	0.000	0.299	15.741
Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	348	249	706	594	0	213	314
normalized size	1	1.00	2.27	1.63	4.61	3.88	0.00	1.39	2.05
time (sec)	N/A	0.246	6.229	0.826	0.458	0.465	0.000	0.302	15.280
Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	267	422	406	239	996	314	830
normalized size	1	1.00	0.82	1.29	1.24	0.73	3.05	0.96	2.54
time (sec)	N/A	0.579	2.000	0.525	0.363	0.469	5.370	0.182	15.576
Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	185	274	264	160	571	198	547
normalized size	1	1.00	0.87	1.29	1.24	0.75	2.68	0.93	2.57
time (sec)	N/A	0.360	1.123	0.408	0.371	0.469	2.308	0.154	15.413

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	104	147	143	84	277	101	134
normalized size	1	1.00	0.94	1.32	1.29	0.76	2.50	0.91	1.21
time (sec)	N/A	0.157	0.439	0.304	0.374	0.471	1.002	0.151	13.314
Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	45	59	57	43	94	48	100
normalized size	1	1.00	0.94	1.23	1.19	0.90	1.96	1.00	2.08
time (sec)	N/A	0.023	0.098	0.157	0.446	0.435	0.351	0.128	13.263
Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	196	294	0	292	0	141	3074
normalized size	1	1.00	2.00	3.00	0.00	2.98	0.00	1.44	31.37
time (sec)	N/A	0.273	0.653	0.404	0.000	0.488	0.000	0.154	16.580
Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	217	434	0	655	0	204	5102
normalized size	1	1.00	1.75	3.50	0.00	5.28	0.00	1.65	41.15
time (sec)	N/A	0.325	1.271	0.493	0.000	0.487	0.000	0.196	20.316
Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	345	2021	0	967	0	594	554
normalized size	1	1.00	1.96	11.48	0.00	5.49	0.00	3.38	3.15
time (sec)	N/A	0.421	2.711	0.515	0.000	0.528	0.000	0.232	15.589
Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	464	464	437	745	724	364	1865	474	1291
normalized size	1	1.00	0.94	1.61	1.56	0.78	4.02	1.02	2.78
time (sec)	N/A	0.952	3.128	0.639	0.465	0.496	11.952	0.217	15.987

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	296	496	478	245	1129	311	765
normalized size	1	1.00	0.88	1.48	1.42	0.73	3.36	0.93	2.28
time (sec)	N/A	0.703	1.567	0.515	0.440	0.456	6.002	0.184	15.705
Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	278	268	144	571	172	492
normalized size	1	1.00	0.96	1.67	1.61	0.87	3.44	1.04	2.96
time (sec)	N/A	0.271	0.755	0.409	0.420	0.448	2.760	0.173	14.603
Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	106	117	114	70	199	88	91
normalized size	1	1.00	1.13	1.24	1.21	0.74	2.12	0.94	0.97
time (sec)	N/A	0.061	0.325	0.297	0.439	0.463	0.949	0.158	13.161
Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	177	713	0	452	0	314	7371
normalized size	1	1.00	1.04	4.17	0.00	2.64	0.00	1.84	43.11
time (sec)	N/A	0.522	0.623	0.430	0.000	0.512	0.000	0.193	20.069
Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	192	848	0	731	0	498	8706
normalized size	1	1.00	0.97	4.28	0.00	3.69	0.00	2.52	43.97
time (sec)	N/A	0.581	0.996	0.512	0.000	0.575	0.000	0.201	21.578
Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	226	1916	0	1483	0	703	8632
normalized size	1	1.00	1.05	8.91	0.00	6.90	0.00	3.27	40.15
time (sec)	N/A	0.623	1.366	0.544	0.000	0.553	0.000	0.242	22.497

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	604	604	528	1077	1056	432	2878	566	1395
normalized size	1	1.00	0.87	1.78	1.75	0.72	4.76	0.94	2.31
time (sec)	N/A	1.486	4.763	0.653	0.627	0.507	22.093	0.252	16.176
Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	355	725	704	299	1804	380	976
normalized size	1	1.00	0.77	1.57	1.52	0.65	3.90	0.82	2.11
time (sec)	N/A	1.128	2.425	0.625	0.461	0.491	10.854	0.213	15.626
Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	156	414	398	178	960	217	550
normalized size	1	1.00	0.78	2.06	1.98	0.89	4.78	1.08	2.74
time (sec)	N/A	0.335	0.913	0.512	0.430	0.473	5.283	0.177	14.623
Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	117	120	178	171	93	371	116	330
normalized size	1	0.92	0.94	1.40	1.35	0.73	2.92	0.91	2.60
time (sec)	N/A	0.101	0.497	0.410	0.381	0.453	2.085	0.165	14.256
Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	233	1357	0	627	0	617	10256
normalized size	1	1.00	0.95	5.52	0.00	2.55	0.00	2.51	41.69
time (sec)	N/A	0.895	0.981	0.479	0.000	0.549	0.000	0.211	21.761
Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	244	1534	0	1027	0	588	11993
normalized size	1	1.00	0.86	5.42	0.00	3.63	0.00	2.08	42.38
time (sec)	N/A	0.941	1.497	0.555	0.000	0.594	0.000	0.213	23.883

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	830	2906	0	1670	0	986	13891
normalized size	1	1.00	2.72	9.53	0.00	5.48	0.00	3.23	45.54
time (sec)	N/A	0.934	3.145	0.564	0.000	0.609	0.000	0.303	25.413
Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	788	1110	1124	470	0	479	839
normalized size	1	1.00	3.58	5.05	5.11	2.14	0.00	2.18	3.81
time (sec)	N/A	0.361	1.272	0.437	0.727	0.483	0.000	0.198	14.054
Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	141	200	524	606	303	5763	222	297
normalized size	1	0.99	1.40	3.66	4.24	2.12	40.30	1.55	2.08
time (sec)	N/A	0.206	0.465	0.416	0.725	0.471	9.445	0.156	16.666
Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	126	179	256	154	1307	160	122
normalized size	1	1.00	1.88	2.67	3.82	2.30	19.51	2.39	1.82
time (sec)	N/A	0.203	0.469	0.360	0.492	0.470	4.181	0.141	13.681
Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	79	65	78	66	109	40	35
normalized size	1	1.00	2.26	1.86	2.23	1.89	3.11	1.14	1.00
time (sec)	N/A	0.049	0.159	0.227	0.518	0.439	1.832	0.146	12.920
Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	148	176	0	595	0	113	154
normalized size	1	1.00	1.47	1.74	0.00	5.89	0.00	1.12	1.52
time (sec)	N/A	0.170	0.327	0.545	0.000	0.482	0.000	0.168	13.322

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	209	615	0	1538	0	443	437
normalized size	1	1.00	1.15	3.40	0.00	8.50	0.00	2.45	2.41
time (sec)	N/A	0.349	1.222	0.576	0.000	0.506	0.000	0.193	15.531
Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	313	2482	0	3303	0	753	1076
normalized size	1	1.00	1.11	8.77	0.00	11.67	0.00	2.66	3.80
time (sec)	N/A	0.550	1.606	0.559	0.000	0.614	0.000	0.298	17.439
Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	547	946	1382	584	0	494	663
normalized size	1	1.00	2.40	4.15	6.06	2.56	0.00	2.17	2.91
time (sec)	N/A	0.523	3.571	0.427	0.570	0.462	0.000	0.217	16.346
Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	338	489	831	375	5358	277	365
normalized size	1	1.00	2.56	3.70	6.30	2.84	40.59	2.10	2.77
time (sec)	N/A	0.510	1.675	0.414	0.575	0.458	17.795	0.193	16.038
Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	180	252	454	208	1062	141	94
normalized size	1	1.00	2.12	2.96	5.34	2.45	12.49	1.66	1.11
time (sec)	N/A	0.211	0.339	0.431	0.502	0.432	8.343	0.154	13.738
Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	70	214	117	372	68	97
normalized size	1	1.00	0.66	1.08	3.29	1.80	5.72	1.05	1.49
time (sec)	N/A	0.052	0.057	0.330	0.350	0.415	4.585	0.185	13.388

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	229	327	0	1285	0	259	302
normalized size	1	1.00	1.51	2.15	0.00	8.45	0.00	1.70	1.99
time (sec)	N/A	0.419	0.628	0.561	0.000	0.500	0.000	0.216	14.783
Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	313	770	0	3123	0	425	844
normalized size	1	1.00	1.14	2.80	0.00	11.36	0.00	1.55	3.07
time (sec)	N/A	0.672	2.830	0.554	0.000	0.593	0.000	0.232	16.763
Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	1522	2641	0	4997	0	944	1686
normalized size	1	1.00	3.94	6.84	0.00	12.95	0.00	2.45	4.37
time (sec)	N/A	0.961	6.366	0.586	0.000	0.695	0.000	0.306	17.693
Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	366	936	1682	649	11456	596	593
normalized size	1	1.00	1.63	4.16	7.48	2.88	50.92	2.65	2.64
time (sec)	N/A	0.809	2.971	0.434	0.584	0.481	56.435	0.222	15.695
Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	514	617	1132	432	3468	382	286
normalized size	1	1.00	3.13	3.76	6.90	2.63	21.15	2.33	1.74
time (sec)	N/A	0.461	0.898	0.418	0.549	0.459	33.403	0.193	16.544
Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	176	151	733	271	1819	223	245
normalized size	1	1.00	1.39	1.19	5.77	2.13	14.32	1.76	1.93
time (sec)	N/A	0.227	0.689	0.439	0.456	0.428	15.102	0.178	14.259

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	63	114	387	190	1015	130	150
normalized size	1	1.00	0.62	1.12	3.79	1.86	9.95	1.27	1.47
time (sec)	N/A	0.075	0.093	0.324	0.394	0.416	8.342	0.166	13.804
Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	502	606	0	2292	0	578	591
normalized size	1	1.00	2.19	2.65	0.00	10.01	0.00	2.52	2.58
time (sec)	N/A	0.724	1.251	0.540	0.000	0.543	0.000	0.261	17.184
Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	1253	1049	0	4486	0	772	1349
normalized size	1	1.00	3.29	2.75	0.00	11.77	0.00	2.03	3.54
time (sec)	N/A	1.081	6.384	0.570	0.000	0.660	0.000	0.303	17.669
Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	548	2918	0	7283	0	1272	2387
normalized size	1	1.00	1.08	5.74	0.00	14.34	0.00	2.50	4.70
time (sec)	N/A	1.447	4.960	0.650	0.000	0.849	0.000	0.593	19.882
Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	305	242	0	467	0	0	-1
normalized size	1	1.00	1.19	0.95	0.00	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.460	1.248	1.400	0.000	0.450	0.000	0.000	0.000
Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	176	161	0	306	0	0	-1
normalized size	1	1.00	0.92	0.84	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.743	1.437	0.000	0.428	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	117	102	0	175	0	0	-1
normalized size	1	1.00	0.99	0.86	0.00	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.249	0.359	1.362	0.000	0.449	0.000	0.000	0.000
Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	82	58	0	85	0	0	-1
normalized size	1	1.00	1.32	0.94	0.00	1.37	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.123	1.019	0.000	0.439	0.000	0.000	0.000
Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	903	139	0	651	0	0	-1
normalized size	1	1.00	9.03	1.39	0.00	6.51	0.00	0.00	-0.01
time (sec)	N/A	0.246	9.035	1.787	0.000	1.259	0.000	0.000	0.000
Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	901	274	0	1012	0	0	-1
normalized size	1	1.00	7.15	2.17	0.00	8.03	0.00	0.00	-0.01
time (sec)	N/A	0.262	8.797	2.019	0.000	1.447	0.000	0.000	0.000
Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	967	628	0	1750	0	0	-1
normalized size	1	1.00	5.04	3.27	0.00	9.11	0.00	0.00	-0.01
time (sec)	N/A	0.370	10.267	2.426	0.000	2.280	0.000	0.000	0.000
Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	390	312	0	637	0	0	-1
normalized size	1	1.00	1.04	0.83	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.920	4.607	1.369	0.000	0.474	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	267	207	0	430	0	0	-1
normalized size	1	1.00	0.91	0.70	0.00	1.46	0.00	0.00	-0.00
time (sec)	N/A	0.712	2.265	1.435	0.000	0.451	0.000	0.000	0.000
Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	144	150	0	257	0	0	-1
normalized size	1	1.00	0.87	0.91	0.00	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.316	1.075	1.283	0.000	0.448	0.000	0.000	0.000
Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	77	0	137	0	0	-1
normalized size	1	1.00	1.00	0.76	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.397	1.085	0.000	0.431	0.000	0.000	0.000
Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	356	291	0	880	0	0	-1
normalized size	1	1.00	2.33	1.90	0.00	5.75	0.00	0.00	-0.01
time (sec)	N/A	0.503	3.535	1.734	0.000	1.474	0.000	0.000	0.000
Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	381	592	0	1428	0	0	-1
normalized size	1	1.00	1.99	3.10	0.00	7.48	0.00	0.00	-0.01
time (sec)	N/A	0.552	5.013	2.315	0.000	1.677	0.000	0.000	0.000
Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	416	895	0	2208	0	0	-1
normalized size	1	1.00	1.88	4.05	0.00	9.99	0.00	0.00	-0.00
time (sec)	N/A	0.610	5.341	2.825	0.000	2.593	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1565	374	0	863	0	0	-1
normalized size	1	1.00	2.93	0.70	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	1.203	6.869	1.395	0.000	0.505	0.000	0.000	0.000
Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	891	257	0	593	0	0	-1
normalized size	1	1.00	2.08	0.60	0.00	1.38	0.00	0.00	-0.00
time (sec)	N/A	1.066	6.608	1.427	0.000	0.474	0.000	0.000	0.000
Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	202	152	0	361	0	0	-1
normalized size	1	1.00	0.95	0.72	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.368	4.243	1.394	0.000	0.464	0.000	0.000	0.000
Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	119	99	0	191	0	0	-1
normalized size	1	1.00	0.86	0.72	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.112	1.512	1.241	0.000	0.445	0.000	0.000	0.000
Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	450	543	0	1314	0	0	-1
normalized size	1	1.00	2.06	2.49	0.00	6.03	0.00	0.00	-0.00
time (sec)	N/A	0.885	5.822	2.052	0.000	2.273	0.000	0.000	0.000
Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	460	932	0	2046	0	0	-1
normalized size	1	1.00	1.74	3.52	0.00	7.72	0.00	0.00	-0.00
time (sec)	N/A	0.938	5.966	2.545	0.000	2.594	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	504	1587	0	3046	0	0	-1
normalized size	1	1.00	1.64	5.15	0.00	9.89	0.00	0.00	-0.00
time (sec)	N/A	0.972	8.240	2.970	0.000	2.979	0.000	0.000	0.000
Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	375	610	0	629	0	0	-1
normalized size	1	1.00	1.32	2.15	0.00	2.21	0.00	0.00	-0.00
time (sec)	N/A	1.001	0.863	1.949	0.000	0.489	0.000	0.000	0.000
Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	246	396	0	448	0	0	-1
normalized size	1	1.00	1.23	1.98	0.00	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.585	0.538	1.756	0.000	0.482	0.000	0.000	0.000
Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	135	232	0	303	0	0	-1
normalized size	1	1.00	1.04	1.78	0.00	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.270	0.480	1.527	0.000	0.468	0.000	0.000	0.000
Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	106	128	0	210	0	0	151
normalized size	1	1.00	1.34	1.62	0.00	2.66	0.00	0.00	1.91
time (sec)	N/A	0.070	0.218	1.292	0.000	0.448	0.000	0.000	1.056
Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	238	199	0	744	0	0	-1
normalized size	1	1.00	1.75	1.46	0.00	5.47	0.00	0.00	-0.01
time (sec)	N/A	0.283	3.343	2.073	0.000	1.492	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	374	899	0	2159	0	0	-1
normalized size	1	1.00	1.81	4.34	0.00	10.43	0.00	0.00	-0.00
time (sec)	N/A	0.617	6.970	2.719	0.000	3.773	0.000	0.000	0.000
Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	847	2277	0	4180	0	0	-1
normalized size	1	1.00	2.74	7.37	0.00	13.53	0.00	0.00	-0.00
time (sec)	N/A	1.054	10.778	3.580	0.000	7.777	0.000	0.000	0.000
Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	684	1030	0	784	0	0	-1
normalized size	1	1.00	2.42	3.64	0.00	2.77	0.00	0.00	-0.00
time (sec)	N/A	1.000	1.067	1.868	0.000	0.494	0.000	0.000	0.000
Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	357	694	0	584	0	0	-1
normalized size	1	1.00	1.76	3.42	0.00	2.88	0.00	0.00	-0.00
time (sec)	N/A	0.575	0.742	1.590	0.000	0.491	0.000	0.000	0.000
Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	246	389	0	407	0	0	-1
normalized size	1	1.00	1.85	2.92	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.456	1.321	0.000	0.484	0.000	0.000	0.000
Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	150	176	0	293	0	0	-1
normalized size	1	1.00	1.72	2.02	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.205	1.091	0.000	0.435	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	419	624	0	1561	0	0	-1
normalized size	1	1.00	2.24	3.34	0.00	8.35	0.00	0.00	-0.01
time (sec)	N/A	0.590	3.137	2.007	0.000	3.290	0.000	0.000	0.000
Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	542	2049	0	3403	0	0	-1
normalized size	1	1.00	1.86	7.02	0.00	11.65	0.00	0.00	-0.00
time (sec)	N/A	1.018	9.543	3.069	0.000	7.667	0.000	0.000	0.000
Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	1395	4707	0	5864	0	0	-1
normalized size	1	1.00	3.47	11.71	0.00	14.59	0.00	0.00	-0.00
time (sec)	N/A	1.562	13.484	3.972	0.000	15.273	0.000	0.000	0.000
Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	523	1438	0	980	0	0	-1
normalized size	1	1.00	1.70	4.67	0.00	3.18	0.00	0.00	-0.00
time (sec)	N/A	1.059	1.791	2.466	0.000	0.503	0.000	0.000	0.000
Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	544	982	0	744	0	0	-1
normalized size	1	1.00	2.48	4.48	0.00	3.40	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.140	2.036	0.000	0.483	0.000	0.000	0.000
Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	267	449	0	536	0	0	-1
normalized size	1	1.00	1.77	2.97	0.00	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.770	1.639	0.000	0.479	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	227	279	0	392	0	0	-1
normalized size	1	1.00	1.80	2.21	0.00	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.372	1.699	0.000	0.457	0.000	0.000	0.000
Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	550	1418	0	2577	0	0	-1
normalized size	1	1.00	2.11	5.43	0.00	9.87	0.00	0.00	-0.00
time (sec)	N/A	0.984	5.610	2.665	0.000	6.574	0.000	0.000	0.000
Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	1318	4092	0	5151	0	0	-1
normalized size	1	1.00	3.34	10.36	0.00	13.04	0.00	0.00	-0.00
time (sec)	N/A	1.536	12.375	4.202	0.000	14.831	0.000	0.000	0.000
Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	2103	7322	0	8555	0	0	-1
normalized size	1	1.00	4.05	14.11	0.00	16.48	0.00	0.00	-0.00
time (sec)	N/A	2.152	13.857	5.811	0.000	45.087	0.000	0.000	0.000
Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	26.082	3.868	0.000	0.531	0.000	0.000	0.000
Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	9.073	3.368	0.000	0.475	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	4.652	1.057	0.000	0.472	0.000	0.000	0.000
Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.341	28.141	10.288	0.000	0.482	0.000	0.000	0.000
Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	245	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.915	26.650	1.285	0.000	0.490	0.000	0.000	0.000
Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.265	8.255	1.237	0.000	0.471	0.000	0.000	0.000
Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	244	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	5.568	0.972	0.000	0.469	0.000	0.000	0.000
Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	603	0	0	0	0	0	-1
normalized size	1	1.00	2.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	17.227	0.983	0.000	0.489	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	300	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.987	7.726	9.533	0.000	0.469	0.000	0.000	0.000
Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	198	212	0	0	0	0	0	-1
normalized size	1	0.99	1.07	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.362	3.589	7.608	0.000	0.459	0.000	0.000	0.000
Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	275	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	1.816	0.018	0.000	0.453	0.000	0.000	0.000
Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	473	0	0	0	0	0	-1
normalized size	1	1.00	2.48	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	7.078	4.046	0.000	0.462	0.000	0.000	0.000
Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	651	0	0	0	0	0	-1
normalized size	1	1.00	2.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	2.496	14.319	0.000	0.479	0.000	0.000	0.000
Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	651	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.347	2.533	12.570	0.000	0.471	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	3281	0	0	0	0	0	-1
normalized size	1	1.00	11.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.628	8.136	1.204	0.000	0.515	0.000	0.000	0.000
Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.555	11.894	1.134	0.000	0.481	0.000	0.000	0.000
Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	672	0	0	0	0	0	-1
normalized size	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	5.714	0.891	0.000	0.477	0.000	0.000	0.000
Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	672	0	0	0	0	0	-1
normalized size	1	1.00	2.33	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	6.224	0.917	0.000	0.507	0.000	0.000	0.000
Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	682	0	0	0	0	0	-1
normalized size	1	1.00	2.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.430	6.082	1.351	0.000	0.484	0.000	0.000	0.000
Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	573	0	0	0	0	0	-1
normalized size	1	1.00	2.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.483	6.936	3.466	0.000	0.486	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	10.115	1.431	0.000	0.494	0.000	0.000	0.000
Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	4.874	2.750	0.000	0.480	0.000	0.000	0.000
Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	56	0	0	98
normalized size	1	1.00	1.00	0.00	0.00	1.44	0.00	0.00	2.51
time (sec)	N/A	0.172	0.678	6.471	0.000	0.536	0.000	0.000	14.774
Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-1)	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	57	0	0	99
normalized size	1	1.00	1.00	0.00	0.00	1.42	0.00	0.00	2.48
time (sec)	N/A	0.172	0.743	6.558	0.000	0.548	0.000	0.000	14.621
Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	188	1246	0	1308	0	776	16312
normalized size	1	1.00	0.94	6.26	0.00	6.57	0.00	3.90	81.97
time (sec)	N/A	0.580	1.596	0.461	0.000	0.610	0.000	6.894	27.619
Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	840	840	2042	6776580	0	0	0	0	-1
normalized size	1	1.00	2.43	8067.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.156	6.750	100.983	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	1901	3150101	0	0	0	0	-1
normalized size	1	1.00	3.02	5000.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.886	10.186	159.526	0.000	2.233	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	1949	99082	0	0	0	0	-1
normalized size	1	1.00	4.67	237.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	6.563	1.846	0.000	0.522	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	2266	198381	0	0	0	0	-1
normalized size	1	1.00	4.17	364.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.377	7.426	3.294	0.000	0.643	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	858	858	2837	827062	0	0	0	0	-1
normalized size	1	1.00	3.31	963.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.616	8.770	19.791	0.000	0.906	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.093	17.694	1.284	0.000	0.873	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [.2857]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	33	0.152
2	A	6	5	1.00	33	0.152
3	A	5	4	1.00	31	0.129
4	A	4	3	1.00	33	0.091
5	A	5	3	1.00	33	0.091
6	A	6	3	1.00	33	0.091
7	A	6	5	1.00	35	0.143
8	A	5	5	1.00	35	0.143
9	A	4	4	1.00	35	0.114
10	A	9	9	1.00	35	0.257
11	A	10	10	1.00	35	0.286
12	A	9	5	1.00	33	0.152
13	A	4	3	1.00	34	0.088
14	A	1	1	1.00	43	0.023
15	A	1	1	1.00	37	0.027
16	A	6	6	1.00	31	0.194
17	A	7	6	1.00	34	0.176
18	A	6	6	1.00	34	0.176
19	A	5	5	1.08	34	0.147
20	A	4	4	1.00	32	0.125
21	A	4	3	1.00	34	0.088
22	A	4	4	1.00	34	0.118
23	A	4	4	1.00	34	0.118
24	A	5	5	1.00	34	0.147
25	A	6	5	1.00	34	0.147
26	A	8	6	1.00	36	0.167
27	A	7	6	1.00	36	0.167
28	A	6	5	1.00	36	0.139
29	A	5	4	1.00	36	0.111
30	A	5	5	1.00	34	0.147
31	A	5	5	1.00	36	0.139
32	A	5	5	1.00	36	0.139
33	A	5	4	1.00	36	0.111
34	A	3	3	1.00	36	0.083
35	A	4	4	1.00	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
36	A	5	4	1.00	36	0.111
37	A	6	4	1.00	36	0.111
38	A	9	6	1.00	36	0.167
39	A	8	6	1.00	36	0.167
40	A	7	5	1.00	36	0.139
41	A	6	4	1.00	36	0.111
42	A	6	5	1.00	36	0.139
43	A	6	6	1.00	34	0.176
44	A	6	6	1.00	36	0.167
45	A	6	6	1.00	36	0.167
46	A	6	5	1.00	36	0.139
47	A	6	4	1.00	36	0.111
48	A	3	3	1.00	36	0.083
49	A	4	4	1.00	36	0.111
50	A	5	4	1.00	36	0.111
51	A	6	4	1.00	36	0.111
52	A	7	7	1.00	36	0.194
53	A	6	6	1.00	36	0.167
54	A	5	5	1.00	36	0.139
55	A	4	3	1.00	34	0.088
56	A	4	4	1.00	36	0.111
57	A	4	4	1.00	36	0.111
58	A	5	5	1.00	36	0.139
59	A	6	5	1.00	36	0.139
60	A	8	7	1.00	36	0.194
61	A	7	6	1.00	36	0.167
62	A	6	6	1.00	36	0.167
63	A	5	5	1.00	36	0.139
64	A	4	4	1.00	34	0.118
65	A	4	4	1.00	36	0.111
66	A	4	3	1.00	36	0.083
67	A	4	3	1.00	36	0.083
68	A	5	4	1.00	36	0.111
69	A	6	4	1.00	36	0.111
70	A	8	6	1.00	36	0.167
71	A	7	6	1.00	36	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	6	5	1.00	36	0.139
73	A	5	4	1.00	36	0.111
74	A	4	4	1.00	34	0.118
75	A	5	5	1.00	36	0.139
76	A	4	3	1.00	36	0.083
77	A	4	3	1.00	36	0.083
78	A	4	3	1.00	36	0.083
79	A	5	4	1.00	36	0.111
80	A	6	4	1.00	36	0.111
81	A	6	4	1.00	36	0.111
82	A	5	4	1.00	36	0.111
83	A	4	4	1.00	36	0.111
84	A	3	3	1.00	36	0.083
85	A	5	5	1.00	36	0.139
86	A	5	5	1.00	36	0.139
87	A	5	5	1.00	36	0.139
88	A	6	6	1.00	36	0.167
89	A	6	4	1.00	38	0.105
90	A	5	4	1.00	38	0.105
91	A	4	4	1.00	38	0.105
92	A	3	3	1.00	38	0.079
93	A	6	5	1.00	38	0.132
94	A	6	5	1.00	38	0.132
95	A	6	6	1.00	38	0.158
96	A	6	5	1.00	38	0.132
97	A	7	6	1.00	38	0.158
98	A	6	4	1.00	38	0.105
99	A	5	4	1.00	38	0.105
100	A	4	4	1.00	38	0.105
101	A	3	3	1.00	38	0.079
102	A	7	5	1.00	38	0.132
103	A	7	5	1.00	38	0.132
104	A	7	6	1.00	38	0.158
105	A	7	6	1.00	38	0.158
106	A	7	5	1.00	38	0.132
107	A	8	6	1.00	38	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	6	4	1.00	38	0.105
109	A	5	4	1.00	38	0.105
110	A	4	4	1.00	38	0.105
111	A	3	3	1.00	38	0.079
112	A	4	4	1.00	38	0.105
113	A	5	5	1.00	38	0.132
114	A	6	6	1.00	38	0.158
115	A	7	4	1.00	38	0.105
116	A	6	4	1.00	38	0.105
117	A	5	4	1.00	38	0.105
118	A	4	4	1.00	38	0.105
119	A	3	3	1.00	38	0.079
120	A	5	5	1.00	38	0.132
121	A	6	6	1.00	38	0.158
122	A	7	7	1.00	38	0.184
123	A	7	4	1.00	38	0.105
124	A	6	4	1.00	38	0.105
125	A	5	4	1.00	38	0.105
126	A	4	4	1.00	38	0.105
127	A	3	3	1.00	38	0.079
128	A	6	5	1.00	38	0.132
129	A	7	7	1.00	38	0.184
130	A	8	7	1.00	38	0.184
131	A	3	2	1.00	40	0.050
132	A	3	2	1.00	40	0.050
133	A	3	2	1.00	40	0.050
134	A	3	2	1.00	40	0.050
135	A	5	5	1.00	40	0.125
136	A	5	5	1.00	40	0.125
137	A	3	2	1.00	40	0.050
138	A	3	2	1.00	40	0.050
139	A	3	3	1.00	40	0.075
140	A	3	3	1.00	40	0.075
141	A	3	3	1.00	40	0.075
142	A	3	2	1.00	40	0.050
143	A	5	5	1.00	40	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	5	5	1.00	40	0.125
145	A	5	5	1.00	40	0.125
146	A	2	2	1.00	40	0.050
147	A	3	3	1.00	40	0.075
148	A	3	3	1.00	40	0.075
149	A	4	3	1.00	40	0.075
150	A	4	3	1.00	40	0.075
151	A	3	3	1.00	40	0.075
152	A	3	2	1.00	40	0.050
153	A	6	5	1.00	40	0.125
154	A	6	5	1.00	40	0.125
155	A	6	6	1.00	40	0.150
156	A	6	5	1.00	40	0.125
157	A	2	2	1.00	40	0.050
158	A	3	3	1.00	40	0.075
159	A	4	3	1.00	40	0.075
160	A	5	3	1.00	40	0.075
161	A	5	3	1.00	40	0.075
162	A	4	3	1.00	40	0.075
163	A	3	3	1.00	40	0.075
164	A	3	2	1.00	40	0.050
165	A	7	5	1.00	40	0.125
166	A	7	5	1.00	40	0.125
167	A	7	6	1.00	40	0.150
168	A	7	6	1.00	40	0.150
169	A	7	5	1.00	40	0.125
170	A	2	2	1.00	40	0.050
171	A	3	3	1.00	40	0.075
172	A	4	3	1.00	40	0.075
173	A	5	3	1.00	40	0.075
174	A	6	5	1.00	40	0.125
175	A	5	5	1.00	40	0.125
176	A	5	5	1.00	40	0.125
177	A	7	4	1.00	40	0.100
178	A	3	3	1.00	40	0.075
179	A	4	4	1.00	40	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	7	5	1.00	40	0.125
181	A	6	5	1.00	40	0.125
182	A	5	5	1.00	40	0.125
183	A	5	5	1.00	40	0.125
184	A	3	3	1.00	40	0.075
185	A	4	4	1.00	40	0.100
186	A	5	4	1.00	40	0.100
187	A	8	6	1.00	40	0.150
188	A	7	6	1.00	40	0.150
189	A	6	6	1.00	40	0.150
190	A	5	5	1.00	40	0.125
191	A	3	2	1.00	40	0.050
192	A	4	4	1.00	40	0.100
193	A	5	4	1.00	40	0.100
194	A	6	4	1.00	40	0.100
195	A	5	5	1.00	36	0.139
196	A	5	5	1.00	36	0.139
197	A	5	5	1.00	36	0.139
198	A	5	5	1.00	34	0.147
199	A	3	3	1.00	23	0.130
200	A	5	5	1.00	36	0.139
201	A	5	5	1.00	36	0.139
202	A	5	5	1.00	36	0.139
203	A	4	4	1.00	38	0.105
204	A	4	4	1.00	38	0.105
205	A	4	3	1.00	38	0.079
206	A	3	3	1.16	38	0.079
207	A	3	2	1.00	38	0.053
208	A	4	4	1.00	38	0.105
209	A	4	4	1.00	38	0.105
210	A	4	4	1.00	38	0.105
211	A	4	3	1.00	40	0.075
212	A	3	3	1.00	40	0.075
213	A	2	2	1.00	40	0.050
214	A	5	5	1.00	40	0.125
215	A	5	5	1.00	38	0.132

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	5	5	1.00	40	0.125
217	A	5	5	1.00	40	0.125
218	A	2	2	1.00	46	0.043
219	A	2	2	1.00	45	0.044
220	A	2	2	1.00	44	0.045
221	A	2	2	1.00	43	0.047
222	A	1	1	1.00	47	0.021
223	A	1	1	1.00	46	0.022
224	A	13	4	1.00	32	0.125
225	A	12	4	1.00	32	0.125
226	A	10	5	1.00	30	0.167
227	A	5	5	1.00	24	0.208
228	A	7	5	1.00	30	0.167
229	A	9	7	1.00	32	0.219
230	A	7	6	1.00	32	0.188
231	A	7	4	1.00	32	0.125
232	A	10	5	1.00	32	0.156
233	A	12	6	1.00	32	0.188
234	A	12	4	1.00	32	0.125
235	A	11	6	1.00	32	0.188
236	A	9	4	1.00	32	0.125
237	A	8	3	1.00	32	0.094
238	A	8	3	1.00	30	0.100
239	A	3	3	1.00	24	0.125
240	A	9	4	1.00	30	0.133
241	A	15	9	1.00	32	0.281
242	A	13	7	1.00	32	0.219
243	A	15	7	1.00	32	0.219
244	A	5	4	1.00	33	0.121
245	A	4	4	1.00	33	0.121
246	A	3	3	1.00	31	0.097
247	A	1	1	1.00	21	0.048
248	A	6	6	1.00	33	0.182
249	A	6	6	1.00	33	0.182
250	A	7	7	1.00	33	0.212
251	A	6	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
252	A	5	5	1.00	35	0.143
253	A	4	4	1.00	33	0.121
254	A	2	2	1.00	23	0.087
255	A	7	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	7	7	1.00	35	0.200
258	A	7	5	1.00	35	0.143
259	A	6	5	1.00	35	0.143
260	A	10	8	1.00	33	0.242
261	A	8	6	0.92	23	0.261
262	A	8	7	1.00	35	0.200
263	A	8	8	1.00	35	0.229
264	A	8	7	1.00	35	0.200
265	A	3	3	1.00	35	0.086
266	A	2	2	0.99	35	0.057
267	A	4	4	1.00	33	0.121
268	A	2	2	1.00	23	0.087
269	A	5	5	1.00	35	0.143
270	A	6	6	1.00	35	0.171
271	A	7	6	1.00	35	0.171
272	A	3	2	1.00	35	0.057
273	A	5	5	1.00	35	0.143
274	A	4	4	1.00	33	0.121
275	A	2	2	1.00	23	0.087
276	A	6	5	1.00	35	0.143
277	A	7	6	1.00	35	0.171
278	A	8	6	1.00	35	0.171
279	A	6	5	1.00	35	0.143
280	A	5	5	1.00	35	0.143
281	A	4	4	1.00	33	0.121
282	A	3	3	1.00	23	0.130
283	A	7	5	1.00	35	0.143
284	A	8	6	1.00	35	0.171
285	A	9	6	1.00	35	0.171
286	A	5	5	1.00	37	0.135
287	A	4	4	1.00	37	0.108

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	4	4	1.00	35	0.114
289	A	2	2	1.00	25	0.080
290	A	3	3	1.00	37	0.081
291	A	3	3	1.00	37	0.081
292	A	4	4	1.00	37	0.108
293	A	6	6	1.00	37	0.162
294	A	5	5	1.00	37	0.135
295	A	5	5	1.00	35	0.143
296	A	3	3	1.00	25	0.120
297	A	4	4	1.00	37	0.108
298	A	4	4	1.00	37	0.108
299	A	4	4	1.00	37	0.108
300	A	7	6	1.00	37	0.162
301	A	6	5	1.00	37	0.135
302	A	6	5	1.00	35	0.143
303	A	4	3	1.00	25	0.120
304	A	5	4	1.00	37	0.108
305	A	5	5	1.00	37	0.135
306	A	5	4	1.00	37	0.108
307	A	7	6	1.00	37	0.162
308	A	6	6	1.00	37	0.162
309	A	5	5	1.00	35	0.143
310	A	3	3	1.00	25	0.120
311	A	5	5	1.00	37	0.135
312	A	6	6	1.00	37	0.162
313	A	7	6	1.00	37	0.162
314	A	7	7	1.00	37	0.189
315	A	6	6	1.00	37	0.162
316	A	5	5	1.00	35	0.143
317	A	3	3	1.00	25	0.120
318	A	6	6	1.00	37	0.162
319	A	7	7	1.00	37	0.189
320	A	8	7	1.00	37	0.189
321	A	7	6	1.00	37	0.162
322	A	6	6	1.00	37	0.162
323	A	5	5	1.00	35	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	4	4	1.00	25	0.160
325	A	7	6	1.00	37	0.162
326	A	8	7	1.00	37	0.189
327	A	9	7	1.00	37	0.189
328	A	7	4	1.00	35	0.114
329	A	8	6	1.00	33	0.182
330	A	7	5	1.00	35	0.143
331	A	7	4	1.00	35	0.114
332	A	11	7	1.00	37	0.189
333	A	4	4	1.00	37	0.108
334	A	7	7	1.00	37	0.189
335	A	7	4	1.00	37	0.108
336	A	6	6	1.00	35	0.171
337	A	5	5	0.99	33	0.152
338	A	3	3	1.00	23	0.130
339	A	6	6	1.00	35	0.171
340	A	7	7	1.00	35	0.200
341	A	8	7	1.00	35	0.200
342	A	9	5	1.00	37	0.135
343	A	9	5	1.00	37	0.135
344	A	9	5	1.00	37	0.135
345	A	9	5	1.00	37	0.135
346	A	9	5	1.00	35	0.143
347	A	7	6	1.00	39	0.154
348	A	4	4	1.00	36	0.111
349	A	4	4	1.00	40	0.100
350	A	1	1	1.00	55	0.018
351	A	1	1	1.00	51	0.020
352	A	6	6	1.00	35	0.171
353	A	7	7	1.00	39	0.180
354	A	5	5	1.00	39	0.128
355	A	3	3	1.00	39	0.077
356	A	4	4	1.00	39	0.103
357	A	5	5	1.00	39	0.128
358	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1
$$\int (d \sin(e + fx))^n (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) dx$$

Optimal. Leaf size=373

$$\frac{a^3(A(4n + 11) + B(4n + 9)) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n + 2)(n + 3)\sqrt{\cos^2(e + fx)}} + \frac{a^3(A(4n^2 + 21n + 11) + B(4n^2 + 21n + 9)) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n + 2)(n + 3)\sqrt{\cos^2(e + fx)}}$$

[Out] $-a^3(B(2n^2+14n+27)+A(2n^2+15n+28))\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(4+n)/(n^2+5*n+6)-a*B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}*(a+a*\sin(f*x+e))^{2/d/f/(4+n)-(A*(4+n)+B*(6+n))*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}*(a^3+a^3*\sin(f*x+e))/d/f/(3+n)/(4+n)+a^3*(B*(4*n^2+19*n+15)+A*(4*n^2+21*n+20))*\cos(f*x+e)*\text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(1+n)}/d/f/(4+n)/(n^2+3*n+2)/(\cos(f*x+e)^2)^{(1/2)}+a^3*(B*(9+4*n)+A*(11+4*n))*\cos(f*x+e)*\text{hypergeom}([1/2, 1+1/2*n], [1/2*n+2], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(2+n)}/d^2/f/(2+n)/(3+n)/(\cos(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.84, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^3(A(4n + 11) + B(4n + 9)) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n + 2)(n + 3)\sqrt{\cos^2(e + fx)}} + \frac{a^3(A(4n^2 + 21n + 11) + B(4n^2 + 21n + 9)) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n + 2)(n + 3)\sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d*\text{Sin}[e + f*x])^n*(a + a*\text{Sin}[e + f*x])^3*(A + B*\text{Sin}[e + f*x]),x]$

[Out] $-((a^3*(B*(27 + 14*n + 2*n^2) + A*(28 + 15*n + 2*n^2))*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(2 + n)*(3 + n)*(4 + n))) + (a^3*(B*(15 + 19*n + 4*n^2) + A*(20 + 21*n + 4*n^2))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, (1 + n)/2, (3 + n)/2, \text{Sin}[e + f*x]^2]*(d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(1 + n)*(2 + n)*(4 + n)*\text{Sqrt}[\text{Cos}[e + f*x]^2]) + (a^3*(B*(9 + 4*n) + A*(11 + 4*n))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, (2 + n)/2, (4 + n)/2, \text{Sin}[e + f*x]^2]*(d*\text{Sin}[e + f*x])^{(2 + n)})/(d^2*f*(2 + n)*(3 + n)*\text{Sqrt}[\text{Cos}[e + f*x]^2]) - (a*B*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)}*(a + a*\text{Sin}[e + f*x])^2)/(d*f*(4 + n)) - ((A*(4 + n) + B*(6 + n))*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)}*(a^3 + a^3*\text{Sin}[e + f*x]))/(d*f*(3 + n)*(4 + n))$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c$

+ d*x]^2))/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) dx &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}(a + a \sin(e + fx))}{df(4 + n)} \\ &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}(a + a \sin(e + fx))}{df(4 + n)} \\ &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}(a + a \sin(e + fx))}{df(4 + n)} \\ &= -\frac{a^3 (B (27 + 14n + 2n^2) + A (28 + 15n + 2n^2)) c}{df(2 + n)(3 + n)(4 + n)} \\ &= -\frac{a^3 (B (27 + 14n + 2n^2) + A (28 + 15n + 2n^2)) c}{df(2 + n)(3 + n)(4 + n)} \\ &= -\frac{a^3 (B (27 + 14n + 2n^2) + A (28 + 15n + 2n^2)) c}{df(2 + n)(3 + n)(4 + n)} \end{aligned}$$

Mathematica [A] time = 2.35, size = 248, normalized size = 0.66

$$a^3 \sin(e + fx) \cos(e + fx) (d \sin(e + fx))^n \left(\sin(e + fx) \left(\frac{{}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{n+2} + \sin(e + fx) \left(\frac{{}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{n+2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]

[Out] (a^3*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*((3*A + B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Sin[e + f*x]*((3*(A + B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e + f*x]^2])/(3 + n) + Sin[e + f*x]*(((A + 3*B)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e + f*x]^2])/(4 + n) + (B*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(5 + n)))))/(f*Sqrt[Cos[e + f*x]^2])

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ba^3 \cos(fx + e)^4 - (3A + 5B)a^3 \cos(fx + e)^2 + 4(A + B)a^3 - \left((A + 3B)a^3 \cos(fx + e)^2 - 4(A + B)a^3\right) \sin(fx + e)\right) (d \sin(fx + e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*(d*sin(f*x + e))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^3 (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x)

maple [F] time = 8.25, size = 0, normalized size = 0.00

$$\int (d \sin(fx + e))^n (a + a \sin(fx + e))^3 (A + B \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^3 (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin(e + f x))^n (A + B \sin(e + f x)) (a + a \sin(e + f x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3,x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)

[Out] Timed out

3.2 $\int (d \sin(e + fx))^n (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) dx$

Optimal. Leaf size=277

$$\frac{a^2(2A(n+3) + B(2n+5)) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{d^2 f(n+2)(n+3)\sqrt{\cos^2(e+fx)}} + \frac{a^2(A(2n+3) + 2B(n+2)) \cos(e+fx)(d \sin(e+fx))^{n+1}}{d^2 f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}$$

[Out] $-a^2(A(3+n)+B(4+n))\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(2+n)/(3+n)-B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}*(a^2+a^2*\sin(f*x+e))/d/f/(3+n)+a^2*(2*B*(1+n)+A*(3+2*n))*\cos(f*x+e)*\text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(1+n)}/d/f/(1+n)/(2+n)/(\cos(f*x+e)^2)^{(1/2)}+a^2*(2*A*(3+n)+B*(5+2*n))*\cos(f*x+e)*\text{hypergeom}([1/2, 1+1/2*n], [1/2*n+2], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(2+n)}/d^2/f/(2+n)/(3+n)/(\cos(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2976, 2968, 3023, 2748, 2643}

$$\frac{a^2(2A(n+3) + B(2n+5)) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{d^2 f(n+2)(n+3)\sqrt{\cos^2(e+fx)}} + \frac{a^2(A(2n+3) + 2B(n+2)) \cos(e+fx)(d \sin(e+fx))^{n+1}}{d^2 f(n+2)(n+3)\sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]

[Out] $-((a^2*(A*(3+n) + B*(4+n))*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)})/(d*f*(2+n)*(3+n))) + (a^2*(2*B*(1+n) + A*(3+2*n))*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, (1+n)/2, (3+n)/2, \text{Sin}[e+f*x]^2]*(d*\text{Sin}[e+f*x])^{(1+n)})/(d*f*(1+n)*(2+n)*\text{Sqrt}[\text{Cos}[e+f*x]^2]) + (a^2*(2*A*(3+n) + B*(5+2*n))*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, (2+n)/2, (4+n)/2, \text{Sin}[e+f*x]^2]*(d*\text{Sin}[e+f*x])^{(2+n)})/(d^2*f*(2+n)*(3+n)*\text{Sqrt}[\text{Cos}[e+f*x]^2]) - (B*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)}*(a^2 + a^2*\text{Sin}[e+f*x]))/(d*f*(3+n))$

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2748

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2968

Int[((a_.) + (b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((A_.) + (B_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_.) + (b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((A_.) + (B_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := -Si

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mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) dx &= -\frac{B \cos(e + fx) (d \sin(e + fx))^{1+n} (a^2 + a^2 \sin(e + fx))}{df(3 + n)} \\ &= -\frac{B \cos(e + fx) (d \sin(e + fx))^{1+n} (a^2 + a^2 \sin(e + fx))}{df(3 + n)} \\ &= -\frac{a^2 (A(3 + n) + B(4 + n)) \cos(e + fx) (d \sin(e + fx))^{1+n}}{df(2 + n)(3 + n)} \\ &= -\frac{a^2 (A(3 + n) + B(4 + n)) \cos(e + fx) (d \sin(e + fx))^{1+n}}{df(2 + n)(3 + n)} \\ &= -\frac{a^2 (A(3 + n) + B(4 + n)) \cos(e + fx) (d \sin(e + fx))^{1+n}}{df(2 + n)(3 + n)} \end{aligned}$$

Mathematica [A] time = 1.55, size = 204, normalized size = 0.74

$$\frac{a^2 \sin(e + fx) \cos(e + fx) (d \sin(e + fx))^n \left(\sin(e + fx) \left(\frac{(2A+B) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{n+2} \right) + \sin(e + fx) \left(\frac{(A+2B) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n}{2}; \sin^2(e + fx)\right)}{n} \right) \right)}{f \sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]
[Out] (a^2*Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*((A*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/(1 + n) + Sin[e + f*x]*((2*A + B)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2])/(2 + n) + Sin[e + f*x]*((A + 2*B)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Sin[e + f*x]^2])/(3 + n) + (B*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(4 + n)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral} \left(-\left((A + 2B)a^2 \cos^2(fx + e) - 2(A + B)a^2 + (Ba^2 \cos^2(fx + e) - 2(A + B)a^2) \sin(fx + e) \right) (d \sin(fx + e))^n \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e))^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x)

maple [F] time = 7.54, size = 0, normalized size = 0.00

$$\int (d \sin(fx + e))^n (a + a \sin(fx + e))^2 (A + B \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin(e + fx))^n (A + B \sin(e + fx)) (a + a \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)

[Out] Timed out

3.3 $\int (d \sin(e + fx))^n (a + a \sin(e + fx))(A + B \sin(e + fx)) dx$

Optimal. Leaf size=191

$$\frac{a(A + B) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n+2) \sqrt{\cos^2(e + fx)}} + \frac{a(A(n+2) + B(n+1)) \cos(e + fx)(d \sin(e + fx))^{n+1}}{df(n+1)(n+2)}$$

[Out] $-a*B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(2+n)+a*(B*(1+n)+A*(2+n))*\cos(f*x+e)*\text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(1+n)}/d/f/(1+n)/(2+n)/(\cos(f*x+e)^2)^{(1/2)}+a*(A+B)*\cos(f*x+e)*\text{hypergeom}([1/2, 1+1/2*n], [1/2*n+2], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(2+n)}/d^2/f/(2+n)/(\cos(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.22, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2968, 3023, 2748, 2643}

$$\frac{a(A + B) \cos(e + fx)(d \sin(e + fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e + fx)\right)}{d^2 f(n+2) \sqrt{\cos^2(e + fx)}} + \frac{a(A(n+2) + B(n+1)) \cos(e + fx)(d \sin(e + fx))^{n+1}}{df(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d*\text{Sin}[e + f*x])^n*(a + a*\text{Sin}[e + f*x])*(A + B*\text{Sin}[e + f*x]),x]$

[Out] $-((a*B*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(2 + n))) + (a*(B*(1 + n) + A*(2 + n))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, (1 + n)/2, (3 + n)/2, \text{Sin}[e + f*x]^2]*(d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(1 + n)*(2 + n)*\text{Sqrt}[\text{Cos}[e + f*x]^2]) + (a*(A + B)*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, (2 + n)/2, (4 + n)/2, \text{Sin}[e + f*x]^2]*(d*\text{Sin}[e + f*x])^{(2 + n)})/(d^2*f*(2 + n)*\text{Sqrt}[\text{Cos}[e + f*x]^2])$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\amp; \text{IntegerQ}[2*n]$

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2968

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x \&\amp; \text{NeQ}[b*c - a*d, 0]$

Rule 3023

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 2)), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +$

2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))(A + B \sin(e + fx)) dx &= \int (d \sin(e + fx))^n (aA + (aA + aB) \sin(e + fx)) dx \\ &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}}{df(2+n)} + \frac{\int (d \sin(e + fx))^{1+n} dx}{df(2+n)} \\ &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}}{df(2+n)} + \frac{(a(A + B) \sin(e + fx))^{1+n}}{df(2+n)} \\ &= -\frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}}{df(2+n)} + \frac{a(B(1 + \sin(e + fx)))^{1+n}}{df(2+n)} \end{aligned}$$

Mathematica [C] time = 3.85, size = 392, normalized size = 2.05

$$a2^{-n-2}e^{ifnx} (1 - e^{2i(e+fx)})^{-n} (-ie^{-i(e+fx)} (-1 + e^{2i(e+fx)}))^n (\sin(e + fx) + 1) \sin^{-n}(e + fx)(d \sin(e + fx))^n \left(\frac{2(A + B \sin(e + fx))^{1+n}}{df(2+n)} - \frac{aB \cos(e + fx)(d \sin(e + fx))^{1+n}}{df(2+n)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]
[Out] -((2^(-2 - n)*a*E^(I*f*n*x)*((( -I)*(-1 + E^((2*I)*(e + f*x)))))/E^(I*(e + f*x)))^n*((2*(A + B)*Hypergeometric2F1[(-1 - n)/2, -n, (1 - n)/2, E^((2*I)*(e + f*x))])/(E^(I*(e + f*(1 + n)*x))*(1 + n)) - (2*(A + B)*E^(I*(e - f*(-1 + n)*x))*Hypergeometric2F1[(1 - n)/2, -n, (3 - n)/2, E^((2*I)*(e + f*x))])/( -1 + n) + I*((B*Hypergeometric2F1[-1 - n/2, -n, -1/2*n, E^((2*I)*(e + f*x))])/(E^(I*(2*e + f*(2 + n)*x))*(2 + n)) + (B*E^((2*I)*(e + f*x))*n*Hypergeometric2F1[1 - n/2, -n, 2 - n/2, E^((2*I)*(e + f*x))]) - 2*(2*A + B)*(-2 + n)*Hypergeometric2F1[-n, -1/2*n, 1 - n/2, E^((2*I)*(e + f*x))])/(E^(I*f*n*x)*(-2 + n)*n))*(d*Sin[e + f*x])^n*(1 + Sin[e + f*x])/((1 - E^((2*I)*(e + f*x)))^n*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sin[e + f*x]^n)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral} \left(-\left(Ba \cos(fx + e)^2 - (A + B)a \sin(fx + e) - (A + B)a \right) (d \sin(fx + e))^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(f*x + e))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)(d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

maple [F] time = 5.40, size = 0, normalized size = 0.00

$$\int (d \sin(fx + e))^n (a + a \sin(fx + e)) (A + B \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)(d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d \sin(e + fx))^n (A + B \sin(e + fx)) (a + a \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

[Out] Timed out

$$3.4 \quad \int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=202

$$\frac{(n+1)(A-B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{ad^2 f(n+2) \sqrt{\cos^2(e+fx)}} + \frac{(-An+Bn+B) \cos(e+fx)(d \sin(e+fx))^{n+1}}{adf(n+1) \sqrt{\cos^2(e+fx)}}$$

[Out] (A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))+(-A*n+B*n+B)*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n], [3/2+1/2*n], sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+(A-B)*(1+n)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n], [1/2*n+2], sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)

Rubi [A] time = 0.22, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(A-B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{ad^2 f(n+2) \sqrt{\cos^2(e+fx)}} + \frac{(-An+Bn+B) \cos(e+fx)(d \sin(e+fx))^{n+1}}{adf(n+1) \sqrt{\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]

[Out] ((B - A*n + B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(a*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((A - B)*(1 + n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(a*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(d*f*(a + a*Sin[e + f*x]))

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n_, x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^m_*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2978

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^m_*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^n_, x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{a + a \sin(e + fx)} dx &= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{df(a + a \sin(e + fx))} + \frac{\int (d \sin(e + fx))^n (ad(B - A))}{df(a + a \sin(e + fx))} \\ &= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{df(a + a \sin(e + fx))} + \frac{((A - B)(1 + n)) \int (d \sin(e + fx))^n}{ad} \\ &= \frac{(B - An + Bn) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(e + fx)\right) (d \sin(e + fx))^{1+n}}{adf(1 + n)\sqrt{\cos^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.89, size = 157, normalized size = 0.78

$$\frac{\sin(e + fx) \cos(e + fx) (d \sin(e + fx))^n \left(\frac{(n+1)(A-B) \sin(e+fx) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{(n+2)\sqrt{\cos^2(e+fx)}} + \frac{(-An+Bn+B) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \sin^2(e+fx)\right)}{(n+1)\sqrt{\cos^2(e+fx)}} \right)}{af}$$

Antiderivative was successfully verified.

[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]

[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(((B - A*n + B*n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2])/((1 + n)*Sqrt[Cos[e + f*x]^2]) + ((A - B)*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[Cos[e + f*x]^2]) + (A - B)/(1 + Sin[e + f*x]))/(a*f)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{a \sin(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

maple [F] time = 1.97, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(fx + e))^n (A + B \sin(fx + e))}{a + a \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e))^n}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{a + a \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)),x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] Timed out

$$3.5 \quad \int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=279

$$\frac{(n+1)(2A(1-n)+2Bn+B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{3a^2 d^2 f(n+2) \sqrt{\cos^2(e+fx)}} \frac{n(-2An+A+2Bn)}{}$$

[Out] $\frac{1}{3}*(B+2*A*(1-n)+2*B*n)*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/a^2/d/f/(1+\sin(f*x+e))+1/3*(A-B)*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(a+a*\sin(f*x+e))^2-1/3*n*(A-2*A*n+2*B*(1+n))*\cos(f*x+e)*\text{hypergeom}([1/2, 1/2+1/2*n], [3/2+1/2*n], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(1+n)}/a^2/d/f/(1+n)/(\cos(f*x+e)^2)^{(1/2)}+1/3*(1+n)*(B+2*A*(1-n)+2*B*n)*\cos(f*x+e)*\text{hypergeom}([1/2, 1+1/2*n], [1/2*n+2], \sin(f*x+e)^2)*(d*\sin(f*x+e))^{(2+n)}/a^2/d^2/f/(2+n)/(\cos(f*x+e)^2)^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2978, 2748, 2643}

$$\frac{(n+1)(2A(1-n)+2Bn+B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{3a^2 d^2 f(n+2) \sqrt{\cos^2(e+fx)}} \frac{n(-2An+A+2Bn)}{}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\frac{(d*\text{Sin}[e+f*x])^n*(A+B*\text{Sin}[e+f*x])}{(a+a*\text{Sin}[e+f*x])^2}, x]$

[Out] $-(n*(A-2*A*n+2*B*(1+n))*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, (1+n)/2, (3+n)/2, \text{Sin}[e+f*x]^2]*(d*\text{Sin}[e+f*x])^{(1+n)})/(3*a^2*d*f*(1+n)*\text{Sqrt}[\text{Cos}[e+f*x]^2]) + ((1+n)*(B+2*A*(1-n)+2*B*n)*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, (2+n)/2, (4+n)/2, \text{Sin}[e+f*x]^2]*(d*\text{Sin}[e+f*x])^{(2+n)})/(3*a^2*d^2*f*(2+n)*\text{Sqrt}[\text{Cos}[e+f*x]^2]) + ((B+2*A*(1-n)+2*B*n)*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)})/(3*a^2*d*f*(1+\text{Sin}[e+f*x])) + ((A-B)*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)})/(3*d*f*(a+a*\text{Sin}[e+f*x])^2)$

Rule 2643

$\text{Int}[\frac{(b_*)*\sin[(c_*) + (d_*)*(x_*)]^n}{(a_*)^2}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{n+1}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2]}{(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2])}, x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2*n]$

Rule 2748

$\text{Int}[\frac{(b_*)*\sin[(e_*) + (f_*)*(x_*)]^m*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])}{(a_*)^2}, x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e+f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e+f*x])^{m+1}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2978

$\text{Int}[\frac{(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^m*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)])}{(c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]^n}, x_Symbol] \rightarrow \text{Simp}[\frac{(b*(A*b-a*B)*\text{Cos}[e+f*x]*(a+b*\text{Sin}[e+f*x])^m*(c+d*\text{Sin}[e+f*x])^{n+1})}{(a*f*(2*m+1)*(b*c-a*d))}, x] + \text{Dist}[1/(a*(2*m+1)*(b*c-a*d)), \text{Int}[(a+b*\text{Sin}[e+f*x])^{m+1}*(c+d*\text{Sin}[e+f*x])^n*\text{Simp}[B*(a*c*m+b*d*(n+1))+A*(b*c*(m+1)-a*d*(2*m+n+2))+d*(A*b-a*B)*(m+n+2)*\text{Sin}[e+f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c-a*d, 0] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{NeQ}[c^2-d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}]$

&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^2} dx &= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{3df(a + a \sin(e + fx))^2} + \frac{\int \frac{(d \sin(e + fx))^n (ad(2A+B-An) + a^2 \sin(e + fx))}{a + a \sin(e + fx)} dx}{3a} \\ &= \frac{(B + 2A(1 - n) + 2Bn) \cos(e + fx) (d \sin(e + fx))^{1+n}}{3a^2 df (1 + \sin(e + fx))} + \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{3a} \\ &= \frac{(B + 2A(1 - n) + 2Bn) \cos(e + fx) (d \sin(e + fx))^{1+n}}{3a^2 df (1 + \sin(e + fx))} + \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{3a} \\ &= -\frac{n(A - 2An + 2B(1 + n)) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(e + fx)\right)}{3a^2 df (1 + n) \sqrt{\cos^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.33, size = 212, normalized size = 0.76

$$\frac{\sin(e + fx) \cos(e + fx) (d \sin(e + fx))^n \left(-\frac{n(-2An + A + 2B(n+1)) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \sin^2(e + fx)\right)}{(n+1)\sqrt{\cos^2(e + fx)}} + \frac{(n+1)(-2A(n-1) + 2Bn + B) \sin(e + fx)}{(n+2)\sqrt{\cos^2(e + fx)}} \right)}{3a^2 f}$$

Antiderivative was successfully verified.

[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2, x]

[Out] (Cos[e + f*x]*Sin[e + f*x]*(d*Sin[e + f*x])^n*(-((n*(A - 2*A*n + 2*B*(1 + n)))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]))/((1 + n)*Sqrt[Cos[e + f*x]^2])) + ((1 + n)*(B - 2*A*(-1 + n) + 2*B*n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/((2 + n)*Sqrt[Cos[e + f*x]^2]) + (A - B)/(1 + Sin[e + f*x])^2 + ((-A + B)*n)/(1 + Sin[e + f*x]) + (2*A + B - A*n + B*n)/(1 + Sin[e + f*x]))/(3*a^2*f)

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)

maple [F] time = 11.46, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(fx + e))^n (A + B \sin(fx + e))}{(a + a \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)

[Out] Timed out

$$3.6 \quad \int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=362

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{15a^3 d^2 f(n+2) \sqrt{\cos^2(e+fx)}} n(A(4$$

[Out] 1/5*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^3+1/15*(A*(5-2*n)+2*B*n)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/a/d/f/(a+a*sin(f*x+e))^2+1/15*(1-n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a^3+a^3*sin(f*x+e))-1/15*n*(B*(-4*n^2-n+3)+A*(4*n^2-9*n+2))*cos(f*x+e)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],sin(f*x+e)^2)*(d*sin(f*x+e))^(1+n)/a^3/d/f/(1+n)/(cos(f*x+e)^2)^(1/2)+1/15*(1-n)*(1+n)*(-4*A*n+4*B*n+7*A+3*B)*cos(f*x+e)*hypergeom([1/2, 1+1/2*n],[1/2*n+2],sin(f*x+e)^2)*(d*sin(f*x+e))^(2+n)/a^3/d^2/f/(2+n)/(cos(f*x+e)^2)^(1/2)

Rubi [A] time = 0.85, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2978, 2748, 2643}

$$\frac{(1-n)(n+1)(-4An+7A+4Bn+3B) \cos(e+fx)(d \sin(e+fx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \sin^2(e+fx)\right)}{15a^3 d^2 f(n+2) \sqrt{\cos^2(e+fx)}} n(A(4$$

Antiderivative was successfully verified.

[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]

[Out] -(n*(B*(3 - n - 4*n^2) + A*(2 - 9*n + 4*n^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(1 + n))/(15*a^3*d*f*(1 + n)*Sqrt[Cos[e + f*x]^2]) + ((1 - n)*(1 + n)*(7*A + 3*B - 4*A*n + 4*B*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*(d*Sin[e + f*x])^(2 + n))/(15*a^3*d^2*f*(2 + n)*Sqrt[Cos[e + f*x]^2]) + ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(5*d*f*(a + a*Sin[e + f*x])^3) + ((A*(5 - 2*n) + 2*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*a*d*f*(a + a*Sin[e + f*x])^2) + ((1 - n)*(7*A + 3*B - 4*A*n + 4*B*n)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(15*d*f*(a^3 + a^3*Sin[e + f*x]))

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2978

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2

```
) * Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^3} dx = \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{5df(a + a \sin(e + fx))^3} + \frac{\int \frac{(d \sin(e + fx))^n (ad(4A+B-An+Bn) + (a+a \sin(e + fx))^3)}{5a^2} dx}{5a^2}$$

$$= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{5df(a + a \sin(e + fx))^3} + \frac{(A(5 - 2n) + 2Bn) \cos(e + fx) (d \sin(e + fx))^{1+n}}{15adf(a + a \sin(e + fx))^3}$$

$$= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{5df(a + a \sin(e + fx))^3} + \frac{(A(5 - 2n) + 2Bn) \cos(e + fx) (d \sin(e + fx))^{1+n}}{15adf(a + a \sin(e + fx))^3}$$

$$= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{5df(a + a \sin(e + fx))^3} + \frac{(A(5 - 2n) + 2Bn) \cos(e + fx) (d \sin(e + fx))^{1+n}}{15adf(a + a \sin(e + fx))^3}$$

$$= \frac{n(B(3 - n - 4n^2) + A(2 - 9n + 4n^2)) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(e + fx)\right)}{15a^3 df(1+n) \sqrt{\cos^2(e + fx)}}$$

Mathematica [A] time = 4.57, size = 260, normalized size = 0.72

$$(d \sin(e + fx))^n \frac{2 \sin(e + fx) \cos(e + fx) \left(n(A(-4n^2 + 9n - 2) + B(4n^2 + n - 3)) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \sin^2(e + fx)\right) + \frac{(n-1)(n+1)^2(A(4n-7) - B(4n+3)) \sin(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \sin^2(e + fx)\right)}{n+2} \right)}{(n+1) \sqrt{\cos^2(e + fx)}}}{30a^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3, x]
```

```
[Out] ((d*Sin[e + f*x])^n*((2*Cos[e + f*x]*Sin[e + f*x]*(n*(A*(-2 + 9*n - 4*n^2) + B*(-3 + n + 4*n^2))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sin[e + f*x]^2] + ((-1 + n)*(1 + n)^2*(A*(-7 + 4*n) - B*(3 + 4*n))*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Sin[e + f*x]^2]*Sin[e + f*x])/(2 + n)))/((1 + n)*Sqrt[Cos[e + f*x]^2]) + (3*(A - B)*Sin[2*(e + f*x)]/(1 + Sin[e + f*x])^3 + ((A*(5 - 2*n) + 2*B*n)*Sin[2*(e + f*x)]/(1 + Sin[e + f*x])^2 + ((-1 + n)*(A*(-7 + 4*n) - B*(3 + 4*n))*Sin[2*(e + f*x)]/(1 + Sin[e + f*x])))/(30*a^3*f)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{3a^3 \cos(fx + e)^2 - 4a^3 + (a^3 \cos(fx + e)^2 - 4a^3) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x)

maple [F] time = 11.62, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(fx + e))^n (A + B \sin(fx + e))}{(a + a \sin(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)

[Out] Timed out

3.7 $\int (d \sin(e+fx))^n (a+a \sin(e+fx))^{5/2} (A+B \sin(e+fx)) dx$

Optimal. Leaf size=336

$$\frac{2a^3 \left(A(32n^3 + 224n^2 + 478n + 301) + 2B(16n^3 + 104n^2 + 203n + 115) \right) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))}{f(2n+3)(2n+5)(2n+7) \sqrt{a \sin(e+fx) + a}}$$

[Out] $-2*a*B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}*(a+a*\sin(f*x+e))^{(3/2)}/d/f/(7+2*n)-2*a^3*(2*B*(16*n^3+104*n^2+203*n+115)+A*(32*n^3+224*n^2+478*n+301))*\cos(f*x+e)*\text{hypergeom}([1/2, -n], [3/2], 1-\sin(f*x+e))*(d*\sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(7+2*n)/(\sin(f*x+e)^n)/(a+a*\sin(f*x+e))^{(1/2)}-2*a^3*(2*B*(4*n^2+23*n+35)+A*(8*n^2+50*n+77))*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(3+2*n)/(5+2*n)/(7+2*n)/(a+a*\sin(f*x+e))^{(1/2)}-2*a^2*(2*B*(5+n)+A*(7+2*n))*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}*(a+a*\sin(f*x+e))^{(1/2)}/d/f/(5+2*n)/(7+2*n)$

Rubi [A] time = 0.87, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2981, 2776, 67, 65}

$$\frac{2a^3 \left(A(32n^3 + 224n^2 + 478n + 301) + 2B(16n^3 + 104n^2 + 203n + 115) \right) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))}{f(2n+3)(2n+5)(2n+7) \sqrt{a \sin(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d*\text{Sin}[e+f*x])^n*(a+a*\text{Sin}[e+f*x])^{(5/2)}*(A+B*\text{Sin}[e+f*x]),x]$

[Out] $(-2*a^3*(2*B*(115+203*n+104*n^2+16*n^3)+A*(301+478*n+224*n^2+32*n^3))*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1/2, -n, 3/2, 1-\text{Sin}[e+f*x]]*(d*\text{Sin}[e+f*x])^n/(f*(3+2*n)*(5+2*n)*(7+2*n)*\text{Sin}[e+f*x]^n*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]) - (2*a^3*(2*B*(35+23*n+4*n^2)+A*(77+50*n+8*n^2))*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)})/(d*f*(3+2*n)*(5+2*n)*(7+2*n)*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]) - (2*a^2*(2*B*(5+n)+A*(7+2*n))*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)}*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])/(d*f*(5+2*n)*(7+2*n)) - (2*a*B*\text{Cos}[e+f*x]*(d*\text{Sin}[e+f*x])^{(1+n)}*(a+a*\text{Sin}[e+f*x])^{(3/2)})/(d*f*(7+2*n))$

Rule 65

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}*\text{Hypergeometric2F1}[-m, n+1, n+2, 1+(d*x)/c]/(d*(n+1)*(-(d/(b*c)))^m), x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-(d/(b*c)), 0])$

Rule 67

$\text{Int}[(b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(-(b*c)/d)^{\text{IntPart}[m]}*(b*x)^{\text{FracPart}[m]}/(-(d*x)/c)^{\text{FracPart}[m]}, \text{Int}[(-(d*x)/c)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[-(d/(b*c)), 0]$

Rule 2776

$\text{Int}[\text{Sqrt}[a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]]*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(a^2*\text{Cos}[e+fx])/(f*\text{Sqrt}[a+b*\text{Sin}[e+fx]])*\text{Sqrt}[a-b*\text{Sin}[e+fx]], \text{Subst}[\text{Int}[(c + d*x)^n/\text{Sqrt}[a-b*x], x], x, \text{Sin}[e+fx]], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d,$

0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]

Rule 2976

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp
[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) dx &= -\frac{2aB \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(7 + 2n)} \\ &= -\frac{2a^2(2B(5 + n) + A(7 + 2n)) \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(5 + 2n)(7 + 2n)} \\ &= -\frac{2a^3(2B(35 + 23n + 4n^2) + A(77 + 50n + 7n^2)) \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(3 + 2n)(5 + 2n)(7 + 2n)} \\ &= -\frac{2a^3(2B(35 + 23n + 4n^2) + A(77 + 50n + 7n^2)) \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(3 + 2n)(5 + 2n)(7 + 2n)} \\ &= -\frac{2a^3(2B(35 + 23n + 4n^2) + A(77 + 50n + 7n^2)) \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(3 + 2n)(5 + 2n)(7 + 2n)} \\ &= -\frac{2a^3(2B(115 + 203n + 104n^2 + 16n^3) + A(157 + 102n + 16n^2)) \cos(e + fx) (d \sin(e + fx))^{1+n} (a + a \sin(e + fx))^{5/2}}{df(3 + 2n)(5 + 2n)(7 + 2n)} \end{aligned}$$

Mathematica [A] time = 18.38, size = 596, normalized size = 1.77

$$2^{n+1} \tan\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) (a(\sin(e + fx) + 1))^{5/2} \sin^{-n}(e + fx) \left(\frac{\tan\left(\frac{1}{2}(e + fx)\right)}{\tan^2\left(\frac{1}{2}(e + fx)\right) + 1}\right)^n \left(\tan^2\left(\frac{1}{2}(e + fx)\right)\right)^n$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]
),x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(5/2)
*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e
+ f*x)/2]^2)^n*((A*Hypergeometric2F1[(1 + n)/2, 9/2 + n, (3 + n)/2, -Tan[(e
+ f*x)/2]^2]/(1 + n) + (A*Hypergeometric2F1[4 + n/2, 9/2 + n, 5 + n/2, -
Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^7)/(8 + n) + Tan[(e + f*x)/2]*(((5*A +
2*B)*Hypergeometric2F1[(2 + n)/2, 9/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2]
)/(2 + n) + Tan[(e + f*x)/2]*(((11*A + 10*B)*Hypergeometric2F1[(3 + n)/2, 9
/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2]/(3 + n) + Tan[(e + f*x)/2]*((5*(3*
A + 4*B)*Hypergeometric2F1[(4 + n)/2, 9/2 + n, (6 + n)/2, -Tan[(e + f*x)/2]
^2))/((4 + n) + Tan[(e + f*x)/2]*((5*(3*A + 4*B)*Hypergeometric2F1[9/2 + n,
(5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2)]/(5 + n) + Tan[(e + f*x)/2]*(((1
1*A + 10*B)*Hypergeometric2F1[9/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)
/2]^2)]/(6 + n) + ((5*A + 2*B)*Hypergeometric2F1[9/2 + n, (7 + n)/2, (9 + n
)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/((7 + n)))))))/(f*sqrt[Sec[(e +
f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))^5*Sin[e + f*x]^n
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left((A + 2B)a^2 \cos(fx + e)^2 - 2(A + B)a^2 + (Ba^2 \cos(fx + e)^2 - 2(A + B)a^2)\sin(fx + e)\right)\sqrt{a \sin(fx + e)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e
))^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}} (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^
n, x)
```

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int (d \sin(fx + e))^n (a + a \sin(fx + e))^{\frac{5}{2}} (A + B \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}} (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algor
ithm="maxima")
```

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin(e + f x))^n (A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)), x)

[Out] Timed out

3.8 $\int (d \sin(e+fx))^n (a+a \sin(e+fx))^{3/2} (A+B \sin(e+fx)) dx$

Optimal. Leaf size=229

$$\frac{2a^2 \left(A(8n^2 + 30n + 25) + 2B(4n^2 + 13n + 9) \right) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))^n {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; 1 - \sin(e+fx)\right)}{f(2n+3)(2n+5)\sqrt{a \sin(e+fx) + a}}$$

[Out] $-2*a^2*(2*B*(4*n^2+13*n+9)+A*(8*n^2+30*n+25))*\cos(f*x+e)*\text{hypergeom}([1/2, -n], [3/2], 1-\sin(f*x+e))*(d*\sin(f*x+e))^n/f/(3+2*n)/(5+2*n)/(\sin(f*x+e)^n)/(a+a*\sin(f*x+e))^{(1/2)}-2*a^2*(2*B*(3+n)+A*(5+2*n))*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(3+2*n)/(5+2*n)/(a+a*\sin(f*x+e))^{(1/2)}-2*a*B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)*(a+a*\sin(f*x+e))^{(1/2)}/d/f/(5+2*n)}$

Rubi [A] time = 0.49, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2981, 2776, 67, 65}

$$\frac{2a^2 \left(A(8n^2 + 30n + 25) + 2B(4n^2 + 13n + 9) \right) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))^n {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; 1 - \sin(e+fx)\right)}{f(2n+3)(2n+5)\sqrt{a \sin(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d*\text{Sin}[e + f*x])^n*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x]),x]$

[Out] $(-2*a^2*(2*B*(9 + 13*n + 4*n^2) + A*(25 + 30*n + 8*n^2))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, -n, 3/2, 1 - \text{Sin}[e + f*x]]*(d*\text{Sin}[e + f*x])^n)/(f*(3 + 2*n)*(5 + 2*n)*\text{Sin}[e + f*x]^n*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*a^2*(2*B*(3 + n) + A*(5 + 2*n))*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(3 + 2*n)*(5 + 2*n)*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*a*B*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1 + n)*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(d*f*(5 + 2*n))$

Rule 65

$\text{Int}[(b_*)*(x_*)^{(m_*)}((c_*) + (d_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}*\text{Hypergeometric2F1}[-m, n+1, n+2, 1 + (d*x)/c]/(d*(n+1)*(-(d/(b*c)))^m), x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-(d/(b*c)), 0])$

Rule 67

$\text{Int}[(b_*)*(x_*)^{(m_*)}((c_*) + (d_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(-(b*c)/d)^m*\text{IntPart}[m]*(b*x)^{\text{FracPart}[m]}/(-(d*x)/c)^{\text{FracPart}[m]}, \text{Int}[(-(d*x)/c)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[-(d/(b*c)), 0]$

Rule 2776

$\text{Int}[\text{Sqrt}[a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]]*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(a^2*\text{Cos}[e + f*x])/(f*\text{Sqrt}[a + b*\sin[e + f*x]]*\text{Sqrt}[a - b*\sin[e + f*x]]), \text{Subst}[\text{Int}[(c + d*x)^n/\text{Sqrt}[a - b*x], x], x, \text{Sin}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 2976

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)}, x_Symbol] \rightarrow -\text{Si}$


```
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) dx &= -\frac{2aB \cos(e + fx)(d \sin(e + fx))^{1+n} \sqrt{a + a \sin(e + fx)}}{df(5 + 2n)} \\ &= -\frac{2a^2(2B(3 + n) + A(5 + 2n)) \cos(e + fx)(d \sin(e + fx))^{1+n} \sqrt{a + a \sin(e + fx)}}{df(3 + 2n)(5 + 2n)} \\ &= -\frac{2a^2(2B(3 + n) + A(5 + 2n)) \cos(e + fx)(d \sin(e + fx))^{1+n} \sqrt{a + a \sin(e + fx)}}{df(3 + 2n)(5 + 2n)} \\ &= -\frac{2a^2(2B(3 + n) + A(5 + 2n)) \cos(e + fx)(d \sin(e + fx))^{1+n} \sqrt{a + a \sin(e + fx)}}{df(3 + 2n)(5 + 2n)} \\ &= -\frac{2a^2(2B(9 + 13n + 4n^2) + A(25 + 30n + 8n^2)) \cos(e + fx)(d \sin(e + fx))^{1+n} \sqrt{a + a \sin(e + fx)}}{df(3 + 2n)(5 + 2n)} \end{aligned}$$

Mathematica [B] time = 15.40, size = 478, normalized size = 2.09

$$2^{n+1} \tan\left(\frac{1}{2}(e + fx)\right) \sec\left(\frac{1}{2}(e + fx)\right) (a(\sin(e + fx) + 1))^{3/2} \sin^{-n}(e + fx) \left(\frac{\tan\left(\frac{1}{2}(e + fx)\right)}{\tan^2\left(\frac{1}{2}(e + fx)\right) + 1}\right)^n \left(\tan^2\left(\frac{1}{2}(e + fx)\right) - 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]), x]
```

```
[Out] (2^(1 + n)*Sec[(e + f*x)/2]*(d*Sin[e + f*x])^n*(a*(1 + Sin[e + f*x]))^(3/2)*Tan[(e + f*x)/2]*(Tan[(e + f*x)/2]/(1 + Tan[(e + f*x)/2]^2))^n*(1 + Tan[(e + f*x)/2]^2)^(3/2)*((A*Hypergeometric2F1[(1 + n)/2, 7/2 + n, (3 + n)/2, -Tan[(e + f*x)/2]^2])/(1 + n) + Tan[(e + f*x)/2]*((3*A + 2*B)*Hypergeometric2F1[(2 + n)/2, 7/2 + n, (4 + n)/2, -Tan[(e + f*x)/2]^2])/(2 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2F1[(3 + n)/2, 7/2 + n, (5 + n)/2, -Tan[(e + f*x)/2]^2])/(3 + n) + Tan[(e + f*x)/2]*((2*(2*A + 3*B)*Hypergeometric2
```

F1[7/2 + n, (4 + n)/2, (6 + n)/2, -Tan[(e + f*x)/2]^2]/(4 + n) + Tan[(e + f*x)/2]*(((3*A + 2*B)*Hypergeometric2F1[7/2 + n, (5 + n)/2, (7 + n)/2, -Tan[(e + f*x)/2]^2]/(5 + n) + (A*Hypergeometric2F1[7/2 + n, (6 + n)/2, (8 + n)/2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(6 + n)))))/(f*sqrt[Sec[(e + f*x)/2]^2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[e + f*x]^n)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba \cos (fx + e)^2 - (A + B)a \sin (fx + e) - (A + B)a\right)\sqrt{a \sin (fx + e) + a}\left(d \sin (fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (fx + e) + A)\left(a \sin (fx + e) + a\right)^{\frac{3}{2}}\left(d \sin (fx + e)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)

maple [F] time = 1.05, size = 0, normalized size = 0.00

$$\int (d \sin (fx + e))^n\left(a + a \sin (fx + e)\right)^{\frac{3}{2}}(A + B \sin (fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (fx + e) + A)\left(a \sin (fx + e) + a\right)^{\frac{3}{2}}\left(d \sin (fx + e)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin (e + fx))^n(A + B \sin (e + fx))\left(a + a \sin (e + fx)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)

```
[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x)
```

```
[Out] Timed out
```

3.9 $\int (d \sin(e+fx))^n \sqrt{a + a \sin(e+fx)} (A+B \sin(e+fx)) dx$

Optimal. Leaf size=137

$$\frac{2a(A(2n+3) + 2B(n+1)) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))^n {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; 1 - \sin(e+fx)\right)}{f(2n+3)\sqrt{a \sin(e+fx) + a}} \frac{2aB \cos(e+fx)}{df(2n+3)}$$

[Out] $-2*a*(2*B*(1+n)+A*(3+2*n))*\cos(f*x+e)*\text{hypergeom}([1/2, -n], [3/2], 1-\sin(f*x+e))*(d*\sin(f*x+e))^n/f/(3+2*n)/(\sin(f*x+e)^n)/(a+a*\sin(f*x+e))^{(1/2)}-2*a*B*\cos(f*x+e)*(d*\sin(f*x+e))^{(1+n)}/d/f/(3+2*n)/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2981, 2776, 67, 65}

$$\frac{2a(A(2n+3) + 2B(n+1)) \cos(e+fx) \sin^{-n}(e+fx) (d \sin(e+fx))^n {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; 1 - \sin(e+fx)\right)}{f(2n+3)\sqrt{a \sin(e+fx) + a}} \frac{2aB \cos(e+fx)}{df(2n+3)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(d*\text{Sin}[e + f*x])^n*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(A + B*\text{Sin}[e + f*x]),x]$

[Out] $(-2*a*(2*B*(1+n) + A*(3+2*n))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, -n, 3/2, 1 - \text{Sin}[e + f*x]]*(d*\text{Sin}[e + f*x])^n)/(f*(3+2*n)*\text{Sin}[e + f*x]^n*\text{Sqrt}[a + a*\text{Sin}[e + f*x]] - (2*a*B*\text{Cos}[e + f*x]*(d*\text{Sin}[e + f*x])^{(1+n)})/(d*f*(3+2*n)*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 65

$\text{Int}[(b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n+1)}*\text{Hypergeometric2F1}[-m, n+1, n+2, 1 + (d*x)/c]/(d*(n+1)*(-(d/(b*c)))^m), x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-(d/(b*c)), 0])$

Rule 67

$\text{Int}[(b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[(-(b*c)/d)^{\text{IntPart}[m]}*(b*x)^{\text{FracPart}[m]}/(-(d*x)/c)^{\text{FracPart}[m]}, \text{Int}[(-(d*x)/c)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{GtQ}[c, 0] \ \&\& \ !\text{GtQ}[-(d/(b*c)), 0]$

Rule 2776

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]]*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(a^2*\text{Cos}[e + f*x])/(f*\text{Sqrt}[a + b*\sin[e + f*x]]*\text{Sqrt}[a - b*\sin[e + f*x]]), \text{Subst}[\text{Int}[(c + d*x)^n/\text{Sqrt}[a - b*x], x], x, \text{Sin}[e + f*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ !\text{IntegerQ}[2*n]$

Rule 2981

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]]*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*B*\text{Cos}[e + f*x]*(c + d*\sin[e + f*x])^{(n+1)})/(d*f*(2*n+3)*\text{Sqrt}[a + b*\sin[e + f*x]]), x] + \text{Dist}[(A*b*d*(2*n+3) - B*(b*c - 2*a*d*(n+1)))/(b*d*(2*n+3)), \text{Int}[\text{Sqrt}[a + b*\sin[e + f*x]]*(c + d*\sin[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 -$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int (d \sin (fx + e))^n \sqrt{a + a \sin (fx + e)} (A + B \sin (fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (fx + e) + A) \sqrt{a \sin (fx + e) + a} (d \sin (fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d \sin (e + fx))^n (A + B \sin (e + fx)) \sqrt{a + a \sin (e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (d \sin(e + fx))^n (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))**n*(a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e)),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(d*sin(e + f*x))**n*(A + B*sin(e + f*x)), x)

$$3.10 \quad \int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=152

$$\frac{(A-B) \cos(e+fx) \sin^{-n}(e+fx) F_1\left(\frac{1}{2}; -n, 1; \frac{3}{2}; 1 - \sin(e+fx), \frac{1}{2}(1 - \sin(e+fx))\right) (d \sin(e+fx))^n - 2B \cos(e+fx)}{f \sqrt{a \sin(e+fx) + a}}$$

[Out] -(A-B)*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-2*B*cos(f*x+e)*hypergeom([1/2,-n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.40, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$\frac{(A-B) \cos(e+fx) \sin^{-n}(e+fx) F_1\left(\frac{1}{2}; -n, 1; \frac{3}{2}; 1 - \sin(e+fx), \frac{1}{2}(1 - \sin(e+fx))\right) (d \sin(e+fx))^n - 2B \cos(e+fx)}{f \sqrt{a \sin(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]],x]

[Out] -(((A - B)*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - (2*B*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]))

Rule 65

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/((d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 67

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Dist[(-(b*c)/d)^IntPart[m]*(b*x)^FracPart[m]/(-(d*x)/c)^FracPart[m], Int[(-(d*x)/c)^m*(c + d*x)^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]

Rule 130

Int[((e_)*(x_))^(p_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]

Rule 429

Int[((a_) + (b_)*(x_))^(n_)*((c_) + (d_)*(x_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2776

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 2785

```
Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Sin[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n])/(b*Sin[e + f*x]^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)

maple [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(fx + e))^n (A + B \sin(fx + e))}{\sqrt{a + a \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2),x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral((d*sin(e + f*x))^n*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.11 \quad \int \frac{(d \sin(e+fx))^n (A+B \sin(e+fx))}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=226

$$\frac{(-4An + A + B(4n + 3)) \cos(e + fx) \sin^{-n}(e + fx) F_1\left(\frac{1}{2}; -n, 1; \frac{3}{2}; 1 - \sin(e + fx), \frac{1}{2}(1 - \sin(e + fx))\right) (d \sin(e + fx))^n}{4af \sqrt{a \sin(e + fx) + a}}$$

[Out] 1/2*(A-B)*cos(f*x+e)*(d*sin(f*x+e))^(1+n)/d/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A-4*A*n+B*(3+4*n))*AppellF1(1/2,-n,1,3/2,1-sin(f*x+e),1/2-1/2*sin(f*x+e))*cos(f*x+e)*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)-1/2*(A-B)*(1+2*n)*cos(f*x+e)*hypergeom([1/2,-n],[3/2],1-sin(f*x+e))*(d*sin(f*x+e))^n/a/f/(sin(f*x+e)^n)/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.67, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2978, 2987, 2787, 2786, 2785, 130, 429, 2776, 67, 65}

$$\frac{(-4An + A + B(4n + 3)) \cos(e + fx) \sin^{-n}(e + fx) F_1\left(\frac{1}{2}; -n, 1; \frac{3}{2}; 1 - \sin(e + fx), \frac{1}{2}(1 - \sin(e + fx))\right) (d \sin(e + fx))^n}{4af \sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2),x]
 [Out] ((A - B)*Cos[e + f*x]*(d*Sin[e + f*x])^(1 + n))/(2*d*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - 4*A*n + B*(3 + 4*n))*AppellF1[1/2, -n, 1, 3/2, 1 - Sin[e + f*x], (1 - Sin[e + f*x])/2]*Cos[e + f*x]*(d*Sin[e + f*x])^n)/(4*a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]]) - ((A - B)*(1 + 2*n)*Cos[e + f*x]*Hypergeometric2F1[1/2, -n, 3/2, 1 - Sin[e + f*x]]*(d*Sin[e + f*x])^n)/(2*a*f*Sin[e + f*x]^n*Sqrt[a + a*Sin[e + f*x]])

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/((d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rule 67

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(-(b*c)/d)^IntPart[m]*(b*x)^FracPart[m]/(-(d*x)/c)^FracPart[m], Int[(-(d*x)/c)^m*(c + d*x)^n, x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]

Rule 130

Int[((e_.)*(x_))^(p_)*((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := With[{k = Denominator[p]}, Dist[k/e, Subst[Int[x^(k*(p + 1) - 1)*(a + (b*x^k)/e)^m*(c + (d*x^k)/e)^n, x], x, (e*x)^(1/k)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && FractionQ[p] && IntegerQ[m]

Rule 429

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -(b*x^n)/a, -(d*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]

&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 2776

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]

Rule 2785

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2))/Sqrt[x], x], x, a - b*Sin[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]

Rule 2786

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n])/(b*Sin[e + f*x]^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]

Rule 2787

Int[((d_)*sin[(e_) + (f_)*(x_)])^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rule 2978

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2987

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^{3/2}} dx &= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} + \int \frac{(d \sin(e + fx))^n (ad(A + B - An - \sqrt{a + a \sin(e + fx)}))}{\sqrt{a + a \sin(e + fx)}} dx \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} + \frac{((A - B)(1 + 2n)) \int (d \sin(e + fx))^n dx}{2df(a + a \sin(e + fx))^{3/2}} \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A - B)d(1 + 2n)\right) \int (d \sin(e + fx))^n dx\right)}{2df(a + a \sin(e + fx))^{3/2}} \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} + \frac{\left(\left(-\frac{1}{2}a^2(A - B)d(1 + 2n)\right) \int (d \sin(e + fx))^n dx\right)}{2df(a + a \sin(e + fx))^{3/2}} \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} - \frac{(A - B)(1 + 2n) \cos(e + fx)}{2df(a + a \sin(e + fx))^{3/2}} \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} - \frac{(A - B)(1 + 2n) \cos(e + fx)}{2df(a + a \sin(e + fx))^{3/2}} \\
&= \frac{(A - B) \cos(e + fx) (d \sin(e + fx))^{1+n}}{2df(a + a \sin(e + fx))^{3/2}} - \frac{(A + 3B - 4An + 4Bn) \cos(e + fx)}{2df(a + a \sin(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [B] time = 21.90, size = 523, normalized size = 2.31

$$\sec(e + fx)(d \sin(e + fx))^n \left(A \left(a^2 \sqrt{2 - 2 \sin(e + fx)} (\sin(e + fx) + 1)^2 (-\sin(e + fx))^{-n} F_1 \left(1; \frac{1}{2}, -n; 2; \frac{1}{2} (\sin(e + fx) + 1)^2 \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d*Sin[e + f*x])^n*(A + B*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] (Sec[e + f*x]*(d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]))/(-Sin[e + f*x])^n - (4*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] * (-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n) + A*((a^2*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, 1 + Sin[e + f*x]]*Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x])^2)/(-Sin[e + f*x])^n - (4*a*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])] * (1 + Sin[e + f*x])*(2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (1 + Sin[e + f*x])^(-1)]*(1 + Sin[e + f*x])))/((-1 + 4*n^2)*(1 - (1 + Sin[e + f*x])^(-1))^n)))/(8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (d \sin(fx + e))^n}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e))^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(fx + e))^n (A + B \sin(fx + e))}{(a + a \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)

[Out] int((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) (d \sin(fx + e))^n}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e))^n/(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2),x)

[Out] int(((d*sin(e + f*x))^n*(A + B*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d \sin(e + fx))^n (A + B \sin(e + fx))}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))**n*(A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2), x)

[Out] Integral((d*sin(e + f*x))**n*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)

3.12 $\int (d \sin(e+fx))^n (a+a \sin(e+fx))^m (A+B \sin(e+fx)) dx$

Optimal. Leaf size=221

$$\frac{2^{m+\frac{1}{2}}(A-B) \cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}} \sin^{-n}(e+fx)(a \sin(e+fx)+a)^m (d \sin(e+fx))^n F_1\left(\frac{1}{2}; -n, \frac{1}{2}-n\right)}{f}$$

[Out] $-2^{(3/2+m)} B \text{AppellF1}(1/2, -n, -1/2-m, 3/2, 1-\sin(f*x+e), 1/2-1/2*\sin(f*x+e)) * \cos(f*x+e) * (d*\sin(f*x+e))^n * (1+\sin(f*x+e))^{(-1/2-m)} * (a+a*\sin(f*x+e))^m / f / (\sin(f*x+e)^n) - 2^{(1/2+m)} * (A-B) * \text{AppellF1}(1/2, -n, 1/2-m, 3/2, 1-\sin(f*x+e), 1/2-1/2*\sin(f*x+e)) * \cos(f*x+e) * (d*\sin(f*x+e))^n * (1+\sin(f*x+e))^{(-1/2-m)} * (a+a*\sin(f*x+e))^m / f / (\sin(f*x+e)^n)$

Rubi [A] time = 0.45, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2987, 2787, 2786, 2785, 133}

$$\frac{2^{m+\frac{1}{2}}(A-B) \cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}} \sin^{-n}(e+fx)(a \sin(e+fx)+a)^m (d \sin(e+fx))^n F_1\left(\frac{1}{2}; -n, \frac{1}{2}-n\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]

[Out] $-((2^{(3/2+m)} B \text{AppellF1}[1/2, -n, -1/2-m, 3/2, 1-\text{Sin}[e+f*x], (1-\text{Sin}[e+f*x])/2] * \text{Cos}[e+f*x] * (d*\text{Sin}[e+f*x])^n * (1+\text{Sin}[e+f*x])^{(-1/2-m)} * (a+a*\text{Sin}[e+f*x])^m) / (f*\text{Sin}[e+f*x]^n) - (2^{(1/2+m)} * (A-B) * \text{AppellF1}[1/2, -n, 1/2-m, 3/2, 1-\text{Sin}[e+f*x], (1-\text{Sin}[e+f*x])/2] * \text{Cos}[e+f*x] * (d*\text{Sin}[e+f*x])^n * (1+\text{Sin}[e+f*x])^{(-1/2-m)} * (a+a*\text{Sin}[e+f*x])^m) / (f*\text{Sin}[e+f*x]^n)$

Rule 133

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m+1)*AppellF1[m+1, -n, -p, m+2, -(d*x)/c, -(f*x)/e])/(b*(m+1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 2785

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(a - x)^n*(2*a - x)^(m-1/2)]/Sqrt[x], x], x, a - b*Sin[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]

Rule 2786

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(d/b)^n*IntPart[n]*(d*Sin[e + f*x])^FracPart[n])/(b*Sin[e + f*x]^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]

Rule 2787

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]

)]/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rule 2987

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]

Rubi steps

$$\begin{aligned} \int (d \sin(e + fx))^n (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx &= (A - B) \int (d \sin(e + fx))^n (a + a \sin(e + fx))^m \\ &= ((A - B)(1 + \sin(e + fx))^{-m} (a + a \sin(e + fx))^m \\ &= ((A - B) \sin^{-n}(e + fx) (d \sin(e + fx))^n (1 + \sin(e + fx))^m \\ &\quad \left((A - B) \cos(e + fx) \sin^{-n}(e + fx) (d \sin(e + fx))^n \right) \\ &= \frac{\left((A - B) \cos(e + fx) \sin^{-n}(e + fx) (d \sin(e + fx))^n \right)}{2^{\frac{3}{2}+m} BF_1\left(\frac{1}{2}; -n, -\frac{1}{2} - m; \frac{3}{2}; 1 - \sin(e + fx)\right)} \end{aligned}$$

Mathematica [B] time = 22.18, size = 5918, normalized size = 26.78

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d*Sin[e + f*x])^n*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]

[Out] Result too large to show

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

maple [F] time = 6.04, size = 0, normalized size = 0.00

$$\int (d \sin (f x + e))^n (a + a \sin (f x + e))^m (A + B \sin (f x + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

[Out] int((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (f x + e) + A)(a \sin (f x + e) + a)^m (d \sin (f x + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (d \sin (e + f x))^n (A + B \sin (e + f x)) (a + a \sin (e + f x))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)

[Out] int((d*sin(e + f*x))^n*(A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin (e + f x) + 1))^m (d \sin (e + f x))^n (A + B \sin (e + f x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

[Out] Integral((a*(sin(e + f*x) + 1))^m*(d*sin(e + f*x))^n*(A + B*sin(e + f*x)), x)

3.13 $\int (d \sin(e + fx))^n (a - a \sin(e + fx))(a + a \sin(e + fx))^m dx$

Optimal. Leaf size=114

$$\frac{\sec(e + fx)(a - a \sin(e + fx))(\sin(e + fx) + 1)^{\frac{1}{2}-m}(a \sin(e + fx) + a)^m (d \sin(e + fx))^{n+1} F_1\left(n + 1; -\frac{1}{2}, \frac{1}{2} - m; \right)}{df(n + 1)\sqrt{1 - \sin(e + fx)}}$$

[Out] AppellF1(1+n,1/2-m,-1/2,2+n,-sin(f*x+e),sin(f*x+e))*sec(f*x+e)*(d*sin(f*x+e))^(1+n)*(1+sin(f*x+e))^(1/2-m)*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m/d/f/(1+n)/(1-sin(f*x+e))^(1/2)

Rubi [A] time = 0.16, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {3008, 135, 133}

$$\frac{\sec(e + fx)(a - a \sin(e + fx))(\sin(e + fx) + 1)^{\frac{1}{2}-m}(a \sin(e + fx) + a)^m (d \sin(e + fx))^{n+1} F_1\left(n + 1; -\frac{1}{2}, \frac{1}{2} - m; \right)}{df(n + 1)\sqrt{1 - \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]

[Out] (AppellF1[1 + n, -1/2, 1/2 - m, 2 + n, Sin[e + f*x], -Sin[e + f*x]]*Sec[e + f*x]*(d*Sin[e + f*x])^(1 + n)*(1 + Sin[e + f*x])^(1/2 - m)*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m)/(d*f*(1 + n)*Sqrt[1 - Sin[e + f*x]])

Rule 133

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)]]/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])

Rule 135

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n]]/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]

Rule 3008

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(p_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (d \sin(e + fx))^n (a - a \sin(e + fx))(a + a \sin(e + fx))^m dx = \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)})}{f\sqrt{1 - \sin^2(e + fx)}} = \frac{(\sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)})}{f\sqrt{1 - \sin^2(e + fx)}} = \frac{(\sec(e + fx)(1 + \sin(e + fx))^{\frac{1}{2}-m}(a - a \sin(e + fx))}{f\sqrt{1 - \sin^2(e + fx)}} = \frac{F_1\left(1 + n; -\frac{1}{2}, \frac{1}{2} - m; 2 + n; \sin(e + fx), -\sin(e + fx)\right)}{f\sqrt{1 - \sin^2(e + fx)}}$$

Mathematica [F] time = 11.25, size = 0, normalized size = 0.00

$$\int (d \sin(e + fx))^n (a - a \sin(e + fx))(a + a \sin(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m,x]

[Out] Integrate[(d*Sin[e + f*x])^n*(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m, x]

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a \sin(fx + e) - a\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e)\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm="fricas")

[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a \sin(fx + e) - a\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e)\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm="giac")

[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)

maple [F] time = 5.91, size = 0, normalized size = 0.00

$$\int \left(d \sin(fx + e)\right)^n \left(a - a \sin(fx + e)\right) \left(a + a \sin(fx + e)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)

[Out] $\int (d \sin(fx+e))^n (a-a \sin(fx+e)) (a+a \sin(fx+e))^m dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (a \sin(fx+e) - a) (a \sin(fx+e) + a)^m (d \sin(fx+e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x, algorithm="maxima")`

[Out] `-integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e))^n, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (d \sin(e + fx))^n (a + a \sin(e + fx))^m (a - a \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)),x)`

[Out] `int((d*sin(e + f*x))^n*(a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-a \left(\int \left(- (d \sin(e + fx))^n (a \sin(e + fx) + a)^m \right) dx + \int (d \sin(e + fx))^n (a \sin(e + fx) + a)^m \sin(e + fx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sin(f*x+e))^n*(a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m,x)`

[Out] `-a*(Integral(-(d*sin(e + f*x))^n*(a*sin(e + f*x) + a)^m, x) + Integral((d*sin(e + f*x))^n*(a*sin(e + f*x) + a)^m*sin(e + f*x), x))`

$$3.14 \quad \int \sin^n(c + dx)(a + a \sin(c + dx))^{-2-n}(-1 - n - (-2 - n) \sin(c + dx)) dx$$

Optimal. Leaf size=37

$$\frac{\cos(c + dx) \sin^{n+1}(c + dx)(a \sin(c + dx) + a)^{-n-2}}{d}$$

[Out] $-\cos(d*x+c)*\sin(d*x+c)^{(1+n)}*(a+a*\sin(d*x+c))^{(-2-n)}/d$

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.023$, Rules used = {2974}

$$\frac{\cos(c + dx) \sin^{n+1}(c + dx)(a \sin(c + dx) + a)^{-n-2}}{d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^n*(a + a*SIN[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c + d*x]), x]

[Out] $-((\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^{(1 + n)}*(a + a*\text{Sin}[c + d*x])^{(-2 - n)}))/d$

Rule 2974

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*(c + d*SIN[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m + n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]

Rubi steps

$$\int \sin^n(c + dx)(a + a \sin(c + dx))^{-2-n}(-1 - n - (-2 - n) \sin(c + dx)) dx = -\frac{\cos(c + dx) \sin^{1+n}(c + dx)(a + a \sin(c + dx))^{-n-2}}{d}$$

Mathematica [B] time = 1.63, size = 107, normalized size = 2.89

$$\frac{2^n \sin\left(\frac{1}{2}(c + dx)\right) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right) (-\sin(c + dx) + \cos(c + dx) + 1)(a(\sin(c + dx) + 1))^{-n-2}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^n*(a + a*SIN[c + d*x])^(-2 - n)*(-1 - n - (-2 - n)*Sin[c + d*x]), x]

[Out] $-((2^n*\text{Sin}[(c + d*x)/2]*(\text{Cos}[(c + d*x)/2] + \text{Sin}[(c + d*x)/2])*(\text{Cos}[(c + d*x)/4]*(-\text{Sin}[(c + d*x)/4] + \text{Sin}[(3*(c + d*x))/4]))^n*(1 + \text{Cos}[c + d*x] - \text{Sin}[c + d*x])*(a*(1 + \text{Sin}[c + d*x]))^{(-2 - n)}))/d$

fricas [A] time = 0.45, size = 41, normalized size = 1.11

$$\frac{(a \sin(dx + c) + a)^{-n-2} \sin(dx + c)^n \cos(dx + c) \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="fricas")

[Out] -(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n*cos(d*x + c)*sin(d*x + c)/d
giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="giac")

[Out] Timed out

maple [F] time = 4.15, size = 0, normalized size = 0.00

$$\int (\sin^n(dx + c)) (a + a \sin(dx + c))^{-2-n} (-1 - n - (-2 - n) \sin(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)

[Out] int(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((n + 2) \sin(dx + c) - n - 1)(a \sin(dx + c) + a)^{-n-2} \sin(dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^n*(a+a*sin(d*x+c))^(2-n)*(-1-n-(-2-n)*sin(d*x+c)),x,
algorithm="maxima")

[Out] integrate(((n + 2)*sin(d*x + c) - n - 1)*(a*sin(d*x + c) + a)^(-n - 2)*sin(d*x + c)^n, x)

mupad [B] time = 12.98, size = 61, normalized size = 1.65

$$\frac{\sin(c + dx)^n \sin(2c + 2dx)}{a^2 d (a (\sin(c + dx) + 1))^n (2 \sin(c + dx)^2 + 4 \sin(c + dx) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(sin(c + d*x)^n*(n - sin(c + d*x)*(n + 2) + 1))/(a + a*sin(c + d*x))^(n + 2),x)

[Out] -(sin(c + d*x)^n*sin(2*c + 2*d*x))/(a^2*d*(a*(sin(c + d*x) + 1))^n*(4*sin(c + d*x) + 2*sin(c + d*x)^2 + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**n*(a+a*sin(d*x+c))**(-2-n)*(-1-n-(-2-n)*sin(d*x+c)),x)

[Out] Timed out

$$3.15 \quad \int \sin^{-2-m}(c + dx)(a + a \sin(c + dx))^m(1 + m - m \sin(c + dx)) dx$$

Optimal. Leaf size=35

$$\frac{\cos(c + dx) \sin^{-m-1}(c + dx)(a \sin(c + dx) + a)^m}{d}$$

[Out] $-\cos(d*x+c)*\sin(d*x+c)^{-1-m}*(a+a*\sin(d*x+c))^m/d$

Rubi [A] time = 0.09, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$, Rules used = {2974}

$$\frac{\cos(c + dx) \sin^{-m-1}(c + dx)(a \sin(c + dx) + a)^m}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x]`

[Out] $-\left(\cos[c + d*x]*\sin[c + d*x]^{-1 - m}*(a + a*\sin[c + d*x])^m\right)/d$

Rule 2974

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[m + n + 2, 0] && EqQ[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]`

Rubi steps

$$\int \sin^{-2-m}(c + dx)(a + a \sin(c + dx))^m(1 + m - m \sin(c + dx)) dx = -\frac{\cos(c + dx) \sin^{-1-m}(c + dx)(a + a \sin(c + dx))^m}{d}$$

Mathematica [A] time = 0.39, size = 35, normalized size = 1.00

$$\frac{\cos(c + dx) \sin^{-m-1}(c + dx)(a(\sin(c + dx) + 1))^m}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[Sin[c + d*x]^(-2 - m)*(a + a*Sin[c + d*x])^m*(1 + m - m*Sin[c + d*x]),x]`

[Out] $-\left(\cos[c + d*x]*\sin[c + d*x]^{-1 - m}*(a*(1 + \sin[c + d*x]))^m\right)/d$

fricas [A] time = 0.45, size = 41, normalized size = 1.17

$$\frac{(a \sin(dx + c) + a)^m \sin(dx + c)^{-m-2} \cos(dx + c) \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algorithm="fricas")`


```

n(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/
2*d*x + 1/2*c) + 2) - 1/4*pi*m)*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
+ 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*s
gn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*
x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/
2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^
2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(
tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))*tan(1/2*d
*x + 1/2*c) + cos(2*pi*m*floor(-1/8*sgn(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*
c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2
*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) + 5/8) + 1/4*pi*m*sgn(4*tan(d
*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) +
4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2) -
1/4*pi*m)*e^(m*log(sqrt(2)*sqrt(abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^
2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*
x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(
1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*
d*x + 1/2*c) + 2))*tan(d*x + c)^2 + abs(4*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c
)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*
d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c) + 2))*tan(1/2*d*x + 1/2*c)^2 + abs(4
*tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(d*x + c)^2*tan(1/2*d*x + 1/2
*c) + 4*tan(d*x + c)^2 + 2*tan(1/2*d*x + 1/2*c)^2 + 8*tan(1/2*d*x + 1/2*c)
+ 2))*abs(a)/(tan(d*x + c)^2*tan(1/2*d*x + 1/2*c)^2 + tan(d*x + c)^2 + tan(
1/2*d*x + 1/2*c)^2 + 1)) - m*log(4*abs(tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x +
1/2*c)^2 + 1)) - 2*log(4*abs(tan(1/2*d*x + 1/2*c)))/(tan(1/2*d*x + 1/2*c)^2
+ 1))*tan(1/2*d*x + 1/2*c))/(d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x
+ 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*
sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1
/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)
^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(
tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/
2*d*x + 1/2*c)^4 + 2*d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/2*d*x + 1/2*c)^
4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) - 2*a)*sgn(4*a*ta
n(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)
)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*x + 1/2*c)) + 1
/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*ta
n(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d
*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2*tan(1/2*d*x + 1/
2*c)^2 + d*tan(1/2*d*x + 1/2*c)^4 + d*tan(-1/2*pi + 1/4*pi*m*sgn(2*a*tan(1/
2*d*x + 1/2*c)^4 + 4*a*tan(1/2*d*x + 1/2*c)^3 - 4*a*tan(1/2*d*x + 1/2*c) -
2*a)*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + 4*a*tan(
1/2*d*x + 1/2*c)) - 1/4*pi*m*sgn(tan(1/2*d*x + 1/2*c)^2 - 1)*sgn(tan(1/2*d*
x + 1/2*c)) + 1/4*pi*m*sgn(4*a*tan(1/2*d*x + 1/2*c)^3 + 8*a*tan(1/2*d*x + 1
/2*c)^2 + 4*a*tan(1/2*d*x + 1/2*c)) - 1/2*pi*sgn(tan(1/2*d*x + 1/2*c)^2 - 1
)*sgn(tan(1/2*d*x + 1/2*c)) - 1/4*pi*m + pi*floor(d*x/pi + c/pi + 1/2))^2 +
2*d*tan(1/2*d*x + 1/2*c)^2 + d)

```

maple [F] time = 1.66, size = 0, normalized size = 0.00

$$\int (\sin^{-2-m}(dx + c)) (a + a \sin(dx + c))^m (1 + m - m \sin(dx + c)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)

[Out] int(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (m \sin(dx + c) - m - 1)(a \sin(dx + c) + a)^m \sin(dx + c)^{-m-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^(-2-m)*(a+a*sin(d*x+c))^m*(1+m-m*sin(d*x+c)),x, algorithm="maxima")

[Out] -integrate((m*sin(d*x + c) - m - 1)*(a*sin(d*x + c) + a)^m*sin(d*x + c)^(-m - 2), x)

mupad [B] time = 12.85, size = 38, normalized size = 1.09

$$\frac{\sin(2c + 2dx) (a (\sin(c + dx) + 1))^m}{2d \sin(c + dx)^{m+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a*sin(c + d*x))^m*(m - m*sin(c + d*x) + 1))/sin(c + d*x)^(m + 2), x)

[Out] -(sin(2*c + 2*d*x)*(a*(sin(c + d*x) + 1))^m)/(2*d*sin(c + d*x)^(m + 2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**(-2-m)*(a+a*sin(d*x+c))**m*(1+m-m*sin(d*x+c)),x)

[Out] Timed out

$$3.16 \quad \int \frac{\sin^2(e+fx)(A+B \sin(e+fx))}{(a+b \sin(e+fx))^2} dx$$

Optimal. Leaf size=153

$$\frac{a^2(Ab - aB) \cos(e + fx)}{b^2 f (a^2 - b^2) (a + b \sin(e + fx))} - \frac{2a(-2a^3B + a^2Ab + 3ab^2B - 2Ab^3) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(e+fx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{b^3 f (a^2 - b^2)^{3/2}} + \frac{x(Ab - 2aB)}{b^3}$$

[Out] (A*b-2*B*a)*x/b^3-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)*arctan((b+a*tan(1/2*f*x+1/2*e))/(a^2-b^2)^(1/2))/b^3/(a^2-b^2)^(3/2)/f-B*cos(f*x+e)/b^2/f+a^2*(A*b-B*a)*cos(f*x+e)/b^2/(a^2-b^2)/f/(a+b*sin(f*x+e))

Rubi [A] time = 0.39, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$-\frac{2a(a^2Ab - 2a^3B + 3ab^2B - 2Ab^3) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(e+fx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{b^3 f (a^2 - b^2)^{3/2}} + \frac{a^2(Ab - aB) \cos(e + fx)}{b^2 f (a^2 - b^2) (a + b \sin(e + fx))} + \frac{x(Ab - 2aB)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[e + f*x]^2*(A + B*SIN[e + f*x]))/(a + b*SIN[e + f*x])^2,x]

[Out] ((A*b - 2*a*B)*x)/b^3 - (2*a*(a^2*A*b - 2*A*b^3 - 2*a^3*B + 3*a*b^2*B)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]]/(b^3*(a^2 - b^2)^(3/2)*f) - (B*Cos[e + f*x])/(b^2*f) + (a^2*(A*b - a*B)*Cos[e + f*x])/(b^2*(a^2 - b^2)*f*(a + b*SIN[e + f*x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2988

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^n, x_Symbol] := Simp[

```
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1)))]*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(e + fx)(A + B \sin(e + fx))}{(a + b \sin(e + fx))^2} dx &= \frac{a^2(Ab - aB) \cos(e + fx)}{b^2(a^2 - b^2) f(a + b \sin(e + fx))} + \frac{\int \frac{ab(Ab - aB) + (a^2 - b^2)(Ab - aB) \sin(e + fx) + b(a^2 - b^2)}{a + b \sin(e + fx)} dx}{b^2(a^2 - b^2)} \\ &= -\frac{B \cos(e + fx)}{b^2 f} + \frac{a^2(Ab - aB) \cos(e + fx)}{b^2(a^2 - b^2) f(a + b \sin(e + fx))} + \frac{\int \frac{ab^2(Ab - aB) + b(a^2 - b^2)}{a + b \sin(e + fx)} dx}{b^3(a^2 - b^2)} \\ &= \frac{(Ab - 2aB)x}{b^3} - \frac{B \cos(e + fx)}{b^2 f} + \frac{a^2(Ab - aB) \cos(e + fx)}{b^2(a^2 - b^2) f(a + b \sin(e + fx))} - \frac{(a^2 - b^2)}{b^3} \\ &= \frac{(Ab - 2aB)x}{b^3} - \frac{B \cos(e + fx)}{b^2 f} + \frac{a^2(Ab - aB) \cos(e + fx)}{b^2(a^2 - b^2) f(a + b \sin(e + fx))} - \frac{(2a^2 - b^2)}{b^3} \\ &= \frac{(Ab - 2aB)x}{b^3} - \frac{B \cos(e + fx)}{b^2 f} + \frac{a^2(Ab - aB) \cos(e + fx)}{b^2(a^2 - b^2) f(a + b \sin(e + fx))} + \frac{(4a^2 - b^2)}{b^3} \\ &= \frac{(Ab - 2aB)x}{b^3} - \frac{2a(a^2 Ab - 2Ab^3 - 2a^3 B + 3ab^2 B) \tan^{-1}\left(\frac{b + a \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{a^2 - b^2}}\right)}{b^3(a^2 - b^2)^{3/2} f} \end{aligned}$$

Mathematica [A] time = 0.88, size = 147, normalized size = 0.96

$$\frac{\frac{a^2 b(Ab - aB) \cos(e + fx)}{(a - b)(a + b)(a + b \sin(e + fx))} + \frac{2a(2a^3 B - a^2 Ab - 3ab^2 B + 2Ab^3) \tan^{-1}\left(\frac{a \tan\left(\frac{1}{2}(e + fx)\right) + b}{\sqrt{a^2 - b^2}}\right)}{(a^2 - b^2)^{3/2}}}{b^3 f} + (e + fx)(Ab - 2aB) - bB \cos(e + fx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[e + f*x]^2*(A + B*Sin[e + f*x]))/(a + b*Sin[e + f*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*(e + f*x) + (2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B)
)*ArcTan[(b + a*Tan[(e + f*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - b*B
```

*Cos[e + f*x] + (a^2*b*(A*b - a*B)*Cos[e + f*x])/((a - b)*(a + b)*(a + b*Sin[e + f*x]))/(b^3*f)

fricas [B] time = 0.52, size = 804, normalized size = 5.25

$$\frac{2(2Ba^6 - Aa^5b - 4Ba^4b^2 + 2Aa^3b^3 + 2Ba^2b^4 - Aab^5)fx + (2Ba^5 - Aa^4b - 3Ba^3b^2 + 2Aa^2b^3 + (2Ba^4b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/2*(2*(2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2 + 2*(a*cos(f*x + e)*sin(f*x + e) + b*cos(f*x + e))*sqrt(-a^2 + b^2)))/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)) + 2*(2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + 2*((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f), -((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*f*x + (2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3 + (2*B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*sin(f*x + e))*sqrt(a^2 - b^2)*arctan(-(a*sin(f*x + e) + b)/(sqrt(a^2 - b^2)*cos(f*x + e))) + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*cos(f*x + e) + ((2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*f*x + (B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6)*cos(f*x + e))*sin(f*x + e))/((a^4*b^4 - 2*a^2*b^6 + b^8)*f*sin(f*x + e) + (a^5*b^3 - 2*a^3*b^5 + a*b^7)*f)]

giac [B] time = 0.18, size = 371, normalized size = 2.42

$$\frac{2(2Ba^4 - Aa^3b - 3Ba^2b^2 + 2Aab^3) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(a) + \arctan \left(\frac{a \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + b}{\sqrt{a^2 - b^2}} \right) \right)}{(a^2b^3 - b^5)\sqrt{a^2 - b^2}} - \frac{2 \left(Ba^2b \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - Aab^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 + 2Ba^3 \tan \left(\frac{1}{2} \right)}{(a \tan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="giac")

[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*f*x + 1/2*e) + b)/sqrt(a^2 - b^2)))/((a^2*b^3 - b^5)*sqrt(a^2 - b^2)) - 2*(B*a^2*b*tan(1/2*f*x + 1/2*e)^3 - A*a*b^2*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*b*tan(1/2*f*x + 1/2*e)^2 - B*a*b^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*b*tan(1/2*f*x + 1/2*e) - A*a*b^2*tan(1/2*f*x + 1/2*e) - 2*B*b^3*tan(1/2*f*x + 1/2*e) + 2*B*a^3 - A*a^2*b - B*a*b^2)/((a*tan(1/2*f*x + 1/2*e)^4 + 2*b*tan(1/2*f*x + 1/2*e)^3 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 2*b*tan(1/2*f*x + 1/2*e) + a)*(a^2*b^2 - b^4)) - (2*B*a - A*b)*(f*x + e)/b^3)/f

maple [B] time = 0.37, size = 493, normalized size = 3.22

$$\frac{2B}{fb^2 \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)} + \frac{2A \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{fb^2} - \frac{4B \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)a}{fb^3} + \frac{2a \tan\left(\frac{fx}{2}\right)}{f \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)a + 2 \tan\left(\frac{fx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x)

[Out]
$$\begin{aligned} & -2/f/b^2*B/(1+\tan(1/2*f*x+1/2*e)^2)+2/f/b^2*A*\arctan(\tan(1/2*f*x+1/2*e))-4/ \\ & f/b^3*B*\arctan(\tan(1/2*f*x+1/2*e))*a+2/f*a/(\tan(1/2*f*x+1/2*e)^2*a+2*\tan(1/ \\ & 2*f*x+1/2*e)*b+a)/(a^2-b^2)*\tan(1/2*f*x+1/2*e)*A-2/f*a^2/b/(\tan(1/2*f*x+1/2 \\ & *e)^2*a+2*\tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*\tan(1/2*f*x+1/2*e)*B+2/f*a^2/b/ \\ & (\tan(1/2*f*x+1/2*e)^2*a+2*\tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*A-2/f*a^3/b^2/(\\ & \tan(1/2*f*x+1/2*e)^2*a+2*\tan(1/2*f*x+1/2*e)*b+a)/(a^2-b^2)*B-2/f*a^3/b^2/(a \\ & ^2-b^2)^{(3/2)}*\arctan(1/2*(2*a*\tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^{(1/2)})*A+4/ \\ & f*a/(a^2-b^2)^{(3/2)}*\arctan(1/2*(2*a*\tan(1/2*f*x+1/2*e)+2*b)/(a^2-b^2)^{(1/2)} \\ &)*A+4/f*a^4/b^3/(a^2-b^2)^{(3/2)}*\arctan(1/2*(2*a*\tan(1/2*f*x+1/2*e)+2*b)/(a^ \\ & 2-b^2)^{(1/2)})*B-6/f*a^2/b/(a^2-b^2)^{(3/2)}*\arctan(1/2*(2*a*\tan(1/2*f*x+1/2*e \\ &)+2*b)/(a^2-b^2)^{(1/2)})*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)^2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see 'assume?' for more details)Is 4*b^2-4*a^2 positive or negative?

mupad [B] time = 17.18, size = 3718, normalized size = 24.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(e + f*x)^2*(A + B*sin(e + f*x)))/(a + b*sin(e + f*x))^2,x)

[Out]
$$\begin{aligned} & ((2*(A*a^2*b - 2*B*a^3 + B*a*b^2))/(b^2*(a^2 - b^2)) - (2*\tan(e/2 + (f*x)/2) \\ &)^3*(B*a^2 - A*a*b))/(b*(a^2 - b^2)) + (2*\tan(e/2 + (f*x)/2)*(2*B*b^2 - 3*B \\ & *a^2 + A*a*b))/(b*(a^2 - b^2)) + (2*\tan(e/2 + (f*x)/2)^2*(A*a^2*b - 2*B*a^3 \\ & + B*a*b^2))/(b^2*(a^2 - b^2)))/(f*(a + 2*b*\tan(e/2 + (f*x)/2) + 2*a*\tan(e/ \\ & 2 + (f*x)/2)^2 + a*\tan(e/2 + (f*x)/2)^4 + 2*b*\tan(e/2 + (f*x)/2)^3)) + (\log \\ & (\tan(e/2 + (f*x)/2) + 1i)*(A*b - 2*B*a)*1i)/(b^3*f) - (\log(\tan(e/2 + (f*x)/ \\ & 2) - 1i)*(A*b*1i - B*a*2i))/(b^3*f) - (a*\operatorname{atan}(((a*(-(a + b))^3*(a - b)^3)^{(1 \\ & /2))*((32*(A^2*a^2*b^8 - 2*A^2*a^4*b^6 + A^2*a^6*b^4 + 4*B^2*a^4*b^6 - 8*B^2 \\ & *a^6*b^4 + 4*B^2*a^8*b^2 - 4*A*B*a^3*b^7 + 8*A*B*a^5*b^5 - 4*A*B*a^7*b^3)))/ \\ & (b^9 - 2*a^2*b^7 + a^4*b^5) + (32*\tan(e/2 + (f*x)/2)*(2*A^2*a*b^10 - 9*A^2* \\ & a^3*b^8 + 8*A^2*a^5*b^6 - 2*A^2*a^7*b^4 + 8*B^2*a^3*b^8 - 29*B^2*a^5*b^6 + \\ & 28*B^2*a^7*b^4 - 8*B^2*a^9*b^2 - 8*A*B*a^2*b^9 + 32*A*B*a^4*b^7 - 30*A*B*a^ \\ & 6*b^5 + 8*A*B*a^8*b^3)))/(b^10 - 2*a^2*b^8 + a^4*b^6) + (a*(-(a + b))^3*(a - \\ & b)^3)^{(1/2))*((32*\tan(e/2 + (f*x)/2)*(4*A*a^2*b^11 - 6*A*a^4*b^9 + 2*A*a^6*b \\ & ^7 - 6*B*a^3*b^10 + 10*B*a^5*b^8 - 4*B*a^7*b^6)))/(b^10 - 2*a^2*b^8 + a^4*b^ \\ & 6) - (32*(A*a^3*b^9 + 2*B*a^2*b^10 - 3*B*a^4*b^8 + B*a^6*b^6 - A*a*b^11)))/(\end{aligned}$$

$$\frac{(Aa^2b - 3Bab^2)/(b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)) * (-(a + b)^3 * (a - b)^3)^{1/2} * (2Ab^3 + 2Ba^3 - Aa^2b - 3Bab^2) * 2i}{f * (b^9 - 3a^2b^7 + 3a^4b^5 - a^6b^3)}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(f*x+e)**2*(A+B*sin(f*x+e))/(a+b*sin(f*x+e))**2,x)

[Out] Timed out

$$3.17 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^4 dx$$

Optimal. Leaf size=182

$$\frac{7ac^4(2A - B) \cos^3(e + fx)}{24f} + \frac{7a(2A - B) \cos^3(e + fx)(c^4 - c^4 \sin(e + fx))}{40f} + \frac{7ac^4(2A - B) \sin(e + fx) \cos(e + fx)}{16f}$$

[Out] $7/16*a*(2*A-B)*c^4*x+7/24*a*(2*A-B)*c^4*\cos(f*x+e)^3/f+7/16*a*(2*A-B)*c^4*\cos(f*x+e)*\sin(f*x+e)/f-1/6*a*B*c*\cos(f*x+e)^3*(c-c*\sin(f*x+e))^3/f+1/10*a*(2*A-B)*\cos(f*x+e)^3*(c^2-c^2*\sin(f*x+e))^2/f+7/40*a*(2*A-B)*\cos(f*x+e)^3*(c^4-c^4*\sin(f*x+e))/f$

Rubi [A] time = 0.30, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{7ac^4(2A - B) \cos^3(e + fx)}{24f} + \frac{a(2A - B) \cos^3(e + fx)(c^2 - c^2 \sin(e + fx))^2}{10f} + \frac{7a(2A - B) \cos^3(e + fx)(c^4 - c^4 \sin(e + fx))}{40f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]

[Out] $(7*a*(2*A - B)*c^4*x)/16 + (7*a*(2*A - B)*c^4*\cos[e + f*x]^3)/(24*f) + (7*a*(2*A - B)*c^4*\cos[e + f*x]*\sin[e + f*x])/(16*f) - (a*B*c*\cos[e + f*x]^3*(c - c*\sin[e + f*x])^3)/(6*f) + (a*(2*A - B)*\cos[e + f*x]^3*(c^2 - c^2*\sin[e + f*x])^2)/(10*f) + (7*a*(2*A - B)*\cos[e + f*x]^3*(c^4 - c^4*\sin[e + f*x]))/(40*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

```
Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_))*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^4 dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^4 dx \\ &= -\frac{aBc \cos^3(e + fx)(c - c \sin(e + fx))^3}{6f} + \frac{1}{2}(a(2A - B)c^4 \cos^3(e + fx) - aBc \cos^3(e + fx)) \\ &= -\frac{aBc \cos^3(e + fx)(c - c \sin(e + fx))^3}{6f} + \frac{a(2A - B)c^4 \cos^3(e + fx)}{6f} \\ &= -\frac{aBc \cos^3(e + fx)(c - c \sin(e + fx))^3}{6f} + \frac{a(2A - B)c^4 \cos^3(e + fx)}{6f} \\ &= \frac{7a(2A - B)c^4 \cos^3(e + fx)}{24f} - \frac{aBc \cos^3(e + fx)}{6f} \\ &= \frac{7a(2A - B)c^4 \cos^3(e + fx)}{24f} + \frac{7a(2A - B)c^4 \cos^3(e + fx)}{24f} \\ &= \frac{7}{16}a(2A - B)c^4 x + \frac{7a(2A - B)c^4 \cos^3(e + fx)}{24f} \end{aligned}$$

Mathematica [A] time = 0.98, size = 131, normalized size = 0.72

$$\frac{ac^4(120(7A - 5B) \cos(e + fx) + 20(13A - 7B) \cos(3(e + fx)) + 240A \sin(2(e + fx)) - 90A \sin(4(e + fx)) - 12A \sin(6(e + fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4, x]
```

```
[Out] (a*c^4*(840*A*f*x - 420*B*f*x + 120*(7*A - 5*B)*Cos[e + f*x] + 20*(13*A - 7*B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] + 36*B*Cos[5*(e + f*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] - 90*A*Sin[4*(e + f*x)] + 105*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 123, normalized size = 0.68

$$\frac{48(A - 3B)ac^4 \cos^5(fx + e) - 320(A - B)ac^4 \cos^3(fx + e) - 105(2A - B)ac^4 fx + 5(8Bac^4 \cos^5(fx + e) + 12Aac^4 \cos^3(fx + e) - 12Bac^4 \cos^3(fx + e))}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out]
$$-1/240*(48*(A - 3*B)*a*c^4*\cos(f*x + e)^5 - 320*(A - B)*a*c^4*\cos(f*x + e)^3 - 105*(2*A - B)*a*c^4*f*x + 5*(8*B*a*c^4*\cos(f*x + e)^5 + 2*(18*A - 25*B)*a*c^4*\cos(f*x + e)^3 - 21*(2*A - B)*a*c^4*\cos(f*x + e))*\sin(f*x + e))/f$$

giac [A] time = 0.17, size = 184, normalized size = 1.01

$$-\frac{Bac^4 \sin(6fx + 6e)}{192f} + \frac{7}{16} (2Aac^4 - Bac^4)x - \frac{(Aac^4 - 3Bac^4) \cos(5fx + 5e)}{80f} + \frac{(13Aac^4 - 7Bac^4) \cos(3fx + 3e)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out]
$$-1/192*B*a*c^4*\sin(6*f*x + 6*e)/f + 7/16*(2*A*a*c^4 - B*a*c^4)*x - 1/80*(A*a*c^4 - 3*B*a*c^4)*\cos(5*f*x + 5*e)/f + 1/48*(13*A*a*c^4 - 7*B*a*c^4)*\cos(3*f*x + 3*e)/f + 1/8*(7*A*a*c^4 - 5*B*a*c^4)*\cos(f*x + e)/f - 1/64*(6*A*a*c^4 - 7*B*a*c^4)*\sin(4*f*x + 4*e)/f + 1/64*(16*A*a*c^4 + B*a*c^4)*\sin(2*f*x + 2*e)/f$$

maple [B] time = 0.59, size = 342, normalized size = 1.88

$$-\frac{Ac^4a\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)}{5} - 3Ac^4a\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) - \frac{2Ac^4a(2+\sin^2(fx+e))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)

[Out]
$$1/f*(-1/5*A*c^4*a*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)-3*A*c^4*a*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-2/3*A*c^4*a*(2+\sin(f*x+e)^2)*\cos(f*x+e)+2*A*c^4*a*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)+3*A*c^4*a*\cos(f*x+e)+B*c^4*a*(-1/6*(\sin(f*x+e)^5+5/4*\sin(f*x+e)^3+15/8*\sin(f*x+e))*\cos(f*x+e)+5/16*f*x+5/16*e)+3/5*B*c^4*a*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)+2*B*c^4*a*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-2/3*B*c^4*a*(2+\sin(f*x+e)^2)*\cos(f*x+e)-3*B*c^4*a*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)+A*c^4*a*(f*x+e)-B*c^4*a*\cos(f*x+e))$$

maxima [A] time = 0.60, size = 336, normalized size = 1.85

$$\frac{64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aac^4 - 640\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aac^4}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out]
$$-1/960*(64*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a*c^4 - 640*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a*c^4 + 90*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a*c^4 - 480*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a*c^4 - 960*(f*x + e)*A*a*c^4 - 192*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a*c^4 - 640*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*$$

$$B*a*c^4 - 5*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B*a*c^4 - 60*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a*c^4 + 720*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a*c^4 - 2880*A*a*c^4*\cos(f*x + e) + 960*B*a*c^4*\cos(f*x + e))/f$$

mupad [B] time = 14.83, size = 454, normalized size = 2.49

$$\tan\left(\frac{e}{2} + \frac{f*x}{2}\right) \left(\frac{Aac^4}{4} + \frac{7Bac^4}{8}\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^{10} (6Aac^4 - 2Bac^4) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 (12Aac^4 - 4Bac^4) - \tan$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^4,x)

[Out] (tan(e/2 + (f*x)/2)*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^10*(6*A*a*c^4 - 2*B*a*c^4) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^4 - 4*B*a*c^4) - tan(e/2 + (f*x)/2)^11*((A*a*c^4)/4 + (7*B*a*c^4)/8) + tan(e/2 + (f*x)/2)^8*(2*2*A*a*c^4 - 18*B*a*c^4) + tan(e/2 + (f*x)/2)^5*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) - tan(e/2 + (f*x)/2)^7*((13*A*a*c^4)/2 - (37*B*a*c^4)/4) + tan(e/2 + (f*x)/2)^2*((38*A*a*c^4)/5 - (34*B*a*c^4)/5) + tan(e/2 + (f*x)/2)^6*((68*A*a*c^4)/3 - (44*B*a*c^4)/3) + tan(e/2 + (f*x)/2)^3*((27*A*a*c^4)/4 - (73*B*a*c^4)/24) - tan(e/2 + (f*x)/2)^9*((27*A*a*c^4)/4 - (73*B*a*c^4)/24) + (34*A*a*c^4)/15 - (22*B*a*c^4)/15)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (7*a*c^4*atan((7*a*c^4*tan(e/2 + (f*x)/2)*(2*A - B))/(8*((7*A*a*c^4)/4 - (7*B*a*c^4)/8)))*(2*A - B))/(8*f)

sympy [A] time = 7.88, size = 853, normalized size = 4.69

$$\left\{ \begin{array}{l} -\frac{9Aac^4x\sin^4(e+fx)}{8} - \frac{9Aac^4x\sin^2(e+fx)\cos^2(e+fx)}{4} + Aac^4x\sin^2(e+fx) - \frac{9Aac^4x\cos^4(e+fx)}{8} + Aac^4x\cos^2(e+fx) + A \\ x(A+B\sin(e))(a\sin(e)+a)(-c\sin(e)+c)^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4,x)

[Out] Piecewise((-9*A*a*c**4*x*sin(e + f*x)**4/8 - 9*A*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a*c**4*x*sin(e + f*x)**2 - 9*A*a*c**4*x*cos(e + f*x)**4/8 + A*a*c**4*x*cos(e + f*x)**2 + A*a*c**4*x - A*a*c**4*sin(e + f*x)**4*cos(e + f*x)/f + 15*A*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 9*A*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a*c**4*sin(e + f*x)*cos(e + f*x)/f - 8*A*a*c**4*cos(e + f*x)**5/(15*f) - 4*A*a*c**4*cos(e + f*x)**3/(3*f) + 3*A*a*c**4*cos(e + f*x)/f + 5*B*a*c**4*x*sin(e + f*x)**6/16 + 15*B*a*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a*c**4*x*sin(e + f*x)**4/4 + 15*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 3*B*a*c**4*x*sin(e + f*x)**2/2 + 5*B*a*c**4*x*cos(e + f*x)**6/16 + 3*B*a*c**4*x*cos(e + f*x)**4/4 - 3*B*a*c**4*x*cos(e + f*x)**2/2 - 11*B*a*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 3*B*a*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B*a*c**4*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 2*B*a*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 5*B*a*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)**3/(4*f) + 3*B*a*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 8*B*a*c**4*cos(e + f*x)**5/(5*f) - 4*B*a*c**4*cos(e + f*x)**3/(3*f) - B*a*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**4, True))

$$3.18 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx$$

Optimal. Leaf size=142

$$\frac{ac^3(5A - 2B) \cos^3(e + fx)}{12f} + \frac{a(5A - 2B) \cos^3(e + fx) (c^3 - c^3 \sin(e + fx))}{20f} + \frac{ac^3(5A - 2B) \sin(e + fx) \cos(e + fx)}{8f}$$

[Out] 1/8*a*(5*A-2*B)*c^3*x+1/12*a*(5*A-2*B)*c^3*cos(f*x+e)^3/f+1/8*a*(5*A-2*B)*c^3*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^2/f+1/20*a*(5*A-2*B)*cos(f*x+e)^3*(c^3-c^3*sin(f*x+e))/f

Rubi [A] time = 0.25, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{ac^3(5A - 2B) \cos^3(e + fx)}{12f} + \frac{a(5A - 2B) \cos^3(e + fx) (c^3 - c^3 \sin(e + fx))}{20f} + \frac{ac^3(5A - 2B) \sin(e + fx) \cos(e + fx)}{8f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]

[Out] (a*(5*A - 2*B)*c^3*x)/8 + (a*(5*A - 2*B)*c^3*Cos[e + f*x]^3)/(12*f) + (a*(5*A - 2*B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^2)/(5*f) + (a*(5*A - 2*B)*Cos[e + f*x]^3*(c^3 - c^3*Sin[e + f*x]))/(20*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := -Simp[(d*(

$g \cos[e + f x]^{(p+1)} (a + b \sin[e + f x])^m / (f g (m + p + 1)), x] + \text{Dist}[(a d^m + b c (m + p + 1)) / (b (m + p + 1)), \text{Int}[(g \cos[e + f x])^p (a + b \sin[e + f x])^m, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

$\text{Int}[(a + b \sin[e + f x])^m ((A + B \sin[e + f x]) + (c + d \sin[e + f x])^n), x] := \text{Dist}[a^m c^m, \text{Int}[\cos[e + f x]^{(2m)} (c + d \sin[e + f x])^{(n-m)} (A + B \sin[e + f x]), x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b c + a d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx \\ &= -\frac{aBc \cos^3(e + fx)(c - c \sin(e + fx))^2}{5f} + \frac{1}{5}(a(5A - 2B)c^3 \cos^3(e + fx) - aBc \cos^3(e + fx)) \\ &= -\frac{aBc \cos^3(e + fx)(c - c \sin(e + fx))^2}{5f} + \frac{a(5A - 2B)c^3 \cos^3(e + fx)}{5f} - \frac{aBc \cos^3(e + fx)}{5f} \\ &= \frac{a(5A - 2B)c^3 \cos^3(e + fx)}{12f} - \frac{aBc \cos^3(e + fx)}{5f} \\ &= \frac{a(5A - 2B)c^3 \cos^3(e + fx)}{12f} + \frac{a(5A - 2B)c^3 \cos^3(e + fx)}{12f} \\ &= \frac{1}{8}a(5A - 2B)c^3 x + \frac{a(5A - 2B)c^3 \cos^3(e + fx)}{12f} \end{aligned}$$

Mathematica [A] time = 0.84, size = 95, normalized size = 0.67

$$\frac{ac^3(15(-(A - 2B) \sin(4(e + fx)) + 4fx(5A - 2B) + 8A \sin(2(e + fx))) + 60(4A - 3B) \cos(e + fx) + 10(8A - 5B) \sin(e + fx))}{480f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3, x]

[Out] (a*c^3*(60*(4*A - 3*B)*Cos[e + f*x] + 10*(8*A - 5*B)*Cos[3*(e + f*x)] + 6*B*Cos[5*(e + f*x)] + 15*(4*(5*A - 2*B)*f*x + 8*A*Sin[2*(e + f*x)] - (A - 2*B)*Sin[4*(e + f*x)])))/(480*f)

fricas [A] time = 0.44, size = 102, normalized size = 0.72

$$\frac{24Bac^3 \cos(fx + e)^5 + 80(A - B)ac^3 \cos(fx + e)^3 + 15(5A - 2B)ac^3 fx - 15(2(A - 2B)ac^3 \cos(fx + e)^3 - (5A - 2B)ac^3 \cos(fx + e)) \sin(fx + e)}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 1/120*(24*B*a*c^3*cos(f*x + e)^5 + 80*(A - B)*a*c^3*cos(f*x + e)^3 + 15*(5*A - 2*B)*a*c^3*f*x - 15*(2*(A - 2*B)*a*c^3*cos(f*x + e)^3 - (5*A - 2*B)*a*c^3*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.18, size = 145, normalized size = 1.02

$$\frac{Bac^3 \cos(5fx + 5e)}{80f} + \frac{Aac^3 \sin(2fx + 2e)}{4f} + \frac{1}{8} (5Aac^3 - 2Bac^3)x + \frac{(8Aac^3 - 5Bac^3) \cos(3fx + 3e)}{48f} + \frac{(4Aac^3 - 2Bac^3) \sin(4fx + 4e)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] 1/80*B*a*c^3*cos(5*f*x + 5*e)/f + 1/4*A*a*c^3*sin(2*f*x + 2*e)/f + 1/8*(5*A*a*c^3 - 2*B*a*c^3)*x + 1/48*(8*A*a*c^3 - 5*B*a*c^3)*cos(3*f*x + 3*e)/f + 1/8*(4*A*a*c^3 - 3*B*a*c^3)*cos(f*x + e)/f - 1/32*(A*a*c^3 - 2*B*a*c^3)*sin(4*f*x + 4*e)/f

maple [A] time = 0.48, size = 208, normalized size = 1.46

$$-Ac^3a \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) - \frac{2Ac^3a(2+\sin^2(fx+e))\cos(fx+e)}{3} + 2Ac^3a \cos(fx+e) + \frac{Bc^3a \left(\frac{8}{3} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

[Out] 1/f*(-A*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/3*A*c^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)+2*A*c^3*a*cos(f*x+e)+1/5*B*c^3*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+2*B*c^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*B*c^3*a*(-1/2*sin(f*x+e))*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^3*a*(f*x+e)-B*c^3*a*cos(f*x+e))

maxima [A] time = 0.33, size = 200, normalized size = 1.41

$$320 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) Aac^3 - 15 \left(12fx + 12e + \sin(4fx + 4e) - 8 \sin(2fx + 2e) \right) Aac^3 + 480 \left(\cos(fx + e) - 3 \cos(fx + e) \right) Bc^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^3 - 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a*c^3 + 480*(f*x + e)*A*a*c^3 + 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a*c^3 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^3 - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3 + 960*A*a*c^3*cos(f*x + e) - 480*B*a*c^3*cos(f*x + e))/f

mupad [B] time = 13.80, size = 389, normalized size = 2.74

$$\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{3Aac^3}{4} + \frac{Bac^3}{2}\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 (4Aac^3 - 2Bac^3) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 \left(\frac{7Aac^3}{2} - 3Bac^3\right) - \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3,x)`

[Out] $(\tan(e/2 + (f*x)/2)*((3*A*a*c^3)/4 + (B*a*c^3)/2) + \tan(e/2 + (f*x)/2)^8*(4*A*a*c^3 - 2*B*a*c^3) + \tan(e/2 + (f*x)/2)^3*((7*A*a*c^3)/2 - 3*B*a*c^3) - \tan(e/2 + (f*x)/2)^7*((7*A*a*c^3)/2 - 3*B*a*c^3) - \tan(e/2 + (f*x)/2)^9*((3*A*a*c^3)/4 + (B*a*c^3)/2) + \tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 - 8*B*a*c^3) + \tan(e/2 + (f*x)/2)^2*((8*A*a*c^3)/3 - (8*B*a*c^3)/3) + \tan(e/2 + (f*x)/2)^4*((16*A*a*c^3)/3 - (4*B*a*c^3)/3) + (4*A*a*c^3)/3 - (14*B*a*c^3)/15)/(f*(5*\tan(e/2 + (f*x)/2)^2 + 10*\tan(e/2 + (f*x)/2)^4 + 10*\tan(e/2 + (f*x)/2)^6 + 5*\tan(e/2 + (f*x)/2)^8 + \tan(e/2 + (f*x)/2)^{10} + 1)) + (a*c^3*\operatorname{atan}(a*c^3*\tan(e/2 + (f*x)/2)*(5*A - 2*B))/(4*((5*A*a*c^3)/4 - (B*a*c^3)/2)))*(5*A - 2*B))/(4*f) - (a*c^3*(5*A - 2*B)*(atan(\tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)$

sympy [A] time = 4.09, size = 486, normalized size = 3.42

$$\left\{ \begin{array}{l} -\frac{3Aac^3x\sin^4(e+fx)}{8} - \frac{3Aac^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aac^3x\cos^4(e+fx)}{8} + Aac^3x + \frac{5Aac^3\sin^3(e+fx)\cos(e+fx)}{8f} - \frac{2Aac^3\sin^2(e+fx)}{f} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)`

[Out] `Piecewise((-3*A*a*c**3*x*sin(e + f*x)**4/8 - 3*A*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - 3*A*a*c**3*x*cos(e + f*x)**4/8 + A*a*c**3*x + 5*A*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A*a*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 3*A*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 4*A*a*c**3*cos(e + f*x)**3/(3*f) + 2*A*a*c**3*cos(e + f*x)/f + 3*B*a*c**3*x*sin(e + f*x)**4/4 + 3*B*a*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a*c**3*x*sin(e + f*x)**2 + 3*B*a*c**3*x*cos(e + f*x)**4/4 - B*a*c**3*x*cos(e + f*x)**2 + B*a*c**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*B*a*c**3*cos(e + f*x)**5/(15*f) - B*a*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**3, True))`

3.19 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx$

Optimal. Leaf size=97

$$\frac{ac^2(A-B)\cos^3(e+fx)}{3f} + \frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{1}{8}ac^2x(4A-B) + \frac{aBc^2\sin(e+fx)\cos^3(e+fx)}{4f}$$

[Out] $1/8*a*(4*A-B)*c^2*x+1/3*a*(A-B)*c^2*\cos(f*x+e)^3/f+1/8*a*(4*A-B)*c^2*\cos(f*x+e)*\sin(f*x+e)/f+1/4*a*B*c^2*\cos(f*x+e)^3*\sin(f*x+e)/f$

Rubi [A] time = 0.19, antiderivative size = 105, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{ac^2(4A-B)\cos^3(e+fx)}{12f} + \frac{ac^2(4A-B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{1}{8}ac^2x(4A-B) - \frac{aB\cos^3(e+fx)(c^2 - c^2\sin(e+fx))}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]

[Out] $(a*(4*A - B)*c^2*x)/8 + (a*(4*A - B)*c^2*\cos[e + f*x]^3)/(12*f) + (a*(4*A - B)*c^2*\cos[e + f*x]*\sin[e + f*x])/(8*f) - (a*B*\cos[e + f*x]^3*(c^2 - c^2*\sin[e + f*x]))/(4*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p+1))/(f*g*(p+1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(f*g*(m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(b*(m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m+p+1, 0]

Rule 2967

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && (LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx \\ &= -\frac{aB \cos^3(e + fx)(c^2 - c^2 \sin(e + fx))}{4f} + \frac{1}{4}(a(4A - B)c^2 \cos^3(e + fx) - aB \cos^3(e + fx)(c^2 - c^2 \sin(e + fx))) \\ &= \frac{a(4A - B)c^2 \cos^3(e + fx)}{12f} - \frac{aB \cos^3(e + fx)(c^2 - c^2 \sin(e + fx))}{4f} \\ &= \frac{a(4A - B)c^2 \cos^3(e + fx)}{12f} + \frac{a(4A - B)c^2 \cos(e + fx)}{8f} \\ &= \frac{1}{8}a(4A - B)c^2 x + \frac{a(4A - B)c^2 \cos^3(e + fx)}{12f} + \frac{a(4A - B)c^2 \cos(e + fx)}{8f} \end{aligned}$$

Mathematica [A] time = 0.67, size = 74, normalized size = 0.76

$$\frac{ac^2(3(8A \sin(2(e + fx)) + 16Afx + B \sin(4(e + fx)) - 4Bfx) + 24(A - B) \cos(e + fx) + 8(A - B) \cos(3(e + fx)))}{96f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2, x]

[Out] (a*c^2*(24*(A - B)*Cos[e + f*x] + 8*(A - B)*Cos[3*(e + f*x)] + 3*(16*A*f*x - 4*B*f*x + 8*A*Sin[2*(e + f*x)] + B*Sin[4*(e + f*x)])))/(96*f)

fricas [A] time = 0.45, size = 82, normalized size = 0.85

$$\frac{8(A - B)ac^2 \cos^3(fx + e) + 3(4A - B)ac^2 fx + 3(2Bac^2 \cos^3(fx + e) + (4A - B)ac^2 \cos(fx + e)) \sin(fx + e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/24*(8*(A - B)*a*c^2*cos(f*x + e)^3 + 3*(4*A - B)*a*c^2*f*x + 3*(2*B*a*c^2*cos(f*x + e)^3 + (4*A - B)*a*c^2*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.14, size = 114, normalized size = 1.18

$$\frac{Bac^2 \sin(4fx + 4e)}{32f} + \frac{Aac^2 \sin(2fx + 2e)}{4f} + \frac{1}{8}(4Aac^2 - Bac^2)x + \frac{(Aac^2 - Bac^2) \cos(3fx + 3e)}{12f} + \frac{(Aac^2 - Bac^2) \sin(3fx + 3e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/32*B*a*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a*c^2*sin(2*f*x + 2*e)/f + 1/8*(4*A*a*c^2 - B*a*c^2)*x + 1/12*(A*a*c^2 - B*a*c^2)*cos(3*f*x + 3*e)/f + 1/4*(A*a*c^2 - B*a*c^2)*cos(f*x + e)/f

maple [B] time = 0.39, size = 185, normalized size = 1.91

$$-\frac{Ac^2a(2+\sin^2(fx+e))\cos(fx+e)}{3} - Ac^2a\left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + Ac^2a\cos(fx+e) + Bc^2a\left(-\frac{\sin^3(fx+e)+\dots}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)

[Out] 1/f*(-1/3*A*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-A*c^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*cos(f*x+e)+B*c^2*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/3*B*c^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)-B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+A*c^2*a*(f*x+e)-B*c^2*a*cos(f*x+e))

maxima [B] time = 0.33, size = 179, normalized size = 1.85

$$32\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aac^2 - 24(2fx+2e - \sin(2fx+2e))Aac^2 + 96(fx+e)Aac^2 - 32(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2 + 96*(f*x + e)*A*a*c^2 - 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c^2 + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c^2 - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 + 96*A*a*c^2*cos(f*x + e) - 96*B*a*c^2*cos(f*x + e))/f

mupad [B] time = 13.38, size = 345, normalized size = 3.56

$$\tan\left(\frac{e}{2} + \frac{fx}{2}\right)\left(Aac^2 + \frac{Bac^2}{4}\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4\left(2Aac^2 - 2Bac^2\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6\left(2Aac^2 - 2Bac^2\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8\left(2Aac^2 - 2Bac^2\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2,x)

[Out] (tan(e/2 + (f*x)/2)*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^4*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 - 2*B*a*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^2 - 2*B*a*c^2) - tan(e/2 + (f*x)/2)^2*((2*A*a*c^2)/3 - (2*B*a*c^2)/3) - tan(e/2 + (f*x)/2)^7*(A*a*c^2 + (B*a*c^2)/4) + tan(e/2 + (f*x)/2)^3*(A*a*c^2 - (7*B*a*c^2)/4) - tan(e/2 + (f*x)/2)^5*(A*a*c^2 - (7*B*a*c^2)/4) + (2*A*a*c^2)/3 - (2*B*a*c^2)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1)) + (a*c^2*atan((a*c^2*tan(e/2 + (f*x)/2)*(4*A - B))/(4*(A*a*c^2 - (B*a*c^2)/4)))*(4*A - B))/(4*f) - (a*c^2*(4*A - B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)

sympy [A] time = 2.13, size = 396, normalized size = 4.08

$$\left\{ \begin{array}{l} -\frac{Aac^2x\sin^2(e+fx)}{2} - \frac{Aac^2x\cos^2(e+fx)}{2} + Aac^2x - \frac{Aac^2\sin^2(e+fx)\cos(e+fx)}{f} + \frac{Aac^2\sin(e+fx)\cos(e+fx)}{2f} - \frac{2Aac^2\cos^3(e+fx)}{3f} \\ x(A + B\sin(e))(a\sin(e) + a)(-c\sin(e) + c)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)

[Out] Piecewise((-A*a*c**2*x*sin(e + f*x)**2/2 - A*a*c**2*x*cos(e + f*x)**2/2 + A*a*c**2*x - A*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f + A*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c**2*cos(e + f*x)**3/(3*f) + A*a*c**2*cos(e + f*x)/f + 3*B*a*c**2*x*sin(e + f*x)**4/8 + 3*B*a*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - B*a*c**2*x*sin(e + f*x)**2/2 + 3*B*a*c**2*x*cos(e + f*x)**4/8 - B*a*c**2*x*cos(e + f*x)**2/2 - 5*B*a*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) + B*a*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a*c**2*cos(e + f*x)**3/(3*f) - B*a*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c)**2, True))

3.20 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx)) dx$

Optimal. Leaf size=49

$$\frac{aAc \sin(e + fx) \cos(e + fx)}{2f} + \frac{1}{2}aAcx - \frac{aBc \cos^3(e + fx)}{3f}$$

[Out] $1/2*a*A*c*x-1/3*a*B*c*\cos(f*x+e)^3/f+1/2*a*A*c*\cos(f*x+e)*\sin(f*x+e)/f$

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2967, 2669, 2635, 8}

$$\frac{aAc \sin(e + fx) \cos(e + fx)}{2f} + \frac{1}{2}aAcx - \frac{aBc \cos^3(e + fx)}{3f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x]),x]$

[Out] $(a*A*c*x)/2 - (a*B*c*\text{Cos}[e + f*x]^3)/(3*f) + (a*A*c*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(2*f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_.*\text{sin}[c_.] + (d_.*(x_)))]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^{2*(n-1)})/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \text{GtQ}[n, 1] \ \&\& \text{IntegerQ}[2*n]$

Rule 2669

$\text{Int}[(\text{cos}[e_.] + (f_.*(x_))]*(g_.)]^{(p_)}*((a_.) + (b_.*\text{sin}[e_.] + (f_.*(x_)))]), x_Symbol] \rightarrow -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)})/(f*g*(p+1)), x] + \text{Dist}[a, \text{Int}[(g*\text{Cos}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x \ \&\& (\text{IntegerQ}[2*p] \ \|\ \text{NeQ}[a^2 - b^2, 0])$

Rule 2967

$\text{Int}[(a_.) + (b_.*\text{sin}[e_.] + (f_.*(x_)))]^{(m_)}*((A_.) + (B_.*\text{sin}[e_.] + (f_.*(x_)))]^{(n_)}, x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \text{EqQ}[b*c + a*d, 0] \ \&\& \text{EqQ}[a^2 - b^2, 0] \ \&\& \text{IntegerQ}[m] \ \&\& !(\text{IntegerQ}[n] \ \&\& ((\text{LtQ}[m, 0] \ \& \ \& \text{GtQ}[n, 0]) \ \|\ \text{LtQ}[0, n, m] \ \|\ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx)) dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx)) dx \\
&= -\frac{aBc \cos^3(e + fx)}{3f} + (aAc) \int \cos^2(e + fx) dx \\
&= -\frac{aBc \cos^3(e + fx)}{3f} + \frac{aAc \cos(e + fx) \sin(e + fx)}{2f} \\
&= \frac{1}{2} aAcx - \frac{aBc \cos^3(e + fx)}{3f} + \frac{aAc \cos(e + fx) \sin(e + fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 0.98

$$-\frac{ac(-3A(\sin(2(e + fx)) - 2e + 2fx) + 3B \cos(e + fx) + B \cos(3(e + fx)))}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]

[Out] -1/12*(a*c*(3*B*Cos[e + f*x] + B*Cos[3*(e + f*x)] - 3*A*(-2*e + 2*f*x + Sin[2*(e + f*x)])))/f

fricas [A] time = 0.45, size = 43, normalized size = 0.88

$$-\frac{2Bac \cos(fx + e)^3 - 3Aacfx - 3Aac \cos(fx + e) \sin(fx + e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] -1/6*(2*B*a*c*cos(f*x + e)^3 - 3*A*a*c*f*x - 3*A*a*c*cos(f*x + e)*sin(f*x + e))/f

giac [A] time = 0.14, size = 58, normalized size = 1.18

$$\frac{1}{2} Aacx - \frac{Bac \cos(3fx + 3e)}{12f} - \frac{Bac \cos(fx + e)}{4f} + \frac{Aac \sin(2fx + 2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] 1/2*A*a*c*x - 1/12*B*a*c*cos(3*f*x + 3*e)/f - 1/4*B*a*c*cos(f*x + e)/f + 1/4*A*a*c*sin(2*f*x + 2*e)/f

maple [A] time = 0.29, size = 74, normalized size = 1.51

$$\frac{\frac{Bac(2+\sin^2(fx+e))\cos(fx+e)}{3} - Aac \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - Bac \cos(fx + e) + Aac(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] $1/f*(1/3*B*a*c*(2+\sin(f*x+e))^2*\cos(f*x+e)-A*a*c*(-1/2*\sin(f*x+e)*\cos(f*x+e))+1/2*f*x+1/2*e)-B*a*c*\cos(f*x+e)+A*a*c*(f*x+e)$

maxima [A] time = 0.38, size = 73, normalized size = 1.49

$$\frac{3(2fx + 2e - \sin(2fx + 2e))Aac - 12(fx + e)Aac + 4(\cos(fx + e)^3 - 3\cos(fx + e))Bac + 12Bac \cos(fx + e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] $-1/12*(3*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a*c - 12*(f*x + e)*A*a*c + 4*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a*c + 12*B*a*c*\cos(f*x + e))/f$

mupad [B] time = 14.33, size = 122, normalized size = 2.49

$$\frac{Aacx \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + \left(\frac{ac(12B-9A(e+fx))}{6} + \frac{3Aac(e+fx)}{2}\right) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - Aac \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + \frac{ac(4B-3A)}{2}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x)),x)

[Out] $(A*a*c*x)/2 - (\tan(e/2 + (f*x)/2)^4*((a*c*(12*B - 9*A*(e + f*x)))/6 + (3*A*a*c*(e + f*x))/2) + (a*c*(4*B - 3*A*(e + f*x)))/6 - A*a*c*\tan(e/2 + (f*x)/2) + (A*a*c*(e + f*x))/2 + A*a*c*\tan(e/2 + (f*x)/2)^5/(f*(\tan(e/2 + (f*x)/2)^2 + 1)^3)$

sympy [A] time = 0.79, size = 138, normalized size = 2.82

$$\left\{ \begin{array}{l} -\frac{Aacx \sin^2(e+fx)}{2} - \frac{Aacx \cos^2(e+fx)}{2} + Aacx + \frac{Aac \sin(e+fx) \cos(e+fx)}{2f} + \frac{Bac \sin^2(e+fx) \cos(e+fx)}{f} + \frac{2Bac \cos^3(e+fx)}{3f} - \frac{Bac}{3f} \\ x(A + B \sin(e))(a \sin(e) + a)(-c \sin(e) + c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] Piecewise((-A*a*c*x*sin(e + f*x)**2/2 - A*a*c*x*cos(e + f*x)**2/2 + A*a*c*x + A*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a*c*sin(e + f*x)**2*cos(e + f*x)/f + 2*B*a*c*cos(e + f*x)**3/(3*f) - B*a*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)*(-c*sin(e) + c), True))

$$3.21 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx$$

Optimal. Leaf size=56

$$\frac{2a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))} - \frac{ax(A+2B)}{c} + \frac{aB \cos(e+fx)}{cf}$$

[Out] $-a*(A+2*B)*x/c+a*B*\cos(f*x+e)/c/f+2*a*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))$

Rubi [A] time = 0.17, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2967, 2857, 2638}

$$\frac{2a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))} - \frac{ax(A+2B)}{c} + \frac{aB \cos(e+fx)}{cf}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])*(A + B*\text{Sin}[e + f*x])]/(c - c*\text{Sin}[e + f*x]),x]$

[Out] $-(a*(A + 2*B)*x)/c + (a*B*\text{Cos}[e + f*x])/(c*f) + (2*a*(A + B)*\text{Cos}[e + f*x])/f*(c - c*\text{Sin}[e + f*x])$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2857

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^2*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(2*(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b^2*f*(2*m+3)), x] + \text{Dist}[1/(b^3*(2*m+3)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+2)}*(b*c + 2*a*d*(m+1) - b*d*(2*m+3)*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -3/2]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx &= (ac) \int \frac{\cos^2(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx \\ &= \frac{2a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))} + \frac{a \int (-Ac - 2Bc - Bc \sin(e+fx)) dx}{c^2} \\ &= -\frac{a(A+2B)x}{c} + \frac{2a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))} - \frac{(aB) \int \sin(e+fx) dx}{c} \\ &= -\frac{a(A+2B)x}{c} + \frac{aB \cos(e+fx)}{cf} + \frac{2a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))} \end{aligned}$$

Mathematica [B] time = 0.93, size = 125, normalized size = 2.23

$$\frac{a(\sin(e + fx) + 1) \left(\frac{4(A+B) \sin\left(\frac{fx}{2}\right)}{f(\cos(\frac{e}{2}) - \sin(\frac{e}{2}))(\cos(\frac{1}{2}(e+fx)) - \sin(\frac{1}{2}(e+fx)))} - (x(A + 2B)) - \frac{B \sin(e) \sin(fx)}{f} + \frac{B \cos(e) \cos(fx)}{f} \right)}{c \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]), x]

[Out] (a*(-((A + 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A + B)*Sin[(f*x)/2])/(f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])))*(1 + Sin[e + f*x]))/(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)

fricas [B] time = 0.42, size = 116, normalized size = 2.07

$$\frac{(A + 2B)afx - Ba \cos(fx + e)^2 - 2(A + B)a + ((A + 2B)afx - (2A + 3B)a) \cos(fx + e) - ((A + 2B)afx - (2A + 3B)a) \cos(fx + e) - ((A + 2B)afx - (2A + 3B)a) \cos(fx + e)}{cf \cos(fx + e) - cf \sin(fx + e) + cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] -((A + 2*B)*a*f*x - B*a*cos(f*x + e)^2 - 2*(A + B)*a + ((A + 2*B)*a*f*x - (2*A + 3*B)*a)*cos(f*x + e) - ((A + 2*B)*a*f*x - B*a*cos(f*x + e) + 2*(A + B)*a)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)

giac [B] time = 0.15, size = 124, normalized size = 2.21

$$\frac{\frac{(Aa+2Ba)(fx+e)}{c} + \frac{2 \left(2Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 2Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2Aa + 3Ba \right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right) c}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] -((A*a + 2*B*a)*(f*x + e)/c + 2*(2*A*a*tan(1/2*f*x + 1/2*e)^2 + 2*B*a*tan(1/2*f*x + 1/2*e)^2 - B*a*tan(1/2*f*x + 1/2*e) + 2*A*a + 3*B*a)/((tan(1/2*f*x + 1/2*e)^3 - tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) - 1)*c))/f

maple [A] time = 0.41, size = 113, normalized size = 2.02

$$\frac{\frac{4aA}{fc \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)} - \frac{4aB}{fc \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)} + \frac{2aB}{fc \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)} - \frac{2a \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right) A}{fc} - \frac{4a \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right) B}{fc}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] -4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*A-4/f*a/c/(tan(1/2*f*x+1/2*e)-1)*B+2/f*a/c*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*a/c*arctan(tan(1/2*f*x+1/2*e))*A-4/f*a/c*arctan(tan(1/2*f*x+1/2*e))*B

maxima [B] time = 0.59, size = 265, normalized size = 4.73

$$\frac{2 \left(Ba \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2}{c - \frac{c \sin(fx+e)}{\cos(fx+e)+1} + \frac{c \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} \right) + Aa \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} - \frac{1}{c - \frac{c \sin(fx+e)}{\cos(fx+e)+1}} \right) + Ba \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} \right) \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -2*(B*a*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + A*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) + B*a*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x + e) + 1))) - A*a/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)))/f

mupad [B] time = 12.64, size = 111, normalized size = 1.98

$$\frac{(4Aa + 4Ba) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 2Ba \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 4Aa + 6Ba}{f \left(-c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - c \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + c \right)} - \frac{Aafx + 2Bafx}{cf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x)),x)

[Out] (4*A*a + 6*B*a + tan(e/2 + (f*x)/2)^2*(4*A*a + 4*B*a) - 2*B*a*tan(e/2 + (f*x)/2))/(f*(c - c*tan(e/2 + (f*x)/2) + c*tan(e/2 + (f*x)/2)^2 - c*tan(e/2 + (f*x)/2)^3)) - (A*a*f*x + 2*B*a*f*x)/(c*f)

sympy [A] time = 4.03, size = 828, normalized size = 14.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] Piecewise((-A*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - A*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + A*a*f*x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*A*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)**3/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*x*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 2*B*a*f*x*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*f*x/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) - 4*B*a*tan(e/2 + f*x/2)**2/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) + 2*B*a*tan(e/2 + f*x/2)/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*x/2) - c*f) -

```
6*B*a/(c*f*tan(e/2 + f*x/2)**3 - c*f*tan(e/2 + f*x/2)**2 + c*f*tan(e/2 + f*
x/2) - c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c), T
rue))
```

$$3.22 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=72

$$-\frac{a(A+7B) \cos(e+fx)}{3c^2 f(1-\sin(e+fx))} + \frac{2a(A+B) \cos(e+fx)}{3f(c-c \sin(e+fx))^2} + \frac{aBx}{c^2}$$

[Out] a*B*x/c^2-1/3*a*(A+7*B)*cos(f*x+e)/c^2/f/(1-sin(f*x+e))+2/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^2

Rubi [A] time = 0.22, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2967, 2857, 2735, 2648}

$$-\frac{a(A+7B) \cos(e+fx)}{3c^2 f(1-\sin(e+fx))} + \frac{2a(A+B) \cos(e+fx)}{3f(c-c \sin(e+fx))^2} + \frac{aBx}{c^2}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] (a*B*x)/c^2 - (a*(A + 7*B)*Cos[e + f*x])/(3*c^2*f*(1 - Sin[e + f*x])) + (2*a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^2)

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^2} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx \\
&= \frac{2a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^2} + \frac{a \int \frac{-Ac - 4Bc - 3Bc \sin(e + fx)}{c - c \sin(e + fx)} dx}{3c^2} \\
&= \frac{aBx}{c^2} + \frac{2a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^2} - \frac{(a(A + 7B)) \int \frac{1}{c - c \sin(e + fx)} dx}{3c} \\
&= \frac{aBx}{c^2} + \frac{2a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^2} - \frac{a(A + 7B) \cos(e + fx)}{3f(c^2 - c^2 \sin(e + fx))}
\end{aligned}$$

Mathematica [B] time = 0.63, size = 160, normalized size = 2.22

$$\frac{a \left(-6(A + 3B) \cos \left(e + \frac{fx}{2} \right) + 2A \cos \left(e + \frac{3fx}{2} \right) + 9Bfx \sin \left(e + \frac{fx}{2} \right) + 3Bfx \sin \left(e + \frac{3fx}{2} \right) + 14B \cos \left(e + \frac{3fx}{2} \right) \right)}{6c^2 f \left(\cos \left(\frac{e}{2} \right) - \sin \left(\frac{e}{2} \right) \right) \left(\cos \left(\frac{1}{2}(e + fx) \right) - \sin \left(\frac{1}{2}(e + fx) \right) \right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2, x]

[Out] -1/6*(a*(-9*B*f*x*Cos[(f*x)/2] - 6*(A + 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f*x)/2] + 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[(f*x)/2] + 9*B*f*x*Sin[e + (f*x)/2] + 3*B*f*x*Sin[e + (3*f*x)/2]))/(c^2*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3)

fricas [B] time = 0.44, size = 162, normalized size = 2.25

$$\frac{6Bafx - (3Bafx + (A + 7B)a) \cos^2(fx + e) + 2(A + B)a + (3Bafx + (A - 5B)a) \cos(fx + e) - (6Bafx + (A + 7B)a) \sin^2(fx + e)}{3(c^2 f \cos^2(fx + e) - c^2 f \cos(fx + e) - 2c^2 f + (c^2 f \cos(fx + e) - c^2 f \sin(fx + e)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(6*B*a*f*x - (3*B*a*f*x + (A + 7*B)*a)*cos(f*x + e)^2 + 2*(A + B)*a + (3*B*a*f*x + (A - 5*B)*a)*cos(f*x + e) - (6*B*a*f*x - 2*(A + B)*a + (3*B*a*f*x - (A + 7*B)*a)*cos(f*x + e))*sin(f*x + e)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))

giac [A] time = 0.15, size = 92, normalized size = 1.28

$$\frac{\frac{3(fx+e)Ba}{c^2} - \frac{2 \left(3Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 3Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 12Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + Aa - 5Ba \right)}{c^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/3*(3*(f*x + e)*B*a/c^2 - 2*(3*A*a*tan(1/2*f*x + 1/2*e)^2 - 3*B*a*tan(1/2*f*x + 1/2*e)^2 + 12*B*a*tan(1/2*f*x + 1/2*e) + A*a - 5*B*a)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f

maple [B] time = 0.44, size = 160, normalized size = 2.22

$$\frac{2aA}{c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)} + \frac{2aB}{c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)} - \frac{8aA}{3c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)^3} - \frac{8aB}{3c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)^3} - \frac{c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)}{c^2 f \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] -2*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)*A+2*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)*B-8/3*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-8/3*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*B-4*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-4*a/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*B+2*a/c^2/f*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [B] time = 0.44, size = 456, normalized size = 6.33

$$2 \left(Ba \left(\frac{\frac{9 \sin(fx+e)}{\cos(fx+e)+1} - \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 4}{c^2 - \frac{3c^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c^2} \right) - \frac{Aa \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2 \right)}{c^2 - \frac{3c^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{c^2 - \frac{3c^2 \sin(fx+e)}{\cos(fx+e)+1}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*(B*a*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + A*a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*a*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f
```

mupad [B] time = 12.51, size = 132, normalized size = 1.83

$$\frac{Bax \left(\frac{a(6A-6B+9B(e+fx))}{3} - 3Ba(e+fx) \right) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + \left(\frac{a(24B-9B(e+fx))}{3} + 3Ba(e+fx) \right) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{c^2} - \frac{c^2 f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1 \right)^3}{c^2 f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^2,x)
```

```
[Out] (B*a*x)/c^2 - ((a*(2*A - 10*B + 3*B*(e + f*x)))/3 + tan(e/2 + (f*x)/2)^2*((a*(6*A - 6*B + 9*B*(e + f*x)))/3 - 3*B*a*(e + f*x)) + tan(e/2 + (f*x)/2)*((a*(24*B - 9*B*(e + f*x)))/3 + 3*B*a*(e + f*x)) - B*a*(e + f*x))/(c^2*f*(tan(e/2 + (f*x)/2) - 1)^3)
```


sympy [A] time = 8.11, size = 700, normalized size = 9.72

$$\left\{ \begin{array}{l} \frac{6Aa \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3c^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) - 9c^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9c^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3c^2f} - \frac{2Aa}{3c^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) - 9c^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9c^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3c^2f} + \frac{x(A+B \sin(e))(a \sin(e)+a)}{(-c \sin(e)+c)^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)

[Out] Piecewise((-6*A*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 2*A*a/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 3*B*a*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 9*B*a*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 9*B*a*f*x*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 3*B*a*f*x/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 6*B*a*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 24*B*a*tan(e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 10*B*a/(3*c**2*f*tan(e/2 + f*x/2)**3 - 9*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**2, True))

$$3.23 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=104

$$-\frac{a(A-4B) \cos(e+fx)}{15f(c^3-c^3 \sin(e+fx))} - \frac{ac(A+11B) \cos(e+fx)}{15f(c^2-c^2 \sin(e+fx))^2} + \frac{2a(A+B) \cos(e+fx)}{5f(c-c \sin(e+fx))^3}$$

[Out] $2/5*a*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))^3-1/15*a*(A+11*B)*c*\cos(f*x+e)/f/(c^2-c^2*\sin(f*x+e))^2-1/15*a*(A-4*B)*\cos(f*x+e)/f/(c^3-c^3*\sin(f*x+e))$

Rubi [A] time = 0.24, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2967, 2857, 2750, 2648}

$$-\frac{a(A-4B) \cos(e+fx)}{15f(c^3-c^3 \sin(e+fx))} - \frac{ac(A+11B) \cos(e+fx)}{15f(c^2-c^2 \sin(e+fx))^2} + \frac{2a(A+B) \cos(e+fx)}{5f(c-c \sin(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] $(2*a*(A+B)*\text{Cos}[e+f*x])/(5*f*(c-c*\text{Sin}[e+f*x])^3) - (a*(A+11*B)*c*\text{Cos}[e+f*x])/(15*f*(c^2-c^2*\text{Sin}[e+f*x])^2) - (a*(A-4*B)*\text{Cos}[e+f*x])/(15*f*(c^3-c^3*\text{Sin}[e+f*x]))$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx = (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^4} dx$$

$$= \frac{2a(A + B) \cos(e + fx)}{5f(c - c \sin(e + fx))^3} + \frac{a \int \frac{-Ac - 6Bc - 5Bc \sin(e + fx)}{(c - c \sin(e + fx))^2} dx}{5c^2}$$

$$= \frac{2a(A + B) \cos(e + fx)}{5f(c - c \sin(e + fx))^3} - \frac{a(A + 11B) \cos(e + fx)}{15cf(c - c \sin(e + fx))^2} - \frac{a(A - 4B)}{15f(c^3 - c^2 \sin(e + fx))}$$

$$= \frac{2a(A + B) \cos(e + fx)}{5f(c - c \sin(e + fx))^3} - \frac{a(A + 11B) \cos(e + fx)}{15cf(c - c \sin(e + fx))^2} - \frac{a(A - 4B)}{15f(c^3 - c^2 \sin(e + fx))}$$

Mathematica [A] time = 0.74, size = 147, normalized size = 1.41

$$\frac{a \left(15(A - B) \cos\left(e + \frac{fx}{2}\right) - 5(A - B) \cos\left(e + \frac{3fx}{2}\right) + A \sin\left(2e + \frac{5fx}{2}\right) + 5A \sin\left(\frac{fx}{2}\right) + 15B \sin\left(2e + \frac{3fx}{2}\right) - 4B \sin\left(\frac{fx}{2}\right) \right)}{30c^3 f \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3, x]

[Out] (a*(15*(A - B)*Cos[e + (f*x)/2] - 5*(A - B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f*x)/2] + 25*B*Sin[(f*x)/2] + 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x)/2] - 4*B*Sin[2*e + (5*f*x)/2]))/(30*c^3*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)

fricas [A] time = 0.41, size = 183, normalized size = 1.76

$$\frac{(A - 4B)a \cos^3(fx + e) - (2A + 7B)a \cos^2(fx + e) + 3(A + B)a \cos(fx + e) + 6(A + B)a + ((A - 4B)a \cos^3(fx + e) - (2A + 7B)a \cos^2(fx + e) + 3(A + B)a \cos(fx + e) + 6(A + B)a)}{15 \left(c^3 f \cos^3(fx + e) + 3c^3 f \cos^2(fx + e) - 2c^3 f \cos(fx + e) - 4c^3 f - \left(c^3 f \cos^3(fx + e) + 3c^3 f \cos^2(fx + e) - 2c^3 f \cos(fx + e) - 4c^3 f - \left(c^3 f \cos^3(fx + e) + 3c^3 f \cos^2(fx + e) - 2c^3 f \cos(fx + e) - 4c^3 f \right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*((A - 4*B)*a*cos(f*x + e)^3 - (2*A + 7*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*cos(f*x + e) + 6*(A + B)*a + ((A - 4*B)*a*cos(f*x + e)^2 + 3*(A + B)*a*cos(f*x + e) + 6*(A + B)*a)*sin(f*x + e))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))

giac [A] time = 0.18, size = 139, normalized size = 1.34

$$\frac{2 \left(15 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 15 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 15 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 25 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 25 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 \right)}{15 c^3 f \left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] $-2/15*(15*A*a*\tan(1/2*f*x + 1/2*e)^4 - 15*A*a*\tan(1/2*f*x + 1/2*e)^3 + 15*B*a*\tan(1/2*f*x + 1/2*e)^3 + 25*A*a*\tan(1/2*f*x + 1/2*e)^2 + 5*B*a*\tan(1/2*f*x + 1/2*e)^2 - 5*A*a*\tan(1/2*f*x + 1/2*e) + 5*B*a*\tan(1/2*f*x + 1/2*e) + 4*A*a - B*a)/(c^3*f*(\tan(1/2*f*x + 1/2*e) - 1)^5)$

maple [A] time = 0.48, size = 115, normalized size = 1.11

$$2a \left(\frac{\frac{14A+10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{8A+8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}}{f c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)`

[Out] $2/f*a/c^3*(-1/3*(14*A+10*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/5*(8*A+8*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/4*(16*A+16*B)/(\tan(1/2*f*x+1/2*e)-1)^4-A/(\tan(1/2*f*x+1/2*e)-1)-1/2*(6*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2)$

maxima [B] time = 0.53, size = 737, normalized size = 7.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")`

[Out] $-2/15*(A*a*(20*\sin(f*x + e)/(\cos(f*x + e) + 1) - 40*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 3*A*a*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 3*B*a*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 2*B*a*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5))/f$

mupad [B] time = 12.98, size = 172, normalized size = 1.65

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{11Aa \cos(e+fx)}{2} - \frac{Ba}{4} - \frac{41Aa}{4} + \frac{Ba \cos(e+fx)}{2} + 5Aa \sin(e+fx) - 5Ba \sin(e+fx) + \frac{3Aa \cos(2e+fx)}{4} \right) \\ \frac{15c^3 f \left(\frac{5\sqrt{2} \cos\left(\frac{3e}{2} - \frac{\pi}{4} + \frac{3fx}{2}\right)}{4} - \frac{5\sqrt{2} \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{2} + \frac{\sqrt{2} \cos\left(\frac{5e}{2} + \frac{3fx}{2}\right)}{4} \right)}{15c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^3,x)`

[Out] $(2*\cos(e/2 + (f*x)/2)*((11*A*a*\cos(e + f*x))/2 - (B*a)/4 - (41*A*a)/4 + (B*a*\cos(e + f*x))/2 + 5*A*a*\sin(e + f*x) - 5*B*a*\sin(e + f*x) + (3*A*a*\cos(2e + f*x))/2) + 15*c^3*f*\left(\frac{5*\sqrt{2}*\cos\left(\frac{3e}{2} - \frac{\pi}{4} + \frac{3fx}{2}\right)}{4} - \frac{5*\sqrt{2}*\cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{2} + \frac{\sqrt{2}*\cos\left(\frac{5e}{2} + \frac{3fx}{2}\right)}{4}\right))/15$

$$\frac{e + 2fx}{4} + \frac{3B \cos(2e + 2fx)}{4} - \frac{5A \sin(2e + 2fx)}{4} + \frac{5B \sin(2e + 2fx)}{4} \Big/ \left(\frac{15c^3 f \left((5\sqrt{2}) \cos\left(\frac{3e}{2} - \frac{\pi}{4} + \frac{3fx}{2}\right) \right)}{4} - \frac{5\sqrt{2} \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{2} + \frac{2\sqrt{2} \cos\left(\frac{5e}{2} + \frac{\pi}{4} + \frac{5fx}{2}\right)}{4} \right)$$

sympy [A] time = 15.78, size = 1035, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)

[Out] Piecewise((-30*A*a*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 30*A*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 50*A*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 10*A*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 8*A*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 30*B*a*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 10*B*a*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 2*B*a/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**3, True))

$$3.24 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=142

$$\frac{a(2A-5B) \cos(e+fx)}{105f(c^4-c^4 \sin(e+fx))} - \frac{a(2A-5B) \cos(e+fx)}{105f(c^2-c^2 \sin(e+fx))^2} - \frac{a(A+15B) \cos(e+fx)}{35cf(c-c \sin(e+fx))^3} + \frac{2a(A+B) \cos(e+fx)}{7f(c-c \sin(e+fx))^4}$$

[Out] 2/7*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^4-1/35*a*(A+15*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^3-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^2-1/105*a*(2*A-5*B)*cos(f*x+e)/f/(c^4-c^4*sin(f*x+e))

Rubi [A] time = 0.29, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2967, 2857, 2750, 2650, 2648}

$$\frac{a(2A-5B) \cos(e+fx)}{105f(c^4-c^4 \sin(e+fx))} - \frac{a(2A-5B) \cos(e+fx)}{105f(c^2-c^2 \sin(e+fx))^2} - \frac{a(A+15B) \cos(e+fx)}{35cf(c-c \sin(e+fx))^3} + \frac{2a(A+B) \cos(e+fx)}{7f(c-c \sin(e+fx))^4}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]

[Out] (2*a*(A + B)*Cos[e + f*x])/(7*f*(c - c*Sin[e + f*x])^4) - (a*(A + 15*B)*Cos[e + f*x])/(35*c*f*(c - c*Sin[e + f*x])^3) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^2 - c^2*Sin[e + f*x])^2) - (a*(2*A - 5*B)*Cos[e + f*x])/(105*f*(c^4 - c^4*Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^4} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^5} dx \\ &= \frac{2a(A + B) \cos(e + fx)}{7f(c - c \sin(e + fx))^4} + \frac{a \int \frac{-Ac - 8Bc - 7Bc \sin(e + fx)}{(c - c \sin(e + fx))^3} dx}{7c^2} \\ &= \frac{2a(A + B) \cos(e + fx)}{7f(c - c \sin(e + fx))^4} - \frac{a(A + 15B) \cos(e + fx)}{35cf(c - c \sin(e + fx))^3} - \frac{a(2A - 5B)}{105f(c^2 - c^2 \sin^2(e + fx))} \\ &= \frac{2a(A + B) \cos(e + fx)}{7f(c - c \sin(e + fx))^4} - \frac{a(A + 15B) \cos(e + fx)}{35cf(c - c \sin(e + fx))^3} - \frac{a(2A - 5B)}{105f(c^2 - c^2 \sin^2(e + fx))} \\ &= \frac{2a(A + B) \cos(e + fx)}{7f(c - c \sin(e + fx))^4} - \frac{a(A + 15B) \cos(e + fx)}{35cf(c - c \sin(e + fx))^3} - \frac{a(2A - 5B)}{105f(c^2 - c^2 \sin^2(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.90, size = 174, normalized size = 1.23

$$\frac{a \left(35(4A - B) \cos\left(e + \frac{fx}{2}\right) + 14A \sin\left(2e + \frac{5fx}{2}\right) - 42A \cos\left(e + \frac{3fx}{2}\right) + 2A \cos\left(3e + \frac{7fx}{2}\right) + 70A \sin\left(\frac{fx}{2}\right) + 140B \sin\left(\frac{fx}{2}\right) \right)}{420c^4 f \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4, x]
```

```
[Out] (a*(35*(4*A - B)*Cos[e + (f*x)/2] - 42*A*Cos[e + (3*f*x)/2] + 2*A*Cos[3*e + (7*f*x)/2] - 5*B*Cos[3*e + (7*f*x)/2] + 70*A*Sin[(f*x)/2] + 140*B*Sin[(f*x)/2] + 105*B*Sin[2*e + (3*f*x)/2] + 14*A*Sin[2*e + (5*f*x)/2] - 35*B*Sin[2*e + (5*f*x)/2]))/(420*c^4*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)
```

fricas [A] time = 0.42, size = 251, normalized size = 1.77

$$\frac{(2A - 5B)a \cos^4(fx + e) + 4(2A - 5B)a \cos^3(fx + e) - 3(3A + 10B)a \cos^2(fx + e) + 15(A + B)a \cos(fx + e)}{105(c^4 f \cos^4(fx + e) - 3c^4 f \cos^3(fx + e) - 8c^4 f \cos^2(fx + e) + 4c^4 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/105*((2*A - 5*B)*a*cos(f*x + e)^4 + 4*(2*A - 5*B)*a*cos(f*x + e)^3 - 3*(3*A + 10*B)*a*cos(f*x + e)^2 + 15*(A + B)*a*cos(f*x + e) + 30*(A + B)*a - ((2*A - 5*B)*a*cos(f*x + e)^3 - 3*(2*A - 5*B)*a*cos(f*x + e)^2 - 15*(A + B)*a*cos(f*x + e) - 30*(A + B)*a)*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f
```

$\cos(fx + e)^3 - 8c^4f\cos(fx + e)^2 + 4c^4f^2\cos(fx + e) + 8c^4f^3 + (c^4f^3\cos(fx + e)^3 + 4c^4f^2\cos(fx + e)^2 - 4c^4f\cos(fx + e) - 8c^4f^3)\sin(fx + e)$

giac [A] time = 0.20, size = 187, normalized size = 1.32

$$2 \left(105 Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 210 Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 105 Ba \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 455 Aa \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] $-2/105*(105*A*a*\tan(1/2*f*x + 1/2*e)^6 - 210*A*a*\tan(1/2*f*x + 1/2*e)^5 + 105*B*a*\tan(1/2*f*x + 1/2*e)^5 + 455*A*a*\tan(1/2*f*x + 1/2*e)^4 - 35*B*a*\tan(1/2*f*x + 1/2*e)^4 - 350*A*a*\tan(1/2*f*x + 1/2*e)^3 + 140*B*a*\tan(1/2*f*x + 1/2*e)^3 + 273*A*a*\tan(1/2*f*x + 1/2*e)^2 - 56*A*a*\tan(1/2*f*x + 1/2*e) + 35*B*a*\tan(1/2*f*x + 1/2*e) + 23*A*a - 5*B*a)/(c^4*f*(\tan(1/2*f*x + 1/2*e) - 1)^7)$

maple [A] time = 0.47, size = 159, normalized size = 1.12

$$2a \left(\frac{28A+14B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{68A+60B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{16A+16B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{8A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{48A+48B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{56A+56B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} \right) / f c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)

[Out] $2/f*a/c^4*(-1/3*(28*A+14*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/5*(68*A+60*B)/(\tan(1/2*f*x+1/2*e)-1)^5-A/(\tan(1/2*f*x+1/2*e)-1)-1/7*(16*A+16*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/2*(8*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/6*(48*A+48*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/4*(56*A+40*B)/(\tan(1/2*f*x+1/2*e)-1)^4)$

maxima [B] time = 0.39, size = 1080, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out] $2/105*(A*a*(91*\sin(f*x + e)/(\cos(f*x + e) + 1) - 168*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 280*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 175*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 105*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + B*a*(91*\sin(f*x + e)/(\cos(f*x + e) + 1) - 168*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 280*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 175*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 105*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7)$

$$e) + 1)^7) - 3Aa(49\sin(fx + e)/(\cos(fx + e) + 1) - 147\sin(fx + e)^2/(\cos(fx + e) + 1)^2 + 210\sin(fx + e)^3/(\cos(fx + e) + 1)^3 - 210\sin(fx + e)^4/(\cos(fx + e) + 1)^4 + 105\sin(fx + e)^5/(\cos(fx + e) + 1)^5 - 35\sin(fx + e)^6/(\cos(fx + e) + 1)^6 - 12)/(c^4 - 7c^4\sin(fx + e)/(\cos(fx + e) + 1) + 21c^4\sin(fx + e)^2/(\cos(fx + e) + 1)^2 - 35c^4\sin(fx + e)^3/(\cos(fx + e) + 1)^3 + 35c^4\sin(fx + e)^4/(\cos(fx + e) + 1)^4 - 21c^4\sin(fx + e)^5/(\cos(fx + e) + 1)^5 + 7c^4\sin(fx + e)^6/(\cos(fx + e) + 1)^6 - c^4\sin(fx + e)^7/(\cos(fx + e) + 1)^7) - 4Ba(14\sin(fx + e)/(\cos(fx + e) + 1) - 42\sin(fx + e)^2/(\cos(fx + e) + 1)^2 + 35\sin(fx + e)^3/(\cos(fx + e) + 1)^3 - 35\sin(fx + e)^4/(\cos(fx + e) + 1)^4 - 2)/(c^4 - 7c^4\sin(fx + e)/(\cos(fx + e) + 1) + 21c^4\sin(fx + e)^2/(\cos(fx + e) + 1)^2 - 35c^4\sin(fx + e)^3/(\cos(fx + e) + 1)^3 + 35c^4\sin(fx + e)^4/(\cos(fx + e) + 1)^4 - 21c^4\sin(fx + e)^5/(\cos(fx + e) + 1)^5 + 7c^4\sin(fx + e)^6/(\cos(fx + e) + 1)^6 - c^4\sin(fx + e)^7/(\cos(fx + e) + 1)^7))/f$$

mupad [B] time = 13.22, size = 228, normalized size = 1.61

$$\frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{15Ba}{4} - \frac{171Aa}{2} + \frac{353Aa \cos(e+fx)}{8} + \frac{5Ba \cos(e+fx)}{4} + \frac{595Aa \sin(e+fx)}{8} - 35Ba \sin(e+fx) + \frac{43Aa \cos(2e+2fx)}{2} - \frac{25Aa \cos(3e+3fx)}{8} - \frac{5Ba \cos(2e+2fx)}{4} + \frac{5Ba \cos(3e+3fx)}{4} - \frac{77Aa \sin(2e+2fx)}{4} - \frac{21Aa \sin(3e+3fx)}{8} + \frac{35Ba \sin(2e+2fx)}{4} \right)}{105c^4 f \left(\frac{35\sqrt{2} \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{8} - \frac{21\sqrt{2} \cos\left(\frac{7e}{2} - \frac{\pi}{4} + \frac{7fx}{2}\right)}{8} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^4,x)

[Out] $-(2\cos(e/2 + (fx)/2))*((15*B*a)/4 - (171*A*a)/2 + (353*A*a*\cos(e + f*x))/8 + (5*B*a*\cos(e + f*x))/4 + (595*A*a*\sin(e + f*x))/8 - 35*B*a*\sin(e + f*x) + (43*A*a*\cos(2*e + 2*f*x))/2 - (25*A*a*\cos(3*e + 3*f*x))/8 - (5*B*a*\cos(2*e + 2*f*x))/4 + (5*B*a*\cos(3*e + 3*f*x))/4 - (77*A*a*\sin(2*e + 2*f*x))/4 - (21*A*a*\sin(3*e + 3*f*x))/8 + (35*B*a*\sin(2*e + 2*f*x))/4)/(105*c^4*f*((35*2^(1/2)*\cos(e/2 + \pi/4 + (fx)/2))/8 - (21*2^(1/2)*\cos((3*e)/2 - \pi/4 + (3*f*x)/2))/8 - (7*2^(1/2)*\cos((5*e)/2 + \pi/4 + (5*f*x)/2))/8 + (2^(1/2)*\cos((7*e)/2 - \pi/4 + (7*f*x)/2))/8))$

sympy [A] time = 29.47, size = 1831, normalized size = 12.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)

[Out] $\text{Piecewise}((-210*A*a*\tan(e/2 + fx/2)**6/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f) + 420*A*a*\tan(e/2 + fx/2)**5/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f) - 910*A*a*\tan(e/2 + fx/2)**4/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f) + 700*A*a*\tan(e/2 + fx/2)**3/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f) - 546*A*a*\tan(e/2 + fx/2)**2/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f) - 105*c**4*f/(105*c**4*f*\tan(e/2 + fx/2)**7 - 735*c**4*f*\tan(e/2 + fx/2)**6 + 2205*c**4*f*\tan(e/2 + fx/2)**5 - 3675*c**4*f*\tan(e/2 + fx/2)**4 + 3675*c**4*f*\tan(e/2 + fx/2)**3 - 2205*c**4*f*\tan(e/2 + fx/2)**2 + 735*c**4*f*\tan(e/2 + fx/2) - 105*c**4*f))$

```

)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 112*A*a*tan(e/2 + f*x/2)/(105*c**4*f*t
an(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)
**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c
**4*f) - 46*A*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/
2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c*
**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 210*B*a*tan(e/2 + f*x/2)**5/(105*c**4
*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e
/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*
x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 1
05*c**4*f) + 70*B*a*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 7
35*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4
*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(
e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 280*B*a*tan(e
/2 + f*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 70*B*a*tan(e/2 + f*x/2)/(105*c**4*f*ta
n(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 +
f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)
**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c
**4*f) + 10*B*a/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2
)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 +
3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**
4*f*tan(e/2 + f*x/2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e)
+ a)/(-c*sin(e) + c)**4, True))

```

$$3.25 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx$$

Optimal. Leaf size=176

$$\frac{2a(A-2B) \cos(e+fx)}{315f(c^5-c^5 \sin(e+fx))} - \frac{2ac(A-2B) \cos(e+fx)}{315f(c^3-c^3 \sin(e+fx))^2} - \frac{ac(A-2B) \cos(e+fx)}{105f(c^2-c^2 \sin(e+fx))^3} - \frac{a(A+19B) \cos(e+fx)}{63cf(c-c \sin(e+fx))}$$

[Out] 2/9*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^5-1/63*a*(A+19*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^4-1/105*a*(A-2*B)*c*cos(f*x+e)/f/(c^2-c^2*sin(f*x+e))^3-2/315*a*(A-2*B)*c*cos(f*x+e)/f/(c^3-c^3*sin(f*x+e))^2-2/315*a*(A-2*B)*cos(f*x+e)/f/(c^5-c^5*sin(f*x+e))

Rubi [A] time = 0.31, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2967, 2857, 2750, 2650, 2648}

$$\frac{2a(A-2B) \cos(e+fx)}{315f(c^5-c^5 \sin(e+fx))} - \frac{2ac(A-2B) \cos(e+fx)}{315f(c^3-c^3 \sin(e+fx))^2} - \frac{ac(A-2B) \cos(e+fx)}{105f(c^2-c^2 \sin(e+fx))^3} - \frac{a(A+19B) \cos(e+fx)}{63cf(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]

[Out] (2*a*(A + B)*Cos[e + f*x])/(9*f*(c - c*Sin[e + f*x])^5) - (a*(A + 19*B)*Cos[e + f*x])/(63*c*f*(c - c*Sin[e + f*x])^4) - (a*(A - 2*B)*c*Cos[e + f*x])/(105*f*(c^2 - c^2*Sin[e + f*x])^3) - (2*a*(A - 2*B)*c*Cos[e + f*x])/(315*f*(c^3 - c^3*Sin[e + f*x])^2) - (2*a*(A - 2*B)*Cos[e + f*x])/(315*f*(c^5 - c^5*Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^5} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^6} dx \\ &= \frac{2a(A + B) \cos(e + fx)}{9f(c - c \sin(e + fx))^5} + \frac{a \int \frac{-Ac - 10Bc - 9Bc \sin(e + fx)}{(c - c \sin(e + fx))^4} dx}{9c^2} \\ &= \frac{2a(A + B) \cos(e + fx)}{9f(c - c \sin(e + fx))^5} - \frac{a(A + 19B) \cos(e + fx)}{63cf(c - c \sin(e + fx))^4} - \frac{a(A - 2B)}{105c^2f(c - c \sin(e + fx))^3} \\ &= \frac{2a(A + B) \cos(e + fx)}{9f(c - c \sin(e + fx))^5} - \frac{a(A + 19B) \cos(e + fx)}{63cf(c - c \sin(e + fx))^4} - \frac{a(A - 2B)}{105c^2f(c - c \sin(e + fx))^3} \\ &= \frac{2a(A + B) \cos(e + fx)}{9f(c - c \sin(e + fx))^5} - \frac{a(A + 19B) \cos(e + fx)}{63cf(c - c \sin(e + fx))^4} - \frac{a(A - 2B)}{105c^2f(c - c \sin(e + fx))^3} \\ &= \frac{2a(A + B) \cos(e + fx)}{9f(c - c \sin(e + fx))^5} - \frac{a(A + 19B) \cos(e + fx)}{63cf(c - c \sin(e + fx))^4} - \frac{a(A - 2B)}{105c^2f(c - c \sin(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 0.89, size = 200, normalized size = 1.14

$$\frac{a \left(-42(2A + B) \cos\left(e + \frac{3fx}{2}\right) + 36A \sin\left(2e + \frac{5fx}{2}\right) - A \sin\left(4e + \frac{9fx}{2}\right) + 315A \cos\left(e + \frac{fx}{2}\right) + 9A \cos\left(3e + \frac{7fx}{2}\right) \right)}{1260c^5f \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5, x]
```

```
[Out] (a*(315*A*Cos[e + (f*x)/2] - 42*(2*A + B)*Cos[e + (3*f*x)/2] + 9*A*Cos[3*e + (7*f*x)/2] - 18*B*Cos[3*e + (7*f*x)/2] + 189*A*Sin[(f*x)/2] + 252*B*Sin[(f*x)/2] + 210*B*Sin[2*e + (3*f*x)/2] + 36*A*Sin[2*e + (5*f*x)/2] - 72*B*Sin[2*e + (5*f*x)/2] - A*Sin[4*e + (9*f*x)/2] + 2*B*Sin[4*e + (9*f*x)/2]))/(1260*c^5*f*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9)
```

fricas [A] time = 0.43, size = 305, normalized size = 1.73

$$\frac{2(A - 2B)a \cos^5(fx + e) - 8(A - 2B)a \cos^4(fx + e) - 25(A - 2B)a \cos^3(fx + e) + 5(4A + 13B)a \cos^2(fx + e) - 20c^5f \cos(fx + e)}{315 \left(c^5f \cos^5(fx + e) + 5c^5f \cos^4(fx + e) - 8c^5f \cos^3(fx + e) - 20c^5f \cos^2(fx + e) - 20c^5f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(2*(A - 2*B)*a*cos(f*x + e)^5 - 8*(A - 2*B)*a*cos(f*x + e)^4 - 25*(A
- 2*B)*a*cos(f*x + e)^3 + 5*(4*A + 13*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*c
os(f*x + e) - 70*(A + B)*a + (2*(A - 2*B)*a*cos(f*x + e)^4 + 10*(A - 2*B)*a
*cos(f*x + e)^3 - 15*(A - 2*B)*a*cos(f*x + e)^2 - 35*(A + B)*a*cos(f*x + e)
- 70*(A + B)*a)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)
^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e
) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*co
s(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [A] time = 0.21, size = 267, normalized size = 1.52

$$2 \left(315 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 - 945 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 315 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 2625 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 315 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 3465 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 945 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 3843 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 441 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 2247 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 609 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 1143 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 81 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 207 A a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 99 B a \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 58 A a - 11 B a \right) / (c^5 f (\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1)^9)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="giac")
```

```
[Out] -2/315*(315*A*a*tan(1/2*f*x + 1/2*e)^8 - 945*A*a*tan(1/2*f*x + 1/2*e)^7 + 3
15*B*a*tan(1/2*f*x + 1/2*e)^7 + 2625*A*a*tan(1/2*f*x + 1/2*e)^6 - 315*B*a*t
an(1/2*f*x + 1/2*e)^6 - 3465*A*a*tan(1/2*f*x + 1/2*e)^5 + 945*B*a*tan(1/2*f
*x + 1/2*e)^5 + 3843*A*a*tan(1/2*f*x + 1/2*e)^4 - 441*B*a*tan(1/2*f*x + 1/2
*e)^4 - 2247*A*a*tan(1/2*f*x + 1/2*e)^3 + 609*B*a*tan(1/2*f*x + 1/2*e)^3 +
1143*A*a*tan(1/2*f*x + 1/2*e)^2 - 81*B*a*tan(1/2*f*x + 1/2*e)^2 - 207*A*a*t
an(1/2*f*x + 1/2*e) + 99*B*a*tan(1/2*f*x + 1/2*e) + 58*A*a - 11*B*a)/(c^5*f
*(tan(1/2*f*x + 1/2*e) - 1)^9)
```

maple [A] time = 0.51, size = 203, normalized size = 1.15

$$2a \left(-\frac{46A+18B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{128A+128B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8} - \frac{236A+168B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{32A+32B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{10A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{24A+24B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} \right) / f c^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)
```

```
[Out] 2/f*a/c^5*(-1/3*(46*A+18*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(128*A+128*B)/(tan
(1/2*f*x+1/2*e)-1)^8-1/5*(236*A+168*B)/(tan(1/2*f*x+1/2*e)-1)^5-A/(tan(1/2*
f*x+1/2*e)-1)-1/9*(32*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/2*(10*A+2*B)/(tan(
1/2*f*x+1/2*e)-1)^2-1/7*(248*A+232*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(296*A+2
48*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(128*A+72*B)/(tan(1/2*f*x+1/2*e)-1)^4)
```

maxima [B] time = 0.40, size = 1425, normalized size = 8.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm
="maxima")
```

```
[Out] -2/315*(A*a*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x
+ e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 33
60*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*
```

$$c^5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 126c^5 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 126c^5 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 + 84c^5 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 - 36c^5 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 + 9c^5 \sin(fx + e)^8 / (\cos(fx + e) + 1)^8 - c^5 \sin(fx + e)^9 / (\cos(fx + e) + 1)^9 - 5Aa(45 \sin(fx + e) / (\cos(fx + e) + 1) - 117 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 273 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 - 315 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + 315 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 - 147 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 + 63 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 - 5) / (c^5 - 9c^5 \sin(fx + e) / (\cos(fx + e) + 1) + 36c^5 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 - 84c^5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 126c^5 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 126c^5 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 + 84c^5 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 - 36c^5 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 + 9c^5 \sin(fx + e)^8 / (\cos(fx + e) + 1)^8 - c^5 \sin(fx + e)^9 / (\cos(fx + e) + 1)^9) - 5Ba(45 \sin(fx + e) / (\cos(fx + e) + 1) - 117 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 273 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 - 315 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + 315 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 - 147 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 + 63 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 - 5) / (c^5 - 9c^5 \sin(fx + e) / (\cos(fx + e) + 1) + 36c^5 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 - 84c^5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 126c^5 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 126c^5 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 + 84c^5 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 - 36c^5 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 + 9c^5 \sin(fx + e)^8 / (\cos(fx + e) + 1)^8 - c^5 \sin(fx + e)^9 / (\cos(fx + e) + 1)^9) + 14Ba(9 \sin(fx + e) / (\cos(fx + e) + 1) - 36 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 54 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 - 81 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + 45 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 - 30 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 - 1) / (c^5 - 9c^5 \sin(fx + e) / (\cos(fx + e) + 1) + 36c^5 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 - 84c^5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 126c^5 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 - 126c^5 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 + 84c^5 \sin(fx + e)^6 / (\cos(fx + e) + 1)^6 - 36c^5 \sin(fx + e)^7 / (\cos(fx + e) + 1)^7 + 9c^5 \sin(fx + e)^8 / (\cos(fx + e) + 1)^8 - c^5 \sin(fx + e)^9 / (\cos(fx + e) + 1)^9) / f$$

mupad [B] time = 13.34, size = 310, normalized size = 1.76

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{1357Aa}{4} - \frac{461Ba}{16} - \frac{635Aa \cos(e+fx)}{4} + \frac{5Ba \cos(e+fx)}{2} - \frac{1575Aa \sin(e+fx)}{4} + \frac{945Ba \sin(e+fx)}{8} - \frac{625Aa \cos(2e+2fx)}{4} + \frac{121Aa \cos(3e+3fx)}{4} + \frac{7Aa \cos(4e+4fx)}{2} + \frac{95Ba \cos(2e+2fx)}{4} - \frac{8Ba \cos(3e+3fx)}{4} - \frac{7Ba \cos(4e+4fx)}{16} + \frac{399Aa \sin(2e+2fx)}{4} + \frac{141Aa \sin(3e+3fx)}{4} - \frac{15Aa \sin(4e+4fx)}{4} - \frac{231Ba \sin(2e+2fx)}{4} - \frac{39Ba \sin(3e+3fx)}{8} + \frac{15Ba \sin(4e+4fx)}{16} \right) / (315c^5 f ((63 \cdot 2^{1/2} \cos(e/2 + \pi/4 + (fx)/2)) / 8 - (21 \cdot 2^{1/2} \cos((3e)/2 - \pi/4 + (3fx)/2)) / 4 - (9 \cdot 2^{1/2} \cos((5e)/2 + \pi/4 + (5fx)/2)) / 4 + (9 \cdot 2^{1/2} \cos((7e)/2 - \pi/4 + (7fx)/2)) / 16 + (2^{1/2} \cos((9e)/2 + \pi/4 + (9fx)/2)) / 16)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^5,x)

[Out] (2*cos(e/2 + (f*x)/2)*((1357*A*a)/4 - (461*B*a)/16 - (635*A*a*cos(e + f*x))/4 + (5*B*a*cos(e + f*x))/2 - (1575*A*a*sin(e + f*x))/4 + (945*B*a*sin(e + f*x))/8 - (625*A*a*cos(2*e + 2*f*x))/4 + (121*A*a*cos(3*e + 3*f*x))/4 + (7*A*a*cos(4*e + 4*f*x))/2 + (95*B*a*cos(2*e + 2*f*x))/4 - 8*B*a*cos(3*e + 3*f*x) - (7*B*a*cos(4*e + 4*f*x))/16 + (399*A*a*sin(2*e + 2*f*x))/4 + (141*A*a*sin(3*e + 3*f*x))/4 - (15*A*a*sin(4*e + 4*f*x))/4 - (231*B*a*sin(2*e + 2*f*x))/4 - (39*B*a*sin(3*e + 3*f*x))/8 + (15*B*a*sin(4*e + 4*f*x))/16))/(315*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/16 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))

sympy [A] time = 52.38, size = 3232, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.


```

f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan
(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1218*B*a*tan(e/2
+ f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)
**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 +
39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 2646
0*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5
*f*tan(e/2 + f*x/2) - 315*c**5*f) + 162*B*a*tan(e/2 + f*x/2)**2/(315*c**5*f
*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e
/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 +
f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)
)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 31
5*c**5*f) - 198*B*a*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835
*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5
*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*ta
n(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2
+ f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 22*B*a/(315*c**
5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*ta
n(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2
+ f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*
x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) -
315*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)/(-c*sin(e) + c)**
5, True))

```


$$3.26 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c - c \sin(e + fx))^5 dx$$

Optimal. Leaf size=229

$$\frac{3a^2c^5(8A - 3B) \cos^5(e + fx)}{80f} + \frac{3a^2(8A - 3B) \cos^5(e + fx) (c^5 - c^5 \sin(e + fx))}{112f} + \frac{3a^2c^5(8A - 3B) \sin(e + fx) c}{64f}$$

[Out] $9/128*a^2*(8*A-3*B)*c^5*x+3/80*a^2*(8*A-3*B)*c^5*\cos(f*x+e)^5/f+9/128*a^2*(8*A-3*B)*c^5*\cos(f*x+e)*\sin(f*x+e)/f+3/64*a^2*(8*A-3*B)*c^5*\cos(f*x+e)^3*\sin(f*x+e)/f+1/56*a^2*(8*A-3*B)*c^3*\cos(f*x+e)^5*(c-c*\sin(f*x+e))^2/f-1/8*a^2*B*c^2*\cos(f*x+e)^5*(c-c*\sin(f*x+e))^3/f+3/112*a^2*(8*A-3*B)*\cos(f*x+e)^5*(c^5-c^5*\sin(f*x+e))/f$

Rubi [A] time = 0.37, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{3a^2c^5(8A - 3B) \cos^5(e + fx)}{80f} + \frac{3a^2c^5(8A - 3B) \sin(e + fx) \cos^3(e + fx)}{64f} + \frac{a^2c^3(8A - 3B) \cos^5(e + fx)(c - c \sin(e + fx))}{56f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]

[Out] $(9*a^2*(8*A - 3*B)*c^5*x)/128 + (3*a^2*(8*A - 3*B)*c^5*\cos[e + f*x]^5)/(80*f) + (9*a^2*(8*A - 3*B)*c^5*\cos[e + f*x]*\sin[e + f*x])/(128*f) + (3*a^2*(8*A - 3*B)*c^5*\cos[e + f*x]^3*\sin[e + f*x])/(64*f) + (a^2*(8*A - 3*B)*c^3*\cos[e + f*x]^5*(c - c*\sin[e + f*x])^2)/(56*f) - (a^2*B*c^2*\cos[e + f*x]^5*(c - c*\sin[e + f*x])^3)/(8*f) + (3*a^2*(8*A - 3*B)*\cos[e + f*x]^5*(c^5 - c^5*\sin[e + f*x]))/(112*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^5 dx = (a^2 c^2) \int \cos^4(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^5 dx$$

$$= -\frac{a^2 B c^2 \cos^5(e + fx) (c - c \sin(e + fx))^3}{8f} + \frac{1}{8} \left(\frac{a^2 (8A - 3B) c^3 \cos^5(e + fx) (c - c \sin(e + fx))^2}{56f} - \frac{a^2 (8A - 3B) c^3 \cos^5(e + fx) (c - c \sin(e + fx))^2}{56f} \right)$$

$$= \frac{3a^2 (8A - 3B) c^5 \cos^5(e + fx)}{80f} + \frac{a^2 (8A - 3B) c^3}{80f}$$

$$= \frac{3a^2 (8A - 3B) c^5 \cos^5(e + fx)}{80f} + \frac{3a^2 (8A - 3B)}{80f}$$

$$= \frac{3a^2 (8A - 3B) c^5 \cos^5(e + fx)}{80f} + \frac{9a^2 (8A - 3B)}{80f}$$

$$= \frac{9}{128} a^2 (8A - 3B) c^5 x + \frac{3a^2 (8A - 3B) c^5 \cos^5(e + fx)}{80f}$$

Mathematica [A] time = 2.04, size = 219, normalized size = 0.96

$$\frac{(a \sin(e + fx) + a)^2 (c - c \sin(e + fx))^5 (2520(8A - 3B)(e + fx) + 560(19A - 3B) \sin(2(e + fx)) - 280(2A - 7B) \cos(2(e + fx)))}{(35840 f (\cos((e + fx)/2) - \sin((e + fx)/2))^{10} (\cos((e + fx)/2) + \sin((e + fx)/2))^4}$$

358

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5, x]
```

```
[Out] ((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5*(2520*(8*A - 3*B)*(e + f*x) + 560*(27*A - 17*B)*Cos[e + f*x] + 560*(13*A - 7*B)*Cos[3*(e + f*x)] + 112*(11*A - B)*Cos[5*(e + f*x)] - 80*(A - 3*B)*Cos[7*(e + f*x)] + 560*(19*A - 3*B)*Sin[2*(e + f*x)] - 280*(2*A - 7*B)*Sin[4*(e + f*x)] - 560*(A - B)*Sin[6*(e + f*x)] - 35*B*Sin[8*(e + f*x)]))/(35840*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```

fricas [A] time = 0.46, size = 158, normalized size = 0.69

$$640(A - 3B)a^2c^5 \cos(fx + e)^7 - 3584(A - B)a^2c^5 \cos(fx + e)^5 - 315(8A - 3B)a^2c^5 fx + 35(16Ba^2c^5 \cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] -1/4480*(640*(A - 3*B)*a^2*c^5*cos(f*x + e)^7 - 3584*(A - B)*a^2*c^5*cos(f*x + e)^5 - 315*(8*A - 3*B)*a^2*c^5*f*x + 35*(16*B*a^2*c^5*cos(f*x + e)^7 + 8*(8*A - 11*B)*a^2*c^5*cos(f*x + e)^5 - 6*(8*A - 3*B)*a^2*c^5*cos(f*x + e)^3 - 9*(8*A - 3*B)*a^2*c^5*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.23, size = 278, normalized size = 1.21

$$-\frac{Ba^2c^5 \sin(8fx + 8e)}{1024f} + \frac{9}{128}(8Aa^2c^5 - 3Ba^2c^5)x - \frac{(Aa^2c^5 - 3Ba^2c^5) \cos(7fx + 7e)}{448f} + \frac{(11Aa^2c^5 - Ba^2c^5) \cos(5fx + 5e)}{320f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] -1/1024*B*a^2*c^5*sin(8*f*x + 8*e)/f + 9/128*(8*A*a^2*c^5 - 3*B*a^2*c^5)*x - 1/448*(A*a^2*c^5 - 3*B*a^2*c^5)*cos(7*f*x + 7*e)/f + 1/320*(11*A*a^2*c^5 - B*a^2*c^5)*cos(5*f*x + 5*e)/f + 1/64*(13*A*a^2*c^5 - 7*B*a^2*c^5)*cos(3*f*x + 3*e)/f + 1/64*(27*A*a^2*c^5 - 17*B*a^2*c^5)*cos(f*x + e)/f - 1/64*(A*a^2*c^5 - B*a^2*c^5)*sin(6*f*x + 6*e)/f - 1/128*(2*A*a^2*c^5 - 7*B*a^2*c^5)*sin(4*f*x + 4*e)/f + 1/64*(19*A*a^2*c^5 - 3*B*a^2*c^5)*sin(2*f*x + 2*e)/f

maple [B] time = 0.67, size = 569, normalized size = 2.48

$$a^2Ac^5 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - \frac{Ba^2c^5(2+\sin^2(fx+e))\cos(fx+e)}{3} + 3a^2Ac^5 \cos(fx+e) + a^2Ac^5 (fx+e) - E$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)

[Out] 1/f*(a^2*A*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c^5*cos(f*x+e)+a^2*A*c^5*(f*x+e)-B*a^2*c^5*cos(f*x+e)-3*B*a^2*c^5*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+1/7*a^2*A*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+3*a^2*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/5*a^2*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-5*a^2*A*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-5/3*a^2*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-3/7*B*a^2*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-B*a^2*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+B*a^2*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+5*B*a^2*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))

maxima [B] time = 0.54, size = 571, normalized size = 2.49

$$3072 \left(5 \cos(fx + e)^7 - 21 \cos(fx + e)^5 + 35 \cos(fx + e)^3 - 35 \cos(fx + e) \right) Aa^2c^5 - 7168 \left(3 \cos(fx + e) \right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out] -1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*A*a^2*c^5 - 7168*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^5 - 179200*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^5 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^5 + 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^5 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^5 - 107520*(f*x + e)*A*a^2*c^5 - 9216*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^5 - 35840*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^5 - 35840*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^5 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^2*c^5 + 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^5 - 16800*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^5 + 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^5 - 322560*A*a^2*c^5*cos(f*x + e) + 107520*B*a^2*c^5*cos(f*x + e))/f

mupad [B] time = 15.13, size = 661, normalized size = 2.89

$$\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} (6Aa^2c^5 - 2Ba^2c^5) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} (30Aa^2c^5 - 10Ba^2c^5) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} (22Aa^2c^5 - 18Ba^2c^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5,x)

[Out] (tan(e/2 + (f*x)/2)^14*(6*A*a^2*c^5 - 2*B*a^2*c^5) + tan(e/2 + (f*x)/2)^10*(30*A*a^2*c^5 - 10*B*a^2*c^5) + tan(e/2 + (f*x)/2)^12*(22*A*a^2*c^5 - 18*B*a^2*c^5) + tan(e/2 + (f*x)/2)^8*(46*A*a^2*c^5 - 26*B*a^2*c^5) + tan(e/2 + (f*x)/2)^4*((74*A*a^2*c^5)/5 - (14*B*a^2*c^5)/5) - tan(e/2 + (f*x)/2)^15*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^2*((158*A*a^2*c^5)/35 - (138*B*a^2*c^5)/35) + tan(e/2 + (f*x)/2)^6*((218*A*a^2*c^5)/5 - (158*B*a^2*c^5)/5) + tan(e/2 + (f*x)/2)^3*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^13*((75*A*a^2*c^5)/8 - (305*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^5*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^11*((55*A*a^2*c^5)/8 - (437*B*a^2*c^5)/64) - tan(e/2 + (f*x)/2)^7*((13*A*a^2*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)^9*((13*A*a^2*c^5)/8 - (919*B*a^2*c^5)/64) + tan(e/2 + (f*x)/2)*((7*A*a^2*c^5)/8 + (27*B*a^2*c^5)/64) + (46*A*a^2*c^5)/35 - (26*B*a^2*c^5)/35)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e/2 + (f*x)/2)^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*tan(e/2 + (f*x)/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*x)/2)^16 + 1)) + (9*a^2*c^5*atan((9*a^2*c^5*tan(e/2 + (f*x)/2)*(8*A - 3*B))/(64*((9*A*a^2*c^5)/8 - (27*B*a^2*c^5)/64)))*(8*A - 3*B))/(64*f)

sympy [A] time = 25.00, size = 1586, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)

[Out] Piecewise((15*A*a**2*c**5*x*sin(e + f*x)**6/16 + 45*A*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 - 15*A*a**2*c**5*x*sin(e + f*x)**4/8 + 45*A*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 15*A*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**2*c**5*x*sin(e + f*x)**2/2 + 15*A*a**2*c**5*x*cos(e + f*x)**6/16 - 15*A*a**2*c**5*x*cos(e + f*x)**4/8 + A*a**2*c**5*x*cos(e + f*x)**2/2 + A*a**2*c**5*x + A*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f - 33*A*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + A*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) + 25*A*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 4*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*A*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 15*A*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**5*cos(e + f*x)**7/(35*f) + 8*A*a**2*c**5*cos(e + f*x)**5/(15*f) - 10*A*a**2*c**5*cos(e + f*x)**3/(3*f) + 3*A*a**2*c**5*cos(e + f*x)/f - 35*B*a**2*c**5*x*sin(e + f*x)**8/128 - 35*B*a**2*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 5*B*a**2*c**5*x*sin(e + f*x)**6/16 - 105*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 15*B*a**2*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 15*B*a**2*c**5*x*sin(e + f*x)**4/8 - 35*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 15*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 15*B*a**2*c**5*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - 3*B*a**2*c**5*x*sin(e + f*x)**2/2 - 35*B*a**2*c**5*x*cos(e + f*x)**8/128 - 5*B*a**2*c**5*x*cos(e + f*x)**6/16 + 15*B*a**2*c**5*x*cos(e + f*x)**4/8 - 3*B*a**2*c**5*x*cos(e + f*x)**2/2 + 93*B*a**2*c**5*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 3*B*a**2*c**5*sin(e + f*x)**6*cos(e + f*x)/f + 511*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 11*B*a**2*c**5*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 6*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + 5*B*a**2*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 25*B*a**2*c**5*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 24*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 20*B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B*a**2*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 5*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 15*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*B*a**2*c**5*sin(e + f*x)*cos(e + f*x)/(2*f) - 48*B*a**2*c**5*cos(e + f*x)**7/(35*f) + 8*B*a**2*c**5*cos(e + f*x)**5/(3*f) - 2*B*a**2*c**5*cos(e + f*x)**3/(3*f) - B*a**2*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**5, True))

$$3.27 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^4 dx$$

Optimal. Leaf size=189

$$\frac{a^2 c^4 (7A - 2B) \cos^5(e + fx)}{30f} + \frac{a^2 (7A - 2B) \cos^5(e + fx) (c^4 - c^4 \sin(e + fx))}{42f} + \frac{a^2 c^4 (7A - 2B) \sin(e + fx) \cos^3(e + fx)}{24f}$$

[Out] 1/16*a^2*(7*A-2*B)*c^4*x+1/30*a^2*(7*A-2*B)*c^4*cos(f*x+e)^5/f+1/16*a^2*(7*A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(7*A-2*B)*c^4*cos(f*x+e)^3*sin(f*x+e)/f-1/7*a^2*B*cos(f*x+e)^5*(c^2-c^2*sin(f*x+e))^2/f+1/42*a^2*(7*A-2*B)*cos(f*x+e)^5*(c^4-c^4*sin(f*x+e))/f

Rubi [A] time = 0.30, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^2 c^4 (7A - 2B) \cos^5(e + fx)}{30f} + \frac{a^2 (7A - 2B) \cos^5(e + fx) (c^4 - c^4 \sin(e + fx))}{42f} + \frac{a^2 c^4 (7A - 2B) \sin(e + fx) \cos^3(e + fx)}{24f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]

[Out] (a^2*(7*A - 2*B)*c^4*x)/16 + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^5)/(30*f) + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(7*A - 2*B)*c^4*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^2 - c^2*Sin[e + f*x])^2)/(7*f) + (a^2*(7*A - 2*B)*Cos[e + f*x]^5*(c^4 - c^4*Sin[e + f*x]))/(42*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^4 dx &= (a^2 c^2) \int \cos^4(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^4 dx \\ &= -\frac{a^2 B \cos^5(e + fx) (c^2 - c^2 \sin(e + fx))^2}{7f} + \frac{a^2 B \cos^5(e + fx) (c^2 - c^2 \sin(e + fx))^2}{7f} \\ &= \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} - \frac{a^2 B \cos^5(e + fx)}{30f} \\ &= \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} + \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} \\ &= \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} + \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} \\ &= \frac{1}{16} a^2 (7A - 2B) c^4 x + \frac{a^2 (7A - 2B) c^4 \cos^5(e + fx)}{30f} \end{aligned}$$

Mathematica [A] time = 1.59, size = 163, normalized size = 0.86

$$\frac{a^2 c^4 (105(16A - 11B) \cos(e + fx) + 105(8A - 5B) \cos(3(e + fx)) + 1785A \sin(2(e + fx)) + 105A \sin(4(e + fx)))}{6720f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4, x]
```

```
[Out] (a^2*c^4*(2940*A*e - 840*B*e + 2940*A*f*x - 840*B*f*x + 105*(16*A - 11*B)*Cos[e + f*x] + 105*(8*A - 5*B)*Cos[3*(e + f*x)] + 168*A*Cos[5*(e + f*x)] - 63*B*Cos[5*(e + f*x)] + 15*B*Cos[7*(e + f*x)] + 1785*A*Sin[2*(e + f*x)] - 210*B*Sin[2*(e + f*x)] + 105*A*Sin[4*(e + f*x)] + 210*B*Sin[4*(e + f*x)] - 35*A*Sin[6*(e + f*x)] + 70*B*Sin[6*(e + f*x)])/(6720*f)
```

fricas [A] time = 0.45, size = 135, normalized size = 0.71

$$\frac{240 B a^2 c^4 \cos^7(fx + e) + 672 (A - B) a^2 c^4 \cos^5(fx + e) + 105 (7A - 2B) a^2 c^4 fx - 35 (8(A - 2B) a^2 c^4 \cos^5(fx + e) + 105A \sin(2(e + fx)) + 105B \sin(4(e + fx)))}{6720f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] 1/1680*(240*B*a^2*c^4*cos(f*x + e)^7 + 672*(A - B)*a^2*c^4*cos(f*x + e)^5 + 105*(7*A - 2*B)*a^2*c^4*f*x - 35*(8*(A - 2*B))*a^2*c^4*cos(f*x + e)^5 - 2*(7*A - 2*B)*a^2*c^4*cos(f*x + e)^3 - 3*(7*A - 2*B)*a^2*c^4*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.21, size = 244, normalized size = 1.29

$$\frac{Ba^2c^4 \cos(7fx + 7e)}{448f} + \frac{1}{16} (7Aa^2c^4 - 2Ba^2c^4)x + \frac{(8Aa^2c^4 - 3Ba^2c^4) \cos(5fx + 5e)}{320f} + \frac{(8Aa^2c^4 - 5Ba^2c^4) \cos(3fx + 3e)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/448*B*a^2*c^4*cos(7*f*x + 7*e)/f + 1/16*(7*A*a^2*c^4 - 2*B*a^2*c^4)*x + 1/320*(8*A*a^2*c^4 - 3*B*a^2*c^4)*cos(5*f*x + 5*e)/f + 1/64*(8*A*a^2*c^4 - 5*B*a^2*c^4)*cos(3*f*x + 3*e)/f + 1/64*(16*A*a^2*c^4 - 11*B*a^2*c^4)*cos(f*x + e)/f - 1/192*(A*a^2*c^4 - 2*B*a^2*c^4)*sin(6*f*x + 6*e)/f + 1/64*(A*a^2*c^4 + 2*B*a^2*c^4)*sin(4*f*x + 4*e)/f + 1/64*(17*A*a^2*c^4 - 2*B*a^2*c^4)*sin(2*f*x + 2*e)/f

maple [B] time = 0.57, size = 463, normalized size = 2.45

$$a^2A^4(fx + e) + 4Ba^2c^4 \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + \frac{Ba^2c^4(2+\sin^2(fx+e))\cos(fx+e)}{3} + 2a^2Ac^4 \cos(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)

[Out] 1/f*(a^2*A*c^4*(f*x+e)+4*B*a^2*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/3*B*a^2*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^2*A*c^4*cos(f*x+e)-2*B*a^2*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-4/3*a^2*A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-1/7*B*a^2*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^2*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+1/5*B*a^2*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c^4*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^2*A*c^4*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+2/5*a^2*A*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^4*cos(f*x+e))

maxima [B] time = 0.34, size = 460, normalized size = 2.43

$$\frac{896 \left(3 \cos(fx + e)^5 - 10 \cos(fx + e)^3 + 15 \cos(fx + e) \right) Aa^2c^4 + 8960 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) Aa^2c^4}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")


```
[Out] 1/6720*(896*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^2*c^4 + 8960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c^4 + 35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^2*c^4 - 210*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*c^4 - 1680*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^4 + 6720*(f*x + e)*A*a^2*c^4 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^2*c^4 + 448*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*c^4 - 2240*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^4 - 70*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^2*c^4 + 840*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c^4 - 3360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^4 + 13440*A*a^2*c^4*cos(f*x + e) - 6720*B*a^2*c^4*cos(f*x + e))/f
```

mupad [B] time = 14.89, size = 553, normalized size = 2.93

$$\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} (4Aa^2c^4 - 2Ba^2c^4) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 (12Aa^2c^4 - 2Ba^2c^4) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{10} (8Aa^2c^4 - 8Ba^2c^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^4,x)
[Out] (tan(e/2 + (f*x)/2)^12*(4*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^8*(12*A*a^2*c^4 - 2*B*a^2*c^4) + tan(e/2 + (f*x)/2)^10*(8*A*a^2*c^4 - 8*B*a^2*c^4) + tan(e/2 + (f*x)/2)^2*((8*A*a^2*c^4)/5 - (8*B*a^2*c^4)/5) - tan(e/2 + (f*x)/2)^13*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + tan(e/2 + (f*x)/2)^6*(16*A*a^2*c^4 - 16*B*a^2*c^4) + tan(e/2 + (f*x)/2)^3*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)/3) - tan(e/2 + (f*x)/2)^11*((29*A*a^2*c^4)/6 - (11*B*a^2*c^4)/3) + tan(e/2 + (f*x)/2)^4*((44*A*a^2*c^4)/5 - (14*B*a^2*c^4)/5) + tan(e/2 + (f*x)/2)^5*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) - tan(e/2 + (f*x)/2)^9*((23*A*a^2*c^4)/24 + (31*B*a^2*c^4)/12) + tan(e/2 + (f*x)/2)*((9*A*a^2*c^4)/8 + (B*a^2*c^4)/4) + (4*A*a^2*c^4)/5 - (18*B*a^2*c^4)/35)/(f*(7*tan(e/2 + (f*x)/2)^2 + 21*tan(e/2 + (f*x)/2)^4 + 35*tan(e/2 + (f*x)/2)^6 + 35*tan(e/2 + (f*x)/2)^8 + 21*tan(e/2 + (f*x)/2)^10 + 7*tan(e/2 + (f*x)/2)^12 + tan(e/2 + (f*x)/2)^14 + 1)) + (a^2*c^4*atan((a^2*c^4*tan(e/2 + (f*x)/2)*(7*A - 2*B))/(8*((7*A*a^2*c^4)/8 - (B*a^2*c^4)/4)))*(7*A - 2*B))/(8*f)
```

sympy [A] time = 15.02, size = 1210, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
[Out] Piecewise((5*A*a**2*c**4*x*sin(e + f*x)**6/16 + 15*A*a**2*c**4*x*sin(e + f*x)**4*cos(e + f*x)**2/16 - 3*A*a**2*c**4*x*sin(e + f*x)**4/8 + 15*A*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*A*a**2*c**4*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - A*a**2*c**4*x*sin(e + f*x)**2/2 + 5*A*a**2*c**4*x*cos(e + f*x)**6/16 - 3*A*a**2*c**4*x*cos(e + f*x)**4/8 - A*a**2*c**4*x*cos(e + f*x)**2/2 + A*a**2*c**4*x - 11*A*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 2*A*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) + 5*A*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 4*A*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*A*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*A*a**2*c**4*cos(e + f*x)**5/(15*f) - 8*A*a**2*c**4*cos(e + f*x)**3/(3*f) + 2*A*a**2*c**4*cos(e + f*x)/f - 5*B*a**2*c
```

```

**4*x**sin(e + f*x)**6/8 - 15*B*a**2*c**4*x**sin(e + f*x)**4*cos(e + f*x)**2/
8 + 3*B*a**2*c**4*x**sin(e + f*x)**4/2 - 15*B*a**2*c**4*x**sin(e + f*x)**2*cos
(e + f*x)**4/8 + 3*B*a**2*c**4*x**sin(e + f*x)**2*cos(e + f*x)**2 - B*a**2*c
**4*x**sin(e + f*x)**2 - 5*B*a**2*c**4*x*cos(e + f*x)**6/8 + 3*B*a**2*c**4*
x*cos(e + f*x)**4/2 - B*a**2*c**4*x*cos(e + f*x)**2 - B*a**2*c**4*sin(e + f
*x)**6*cos(e + f*x)/f + 11*B*a**2*c**4*sin(e + f*x)**5*cos(e + f*x)/(8*f) -
2*B*a**2*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f + B*a**2*c**4*sin(e + f*x)
**4*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) -
5*B*a**2*c**4*sin(e + f*x)**3*cos(e + f*x)/(2*f) - 8*B*a**2*c**4*sin(e + f*
x)**2*cos(e + f*x)**5/(5*f) + 4*B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)**3
/(3*f) + B*a**2*c**4*sin(e + f*x)**2*cos(e + f*x)/f + 5*B*a**2*c**4*sin(e +
f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**2*c**4*sin(e + f*x)*cos(e + f*x)**3/(2
*f) + B*a**2*c**4*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**2*c**4*cos(e + f*x)
**7/(35*f) + 8*B*a**2*c**4*cos(e + f*x)**5/(15*f) + 2*B*a**2*c**4*cos(e + f
*x)**3/(3*f) - B*a**2*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*
sin(e) + a)**2*(-c*sin(e) + c)**4, True))

```

$$3.28 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^3 dx$$

Optimal. Leaf size=147

$$\frac{a^2 c^3 (6A - B) \cos^5(e + fx)}{30f} + \frac{a^2 c^3 (6A - B) \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{a^2 c^3 (6A - B) \sin(e + fx) \cos(e + fx)}{16f} + \frac{1}{16}$$

[Out] 1/16*a^2*(6*A-B)*c^3*x+1/30*a^2*(6*A-B)*c^3*cos(f*x+e)^5/f+1/16*a^2*(6*A-B)*c^3*cos(f*x+e)*sin(f*x+e)/f+1/24*a^2*(6*A-B)*c^3*cos(f*x+e)^3*sin(f*x+e)/f-1/6*a^2*B*cos(f*x+e)^5*(c^3-c^3*sin(f*x+e))/f

Rubi [A] time = 0.22, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^2 c^3 (6A - B) \cos^5(e + fx)}{30f} + \frac{a^2 c^3 (6A - B) \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{a^2 c^3 (6A - B) \sin(e + fx) \cos(e + fx)}{16f} + \frac{1}{16}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]

[Out] (a^2*(6*A - B)*c^3*x)/16 + (a^2*(6*A - B)*c^3*Cos[e + f*x]^5)/(30*f) + (a^2*(6*A - B)*c^3*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^2*(6*A - B)*c^3*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (a^2*B*Cos[e + f*x]^5*(c^3 - c^3*Sin[e + f*x]))/(6*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2860

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^3 dx &= (a^2 c^2) \int \cos^4(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^3 dx \\ &= -\frac{a^2 B \cos^5(e + fx) (c^3 - c^3 \sin(e + fx))}{6f} + \frac{1}{6} (a^2 c^3 \cos^5(e + fx) - a^2 B \cos^5(e + fx)) \\ &= \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} - \frac{a^2 B \cos^5(e + fx)}{6f} \\ &= \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} + \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} \\ &= \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} + \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} \\ &= \frac{1}{16} a^2 (6A - B) c^3 x + \frac{a^2 (6A - B) c^3 \cos^5(e + fx)}{30f} \end{aligned}$$

Mathematica [A] time = 1.09, size = 137, normalized size = 0.93

$$\frac{a^2 c^3 (120(A - B) \cos(e + fx) + 60(A - B) \cos(3(e + fx)) + 240A \sin(2(e + fx)) + 30A \sin(4(e + fx)) + 12A \cos(6(e + fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^2*c^3*(360*A*e - 60*B*e + 360*A*f*x - 60*B*f*x + 120*(A - B)*Cos[e + f*x]
+ 60*(A - B)*Cos[3*(e + f*x)] + 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] - 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] + 15*B*Sin[4*(e + f*x)] + 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 114, normalized size = 0.78

$$\frac{48(A - B)a^2 c^3 \cos^5(fx + e) + 15(6A - B)a^2 c^3 fx + 5(8Ba^2 c^3 \cos^5(fx + e) + 2(6A - B)a^2 c^3 \cos^3(fx + e) + 3(6A - B)a^2 c^3 \cos(fx + e)) \sin(fx + e)}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorit
hm="fricas")
```

```
[Out] 1/240*(48*(A - B)*a^2*c^3*cos(f*x + e)^5 + 15*(6*A - B)*a^2*c^3*f*x + 5*(8*
B*a^2*c^3*cos(f*x + e)^5 + 2*(6*A - B)*a^2*c^3*cos(f*x + e)^3 + 3*(6*A - B)
*a^2*c^3*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 208, normalized size = 1.41

$$\frac{Ba^2 c^3 \sin(6fx + 6e)}{192f} + \frac{1}{16} (6Aa^2 c^3 - Ba^2 c^3) x + \frac{(Aa^2 c^3 - Ba^2 c^3) \cos(5fx + 5e)}{80f} + \frac{(Aa^2 c^3 - Ba^2 c^3) \cos(3fx + 3e)}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] $\frac{1}{192}B*a^2*c^3*\sin(6*f*x + 6*e)/f + \frac{1}{16}*(6*A*a^2*c^3 - B*a^2*c^3)*x + \frac{1}{8}0*(A*a^2*c^3 - B*a^2*c^3)*\cos(5*f*x + 5*e)/f + \frac{1}{16}*(A*a^2*c^3 - B*a^2*c^3)*\cos(3*f*x + 3*e)/f + \frac{1}{8}*(A*a^2*c^3 - B*a^2*c^3)*\cos(f*x + e)/f + \frac{1}{64}*(2*A*a^2*c^3 + B*a^2*c^3)*\sin(4*f*x + 4*e)/f + \frac{1}{64}*(16*A*a^2*c^3 - B*a^2*c^3)*\sin(2*f*x + 2*e)/f$

maple [B] time = 0.57, size = 365, normalized size = 2.48

$$\frac{a^2Ac^3\left(\frac{8}{3}+\sin^4(fx+e)+\frac{4(\sin^2(fx+e))}{3}\right)\cos(fx+e)}{5} + a^2Ac^3\left(-\frac{\left(\sin^3(fx+e)+\frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) - \frac{2a^2Ac^3(2+\sin^2(fx+e))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

[Out] $\frac{1}{f}*(\frac{1}{5}a^2Ac^3*(\frac{8}{3}+\sin(f*x+e)^4+\frac{4}{3}\sin(f*x+e)^2)*\cos(f*x+e)+a^2Ac^3*(-\frac{1}{4}*(\sin(f*x+e)^3+\frac{3}{2}\sin(f*x+e))*\cos(f*x+e)+\frac{3}{8}f*x+\frac{3}{8}e)-\frac{2}{3}a^2Ac^3*(2+\sin(f*x+e)^2)*\cos(f*x+e)-2a^2Ac^3*(-\frac{1}{2}\sin(f*x+e)*\cos(f*x+e)+\frac{1}{2}f*x+\frac{1}{2}e)-B*a^2*c^3*(-\frac{1}{6}*(\sin(f*x+e)^5+\frac{5}{4}\sin(f*x+e)^3+\frac{15}{8}\sin(f*x+e))*\cos(f*x+e)+\frac{5}{16}f*x+\frac{5}{16}e)-\frac{1}{5}B*a^2*c^3*(\frac{8}{3}+\sin(f*x+e)^4+\frac{4}{3}\sin(f*x+e)^2)*\cos(f*x+e)+2B*a^2*c^3*(-\frac{1}{4}*(\sin(f*x+e)^3+\frac{3}{2}\sin(f*x+e))*\cos(f*x+e)+\frac{3}{8}f*x+\frac{3}{8}e)+\frac{2}{3}B*a^2*c^3*(2+\sin(f*x+e)^2)*\cos(f*x+e)+a^2Ac^3*\cos(f*x+e)-B*a^2*c^3*(-\frac{1}{2}\sin(f*x+e)*\cos(f*x+e)+\frac{1}{2}f*x+\frac{1}{2}e)+a^2Ac^3*(f*x+e)-B*a^2*c^3*\cos(f*x+e))$

maxima [B] time = 0.47, size = 360, normalized size = 2.45

$$64\left(3\cos(fx+e)^5 - 10\cos(fx+e)^3 + 15\cos(fx+e)\right)Aa^2c^3 + 640\left(\cos(fx+e)^3 - 3\cos(fx+e)\right)Aa^2c^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{960}*(64*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a^2*c^3 + 640*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^2*c^3 + 30*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^2*c^3 - 480*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 - 64*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a^2*c^3 - 640*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2*c^3 - 5*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B*a^2*c^3 + 60*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^2*c^3 - 240*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^2*c^3 + 960*A*a^2*c^3*\cos(f*x + e) - 960*B*a^2*c^3*\cos(f*x + e))/f$

mupad [B] time = 14.15, size = 542, normalized size = 3.69

$$\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 (4Aa^2c^3 - 4Ba^2c^3) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 (2Aa^2c^3 - 2Ba^2c^3) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 (4Aa^2c^3 - 4Ba^2c^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3,x)
[Out] (tan(e/2 + (f*x)/2)^4*(4*A*a^2*c^3 - 4*B*a^2*c^3) + tan(e/2 + (f*x)/2)^8*(2
*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c^3 - 4*B*a^2*c^3
) + tan(e/2 + (f*x)/2)^10*(2*A*a^2*c^3 - 2*B*a^2*c^3) + tan(e/2 + (f*x)/2)^
2*((2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5) + tan(e/2 + (f*x)/2)^5*((A*a^2*c^3)/2
+ (13*B*a^2*c^3)/4) - tan(e/2 + (f*x)/2)^7*((A*a^2*c^3)/2 + (13*B*a^2*c^3)
/4) - tan(e/2 + (f*x)/2)^11*((5*A*a^2*c^3)/4 + (B*a^2*c^3)/8) + tan(e/2 + (
f*x)/2)^3*((7*A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) - tan(e/2 + (f*x)/2)^9*((7*
A*a^2*c^3)/4 - (47*B*a^2*c^3)/24) + tan(e/2 + (f*x)/2)*((5*A*a^2*c^3)/4 + (
B*a^2*c^3)/8) + (2*A*a^2*c^3)/5 - (2*B*a^2*c^3)/5)/(f*(6*tan(e/2 + (f*x)/2)
^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)
/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) + (a^2*c^3*at
an((a^2*c^3*tan(e/2 + (f*x)/2)*(6*A - B))/(8*((3*A*a^2*c^3)/4 - (B*a^2*c^3)
/8)))*(6*A - B))/(8*f) - (a^2*c^3*(6*A - B)*(atan(tan(e/2 + (f*x)/2)) - (f*
x)/2))/(8*f)
```

sympy [A] time = 9.14, size = 910, normalized size = 6.19

$$\left\{ \begin{array}{l} \frac{3Aa^2c^3x\sin^4(e+fx)}{8} + \frac{3Aa^2c^3x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^3x\sin^2(e+fx) + \frac{3Aa^2c^3x\cos^4(e+fx)}{8} - Aa^2c^3x\cos^2(e+fx) \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)
[Out] Piecewise(((3*A*a**2*c**3*x*sin(e + f*x)**4/8 + 3*A*a**2*c**3*x*sin(e + f*x)
**2*cos(e + f*x)**2/4 - A*a**2*c**3*x*sin(e + f*x)**2 + 3*A*a**2*c**3*x*cos
(e + f*x)**4/8 - A*a**2*c**3*x*cos(e + f*x)**2 + A*a**2*c**3*x + A*a**2*c**
3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**2*c**3*sin(e + f*x)**3*cos(e + f*
x)/(8*f) + 4*A*a**2*c**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 2*A*a**2*c
**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**2*c**3*sin(e + f*x)*cos(e + f*x)
**3/(8*f) + A*a**2*c**3*sin(e + f*x)*cos(e + f*x)/f + 8*A*a**2*c**3*cos(e
+ f*x)**5/(15*f) - 4*A*a**2*c**3*cos(e + f*x)**3/(3*f) + A*a**2*c**3*cos(e
+ f*x)/f - 5*B*a**2*c**3*x*sin(e + f*x)**6/16 - 15*B*a**2*c**3*x*sin(e + f*
x)**4*cos(e + f*x)**2/16 + 3*B*a**2*c**3*x*sin(e + f*x)**4/4 - 15*B*a**2*c*
**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**2*c**3*x*sin(e + f*x)**2*c
os(e + f*x)**2/2 - B*a**2*c**3*x*sin(e + f*x)**2/2 - 5*B*a**2*c**3*x*cos(e
+ f*x)**6/16 + 3*B*a**2*c**3*x*cos(e + f*x)**4/4 - B*a**2*c**3*x*cos(e + f*
x)**2/2 + 11*B*a**2*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**2*c**3*
sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)
**3/(6*f) - 5*B*a**2*c**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**2*c**
3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**2*c**3*sin(e + f*x)**2*cos
(e + f*x)/f + 5*B*a**2*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**2*
c**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**2*c**3*sin(e + f*x)*cos(e +
f*x)/(2*f) - 8*B*a**2*c**3*cos(e + f*x)**5/(15*f) + 4*B*a**2*c**3*cos(e + f
*x)**3/(3*f) - B*a**2*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a
sin(e) + a)**2*(-c*sin(e) + c)**3, True))
```

$$3.29 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^2 dx$$

Optimal. Leaf size=89

$$\frac{a^2 Ac^2 \sin(e + fx) \cos^3(e + fx)}{4f} + \frac{3a^2 Ac^2 \sin(e + fx) \cos(e + fx)}{8f} + \frac{3}{8} a^2 Ac^2 x - \frac{a^2 Bc^2 \cos^5(e + fx)}{5f}$$

[Out] $3/8*a^2*A*c^2*x-1/5*a^2*B*c^2*\cos(f*x+e)^5/f+3/8*a^2*A*c^2*\cos(f*x+e)*\sin(f*x+e)/f+1/4*a^2*A*c^2*\cos(f*x+e)^3*\sin(f*x+e)/f$

Rubi [A] time = 0.14, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^2 Ac^2 \sin(e + fx) \cos^3(e + fx)}{4f} + \frac{3a^2 Ac^2 \sin(e + fx) \cos(e + fx)}{8f} + \frac{3}{8} a^2 Ac^2 x - \frac{a^2 Bc^2 \cos^5(e + fx)}{5f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^2*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^2,x]$

[Out] $(3*a^2*A*c^2*x)/8 - (a^2*B*c^2*\text{Cos}[e + f*x]^5)/(5*f) + (3*a^2*A*c^2*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(8*f) + (a^2*A*c^2*\text{Cos}[e + f*x]^3*\text{Sin}[e + f*x])/(4*f)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2669

$\text{Int}[(\text{cos}[(e_*) + (f_*)*(x_)]*(g_*))^{(p_)*((a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)})/(f*g*(p+1)), x] + \text{Dist}[a, \text{Int}[(g*\text{Cos}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, e, f, g, p, x\} \&\& (\text{IntegerQ}[2*p] \parallel \text{NeQ}[a^2 - b^2, 0])$

Rule 2967

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_)]*(c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n, x\} \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \& \text{GtQ}[n, 0]) \parallel \text{LtQ}[0, n, m] \parallel \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^2 dx &= (a^2 c^2) \int \cos^4(e + fx) (A + B \sin(e + fx)) dx \\
&= -\frac{a^2 B c^2 \cos^5(e + fx)}{5f} + (a^2 A c^2) \int \cos^4(e + fx) dx \\
&= -\frac{a^2 B c^2 \cos^5(e + fx)}{5f} + \frac{a^2 A c^2 \cos^3(e + fx) \sin(e + fx)}{4f} \\
&= -\frac{a^2 B c^2 \cos^5(e + fx)}{5f} + \frac{3a^2 A c^2 \cos(e + fx) \sin^3(e + fx)}{8f} \\
&= \frac{3}{8} a^2 A c^2 x - \frac{a^2 B c^2 \cos^5(e + fx)}{5f} + \frac{3a^2 A c^2 \cos(e + fx) \sin^3(e + fx)}{8f}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 54, normalized size = 0.61

$$\frac{a^2 c^2 (5A(12(e + fx) + 8 \sin(2(e + fx)) + \sin(4(e + fx))) - 32B \cos^5(e + fx))}{160f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]

[Out] (a^2*c^2*(-32*B*Cos[e + f*x]^5 + 5*A*(12*(e + f*x) + 8*Sin[2*(e + f*x)] + Sin[4*(e + f*x)])))/(160*f)

fricas [A] time = 0.42, size = 75, normalized size = 0.84

$$\frac{8 B a^2 c^2 \cos(fx + e)^5 - 15 A a^2 c^2 fx - 5 \left(2 A a^2 c^2 \cos(fx + e)^3 + 3 A a^2 c^2 \cos(fx + e) \right) \sin(fx + e)}{40 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/40*(8*B*a^2*c^2*cos(f*x + e)^5 - 15*A*a^2*c^2*f*x - 5*(2*A*a^2*c^2*cos(f*x + e)^3 + 3*A*a^2*c^2*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.15, size = 118, normalized size = 1.33

$$\frac{3}{8} A a^2 c^2 x - \frac{B a^2 c^2 \cos(5fx + 5e)}{80f} - \frac{B a^2 c^2 \cos(3fx + 3e)}{16f} - \frac{B a^2 c^2 \cos(fx + e)}{8f} + \frac{A a^2 c^2 \sin(4fx + 4e)}{32f} + \frac{A a^2 c^2 \sin^3(fx + e)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] 3/8*A*a^2*c^2*x - 1/80*B*a^2*c^2*cos(5*f*x + 5*e)/f - 1/16*B*a^2*c^2*cos(3*f*x + 3*e)/f - 1/8*B*a^2*c^2*cos(f*x + e)/f + 1/32*A*a^2*c^2*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c^2*sin(2*f*x + 2*e)/f

maple [B] time = 0.48, size = 166, normalized size = 1.87

$$\frac{B a^2 c^2 \left(\frac{8}{3} + \sin^4(fx + e) + \frac{4 \sin^2(fx + e)}{3} \right) \cos(fx + e)}{5} + a^2 A c^2 \left(-\frac{\left(\sin^3(fx + e) + \frac{3 \sin(fx + e)}{2} \right) \cos(fx + e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + \frac{2B a^2 c^2 (2 + \sin^2(fx + e)) \cos(fx + e)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)`

[Out] $1/f*(-1/5*B*a^2*c^2*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)+a^2*A*c^2*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)+2/3*B*a^2*c^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)-2*a^2*A*c^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^2*\cos(f*x+e)+a^2*A*c^2*(f*x+e))$

maxima [B] time = 0.49, size = 164, normalized size = 1.84

$15(12fx + 12e + \sin(4fx + 4e) - 8\sin(2fx + 2e))Aa^2c^2 - 240(2fx + 2e - \sin(2fx + 2e))Aa^2c^2 + 480$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")`

[Out] $1/480*(15*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^2*c^2 - 240*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2 - 32*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a^2*c^2 - 320*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2*c^2 - 480*B*a^2*c^2*\cos(f*x + e))/f$

mupad [B] time = 14.21, size = 238, normalized size = 2.67

$$\frac{3Aa^2c^2x}{8} \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 \left(\frac{a^2c^2(80B-75A(e+fx))}{40} + \frac{15Aa^2c^2(e+fx)}{8}\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \left(\frac{a^2c^2(160B-150A(e+fx))}{40} + \frac{15Aa^2c^2(e+fx)}{8}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2,x)`

[Out] $(3*A*a^2*c^2*x)/8 - (\tan(e/2 + (f*x)/2)^8*((a^2*c^2*(80*B - 75*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/8) + \tan(e/2 + (f*x)/2)^4*((a^2*c^2*(160*B - 150*A*(e + f*x)))/40 + (15*A*a^2*c^2*(e + f*x))/4) + (a^2*c^2*(16*B - 15*A*(e + f*x)))/40 + (3*A*a^2*c^2*(e + f*x))/8 - (A*a^2*c^2*\tan(e/2 + (f*x)/2)^3)/2 + (A*a^2*c^2*\tan(e/2 + (f*x)/2)^7)/2 + (5*A*a^2*c^2*\tan(e/2 + (f*x)/2)^9)/4 - (5*A*a^2*c^2*\tan(e/2 + (f*x)/2))/4/(f*(\tan(e/2 + (f*x)/2)^2 + 1)^5)$

sympy [A] time = 3.70, size = 372, normalized size = 4.18

$$\left\{ \begin{array}{l} \frac{3Aa^2c^2x\sin^4(e+fx)}{8} + \frac{3Aa^2c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^2c^2x\sin^2(e+fx) + \frac{3Aa^2c^2x\cos^4(e+fx)}{8} - Aa^2c^2x\cos^2(e+fx) \\ x(A+B\sin(e))(a\sin(e)+a)^2(-c\sin(e)+c)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)`

[Out] `Piecewise((3*A*a**2*c**2*x*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)**4/8 + 3*A*a**2*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - A*a**2*c**2*x*sin(e + f*x)**2 + 3*A*a**2*c**2*x*cos(e + f*x)**4/8 - A*a**2*c**2*x*cos(e + f*x)**2 + A*a**2*c**2*x - 5*A*a**2*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*A*a**2*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**2*c**2*sin(e + f*x)*cos(e + f*x)/f - B*a**2*c**2*sin(e + f*x)**4/8), (0, 1))`

```

e + f*x)**4*cos(e + f*x)/f - 4*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)**3/
(3*f) + 2*B*a**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 8*B*a**2*c**2*cos(e
+ f*x)**5/(15*f) + 4*B*a**2*c**2*cos(e + f*x)**3/(3*f) - B*a**2*c**2*cos(e
+ f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2*(-c*sin(e) + c)**2
, True))

```

$$3.30 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx)) dx$$

Optimal. Leaf size=98

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f} + \frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{1}{8}a^2cx(4A+B) - \frac{Bc\cos^3(e+fx)(a^2\sin(e+fx))}{4f}$$

[Out] $1/8*a^2*(4*A+B)*c*x - 1/12*a^2*(4*A+B)*c*\cos(f*x+e)^3/f + 1/8*a^2*(4*A+B)*c*\cos(f*x+e)*\sin(f*x+e)/f - 1/4*B*c*\cos(f*x+e)^3*(a^2+a^2*\sin(f*x+e))/f$

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^2c(4A+B)\cos^3(e+fx)}{12f} + \frac{a^2c(4A+B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{1}{8}a^2cx(4A+B) - \frac{Bc\cos^3(e+fx)(a^2\sin(e+fx))}{4f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]

[Out] $(a^2*(4*A + B)*c*x)/8 - (a^2*(4*A + B)*c*\cos[e + f*x]^3)/(12*f) + (a^2*(4*A + B)*c*\cos[e + f*x]*\sin[e + f*x])/(8*f) - (B*c*\cos[e + f*x]^3*(a^2 + a^2*\sin[e + f*x]))/(4*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2860

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_))*((A_.) + (B_)*sin[(e_.) + (f_)*(x_)])*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]^(n_)), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c - c \sin(e + fx)) dx &= (ac) \int \cos^2(e + fx)(a + a \sin(e + fx))(A + B \sin(e + fx)) dx \\ &= -\frac{Bc \cos^3(e + fx)(a^2 + a^2 \sin(e + fx))}{4f} + \frac{1}{4}(a^2(4A + B)c \cos^3(e + fx) - Bc \cos^3(e + fx)) \\ &= -\frac{a^2(4A + B)c \cos^3(e + fx)}{12f} - \frac{Bc \cos^3(e + fx)}{4f} \\ &= -\frac{a^2(4A + B)c \cos^3(e + fx)}{12f} + \frac{a^2(4A + B)c \cos^3(e + fx)}{12f} \\ &= \frac{1}{8}a^2(4A + B)cx - \frac{a^2(4A + B)c \cos^3(e + fx)}{12f} + \frac{a^2(4A + B)c \cos^3(e + fx)}{12f} \end{aligned}$$

Mathematica [A] time = 0.80, size = 67, normalized size = 0.68

$$\frac{a^2c(24(A + B) \cos(e + fx) + 8(A + B) \cos(3(e + fx)) - 12fx(4A + B) - 24A \sin(2(e + fx)) + 3B \sin(4(e + fx)))}{96f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]), x]

[Out] -1/96*(a^2*c*(-12*(4*A + B)*f*x + 24*(A + B)*Cos[e + f*x] + 8*(A + B)*Cos[3*(e + f*x)] - 24*A*Sin[2*(e + f*x)] + 3*B*Sin[4*(e + f*x)]))/f

fricas [A] time = 0.43, size = 77, normalized size = 0.79

$$\frac{8(A + B)a^2c \cos(fx + e)^3 - 3(4A + B)a^2cfx + 3(2Ba^2c \cos(fx + e)^3 - (4A + B)a^2c \cos(fx + e)) \sin(fx + e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] -1/24*(8*(A + B)*a^2*c*cos(f*x + e)^3 - 3*(4*A + B)*a^2*c*f*x + 3*(2*B*a^2*c*cos(f*x + e)^3 - (4*A + B)*a^2*c*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.15, size = 111, normalized size = 1.13

$$-\frac{Ba^2c \sin(4fx + 4e)}{32f} + \frac{Aa^2c \sin(2fx + 2e)}{4f} + \frac{1}{8}(4Aa^2c + Ba^2c)x - \frac{(Aa^2c + Ba^2c) \cos(3fx + 3e)}{12f} - \frac{(Aa^2c + Ba^2c) \cos(fx + e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] -1/32*B*a^2*c*sin(4*f*x + 4*e)/f + 1/4*A*a^2*c*sin(2*f*x + 2*e)/f + 1/8*(4*A*a^2*c + B*a^2*c)*x - 1/12*(A*a^2*c + B*a^2*c)*cos(3*f*x + 3*e)/f - 1/4*(A*a^2*c + B*a^2*c)*cos(f*x + e)/f

maple [B] time = 0.39, size = 186, normalized size = 1.90

$$\frac{a^2 A c (2 + \sin^2(fx+e)) \cos(fx+e)}{3} - a^2 A c \left(-\frac{\sin(fx+e) \cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - B a^2 c \left(-\frac{\left(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] 1/f*(1/3*a^2*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)-a^2*A*c*cos(f*x+e)+B*a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*c*(f*x+e)-B*a^2*c*cos(f*x+e))

maxima [A] time = 0.37, size = 179, normalized size = 1.83

$$\frac{32 \left(\cos(fx+e)^3 - 3 \cos(fx+e) \right) A a^2 c + 24 (2fx + 2e - \sin(2fx + 2e)) A a^2 c - 96 (fx+e) A a^2 c + 32 (c \cos(fx+e) - c^2)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -1/96*(32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c - 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e)))*B*a^2*c - 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 96*A*a^2*c*cos(f*x + e) + 96*B*a^2*c*cos(f*x + e))/f

mupad [B] time = 13.70, size = 339, normalized size = 3.46

$$\frac{a^2 c \operatorname{atan} \left(\frac{a^2 c \tan \left(\frac{e}{2} + \frac{fx}{2} \right) (4A+B)}{4 \left(A a^2 c + \frac{B a^2 c}{4} \right)} \right) (4A+B)}{4f} - \frac{a^2 c (4A+B) \left(\operatorname{atan} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right) - \frac{fx}{2} \right) \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^4 (2Aa^2c + B a^2 c)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x)),x)

[Out] (a^2*c*atan((a^2*c*tan(e/2 + (f*x)/2)*(4*A + B))/(4*(A*a^2*c + (B*a^2*c)/4)))*(4*A + B))/(4*f) - (a^2*c*(4*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f) - (tan(e/2 + (f*x)/2)^4*(2*A*a^2*c + 2*B*a^2*c) - tan(e/2 + (f*x)/2)*(A*a^2*c - (B*a^2*c)/4) + tan(e/2 + (f*x)/2)^6*(2*A*a^2*c + 2*B*a^2*c) + tan(e/2 + (f*x)/2)^2*((2*A*a^2*c)/3 + (2*B*a^2*c)/3) + tan(e/2 + (f*x)/2)^7*(A*a^2*c - (B*a^2*c)/4) - tan(e/2 + (f*x)/2)^3*(A*a^2*c + (7*B*a^2*c)/4) + tan(e/2 + (f*x)/2)^5*(A*a^2*c + (7*B*a^2*c)/4) + (2*A*a^2*c)/3 + (2*B*a^2*c)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))

sympy [A] time = 2.43, size = 396, normalized size = 4.04

$$\left\{ \begin{array}{l} -\frac{A a^2 c x \sin^2(e+fx)}{2} - \frac{A a^2 c x \cos^2(e+fx)}{2} + A a^2 c x + \frac{A a^2 c \sin^2(e+fx) \cos(e+fx)}{f} + \frac{A a^2 c \sin(e+fx) \cos(e+fx)}{2f} + \frac{2 A a^2 c \cos^3(e+fx)}{3f} \\ x (A + B \sin(e)) (a \sin(e) + a)^2 (-c \sin(e) + c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-A*a**2*c*x*sin(e + f*x)**2/2 - A*a**2*c*x*cos(e + f*x)**2/2 + A
*a**2*c*x + A*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + A*a**2*c*sin(e + f*x)
*cos(e + f*x)/(2*f) + 2*A*a**2*c*cos(e + f*x)**3/(3*f) - A*a**2*c*cos(e + f
*x)/f - 3*B*a**2*c*x*sin(e + f*x)**4/8 - 3*B*a**2*c*x*sin(e + f*x)**2*cos(e
+ f*x)**2/4 + B*a**2*c*x*sin(e + f*x)**2/2 - 3*B*a**2*c*x*cos(e + f*x)**4/
8 + B*a**2*c*x*cos(e + f*x)**2/2 + 5*B*a**2*c*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*B*a**2*c*sin(e + f*x)*c
os(e + f*x)**3/(8*f) - B*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) + 2*B*a**2*
c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin
(e))*(a*sin(e) + a)**2*(-c*sin(e) + c), True))
```

$$3.31 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx$$

Optimal. Leaf size=117

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{f(c-c \sin(e+fx))^3} + \frac{3a^2(2A+3B) \cos(e+fx)}{2cf} + \frac{a^2(2A+3B) \cos^3(e+fx)}{2f(c-c \sin(e+fx))} - \frac{3a^2 x(2A+3B)}{2c}$$

[Out] $-3/2*a^2*(2*A+3*B)*x/c+3/2*a^2*(2*A+3*B)*\cos(f*x+e)/c/f+a^2*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^3+1/2*a^2*(2*A+3*B)*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))$

Rubi [A] time = 0.29, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2679, 2682, 8}

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{f(c-c \sin(e+fx))^3} + \frac{3a^2(2A+3B) \cos(e+fx)}{2cf} + \frac{a^2(2A+3B) \cos^3(e+fx)}{2f(c-c \sin(e+fx))} - \frac{3a^2 x(2A+3B)}{2c}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]), x]

[Out] $(-3*a^2*(2*A + 3*B)*x)/(2*c) + (3*a^2*(2*A + 3*B)*\text{Cos}[e + f*x])/(2*c*f) + (a^2*(A + B)*c^2*\text{Cos}[e + f*x]^5)/(f*(c - c*\text{Sin}[e + f*x])^3) + (a^2*(2*A + 3*B)*\text{Cos}[e + f*x]^3)/(2*f*(c - c*\text{Sin}[e + f*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{c - c \sin(e + fx)} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{f (c - c \sin(e + fx))^3} - (a^2 (2A + 3B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{f (c - c \sin(e + fx))^3} + \frac{a^2 (2A + 3B) \cos^3(e + fx)}{2f (c - c \sin(e + fx))} - \frac{1}{2} (3a^2 (A + B) \cos(e + fx) - 4(A + 3B) \cos(e + fx) + 4cf \sin(e + fx) - 1) \\ &= \frac{3a^2 (2A + 3B) \cos(e + fx)}{2cf} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{f (c - c \sin(e + fx))^3} + \frac{a^2 (2A + 3B) \cos^3(e + fx)}{2f (c - c \sin(e + fx))} \\ &= -\frac{3a^2 (2A + 3B)x}{2c} + \frac{3a^2 (2A + 3B) \cos(e + fx)}{2cf} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{f (c - c \sin(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 1.30, size = 191, normalized size = 1.63

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) (6(2A + 3B)(e + fx) - 4(A + 3B) \cos(e + fx)) - 4cf \sin(e + fx) - 1 \right)}{4cf \sin(e + fx) - 1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(Cos[(e + f*x)/2]*(6*(2*A + 3*B)*(e + f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(4*A*(8 + 3*e + 3*f*x) + 2*B*(16 + 9*e + 9*f*x) - 4*(A + 3*B)*Cos[e + f*x] - B*Sin[2*(e + f*x)])))/(4*c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x]))
```

fricas [A] time = 0.43, size = 177, normalized size = 1.51

$$\frac{Ba^2 \cos(fx + e)^3 - 3(2A + 3B)a^2 fx + 2(A + 3B)a^2 \cos(fx + e)^2 + 8(A + B)a^2 - (3(2A + 3B)a^2 fx - (10A + 13B)a^2) \cos(fx + e) + (3(2A + 3B)a^2 fx + B a^2 \cos(fx + e)^2 - (2A + 5B)a^2 c \cos(fx + e) + 8(A + B)a^2) \sin(fx + e)}{2(cf \cos(fx + e) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*a^2*cos(f*x + e)^3 - 3*(2*A + 3*B)*a^2*f*x + 2*(A + 3*B)*a^2*cos(f*x + e)^2 + 8*(A + B)*a^2 - (3*(2*A + 3*B)*a^2*f*x - (10*A + 13*B)*a^2)*cos(f*x + e) + (3*(2*A + 3*B)*a^2*f*x + B*a^2*cos(f*x + e)^2 - (2*A + 5*B)*a^2*c*os(f*x + e) + 8*(A + B)*a^2)*sin(f*x + e))/(c*f*cos(f*x + e) - c*f*sin(f*x + e) + c*f)
```


giac [A] time = 0.18, size = 163, normalized size = 1.39

$$\frac{3(2Aa^2+3Ba^2)(fx+e)}{c} + \frac{16(Aa^2+Ba^2)}{c(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1)} + \frac{2\left(Ba^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)^3 - 2Aa^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)^2 - 6Ba^2 \tan(\frac{1}{2}fx+\frac{1}{2}e) - Ba^2 \tan(\frac{1}{2}fx+\frac{1}{2}e)\right)}{\left(\tan(\frac{1}{2}fx+\frac{1}{2}e)^2 + 1\right)^2 c}$$

$$2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] $-1/2*(3*(2*A*a^2 + 3*B*a^2)*(f*x + e)/c + 16*(A*a^2 + B*a^2)/(c*(\tan(1/2*f*x + 1/2*e) - 1))) + 2*(B*a^2*\tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*\tan(1/2*f*x + 1/2*e)^2 - 6*B*a^2*\tan(1/2*f*x + 1/2*e) - B*a^2*\tan(1/2*f*x + 1/2*e) - 2*A*a^2 - 6*B*a^2)/((\tan(1/2*f*x + 1/2*e)^2 + 1)^2*c)/f$

maple [B] time = 0.43, size = 299, normalized size = 2.56

$$\frac{8a^2A}{fc\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{8a^2B}{fc\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{a^2\left(\tan^3\left(\frac{fx}{2} + \frac{e}{2}\right)\right)B}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} + \frac{2a^2\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)A}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} + \frac{6a^2\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)A}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] $-8/f*a^2/c/(\tan(1/2*f*x+1/2*e)-1)*A-8/f*a^2/c/(\tan(1/2*f*x+1/2*e)-1)*B-1/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^3*B+2/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^2*A+6/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^2*B+1/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*B*\tan(1/2*f*x+1/2*e)+2/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*A+6/f*a^2/c/(1+\tan(1/2*f*x+1/2*e)^2)^2*B-9/f*a^2/c*\arctan(\tan(1/2*f*x+1/2*e))*B-6/f*a^2/c*\arctan(\tan(1/2*f*x+1/2*e))*A$

maxima [B] time = 0.48, size = 624, normalized size = 5.33

$$2Aa^2 \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2}{c - \frac{c \sin(fx+e)}{\cos(fx+e)+1} + \frac{c \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} \right) + 4Ba^2 \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 2}{c - \frac{c \sin(fx+e)}{\cos(fx+e)+1} + \frac{c \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{c \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{c} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] $-(2*A*a^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 2)/(c - c*\sin(f*x + e)/(\cos(f*x + e) + 1) + c*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - c*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/c) + 4*B*a^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) - \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 2)/(c - c*\sin(f*x + e)/(\cos(f*x + e) + 1) + c*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - c*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/c) + B*a^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) - 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 4)/(c - c*\sin(f*x + e)/(\cos(f*x + e) + 1) + 2*c*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 2*c*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + c*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4))$


```

- 2*c*f) + 9*B*a**2*f*x*tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*
c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2
)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*f*x*tan(e/2 + f*x/2)**3/
(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*
x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 18*
B*a**2*f*x*tan(e/2 + f*x/2)**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*
tan(e/2 + f*x/2) - 2*c*f) - 9*B*a**2*f*x*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 +
f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*t
an(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 9*B*a**2*f*x/(2*c*f*
tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3
- 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 18*B*a**2*
tan(e/2 + f*x/2)**4/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4
+ 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f
*x/2) - 2*c*f) + 14*B*a**2*tan(e/2 + f*x/2)**3/(2*c*f*tan(e/2 + f*x/2)**5 -
2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*
x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) - 42*B*a**2*tan(e/2 + f*x/2)**2/(
2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)**4 + 4*c*f*tan(e/2 + f*x
/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2 + f*x/2) - 2*c*f) + 10*B
*a**2*tan(e/2 + f*x/2)/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 + f*x/2)*
**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*tan(e/2
+ f*x/2) - 2*c*f) - 28*B*a**2/(2*c*f*tan(e/2 + f*x/2)**5 - 2*c*f*tan(e/2 +
f*x/2)**4 + 4*c*f*tan(e/2 + f*x/2)**3 - 4*c*f*tan(e/2 + f*x/2)**2 + 2*c*f*t
an(e/2 + f*x/2) - 2*c*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-
c*sin(e) + c), True))

```

$$3.32 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=109

$$\frac{a^2(A+4B) \cos(e+fx)}{c^2 f} + \frac{a^2 c^2 (A+B) \cos^5(e+fx)}{3f(c-c \sin(e+fx))^4} + \frac{a^2 x (A+4B)}{c^2} - \frac{2a^2(A+4B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^2}$$

[Out] $a^2*(A+4*B)*x/c^2 - a^2*(A+4*B)*\cos(f*x+e)/c^2/f + 1/3*a^2*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^4 - 2/3*a^2*(A+4*B)*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^2$

Rubi [A] time = 0.28, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{a^2(A+4B) \cos(e+fx)}{c^2 f} + \frac{a^2 c^2 (A+B) \cos^5(e+fx)}{3f(c-c \sin(e+fx))^4} + \frac{a^2 x (A+4B)}{c^2} - \frac{2a^2(A+4B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] $(a^2*(A+4*B)*x)/c^2 - (a^2*(A+4*B)*\text{Cos}[e+f*x])/(c^2*f) + (a^2*(A+B)*c^2*\text{Cos}[e+f*x]^5)/(3*f*(c-c*\text{Sin}[e+f*x])^4) - (2*a^2*(A+4*B)*\text{Cos}[e+f*x]^3)/(3*f*(c-c*\text{Sin}[e+f*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegersQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(a*b*(2*m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m+p], 0]) && NeQ[2*m+p+1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[

$e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \& \ \text{GtQ}[n, 0]) \ || \ \text{LtQ}[0, n, m] \ || \ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^2} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^4} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{3f (c - c \sin(e + fx))^4} - \frac{1}{3} (a^2 (A + 4B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^3} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{3f (c - c \sin(e + fx))^4} - \frac{2a^2 (A + 4B) \cos^3(e + fx)}{3f (c - c \sin(e + fx))^2} + \frac{a^2 (A + 4B) \cos(e + fx)}{c^2 f} \\ &= -\frac{a^2 (A + 4B) \cos(e + fx)}{c^2 f} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{3f (c - c \sin(e + fx))^4} - \frac{2a^2 (A + 4B) \cos^3(e + fx)}{3f (c - c \sin(e + fx))^2} \\ &= \frac{a^2 (A + 4B) x}{c^2} - \frac{a^2 (A + 4B) \cos(e + fx)}{c^2 f} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{3f (c - c \sin(e + fx))^4} \end{aligned}$$

Mathematica [B] time = 0.63, size = 238, normalized size = 2.18

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(8(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 3(A + 4B)(e + fx) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{3f (c - c \sin(e + fx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 3*(A + 4*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 8*(A + B)*Sin[(e + f*x)/2] - 8*(2*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^2)

fricas [B] time = 0.44, size = 237, normalized size = 2.17

$$\frac{3Ba^2 \cos(fx + e)^3 + 6(A + 4B)a^2 fx + 4(A + B)a^2 - (3(A + 4B)a^2 fx + (8A + 23B)a^2) \cos(fx + e)^2 + (3(A + 4B)a^2 fx + (8A + 23B)a^2) \cos(fx + e)}{3(c^2 f \cos(fx + e) - c^2 f \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(3*B*a^2*cos(f*x + e)^3 + 6*(A + 4*B)*a^2*f*x + 4*(A + B)*a^2 - (3*(A + 4*B)*a^2*f*x + (8*A + 23*B)*a^2)*cos(f*x + e)^2 + (3*(A + 4*B)*a^2*f*x - 2*(2*A + 11*B)*a^2)*cos(f*x + e) - (6*(A + 4*B)*a^2*f*x - 3*B*a^2*cos(f*x + e)^2 - 4*(A + B)*a^2 + (3*(A + 4*B)*a^2*f*x - 2*(4*A + 13*B)*a^2)*cos(f*x + e))*sin(f*x + e)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))

giac [A] time = 0.16, size = 135, normalized size = 1.24

$$\frac{3(Aa^2+4Ba^2)(fx+e)}{c^2} - \frac{6Ba^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)c^2} + \frac{8\left(3Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Aa^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-9Ba^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Aa^2+4Ba^2\right)}{c^2\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)^3}$$

$$3f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/3*(3*(A*a^2 + 4*B*a^2)*(f*x + e)/c^2 - 6*B*a^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*c^2) + 8*(3*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*a^2*tan(1/2*f*x + 1/2*e) - 9*B*a^2*tan(1/2*f*x + 1/2*e) + A*a^2 + 4*B*a^2)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f

maple [A] time = 0.46, size = 198, normalized size = 1.82

$$\frac{16a^2A}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{16a^2B}{3c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{8a^2A}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{8a^2B}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} + \frac{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}{c^2f\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)

[Out] -16/3*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-16/3*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*B-8*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-8*a^2/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*B+8*a^2/c^2/f*B/(tan(1/2*f*x+1/2*e)-1)-2*a^2/c^2/f*B/(1+tan(1/2*f*x+1/2*e)^2)+2*a^2/c^2/f*arctan(tan(1/2*f*x+1/2*e))*A+8*a^2/c^2/f*arctan(tan(1/2*f*x+1/2*e))*B

maxima [B] time = 0.44, size = 839, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 2/3*(2*B*a^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) - 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 5)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + A*a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) + 2*B*a^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 4)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^2) - A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 2*A*a^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) - 1)/(c^2 - 3*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*c^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c^2


```

12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) + 48*B*a**2*f*x*tan(e/2 + f*x/2)**3/(3*c**2*f*t
an(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x
/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2
*f) - 48*B*a**2*f*x*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c
**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e
/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 36*B*a**2*f*x*tan(
e/2 + f*x/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 +
12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*t
an(e/2 + f*x/2) - 3*c**2*f) - 12*B*a**2*f*x/(3*c**2*f*tan(e/2 + f*x/2)**5 -
9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*t
an(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) + 24*B*a**2*tan(
e/2 + f*x/2)**4/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**
4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*
f*tan(e/2 + f*x/2) - 3*c**2*f) - 78*B*a**2*tan(e/2 + f*x/2)**3/(3*c**2*f*ta
n(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/
2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*
f) + 74*B*a**2*tan(e/2 + f*x/2)**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f
*tan(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 +
f*x/2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f) - 90*B*a**2*tan(e/2 + f*x
/2)/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan(e/2 + f*x/2)**4 + 12*c**2*
f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/2)**2 + 9*c**2*f*tan(e/2 +
f*x/2) - 3*c**2*f) + 38*B*a**2/(3*c**2*f*tan(e/2 + f*x/2)**5 - 9*c**2*f*tan
(e/2 + f*x/2)**4 + 12*c**2*f*tan(e/2 + f*x/2)**3 - 12*c**2*f*tan(e/2 + f*x/
2)**2 + 9*c**2*f*tan(e/2 + f*x/2) - 3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(a*sin(e) + a)**2/(-c*sin(e) + c)**2, True))

```


$$3.33 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=112

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{5f(c-c \sin(e+fx))^5} + \frac{2a^2 B \cos(e+fx)}{f(c^3 - c^3 \sin(e+fx))} - \frac{a^2 B x}{c^3} - \frac{2a^2 B \cos^3(e+fx)}{3f(c-c \sin(e+fx))^3}$$

[Out] $-a^2 B x / c^3 + 1/5 a^2 (A+B) c^2 \cos(f x + e)^5 / f / (c - c \sin(f x + e))^5 - 2/3 a^2 B \cos(f x + e)^3 / f / (c - c \sin(f x + e))^3 + 2 a^2 B \cos(f x + e) / f / (c^3 - c^3 \sin(f x + e))$

Rubi [A] time = 0.28, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{5f(c-c \sin(e+fx))^5} + \frac{2a^2 B \cos(e+fx)}{f(c^3 - c^3 \sin(e+fx))} - \frac{a^2 B x}{c^3} - \frac{2a^2 B \cos^3(e+fx)}{3f(c-c \sin(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] $-((a^2 B x) / c^3) + (a^2 (A + B) c^2 \cos[e + f x]^5) / (5 f (c - c \sin[e + f x])^5) - (2 a^2 B \cos[e + f x]^3) / (3 f (c - c \sin[e + f x])^3) + (2 a^2 B \cos[e + f x]) / (f (c^3 - c^3 \sin[e + f x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegerQ[2*m, 2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^5} dx \\
&= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} - (a^2 B c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^4} dx \\
&= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} - \frac{2a^2 B \cos^3(e + fx)}{3f(c - c \sin(e + fx))^3} + \frac{(a^2 B) \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^2} dx}{f} \\
&= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} - \frac{2a^2 B \cos^3(e + fx)}{3f(c - c \sin(e + fx))^3} + \frac{2a^2 B \cos(e + fx)}{f(c^3 - c^3 \sin^2(e + fx))} \\
&= -\frac{a^2 B x}{c^3} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} - \frac{2a^2 B \cos^3(e + fx)}{3f(c - c \sin(e + fx))^3} + \frac{2a^2 B \cos(e + fx)}{f(c^3 - c^3 \sin^2(e + fx))}
\end{aligned}$$

Mathematica [B] time = 0.74, size = 278, normalized size = 2.48

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(24(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 2(3A + 43B) \sin\left(\frac{1}{2}(e + fx)\right) \right)}{15f(c - c \sin(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 4*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 15*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 24*(A + B)*Sin[(e + f*x)/2] - 8*(3*A + 8*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 2*(3*A + 43*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^3)

fricas [B] time = 0.44, size = 277, normalized size = 2.47

$$\frac{60Ba^2fx - (15Ba^2fx - (3A + 43B)a^2) \cos^3(fx + e) - 12(A + B)a^2 - (45Ba^2fx - (9A - 11B)a^2) \cos^2(fx + e)}{15(c^3f \cos^3(fx + e) + 3c^3f \cos^2(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*(60*B*a^2*f*x - (15*B*a^2*f*x - (3*A + 43*B)*a^2)*cos(f*x + e)^3 - 12*(A + B)*a^2 - (45*B*a^2*f*x - (9*A - 11*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x - (A + 11*B)*a^2)*cos(f*x + e) - (60*B*a^2*f*x + 12*(A + B)*a^2 - (15*B*a^2*f*x + (3*A + 43*B)*a^2)*cos(f*x + e)^2 + 6*(5*B*a^2*f*x + (A - 9*B)*a^2)*cos(f*x + e))*sin(f*x + e)/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e))^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e)

giac [A] time = 0.18, size = 159, normalized size = 1.42

$$\frac{15(fx+e)Ba^2}{c^3} + \frac{2\left(15Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 15Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 60Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 30Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 170Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 100Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 100Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 100Aa^2 + 100Ba^2\right)}{c^3\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^5}$$

15f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out]
$$-1/15*(15*(f*x + e)*B*a^2/c^3 + 2*(15*A*a^2*\tan(1/2*f*x + 1/2*e)^4 + 15*B*a^2*\tan(1/2*f*x + 1/2*e)^4 - 60*B*a^2*\tan(1/2*f*x + 1/2*e)^3 + 30*A*a^2*\tan(1/2*f*x + 1/2*e)^2 + 170*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 100*B*a^2*\tan(1/2*f*x + 1/2*e) + 3*A*a^2 + 23*B*a^2)/(c^3*(\tan(1/2*f*x + 1/2*e) - 1)^5)/f$$

maple [B] time = 0.50, size = 249, normalized size = 2.22

$$\frac{2a^2A}{c^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} \frac{2a^2B}{c^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} \frac{32a^2A}{5c^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^5} \frac{32a^2B}{5c^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^5} c^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)

[Out]
$$-2*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)*A-2*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)*B-32/5*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^5*A-32/5*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^5*B-16*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^3*A-32/3*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^3*B-16*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^4*A-16*a^2/c^3/f/(\tan(1/2*f*x+1/2*e)-1)^4*B-8*a^2/c^3/f*A/(\tan(1/2*f*x+1/2*e)-1)^2-2*a^2/c^3/f*B*a^2*\operatorname{rctan}(\tan(1/2*f*x+1/2*e))$$

maxima [B] time = 0.63, size = 1139, normalized size = 10.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out]
$$-2/15*(B*a^2*((95*\sin(f*x + e))/(\cos(f*x + e) + 1) - 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/c^3) + A*a^2*(20*\sin(f*x + e))/(\cos(f*x + e) + 1) - 40*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 6*A*a^2*(5*\sin(f*x + e))/(\cos(f*x + e) + 1) - 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 2*A*a^2*(5*\sin(f*x + e))/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 4*B*a^2*(5$$


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*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 340*B*a**2*tan(e/2 + f
*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 1
50*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) + 200*B*a**2*tan(e/2 + f*x/2)/(15*c**3*f*tan(
e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/2)**4 + 150*c**3*f*tan(e/2 + f*x/
2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c*
**3*f) - 46*B*a**2/(15*c**3*f*tan(e/2 + f*x/2)**5 - 75*c**3*f*tan(e/2 + f*x/
2)**4 + 150*c**3*f*tan(e/2 + f*x/2)**3 - 150*c**3*f*tan(e/2 + f*x/2)**2 + 7
5*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin
(e) + a)**2/(-c*sin(e) + c)**3, True))

```

$$3.34 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=75

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{7f(c-c \sin(e+fx))^6} + \frac{a^2c(A-6B) \cos^5(e+fx)}{35f(c-c \sin(e+fx))^5}$$

[Out] $1/7*a^2*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^6+1/35*a^2*(A-6*B)*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^5$

Rubi [A] time = 0.23, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2859, 2671}

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{7f(c-c \sin(e+fx))^6} + \frac{a^2c(A-6B) \cos^5(e+fx)}{35f(c-c \sin(e+fx))^5}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]

[Out] $(a^2*(A + B)*c^2*\cos[e + f*x]^5)/(7*f*(c - c*\sin[e + f*x])^6) + (a^2*(A - 6*B)*c*\cos[e + f*x]^5)/(35*f*(c - c*\sin[e + f*x])^5)$

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx &= (a^2c^2) \int \frac{\cos^4(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6} dx \\ &= \frac{a^2(A+B)c^2 \cos^5(e+fx)}{7f(c-c \sin(e+fx))^6} + \frac{1}{7} (a^2(A-6B)c) \int \frac{\cos^4(e+fx)}{(c-c \sin(e+fx))^5} dx \\ &= \frac{a^2(A+B)c^2 \cos^5(e+fx)}{7f(c-c \sin(e+fx))^6} + \frac{a^2(A-6B)c \cos^5(e+fx)}{35f(c-c \sin(e+fx))^5} \end{aligned}$$

Mathematica [B] time = 0.95, size = 191, normalized size = 2.55

$$a^2 \left(-35(A + 4B) \cos\left(\frac{1}{2}(e + fx)\right) + 7(2A + 13B) \cos\left(\frac{3}{2}(e + fx)\right) - 70A \sin\left(\frac{1}{2}(e + fx)\right) - 35A \sin\left(\frac{3}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]

[Out] -1/140*(a^2*(-35*(A + 4*B)*Cos[(e + f*x)/2] + 7*(2*A + 13*B)*Cos[(3*(e + f*x))/2] + 35*B*Cos[(5*(e + f*x))/2] + A*Cos[(7*(e + f*x))/2] - 6*B*Cos[(7*(e + f*x))/2] - 70*A*Sin[(e + f*x)/2] + 70*B*Sin[(e + f*x)/2] - 35*A*Sin[(3*(e + f*x))/2] + 35*B*Sin[(3*(e + f*x))/2] + 7*A*Sin[(5*(e + f*x))/2] - 7*B*Sin[(5*(e + f*x))/2]))/(c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7)

fricas [B] time = 0.43, size = 263, normalized size = 3.51

$$\frac{(A - 6B)a^2 \cos^4(fx + e) + (4A + 11B)a^2 \cos^3(fx + e) + (13A + 27B)a^2 \cos^2(fx + e) - 10(A + B)a^2 \cos(fx + e) - 20(A + B)a^2}{35 \left(c^4 f \cos^4(fx + e) - 3c^4 f \cos^3(fx + e) - 8c^4 f \cos^2(fx + e) + 4c^4 f \cos(fx + e) - 8c^4 f \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] -1/35*((A - 6*B)*a^2*cos(f*x + e)^4 + (4*A + 11*B)*a^2*cos(f*x + e)^3 + (13*A + 27*B)*a^2*cos(f*x + e)^2 - 10*(A + B)*a^2*cos(f*x + e) - 20*(A + B)*a^2 - ((A - 6*B)*a^2*cos(f*x + e)^3 - (3*A + 17*B)*a^2*cos(f*x + e)^2 + 10*(A + B)*a^2*cos(f*x + e) + 20*(A + B)*a^2*sin(f*x + e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))

giac [B] time = 0.19, size = 229, normalized size = 3.05

$$2 \left(35 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 35 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 35 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 140 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] -2/35*(35*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 35*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 35*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 140*A*a^2*tan(1/2*f*x + 1/2*e)^4 + 35*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 70*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 70*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 91*A*a^2*tan(1/2*f*x + 1/2*e)^2 + 14*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^2*tan(1/2*f*x + 1/2*e) + 7*B*a^2*tan(1/2*f*x + 1/2*e) + 6*A*a^2 - B*a^2)/(c^4*f*(tan(1/2*f*x + 1/2*e) - 1)^7)

maple [B] time = 0.47, size = 161, normalized size = 2.15

$$2a^2 \left(\frac{128A+112B}{5 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^5} - \frac{96A+64B}{4 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^4} - \frac{96A+96B}{6 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^3} - \frac{42A+18B}{3 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^2} - \frac{A}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{32A+32B}{7 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)} - \frac{1}{2 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)} \right) f c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)`

[Out] $2/f*a^2/c^4*(-1/5*(128*A+112*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/4*(96*A+64*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/6*(96*A+96*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/3*(42*A+18*B)/(\tan(1/2*f*x+1/2*e)-1)^3-A/(\tan(1/2*f*x+1/2*e)-1)-1/7*(32*A+32*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/2*(10*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2)$

maxima [B] time = 0.61, size = 1571, normalized size = 20.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")`

[Out] $2/105*(2*A*a^2*(91*\sin(f*x + e)/(\cos(f*x + e) + 1) - 168*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 280*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 175*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 105*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + B*a^2*(91*\sin(f*x + e)/(\cos(f*x + e) + 1) - 168*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 280*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 175*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 105*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) - 3*A*a^2*(49*\sin(f*x + e)/(\cos(f*x + e) + 1) - 147*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 210*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 210*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 105*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 35*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 12)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) - 4*A*a^2*(14*\sin(f*x + e)/(\cos(f*x + e) + 1) - 42*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 35*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 35*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) - 8*B*a^2*(14*\sin(f*x + e)/(\cos(f*x + e) + 1) - 42*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 35*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 35*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 6*B*a^2*(7*\sin(f*x + e)/(\cos(f*x + e) + 1) - 21*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 35*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 35*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 35*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 21*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 7*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7))/f$

mupad [B] time = 13.35, size = 269, normalized size = 3.59

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{109 A a^2}{4} + \frac{11 B a^2}{4} - \frac{27 A a^2 \cos(2e+2fx)}{4} + \frac{5 A a^2 \cos(3e+3fx)}{8} - \frac{13 B a^2 \cos(2e+2fx)}{4} + \frac{5 B a^2 \cos(3e+3fx)}{8} \right) \\ \hline 35 c^4 f \left(\frac{35 \sqrt{2} \cos\left(\frac{e}{2} + \frac{\pi}{4} + \frac{fx}{2}\right)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^4,x)

[Out] (2*cos(e/2 + (f*x)/2)*((109*A*a^2)/4 + (11*B*a^2)/4 - (27*A*a^2*cos(2*e + 2*f*x))/4 + (5*A*a^2*cos(3*e + 3*f*x))/8 - (13*B*a^2*cos(2*e + 2*f*x))/4 + (5*B*a^2*cos(3*e + 3*f*x))/8 + (7*A*a^2*sin(2*e + 2*f*x))/2 + (7*A*a^2*sin(3*e + 3*f*x))/8 - (7*B*a^2*sin(2*e + 2*f*x))/2 - (7*B*a^2*sin(3*e + 3*f*x))/8 - (121*A*a^2*cos(e + f*x))/8 - (9*B*a^2*cos(e + f*x))/8 - (105*A*a^2*sin(e + f*x))/8 + (105*B*a^2*sin(e + f*x))/8))/(35*c^4*f*((35*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/8 - (7*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/8 + (2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/8))

sympy [A] time = 45.07, size = 2008, normalized size = 26.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**4,x)

[Out] Piecewise((-70*A*a**2*tan(e/2 + f*x/2)**6/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*A*a**2*tan(e/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 280*A*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 140*A*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 182*A*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 14*A*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**5/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 12*A*a**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**4/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 70*B*a**2*tan(e/2 + f*x/2)**1/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 70*B*a**2/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f))

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+ f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)
**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 140*B*a**2*tan(e/2 + f*x/2)
)**3/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*
c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*
tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 +
f*x/2) - 35*c**4*f) - 28*B*a**2*tan(e/2 + f*x/2)**2/(35*c**4*f*tan(e/2 + f
*x/2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5
- 1225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c
**4*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) - 14*B
*a**2*tan(e/2 + f*x/2)/(35*c**4*f*tan(e/2 + f*x/2)**7 - 245*c**4*f*tan(e/2
+ f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1225*c**4*f*tan(e/2 + f*x/2)
**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4*f*tan(e/2 + f*x/2)**2 + 24
5*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f) + 2*B*a**2/(35*c**4*f*tan(e/2 + f*x/
2)**7 - 245*c**4*f*tan(e/2 + f*x/2)**6 + 735*c**4*f*tan(e/2 + f*x/2)**5 - 1
225*c**4*f*tan(e/2 + f*x/2)**4 + 1225*c**4*f*tan(e/2 + f*x/2)**3 - 735*c**4
*f*tan(e/2 + f*x/2)**2 + 245*c**4*f*tan(e/2 + f*x/2) - 35*c**4*f), Ne(f, 0)
), (x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**4, True))

```

$$3.35 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx$$

Optimal. Leaf size=115

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{9f(c-c \sin(e+fx))^7} + \frac{a^2(2A-7B) \cos^5(e+fx)}{315f(c-c \sin(e+fx))^5} + \frac{a^2c(2A-7B) \cos^5(e+fx)}{63f(c-c \sin(e+fx))^6}$$

[Out] 1/9*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+1/63*a^2*(2*A-7*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+1/315*a^2*(2*A-7*B)*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^5

Rubi [A] time = 0.29, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{9f(c-c \sin(e+fx))^7} + \frac{a^2(2A-7B) \cos^5(e+fx)}{315f(c-c \sin(e+fx))^5} + \frac{a^2c(2A-7B) \cos^5(e+fx)}{63f(c-c \sin(e+fx))^6}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]

[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(9*f*(c - c*Sin[e + f*x])^7) + (a^2*(2*A - 7*B)*c*Cos[e + f*x]^5)/(63*f*(c - c*Sin[e + f*x])^6) + (a^2*(2*A - 7*B)*Cos[e + f*x]^5)/(315*f*(c - c*Sin[e + f*x])^5)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && (LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^5} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^7} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{9 f (c - c \sin(e + fx))^7} + \frac{1}{9} (a^2 (2A - 7B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^6} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{9 f (c - c \sin(e + fx))^7} + \frac{a^2 (2A - 7B) c \cos^5(e + fx)}{63 f (c - c \sin(e + fx))^6} + \frac{1}{63} (a^2 (2A - 7B) c) \int \frac{\cos^3(e + fx)}{(c - c \sin(e + fx))^5} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{9 f (c - c \sin(e + fx))^7} + \frac{a^2 (2A - 7B) c \cos^5(e + fx)}{63 f (c - c \sin(e + fx))^6} + \frac{a^2 (2A - 7B) c \cos^5(e + fx)}{315 f (c - c \sin(e + fx))^5} \end{aligned}$$

Mathematica [B] time = 1.24, size = 261, normalized size = 2.27

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(315(2A + 3B) \cos\left(\frac{1}{2}(e + fx)\right) - 63(4A + 11B) \cos\left(\frac{3}{2}(e + fx)\right) \right)}{(c - c \sin(e + fx))^5}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]

[Out] -1/2520*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(315*(2*A + 3*B)*Cos[(e + f*x)/2] - 63*(4*A + 11*B)*Cos[(3*(e + f*x))/2] - 315*B*Cos[(5*(e + f*x))/2] - 18*A*Cos[(7*(e + f*x))/2] + 63*B*Cos[(7*(e + f*x))/2] + 882*A*Sin[(e + f*x)/2] + 63*B*Sin[(e + f*x)/2] + 420*A*Sin[(3*(e + f*x))/2] + 105*B*Sin[(3*(e + f*x))/2] - 72*A*Sin[(5*(e + f*x))/2] - 63*B*Sin[(5*(e + f*x))/2] + 2*A*Sin[(9*(e + f*x))/2] - 7*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^5)

fricas [B] time = 0.41, size = 335, normalized size = 2.91

$$\frac{(2A - 7B)a^2 \cos^5(fx + e) - 4(2A - 7B)a^2 \cos^4(fx + e) - 5(5A + 14B)a^2 \cos^3(fx + e) - 5(17A + 35B)a^2 \cos^2(fx + e) + 70(A + B)a^2 \cos(fx + e) + 140(A + B)a^2 + ((2A - 7B)a^2 \cos^4(fx + e) + 5(2A - 7B)a^2 \cos^3(fx + e) - 15(A + 7B)a^2 \cos^2(fx + e) + 70(A + B)a^2 \cos(fx + e) + 140(A + B)a^2) \sin(fx + e)}{(c^5 f \cos(fx + e)^5 + 5c^5 f \cos(fx + e)^4 - 8c^5 f \cos(fx + e)^3 - 20c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f - (c^5 f \cos(fx + e)^4 - 4c^5 f \cos(fx + e)^3 - 12c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f) \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] 1/315*((2*A - 7*B)*a^2*cos(f*x + e)^5 - 4*(2*A - 7*B)*a^2*cos(f*x + e)^4 - 5*(5*A + 14*B)*a^2*cos(f*x + e)^3 - 5*(17*A + 35*B)*a^2*cos(f*x + e)^2 + 70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2 + ((2*A - 7*B)*a^2*cos(f*x + e)^4 + 5*(2*A - 7*B)*a^2*cos(f*x + e)^3 - 15*(A + 7*B)*a^2*cos(f*x + e)^2 + 70*(A + B)*a^2*cos(f*x + e) + 140*(A + B)*a^2)*sin(f*x + e)/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))

giac [B] time = 0.23, size = 301, normalized size = 2.62

$$2 \left(315 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 - 630 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 315 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 2310 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 + 105 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 2520 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 945 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 3402 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 63 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 1638 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 693 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 1062 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 63 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 108 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 63 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 47 A a^2 - 7 B a^2 \right) / (c^5 f (\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1)^9)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out]
$$\frac{-2/315*(315*A*a^2*\tan(1/2*f*x + 1/2*e)^8 - 630*A*a^2*\tan(1/2*f*x + 1/2*e)^7 + 315*B*a^2*\tan(1/2*f*x + 1/2*e)^7 + 2310*A*a^2*\tan(1/2*f*x + 1/2*e)^6 + 105*B*a^2*\tan(1/2*f*x + 1/2*e)^6 - 2520*A*a^2*\tan(1/2*f*x + 1/2*e)^5 + 945*B*a^2*\tan(1/2*f*x + 1/2*e)^5 + 3402*A*a^2*\tan(1/2*f*x + 1/2*e)^4 + 63*B*a^2*\tan(1/2*f*x + 1/2*e)^4 - 1638*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 693*B*a^2*\tan(1/2*f*x + 1/2*e)^3 + 1062*A*a^2*\tan(1/2*f*x + 1/2*e)^2 + 63*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 108*A*a^2*\tan(1/2*f*x + 1/2*e) + 63*B*a^2*\tan(1/2*f*x + 1/2*e) + 47*A*a^2 - 7*B*a^2)/(c^5*f*(\tan(1/2*f*x + 1/2*e) - 1)^9)}$$

maple [A] time = 0.53, size = 205, normalized size = 1.78

$$2a^2 \left(\frac{404A+276B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{64A+64B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{64A+22B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{480A+448B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{12A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} \right) / fc^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)

[Out]
$$2/f*a^2/c^5*(-1/5*(404*A+276*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/9*(64*A+64*B)/(\tan(1/2*f*x+1/2*e)-1)^9-1/3*(64*A+22*B)/(\tan(1/2*f*x+1/2*e)-1)^3-A/(\tan(1/2*f*x+1/2*e)-1)-1/7*(480*A+448*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/2*(12*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/4*(200*A+104*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/6*(544*A+448*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/8*(256*A+256*B)/(\tan(1/2*f*x+1/2*e)-1)^8)$$

maxima [B] time = 0.42, size = 2087, normalized size = 18.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2/315*(A*a^2*(432*\sin(f*x + e)/(\cos(f*x + e) + 1) - 1728*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3612*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 5418*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5040*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 3360*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 1260*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 315*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 36*c^5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 84*c^5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 126*c^5*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 126*c^5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 84*c^5*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 36*c^5*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 9*c^5*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - c^5*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9) - 10*A*a^2*(45*\sin(f*x + e)/(\cos(f*x + e) + 1) - 117*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 273*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 315*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 315*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 \end{aligned}$$

```

- 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 +
126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^2*(45*sin(f*x + e)/(cos(
f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 +
63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos
(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*
x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4
- 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos
(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f
*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) -
10*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(co
s(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^
5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1
)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(
cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) + 14*A*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/
(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/
(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*s
in(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^
9) + 28*B*a^2*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4
/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x
+ e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) +
1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) +
1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(
cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f

```

mupad [B] time = 13.36, size = 331, normalized size = 2.88

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{265 A a^2 \cos(2e+2fx)}{2} - \frac{49 B a^2}{8} - \frac{4967 A a^2}{16} - \frac{89 A a^2 \cos(3e+3fx)}{4} - \frac{49 A a^2 \cos(4e+4fx)}{16} + \frac{35 B a^2 \cos(2e+2fx)}{4} \right)$$

315c

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^5,x)

[Out] -(2*cos(e/2 + (f*x)/2)*((265*A*a^2*cos(2*e + 2*f*x))/2 - (49*B*a^2)/8 - (4967*A*a^2)/16 - (89*A*a^2*cos(3*e + 3*f*x))/4 - (49*A*a^2*cos(4*e + 4*f*x))/16 + (35*B*a^2*cos(2*e + 2*f*x))/4 - (7*B*a^2*cos(3*e + 3*f*x))/8 + (7*B*a^2*cos(4*e + 4*f*x))/8 - (567*A*a^2*sin(2*e + 2*f*x))/8 - (243*A*a^2*sin(3*e + 3*f*x))/8 + (45*A*a^2*sin(4*e + 4*f*x))/16 + (63*B*a^2*sin(2*e + 2*f*x))/2 + (63*B*a^2*sin(3*e + 3*f*x))/8 + (625*A*a^2*cos(e + f*x))/4 + (35*B*a^2

$$\frac{\cos(e + fx)}{8} + \frac{(2205A^2 \sin(e + fx))}{8} - \frac{(945B^2 \sin(e + fx))}{8} \Big/ \frac{(315c^5 f ((63 \cdot 2^{1/2} \cos(e/2 + \pi/4 + (fx)/2)))}{8} - \frac{(21 \cdot 2^{1/2} \cos((3e)/2 - \pi/4 + (3fx)/2))}{4} - \frac{(9 \cdot 2^{1/2} \cos((5e)/2 + \pi/4 + (5fx)/2))}{4} + \frac{(9 \cdot 2^{1/2} \cos((7e)/2 - \pi/4 + (7fx)/2))}{16} + \frac{(2^{1/2} \cos((9e)/2 + \pi/4 + (9fx)/2))}{16}$$

`sympy [A]` time = 78.09, size = 3262, normalized size = 28.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**5,x)`

[Out] `Piecewise((-630A*a**2*tan(e/2 + f*x/2)**8/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 1260A*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 4620A*a**2*tan(e/2 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 5040A*a**2*tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 6804A*a**2*tan(e/2 + f*x/2)**4/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 3276A*a**2*tan(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) + 216A*a**2*tan(e/2 + f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 630B*a**2*tan(e/2 + f*x/2)**7/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f)`

```

**5*f) - 210*B*a**2*tan(e/2 + f*x/2)**6/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2
835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c
**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f
*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(
e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 1890*B*a**2*
tan(e/2 + f*x/2)**5/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 +
f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/
2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4
+ 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 28
35*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2 + f*x/2)**4/(
315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c*
**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*
tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e
/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f
*x/2) - 315*c**5*f) - 1386*B*a**2*tan(e/2 + f*x/2)**3/(315*c**5*f*tan(e/2 +
f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)
)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5
- 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 113
40*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f)
- 126*B*a**2*tan(e/2 + f*x/2)**2/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**
5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*t
an(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/
2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f
*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f) - 126*B*a**2*tan(e/2
+ f*x/2)/(315*c**5*f*tan(e/2 + f*x/2)**9 - 2835*c**5*f*tan(e/2 + f*x/2)**8
+ 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460*c**5*f*tan(e/2 + f*x/2)**6 + 396
90*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5*f*tan(e/2 + f*x/2)**4 + 26460*c*
**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*tan(e/2 + f*x/2)**2 + 2835*c**5*f*t
an(e/2 + f*x/2) - 315*c**5*f) + 14*B*a**2/(315*c**5*f*tan(e/2 + f*x/2)**9 -
2835*c**5*f*tan(e/2 + f*x/2)**8 + 11340*c**5*f*tan(e/2 + f*x/2)**7 - 26460
*c**5*f*tan(e/2 + f*x/2)**6 + 39690*c**5*f*tan(e/2 + f*x/2)**5 - 39690*c**5
*f*tan(e/2 + f*x/2)**4 + 26460*c**5*f*tan(e/2 + f*x/2)**3 - 11340*c**5*f*ta
n(e/2 + f*x/2)**2 + 2835*c**5*f*tan(e/2 + f*x/2) - 315*c**5*f), Ne(f, 0)),
(x*(A + B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**5, True))

```


$$3.36 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6} dx$$

Optimal. Leaf size=156

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{11f(c-c \sin(e+fx))^8} + \frac{2a^2(3A-8B) \cos^5(e+fx)}{3465cf(c-c \sin(e+fx))^5} + \frac{2a^2(3A-8B) \cos^5(e+fx)}{693f(c-c \sin(e+fx))^6} + \frac{a^2c(3A-8B) \cos^5(e+fx)}{99f(c-c \sin(e+fx))^7}$$

[Out] 1/11*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/99*a^2*(3*A-8*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/693*a^2*(3*A-8*B)*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^6+2/3465*a^2*(3*A-8*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^5

Rubi [A] time = 0.37, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{11f(c-c \sin(e+fx))^8} + \frac{2a^2(3A-8B) \cos^5(e+fx)}{3465cf(c-c \sin(e+fx))^5} + \frac{2a^2(3A-8B) \cos^5(e+fx)}{693f(c-c \sin(e+fx))^6} + \frac{a^2c(3A-8B) \cos^5(e+fx)}{99f(c-c \sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]

[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(11*f*(c - c*Sin[e + f*x])^8) + (a^2*(3*A - 8*B)*c*Cos[e + f*x]^5)/(99*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(3*A - 8*B)*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(3*A - 8*B)*Cos[e + f*x]^5)/(3465*c*f*(c - c*Sin[e + f*x])^5)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^ (p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^ (m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^ (n_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^ (n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^6} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^8} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{11 f (c - c \sin(e + fx))^8} + \frac{1}{11} (a^2 (3A - 8B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^7} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{11 f (c - c \sin(e + fx))^8} + \frac{a^2 (3A - 8B) c \cos^5(e + fx)}{99 f (c - c \sin(e + fx))^7} + \frac{1}{99} (2a^2 (3A - 8B) c) \int \frac{\cos^3(e + fx)}{(c - c \sin(e + fx))^6} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{11 f (c - c \sin(e + fx))^8} + \frac{a^2 (3A - 8B) c \cos^5(e + fx)}{99 f (c - c \sin(e + fx))^7} + \frac{2a^2 (3A - 8B) c \cos^5(e + fx)}{693 f (c - c \sin(e + fx))^6} \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{11 f (c - c \sin(e + fx))^8} + \frac{a^2 (3A - 8B) c \cos^5(e + fx)}{99 f (c - c \sin(e + fx))^7} + \frac{2a^2 (3A - 8B) c \cos^5(e + fx)}{693 f (c - c \sin(e + fx))^6} \end{aligned}$$

Mathematica [A] time = 1.60, size = 285, normalized size = 1.83

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(231(27A + 28B) \cos\left(\frac{1}{2}(e + fx)\right) - 2475(A + 2B) \cos\left(\frac{1}{2}(e + fx)\right) \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(231*(27*A + 28*B)*Cos[(e + f*x)/2] - 2475*(A + 2*B)*Cos[(3*(e + f*x))/2] - 2310*B*Cos[(5*(e + f*x))/2] - 165*A*Cos[(7*(e + f*x))/2] + 440*B*Cos[(7*(e + f*x))/2] + 3*A*Cos[(11*(e + f*x))/2] - 8*B*Cos[(11*(e + f*x))/2] + 7623*A*Sin[(e + f*x)/2] + 2772*B*Sin[(e + f*x)/2] + 3465*A*Sin[(3*(e + f*x))/2] + 2310*B*Sin[(3*(e + f*x))/2] - 495*A*Sin[(5*(e + f*x))/2] - 990*B*Sin[(5*(e + f*x))/2] + 33*A*Sin[(9*(e + f*x))/2] - 88*B*Sin[(9*(e + f*x))/2]))/(27720*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^6)

fricas [B] time = 0.42, size = 407, normalized size = 2.61

$$\frac{2(3A - 8B)a^2 \cos^6(fx + e) + 12(3A - 8B)a^2 \cos^5(fx + e) - 25(3A - 8B)a^2 \cos^4(fx + e) - 35(6A + 17B)a^2 \cos^3(fx + e) - 35(21A + 43B)a^2 \cos^2(fx + e) + 630(A + B)a^2 \cos(fx + e) + 1260(A + B)a^2 - (2(3A - 8B)a^2 \cos^5(fx + e) - 10(3A - 8B)a^2 \cos^4(fx + e) - 35(3A - 8B)a^2 \cos^3(fx + e) + 35(3A + 25B)a^2 \cos^2(fx + e) - 630(A + B)a^2 \cos(fx + e) - 1260(A + B)a^2) \sin(fx + e)}{3465(c^6 f \cos^6(fx + e) - 5c^6 f \cos^5(fx + e) - 18c^6 f \cos^4(fx + e) + 20c^6 f \cos^3(fx + e) + 48c^6 f \cos^2(fx + e) - 16c^6 f \cos(fx + e) - 1260(A + B)a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="fricas")

[Out] -1/3465*(2*(3*A - 8*B)*a^2*cos(f*x + e)^6 + 12*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 25*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(6*A + 17*B)*a^2*cos(f*x + e)^3 - 35*(21*A + 43*B)*a^2*cos(f*x + e)^2 + 630*(A + B)*a^2*cos(f*x + e) + 1260*(A + B)*a^2 - (2*(3*A - 8*B)*a^2*cos(f*x + e)^5 - 10*(3*A - 8*B)*a^2*cos(f*x + e)^4 - 35*(3*A - 8*B)*a^2*cos(f*x + e)^3 + 35*(3*A + 25*B)*a^2*cos(f*x + e)^2 - 630*(A + B)*a^2*cos(f*x + e) - 1260*(A + B)*a^2)*sin(f*x + e)/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 1260*(A + B)*a^2)

$32c^6f + (c^6f\cos(fx + e))^5 + 6c^6f\cos(fx + e)^4 - 12c^6f\cos(fx + e)^3 - 32c^6f\cos(fx + e)^2 + 16c^6f\cos(fx + e) + 32c^6f)\sin(fx + e)$

giac [B] time = 0.25, size = 373, normalized size = 2.39

$$2 \left(3465 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{10} - 10395 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9 + 3465 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^9 + 41580 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 - 1155 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 - 69300 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 + 16170 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 + 112266 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 6006 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 98406 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 22176 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 81180 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 3960 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 33660 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 8910 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 14685 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 110 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1551 Aa^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 671 Ba^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 456 Aa^2 - 61 Ba^2 \right) / (c^6 f (\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1)^{11})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="giac")

[Out] $-2/3465*(3465*A*a^2*\tan(1/2*f*x + 1/2*e)^{10} - 10395*A*a^2*\tan(1/2*f*x + 1/2*e)^9 + 3465*B*a^2*\tan(1/2*f*x + 1/2*e)^9 + 41580*A*a^2*\tan(1/2*f*x + 1/2*e)^8 - 1155*B*a^2*\tan(1/2*f*x + 1/2*e)^8 - 69300*A*a^2*\tan(1/2*f*x + 1/2*e)^7 + 16170*B*a^2*\tan(1/2*f*x + 1/2*e)^7 + 112266*A*a^2*\tan(1/2*f*x + 1/2*e)^6 - 6006*B*a^2*\tan(1/2*f*x + 1/2*e)^6 - 98406*A*a^2*\tan(1/2*f*x + 1/2*e)^5 + 22176*B*a^2*\tan(1/2*f*x + 1/2*e)^5 + 81180*A*a^2*\tan(1/2*f*x + 1/2*e)^4 - 3960*B*a^2*\tan(1/2*f*x + 1/2*e)^4 - 33660*A*a^2*\tan(1/2*f*x + 1/2*e)^3 + 8910*B*a^2*\tan(1/2*f*x + 1/2*e)^3 + 14685*A*a^2*\tan(1/2*f*x + 1/2*e)^2 + 110*B*a^2*\tan(1/2*f*x + 1/2*e)^2 - 1551*A*a^2*\tan(1/2*f*x + 1/2*e) + 671*B*a^2*\tan(1/2*f*x + 1/2*e) + 456*A*a^2 - 61*B*a^2)/(c^6*f*(\tan(1/2*f*x + 1/2*e) - 1)^{11})$

maple [A] time = 0.51, size = 249, normalized size = 1.60

$$2a^2 \left(-\frac{932A+528B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{128A+128B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}} - \frac{1536A+1472B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{352A+152B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{640A+640B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}} - \frac{90A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} \right) / f c^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)

[Out] $2/f*a^2/c^6*(-1/5*(932*A+528*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/11*(128*A+128*B)/(\tan(1/2*f*x+1/2*e)-1)^{11}-1/9*(1536*A+1472*B)/(\tan(1/2*f*x+1/2*e)-1)^9-1/4*(352*A+152*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/10*(640*A+640*B)/(\tan(1/2*f*x+1/2*e)-1)^{10}-1/3*(90*A+26*B)/(\tan(1/2*f*x+1/2*e)-1)^3-A/(\tan(1/2*f*x+1/2*e)-1)-1/8*(2304*A+2048*B)/(\tan(1/2*f*x+1/2*e)-1)^8-1/7*(2376*A+1896*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/6*(1752*A+1208*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/2*(14*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2)$

maxima [B] time = 0.45, size = 2604, normalized size = 16.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="maxima")

[Out] $-2/3465*(5*A*a^2*(913*\sin(f*x + e)/(\cos(f*x + e) + 1) - 4565*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 12540*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 25080*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 33726*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 33726*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 23100*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 11550*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 3465*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 693*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} -$

$$\begin{aligned}
& 146)/(c^6 - 11*c^6*\sin(f*x + e))/(cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/ \\
& (cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6* \\
& 6*\sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(cos(f*x + e) \\
&) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + \\
& e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 5 \\
& 5*c^6*\sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^10/(cos(f*x \\
& + e) + 1)^10 - c^6*\sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 6*A*a^2*(671*s \\
& in(f*x + e)/(cos(f*x + e) + 1) - 2200*\sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + \\
& 6600*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*\sin(f*x + e)^4/(cos(f*x + \\
& e) + 1)^4 + 15246*\sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*\sin(f*x + e) \\
& ^6/(cos(f*x + e) + 1)^6 + 9240*\sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3465*s \\
& in(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*\sin(f*x + e)^9/(cos(f*x + e) + 1) \\
& ^9 - 61)/(c^6 - 11*c^6*\sin(f*x + e))/(cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e \\
&)^2/(cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 33 \\
& 0*c^6*\sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(cos(f*x \\
& + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*\sin(f* \\
& x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(cos(f*x + e) + 1)^8 \\
& - 55*c^6*\sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^10/(cos \\
& (f*x + e) + 1)^10 - c^6*\sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 3*B*a^2*(6 \\
& 71*\sin(f*x + e)/(cos(f*x + e) + 1) - 2200*\sin(f*x + e)^2/(cos(f*x + e) + 1) \\
& ^2 + 6600*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 10890*\sin(f*x + e)^4/(cos(f \\
& *x + e) + 1)^4 + 15246*\sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 12936*\sin(f*x \\
& + e)^6/(cos(f*x + e) + 1)^6 + 9240*\sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 34 \\
& 65*\sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 1155*\sin(f*x + e)^9/(cos(f*x + e) \\
& + 1)^9 - 61)/(c^6 - 11*c^6*\sin(f*x + e))/(cos(f*x + e) + 1) + 55*c^6*\sin(f*x \\
& + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 \\
& + 330*c^6*\sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(cos \\
& (f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*si \\
& n(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(cos(f*x + e) + \\
& 1)^8 - 55*c^6*\sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^10/ \\
& (cos(f*x + e) + 1)^10 - c^6*\sin(f*x + e)^11/(cos(f*x + e) + 1)^11) - 2*B*a^ \\
& 2*(341*\sin(f*x + e)/(cos(f*x + e) + 1) - 1705*\sin(f*x + e)^2/(cos(f*x + e) \\
& + 1)^2 + 5115*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6765*\sin(f*x + e)^4/(co \\
& s(f*x + e) + 1)^4 + 9471*\sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 4851*\sin(f*x \\
& + e)^6/(cos(f*x + e) + 1)^6 + 3465*\sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 3 \\
& 1)/(c^6 - 11*c^6*\sin(f*x + e))/(cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(c \\
& os(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 330*c^6* \\
& sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(cos(f*x + e) \\
& + 1)^5 + 462*c^6*\sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e) \\
& ^7/(cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 55* \\
& c^6*\sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^10/(cos(f*x + \\
& e) + 1)^10 - c^6*\sin(f*x + e)^11/(cos(f*x + e) + 1)^11) + 4*A*a^2*(253*\sin \\
& (f*x + e)/(cos(f*x + e) + 1) - 1265*\sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2 \\
& 640*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5280*\sin(f*x + e)^4/(cos(f*x + e) \\
& + 1)^4 + 5313*\sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 5313*\sin(f*x + e)^6/(c \\
& os(f*x + e) + 1)^6 + 2310*\sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 1155*\sin(f* \\
& x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*\sin(f*x + e))/(cos(f*x + e \\
&) + 1) + 55*c^6*\sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^ \\
& 3/(cos(f*x + e) + 1)^3 + 330*c^6*\sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 462* \\
& c^6*\sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(cos(f*x + \\
& e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x \\
& + e)^8/(cos(f*x + e) + 1)^8 - 55*c^6*\sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + \\
& 11*c^6*\sin(f*x + e)^10/(cos(f*x + e) + 1)^10 - c^6*\sin(f*x + e)^11/(cos(f*x \\
& + e) + 1)^11) + 8*B*a^2*(253*\sin(f*x + e)/(cos(f*x + e) + 1) - 1265*\sin(f* \\
& x + e)^2/(cos(f*x + e) + 1)^2 + 2640*\sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - \\
& 5280*\sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5313*\sin(f*x + e)^5/(cos(f*x + e \\
&) + 1)^5 - 5313*\sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2310*\sin(f*x + e)^7/(\\
& cos(f*x + e) + 1)^7 - 1155*\sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 23)/(c^6 - \\
& 11*c^6*\sin(f*x + e)/(cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(cos(f*x +
\end{aligned}$$

$$e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - c^6*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11)/f$$

mupad [B] time = 13.54, size = 423, normalized size = 2.71

$$2 \cos\left(\frac{e}{2} + \frac{f x}{2}\right) \left(\frac{38163 A a^2}{8} - \frac{1283 B a^2}{8} - \frac{11931 A a^2 \cos(2e+2fx)}{4} + \frac{9609 A a^2 \cos(3e+3fx)}{16} + \frac{1383 A a^2 \cos(4e+4fx)}{8} - \frac{225 A a^2 \cos(5e+5fx)}{16} + \frac{631 B a^2 \cos(2e+2fx)}{4} - \frac{1583 B a^2 \cos(3e+3fx)}{32} - \frac{223 B a^2 \cos(4e+4fx)}{8} + \frac{45 B a^2 \cos(5e+5fx)}{32} + 1386 A a^2 \sin(2e+2fx) + \frac{14949 A a^2 \sin(3e+3fx)}{16} - \frac{561 A a^2 \sin(4e+4fx)}{4} - \frac{231 A a^2 \sin(5e+5fx)}{16} - \frac{3003 B a^2 \sin(2e+2fx)}{8} - \frac{4653 B a^2 \sin(3e+3fx)}{32} + \frac{209 B a^2 \sin(4e+4fx)}{16} + \frac{77 B a^2 \sin(5e+5fx)}{32} - 2091 A a^2 \cos(e+fx) + \frac{281 B a^2 \cos(e+fx)}{16} - \frac{22869 A a^2 \sin(e+fx)}{4} + \frac{23331 B a^2 \sin(e+fx)}{16} \right) / (3465 c^6 f ((231 * 2^{1/2}) \cos(e/2 + \pi/4 + (f x)/2)) / 16 - (165 * 2^{1/2}) \cos((3e)/2 - \pi/4 + (3f x)/2)) / 16 - (165 * 2^{1/2}) \cos((5e)/2 + \pi/4 + (5f x)/2)) / 32 + (55 * 2^{1/2}) \cos((7e)/2 - \pi/4 + (7f x)/2)) / 32 + (11 * 2^{1/2}) \cos((9e)/2 + \pi/4 + (9f x)/2)) / 32 - (2^{1/2}) \cos((11e)/2 - \pi/4 + (11f x)/2)) / 32)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^6,x)

[Out] (2*cos(e/2 + (f*x)/2)*((38163*A*a^2)/8 - (1283*B*a^2)/8 - (11931*A*a^2*cos(2*e + 2*f*x))/4 + (9609*A*a^2*cos(3*e + 3*f*x))/16 + (1383*A*a^2*cos(4*e + 4*f*x))/8 - (225*A*a^2*cos(5*e + 5*f*x))/16 + (631*B*a^2*cos(2*e + 2*f*x))/4 - (1583*B*a^2*cos(3*e + 3*f*x))/32 - (223*B*a^2*cos(4*e + 4*f*x))/8 + (45*B*a^2*cos(5*e + 5*f*x))/32 + 1386*A*a^2*sin(2*e + 2*f*x) + (14949*A*a^2*sin(3*e + 3*f*x))/16 - (561*A*a^2*sin(4*e + 4*f*x))/4 - (231*A*a^2*sin(5*e + 5*f*x))/16 - (3003*B*a^2*sin(2*e + 2*f*x))/8 - (4653*B*a^2*sin(3*e + 3*f*x))/32 + (209*B*a^2*sin(4*e + 4*f*x))/16 + (77*B*a^2*sin(5*e + 5*f*x))/32 - 2091*A*a^2*cos(e + f*x) + (281*B*a^2*cos(e + f*x))/16 - (22869*A*a^2*sin(e + f*x))/4 + (23331*B*a^2*sin(e + f*x))/16)/(3465*c^6*f*((231*2^(1/2))*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2))*cos((3*e)/2 - pi/4 + (3*f*x)/2))/16 - (165*2^(1/2))*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2))*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2))*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 - (2^(1/2))*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32)

sympy [A] time = 125.92, size = 4816, normalized size = 30.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**6,x)

[Out] Piecewise((-6930*A*a**2*tan(e/2 + f*x/2)**10/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 20790*A*a**2*tan(e/2 + f*x/2)**9/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 83160*A*a**2*tan(e/2 + f*x/2)**8/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 138600*A*a**2*tan(e/2 + f*x/2)**7/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f


```

+ f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 +
f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f
*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*
x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2
)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 12012*B*a**2*tan(e/2
+ f*x/2)**6/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/
2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)
**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)*
**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**
4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 +
38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) - 44352*B*a**2*tan(e/2 + f*x/
2)**5/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10
+ 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 +
1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1
600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 57
1725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115
*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f) + 7920*B*a**2*tan(e/2 + f*x/2)**4/(
3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 1905
75*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450
*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*
c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c*
**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f
*tan(e/2 + f*x/2) - 3465*c**6*f) - 17820*B*a**2*tan(e/2 + f*x/2)**3/(3465*c
**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**
6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*
f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f
*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*t
an(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e
/2 + f*x/2) - 3465*c**6*f) - 220*B*a**2*tan(e/2 + f*x/2)**2/(3465*c**6*f*ta
n(e/2 + f*x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(
e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/
2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2
+ f*x/2)**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 +
f*x/2)**3 - 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x
/2) - 3465*c**6*f) - 1342*B*a**2*tan(e/2 + f*x/2)/(3465*c**6*f*tan(e/2 + f*
x/2)**11 - 38115*c**6*f*tan(e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/
2)**9 - 571725*c**6*f*tan(e/2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)
**7 - 1600830*c**6*f*tan(e/2 + f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)*
**5 - 1143450*c**6*f*tan(e/2 + f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3
- 190575*c**6*f*tan(e/2 + f*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465
*c**6*f) + 122*B*a**2/(3465*c**6*f*tan(e/2 + f*x/2)**11 - 38115*c**6*f*tan(
e/2 + f*x/2)**10 + 190575*c**6*f*tan(e/2 + f*x/2)**9 - 571725*c**6*f*tan(e/
2 + f*x/2)**8 + 1143450*c**6*f*tan(e/2 + f*x/2)**7 - 1600830*c**6*f*tan(e/2
+ f*x/2)**6 + 1600830*c**6*f*tan(e/2 + f*x/2)**5 - 1143450*c**6*f*tan(e/2
+ f*x/2)**4 + 571725*c**6*f*tan(e/2 + f*x/2)**3 - 190575*c**6*f*tan(e/2 + f
*x/2)**2 + 38115*c**6*f*tan(e/2 + f*x/2) - 3465*c**6*f), Ne(f, 0)), (x*(A +
B*sin(e))*(a*sin(e) + a)**2/(-c*sin(e) + c)**6, True))

```

$$3.37 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^7} dx$$

Optimal. Leaf size=197

$$\frac{2a^2(4A-9B) \cos^5(e+fx)}{15015c^2 f(c-c \sin(e+fx))^5} + \frac{a^2c^2(A+B) \cos^5(e+fx)}{13f(c-c \sin(e+fx))^9} + \frac{2a^2(4A-9B) \cos^5(e+fx)}{3003cf(c-c \sin(e+fx))^6} + \frac{a^2(4A-9B) \cos^5(e+fx)}{429f(c-c \sin(e+fx))^7}$$

[Out] 1/13*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^9+1/143*a^2*(4*A-9*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^8+1/429*a^2*(4*A-9*B)*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^7+2/3003*a^2*(4*A-9*B)*cos(f*x+e)^5/c/f/(c-c*sin(f*x+e))^6+2/15015*a^2*(4*A-9*B)*cos(f*x+e)^5/c^2/f/(c-c*sin(f*x+e))^5

Rubi [A] time = 0.46, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{2a^2(4A-9B) \cos^5(e+fx)}{15015c^2 f(c-c \sin(e+fx))^5} + \frac{a^2c^2(A+B) \cos^5(e+fx)}{13f(c-c \sin(e+fx))^9} + \frac{2a^2(4A-9B) \cos^5(e+fx)}{3003cf(c-c \sin(e+fx))^6} + \frac{a^2(4A-9B) \cos^5(e+fx)}{429f(c-c \sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]

[Out] (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(13*f*(c - c*Sin[e + f*x])^9) + (a^2*(4*A - 9*B)*c*Cos[e + f*x]^5)/(143*f*(c - c*Sin[e + f*x])^8) + (a^2*(4*A - 9*B)*Cos[e + f*x]^5)/(429*f*(c - c*Sin[e + f*x])^7) + (2*a^2*(4*A - 9*B)*Cos[e + f*x]^5)/(3003*c*f*(c - c*Sin[e + f*x])^6) + (2*a^2*(4*A - 9*B)*Cos[e + f*x]^5)/(15015*c^2*f*(c - c*Sin[e + f*x])^5)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

$e + f*x]), x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, n\}, x] \&\& EqQ[b*c + a*d, 0] \&\& EqQ[a^2 - b^2, 0] \&\& IntegerQ[m] \&\& !(IntegerQ[n] \&\& ((LtQ[m, 0] \& \& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^7} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{13 f (c - c \sin(e + fx))^9} + \frac{1}{13} (a^2 (4A - 9B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{13 f (c - c \sin(e + fx))^9} + \frac{a^2 (4A - 9B) c \cos^5(e + fx)}{143 f (c - c \sin(e + fx))^8} + \frac{1}{143} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{13 f (c - c \sin(e + fx))^9} + \frac{a^2 (4A - 9B) c \cos^5(e + fx)}{143 f (c - c \sin(e + fx))^8} + \frac{a^2}{42} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{13 f (c - c \sin(e + fx))^9} + \frac{a^2 (4A - 9B) c \cos^5(e + fx)}{143 f (c - c \sin(e + fx))^8} + \frac{a^2}{42} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{13 f (c - c \sin(e + fx))^9} + \frac{a^2 (4A - 9B) c \cos^5(e + fx)}{143 f (c - c \sin(e + fx))^8} + \frac{a^2}{42} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^9} dx \end{aligned}$$

Mathematica [A] time = 3.65, size = 313, normalized size = 1.59

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(6006(8A + 7B) \cos\left(\frac{1}{2}(e + fx)\right) - 1716(11A + 19B) \right)}{15015 c^7 f \cos^7(fx + e) + 7 c^7 f \cos^6(fx + e) - 49(4A - 9B) a^2 \cos^5(fx + e) + 70(4A - 9B) a^2 \cos^4(fx + e) - 12(4A - 9B) a^2 \cos^3(fx + e) + 49(4A - 9B) a^2 \cos^2(fx + e) - 70(4A - 9B) a^2 \cos(fx + e) + 70(4A - 9B) a^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]

[Out] -1/240240*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6006*(8*A + 7*B)*Cos[(e + f*x)/2] - 1716*(11*A + 19*B)*Cos[(3*(e + f*x))/2] - 15015*B*Cos[(5*(e + f*x))/2] - 1144*A*Cos[(7*(e + f*x))/2] + 2574*B*Cos[(7*(e + f*x))/2] + 52*A*Cos[(11*(e + f*x))/2] - 117*B*Cos[(11*(e + f*x))/2] + 54912*A*Sin[(e + f*x)/2] + 26598*B*Sin[(e + f*x)/2] + 24024*A*Sin[(3*(e + f*x))/2] + 21021*B*Sin[(3*(e + f*x))/2] - 2860*A*Sin[(5*(e + f*x))/2] - 8580*B*Sin[(5*(e + f*x))/2] + 312*A*Sin[(9*(e + f*x))/2] - 702*B*Sin[(9*(e + f*x))/2] - 4*A*Sin[(13*(e + f*x))/2] + 9*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(-1 + Sin[e + f*x])^7)

fricas [B] time = 0.44, size = 475, normalized size = 2.41

$$\frac{2(4A - 9B)a^2 \cos^7(fx + e) - 12(4A - 9B)a^2 \cos^6(fx + e) - 49(4A - 9B)a^2 \cos^5(fx + e) + 70(4A - 9B)a^2 \cos^4(fx + e) - 12(4A - 9B)a^2 \cos^3(fx + e) + 49(4A - 9B)a^2 \cos^2(fx + e) - 70(4A - 9B)a^2 \cos(fx + e) + 70(4A - 9B)a^2}{15015 \left(c^7 f \cos^7(fx + e) + 7 c^7 f \cos^6(fx + e) - 49(4A - 9B) a^2 \cos^5(fx + e) + 70(4A - 9B) a^2 \cos^4(fx + e) - 12(4A - 9B) a^2 \cos^3(fx + e) + 49(4A - 9B) a^2 \cos^2(fx + e) - 70(4A - 9B) a^2 \cos(fx + e) + 70(4A - 9B) a^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="fricas")

[Out] 1/15015*(2*(4*A - 9*B)*a^2*cos(f*x + e)^7 - 12*(4*A - 9*B)*a^2*cos(f*x + e)^6 - 49*(4*A - 9*B)*a^2*cos(f*x + e)^5 + 70*(4*A - 9*B)*a^2*cos(f*x + e)^4 - 12*(4*A - 9*B)*a^2*cos(f*x + e)^3 + 49*(4*A - 9*B)*a^2*cos(f*x + e)^2 - 70*(4*A - 9*B)*a^2*cos(f*x + e) + 70*(4*A - 9*B)*a^2)

$$+ 105*(7*A + 20*B)*a^2*\cos(f*x + e)^3 + 105*(25*A + 51*B)*a^2*\cos(f*x + e)^2 - 2310*(A + B)*a^2*\cos(f*x + e) - 4620*(A + B)*a^2 + (2*(4*A - 9*B)*a^2*\cos(f*x + e)^6 + 14*(4*A - 9*B)*a^2*\cos(f*x + e)^5 - 35*(4*A - 9*B)*a^2*\cos(f*x + e)^4 - 105*(4*A - 9*B)*a^2*\cos(f*x + e)^3 + 105*(3*A + 29*B)*a^2*\cos(f*x + e)^2 - 2310*(A + B)*a^2*\cos(f*x + e) - 4620*(A + B)*a^2)*\sin(f*x + e) / (c^7*f*\cos(f*x + e)^7 + 7*c^7*f*\cos(f*x + e)^6 - 18*c^7*f*\cos(f*x + e)^5 - 56*c^7*f*\cos(f*x + e)^4 + 48*c^7*f*\cos(f*x + e)^3 + 112*c^7*f*\cos(f*x + e)^2 - 32*c^7*f*\cos(f*x + e) - 64*c^7*f - (c^7*f*\cos(f*x + e)^6 - 6*c^7*f*\cos(f*x + e)^5 - 24*c^7*f*\cos(f*x + e)^4 + 32*c^7*f*\cos(f*x + e)^3 + 80*c^7*f*\cos(f*x + e)^2 - 32*c^7*f*\cos(f*x + e) - 64*c^7*f)*\sin(f*x + e))$$

giac [B] time = 0.33, size = 445, normalized size = 2.26

$$\frac{2 \left(15015 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{12} - 60060 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{11} + 15015 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{11} + 270270 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{10} - 15015 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{10} - 600600 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 + 105105 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 + 1174173 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 - 93093 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 - 1465464 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 234234 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 1559844 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 131274 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 1094808 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 181038 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 659945 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 47190 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 233948 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 45903 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 77454 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1599 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 7904 A a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 2769 B a^2 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1763 A a^2 - 213 B a^2 \right)}{(c^7 f (\tan(\frac{1}{2} f x + \frac{1}{2} e) - 1)^{13})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="giac")

[Out] -2/15015*(15015*A*a^2*tan(1/2*f*x + 1/2*e)^12 - 60060*A*a^2*tan(1/2*f*x + 1/2*e)^11 + 15015*B*a^2*tan(1/2*f*x + 1/2*e)^11 + 270270*A*a^2*tan(1/2*f*x + 1/2*e)^10 - 15015*B*a^2*tan(1/2*f*x + 1/2*e)^10 - 600600*A*a^2*tan(1/2*f*x + 1/2*e)^9 + 105105*B*a^2*tan(1/2*f*x + 1/2*e)^9 + 1174173*A*a^2*tan(1/2*f*x + 1/2*e)^8 - 93093*B*a^2*tan(1/2*f*x + 1/2*e)^8 - 1465464*A*a^2*tan(1/2*f*x + 1/2*e)^7 + 234234*B*a^2*tan(1/2*f*x + 1/2*e)^7 + 1559844*A*a^2*tan(1/2*f*x + 1/2*e)^6 - 131274*B*a^2*tan(1/2*f*x + 1/2*e)^6 - 1094808*A*a^2*tan(1/2*f*x + 1/2*e)^5 + 181038*B*a^2*tan(1/2*f*x + 1/2*e)^5 + 659945*A*a^2*tan(1/2*f*x + 1/2*e)^4 - 47190*B*a^2*tan(1/2*f*x + 1/2*e)^4 - 233948*A*a^2*tan(1/2*f*x + 1/2*e)^3 + 45903*B*a^2*tan(1/2*f*x + 1/2*e)^3 + 77454*A*a^2*tan(1/2*f*x + 1/2*e)^2 - 1599*B*a^2*tan(1/2*f*x + 1/2*e)^2 - 7904*A*a^2*tan(1/2*f*x + 1/2*e) + 2769*B*a^2*tan(1/2*f*x + 1/2*e) + 1763*A*a^2 - 213*B*a^2)/(c^7*f*(tan(1/2*f*x + 1/2*e) - 1)^13)

maple [A] time = 0.57, size = 293, normalized size = 1.49

$$2a^2 \left(-\frac{10896A+9360B}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{1816A+884B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{4480A+4352B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}} - \frac{1536A+1536B}{12\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{12}} - \frac{560A+208B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{16A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)

[Out] 2/f*a^2/c^7*(-1/9*(10896*A+9360*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/5*(1816*A+884*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/11*(4480*A+4352*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/12*(1536*A+1536*B)/(tan(1/2*f*x+1/2*e)-1)^12-1/4*(560*A+208*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/2*(16*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/3*(120*A+30*B)/(tan(1/2*f*x+1/2*e)-1)^3-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(7744*A+5368*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/13*(256*A+256*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/10*(8320*A+7680*B)/(tan(1/2*f*x+1/2*e)-1)^10-1/6*(4320*A+2568*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/8*(10560*A+8256*B)/(tan(1/2*f*x+1/2*e)-1)^8)

maxima [B] time = 0.69, size = 3120, normalized size = 15.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2/45045*(2*A*a^2*(4771*\sin(f*x + e))/(\cos(f*x + e) + 1) - 28626*\sin(f*x + e) \\ &)^2/(\cos(f*x + e) + 1)^2 + 74932*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1873 \\ & 30*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 265122*\sin(f*x + e)^5/(\cos(f*x + e) \\ &) + 1)^5 - 353496*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 276276*\sin(f*x + e) \\ & ^7/(\cos(f*x + e) + 1)^7 - 207207*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 7507 \\ & 5*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 30030*\sin(f*x + e)^10/(\cos(f*x + e) \\ & + 1)^10 - 367)/(c^7 - 13*c^7*\sin(f*x + e))/(\cos(f*x + e) + 1) + 78*c^7*\sin(\\ & f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1) \\ & ^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/ \\ & (\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716* \\ & c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x \\ & + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x \\ & + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^ \\ & 11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(co \\ & s(f*x + e) + 1)^13 + 4*B*a^2*(4771*\sin(f*x + e))/(\cos(f*x + e) + 1) - 28626 \\ & *\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 74932*\sin(f*x + e)^3/(\cos(f*x + e) + \\ & 1)^3 - 187330*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 265122*\sin(f*x + e)^5/ \\ & (\cos(f*x + e) + 1)^5 - 353496*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 276276* \\ & \sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 207207*\sin(f*x + e)^8/(\cos(f*x + e) + \\ & 1)^8 + 75075*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 30030*\sin(f*x + e)^10/(\\ & \cos(f*x + e) + 1)^10 - 367)/(c^7 - 13*c^7*\sin(f*x + e))/(\cos(f*x + e) + 1) + \\ & 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f \\ & *x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin \\ & (f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + \\ & 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e) \\ & ^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286 \\ & *c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f* \\ & x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x \\ & + e)^13/(\cos(f*x + e) + 1)^13 + 15*A*a^2*(3796*\sin(f*x + e))/(\cos(f*x + e) \\ & + 1) - 22776*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 77506*\sin(f*x + e)^3/(c \\ & os(f*x + e) + 1)^3 - 193765*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 339768*si \\ & n(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 453024*\sin(f*x + e)^6/(\cos(f*x + e) + 1 \\ &)^6 + 444444*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 333333*\sin(f*x + e)^8/(c \\ & os(f*x + e) + 1)^8 + 180180*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 72072*\sin \\ & (f*x + e)^10/(\cos(f*x + e) + 1)^10 + 18018*\sin(f*x + e)^11/(\cos(f*x + e) + \\ & 1)^11 - 3003*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 523)/(c^7 - 13*c^7*\sin \\ & (f*x + e))/(\cos(f*x + e) + 1) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - \\ & 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(\\ & f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7* \\ & \sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) \\ & + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e) \\ &)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - \\ & 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(\\ & f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13) - 70*A*a^2*(6 \\ & 11*\sin(f*x + e))/(\cos(f*x + e) + 1) - 2379*\sin(f*x + e)^2/(\cos(f*x + e) + 1) \\ & ^2 + 8723*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 18590*\sin(f*x + e)^4/(\cos(f \\ & *x + e) + 1)^4 + 33462*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 40326*\sin(f*x \\ & + e)^6/(\cos(f*x + e) + 1)^6 + 40326*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 2 \\ & 7027*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 15015*\sin(f*x + e)^9/(\cos(f*x + \\ & e) + 1)^9 - 4719*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 1287*\sin(f*x + e)^ \\ & 11/(\cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*\sin(f*x + e))/(\cos(f*x + e) + 1 \\ &) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(co \\ & s(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7* \\ & \sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) \\ & + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + \\ & e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + \\ & 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos \end{aligned}$$

```
(f*x + e) + 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(
f*x + e)^13/(cos(f*x + e) + 1)^13) - 35*B*a^2*(611*sin(f*x + e)/(cos(f*x +
e) + 1) - 2379*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8723*sin(f*x + e)^3/(c
os(f*x + e) + 1)^3 - 18590*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 33462*sin(
f*x + e)^5/(cos(f*x + e) + 1)^5 - 40326*sin(f*x + e)^6/(cos(f*x + e) + 1)^6
+ 40326*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 27027*sin(f*x + e)^8/(cos(f*
x + e) + 1)^8 + 15015*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 - 4719*sin(f*x +
e)^10/(cos(f*x + e) + 1)^10 + 1287*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 -
47)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 715*c^7
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x + e)^5/(cos(f*x + e
) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1716*c^7*sin(f*x
+ e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos(f*x + e) + 1)^8
- 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*sin(f*x + e)^10/(c
os(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e) + 1)^11 + 13*c^7*
sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^13/(cos(f*x + e) +
1)^13) - 462*B*a^2*(13*sin(f*x + e)/(cos(f*x + e) + 1) - 78*sin(f*x + e)^2
/(cos(f*x + e) + 1)^2 + 286*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 520*sin(f
*x + e)^4/(cos(f*x + e) + 1)^4 + 936*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 -
858*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 858*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7 - 351*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 195*sin(f*x + e)^9/(cos(
f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*sin(f*x + e)/(cos(f*x + e) + 1) + 78*c^7
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 286*c^7*sin(f*x + e)^3/(cos(f*x + e
) + 1)^3 + 715*c^7*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1287*c^7*sin(f*x +
e)^5/(cos(f*x + e) + 1)^5 + 1716*c^7*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
1716*c^7*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1287*c^7*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - 715*c^7*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 286*c^7*si
n(f*x + e)^10/(cos(f*x + e) + 1)^10 - 78*c^7*sin(f*x + e)^11/(cos(f*x + e)
+ 1)^11 + 13*c^7*sin(f*x + e)^12/(cos(f*x + e) + 1)^12 - c^7*sin(f*x + e)^1
3/(cos(f*x + e) + 1)^13))/f
```

mupad [B] time = 14.05, size = 500, normalized size = 2.54

$$2 \cos\left(\frac{e}{2} + \frac{f x}{2}\right) \left(\frac{994249 A a^2}{32} - \frac{63639 B a^2}{32} - \frac{1609013 A a^2 \cos(2e+2fx)}{64} + \frac{85687 A a^2 \cos(3e+3fx)}{16} + \frac{79591 A a^2 \cos(4e+4fx)}{32} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^7,x)

[Out] $-(2*\cos(e/2 + (f*x)/2))*((994249*A*a^2)/32 - (63639*B*a^2)/32 - (1609013*A*a^2*\cos(2*e + 2*f*x))/64 + (85687*A*a^2*\cos(3*e + 3*f*x))/16 + (79591*A*a^2*\cos(4*e + 4*f*x))/32 - (5261*A*a^2*\cos(5*e + 5*f*x))/16 - (1771*A*a^2*\cos(6*e + 6*f*x))/64 + (140553*B*a^2*\cos(2*e + 2*f*x))/64 - (4431*B*a^2*\cos(3*e + 3*f*x))/8 - (10161*B*a^2*\cos(4*e + 4*f*x))/32 + 36*B*a^2*\cos(5*e + 5*f*x) + (231*B*a^2*\cos(6*e + 6*f*x))/64 + (636207*A*a^2*\sin(2*e + 2*f*x))/64 + (309309*A*a^2*\sin(3*e + 3*f*x))/32 - (7007*A*a^2*\sin(4*e + 4*f*x))/4 - (12389*A*a^2*\sin(5*e + 5*f*x))/32 + (1755*A*a^2*\sin(6*e + 6*f*x))/64 - (121407*B*a^2*\sin(2*e + 2*f*x))/64 - (39039*B*a^2*\sin(3*e + 3*f*x))/32 + (3003*B*a^2*\sin(4*e + 4*f*x))/16 + (1599*B*a^2*\sin(5*e + 5*f*x))/32 - (195*B*a^2*\sin(6*e + 6*f*x))/64 - (93221*A*a^2*\cos(e + f*x))/8 + (3291*B*a^2*\cos(e + f*x))/8 - (704847*A*a^2*\sin(e + f*x))/16 + (125697*B*a^2*\sin(e + f*x))/16)/(15015*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**7,x)

[Out] Timed out

$$3.38 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^6 dx$$

Optimal. Leaf size=265

$$\frac{11a^3c^6(10A - 3B) \cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B) \cos^7(e + fx) (c^6 - c^6 \sin(e + fx))}{720f} + \frac{11a^3c^6(10A - 3B) \sin(e + fx)}{480f}$$

[Out] 11/256*a^3*(10*A-3*B)*c^6*x+11/560*a^3*(10*A-3*B)*c^6*cos(f*x+e)^7/f+11/256*a^3*(10*A-3*B)*c^6*cos(f*x+e)*sin(f*x+e)/f+11/384*a^3*(10*A-3*B)*c^6*cos(f*x+e)^3*sin(f*x+e)/f+11/480*a^3*(10*A-3*B)*c^6*cos(f*x+e)^5*sin(f*x+e)/f-1/10*a^3*B*cos(f*x+e)^7*(c^2-c^2*sin(f*x+e))^3/f+1/90*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^3-c^3*sin(f*x+e))^2/f+11/720*a^3*(10*A-3*B)*cos(f*x+e)^7*(c^6-c^6*sin(f*x+e))/f

Rubi [A] time = 0.39, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{11a^3c^6(10A - 3B) \cos^7(e + fx)}{560f} + \frac{a^3(10A - 3B) \cos^7(e + fx) (c^3 - c^3 \sin(e + fx))^2}{90f} + \frac{11a^3(10A - 3B) \cos^7(e + fx)}{720f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]

[Out] (11*a^3*(10*A - 3*B)*c^6*x)/256 + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^7)/(560*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]*Sin[e + f*x])/(256*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^3*Sin[e + f*x])/(384*f) + (11*a^3*(10*A - 3*B)*c^6*Cos[e + f*x]^5*Sin[e + f*x])/(480*f) - (a^3*B*Cos[e + f*x]^7*(c^2 - c^2*Sin[e + f*x]^3))/(10*f) + (a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^3 - c^3*Sin[e + f*x]^2))/(90*f) + (11*a^3*(10*A - 3*B)*Cos[e + f*x]^7*(c^6 - c^6*Sin[e + f*x]))/(720*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2]

*m, 2*p]

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^6 dx &= (a^3 c^3) \int \cos^6(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^6 dx \\ &= -\frac{a^3 B \cos^7(e + fx) (c^2 - c^2 \sin(e + fx))^3}{10f} + \frac{a^3 B \cos^7(e + fx) (c^2 - c^2 \sin(e + fx))^3}{10f} \\ &= -\frac{a^3 B \cos^7(e + fx) (c^2 - c^2 \sin(e + fx))^3}{10f} + \frac{a^3 B \cos^7(e + fx) (c^2 - c^2 \sin(e + fx))^3}{10f} \\ &= \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} - \frac{a^3 B \cos^7(e + fx)}{560f} \\ &= \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} \\ &= \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} \\ &= \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} + \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} \\ &= \frac{11}{256} a^3 (10A - 3B) c^6 x + \frac{11a^3(10A - 3B)c^6 \cos^7(e + fx)}{560f} \end{aligned}$$

Mathematica [A] time = 4.31, size = 255, normalized size = 0.96

$$\frac{(a \sin(e + fx) + a)^3 (c - c \sin(e + fx))^6 (27720(10A - 3B)(e + fx) + 1260(144A - 25B) \sin(2(e + fx)) + 2520)}{560f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^6,x]
```

[Out] $((a + a*\sin[e + f*x])^3*(c - c*\sin[e + f*x])^6*(27720*(10*A - 3*B)*(e + f*x) + 5040*(33*A - 19*B)*\cos[e + f*x] + 3360*(29*A - 15*B)*\cos[3*(e + f*x)] + 10080*(3*A - B)*\cos[5*(e + f*x)] + 360*(9*A + 5*B)*\cos[7*(e + f*x)] - 280*(A - 3*B)*\cos[9*(e + f*x)] + 1260*(144*A - 25*B)*\sin[2*(e + f*x)] + 2520*(6*A + 7*B)*\sin[4*(e + f*x)] - 210*(32*A - 51*B)*\sin[6*(e + f*x)] - 315*(6*A - 5*B)*\sin[8*(e + f*x)] - 126*B*\sin[10*(e + f*x)])/(645120*f*(\cos[(e + f*x)/2] - \sin[(e + f*x)/2])^12*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^6)$

fricas [A] time = 0.50, size = 181, normalized size = 0.68

$$8960(A - 3B)a^3c^6 \cos(fx + e)^9 - 46080(A - B)a^3c^6 \cos(fx + e)^7 - 3465(10A - 3B)a^3c^6 fx + 21(384Ba^3c^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorithm="fricas")

[Out] $-1/80640*(8960*(A - 3*B)*a^3*c^6*\cos(f*x + e)^9 - 46080*(A - B)*a^3*c^6*\cos(f*x + e)^7 - 3465*(10*A - 3*B)*a^3*c^6*f*x + 21*(384*B*a^3*c^6*\cos(f*x + e)^9 + 48*(30*A - 41*B)*a^3*c^6*\cos(f*x + e)^7 - 88*(10*A - 3*B)*a^3*c^6*\cos(f*x + e)^5 - 110*(10*A - 3*B)*a^3*c^6*\cos(f*x + e)^3 - 165*(10*A - 3*B)*a^3*c^6*\cos(f*x + e))*\sin(f*x + e))/f$

giac [A] time = 0.27, size = 347, normalized size = 1.31

$$-\frac{Ba^3c^6 \sin(10fx + 10e)}{5120f} + \frac{11}{256} (10Aa^3c^6 - 3Ba^3c^6)x - \frac{(Aa^3c^6 - 3Ba^3c^6) \cos(9fx + 9e)}{2304f} + \frac{(9Aa^3c^6 + 5Ba^3c^6)}{1792}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorithm="giac")

[Out] $-1/5120*B*a^3*c^6*\sin(10*f*x + 10*e)/f + 11/256*(10*A*a^3*c^6 - 3*B*a^3*c^6)*x - 1/2304*(A*a^3*c^6 - 3*B*a^3*c^6)*\cos(9*f*x + 9*e)/f + 1/1792*(9*A*a^3*c^6 + 5*B*a^3*c^6)*\cos(7*f*x + 7*e)/f + 1/64*(3*A*a^3*c^6 - B*a^3*c^6)*\cos(5*f*x + 5*e)/f + 1/192*(29*A*a^3*c^6 - 15*B*a^3*c^6)*\cos(3*f*x + 3*e)/f + 1/128*(33*A*a^3*c^6 - 19*B*a^3*c^6)*\cos(f*x + e)/f - 1/2048*(6*A*a^3*c^6 - 5*B*a^3*c^6)*\sin(8*f*x + 8*e)/f - 1/3072*(32*A*a^3*c^6 - 51*B*a^3*c^6)*\sin(6*f*x + 6*e)/f + 1/256*(6*A*a^3*c^6 + 7*B*a^3*c^6)*\sin(4*f*x + 4*e)/f + 1/512*(144*A*a^3*c^6 - 25*B*a^3*c^6)*\sin(2*f*x + 2*e)/f$

maple [B] time = 0.85, size = 651, normalized size = 2.46

$$a^3A c^6 (fx + e) - B a^3c^6 \cos(fx + e) - \frac{8a^3A c^6(2+\sin^2(fx+e))\cos(fx+e)}{3} + 8B a^3c^6 \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3f}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x)

[Out] $1/f*(a^3*A*c^6*(f*x+e) - B*a^3*c^6*\cos(f*x+e) - 8/3*a^3*A*c^6*(2+\sin(f*x+e))^2)*\cos(f*x+e) + 8*B*a^3*c^6*(-1/4*(\sin(f*x+e)^3 + 3/2*\sin(f*x+e))*\cos(f*x+e) + 3/8*f*x + 3/8*e) + 3*a^3*A*c^6*\cos(f*x+e) + 6/5*B*a^3*c^6*(8/3 + \sin(f*x+e))^4 + 4/3*\sin(f*x+e)^2)*\cos(f*x+e) + B*a^3*c^6*(-1/10*(\sin(f*x+e)^9 + 9/8*\sin(f*x+e)^7 + 21/16*\sin(f*x+e)^5 + 105/64*\sin(f*x+e)^3 + 315/128*\sin(f*x+e))*\cos(f*x+e) + 63/256*f*x + 63$

$$\begin{aligned} & /256*e)+1/3*B*a^3*c^6*(128/35+\sin(f*x+e))^8+8/7*\sin(f*x+e)^6+48/35*\sin(f*x+e) \\ & ^4+64/35*\sin(f*x+e)^2)*\cos(f*x+e)-8/7*B*a^3*c^6*(16/5+\sin(f*x+e))^6+6/5*\sin \\ & (f*x+e)^4+8/5*\sin(f*x+e)^2)*\cos(f*x+e)-6*B*a^3*c^6*(-1/6*(\sin(f*x+e))^5+5/4* \\ & \sin(f*x+e)^3+15/8*\sin(f*x+e))*\cos(f*x+e)+5/16*f*x+5/16*e)-1/9*a^3*A*c^6*(12 \\ & 8/35+\sin(f*x+e))^8+8/7*\sin(f*x+e)^6+48/35*\sin(f*x+e)^4+64/35*\sin(f*x+e)^2)*c \\ & \cos(f*x+e)-3*a^3*A*c^6*(-1/8*(\sin(f*x+e))^7+7/6*\sin(f*x+e)^5+35/24*\sin(f*x+e) \\ & ^3+35/16*\sin(f*x+e))*\cos(f*x+e)+35/128*f*x+35/128*e)+8*a^3*A*c^6*(-1/6*(\sin \\ & (f*x+e))^5+5/4*\sin(f*x+e)^3+15/8*\sin(f*x+e))*\cos(f*x+e)+5/16*f*x+5/16*e)+6/5 \\ & *a^3*A*c^6*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)-6*a^3*A*c^6*(-1/4 \\ & *(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-3*B*a^3*c^6*(-1/2* \\ & \sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)) \end{aligned}$$

maxima [B] time = 0.36, size = 661, normalized size = 2.49

$$2048 \left(35 \cos (fx + e)^9 - 180 \cos (fx + e)^7 + 378 \cos (fx + e)^5 - 420 \cos (fx + e)^3 + 315 \cos (fx + e) \right) A a^3 c^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^6,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/645120*(2048*(35*\cos(f*x + e)^9 - 180*\cos(f*x + e)^7 + 378*\cos(f*x + e)^5 - 420*\cos(f*x + e)^3 + 315*\cos(f*x + e))*A*a^3*c^6 - 258048*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a^3*c^6 - 1720320*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^3*c^6 + 630*(128*\sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*\sin(8*f*x + 8*e) + 168*\sin(4*f*x + 4*e) - 768*\sin(2*f*x + 2*e))*A*a^3*c^6 - 26880*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*A*a^3*c^6 + 120960*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^3*c^6 - 645120*(f*x + e)*A*a^3*c^6 - 6144*(35*\cos(f*x + e)^9 - 180*\cos(f*x + e)^7 + 378*\cos(f*x + e)^5 - 420*\cos(f*x + e)^3 + 315*\cos(f*x + e))*B*a^3*c^6 - 147456*(5*\cos(f*x + e)^7 - 21*\cos(f*x + e)^5 + 35*\cos(f*x + e)^3 - 35*\cos(f*x + e))*B*a^3*c^6 - 258048*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a^3*c^6 + 63*(32*\sin(2*f*x + 2*e)^5 - 640*\sin(2*f*x + 2*e)^3 - 2520*f*x - 2520*e - 25*\sin(8*f*x + 8*e) - 600*\sin(4*f*x + 4*e) + 2560*\sin(2*f*x + 2*e))*B*a^3*c^6 + 20160*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B*a^3*c^6 - 161280*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^3*c^6 + 483840*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^3*c^6 - 1935360*A*a^3*c^6*\cos(f*x + e) + 645120*B*a^3*c^6*\cos(f*x + e))/f \end{aligned}$$

mupad [B] time = 14.87, size = 812, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6,x)

[Out]
$$\begin{aligned} & (\tan(e/2 + (f*x)/2)^{18}*(6*A*a^3*c^6 - 2*B*a^3*c^6) + \tan(e/2 + (f*x)/2)^{16} \\ & (22*A*a^3*c^6 - 18*B*a^3*c^6) + \tan(e/2 + (f*x)/2)^8*(84*A*a^3*c^6 - 28*B*a \\ & ^3*c^6) + \tan(e/2 + (f*x)/2)^{14}*((136*A*a^3*c^6)/3 - 8*B*a^3*c^6) + \tan(e/2 \\ & + (f*x)/2)^4*((136*A*a^3*c^6)/7 - (24*B*a^3*c^6)/7) + \tan(e/2 + (f*x)/2)^{1 \\ & 0}*(116*A*a^3*c^6 - 60*B*a^3*c^6) - \tan(e/2 + (f*x)/2)^{19}*((73*A*a^3*c^6)/64 \\ & + (33*B*a^3*c^6)/128) + \tan(e/2 + (f*x)/2)^2*((202*A*a^3*c^6)/63 - (58*B*a \\ & ^3*c^6)/21) + \tan(e/2 + (f*x)/2)^{12}*((328*A*a^3*c^6)/3 - 72*B*a^3*c^6) + \tan \\ & (e/2 + (f*x)/2)^7*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) - \tan(e/2 + (f \\ & *x)/2)^{13}*((341*A*a^3*c^6)/16 + (333*B*a^3*c^6)/32) + \tan(e/2 + (f*x)/2)^6* \\ & ((456*A*a^3*c^6)/7 - (344*B*a^3*c^6)/7) + \tan(e/2 + (f*x)/2)^5*((449*A*a^3*c \\ & ^6)/48 - (577*B*a^3*c^6)/160) - \tan(e/2 + (f*x)/2)^{15}*((449*A*a^3*c^6)/48 \\ & - (577*B*a^3*c^6)/160) + \tan(e/2 + (f*x)/2)^3*((2117*A*a^3*c^6)/192 - (705* \end{aligned}$$

$$B*a^3*c^6)/128) - \tan(e/2 + (f*x)/2)^{17}*((2117*A*a^3*c^6)/192 - (705*B*a^3*c^6)/128) + \tan(e/2 + (f*x)/2)^9*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64) - \tan(e/2 + (f*x)/2)^{11}*((699*A*a^3*c^6)/32 - (2749*B*a^3*c^6)/64) + \tan(e/2 + (f*x)/2)*((73*A*a^3*c^6)/64 + (33*B*a^3*c^6)/128) + (58*A*a^3*c^6)/63 - (10*B*a^3*c^6)/21)/(f*(10*\tan(e/2 + (f*x)/2)^2 + 45*\tan(e/2 + (f*x)/2)^4 + 120*\tan(e/2 + (f*x)/2)^6 + 210*\tan(e/2 + (f*x)/2)^8 + 252*\tan(e/2 + (f*x)/2)^{10} + 210*\tan(e/2 + (f*x)/2)^{12} + 120*\tan(e/2 + (f*x)/2)^{14} + 45*\tan(e/2 + (f*x)/2)^{16} + 10*\tan(e/2 + (f*x)/2)^{18} + \tan(e/2 + (f*x)/2)^{20} + 1)) + (11*a^3*c^6*atan((11*a^3*c^6*\tan(e/2 + (f*x)/2)*(10*A - 3*B))/(128*((55*A*a^3*c^6)/64 - (33*B*a^3*c^6)/128)))*(10*A - 3*B))/(128*f)$$

sympy [A] time = 52.76, size = 1948, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**6,x)

[Out] Piecewise((-105*A*a**3*c**6*x*sin(e + f*x)**8/128 - 105*A*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**6*x*sin(e + f*x)**6/2 - 315*A*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/2 - 9*A*a**3*c**6*x*sin(e + f*x)**4/4 - 105*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/2 - 9*A*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - 105*A*a**3*c**6*x*cos(e + f*x)**8/128 + 5*A*a**3*c**6*x*cos(e + f*x)**6/2 - 9*A*a**3*c**6*x*cos(e + f*x)**4/4 + A*a**3*c**6*x - A*a**3*c**6*sin(e + f*x)*8*cos(e + f*x)/f + 279*A*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 8*A*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) + 511*A*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**3/(128*f) - 11*A*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(2*f) - 16*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) + 6*A*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 20*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) + 15*A*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 64*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) + 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f - 8*A*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)/f + 105*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**5/(2*f) + 9*A*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 128*A*a**3*c**6*cos(e + f*x)**9/(315*f) + 16*A*a**3*c**6*cos(e + f*x)**5/(5*f) - 16*A*a**3*c**6*cos(e + f*x)**3/(3*f) + 3*A*a**3*c**6*cos(e + f*x)/f + 63*B*a**3*c**6*x*sin(e + f*x)**10/256 + 315*B*a**3*c**6*x*sin(e + f*x)**8*cos(e + f*x)**2/256 + 315*B*a**3*c**6*x*sin(e + f*x)**6*cos(e + f*x)**4/128 - 15*B*a**3*c**6*x*sin(e + f*x)**6/8 + 315*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**6/128 - 45*B*a**3*c**6*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*B*a**3*c**6*x*sin(e + f*x)**4 + 315*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**8/256 - 45*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 6*B*a**3*c**6*x*sin(e + f*x)**2*cos(e + f*x)**2 - 3*B*a**3*c**6*x*sin(e + f*x)**2/2 + 63*B*a**3*c**6*x*cos(e + f*x)**10/256 - 15*B*a**3*c**6*x*cos(e + f*x)**6/8 + 3*B*a**3*c**6*x*cos(e + f*x)**4 - 3*B*a**3*c**6*x*cos(e + f*x)**2/2 - 193*B*a**3*c**6*sin(e + f*x)**9*cos(e + f*x)/(256*f) + 3*B*a**3*c**6*sin(e + f*x)**8*cos(e + f*x)/f - 237*B*a**3*c**6*sin(e + f*x)**7*cos(e + f*x)**3/(128*f) + 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)**3/f - 8*B*a**3*c**6*sin(e + f*x)**6*cos(e + f*x)/f - 21*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)**5/(10*f) + 33*B*a**3*c**6*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 48*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) - 16*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)**3/f + 6*B*a**3*c**6*sin(e + f*x)**4*cos(e + f*x)/f - 147*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**7/(128*f) + 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)**3/f - 5*B*a**3*c**6*sin(e + f*x)**3*cos(e + f*x)/f + 192*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 64*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 8*B*a**3*c**6*sin(e + f*x)**2*cos(e + f*x)**3/f - 63*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**9/(256*f) + 15*B*a**3*c**6*sin(e

```

+ f*x)*cos(e + f*x)**5/(8*f) - 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)**3/
f + 3*B*a**3*c**6*sin(e + f*x)*cos(e + f*x)/(2*f) + 128*B*a**3*c**6*cos(e +
f*x)**9/(105*f) - 128*B*a**3*c**6*cos(e + f*x)**7/(35*f) + 16*B*a**3*c**6*
cos(e + f*x)**5/(5*f) - B*a**3*c**6*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*si
n(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**6, True))

```

$$3.39 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^5 dx$$

Optimal. Leaf size=222

$$\frac{a^3 c^5 (9A - 2B) \cos^7(e + fx)}{56f} + \frac{a^3 (9A - 2B) \cos^7(e + fx) (c^5 - c^5 \sin(e + fx))}{72f} + \frac{a^3 c^5 (9A - 2B) \sin(e + fx) \cos^5(e + fx)}{48f}$$

[Out] 5/128*a^3*(9*A-2*B)*c^5*x+1/56*a^3*(9*A-2*B)*c^5*cos(f*x+e)^7/f+5/128*a^3*(9*A-2*B)*c^5*cos(f*x+e)*sin(f*x+e)/f+5/192*a^3*(9*A-2*B)*c^5*cos(f*x+e)^3*sin(f*x+e)/f+1/48*a^3*(9*A-2*B)*c^5*cos(f*x+e)^5*sin(f*x+e)/f-1/9*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^2/f+1/72*a^3*(9*A-2*B)*cos(f*x+e)^7*(c^5-c^5*sin(f*x+e))/f

Rubi [A] time = 0.32, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^3 c^5 (9A - 2B) \cos^7(e + fx)}{56f} + \frac{a^3 (9A - 2B) \cos^7(e + fx) (c^5 - c^5 \sin(e + fx))}{72f} + \frac{a^3 c^5 (9A - 2B) \sin(e + fx) \cos^5(e + fx)}{48f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5,x]

[Out] (5*a^3*(9*A - 2*B)*c^5*x)/128 + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^7)/(56*f) + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + (a^3*(9*A - 2*B)*c^5*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) - (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^2)/(9*f) + (a^3*(9*A - 2*B)*Cos[e + f*x]^7*(c^5 - c^5*Sin[e + f*x]))/(72*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^5 dx &= (a^3 c^3) \int \cos^6(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^5 dx \\ &= -\frac{a^3 B c^3 \cos^7(e + fx) (c - c \sin(e + fx))^2}{9f} + \frac{a^3 B c^3 \cos^7(e + fx) (c - c \sin(e + fx))^2}{9f} \\ &= -\frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} - \frac{a^3 B c^3 \cos^7(e + fx)}{56f} \\ &= \frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} + \frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} \\ &= \frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} + \frac{5a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} \\ &= \frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} + \frac{5a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} \\ &= \frac{5}{128} a^3 (9A - 2B) c^5 x + \frac{a^3 (9A - 2B) c^5 \cos^7(e + fx)}{56f} \end{aligned}$$

Mathematica [A] time = 2.57, size = 232, normalized size = 1.05

$$\frac{(a \sin(e + fx) + a)^3 (c - c \sin(e + fx))^5 (2520(9A - 2B)(e + fx) + 2016(8A - B) \sin(2(e + fx)) + 504(5A + 2B) \cos(e + fx) + 336(18A - 11B) \cos(3(e + fx)) + 1008(2A - B) \cos(5(e + fx)) + 36(8A - B) \cos(7(e + fx)) + 28B \cos(9(e + fx)) + 2016(8A - B) \sin(2(e + fx)) + 504(5A + 2B) \sin(4(e + fx)) + 672B \sin(6(e + fx)) - 63(A - 2B) \sin(8(e + fx)))}{(64512 f^* (\cos(e + fx)))^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5, x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5*(2520*(9*A - 2*B)*(e + f*x) + 504*(20*A - 13*B)*Cos[e + f*x] + 336*(18*A - 11*B)*Cos[3*(e + f*x)] + 1008*(2*A - B)*Cos[5*(e + f*x)] + 36*(8*A - B)*Cos[7*(e + f*x)] + 28*B*Cos[9*(e + f*x)] + 2016*(8*A - B)*Sin[2*(e + f*x)] + 504*(5*A + 2*B)*Sin[4*(e + f*x)] + 672*B*Sin[6*(e + f*x)] - 63*(A - 2*B)*Sin[8*(e + f*x)]))/(64512*f*(Cos[e + f*x])^6)
```

$s[(e + f*x)/2] - \text{Sin}[(e + f*x)/2])^{10} * (\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2])^6)$

fricas [A] time = 0.50, size = 158, normalized size = 0.71

$$896 Ba^3c^5 \cos(fx + e)^9 + 2304(A - B)a^3c^5 \cos(fx + e)^7 + 315(9A - 2B)a^3c^5 fx - 21(48(A - 2B)a^3c^5 \cos(fx + e)^7 - 8(9A - 2B)a^3c^5 \cos(fx + e)^5 - 10(9A - 2B)a^3c^5 \cos(fx + e)^3 - 15(9A - 2B)a^3c^5 \cos(fx + e)) \sin(fx + e) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] 1/8064*(896*B*a^3*c^5*cos(f*x + e)^9 + 2304*(A - B)*a^3*c^5*cos(f*x + e)^7 + 315*(9*A - 2*B)*a^3*c^5*f*x - 21*(48*(A - 2*B)*a^3*c^5*cos(f*x + e)^7 - 8*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^5 - 10*(9*A - 2*B)*a^3*c^5*cos(f*x + e)^3 - 15*(9*A - 2*B)*a^3*c^5*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.25, size = 301, normalized size = 1.36

$$\frac{Ba^3c^5 \cos(9fx + 9e)}{2304f} + \frac{Ba^3c^5 \sin(6fx + 6e)}{96f} + \frac{5}{128} (9Aa^3c^5 - 2Ba^3c^5)x + \frac{(8Aa^3c^5 - Ba^3c^5) \cos(7fx + 7e)}{1792f} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] 1/2304*B*a^3*c^5*cos(9*f*x + 9*e)/f + 1/96*B*a^3*c^5*sin(6*f*x + 6*e)/f + 5/128*(9*A*a^3*c^5 - 2*B*a^3*c^5)*x + 1/1792*(8*A*a^3*c^5 - B*a^3*c^5)*cos(7*f*x + 7*e)/f + 1/64*(2*A*a^3*c^5 - B*a^3*c^5)*cos(5*f*x + 5*e)/f + 1/192*(18*A*a^3*c^5 - 11*B*a^3*c^5)*cos(3*f*x + 3*e)/f + 1/128*(20*A*a^3*c^5 - 13*B*a^3*c^5)*cos(f*x + e)/f - 1/1024*(A*a^3*c^5 - 2*B*a^3*c^5)*sin(8*f*x + 8*e)/f + 1/128*(5*A*a^3*c^5 + 2*B*a^3*c^5)*sin(4*f*x + 4*e)/f + 1/32*(8*A*a^3*c^5 - B*a^3*c^5)*sin(2*f*x + 2*e)/f

maple [B] time = 0.76, size = 611, normalized size = 2.75

$$a^3Ac^5(fx + e) - a^3Ac^5 \left(\frac{\left(\sin^7(fx+e) + \frac{7(\sin^5(fx+e))}{6} + \frac{35(\sin^3(fx+e))}{24} + \frac{35\sin(fx+e)}{16} \right) \cos(fx+e)}{8} + \frac{35fx}{128} + \frac{35e}{128} \right) - \frac{2a^3Ac^5 \left(\frac{16}{5} + \sin^6(fx+e) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x)

[Out] 1/f*(a^3*A*c^5*(f*x+e)-a^3*A*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*a^3*A*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+6/5*a^3*A*c^5*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+1/9*B*a^3*c^5*(128/35+sin(f*x+e)^8+8/7*sin(f*x+e)^6+48/35*sin(f*x+e)^4+64/35*sin(f*x+e)^2)*cos(f*x+e)+2*B*a^3*c^5*(-1/8*(sin(f*x+e)^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)-2/7*B*a^3*c^5*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-6*B*a^3*c^5*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-B*a^3*c^5*cos(f*x+e)-2*a^3*A*c^5*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^3*c^5*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)

$-2*a^3*A*c^5*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)+2/3*B*a^3*c^5*(2*\sin(f*x+e)^2)*\cos(f*x+e)+2*a^3*A*c^5*\cos(f*x+e)-2*B*a^3*c^5*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e))$

maxima [B] time = 0.35, size = 617, normalized size = 2.78

$$\frac{18432 \left(5 \cos(fx + e)^7 - 21 \cos(fx + e)^5 + 35 \cos(fx + e)^3 - 35 \cos(fx + e) \right) Aa^3c^5 + 129024 \left(3 \cos(fx + e)^9 - 180 \cos(fx + e)^7 + 378 \cos(fx + e)^5 - 420 \cos(fx + e)^3 + 315 \cos(fx + e) \right) B^2a^3c^5 + 18432 \left(5 \cos(fx + e)^7 - 21 \cos(fx + e)^5 + 35 \cos(fx + e)^3 - 35 \cos(fx + e) \right) B^2a^3c^5 - 215040 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) B^2a^3c^5 + 210 \left(128 \sin(2fx + 2e)^3 + 840fx + 840e + 3 \sin(8fx + 8e) + 168 \sin(4fx + 4e) - 768 \sin(2fx + 2e) \right) B^2a^3c^5 + 3360 \left(4 \sin(2fx + 2e)^3 + 60fx + 60e + 9 \sin(4fx + 4e) - 48 \sin(2fx + 2e) \right) B^2a^3c^5 - 161280 \left(2fx + 2e - \sin(2fx + 2e) \right) B^2a^3c^5 + 322560 \left(fx + e \right) B^2a^3c^5 + 1024 \left(35 \cos(fx + e)^9 - 180 \cos(fx + e)^7 + 378 \cos(fx + e)^5 - 420 \cos(fx + e)^3 + 315 \cos(fx + e) \right) B^2a^3c^5 + 18432 \left(5 \cos(fx + e)^7 - 21 \cos(fx + e)^5 + 35 \cos(fx + e)^3 - 35 \cos(fx + e) \right) B^2a^3c^5 - 215040 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) B^2a^3c^5 + 210 \left(128 \sin(2fx + 2e)^3 + 840fx + 840e + 3 \sin(8fx + 8e) + 168 \sin(4fx + 4e) - 768 \sin(2fx + 2e) \right) B^2a^3c^5 - 10080 \left(4 \sin(2fx + 2e)^3 + 60fx + 60e + 9 \sin(4fx + 4e) - 48 \sin(2fx + 2e) \right) B^2a^3c^5 + 60480 \left(12fx + 12e + \sin(4fx + 4e) - 8 \sin(2fx + 2e) \right) B^2a^3c^5 - 161280 \left(2fx + 2e - \sin(2fx + 2e) \right) B^2a^3c^5 + 645120 Aa^3c^5 \cos(fx + e) - 322560 B^2a^3c^5 \cos(fx + e)}{fx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out] $1/322560*(18432*(5*\cos(f*x + e)^7 - 21*\cos(f*x + e)^5 + 35*\cos(f*x + e)^3 - 35*\cos(f*x + e))*A*a^3*c^5 + 129024*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a^3*c^5 + 645120*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^3*c^5 - 105*(128*\sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*\sin(8*f*x + 8*e) + 168*\sin(4*f*x + 4*e) - 768*\sin(2*f*x + 2*e))*A*a^3*c^5 + 3360*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*A*a^3*c^5 - 161280*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^3*c^5 + 322560*(f*x + e)*A*a^3*c^5 + 1024*(35*\cos(f*x + e)^9 - 180*\cos(f*x + e)^7 + 378*\cos(f*x + e)^5 - 420*\cos(f*x + e)^3 + 315*\cos(f*x + e))*B^2*a^3*c^5 + 18432*(5*\cos(f*x + e)^7 - 21*\cos(f*x + e)^5 + 35*\cos(f*x + e)^3 - 35*\cos(f*x + e))*B^2*a^3*c^5 - 215040*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B^2*a^3*c^5 + 210*(128*\sin(2*f*x + 2*e)^3 + 840*f*x + 840*e + 3*\sin(8*f*x + 8*e) + 168*\sin(4*f*x + 4*e) - 768*\sin(2*f*x + 2*e))*B^2*a^3*c^5 - 10080*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B^2*a^3*c^5 + 60480*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B^2*a^3*c^5 - 161280*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B^2*a^3*c^5 + 645120*A*a^3*c^5*\cos(f*x + e) - 322560*B^2*a^3*c^5*\cos(f*x + e))/f$

mupad [B] time = 14.93, size = 705, normalized size = 3.18

$$\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{16} (4Aa^3c^5 - 2Ba^3c^5) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{14} (8Aa^3c^5 - 8Ba^3c^5) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 \left(\frac{8Aa^3c^5}{7} - \frac{8Ba^3c^5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5,x)

[Out] $(\tan(e/2 + (f*x)/2)^{16}*(4*A*a^3*c^5 - 2*B*a^3*c^5) + \tan(e/2 + (f*x)/2)^{14}*(8*A*a^3*c^5 - 8*B*a^3*c^5) + \tan(e/2 + (f*x)/2)^2*((8*A*a^3*c^5)/7 - (8*B*a^3*c^5)/7) + \tan(e/2 + (f*x)/2)^8*(32*A*a^3*c^5 - 4*B*a^3*c^5) + \tan(e/2 + (f*x)/2)^6*(24*A*a^3*c^5 - 24*B*a^3*c^5) + \tan(e/2 + (f*x)/2)^{12}*(24*A*a^3*c^5 - (16*B*a^3*c^5)/3) + \tan(e/2 + (f*x)/2)^{10}*(40*A*a^3*c^5 - 40*B*a^3*c^5) + \tan(e/2 + (f*x)/2)^4*((88*A*a^3*c^5)/7 - (32*B*a^3*c^5)/7) - \tan(e/2 + (f*x)/2)^{17}*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + \tan(e/2 + (f*x)/2)^5*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) - \tan(e/2 + (f*x)/2)^{13}*((149*A*a^3*c^5)/32 + (83*B*a^3*c^5)/16) + \tan(e/2 + (f*x)/2)^3*((189*A*a^3*c^5)/32 - (191*B*a^3*c^5)/48) - \tan(e/2 + (f*x)/2)^{15}*((189*A*a^3*c^5)/32 - (191*B*a^3*c^5)/48) + \tan(e/2 + (f*x)/2)^7*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/16) - \tan(e/2 + (f*x)/2)^{11}*((409*A*a^3*c^5)/32 - (145*B*a^3*c^5)/16) + \tan(e/2 + (f*x)/2)*((83*A*a^3*c^5)/64 + (5*B*a^3*c^5)/32) + (4*A*a^3*c^5)/7 - (22*B*a^3*c^5)/63)/(f*(9*\tan(e/2 + (f*x)/2)^2 + 36*\tan(e/2 + (f*x)/2)^4 + 84*\tan(e/2 + (f*x)/2)^6 + 126*\tan(e/2 + (f*x)/2)^8 + 126*\tan(e/2 + (f*x)/2)^{10} + 84*\tan(e/2 + (f*x)/2)^{12} + 36*\tan(e/2 + (f*x)/2)^{14} + 9*\tan(e/2 + (f*x)/2)^{16} + 1)$

$$\frac{1}{2})^{16} + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^{18} + 1) + (5*a^3*c^5*\operatorname{atan}\left(\frac{5*a^3*c^5*\tan\left(\frac{e}{2} + \frac{f*x}{2}\right)*(9*A - 2*B)}{64*\left(\frac{45*A*a^3*c^5}{64} - \frac{5*B*a^3*c^5}{32}\right)}\right))*(9*A - 2*B))/64*f)$$

`sympy [A]` time = 37.38, size = 1753, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**5,x)`

[Out] `Piecewise((-35*A*a**3*c**5*x*sin(e + f*x)**8/128 - 35*A*a**3*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 5*A*a**3*c**5*x*sin(e + f*x)**6/8 - 105*A*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 15*A*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/8 - 35*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 15*A*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 - A*a**3*c**5*x*sin(e + f*x)**2 - 35*A*a**3*c**5*x*cos(e + f*x)**8/128 + 5*A*a**3*c**5*x*cos(e + f*x)**6/8 - A*a**3*c**5*x*cos(e + f*x)**2 + A*a**3*c**5*x + 93*A*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(128*f) - 2*A*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)/f + 511*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) - 11*A*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) - 4*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f + 6*A*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)/f + 385*A*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(384*f) - 5*A*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - 16*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 8*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**3/f - 6*A*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f + 35*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(128*f) - 5*A*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) + A*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f - 32*A*a**3*c**5*cos(e + f*x)**7/(35*f) + 16*A*a**3*c**5*cos(e + f*x)**5/(5*f) - 4*A*a**3*c**5*cos(e + f*x)**3/f + 2*A*a**3*c**5*cos(e + f*x)/f + 35*B*a**3*c**5*x*sin(e + f*x)**8/64 + 35*B*a**3*c**5*x*sin(e + f*x)**6*cos(e + f*x)**2/16 - 15*B*a**3*c**5*x*sin(e + f*x)**6/8 + 105*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**4/32 - 45*B*a**3*c**5*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 9*B*a**3*c**5*x*sin(e + f*x)**4/4 + 35*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**6/16 - 45*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**4/8 + 9*B*a**3*c**5*x*sin(e + f*x)**2*cos(e + f*x)**2/2 - B*a**3*c**5*x*sin(e + f*x)**2 + 35*B*a**3*c**5*x*cos(e + f*x)**8/64 - 15*B*a**3*c**5*x*cos(e + f*x)**6/8 + 9*B*a**3*c**5*x*cos(e + f*x)**4/4 - B*a**3*c**5*x*cos(e + f*x)**2 + B*a**3*c**5*sin(e + f*x)**8*cos(e + f*x)/f - 93*B*a**3*c**5*sin(e + f*x)**7*cos(e + f*x)/(64*f) + 8*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)**3/(3*f) - 2*B*a**3*c**5*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)**3/(192*f) + 33*B*a**3*c**5*sin(e + f*x)**5*cos(e + f*x)/(8*f) + 16*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**5*sin(e + f*x)**4*cos(e + f*x)**3/f - 385*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**5/(192*f) + 5*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)**3/f - 15*B*a**3*c**5*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 64*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**7/(35*f) - 16*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**5*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**7/(64*f) + 15*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**5/(8*f) - 9*B*a**3*c**5*sin(e + f*x)*cos(e + f*x)**3/(4*f) + B*a**3*c**5*sin(e + f*x)*cos(e + f*x)/f + 128*B*a**3*c**5*cos(e + f*x)**9/(315*f) - 32*B*a**3*c**5*cos(e + f*x)**7/(35*f) + 4*B*a**3*c**5*cos(e + f*x)**3/(3*f) - B*a**3*c**5*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**5, True))`

$$3.40 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^4 dx$$

Optimal. Leaf size=181

$$\frac{a^3 c^4 (8A - B) \cos^7(e + fx)}{56f} + \frac{a^3 c^4 (8A - B) \sin(e + fx) \cos^5(e + fx)}{48f} + \frac{5a^3 c^4 (8A - B) \sin(e + fx) \cos^3(e + fx)}{192f} +$$

[Out] $5/128*a^3*(8*A-B)*c^4*x+1/56*a^3*(8*A-B)*c^4*\cos(f*x+e)^7/f+5/128*a^3*(8*A-B)*c^4*\cos(f*x+e)*\sin(f*x+e)/f+5/192*a^3*(8*A-B)*c^4*\cos(f*x+e)^3*\sin(f*x+e)/f+1/48*a^3*(8*A-B)*c^4*\cos(f*x+e)^5*\sin(f*x+e)/f-1/8*a^3*B*\cos(f*x+e)^7*(c^4-c^4*\sin(f*x+e))/f$

Rubi [A] time = 0.23, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2669, 2635, 8}

$$\frac{a^3 c^4 (8A - B) \cos^7(e + fx)}{56f} + \frac{a^3 c^4 (8A - B) \sin(e + fx) \cos^5(e + fx)}{48f} + \frac{5a^3 c^4 (8A - B) \sin(e + fx) \cos^3(e + fx)}{192f} +$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]

[Out] $(5*a^3*(8*A - B)*c^4*x)/128 + (a^3*(8*A - B)*c^4*\cos[e + f*x]^7)/(56*f) + (5*a^3*(8*A - B)*c^4*\cos[e + f*x]*\sin[e + f*x])/(128*f) + (5*a^3*(8*A - B)*c^4*\cos[e + f*x]^3*\sin[e + f*x])/(192*f) + (a^3*(8*A - B)*c^4*\cos[e + f*x]^5*\sin[e + f*x])/(48*f) - (a^3*B*\cos[e + f*x]^7*(c^4 - c^4*\sin[e + f*x]))/(8*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2860

Int[(cos[(e_.) + (f_)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^4 dx = (a^3 c^3) \int \cos^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^4 dx$$

$$= -\frac{a^3 B \cos^7(e + fx)(c^4 - c^4 \sin(e + fx))}{8f} + \frac{1}{8} (a^3 c^3) \int \cos^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx$$

$$= \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f} - \frac{a^3 B \cos^7(e + fx)}{56f}$$

$$= \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f} + \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f}$$

$$= \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f} + \frac{5a^3(8A - B)c^4 \cos^7(e + fx)}{56f}$$

$$= \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f} + \frac{5a^3(8A - B)c^4 \cos^7(e + fx)}{56f}$$

$$= \frac{5}{128} a^3(8A - B)c^4 x + \frac{a^3(8A - B)c^4 \cos^7(e + fx)}{56f}$$

Mathematica [A] time = 1.95, size = 209, normalized size = 1.15

$$\frac{(a \sin(e + fx) + a)^3 (c - c \sin(e + fx))^4 (840(8A - B)(e + fx) + 336(15A - B) \sin(2(e + fx)) + 168(6A + B) \sin(4(e + fx)))}{21504f (c - c \sin(e + fx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4,x]
```

```
[Out] ((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4*(840*(8*A - B)*(e + f*x) + 1680*(A - B)*Cos[e + f*x] + 1008*(A - B)*Cos[3*(e + f*x)] + 336*(A - B)*Cos[5*(e + f*x)] + 48*(A - B)*Cos[7*(e + f*x)] + 336*(15*A - B)*Sin[2*(e + f*x)] + 168*(6*A + B)*Sin[4*(e + f*x)] + 112*(A + B)*Sin[6*(e + f*x)] + 21*B*Sin[8*(e + f*x)]))/(21504*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.47, size = 137, normalized size = 0.76

$$\frac{384(A - B)a^3 c^4 \cos^7(fx + e) + 105(8A - B)a^3 c^4 fx + 7(48Ba^3 c^4 \cos^7(fx + e) + 8(8A - B)a^3 c^4 \cos^5(fx + e))}{2688f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="fricas")
```

```
[Out] 1/2688*(384*(A - B)*a^3*c^4*cos(f*x + e)^7 + 105*(8*A - B)*a^3*c^4*f*x + 7*(48*B*a^3*c^4*cos(f*x + e)^7 + 8*(8*A - B)*a^3*c^4*cos(f*x + e)^5 + 10*(8*A
```

$$- B) * a^3 * c^4 * \cos(f * x + e)^3 + 15 * (8 * A - B) * a^3 * c^4 * \cos(f * x + e) * \sin(f * x + e) / f$$

giac [A] time = 0.23, size = 273, normalized size = 1.51

$$\frac{B a^3 c^4 \sin(8 f x + 8 e)}{1024 f} + \frac{5}{128} (8 A a^3 c^4 - B a^3 c^4) x + \frac{(A a^3 c^4 - B a^3 c^4) \cos(7 f x + 7 e)}{448 f} + \frac{(A a^3 c^4 - B a^3 c^4) \cos(5 f x + 5 e)}{64 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/1024*B*a^3*c^4*sin(8*f*x + 8*e)/f + 5/128*(8*A*a^3*c^4 - B*a^3*c^4)*x + 1/448*(A*a^3*c^4 - B*a^3*c^4)*cos(7*f*x + 7*e)/f + 1/64*(A*a^3*c^4 - B*a^3*c^4)*cos(5*f*x + 5*e)/f + 3/64*(A*a^3*c^4 - B*a^3*c^4)*cos(3*f*x + 3*e)/f + 5/64*(A*a^3*c^4 - B*a^3*c^4)*cos(f*x + e)/f + 1/192*(A*a^3*c^4 + B*a^3*c^4)*sin(6*f*x + 6*e)/f + 1/128*(6*A*a^3*c^4 + B*a^3*c^4)*sin(4*f*x + 4*e)/f + 1/64*(15*A*a^3*c^4 - B*a^3*c^4)*sin(2*f*x + 2*e)/f

maple [B] time = 0.67, size = 568, normalized size = 3.14

$$a^3 A c^4 (f x + e) + B a^3 c^4 \left(\frac{\left(\sin^7(f x + e) + \frac{7 \sin^5(f x + e)}{6} + \frac{35 \sin^3(f x + e)}{24} + \frac{35 \sin(f x + e)}{16} \right) \cos(f x + e)}{8} + \frac{35 f x}{128} + \frac{35 e}{128} \right) + \frac{B a^3 c^4 \left(\frac{16}{5} + \sin^6(f x + e) \right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)

[Out] 1/f*(a^3*A*c^4*(f*x+e)+B*a^3*c^4*(-1/8*(sin(f*x+e))^7+7/6*sin(f*x+e)^5+35/24*sin(f*x+e)^3+35/16*sin(f*x+e))*cos(f*x+e)+35/128*f*x+35/128*e)+1/7*B*a^3*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-3*B*a^3*c^4*(-1/6*(sin(f*x+e))^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3/5*B*a^3*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^4*(-1/6*(sin(f*x+e))^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3/5*a^3*A*c^4*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c^4*(-1/4*(sin(f*x+e))^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/7*a^3*A*c^4*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)+a^3*A*c^4*cos(f*x+e)-B*a^3*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^3*A*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^4*(-1/4*(sin(f*x+e))^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*A*c^4*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a^3*c^4*(2+sin(f*x+e)^2)*cos(f*x+e)-B*a^3*c^4*cos(f*x+e))

maxima [B] time = 0.36, size = 571, normalized size = 3.15

$$3072 \left(5 \cos(f x + e)^7 - 21 \cos(f x + e)^5 + 35 \cos(f x + e)^3 - 35 \cos(f x + e) \right) A a^3 c^4 + 21504 \left(3 \cos(f x + e)^6 - 10 \cos(f x + e)^4 + 15 \cos(f x + e)^2 - 3 \cos(f x + e) \right) B a^3 c^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out] 1/107520*(3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*A*a^3*c^4 + 21504*(3*cos(f*x + e)^6 - 10*cos(f*x + e)^4 + 15*cos(f*x + e))*A*a^3*c^4 + 107520*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^4

```
*c^4 - 560*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*A*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*A*a^3*c^4 - 80640*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c
^4 + 107520*(f*x + e)*A*a^3*c^4 - 3072*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^
5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^4 - 21504*(3*cos(f*x + e)^
5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^4 - 107520*(cos(f*x + e)^3
- 3*cos(f*x + e))*B*a^3*c^4 + 35*(128*sin(2*f*x + 2*e)^3 + 840*f*x + 840*e
+ 3*sin(8*f*x + 8*e) + 168*sin(4*f*x + 4*e) - 768*sin(2*f*x + 2*e))*B*a^3*
c^4 - 1680*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*
sin(2*f*x + 2*e))*B*a^3*c^4 + 10080*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*s
in(2*f*x + 2*e))*B*a^3*c^4 - 26880*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c
^4 + 107520*A*a^3*c^4*cos(f*x + e) - 107520*B*a^3*c^4*cos(f*x + e))/f
```

mupad [B] time = 14.78, size = 661, normalized size = 3.65

$$\frac{\tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 (6 A a^3 c^4 - 6 B a^3 c^4) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^{12} (2 A a^3 c^4 - 2 B a^3 c^4) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 (6 A a^3 c^4 - 6 B a^3 c^4)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4,x)
[Out] (tan(e/2 + (f*x)/2)^4*(6*A*a^3*c^4 - 6*B*a^3*c^4) + tan(e/2 + (f*x)/2)^12*(
2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)^6*(6*A*a^3*c^4 - 6*B*a^3*c^
4) + tan(e/2 + (f*x)/2)^14*(2*A*a^3*c^4 - 2*B*a^3*c^4) + tan(e/2 + (f*x)/2)
^2*((2*A*a^3*c^4)/7 - (2*B*a^3*c^4)/7) + tan(e/2 + (f*x)/2)^8*(10*A*a^3*c^4
- 10*B*a^3*c^4) + tan(e/2 + (f*x)/2)^10*(10*A*a^3*c^4 - 10*B*a^3*c^4) - ta
n(e/2 + (f*x)/2)^15*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + tan(e/2 + (f*x)
/2)^3*((61*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^13*((6
1*A*a^3*c^4)/24 - (397*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^5*((113*A*a^3*c
^4)/24 + (895*B*a^3*c^4)/192) - tan(e/2 + (f*x)/2)^11*((113*A*a^3*c^4)/24 +
(895*B*a^3*c^4)/192) + tan(e/2 + (f*x)/2)^7*((85*A*a^3*c^4)/24 - (1765*B*a
^3*c^4)/192) - tan(e/2 + (f*x)/2)^9*((85*A*a^3*c^4)/24 - (1765*B*a^3*c^4)/1
92) + tan(e/2 + (f*x)/2)*((11*A*a^3*c^4)/8 + (5*B*a^3*c^4)/64) + (2*A*a^3*c
^4)/7 - (2*B*a^3*c^4)/7)/(f*(8*tan(e/2 + (f*x)/2)^2 + 28*tan(e/2 + (f*x)/2)
^4 + 56*tan(e/2 + (f*x)/2)^6 + 70*tan(e/2 + (f*x)/2)^8 + 56*tan(e/2 + (f*x)
/2)^10 + 28*tan(e/2 + (f*x)/2)^12 + 8*tan(e/2 + (f*x)/2)^14 + tan(e/2 + (f*
x)/2)^16 + 1)) + (5*a^3*c^4*atan((5*a^3*c^4*tan(e/2 + (f*x)/2)*(8*A - B))/(
64*((5*A*a^3*c^4)/8 - (5*B*a^3*c^4)/64)))*(8*A - B))/(64*f)
```

sympy [A] time = 23.43, size = 1579, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4,x)
[Out] Piecewise((-5*A*a**3*c**4*x*sin(e + f*x)**6/16 - 15*A*a**3*c**4*x*sin(e + f
*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**4*x*sin(e + f*x)**4/8 - 15*A*a**3*c
**4*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**4*x*sin(e + f*x)**2*
cos(e + f*x)**2/4 - 3*A*a**3*c**4*x*sin(e + f*x)**2/2 - 5*A*a**3*c**4*x*cos
(e + f*x)**6/16 + 9*A*a**3*c**4*x*cos(e + f*x)**4/8 - 3*A*a**3*c**4*x*cos(e
+ f*x)**2/2 + A*a**3*c**4*x - A*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f +
11*A*a**3*c**4*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*A*a**3*c**4*sin(e +
f*x)**4*cos(e + f*x)**3/f + 3*A*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f +
5*A*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**4*sin(e
+ f*x)**3*cos(e + f*x)/(8*f) - 8*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)*
```

```

*5/(5*f) + 4*A*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)**3/f - 3*A*a**3*c**4*
sin(e + f*x)**2*cos(e + f*x)/f + 5*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**5
/(16*f) - 9*A*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**4*
sin(e + f*x)*cos(e + f*x)/(2*f) - 16*A*a**3*c**4*cos(e + f*x)**7/(35*f) + 8
*A*a**3*c**4*cos(e + f*x)**5/(5*f) - 2*A*a**3*c**4*cos(e + f*x)**3/f + A*a
**3*c**4*cos(e + f*x)/f + 35*B*a**3*c**4*x*sin(e + f*x)**8/128 + 35*B*a**3*c
**4*x*sin(e + f*x)**6*cos(e + f*x)**2/32 - 15*B*a**3*c**4*x*sin(e + f*x)**6
/16 + 105*B*a**3*c**4*x*sin(e + f*x)**4*cos(e + f*x)**4/64 - 45*B*a**3*c**4
*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*c**4*x*sin(e + f*x)**4/8 +
35*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f*x)**6/32 - 45*B*a**3*c**4*x*sin
(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*c**4*x*sin(e + f*x)**2*cos(e + f
*x)**2/4 - B*a**3*c**4*x*sin(e + f*x)**2/2 + 35*B*a**3*c**4*x*cos(e + f*x)*
*8/128 - 15*B*a**3*c**4*x*cos(e + f*x)**6/16 + 9*B*a**3*c**4*x*cos(e + f*x)
**4/8 - B*a**3*c**4*x*cos(e + f*x)**2/2 - 93*B*a**3*c**4*sin(e + f*x)**7*co
s(e + f*x)/(128*f) + B*a**3*c**4*sin(e + f*x)**6*cos(e + f*x)/f - 511*B*a**
3*c**4*sin(e + f*x)**5*cos(e + f*x)**3/(384*f) + 33*B*a**3*c**4*sin(e + f*x
)**5*cos(e + f*x)/(16*f) + 2*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)**3/f
- 3*B*a**3*c**4*sin(e + f*x)**4*cos(e + f*x)/f - 385*B*a**3*c**4*sin(e + f*
x)**3*cos(e + f*x)**5/(384*f) + 5*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)*
*3/(2*f) - 15*B*a**3*c**4*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 8*B*a**3*c**
4*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**4*sin(e + f*x)**2*cos
(e + f*x)**3/f + 3*B*a**3*c**4*sin(e + f*x)**2*cos(e + f*x)/f - 35*B*a**3*c
**4*sin(e + f*x)*cos(e + f*x)**7/(128*f) + 15*B*a**3*c**4*sin(e + f*x)*cos(
e + f*x)**5/(16*f) - 9*B*a**3*c**4*sin(e + f*x)*cos(e + f*x)**3/(8*f) + B*a
**3*c**4*sin(e + f*x)*cos(e + f*x)/(2*f) + 16*B*a**3*c**4*cos(e + f*x)**7/(
35*f) - 8*B*a**3*c**4*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**4*cos(e + f*x)**3
/f - B*a**3*c**4*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3*(-c*sin(e) + c)**4, True))

```

3.41 $\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx$

Optimal. Leaf size=117

$$\frac{a^3 A c^3 \sin(e + fx) \cos^5(e + fx)}{6f} + \frac{5a^3 A c^3 \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{5a^3 A c^3 \sin(e + fx) \cos(e + fx)}{16f} + \frac{5}{16} a^3 A c^3 x - \frac{5}{16} a^3 A c^3 x$$

[Out] $5/16*a^3*A*c^3*x-1/7*a^3*B*c^3*\cos(f*x+e)^7/f+5/16*a^3*A*c^3*\cos(f*x+e)*\sin(f*x+e)/f+5/24*a^3*A*c^3*\cos(f*x+e)^3*\sin(f*x+e)/f+1/6*a^3*A*c^3*\cos(f*x+e)^5*\sin(f*x+e)/f$

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2669, 2635, 8}

$$\frac{a^3 A c^3 \sin(e + fx) \cos^5(e + fx)}{6f} + \frac{5a^3 A c^3 \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{5a^3 A c^3 \sin(e + fx) \cos(e + fx)}{16f} + \frac{5}{16} a^3 A c^3 x - \frac{5}{16} a^3 A c^3 x$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^3*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^3, x]$

[Out] $(5*a^3*A*c^3*x)/16 - (a^3*B*c^3*\text{Cos}[e + f*x]^7)/(7*f) + (5*a^3*A*c^3*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(16*f) + (5*a^3*A*c^3*\text{Cos}[e + f*x]^3*\text{Sin}[e + f*x])/(24*f) + (a^3*A*c^3*\text{Cos}[e + f*x]^5*\text{Sin}[e + f*x])/(6*f)$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] \text{ /; } \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \text{ :> } -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] \text{ /; } \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2669

$\text{Int}[(\text{cos}[(e_*) + (f_*)*(x_)]*(g_*)^{(p_)*((a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]))}, x_Symbol] \text{ :> } -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)})/(f*g*(p+1)), x] + \text{Dist}[a, \text{Int}[(g*\text{Cos}[e + f*x])^p, x], x] \text{ /; } \text{FreeQ}[\{a, b, e, f, g, p\}, x] \ \&\& \ (\text{IntegerQ}[2*p] \ || \ \text{NeQ}[a^2 - b^2, 0])$

Rule 2967

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_)]*(c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(n_)}}, x_Symbol] \text{ :> } \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}*(A + B*\text{Sin}[e + f*x]), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \& \ \text{GtQ}[n, 0]) \ || \ \text{LtQ}[0, n, m] \ || \ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx &= (a^3 c^3) \int \cos^6(e + fx)(A + B \sin(e + fx)) dx \\
&= -\frac{a^3 B c^3 \cos^7(e + fx)}{7f} + (a^3 A c^3) \int \cos^6(e + fx) dx \\
&= -\frac{a^3 B c^3 \cos^7(e + fx)}{7f} + \frac{a^3 A c^3 \cos^5(e + fx)}{6f} \\
&= -\frac{a^3 B c^3 \cos^7(e + fx)}{7f} + \frac{5a^3 A c^3 \cos^3(e + fx)}{24f} \\
&= -\frac{a^3 B c^3 \cos^7(e + fx)}{7f} + \frac{5a^3 A c^3 \cos(e + fx)}{16f} \\
&= \frac{5}{16} a^3 A c^3 x - \frac{a^3 B c^3 \cos^7(e + fx)}{7f} + \frac{5a^3 A c^3 \cos(e + fx)}{16f}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 64, normalized size = 0.55

$$\frac{a^3 c^3 (7A(45 \sin(2(e + fx)) + 9 \sin(4(e + fx)) + \sin(6(e + fx)) + 60e + 60fx) - 192B \cos^7(e + fx))}{1344f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]

[Out] (a^3*c^3*(-192*B*Cos[e + f*x]^7 + 7*A*(60*e + 60*f*x + 45*Sin[2*(e + f*x)] + 9*Sin[4*(e + f*x)] + Sin[6*(e + f*x)])))/(1344*f)

fricas [A] time = 0.45, size = 92, normalized size = 0.79

$$\frac{48 B a^3 c^3 \cos(fx + e)^7 - 105 A a^3 c^3 fx - 7 \left(8 A a^3 c^3 \cos(fx + e)^5 + 10 A a^3 c^3 \cos(fx + e)^3 + 15 A a^3 c^3 \cos(fx + e) \right)}{336 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/336*(48*B*a^3*c^3*cos(f*x + e)^7 - 105*A*a^3*c^3*f*x - 7*(8*A*a^3*c^3*cos(f*x + e)^5 + 10*A*a^3*c^3*cos(f*x + e)^3 + 15*A*a^3*c^3*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.19, size = 162, normalized size = 1.38

$$\frac{5}{16} A a^3 c^3 x - \frac{B a^3 c^3 \cos(7fx + 7e)}{448 f} - \frac{B a^3 c^3 \cos(5fx + 5e)}{64 f} - \frac{3 B a^3 c^3 \cos(3fx + 3e)}{64 f} - \frac{5 B a^3 c^3 \cos(fx + e)}{64 f} + \frac{A a^3 c^3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] 5/16*A*a^3*c^3*x - 1/448*B*a^3*c^3*cos(7*f*x + 7*e)/f - 1/64*B*a^3*c^3*cos(5*f*x + 5*e)/f - 3/64*B*a^3*c^3*cos(3*f*x + 3*e)/f - 5/64*B*a^3*c^3*cos(f*x + e)/f + 1/192*A*a^3*c^3*sin(6*f*x + 6*e)/f + 3/64*A*a^3*c^3*sin(4*f*x + 4*e)/f + 15/64*A*a^3*c^3*sin(2*f*x + 2*e)/f

maple [B] time = 0.67, size = 263, normalized size = 2.25

$$\frac{B a^3 c^3 \left(\frac{16}{5} + \sin^6(fx+e) + \frac{6(\sin^4(fx+e))}{5} + \frac{8(\sin^2(fx+e))}{5} \right) \cos(fx+e)}{7} - a^3 A c^3 \left(- \frac{\left(\sin^5(fx+e) + \frac{5(\sin^3(fx+e))}{4} + \frac{15 \sin(fx+e)}{8} \right) \cos(fx+e)}{6} + \frac{5fx}{16} + \frac{5e}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

[Out] 1/f*(1/7*B*a^3*c^3*(16/5+sin(f*x+e)^6+6/5*sin(f*x+e)^4+8/5*sin(f*x+e)^2)*cos(f*x+e)-a^3*A*c^3*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)-3/5*B*a^3*c^3*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*c^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^3*cos(f*x+e)+a^3*A*c^3*(f*x+e))

maxima [B] time = 0.38, size = 264, normalized size = 2.26

$$\frac{35 \left(4 \sin(2fx + 2e)^3 + 60fx + 60e + 9 \sin(4fx + 4e) - 48 \sin(2fx + 2e) \right) A a^3 c^3 - 630 (12fx + 12e + \sin(4fx + 4e) - 8 \sin(2fx + 2e)) A a^3 c^3 + 5040 (2fx + 2e - \sin(2fx + 2e)) A a^3 c^3 - 6720 (fx + e) A a^3 c^3 + 192 (5 \cos(fx + e)^7 - 21 \cos(fx + e)^5 + 35 \cos(fx + e)^3 - 35 \cos(fx + e)) B a^3 c^3 + 1344 (3 \cos(fx + e)^5 - 10 \cos(fx + e)^3 + 15 \cos(fx + e)) B a^3 c^3 + 6720 (\cos(fx + e)^3 - 3 \cos(fx + e)) B a^3 c^3 + 6720 B a^3 c^3 \cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out] -1/6720*(35*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*A*a^3*c^3 - 630*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c^3 + 5040*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^3 - 6720*(f*x + e)*A*a^3*c^3 + 192*(5*cos(f*x + e)^7 - 21*cos(f*x + e)^5 + 35*cos(f*x + e)^3 - 35*cos(f*x + e))*B*a^3*c^3 + 1344*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c^3 + 6720*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^3 + 6720*B*a^3*c^3*cos(f*x + e))/f

mupad [B] time = 14.29, size = 325, normalized size = 2.78

$$\frac{5 A a^3 c^3 x \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^{12} \left(\frac{a^3 c^3 (672 B - 735 A (e+fx))}{336} + \frac{35 A a^3 c^3 (e+fx)}{16} \right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 \left(\frac{a^3 c^3 (2016 B - 2205 A (e+fx))}{336} + \frac{105 A a^3 c^3 (e+fx)}{16} \right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^3,x)

[Out] (5*A*a^3*c^3*x)/16 - (tan(e/2 + (f*x)/2)^12*((a^3*c^3*(672*B - 735*A*(e + f*x)))/336 + (35*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^4*((a^3*c^3*(2016*B - 2205*A*(e + f*x)))/336 + (105*A*a^3*c^3*(e + f*x))/16) + tan(e/2 + (f*x)/2)^8*((a^3*c^3*(3360*B - 3675*A*(e + f*x)))/336 + (175*A*a^3*c^3*(e + f*x))/16) + (a^3*c^3*(96*B - 105*A*(e + f*x)))/336 + (5*A*a^3*c^3*(e + f*x))/16 - (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^3)/6 - (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^5)/24 + (85*A*a^3*c^3*tan(e/2 + (f*x)/2)^9)/24 + (7*A*a^3*c^3*tan(e/2 + (f*x)/2)^11)/6 + (11*A*a^3*c^3*tan(e/2 + (f*x)/2)^13)/8 - (11*A*a^3*c^3*tan(e/2 + (f*x)/2))/8/(f*(tan(e/2 + (f*x)/2)^2 + 1)^7)

sympy [A] time = 10.35, size = 682, normalized size = 5.83

$$\left\{ \begin{array}{l} \frac{5Aa^3c^3x\sin^6(e+fx)}{16} - \frac{15Aa^3c^3x\sin^4(e+fx)\cos^2(e+fx)}{16} + \frac{9Aa^3c^3x\sin^4(e+fx)}{8} - \frac{15Aa^3c^3x\sin^2(e+fx)\cos^4(e+fx)}{16} + \frac{9Aa^3c^3x\sin^2(e+fx)}{8} \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3,x)

[Out] Piecewise((-5*A*a**3*c**3*x*sin(e + f*x)**6/16 - 15*A*a**3*c**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*c**3*x*sin(e + f*x)**4/8 - 15*A*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - 3*A*a**3*c**3*x*sin(e + f*x)**2/2 - 5*A*a**3*c**3*x*cos(e + f*x)**6/16 + 9*A*a**3*c**3*x*cos(e + f*x)**4/8 - 3*A*a**3*c**3*x*cos(e + f*x)**2/2 + A*a**3*c**3*x + 11*A*a**3*c**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 3*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) + B*a**3*c**3*sin(e + f*x)**6*cos(e + f*x)/f + 2*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*c**3*sin(e + f*x)**4*cos(e + f*x)/f + 8*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**5/(5*f) - 4*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)**3/f + 3*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f + 16*B*a**3*c**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*c**3*cos(e + f*x)**5/(5*f) + 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a**3*c**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**3, True))

$$3.42 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx$$

Optimal. Leaf size=138

$$-\frac{a^3 c^2 (6A + B) \cos^5(e + fx)}{30f} + \frac{a^3 c^2 (6A + B) \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{a^3 c^2 (6A + B) \sin(e + fx) \cos(e + fx)}{16f} + \frac{1}{16f}$$

[Out] 1/16*a^3*(6*A+B)*c^2*x-1/30*a^3*(6*A+B)*c^2*cos(f*x+e)^5/f+1/16*a^3*(6*A+B)*c^2*cos(f*x+e)*sin(f*x+e)/f+1/24*a^3*(6*A+B)*c^2*cos(f*x+e)^3*sin(f*x+e)/f-1/6*B*c^2*cos(f*x+e)^5*(a^3+a^3*sin(f*x+e))/f

Rubi [A] time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2669, 2635, 8}

$$-\frac{a^3 c^2 (6A + B) \cos^5(e + fx)}{30f} + \frac{a^3 c^2 (6A + B) \sin(e + fx) \cos^3(e + fx)}{24f} + \frac{a^3 c^2 (6A + B) \sin(e + fx) \cos(e + fx)}{16f} + \frac{1}{16f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]

[Out] (a^3*(6*A + B)*c^2*x)/16 - (a^3*(6*A + B)*c^2*Cos[e + f*x]^5)/(30*f) + (a^3*(6*A + B)*c^2*Cos[e + f*x]*Sin[e + f*x])/(16*f) + (a^3*(6*A + B)*c^2*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (B*c^2*Cos[e + f*x]^5*(a^3 + a^3*Sin[e + f*x]))/(6*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2860

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^2 dx &= (a^2 c^2) \int \cos^4(e + fx) (a + a \sin(e + fx)) (A + B \sin(e + fx)) (c - c \sin(e + fx))^2 dx \\ &= -\frac{Bc^2 \cos^5(e + fx) (a^3 + a^3 \sin(e + fx))}{6f} + \frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} - \frac{Bc^2 \cos^5(e + fx)}{6f} \\ &= -\frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} + \frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} \\ &= -\frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} + \frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} \\ &= \frac{1}{16} a^3 (6A + B)c^2 x - \frac{a^3 (6A + B)c^2 \cos^5(e + fx)}{30f} \end{aligned}$$

Mathematica [A] time = 1.09, size = 133, normalized size = 0.96

$$\frac{a^3 c^2 (-120(A + B) \cos(e + fx) - 60(A + B) \cos(3(e + fx)) + 240A \sin(2(e + fx)) + 30A \sin(4(e + fx)) - 12A \sin(6(e + fx)))}{960f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*c^2*(360*A*e + 60*B*e + 360*A*f*x + 60*B*f*x - 120*(A + B)*Cos[e + f*x]
] - 60*(A + B)*Cos[3*(e + f*x)] - 12*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f
*x)] + 240*A*Sin[2*(e + f*x)] + 15*B*Sin[2*(e + f*x)] + 30*A*Sin[4*(e + f*x
)] - 15*B*Sin[4*(e + f*x)] - 5*B*Sin[6*(e + f*x)]))/(960*f)
```

fricas [A] time = 0.44, size = 106, normalized size = 0.77

$$\frac{48(A + B)a^3c^2 \cos(fx + e)^5 - 15(6A + B)a^3c^2 fx + 5(8Ba^3c^2 \cos(fx + e)^5 - 2(6A + B)a^3c^2 \cos(fx + e))}{240f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorith
m="fricas")
```

```
[Out] -1/240*(48*(A + B)*a^3*c^2*cos(f*x + e)^5 - 15*(6*A + B)*a^3*c^2*f*x + 5*(8
*B*a^3*c^2*cos(f*x + e)^5 - 2*(6*A + B)*a^3*c^2*cos(f*x + e)^3 - 3*(6*A + B
)*a^3*c^2*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.19, size = 204, normalized size = 1.48

$$\frac{Ba^3c^2 \sin(6fx + 6e)}{192f} + \frac{1}{16} (6Aa^3c^2 + Ba^3c^2)x - \frac{(Aa^3c^2 + Ba^3c^2) \cos(5fx + 5e)}{80f} - \frac{(Aa^3c^2 + Ba^3c^2) \cos(3fx + 3e)}{16f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] $-\frac{1}{192}B a^3 c^2 \sin(6fx + 6e)/f + \frac{1}{16}(6A a^3 c^2 + B a^3 c^2)x - \frac{1}{80}(A a^3 c^2 + B a^3 c^2) \cos(5fx + 5e)/f - \frac{1}{16}(A a^3 c^2 + B a^3 c^2) \cos(3fx + 3e)/f - \frac{1}{8}(A a^3 c^2 + B a^3 c^2) \cos(fx + e)/f + \frac{1}{64}(2A a^3 c^2 - B a^3 c^2) \sin(4fx + 4e)/f + \frac{1}{64}(16A a^3 c^2 + B a^3 c^2) \sin(2fx + 2e)/f$

maple [B] time = 0.58, size = 364, normalized size = 2.64

$$-\frac{a^3 A c^2 \left(\frac{8}{3} + \sin^4(fx+e) + \frac{4(\sin^2(fx+e))}{3} \right) \cos(fx+e)}{5} + a^3 A c^2 \left(-\frac{\left(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + \frac{2a^3 A c^2 (2 + \sin^2(fx+e))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)

[Out] $\frac{1}{f} \left(-\frac{1}{5} a^3 A c^2 (8/3 + \sin(fx+e)^4 + 4/3 \sin(fx+e)^2) \cos(fx+e) + a^3 A c^2 \left(-\frac{1}{4} (\sin(fx+e)^3 + 3/2 \sin(fx+e)) \cos(fx+e) + 3/8 fx + 3/8 e \right) + \frac{2}{3} a^3 A c^2 (2 + \sin(fx+e)^2) \cos(fx+e) + B a^3 c^2 \left(-\frac{1}{6} (\sin(fx+e)^5 + 5/4 \sin(fx+e)^3 + 15/8 \sin(fx+e)) \cos(fx+e) + 5/16 fx + 5/16 e \right) - \frac{1}{5} B a^3 c^2 (8/3 + \sin(fx+e)^4 + 4/3 \sin(fx+e)^2) \cos(fx+e) - 2 B a^3 c^2 \left(-\frac{1}{4} (\sin(fx+e)^3 + 3/2 \sin(fx+e)) \cos(fx+e) + 3/8 fx + 3/8 e \right) - 2 a^3 A c^2 \left(-\frac{1}{2} \sin(fx+e) \cos(fx+e) + 1/2 fx + 1/2 e \right) + \frac{2}{3} B a^3 c^2 (2 + \sin(fx+e)^2) \cos(fx+e) - a^3 A c^2 \cos(fx+e) + B a^3 c^2 \left(-\frac{1}{2} \sin(fx+e) \cos(fx+e) + 1/2 fx + 1/2 e \right) + a^3 A c^2 (fx+e) - B a^3 c^2 \cos(fx+e) \right)$

maxima [B] time = 0.34, size = 360, normalized size = 2.61

$$\frac{64 \left(3 \cos(fx+e)^5 - 10 \cos(fx+e)^3 + 15 \cos(fx+e) \right) A a^3 c^2 + 640 \left(\cos(fx+e)^3 - 3 \cos(fx+e) \right) A a^3 c^2 - \dots}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] $-\frac{1}{960} (64 (3 \cos(fx+e)^5 - 10 \cos(fx+e)^3 + 15 \cos(fx+e)) A a^3 c^2 + 640 (\cos(fx+e)^3 - 3 \cos(fx+e)) A a^3 c^2 - 30 (12 fx + 12 e + \sin(4fx + 4e) - 8 \sin(2fx + 2e)) A a^3 c^2 + 480 (2 fx + 2e - \sin(2fx + 2e)) A a^3 c^2 - 960 (fx + e) A a^3 c^2 + 64 (3 \cos(fx+e)^5 - 10 \cos(fx+e)^3 + 15 \cos(fx+e)) B a^3 c^2 + 640 (\cos(fx+e)^3 - 3 \cos(fx+e)) B a^3 c^2 - 5 (4 \sin(2fx + 2e)^3 + 60 fx + 60 e + 9 \sin(4fx + 4e) - 48 \sin(2fx + 2e)) B a^3 c^2 + 60 (12 fx + 12 e + \sin(4fx + 4e) - 8 \sin(2fx + 2e)) B a^3 c^2 - 240 (2 fx + 2e - \sin(2fx + 2e)) B a^3 c^2 + 960 A a^3 c^2 \cos(fx+e) + 960 B a^3 c^2 \cos(fx+e)) / f$

mupad [B] time = 14.17, size = 536, normalized size = 3.88

$$\frac{a^3 c^2 \operatorname{atan} \left(\frac{a^3 c^2 \tan \left(\frac{e}{2} + \frac{fx}{2} \right) (6A+B)}{8 \left(\frac{3Aa^3 c^2}{4} + \frac{Ba^3 c^2}{8} \right)} \right) (6A+B)}{8f} - \frac{\tan \left(\frac{e}{2} + \frac{fx}{2} \right)^4 (4Aa^3 c^2 + 4Ba^3 c^2) + \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^8 (2Aa^3 c^2 + 2Ba^3 c^2)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2,x)

[Out] (a^3*c^2*atan((a^3*c^2*tan(e/2 + (f*x)/2)*(6*A + B))/(8*((3*A*a^3*c^2)/4 + (B*a^3*c^2)/8)))*(6*A + B))/(8*f) - (tan(e/2 + (f*x)/2)^4*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^8*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^6*(4*A*a^3*c^2 + 4*B*a^3*c^2) + tan(e/2 + (f*x)/2)^10*(2*A*a^3*c^2 + 2*B*a^3*c^2) + tan(e/2 + (f*x)/2)^2*((2*A*a^3*c^2)/5 + (2*B*a^3*c^2)/5) - tan(e/2 + (f*x)/2)^5*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^7*((A*a^3*c^2)/2 - (13*B*a^3*c^2)/4) + tan(e/2 + (f*x)/2)^11*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/24) + tan(e/2 + (f*x)/2)^9*((7*A*a^3*c^2)/4 + (47*B*a^3*c^2)/24) - tan(e/2 + (f*x)/2)*((5*A*a^3*c^2)/4 - (B*a^3*c^2)/8) + (2*A*a^3*c^2)/5 + (2*B*a^3*c^2)/5)/(f*(6*tan(e/2 + (f*x)/2)^2 + 15*tan(e/2 + (f*x)/2)^4 + 20*tan(e/2 + (f*x)/2)^6 + 15*tan(e/2 + (f*x)/2)^8 + 6*tan(e/2 + (f*x)/2)^10 + tan(e/2 + (f*x)/2)^12 + 1)) - (a^3*c^2*(6*A + B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(8*f)

sympy [A] time = 8.10, size = 910, normalized size = 6.59

$$\left\{ \begin{array}{l} \frac{3Aa^3c^2x\sin^4(e+fx)}{8} + \frac{3Aa^3c^2x\sin^2(e+fx)\cos^2(e+fx)}{4} - Aa^3c^2x\sin^2(e+fx) + \frac{3Aa^3c^2x\cos^4(e+fx)}{8} - Aa^3c^2x\cos^2(e+fx) \\ x(A+B\sin(e))(a\sin(e)+a)^3(-c\sin(e)+c)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)

[Out] Piecewise(((3*A*a**3*c**2*x*sin(e + f*x)**4/8 + 3*A*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - A*a**3*c**2*x*sin(e + f*x)**2 + 3*A*a**3*c**2*x*cos(e + f*x)**4/8 - A*a**3*c**2*x*cos(e + f*x)**2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/f - 8*A*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*A*a**3*c**2*cos(e + f*x)**3/(3*f) - A*a**3*c**2*cos(e + f*x)/f + 5*B*a**3*c**2*x*sin(e + f*x)**6/16 + 15*B*a**3*c**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 - 3*B*a**3*c**2*x*sin(e + f*x)**4/4 + 15*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 - 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c**2*x*sin(e + f*x)**2/2 + 5*B*a**3*c**2*x*cos(e + f*x)**6/16 - 3*B*a**3*c**2*x*cos(e + f*x)**4/4 + B*a**3*c**2*x*cos(e + f*x)**2/2 - 11*B*a**3*c**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - B*a**3*c**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) + 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 2*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 5*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*c**2*cos(e + f*x)**5/(15*f) + 4*B*a**3*c**2*cos(e + f*x)**3/(3*f) - B*a**3*c**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c)**2, True))

$$3.43 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx)) dx$$

Optimal. Leaf size=140

$$\frac{a^3 c(5A + 2B) \cos^3(e + fx)}{12f} - \frac{c(5A + 2B) \cos^3(e + fx) (a^3 \sin(e + fx) + a^3)}{20f} + \frac{a^3 c(5A + 2B) \sin(e + fx) \cos(e + fx)}{8f}$$

[Out] 1/8*a^3*(5*A+2*B)*c*x-1/12*a^3*(5*A+2*B)*c*cos(f*x+e)^3/f+1/8*a^3*(5*A+2*B)*c*cos(f*x+e)*sin(f*x+e)/f-1/5*a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^2/f-1/20*(5*A+2*B)*c*cos(f*x+e)^3*(a^3+a^3*sin(f*x+e))/f

Rubi [A] time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2967, 2860, 2678, 2669, 2635, 8}

$$\frac{a^3 c(5A + 2B) \cos^3(e + fx)}{12f} - \frac{c(5A + 2B) \cos^3(e + fx) (a^3 \sin(e + fx) + a^3)}{20f} + \frac{a^3 c(5A + 2B) \sin(e + fx) \cos(e + fx)}{8f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]

[Out] (a^3*(5*A + 2*B)*c*x)/8 - (a^3*(5*A + 2*B)*c*Cos[e + f*x]^3)/(12*f) + (a^3*(5*A + 2*B)*c*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^2)/(5*f) - ((5*A + 2*B)*c*Cos[e + f*x]^3*(a^3 + a^3*Sin[e + f*x]))/(20*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2669

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2678

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2860

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(

$g \cos[e + f*x]^{(p + 1)} * (a + b * \sin[e + f*x])^m / (f * g * (m + p + 1)), x] + \text{Dist}[(a * d * m + b * c * (m + p + 1)) / (b * (m + p + 1)), \text{Int}[(g * \cos[e + f*x])^p * (a + b * \sin[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a + b * \sin[e + f*x])^m * (A + B * \sin[e + f*x])^n, x_Symbol] := \text{Dist}[a^m * c^m, \text{Int}[\cos[e + f*x]^{(2*m)} * (c + d * \sin[e + f*x])^{(n - m)} * (A + B * \sin[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b * c + a * d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& (\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) \text{ || } \text{LtQ}[0, n, m] \text{ || } \text{LtQ}[m, n, 0])$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx)) dx &= (ac) \int \cos^2(e + fx) (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx)) dx \\ &= -\frac{aBc \cos^3(e + fx) (a + a \sin(e + fx))^2}{5f} + \frac{1}{5} (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx)) \\ &= -\frac{aBc \cos^3(e + fx) (a + a \sin(e + fx))^2}{5f} - \frac{a^3 (5A + 2B)c \cos^3(e + fx)}{12f} - \frac{aBc \cos^3(e + fx)}{12f} \\ &= -\frac{a^3 (5A + 2B)c \cos^3(e + fx)}{12f} + \frac{a^3 (5A + 2B)}{12f} \\ &= \frac{1}{8} a^3 (5A + 2B) cx - \frac{a^3 (5A + 2B)c \cos^3(e + fx)}{12f} \end{aligned}$$

Mathematica [A] time = 0.85, size = 95, normalized size = 0.68

$$\frac{a^3 c (15(-A + 2B) \sin(4(e + fx)) + 4fx(5A + 2B) + 8A \sin(2(e + fx))) - 60(4A + 3B) \cos(e + fx) - 10(8A + 3B) \sin(e + fx)}{480f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a * Sin[e + f*x])^3 * (A + B * Sin[e + f*x]) * (c - c * Sin[e + f*x]), x]

[Out] (a^3 * c * (-60 * (4 * A + 3 * B) * Cos[e + f*x] - 10 * (8 * A + 5 * B) * Cos[3 * (e + f*x)] + 6 * B * Cos[5 * (e + f*x)] + 15 * (4 * (5 * A + 2 * B) * f * x + 8 * A * Sin[2 * (e + f*x)] - (A + 2 * B) * Sin[4 * (e + f*x)])) / (480 * f)

fricas [A] time = 0.43, size = 100, normalized size = 0.71

$$\frac{24Ba^3c \cos(fx + e)^5 - 80(A + B)a^3c \cos(fx + e)^3 + 15(5A + 2B)a^3cfx - 15(2(A + 2B)a^3c \cos(fx + e)^3)}{120f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/120*(24*B*a^3*c*cos(f*x + e)^5 - 80*(A + B)*a^3*c*cos(f*x + e)^3 + 15*(5*A + 2*B)*a^3*c*f*x - 15*(2*(A + 2*B)*a^3*c*cos(f*x + e)^3 - (5*A + 2*B)*a^3*c*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.21, size = 145, normalized size = 1.04

$$\frac{Ba^3c \cos(5fx + 5e)}{80f} + \frac{Aa^3c \sin(2fx + 2e)}{4f} + \frac{1}{8} (5Aa^3c + 2Ba^3c)x - \frac{(8Aa^3c + 5Ba^3c) \cos(3fx + 3e)}{48f} - \frac{(4Aa^3c + 3Ba^3c) \sin(4fx + 4e)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] 1/80*B*a^3*c*cos(5*f*x + 5*e)/f + 1/4*A*a^3*c*sin(2*f*x + 2*e)/f + 1/8*(5*A*a^3*c + 2*B*a^3*c)*x - 1/48*(8*A*a^3*c + 5*B*a^3*c)*cos(3*f*x + 3*e)/f - 1/8*(4*A*a^3*c + 3*B*a^3*c)*cos(f*x + e)/f - 1/32*(A*a^3*c + 2*B*a^3*c)*sin(4*f*x + 4*e)/f

maple [A] time = 0.48, size = 208, normalized size = 1.49

$$-a^3Ac \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + \frac{2a^3Ac(2+\sin^2(fx+e))\cos(fx+e)}{3} + \frac{Ba^3c \left(\frac{8}{3} + \sin^4(fx+e) + \frac{4(\sin^2(fx+e))}{3} \right) \cos(fx+e)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] 1/f*(-a^3*A*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2/3*a^3*A*c*(2+sin(f*x+e)^2)*cos(f*x+e)+1/5*B*a^3*c*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-2*B*a^3*c*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^3*A*c*cos(f*x+e)+2*B*a^3*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*c*(f*x+e)-B*a^3*c*cos(f*x+e))

maxima [A] time = 0.52, size = 200, normalized size = 1.43

$$\frac{320 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) Aa^3c + 15 \left(12fx + 12e + \sin(4fx + 4e) - 8 \sin(2fx + 2e) \right) Aa^3c - 480 \cos(fx + e) Ba^3c}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -1/480*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c - 480*(f*x + e)*A*a^3*c - 3*2*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c - 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c + 960*A*a^3*c*cos(f*x + e) + 480*B*a^3*c*cos(f*x + e))/f

mupad [B] time = 13.66, size = 390, normalized size = 2.79

$$\frac{a^3c \operatorname{atan} \left(\frac{a^3c \tan \left(\frac{e}{2} + \frac{fx}{2} \right) (5A+2B)}{4 \left(\frac{5Aa^3c}{4} + \frac{Ba^3c}{2} \right)} \right) (5A+2B)}{4f} - \frac{\tan \left(\frac{e}{2} + \frac{fx}{2} \right)^8 (4Aa^3c + 2Ba^3c) - \tan \left(\frac{e}{2} + \frac{fx}{2} \right) \left(\frac{3Aa^3c}{4} - \frac{Ba^3c}{2} \right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x)),x)

[Out] (a^3*c*atan((a^3*c*tan(e/2 + (f*x)/2)*(5*A + 2*B))/(4*((5*A*a^3*c)/4 + (B*a^3*c)/2)))*(5*A + 2*B))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^3*c + 2*B*a^3*c) - tan(e/2 + (f*x)/2)*((3*A*a^3*c)/4 - (B*a^3*c)/2) - tan(e/2 + (f*x)/2)^3*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^7*((7*A*a^3*c)/2 + 3*B*a^3*c) + tan(e/2 + (f*x)/2)^9*((3*A*a^3*c)/4 - (B*a^3*c)/2) + tan(e/2 + (f*x)/2)^6*(8*A*a^3*c + 8*B*a^3*c) + tan(e/2 + (f*x)/2)^2*((8*A*a^3*c)/3 + (8*B*a^3*c)/3) + tan(e/2 + (f*x)/2)^4*((16*A*a^3*c)/3 + (4*B*a^3*c)/3) + (4*A*a^3*c)/3 + (14*B*a^3*c)/15)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1)) - (a^3*c*(5*A + 2*B)*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2))/(4*f)

sympy [A] time = 4.21, size = 486, normalized size = 3.47

$$\left\{ \begin{array}{l} \frac{3Aa^3cx\sin^4(e+fx)}{8} - \frac{3Aa^3cx\sin^2(e+fx)\cos^2(e+fx)}{4} - \frac{3Aa^3cx\cos^4(e+fx)}{8} + Aa^3cx + \frac{5Aa^3c\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{2Aa^3c\sin^2(e+fx)}{8f} \\ x(A + B\sin(e))(a\sin(e) + a)^3(-c\sin(e) + c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] Piecewise((-3*A*a**3*c*x*sin(e + f*x)**4/8 - 3*A*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 - 3*A*a**3*c*x*cos(e + f*x)**4/8 + A*a**3*c*x + 5*A*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 2*A*a**3*c*sin(e + f*x)**2*cos(e + f*x)/f + 3*A*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(8*f) + 4*A*a**3*c*cos(e + f*x)**3/(3*f) - 2*A*a**3*c*cos(e + f*x)/f - 3*B*a**3*c*x*sin(e + f*x)**4/4 - 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*x*sin(e + f*x)**2 - 3*B*a**3*c*x*cos(e + f*x)**4/4 + B*a**3*c*x*cos(e + f*x)**2 + B*a**3*c*sin(e + f*x)**4*cos(e + f*x)/f + 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(4*f) + 4*B*a**3*c*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) + 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*sin(e + f*x)*cos(e + f*x)/f + 8*B*a**3*c*cos(e + f*x)**5/(15*f) - B*a**3*c*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3*(-c*sin(e) + c), True))

$$3.44 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx$$

Optimal. Leaf size=156

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f(c^2-c^2\sin(e+fx))^2} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\sin(e+fx)}{2cf}$$

[Out] $-5/2*a^3*(3*A+4*B)*x/c+5/3*a^3*(3*A+4*B)*\cos(f*x+e)^3/c/f-5/2*a^3*(3*A+4*B)*\cos(f*x+e)*\sin(f*x+e)/c/f+a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^4+2*a^3*(3*A+4*B)*c^3*\cos(f*x+e)^5/f/(c^2-c^2*\sin(f*x+e))^2$

Rubi [A] time = 0.31, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{a^3c^3(A+B)\cos^7(e+fx)}{f(c-c\sin(e+fx))^4} + \frac{2a^3c^3(3A+4B)\cos^5(e+fx)}{f(c^2-c^2\sin(e+fx))^2} + \frac{5a^3(3A+4B)\cos^3(e+fx)}{3cf} - \frac{5a^3(3A+4B)\sin(e+fx)}{2cf}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
 [Out] $(-5*a^3*(3*A + 4*B)*x)/(2*c) + (5*a^3*(3*A + 4*B)*\text{Cos}[e + f*x]^3)/(3*c*f) - (5*a^3*(3*A + 4*B)*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(2*c*f) + (a^3*(A + B)*c^3*\text{Cos}[e + f*x]^7)/(f*(c - c*\text{Sin}[e + f*x])^4) + (2*a^3*(3*A + 4*B)*c^3*\text{Cos}[e + f*x]^5)/(f*(c^2 - c^2*\text{Sin}[e + f*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_.) + (f_)*(x_)]*(g_.)^(p_))*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegersQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_)*(x_)]*(g_.)^(p_))/((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])], x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_)*(x_)]*(g_.)^(p_))*((a_.) + (b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((c_.) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m]/(a*f*g*(2*m+p+1))

```

)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]

```

Rule 2967

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{c - c \sin(e + fx)} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^4} dx \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{f (c - c \sin(e + fx))^4} - (a^3 (3A + 4B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^4} dx \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{f (c - c \sin(e + fx))^4} + \frac{2a^3 (3A + 4B) c \cos^5(e + fx)}{f (c - c \sin(e + fx))^2} - (5a^3 (3A + 4B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^4} dx \\
&= \frac{5a^3 (3A + 4B) \cos^3(e + fx)}{3cf} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{f (c - c \sin(e + fx))^4} + \frac{2a^3 (3A + 4B) c \cos^5(e + fx)}{f (c - c \sin(e + fx))^2} \\
&= \frac{5a^3 (3A + 4B) \cos^3(e + fx)}{3cf} - \frac{5a^3 (3A + 4B) \cos(e + fx) \sin(e + fx)}{2cf} \\
&= -\frac{5a^3 (3A + 4B)x}{2c} + \frac{5a^3 (3A + 4B) \cos^3(e + fx)}{3cf} - \frac{5a^3 (3A + 4B) \cos(e + fx) \sin(e + fx)}{2cf}
\end{aligned}$$

Mathematica [A] time = 1.42, size = 223, normalized size = 1.43

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) (30(3A + 4B)(e + fx) - 3(A + 4B) \sin(e + fx)) - \sin\left(\frac{1}{2}(e + fx)\right) (30(3A + 4B)(e + fx) + 3(A + 4B) \cos(e + fx)) \right)}{(c - c \sin(e + fx))^4}$$

Antiderivative was successfully verified.

```

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]
),x]

```

```

[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(Cos[(e + f
*x)/2]*(30*(3*A + 4*B)*(e + f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos[3*(
e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)]) - Sin[(e + f*x)/2]*(24*B*(8 + 5*e
+ 5*f*x) + 6*A*(32 + 15*e + 15*f*x) - 3*(16*A + 31*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A + 4*B)*Sin[2*(e + f*x)])))/(12*c*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x]))

```

fricas [A] time = 0.44, size = 218, normalized size = 1.40

$$\frac{2Ba^3 \cos^4(fx + e) - (3A + 10B)a^3 \cos^3(fx + e) + 15(3A + 4B)a^3 fx - 24(A + 2B)a^3 \cos^2(fx + e) - 48Aa^3 \cos(fx + e) + 48A^2 a^3}{(c - c \sin(e + fx))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out]
$$-1/6*(2*B*a^3*\cos(f*x + e)^4 - (3*A + 10*B)*a^3*\cos(f*x + e)^3 + 15*(3*A + 4*B)*a^3*f*x - 24*(A + 2*B)*a^3*\cos(f*x + e)^2 - 48*(A + B)*a^3 + 3*(5*(3*A + 4*B)*a^3*f*x - (23*A + 28*B)*a^3)*\cos(f*x + e) - (2*B*a^3*\cos(f*x + e)^3 + 15*(3*A + 4*B)*a^3*f*x + 3*(A + 4*B)*a^3*\cos(f*x + e)^2 - 3*(7*A + 12*B)*a^3*\cos(f*x + e) + 48*(A + B)*a^3)*\sin(f*x + e))/(c*f*\cos(f*x + e) - c*f*\sin(f*x + e) + c*f)$$

giac [A] time = 0.22, size = 234, normalized size = 1.50

$$\frac{15(3Aa^3+4Ba^3)(fx+e)}{c} + \frac{96(Aa^3+Ba^3)}{c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)} + \frac{2\left(3Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+12Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5-24Aa^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-42Ba^3\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4\right)}{c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)}$$

6f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out]
$$-1/6*(15*(3*A*a^3 + 4*B*a^3)*(f*x + e)/c + 96*(A*a^3 + B*a^3)/(c*(\tan(1/2*f*x + 1/2*e) - 1)) + 2*(3*A*a^3*\tan(1/2*f*x + 1/2*e)^5 + 12*B*a^3*\tan(1/2*f*x + 1/2*e)^5 - 24*A*a^3*\tan(1/2*f*x + 1/2*e)^4 - 42*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 48*A*a^3*\tan(1/2*f*x + 1/2*e)^2 - 96*B*a^3*\tan(1/2*f*x + 1/2*e)^2 - 3*A*a^3*\tan(1/2*f*x + 1/2*e) - 12*B*a^3*\tan(1/2*f*x + 1/2*e) - 24*A*a^3 - 46*B*a^3)/((\tan(1/2*f*x + 1/2*e)^2 + 1)^3*c))/f$$

maple [B] time = 0.44, size = 449, normalized size = 2.88

$$\frac{16a^3A}{fc\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{16a^3B}{fc\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{a^3\left(\tan^5\left(\frac{fx}{2} + \frac{e}{2}\right)\right)A}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^3} - \frac{4a^3\left(\tan^5\left(\frac{fx}{2} + \frac{e}{2}\right)\right)B}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^3} + \frac{8a^3\left(\tan^4\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{fc\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out]
$$-16/f*a^3/c/(\tan(1/2*f*x+1/2*e)-1)*A-16/f*a^3/c/(\tan(1/2*f*x+1/2*e)-1)*B-1/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^5*A-4/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^5*B+8/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^4*A+14/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^4*B+16/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^2*A+32/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^2*B+1/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)*A+4/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)*B+8/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*A+46/3/f*a^3/c/(1+\tan(1/2*f*x+1/2*e)^2)^3*B-15/f*a^3/c*\arctan(\tan(1/2*f*x+1/2*e))*A-20/f*a^3/c*\arctan(\tan(1/2*f*x+1/2*e))*B$$

maxima [B] time = 0.47, size = 1139, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out]
$$-1/3*(B*a^3*((7*\sin(f*x + e)/(\cos(f*x + e) + 1) - 39*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 24*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 24*\sin(f*x + e)^4/$$

```
(cos(f*x + e) + 1)^4 + 9*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 9*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 16)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + 3*
c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 3*c*sin(f*x + e)^3/(cos(f*x + e) +
1)^3 + 3*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 3*c*sin(f*x + e)^5/(cos(f*
x + e) + 1)^5 + c*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c*sin(f*x + e)^7/(c
os(f*x + e) + 1)^7) + 9*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a
^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c*sin(f*x + e)^2/(cos(f*x + e
) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(co
s(f*x + e) + 1))/c) + 18*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 - 2)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1) + c
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - c*sin(f*x + e)^3/(cos(f*x + e) + 1)^
3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 3*A*a^3*((sin(f*x + e)/(c
os(f*x + e) + 1) - 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*s
in(f*x + e)/(cos(f*x + e) + 1) + 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 -
2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) +
1)^4 - c*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(
f*x + e) + 1))/c) + 9*B*a^3*((sin(f*x + e)/(cos(f*x + e) + 1) - 5*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 3*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 4)/(c - c*sin(f*x + e)/(cos(f*x + e) + 1)
+ 2*c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*c*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - c*sin(f*x + e)^5/(cos(
f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c) + 18*A*a^3*(
arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c*sin(f*x + e)/(cos(f*x
+ e) + 1))) + 6*B*a^3*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c - 1/(c - c
*sin(f*x + e)/(cos(f*x + e) + 1))) - 6*A*a^3/(c - c*sin(f*x + e)/(cos(f*x +
e) + 1)))/f
```

mupad [B] time = 14.12, size = 323, normalized size = 2.07

$$\frac{24 A a^3 - \tan\left(\frac{e}{2} + \frac{f x}{2}\right) \left(7 A a^3 + \frac{34 B a^3}{3}\right) + \frac{94 B a^3}{3} - \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 (9 A a^3 + 18 B a^3) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 (17 A a^3 + 20 B a^3) - \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 (16 A a^3 + 32 B a^3) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 (56 A a^3 + 62 B a^3) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 (63 A a^3 + 76 B a^3)}{f \left(-c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^7 + c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 - 3 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 + 3 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 - 3 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 + 3 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 - 3 c \tan\left(\frac{e}{2} + \frac{f x}{2}\right) + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x)),x)
```

```
[Out] (24*A*a^3 - tan(e/2 + (f*x)/2)*(7*A*a^3 + (34*B*a^3)/3) + (94*B*a^3)/3 - ta
n(e/2 + (f*x)/2)^5*(9*A*a^3 + 18*B*a^3) + tan(e/2 + (f*x)/2)^6*(17*A*a^3 +
20*B*a^3) - tan(e/2 + (f*x)/2)^3*(16*A*a^3 + 32*B*a^3) + tan(e/2 + (f*x)/2)
^4*(56*A*a^3 + 62*B*a^3) + tan(e/2 + (f*x)/2)^2*(63*A*a^3 + 76*B*a^3))/(f*(
c - c*tan(e/2 + (f*x)/2) + 3*c*tan(e/2 + (f*x)/2)^2 - 3*c*tan(e/2 + (f*x)/2
)^3 + 3*c*tan(e/2 + (f*x)/2)^4 - 3*c*tan(e/2 + (f*x)/2)^5 + c*tan(e/2 + (f*
x)/2)^6 - c*tan(e/2 + (f*x)/2)^7)) - (5*a^3*atan((5*a^3*tan(e/2 + (f*x)/2)*
(3*A + 4*B))/(15*A*a^3 + 20*B*a^3))*(3*A + 4*B))/(c*f)
```

sympy [A] time = 15.99, size = 4255, normalized size = 27.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-45*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c*f*tan(e/2 + f*x/2)**7 -
6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f
*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*
```

$$\begin{aligned}
& \tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x*\tan(e/2 + f*x/2)**6/(6*c*f*\tan(e/ \\
& 2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18 \\
& *c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f* \\
& x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 135*A*a**3*f*x*\tan(e/2 + f*x/2) \\
& **5/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 \\
& + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18 \\
& *c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) + 135*A*a**3*f*x \\
& *\tan(e/2 + f*x/2)**4/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 \\
& + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 \\
& + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) \\
& - 135*A*a**3*f*x*\tan(e/2 + f*x/2)**3/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan \\
& (e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 \\
& + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 \\
& + f*x/2) - 6*c*f) + 135*A*a**3*f*x*\tan(e/2 + f*x/2)**2/(6*c*f*\tan(e/2 + f*x \\
& /2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan \\
& (e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 \\
& + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 45*A*a**3*f*x*\tan(e/2 + f*x/2)/(6*c*f* \\
& \tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)** \\
& 5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/ \\
& 2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) + 45*A*a**3*f*x/(6*c*f*\tan(\\
& e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - \\
& 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + \\
& f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 102*A*a**3*\tan(e/2 + f*x/2)** \\
& 6/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + \\
& f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c \\
& *f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) + 54*A*a**3*\tan(e/ \\
& 2 + f*x/2)**5/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c \\
& *f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/ \\
& 2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 336* \\
& A*a**3*\tan(e/2 + f*x/2)**4/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x \\
& /2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan \\
& (e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - \\
& 6*c*f) + 96*A*a**3*\tan(e/2 + f*x/2)**3/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan \\
& (e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)** \\
& 4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 \\
& + f*x/2) - 6*c*f) - 378*A*a**3*\tan(e/2 + f*x/2)**2/(6*c*f*\tan(e/2 + f*x/2) \\
& **7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e \\
& /2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + \\
& 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) + 42*A*a**3*\tan(e/2 + f*x/2)/(6*c*f*\tan(e/2 \\
& + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18* \\
& c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x \\
& /2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 144*A*a**3/(6*c*f*\tan(e/2 + f*x/ \\
& 2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan \\
& (e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 \\
& + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*\tan(e/2 + f*x/2)**7/(6*c* \\
& f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2) \\
& **5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(\\
& e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) + 60*B*a**3*f*x*\tan(e/2 + \\
& f*x/2)**6/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f* \\
& \tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)* \\
& *3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - 6*c*f) - 180*B*a \\
& **3*f*x*\tan(e/2 + f*x/2)**5/(6*c*f*\tan(e/2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f* \\
& x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f*x/2)**4 + 18*c*f* \\
& \tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan(e/2 + f*x/2) - \\
& 6*c*f) + 180*B*a**3*f*x*\tan(e/2 + f*x/2)**4/(6*c*f*\tan(e/2 + f*x/2)**7 - 6 \\
& *c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18*c*f*\tan(e/2 + f* \\
& x/2)**4 + 18*c*f*\tan(e/2 + f*x/2)**3 - 18*c*f*\tan(e/2 + f*x/2)**2 + 6*c*f*\tan \\
& (e/2 + f*x/2) - 6*c*f) - 180*B*a**3*f*x*\tan(e/2 + f*x/2)**3/(6*c*f*\tan(e/ \\
& 2 + f*x/2)**7 - 6*c*f*\tan(e/2 + f*x/2)**6 + 18*c*f*\tan(e/2 + f*x/2)**5 - 18
\end{aligned}$$

```

*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*
x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 180*B*a**3*f*x*tan(e/2 + f*x/2)
**2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
+ f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18
*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 60*B*a**3*f*x*
tan(e/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 1
8*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f
*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 6
0*B*a**3*f*x/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*
f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 120*B
*a**3*tan(e/2 + f*x/2)**6/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/
2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*ta
n(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6
*c*f) + 108*B*a**3*tan(e/2 + f*x/2)**5/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*t
an(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**
4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2
+ f*x/2) - 6*c*f) - 372*B*a**3*tan(e/2 + f*x/2)**4/(6*c*f*tan(e/2 + f*x/2)
**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e
/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 + f*x/2)**2 +
6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 192*B*a**3*tan(e/2 + f*x/2)**3/(6*c*f*tan
(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2 + f*x/2)**5 -
18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*c*f*tan(e/2 +
f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 456*B*a**3*tan(e/2 + f*x/2)*
*2/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/2
+ f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 18*
c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) + 68*B*a**3*tan(e
/2 + f*x/2)/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f
*tan(e/2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)
)**3 - 18*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f) - 188*B*
a**3/(6*c*f*tan(e/2 + f*x/2)**7 - 6*c*f*tan(e/2 + f*x/2)**6 + 18*c*f*tan(e/
2 + f*x/2)**5 - 18*c*f*tan(e/2 + f*x/2)**4 + 18*c*f*tan(e/2 + f*x/2)**3 - 1
8*c*f*tan(e/2 + f*x/2)**2 + 6*c*f*tan(e/2 + f*x/2) - 6*c*f), Ne(f, 0)), (x*
(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c), True))

```

$$3.45 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=163

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{3f(c-c \sin(e+fx))^5} - \frac{5a^3(2A+5B) \cos(e+fx)}{2c^2f} - \frac{5a^3(2A+5B) \cos^3(e+fx)}{6f(c^2-c^2 \sin(e+fx))} + \frac{5a^3x(2A+5B)}{2c^2} - \frac{2a^3c(2A+5B)}{3f(c-c \sin(e+fx))}$$

[Out] $5/2*a^3*(2*A+5*B)*x/c^2-5/2*a^3*(2*A+5*B)*\cos(f*x+e)/c^2/f+1/3*a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^5-2/3*a^3*(2*A+5*B)*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^3-5/6*a^3*(2*A+5*B)*\cos(f*x+e)^3/f/(c^2-c^2*\sin(f*x+e))$

Rubi [A] time = 0.35, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{5a^3(2A+5B) \cos(e+fx)}{2c^2f} + \frac{a^3c^3(A+B) \cos^7(e+fx)}{3f(c-c \sin(e+fx))^5} - \frac{5a^3(2A+5B) \cos^3(e+fx)}{6f(c^2-c^2 \sin(e+fx))} + \frac{5a^3x(2A+5B)}{2c^2} - \frac{2a^3c(2A+5B)}{3f(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] $(5*a^3*(2*A + 5*B)*x)/(2*c^2) - (5*a^3*(2*A + 5*B)*\text{Cos}[e + f*x])/(2*c^2*f) + (a^3*(A + B)*c^3*\text{Cos}[e + f*x]^7)/(3*f*(c - c*\text{Sin}[e + f*x])^5) - (2*a^3*(2*A + 5*B)*c*\text{Cos}[e + f*x]^5)/(3*f*(c - c*\text{Sin}[e + f*x])^3) - (5*a^3*(2*A + 5*B)*\text{Cos}[e + f*x]^3)/(6*f*(c^2 - c^2*\text{Sin}[e + f*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.)^(p_))*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])]^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(a*(m + p)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.)^(p_))*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])]^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.)^(p_))/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859


```
Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m_))*((A_) + (B_)*sin[(e_) + (f_)*(x_)]*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^2} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^5} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{1}{3} (a^3 (2A + 5B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^5} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{2a^3 (2A + 5B) c \cos^5(e + fx)}{3f(c - c \sin(e + fx))^3} + \frac{1}{3} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^5} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{2a^3 (2A + 5B) c \cos^5(e + fx)}{3f(c - c \sin(e + fx))^3} - \frac{5a^3 (2A + 5B) \cos(e + fx)}{2c^2 f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{2a^3 (2A + 5B) c \cos^5(e + fx)}{3f(c - c \sin(e + fx))^3} \\ &= \frac{5a^3 (2A + 5B) \cos(e + fx)}{2c^2 f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{2a^3 (2A + 5B) c \cos^5(e + fx)}{3f(c - c \sin(e + fx))^3} \\ &= \frac{5a^3 (2A + 5B) x}{2c^2} - \frac{5a^3 (2A + 5B) \cos(e + fx)}{2c^2 f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{3f(c - c \sin(e + fx))^5} - \frac{2a^3 (2A + 5B) c \cos^5(e + fx)}{3f(c - c \sin(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 0.88, size = 280, normalized size = 1.72

$$a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(64(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 30(2A + 5B)(e + fx) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(32*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + 30*(2*A + 5*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 12*(A + 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 64*(A + B)*Sin[(e + f*x)/2] - 32*(7*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] - 3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^2)
```

fricas [A] time = 0.45, size = 286, normalized size = 1.75

$$3Ba^3 \cos(fx + e)^4 - 6(A + 4B)a^3 \cos(fx + e)^3 - 30(2A + 5B)a^3 fx - 16(A + B)a^3 + (15(2A + 5B)a^3 fx + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/6*(3*B*a^3*cos(f*x + e)^4 - 6*(A + 4*B)*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x - 16*(A + B)*a^3 + (15*(2*A + 5*B)*a^3*f*x + (62*A + 131*B)*a^3)*cos(f*x + e)^2 - (15*(2*A + 5*B)*a^3*f*x - 2*(26*A + 71*B)*a^3)*cos(f*x + e) - (3*B*a^3*cos(f*x + e)^3 - 30*(2*A + 5*B)*a^3*f*x + 3*(2*A + 9*B)*a^3*cos(f*x + e)^2 + 16*(A + B)*a^3 - (15*(2*A + 5*B)*a^3*f*x - 2*(34*A + 79*B)*a^3)*cos(f*x + e))*sin(f*x + e)/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))

giac [A] time = 0.18, size = 233, normalized size = 1.43

$$\frac{15(2Aa^3+5Ba^3)(fx+e)}{c^2} + \frac{6\left(Ba^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 2Aa^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 10Ba^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - Ba^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) - 2Aa^3 - 10Ba^3\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 1\right)^2 c^2} + \frac{16\left(3Aa^3\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/6*(15*(2*A*a^3 + 5*B*a^3)*(f*x + e)/c^2 + 6*(B*a^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 10*B*a^3*tan(1/2*f*x + 1/2*e)^2 - B*a^3*tan(1/2*f*x + 1/2*e) - 2*A*a^3 - 10*B*a^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*c^2) + 16*(3*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 9*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*tan(1/2*f*x + 1/2*e) - 24*B*a^3*tan(1/2*f*x + 1/2*e) + 5*A*a^3 + 11*B*a^3)/(c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f

maple [B] time = 0.47, size = 399, normalized size = 2.45

$$\frac{8a^3A}{c^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} + \frac{24a^3B}{c^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{32a^3A}{3c^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3} - \frac{32a^3B}{3c^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3} - \frac{16}{c^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)

[Out] 8*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)*A+24*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)*B-32/3*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*A-32/3*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^3*B-16*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*A-16*a^3/c^2/f/(tan(1/2*f*x+1/2*e)-1)^2*B+a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A-10*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)-2*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A-10*a^3/c^2/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B+25*a^3/c^2/f*arctan(tan(1/2*f*x+1/2*e))*B+10*a^3/c^2/f*arctan(tan(1/2*f*x+1/2*e))*A

maxima [B] time = 0.61, size = 1386, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out]
$$\frac{1}{3} \left(B a^3 \left(\frac{75 \sin(fx + e)}{\cos(fx + e) + 1} - 97 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 + 126 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 - 98 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 + 63 \sin^5(fx + e) / (\cos(fx + e) + 1)^5 - 21 \sin^6(fx + e) / (\cos(fx + e) + 1)^6 - 32 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 5c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 7c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 + 7c^2 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 - 5c^2 \sin^5(fx + e) / (\cos(fx + e) + 1)^5 + 3c^2 \sin^6(fx + e) / (\cos(fx + e) + 1)^6 - c^2 \sin^7(fx + e) / (\cos(fx + e) + 1)^7) + 21 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^2 \right) + 4A a^3 \left(\frac{12 \sin(fx + e)}{\cos(fx + e) + 1} - 11 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 + 9 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 - 3 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 - 5 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 4c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 4c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 + 3c^2 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 - c^2 \sin^5(fx + e) / (\cos(fx + e) + 1)^5) + 3 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^2 \right) + 12B a^3 \left(\frac{12 \sin(fx + e)}{\cos(fx + e) + 1} - 11 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 + 9 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 - 3 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 - 5 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 4c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 4c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3 + 3c^2 \sin^4(fx + e) / (\cos(fx + e) + 1)^4 - c^2 \sin^5(fx + e) / (\cos(fx + e) + 1)^5) + 3 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^2 \right) + 6A a^3 \left(\frac{9 \sin(fx + e)}{\cos(fx + e) + 1} - 3 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 4 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 3c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3) + 3 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^2 \right) + 6B a^3 \left(\frac{9 \sin(fx + e)}{\cos(fx + e) + 1} - 3 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 4 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 3c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3) + 3 \arctan(\sin(fx + e) / (\cos(fx + e) + 1)) / c^2 \right) - 2A a^3 \left(\frac{3 \sin(fx + e)}{\cos(fx + e) + 1} - 3 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - 2 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 3c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3) + 6A a^3 \left(\frac{3 \sin(fx + e)}{\cos(fx + e) + 1} - 1 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 3c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3) + 2B a^3 \left(\frac{3 \sin(fx + e)}{\cos(fx + e) + 1} - 1 \right) / (c^2 - 3c^2 \sin(fx + e) / (\cos(fx + e) + 1) + 3c^2 \sin^2(fx + e) / (\cos(fx + e) + 1)^2 - c^2 \sin^3(fx + e) / (\cos(fx + e) + 1)^3) \right) / f$$

mupad [B] time = 14.00, size = 341, normalized size = 2.09

$$\frac{5a^3 \operatorname{atan}\left(\frac{5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(2A+5B)}{10Aa^3+25Ba^3}\right)(2A+5B) - \frac{46Aa^3}{3} - \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(38Aa^3+93Ba^3) + \frac{118Ba^3}{3} + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{c^2 f} - \frac{f \left(-c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)^7}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^2,x)

[Out]
$$\frac{(5a^3 \operatorname{atan}\left(\frac{5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(2A+5B)}{10Aa^3+25Ba^3}\right)(2A+5B)) / (10Aa^3+25Ba^3) * (2A+5B)}{c^2 f} - \left(\frac{46Aa^3}{3} - \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(38Aa^3+93Ba^3) + \frac{118Ba^3}{3} + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)\right)^6 * (8Aa^3+25Ba^3) - \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 * (34Aa^3+77Ba^3) - \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 * (72Aa^3+166Ba^3) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 * \left(\frac{106Aa^3}{3} + \frac{328Ba^3}{3}\right) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 * \left(\frac{128Aa^3}{3} + \frac{359Ba^3}{3}\right) / (f * (5c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 7c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 7c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 5c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + c^2))$$

$\ast x)/2)^5 + 3c^2 \tan(e/2 + (f \ast x)/2)^6 - c^2 \tan(e/2 + (f \ast x)/2)^7 + c^2 - 3c^2 \tan(e/2 + (f \ast x)/2))$

sympy [A] time = 29.89, size = 4665, normalized size = 28.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)

[Out] Piecewise(((30*A*a**3*f*x*tan(e/2 + f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 90*A*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 150*A*a**3*f*x*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 210*A*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 210*A*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 150*A*a**3*f*x*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 90*A*a**3*f*x*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 30*A*a**3*f*x/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 48*A*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 204*A*a**3*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 212*A*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 432*A*a**3*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 256*A*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 228*A*a**3*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 92*A*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 -

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18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*t
an(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*
x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 75*B*a**3*f*x*tan(e/2 +
f*x/2)**7/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 3
0*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*ta
n(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x
/2) - 6*c**2*f) - 225*B*a**3*f*x*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*
x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 4
2*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*ta
n(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 375*B*a**3*f*x
*tan(e/2 + f*x/2)**5/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*
x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 4
2*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*ta
n(e/2 + f*x/2) - 6*c**2*f) - 525*B*a**3*f*x*tan(e/2 + f*x/2)**4/(6*c**2*f*ta
n(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*
x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 3
0*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 525
*B*a**3*f*x*tan(e/2 + f*x/2)**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*ta
n(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 375*B*a**3*f*x*tan(e/2 + f*x/2)**2/
(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*ta
n(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*
x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c*
**2*f) + 225*B*a**3*f*x*tan(e/2 + f*x/2)/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*
c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(
e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2
)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 75*B*a**3*f*x/(6*c**2*f*ta
n(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/
2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*
c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 150*B
*a**3*tan(e/2 + f*x/2)**6/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2
 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**
4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2
*f*tan(e/2 + f*x/2) - 6*c**2*f) - 462*B*a**3*tan(e/2 + f*x/2)**5/(6*c**2*f*
tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f
*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 -
30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) + 65
6*B*a**3*tan(e/2 + f*x/2)**4/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(
e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2
)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c
**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 996*B*a**3*tan(e/2 + f*x/2)**3/(6*c**2
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
718*B*a**3*tan(e/2 + f*x/2)**2/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*ta
n(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*
x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 1
8*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) - 558*B*a**3*tan(e/2 + f*x/2)/(6*c**2
*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 + 30*c**2*f*tan(e/2
 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*tan(e/2 + f*x/2)**3
 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f*x/2) - 6*c**2*f) +
236*B*a**3/(6*c**2*f*tan(e/2 + f*x/2)**7 - 18*c**2*f*tan(e/2 + f*x/2)**6 +
30*c**2*f*tan(e/2 + f*x/2)**5 - 42*c**2*f*tan(e/2 + f*x/2)**4 + 42*c**2*f*
tan(e/2 + f*x/2)**3 - 30*c**2*f*tan(e/2 + f*x/2)**2 + 18*c**2*f*tan(e/2 + f
*x/2) - 6*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e
) + c)**2, True))

```

$$3.46 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=153

$$\frac{a^3(A+6B) \cos(e+fx)}{c^3 f} + \frac{a^3 c^3(A+B) \cos^7(e+fx)}{5f(c-c \sin(e+fx))^6} + \frac{2a^3 c^3(A+6B) \cos^3(e+fx)}{3f(c^3-c^3 \sin(e+fx))^2} - \frac{a^3 x(A+6B)}{c^3} - \frac{2a^3 c(A+6B) \cos(e+fx)}{15f(c-c \sin(e+fx))}$$

[Out] $-a^3*(A+6*B)*x/c^3+a^3*(A+6*B)*\cos(f*x+e)/c^3/f+1/5*a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^6-2/15*a^3*(A+6*B)*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^4+2/3*a^3*(A+6*B)*c^3*\cos(f*x+e)^3/f/(c^3-c^3*\sin(f*x+e))^2$

Rubi [A] time = 0.34, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{a^3(A+6B) \cos(e+fx)}{c^3 f} + \frac{a^3 c^3(A+B) \cos^7(e+fx)}{5f(c-c \sin(e+fx))^6} + \frac{2a^3 c^3(A+6B) \cos^3(e+fx)}{3f(c^3-c^3 \sin(e+fx))^2} - \frac{a^3 x(A+6B)}{c^3} - \frac{2a^3 c(A+6B) \cos(e+fx)}{15f(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] $-((a^3*(A+6*B)*x)/c^3) + (a^3*(A+6*B)*\text{Cos}[e+f*x])/(c^3*f) + (a^3*(A+B)*c^3*\text{Cos}[e+f*x]^7)/(5*f*(c-c*\text{Sin}[e+f*x])^6) - (2*a^3*(A+6*B)*c*\text{Cos}[e+f*x]^5)/(15*f*(c-c*\text{Sin}[e+f*x])^4) + (2*a^3*(A+6*B)*c^3*\text{Cos}[e+f*x]^3)/(3*f*(c^3-c^3*\text{Sin}[e+f*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegersQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(a*b*(2*m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m+p], 0]) && NeQ[2*m+p+1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^6} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{5f(c - c \sin(e + fx))^6} - \frac{1}{5} (a^3 (A + 6B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^5} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{5f(c - c \sin(e + fx))^6} - \frac{2a^3 (A + 6B) c \cos^5(e + fx)}{15f(c - c \sin(e + fx))^4} + \frac{1}{3} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^3} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{5f(c - c \sin(e + fx))^6} - \frac{2a^3 (A + 6B) c \cos^5(e + fx)}{15f(c - c \sin(e + fx))^4} + \frac{2a^3 (A + 6B) \cos^3(e + fx)}{15f(c - c \sin(e + fx))^2} \\ &= \frac{a^3 (A + 6B) \cos(e + fx)}{c^3 f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{5f(c - c \sin(e + fx))^6} - \frac{2a^3 (A + 6B) \cos^3(e + fx)}{15f(c - c \sin(e + fx))^2} \\ &= -\frac{a^3 (A + 6B) x}{c^3} + \frac{a^3 (A + 6B) \cos(e + fx)}{c^3 f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{5f(c - c \sin(e + fx))^6} \end{aligned}$$

Mathematica [B] time = 1.10, size = 316, normalized size = 2.07

$$a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(48(A + B) \sin\left(\frac{1}{2}(e + fx)\right) - 15(A + 6B)(e + fx) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(24*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 4*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - 15*(A + 6*B)*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 48*(A + B)*Sin[(e + f*x)/2] - 8*(11*A + 21*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 4*(23*A + 93*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(15*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^3)
```

fricas [B] time = 0.45, size = 337, normalized size = 2.20

$$15Ba^3 \cos(fx + e)^4 + 60(A + 6B)a^3 fx - 24(A + B)a^3 - (15(A + 6B)a^3 fx - (46A + 231B)a^3) \cos(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")
```


$$\begin{aligned}
& e)^3/(\cos(f*x + e) + 1)^3 + 15*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1 \\
& 1*c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5*c^3*\sin(f*x + e)^6/(\cos(f*x + \\
& e) + 1)^6 - c^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 15*\arctan(\sin(f*x + \\
& e)/(\cos(f*x + e) + 1))/c^3) + A*a^3*((95*\sin(f*x + e)/(\cos(f*x + e) + 1) - \\
& 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) \\
& + 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*\sin(f*x \\
& + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c \\
& ^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) \\
& + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin(f*x + e) \\
& /(\cos(f*x + e) + 1))/c^3) + 3*B*a^3*((95*\sin(f*x + e)/(\cos(f*x + e) + 1) - \\
& 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + \\
& 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 22)/(c^3 - 5*c^3*\sin(f*x + \\
& e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^ \\
& 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) \\
& + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin(f*x + e)/ \\
& (\cos(f*x + e) + 1))/c^3) + A*a^3*(20*\sin(f*x + e)/(\cos(f*x + e) + 1) - 40*s \\
& in(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\
& - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 7)/(c^3 - 5*c^3*\sin(f*x + e)/(\c \\
& os(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(\\
& f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 \\
& - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 9*A*a^3*(5*\sin(f*x + e)/(\cos(\\
& f*x + e) + 1) - 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\c \\
& os(f*x + e) + 1)^3 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c \\
& ^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e \\
&) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\\
& cos(f*x + e) + 1)^5) - 3*B*a^3*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 5*\sin(f \\
& *x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 1) \\
& /(\c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x + e)^2/(\cos(\\
& f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*c^3*\sin(f \\
& x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + \\
& 6*A*a^3*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^2/(\cos(f*x + e \\
&) + 1)^2 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*c^3*\sin(f*x \\
& + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + \\
& 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^5/(\cos(f*x + \\
& e) + 1)^5) + 6*B*a^3*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^2 \\
& /(\cos(f*x + e) + 1)^2 - 1)/(c^3 - 5*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 1 \\
& 0*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 10*c^3*\sin(f*x + e)^3/(\cos(f*x \\
& + e) + 1)^3 + 5*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - c^3*\sin(f*x + e)^ \\
& 5/(\cos(f*x + e) + 1)^5))/f
\end{aligned}$$

mupad [B] time = 14.00, size = 336, normalized size = 2.20

$$\frac{\frac{52 A a^3}{15} - \tan\left(\frac{e}{2} + \frac{f x}{2}\right) \left(\frac{40 A a^3}{3} + 82 B a^3\right) + \frac{94 B a^3}{5} + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 (4 A a^3 + 12 B a^3) - \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 (8 A a^3 + 58 B a^3) - \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \left(\frac{64 A a^3}{3} + 148 B a^3\right) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 \left(\frac{92 A a^3}{3} + 134 B a^3\right) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 \left(\frac{452 A a^3}{15} + \frac{744 B a^3}{5}\right)}{f \left(-c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^7 + 5 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 - 11 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 + 15 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 - 11 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 + 5 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 - 11 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right) + c^3 - 5 c^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right) \left(\frac{2 a^3 \operatorname{atan}\left(\frac{2 a^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right) (A + 6 B)}{2 A a^3 + 12 B a^3}\right) (A + 6 B)}{c^3 f}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^3,x)

[Out] ((52*A*a^3)/15 - tan(e/2 + (f*x)/2)*((40*A*a^3)/3 + 82*B*a^3) + (94*B*a^3)/5 + tan(e/2 + (f*x)/2)^6*(4*A*a^3 + 12*B*a^3) - tan(e/2 + (f*x)/2)^5*(8*A*a^3 + 58*B*a^3) - tan(e/2 + (f*x)/2)^3*((64*A*a^3)/3 + 148*B*a^3) + tan(e/2 + (f*x)/2)^4*((92*A*a^3)/3 + 134*B*a^3) + tan(e/2 + (f*x)/2)^2*((452*A*a^3)/15 + (744*B*a^3)/5))/(f*(11*c^3*tan(e/2 + (f*x)/2)^2 - 15*c^3*tan(e/2 + (f*x)/2)^3 + 15*c^3*tan(e/2 + (f*x)/2)^4 - 11*c^3*tan(e/2 + (f*x)/2)^5 + 5*c^3*tan(e/2 + (f*x)/2)^6 - c^3*tan(e/2 + (f*x)/2)^7 + c^3 - 5*c^3*tan(e/2 + (f*x)/2)*(2*a^3*atan((2*a^3*tan(e/2 + (f*x)/2)*(A + 6*B))/(2*A*a^3 + 12*B*a^3))*(A + 6*B))/(c^3*f))

sympy [A] time = 48.98, size = 4665, normalized size = 30.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)

[Out] Piecewise((-15*A*a**3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 75*A*a**3*f*x*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 165*A*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 225*A*a**3*f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 225*A*a**3*f*x*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 165*A*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 75*A*a**3*f*x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 15*A*a**3*f*x/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 60*A*a**3*tan(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 120*A*a**3*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 460*A*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 320*A*a**3*tan(e/2 + f*x/2)**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 452*A*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 200*A*a**3*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 52*A*a**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 -

```

225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3
*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 90*B*a**
3*f*x*tan(e/2 + f*x/2)**7/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/
2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)
)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75
*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 450*B*a**3*f*x*tan(e/2 + f*x/2)**6/
(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f
*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2
+ f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) - 990*B*a**3*f*x*tan(e/2 + f*x/2)**5/(15*c**3*f*tan(e/2 + f*x/2)
)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225
*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*t
an(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1350*B*a**3*
f*x*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2
+ f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)*
**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c
**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 1350*B*a**3*f*x*tan(e/2 + f*x/2)**3/(
15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*
tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 +
f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) -
15*c**3*f) + 990*B*a**3*f*x*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/2)
)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*
c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 450*B*a**3*f*
x*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x
/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 +
225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f
*tan(e/2 + f*x/2) - 15*c**3*f) + 90*B*a**3*f*x/(15*c**3*f*tan(e/2 + f*x/2)*
**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c
**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan
(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) - 180*B*a**3*tan
(e/2 + f*x/2)**6/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)
)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 22
5*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*t
an(e/2 + f*x/2) - 15*c**3*f) + 870*B*a**3*tan(e/2 + f*x/2)**5/(15*c**3*f*tan
(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*
x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 -
165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) -
2010*B*a**3*tan(e/2 + f*x/2)**4/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f
*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2
+ f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)*
**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 2220*B*a**3*tan(e/2 + f*x/2)
**3/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c*
**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(
e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/
2) - 15*c**3*f) - 2232*B*a**3*tan(e/2 + f*x/2)**2/(15*c**3*f*tan(e/2 + f*x/
2)**7 - 75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 22
5*c**3*f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*
tan(e/2 + f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f) + 1230*B*a**3
*tan(e/2 + f*x/2)/(15*c**3*f*tan(e/2 + f*x/2)**7 - 75*c**3*f*tan(e/2 + f*x/
2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*f*tan(e/2 + f*x/2)**4 + 2
25*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2 + f*x/2)**2 + 75*c**3*f*
tan(e/2 + f*x/2) - 15*c**3*f) - 282*B*a**3/(15*c**3*f*tan(e/2 + f*x/2)**7 -
75*c**3*f*tan(e/2 + f*x/2)**6 + 165*c**3*f*tan(e/2 + f*x/2)**5 - 225*c**3*
f*tan(e/2 + f*x/2)**4 + 225*c**3*f*tan(e/2 + f*x/2)**3 - 165*c**3*f*tan(e/2
+ f*x/2)**2 + 75*c**3*f*tan(e/2 + f*x/2) - 15*c**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e) + c)**3, True))

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$$3.47 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=151

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{7f(c-c \sin(e+fx))^7} - \frac{2a^3B \cos(e+fx)}{f(c^4-c^4 \sin(e+fx))} + \frac{a^3Bx}{c^4} + \frac{2a^3Bc^2 \cos^3(e+fx)}{3f(c^2-c^2 \sin(e+fx))^3} - \frac{2a^3Bc \cos^5(e+fx)}{5f(c-c \sin(e+fx))^5}$$

[Out] $a^3Bx/c^4 + 1/7*a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^7 - 2/5*a^3*B*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^5 + 2/3*a^3*B*c^2*\cos(f*x+e)^3/f/(c^2-c^2*\sin(f*x+e))^3 - 2*a^3*B*\cos(f*x+e)/f/(c^4-c^4*\sin(f*x+e))$

Rubi [A] time = 0.33, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2680, 8}

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{7f(c-c \sin(e+fx))^7} + \frac{2a^3Bc^2 \cos^3(e+fx)}{3f(c^2-c^2 \sin(e+fx))^3} - \frac{2a^3B \cos(e+fx)}{f(c^4-c^4 \sin(e+fx))} + \frac{a^3Bx}{c^4} - \frac{2a^3Bc \cos^5(e+fx)}{5f(c-c \sin(e+fx))^5}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]

[Out] $(a^3Bx)/c^4 + (a^3*(A+B)*c^3*\cos[e+f*x]^7)/(7*f*(c-c*\sin[e+f*x])^7) - (2*a^3*B*c*\cos[e+f*x]^5)/(5*f*(c-c*\sin[e+f*x])^5) + (2*a^3*B*c^2*\cos[e+f*x]^3)/(3*f*(c^2-c^2*\sin[e+f*x])^3) - (2*a^3*B*\cos[e+f*x])/f*(c^4-c^4*\sin[e+f*x])$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegersQ[2*m, 2*p]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(a*b*(2*m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m+p], 0]) && NeQ[2*m+p+1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^4} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^7} dx \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^7} - (a^3 B c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^6} dx \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^7} - \frac{2a^3 B c \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} + (a^3 B) \int \frac{\cos^5(e + fx)}{(c - c \sin(e + fx))^5} dx \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^7} - \frac{2a^3 B c \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} + \frac{2a^3 B}{3cf(c - c \sin(e + fx))^3} \\
&= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^7} - \frac{2a^3 B c \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} + \frac{2a^3 B}{3cf(c - c \sin(e + fx))^3} \\
&= \frac{a^3 B x}{c^4} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^7} - \frac{2a^3 B c \cos^5(e + fx)}{5f(c - c \sin(e + fx))^5} + \frac{2a^3 B}{3cf(c - c \sin(e + fx))^3}
\end{aligned}$$

Mathematica [B] time = 1.21, size = 356, normalized size = 2.36

$$a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(240(A + B) \sin\left(\frac{1}{2}(e + fx)\right) - 2(15A + 337B) \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^4,x]

[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(120*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 12*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 2*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + 105*B*(e + f*x)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7 + 240*(A + B)*Sin[(e + f*x)/2] - 24*(15*A + 29*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 4*(45*A + 199*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] - 2*(15*A + 337*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(105*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^4)

fricas [B] time = 0.44, size = 363, normalized size = 2.40

$$840 B a^3 f x + (105 B a^3 f x + (15 A + 337 B) a^3) \cos(f x + e)^4 + 120 (A + B) a^3 - (315 B a^3 f x + (45 A - 613 B) a^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] 1/105*(840*B*a^3*f*x + (105*B*a^3*f*x + (15*A + 337*B)*a^3)*cos(f*x + e)^4 + 120*(A + B)*a^3 - (315*B*a^3*f*x + (45*A - 613*B)*a^3)*cos(f*x + e)^3 - 24*(35*B*a^3*f*x + (5*A + 26*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x + (A - 13*B)*a^3)*cos(f*x + e) - (840*B*a^3*f*x - 120*(A + B)*a^3 - (105*B*a^3*f*x - (15*A + 337*B)*a^3)*cos(f*x + e)^3 - 12*(35*B*a^3*f*x - (5*A - 23*B)*a^3)*cos(f*x + e)^2 + 60*(7*B*a^3*f*x - (A + 15*B)*a^3)*cos(f*x + e))*sin(f*x + e)

+ e))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))

giac [A] time = 0.24, size = 213, normalized size = 1.41

$$\frac{105(f_{x+e})Ba^3}{c^4} - \frac{2\left(105Aa^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 105Ba^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 840Ba^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 525Aa^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1925Ba^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3\right)}{c^4 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^7} - \frac{105f}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] 1/105*(105*(f*x + e)*B*a^3/c^4 - 2*(105*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 105*B*a^3*tan(1/2*f*x + 1/2*e)^6 + 840*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 525*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 1925*B*a^3*tan(1/2*f*x + 1/2*e)^4 + 3920*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 315*A*a^3*tan(1/2*f*x + 1/2*e)^2 - 2667*B*a^3*tan(1/2*f*x + 1/2*e)^2 + 1064*B*a^3*tan(1/2*f*x + 1/2*e) + 15*A*a^3 - 167*B*a^3)/(c^4*(tan(1/2*f*x + 1/2*e) - 1)^7)/f

maple [B] time = 0.48, size = 374, normalized size = 2.48

$$\frac{2a^3A}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} + \frac{2a^3B}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)} - \frac{12a^3A}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^2} - \frac{4a^3B}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^2} - \frac{40a^3A}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3} - \frac{40a^3B}{c^4f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)

[Out] -2*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)*A+2*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)*B-12*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^2*A-4*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^2*B-40*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^3*A-40/3*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^3*B-128/7*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^7*A-128/7*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^7*B-80*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^4*A-48*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^4*B-64*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^6*A-64*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^6*B-96*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^5*A-416/5*a^3/c^4/f/(tan(1/2*f*x+1/2*e)-1)^5*B+2*a^3/c^4/f*B*arctan(tan(1/2*f*x+1/2*e))

maxima [B] time = 0.58, size = 2118, normalized size = 14.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out] 2/105*(5*B*a^3*((203*sin(f*x + e))/(cos(f*x + e) + 1) - 525*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 686*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 434*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 147*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 21*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 32)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + 21*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/c^4 + 3*A*a^3*(91*sin(f*x + e)/(cos(f*x + e) + 1)

```

- 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)^5/(c
os(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*
c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x +
e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e
)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*
sin(f*x + e)^7/(cos(f*x + e) + 1)^7) + B*a^3*(91*sin(f*x + e)/(cos(f*x + e)
+ 1) - 168*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 280*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 - 175*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f*x + e)
^5/(cos(f*x + e) + 1)^5 - 13)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1)
+ 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f
*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*
x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 -
c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 3*A*a^3*(49*sin(f*x + e)/(cos(f
*x + e) + 1) - 147*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 210*sin(f*x + e)^3
/(cos(f*x + e) + 1)^3 - 210*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 105*sin(f
*x + e)^5/(cos(f*x + e) + 1)^5 - 35*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1
2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(c
os(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*
x + e) + 1)^7) - 12*A*a^3*(14*sin(f*x + e)/(cos(f*x + e) + 1) - 42*sin(f*x
+ e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 35*s
in(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x
+ e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*
c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e)
+ 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^7) - 12*B*a^3*(14*sin(f*x
+ e)/(cos(f*x + e) + 1) - 42*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 35*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 - 35*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 2
)/(c^4 - 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(
f*x + e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^
5 + 7*c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x
+ e) + 1)^7) + 6*A*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4
- 7*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x +
e)^4/(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*
c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e)
+ 1)^7) + 18*B*a^3*(7*sin(f*x + e)/(cos(f*x + e) + 1) - 21*sin(f*x + e)^2/(
cos(f*x + e) + 1)^2 + 35*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(c^4 - 7*
c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 21*c^4*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 - 35*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 35*c^4*sin(f*x + e)^4/
(cos(f*x + e) + 1)^4 - 21*c^4*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 7*c^4*s
in(f*x + e)^6/(cos(f*x + e) + 1)^6 - c^4*sin(f*x + e)^7/(cos(f*x + e) + 1)^
7))/f

```

mupad [B] time = 15.98, size = 316, normalized size = 2.09

$$\frac{B a^3 x \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 \left(\frac{a^3 (1680 B - 2205 B (e + f x))}{105} + 21 B a^3 (e + f x)\right) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \left(\frac{a^3 (7840 B - 3675 B (e + f x))}{105} + 35 B a^3 (e + f x)\right)}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^4,x)

[Out] (B*a^3*x)/c^4 - (tan(e/2 + (f*x)/2)^5*((a^3*(1680*B - 2205*B*(e + f*x)))/105 + 21*B*a^3*(e + f*x)) + tan(e/2 + (f*x)/2)^3*((a^3*(7840*B - 3675*B*(e +


```

5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f) - 1680*B*a**3*tan(e/2 + f*x/2)**5/(105*c**4*f*tan(e/2 + f*
x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5
- 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*
c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 38
50*B*a**3*tan(e/2 + f*x/2)**4/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*
tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2
+ f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/
2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) - 7840*B*a**3*tan(e/2 + f
*x/2)**3/(105*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 +
2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c
**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*ta
n(e/2 + f*x/2) - 105*c**4*f) + 5334*B*a**3*tan(e/2 + f*x/2)**2/(105*c**4*f*
tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2
+ f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2
)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*
c**4*f) - 2128*B*a**3*tan(e/2 + f*x/2)/(105*c**4*f*tan(e/2 + f*x/2)**7 - 73
5*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f*tan(e/2 + f*x/2)**5 - 3675*c**4*
f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/2 + f*x/2)**3 - 2205*c**4*f*tan(e
/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/2) - 105*c**4*f) + 334*B*a**3/(10
5*c**4*f*tan(e/2 + f*x/2)**7 - 735*c**4*f*tan(e/2 + f*x/2)**6 + 2205*c**4*f
*tan(e/2 + f*x/2)**5 - 3675*c**4*f*tan(e/2 + f*x/2)**4 + 3675*c**4*f*tan(e/
2 + f*x/2)**3 - 2205*c**4*f*tan(e/2 + f*x/2)**2 + 735*c**4*f*tan(e/2 + f*x/
2) - 105*c**4*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3/(-c*sin(e)
+ c)**4, True))

```

$$3.48 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx$$

Optimal. Leaf size=77

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{9f(c-c \sin(e+fx))^8} + \frac{a^3c^2(A-8B) \cos^7(e+fx)}{63f(c-c \sin(e+fx))^7}$$

[Out] 1/9*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/63*a^3*(A-8*B)*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7

Rubi [A] time = 0.24, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2859, 2671}

$$\frac{a^3c^2(A-8B) \cos^7(e+fx)}{63f(c-c \sin(e+fx))^7} + \frac{a^3c^3(A+B) \cos^7(e+fx)}{9f(c-c \sin(e+fx))^8}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]

[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(9*f*(c - c*Sin[e + f*x])^8) + (a^3*(A - 8*B)*c^2*Cos[e + f*x]^7)/(63*f*(c - c*Sin[e + f*x])^7)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^5} dx &= (a^3c^3) \int \frac{\cos^6(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^8} dx \\ &= \frac{a^3(A+B)c^3 \cos^7(e+fx)}{9f(c-c \sin(e+fx))^8} + \frac{1}{9} (a^3(A-8B)c^2) \int \frac{\cos^6(e+fx)}{(c-c \sin(e+fx))^7} dx \\ &= \frac{a^3(A+B)c^3 \cos^7(e+fx)}{9f(c-c \sin(e+fx))^8} + \frac{a^3(A-8B)c^2 \cos^7(e+fx)}{63f(c-c \sin(e+fx))^7} \end{aligned}$$

Mathematica [B] time = 2.52, size = 283, normalized size = 3.68

$$\frac{a^3(\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(315(A - B) \cos\left(\frac{1}{2}(e + fx)\right) - 189(A - B) \cos\left(\frac{3}{2}(e + fx)\right) \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^5,x]

[Out] -1/504*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(315*(A - B)*Cos[(e + f*x)/2] - 189*(A - B)*Cos[(3*(e + f*x))/2] - 63*A*Cos[(5*(e + f*x))/2] + 63*B*Cos[(5*(e + f*x))/2] + 9*A*Cos[(7*(e + f*x))/2] - 9*B*Cos[(7*(e + f*x))/2] + 189*A*Sin[(e + f*x)/2] + 693*B*Sin[(e + f*x)/2] + 105*A*Sin[(3*(e + f*x))/2] + 483*B*Sin[(3*(e + f*x))/2] - 27*A*Sin[(5*(e + f*x))/2] - 225*B*Sin[(5*(e + f*x))/2] - 63*B*Sin[(7*(e + f*x))/2] - A*Sin[(9*(e + f*x))/2] + 8*B*Sin[(9*(e + f*x))/2]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^5)

fricas [B] time = 0.43, size = 331, normalized size = 4.30

$$\frac{(A - 8B)a^3 \cos(fx + e)^5 - (4A + 31B)a^3 \cos(fx + e)^4 + (19A + 37B)a^3 \cos(fx + e)^3 + 4(13A + 22B)a^3 \cos(fx + e)^2 - 28(A + B)a^3 \cos(fx + e) - 56(A + B)a^3 + ((A - 8B)a^3 \cos(fx + e)^4 + (5A + 23B)a^3 \cos(fx + e)^3 + 12(2A + 5B)a^3 \cos(fx + e)^2 - 28(A + B)a^3 \cos(fx + e) - 56(A + B)a^3) \sin(fx + e)}{63(c^5 f \cos(fx + e)^5 + 5c^5 f \cos(fx + e)^4 - 8c^5 f \cos(fx + e)^3 - 20c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f - (c^5 f \cos(fx + e)^4 - 4c^5 f \cos(fx + e)^3 - 12c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f) \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] -1/63*((A - 8*B)*a^3*cos(f*x + e)^5 - (4*A + 31*B)*a^3*cos(f*x + e)^4 + (19*A + 37*B)*a^3*cos(f*x + e)^3 + 4*(13*A + 22*B)*a^3*cos(f*x + e)^2 - 28*(A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3 + ((A - 8*B)*a^3*cos(f*x + e)^4 + (5*A + 23*B)*a^3*cos(f*x + e)^3 + 12*(2*A + 5*B)*a^3*cos(f*x + e)^2 - 28*(A + B)*a^3*cos(f*x + e) - 56*(A + B)*a^3)*sin(f*x + e))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f)*sin(f*x + e))

giac [B] time = 0.27, size = 301, normalized size = 3.91

$$\frac{2 \left(63 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 - 63 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 63 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^7 + 483 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 105 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^6 - 315 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 315 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^5 + 693 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 189 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 189 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 189 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 225 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 27 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 9 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 9 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 8 A a^3 - 8 B a^3 \right)}{c^5 f (\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] -2/63*(63*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 63*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 63*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 483*A*a^3*tan(1/2*f*x + 1/2*e)^6 + 105*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 315*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 315*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 693*A*a^3*tan(1/2*f*x + 1/2*e)^4 + 189*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 189*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 189*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 225*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 27*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 9*A*a^3*tan(1/2*f*x + 1/2*e) + 9*B*a^3*tan(1/2*f*x + 1/2*e) + 8*A*a^3 - 8*B*a^3)/(c^5*f*(tan(1/2*f*x + 1/2*e) - 1)^9)

maple [B] time = 0.53, size = 205, normalized size = 2.66

$$2a^3 \left(\frac{992A+800B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{304A+144B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{928A+864B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{86A+26B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{512A+512B}{8\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8} - \frac{14A}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} \right) \frac{1}{fc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x)

[Out] 2/f*a^3/c^5*(-1/6*(992*A+800*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/4*(304*A+144*B)/(tan(1/2*f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(928*A+864*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/3*(86*A+26*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/8*(512*A+512*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/2*(14*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/5*(680*A+440*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/9*(128*A+128*B)/(tan(1/2*f*x+1/2*e)-1)^9)

maxima [B] time = 0.54, size = 2701, normalized size = 35.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out] -2/315*(A*a^3*(432*sin(f*x + e)/(cos(f*x + e) + 1) - 1728*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3612*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 5418*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5040*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 3360*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 1260*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 315*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 83)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 15*A*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 5*B*a^3*(45*sin(f*x + e)/(cos(f*x + e) + 1) - 117*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 273*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 315*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 315*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 147*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 63*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 5)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9) - 10*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)

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cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*
sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) +
1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos
(f*x + e) + 1)^9) - 30*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 63
*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 63*sin(f*x + e)^5/(cos(f*x + e) + 1)
^5 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^
2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c
^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e
)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*
sin(f*x + e)^9/(cos(f*x + e) + 1)^9) + 8*B*a^3*(9*sin(f*x + e)/(cos(f*x + e
) + 1) - 36*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 84*sin(f*x + e)^3/(cos(f*
x + e) + 1)^3 - 126*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 1)/(c^5 - 9*c^5*s
in(f*x + e)/(cos(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2
- 84*c^5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin
(f*x + e)^6/(cos(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)
^7 + 9*c^5*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*
x + e) + 1)^9) + 42*A*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*si
n(f*x + e)^4/(cos(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5
- 30*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(co
s(f*x + e) + 1) + 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f
*x + e)^3/(cos(f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^
4 - 126*c^5*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(co
s(f*x + e) + 1)^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(
f*x + e)^8/(cos(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9)
+ 42*B*a^3*(9*sin(f*x + e)/(cos(f*x + e) + 1) - 36*sin(f*x + e)^2/(cos(f*x
+ e) + 1)^2 + 54*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 81*sin(f*x + e)^4/(c
os(f*x + e) + 1)^4 + 45*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 30*sin(f*x +
e)^6/(cos(f*x + e) + 1)^6 - 1)/(c^5 - 9*c^5*sin(f*x + e)/(cos(f*x + e) + 1)
+ 36*c^5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 84*c^5*sin(f*x + e)^3/(cos(
f*x + e) + 1)^3 + 126*c^5*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 126*c^5*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 + 84*c^5*sin(f*x + e)^6/(cos(f*x + e) + 1)
^6 - 36*c^5*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 9*c^5*sin(f*x + e)^8/(cos
(f*x + e) + 1)^8 - c^5*sin(f*x + e)^9/(cos(f*x + e) + 1)^9))/f

```

mupad [B] time = 13.39, size = 346, normalized size = 4.49

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{1013 A a^3}{16} + \frac{149 B a^3}{16} - \frac{113 A a^3 \cos(2e+2fx)}{4} + \frac{37 A a^3 \cos(3e+3fx)}{8} + \frac{7 A a^3 \cos(4e+4fx)}{16} - \frac{41 B a^3 \cos(2e+2fx)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^5,x)
[Out] (2*cos(e/2 + (f*x)/2)*((1013*A*a^3)/16 + (149*B*a^3)/16 - (113*A*a^3*cos(2*
e + 2*f*x))/4 + (37*A*a^3*cos(3*e + 3*f*x))/8 + (7*A*a^3*cos(4*e + 4*f*x))/
16 - (41*B*a^3*cos(2*e + 2*f*x))/4 + (19*B*a^3*cos(3*e + 3*f*x))/8 + (7*B*a
^3*cos(4*e + 4*f*x))/16 + (63*A*a^3*sin(2*e + 2*f*x))/8 + (9*A*a^3*sin(3*e
+ 3*f*x))/2 - (9*A*a^3*sin(4*e + 4*f*x))/16 - (63*B*a^3*sin(2*e + 2*f*x))/8
- (9*B*a^3*sin(3*e + 3*f*x))/2 + (9*B*a^3*sin(4*e + 4*f*x))/16 - (257*A*a^
3*cos(e + f*x))/8 - (23*B*a^3*cos(e + f*x))/8 - (63*A*a^3*sin(e + f*x))/2 +
(63*B*a^3*sin(e + f*x))/2))/(63*c^5*f*((63*2^(1/2)*cos(e/2 + pi/4 + (f*x)/
2))/8 - (21*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/4 - (9*2^(1/2)*cos((5*
e)/2 + pi/4 + (5*f*x)/2))/4 + (9*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/1
6 + (2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/16))

```



```

- 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938
*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f
*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**
3*tan(e/2 + f*x/2)**4/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)
**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5
292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5
*f*tan(e/2 + f*x/2) - 63*c**5*f) - 378*B*a**3*tan(e/2 + f*x/2)**3/(63*c**5*
f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/
2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x
/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3
- 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f
) - 54*B*a**3*tan(e/2 + f*x/2)**2/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5
*f*tan(e/2 + f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(
e/2 + f*x/2)**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f
*x/2)**4 + 5292*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**
2 + 567*c**5*f*tan(e/2 + f*x/2) - 63*c**5*f) - 18*B*a**3*tan(e/2 + f*x/2)/(
63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 + f*x/2)**8 + 2268*c**5*
f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)**6 + 7938*c**5*f*tan(e
/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 5292*c**5*f*tan(e/2 + f*
x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*f*tan(e/2 + f*x/2) - 6
3*c**5*f) + 2*B*a**3/(63*c**5*f*tan(e/2 + f*x/2)**9 - 567*c**5*f*tan(e/2 +
f*x/2)**8 + 2268*c**5*f*tan(e/2 + f*x/2)**7 - 5292*c**5*f*tan(e/2 + f*x/2)*
**6 + 7938*c**5*f*tan(e/2 + f*x/2)**5 - 7938*c**5*f*tan(e/2 + f*x/2)**4 + 52
92*c**5*f*tan(e/2 + f*x/2)**3 - 2268*c**5*f*tan(e/2 + f*x/2)**2 + 567*c**5*
f*tan(e/2 + f*x/2) - 63*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a
)**3/(-c*sin(e) + c)**5, True))

```

$$3.49 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^6} dx$$

Optimal. Leaf size=118

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{11f(c-c \sin(e+fx))^9} + \frac{a^3c^2(2A-9B) \cos^7(e+fx)}{99f(c-c \sin(e+fx))^8} + \frac{a^3c(2A-9B) \cos^7(e+fx)}{693f(c-c \sin(e+fx))^7}$$

[Out] 1/11*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+1/99*a^3*(2*A-9*B)*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+1/693*a^3*(2*A-9*B)*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3c^2(2A-9B) \cos^7(e+fx)}{99f(c-c \sin(e+fx))^8} + \frac{a^3c^3(A+B) \cos^7(e+fx)}{11f(c-c \sin(e+fx))^9} + \frac{a^3c(2A-9B) \cos^7(e+fx)}{693f(c-c \sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]

[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^9) + (a^3*(2*A - 9*B)*c^2*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^8) + (a^3*(2*A - 9*B)*c*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^7)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^6} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{11 f (c - c \sin(e + fx))^9} + \frac{1}{11} (a^3 (2A - 9B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{11 f (c - c \sin(e + fx))^9} + \frac{a^3 (2A - 9B) c^2 \cos^7(e + fx)}{99 f (c - c \sin(e + fx))^8} + \frac{1}{99} \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^9} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{11 f (c - c \sin(e + fx))^9} + \frac{a^3 (2A - 9B) c^2 \cos^7(e + fx)}{99 f (c - c \sin(e + fx))^8} + \frac{a^3}{6} \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^9} dx \end{aligned}$$

Mathematica [B] time = 2.94, size = 313, normalized size = 2.65

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(462(11A + 3B) \cos\left(\frac{1}{2}(e + fx)\right) - 594(5A + 2B) \cos\left(\frac{3}{2}(e + fx)\right) + 924A \cos\left(\frac{5}{2}(e + fx)\right) - 693B \cos\left(\frac{7}{2}(e + fx)\right) + 110A \cos\left(\frac{9}{2}(e + fx)\right) + 198B \cos\left(\frac{11}{2}(e + fx)\right) - 2A \cos\left(\frac{13}{2}(e + fx)\right) + 9B \cos\left(\frac{15}{2}(e + fx)\right) + 4158A \sin\left(\frac{1}{2}(e + fx)\right) + 5544B \sin\left(\frac{3}{2}(e + fx)\right) + 2310A \sin\left(\frac{5}{2}(e + fx)\right) + 4158B \sin\left(\frac{7}{2}(e + fx)\right) - 594A \sin\left(\frac{9}{2}(e + fx)\right) - 2178B \sin\left(\frac{11}{2}(e + fx)\right) - 693B \sin\left(\frac{13}{2}(e + fx)\right) - 22A \sin\left(\frac{15}{2}(e + fx)\right) + 99B \sin\left(\frac{17}{2}(e + fx)\right) \right)}{(11088c^6 f (\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right))^6 (-1 + \sin(e + fx))^6)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^6,x]

[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(462*(11*A + 3*B)*Cos[(e + f*x)/2] - 594*(5*A + 2*B)*Cos[(3*(e + f*x))/2] - 924*A*Cos[(5*(e + f*x))/2] - 693*B*Cos[(5*(e + f*x))/2] + 110*A*Cos[(7*(e + f*x))/2] + 198*B*Cos[(7*(e + f*x))/2] - 2*A*Cos[(11*(e + f*x))/2] + 9*B*Cos[(11*(e + f*x))/2] + 4158*A*Sin[(e + f*x)/2] + 5544*B*Sin[(e + f*x)/2] + 2310*A*Sin[(3*(e + f*x))/2] + 4158*B*Sin[(3*(e + f*x))/2] - 594*A*Sin[(5*(e + f*x))/2] - 2178*B*Sin[(5*(e + f*x))/2] - 693*B*Sin[(7*(e + f*x))/2] - 22*A*Sin[(9*(e + f*x))/2] + 99*B*Sin[(9*(e + f*x))/2]))/(11088*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^6)

fricas [B] time = 0.45, size = 405, normalized size = 3.43

$$\frac{(2A - 9B)a^3 \cos^6(fx + e) + 6(2A - 9B)a^3 \cos^5(fx + e) - (25A + 234B)a^3 \cos^4(fx + e) + 7(23A + 45B)a^3 \cos^3(fx + e) + 28(16A + 27B)a^3 \cos^2(fx + e) - 252(A + B)a^3 \cos(fx + e) - 504(A + B)a^3 - ((2A - 9B)a^3 \cos^5(fx + e) - 5(2A - 9B)a^3 \cos^4(fx + e) - 4 - 7(5A + 27B)a^3 \cos^3(fx + e) - 28(7A + 18B)a^3 \cos^2(fx + e) + 252(A + B)a^3 \cos(fx + e) + 504(A + B)a^3) \sin(fx + e)}{(c^6 f \cos^6(fx + e) - 5c^6 f \cos^5(fx + e) - 18c^6 f \cos^4(fx + e) + 20c^6 f \cos^3(fx + e) + 48c^6 f \cos^2(fx + e) - 16c^6 f \cos(fx + e) - 32c^6 f + (c^6 f \cos^5(fx + e) + 6c^6 f \cos^4(fx + e) - 12c^6 f \cos^3(fx + e) - 32c^6 f \cos^2(fx + e) + 16c^6 f \cos(fx + e) + 32c^6 f) \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="fricas")

[Out] 1/693*((2*A - 9*B)*a^3*cos(f*x + e)^6 + 6*(2*A - 9*B)*a^3*cos(f*x + e)^5 - (25*A + 234*B)*a^3*cos(f*x + e)^4 + 7*(23*A + 45*B)*a^3*cos(f*x + e)^3 + 28*(16*A + 27*B)*a^3*cos(f*x + e)^2 - 252*(A + B)*a^3*cos(f*x + e) - 504*(A + B)*a^3 - ((2*A - 9*B)*a^3*cos(f*x + e)^5 - 5*(2*A - 9*B)*a^3*cos(f*x + e)^4 - 7*(5*A + 27*B)*a^3*cos(f*x + e)^3 - 28*(7*A + 18*B)*a^3*cos(f*x + e)^2 + 252*(A + B)*a^3*cos(f*x + e) + 504*(A + B)*a^3)*sin(f*x + e)/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))

giac [B] time = 0.31, size = 373, normalized size = 3.16

$$2 \left(693 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{10} - 1386 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 + 693 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^9 + 8085 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="giac")

[Out]
$$\frac{-2/693*(693*A*a^3*\tan(1/2*f*x + 1/2*e)^{10} - 1386*A*a^3*\tan(1/2*f*x + 1/2*e)^9 + 693*B*a^3*\tan(1/2*f*x + 1/2*e)^9 + 8085*A*a^3*\tan(1/2*f*x + 1/2*e)^8 + 693*B*a^3*\tan(1/2*f*x + 1/2*e)^8 - 10626*A*a^3*\tan(1/2*f*x + 1/2*e)^7 + 4158*B*a^3*\tan(1/2*f*x + 1/2*e)^7 + 21252*A*a^3*\tan(1/2*f*x + 1/2*e)^6 + 1386*B*a^3*\tan(1/2*f*x + 1/2*e)^6 - 15246*A*a^3*\tan(1/2*f*x + 1/2*e)^5 + 5544*B*a^3*\tan(1/2*f*x + 1/2*e)^5 + 15444*A*a^3*\tan(1/2*f*x + 1/2*e)^4 + 1188*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 4950*A*a^3*\tan(1/2*f*x + 1/2*e)^3 + 2178*B*a^3*\tan(1/2*f*x + 1/2*e)^3 + 2959*A*a^3*\tan(1/2*f*x + 1/2*e)^2 + 198*B*a^3*\tan(1/2*f*x + 1/2*e)^2 - 176*A*a^3*\tan(1/2*f*x + 1/2*e) + 99*B*a^3*\tan(1/2*f*x + 1/2*e) + 79*A*a^3 - 9*B*a^3)/(c^6*f*(\tan(1/2*f*x + 1/2*e) - 1)^{11})$$

maple [B] time = 0.52, size = 249, normalized size = 2.11

$$2a^3 \left(\frac{A}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{504A+200B}{4\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^4} - \frac{4352A+3840B}{8\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^8} - \frac{4272A+3344B}{7\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^7} - \frac{116A+30B}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3} - \frac{1280A+1280B}{10\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^{10}} - \frac{2960A+1968B}{6\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^6} \right) / f c^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)

[Out]
$$\frac{2/f*a^3/c^6*(-A/(\tan(1/2*f*x+1/2*e)-1)-1/4*(504*A+200*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/8*(4352*A+3840*B)/(\tan(1/2*f*x+1/2*e)-1)^8-1/7*(4272*A+3344*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/3*(116*A+30*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/10*(1280*A+1280*B)/(\tan(1/2*f*x+1/2*e)-1)^{10}-1/6*(2960*A+1968*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/5*(1460*A+780*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/2*(16*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/11*(256*A+256*B)/(\tan(1/2*f*x+1/2*e)-1)^{11}-1/9*(3008*A+2880*B)/(\tan(1/2*f*x+1/2*e)-1)^9)}$$

maxima [B] time = 0.53, size = 3390, normalized size = 28.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x, algorithm="maxima")

[Out]
$$\frac{-2/3465*(5*A*a^3*(913*\sin(f*x + e)/(\cos(f*x + e) + 1) - 4565*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 12540*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 25080*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 33726*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 33726*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 23100*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 11550*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 3465*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 693*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 146)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 11550*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 3465*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 693*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 146)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 11550*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 3465*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 693*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 146)}$$

$$\begin{aligned}
& e^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5 \\
& 5*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/(\cos(f*x \\
& + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 9*A*a^3*(671*s \\
& \sin(f*x + e)/(\cos(f*x + e) + 1) - 2200*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + \\
& 6600*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 10890*\sin(f*x + e)^4/(\cos(f*x + \\
& e) + 1)^4 + 15246*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 12936*\sin(f*x + e) \\
& ^6/(\cos(f*x + e) + 1)^6 + 9240*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3465*s \\
& \sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 1155*\sin(f*x + e)^9/(\cos(f*x + e) + 1) \\
& ^9 - 61)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e) \\
&)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 33 \\
& 0*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x \\
& + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f* \\
& x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 \\
& - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/(\cos \\
& (f*x + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 3*B*a^3*(6 \\
& 71*\sin(f*x + e)/(\cos(f*x + e) + 1) - 2200*\sin(f*x + e)^2/(\cos(f*x + e) + 1) \\
& ^2 + 6600*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 10890*\sin(f*x + e)^4/(\cos(f \\
& *x + e) + 1)^4 + 15246*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 12936*\sin(f*x \\
& + e)^6/(\cos(f*x + e) + 1)^6 + 9240*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 34 \\
& 65*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 1155*\sin(f*x + e)^9/(\cos(f*x + e) \\
& + 1)^9 - 61)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x \\
& + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\
& + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos \\
& (f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*si \\
& n(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + \\
& 1)^8 - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/ \\
& (\cos(f*x + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 2*A*a^ \\
& 3*(341*\sin(f*x + e)/(\cos(f*x + e) + 1) - 1705*\sin(f*x + e)^2/(\cos(f*x + e) \\
& + 1)^2 + 5115*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 6765*\sin(f*x + e)^4/(co \\
& s(f*x + e) + 1)^4 + 9471*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 4851*\sin(f*x \\
& + e)^6/(\cos(f*x + e) + 1)^6 + 3465*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3 \\
& 1)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(c \\
& os(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 330*c^6* \\
& \sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) \\
& + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e) \\
& ^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 55* \\
& c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/(\cos(f*x + \\
& e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 6*B*a^3*(341*\sin \\
& (f*x + e)/(\cos(f*x + e) + 1) - 1705*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5 \\
& 115*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 6765*\sin(f*x + e)^4/(\cos(f*x + e) \\
& + 1)^4 + 9471*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 4851*\sin(f*x + e)^6/(c \\
& os(f*x + e) + 1)^6 + 3465*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 31)/(c^6 - \\
& 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) \\
&) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 330*c^6*\sin(f*x + \\
& e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 4 \\
& 62*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(\cos(f* \\
& x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 55*c^6*\sin(f* \\
& x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{1 \\
& 0} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 12*A*a^3*(253*\sin(f*x + e) \\
& /(\cos(f*x + e) + 1) - 1265*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2640*\sin(f \\
& *x + e)^3/(\cos(f*x + e) + 1)^3 - 5280*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + \\
& 5313*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 5313*\sin(f*x + e)^6/(\cos(f*x + \\
& e) + 1)^6 + 2310*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 1155*\sin(f*x + e)^8/ \\
& (\cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + \\
& 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f* \\
& x + e) + 1)^3 + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f \\
& *x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^ \\
& 6 - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(c \\
& os(f*x + e) + 1)^8 - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*si
\end{aligned}$$

$$\begin{aligned} & n(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} \\ & + 12*B*a^3*(253*\sin(f*x + e)/(\cos(f*x + e) + 1) - 1265*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 \\ & + 2640*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 5280*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 \\ & + 5313*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 5313*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 \\ & + 2310*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 1155*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 23)/(c^6 - 11*c^6* \\ & \sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\ & + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6* \\ & \sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 \\ & - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 11*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} \\ & + 48*B*a^3*(11*\sin(f*x + e)/(\cos(f*x + e) + 1) - 55*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 165*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\ & - 330*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 231*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 231*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1) \\ & / (c^6 - 11*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 55*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 165*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\ & + 330*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 462*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 462*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 \\ & - 330*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 165*c^6*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 55*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 \\ & + 11*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11})/f \end{aligned}$$

mupad [B] time = 13.54, size = 408, normalized size = 3.46

$$2 \cos\left(\frac{e}{2} + \frac{f x}{2}\right) \left(565 A a^3 \cos(2e + 2fx) - \frac{837 B a^3}{16} - 922 A a^3 - \frac{3527 A a^3 \cos(3e + 3fx)}{32} - 29 A a^3 \cos(4e + 4fx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^6,x)
[Out] -(2*cos(e/2 + (f*x)/2)*(565*A*a^3*cos(2*e + 2*f*x) - (837*B*a^3)/16 - 922*A*a^3 - (3527*A*a^3*cos(3*e + 3*f*x))/32 - 29*A*a^3*cos(4*e + 4*f*x) + (81*A*a^3*cos(5*e + 5*f*x))/32 + (225*B*a^3*cos(2*e + 2*f*x))/4 - (207*B*a^3*cos(3*e + 3*f*x))/16 + (9*B*a^3*cos(4*e + 4*f*x))/16 - (9*B*a^3*cos(5*e + 5*f*x))/16 - (1617*A*a^3*sin(2*e + 2*f*x))/8 - (5049*A*a^3*sin(3*e + 3*f*x))/32 + (407*A*a^3*sin(4*e + 4*f*x))/16 + (77*A*a^3*sin(5*e + 5*f*x))/32 + (693*B*a^3*sin(2*e + 2*f*x))/8 + (99*B*a^3*sin(3*e + 3*f*x))/2 - (99*B*a^3*sin(4*e + 4*f*x))/16 + (6635*A*a^3*cos(e + f*x))/16 + 18*B*a^3*cos(e + f*x) + (13629*A*a^3*sin(e + f*x))/16 - (693*B*a^3*sin(e + f*x))/2))/(693*c^6*f*((231*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 - (165*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/16 - (165*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/32 + (55*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 + (11*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 - (2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/32))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^6,x)
[Out] Timed out
```

$$3.50 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^7} dx$$

Optimal. Leaf size=156

$$\frac{a^3 c^3 (A+B) \cos^7(e+fx)}{13 f (c-c \sin(e+fx))^{10}} + \frac{a^3 c^2 (3A-10B) \cos^7(e+fx)}{143 f (c-c \sin(e+fx))^9} + \frac{2 a^3 (3A-10B) \cos^7(e+fx)}{9009 f (c-c \sin(e+fx))^7} + \frac{2 a^3 c (3A-10B) \cos^7(e+fx)}{1287 f (c-c \sin(e+fx))^7}$$

[Out] 1/13*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/143*a^3*(3*A-10*B)*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/1287*a^3*(3*A-10*B)*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+2/9009*a^3*(3*A-10*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^7

Rubi [A] time = 0.38, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3 c^2 (3A-10B) \cos^7(e+fx)}{143 f (c-c \sin(e+fx))^9} + \frac{a^3 c^3 (A+B) \cos^7(e+fx)}{13 f (c-c \sin(e+fx))^{10}} + \frac{2 a^3 (3A-10B) \cos^7(e+fx)}{9009 f (c-c \sin(e+fx))^7} + \frac{2 a^3 c (3A-10B) \cos^7(e+fx)}{1287 f (c-c \sin(e+fx))^7}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7,x]

[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(13*f*(c - c*Sin[e + f*x])^10) + (a^3*(3*A - 10*B)*c^2*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(3*A - 10*B)*c*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(3*A - 10*B)*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^7)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^7} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{10}} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{13 f (c - c \sin(e + fx))^{10}} + \frac{1}{13} (a^3 (3A - 10B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{10}} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{13 f (c - c \sin(e + fx))^{10}} + \frac{a^3 (3A - 10B) c^2 \cos^7(e + fx)}{143 f (c - c \sin(e + fx))^9} + \frac{1}{143} \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{10}} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{13 f (c - c \sin(e + fx))^{10}} + \frac{a^3 (3A - 10B) c^2 \cos^7(e + fx)}{143 f (c - c \sin(e + fx))^9} + \frac{2a^3}{12} \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{10}} dx \\ &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{13 f (c - c \sin(e + fx))^{10}} + \frac{a^3 (3A - 10B) c^2 \cos^7(e + fx)}{143 f (c - c \sin(e + fx))^9} + \frac{2a^3}{12} \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{10}} dx \end{aligned}$$

Mathematica [B] time = 5.29, size = 339, normalized size = 2.17

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(6006(9A + 5B) \cos\left(\frac{1}{2}(e + fx)\right) - 7722(4A + 3B) \cos\left(\frac{1}{2}(e + fx)\right) \right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^7, x]

[Out] -1/144144*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(6006*(9*A + 5*B)*Cos[(e + f*x)/2] - 7722*(4*A + 3*B)*Cos[(3*(e + f*x))/2] - 9009*A*Cos[(5*(e + f*x))/2] - 12012*B*Cos[(5*(e + f*x))/2] + 858*A*Cos[(7*(e + f*x))/2] + 3146*B*Cos[(7*(e + f*x))/2] - 39*A*Cos[(11*(e + f*x))/2] + 130*B*Cos[(11*(e + f*x))/2] + 48906*A*Sin[(e + f*x)/2] + 47190*B*Sin[(e + f*x)/2] + 27027*A*Sin[(3*(e + f*x))/2] + 36036*B*Sin[(3*(e + f*x))/2] - 6864*A*Sin[(5*(e + f*x))/2] - 19162*B*Sin[(5*(e + f*x))/2] - 6006*B*Sin[(7*(e + f*x))/2] - 234*A*Sin[(9*(e + f*x))/2] + 780*B*Sin[(9*(e + f*x))/2] + 3*A*Sin[(13*(e + f*x))/2] - 10*B*Sin[(13*(e + f*x))/2]))/(c^7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(-1 + Sin[e + f*x])^7)

fricas [B] time = 0.45, size = 475, normalized size = 3.04

$$\frac{2(3A - 10B)a^3 \cos^7(fx + e) - 12(3A - 10B)a^3 \cos^6(fx + e) - 49(3A - 10B)a^3 \cos^5(fx + e) + 7(30A + 329B)a^3 \cos^4(fx + e) - 63(27A + 53B)a^3 \cos^3(fx + e) - 252(19A + 32B)a^3 \cos^2(fx + e) + 2772(A + B)a^3 \cos(fx + e) + 5544(A + B)a^3 + (2(3A - 10B)a^3 \cos^7(fx + e) + 14(3A - 10B)a^3 \cos^6(fx + e) - 35(3A - 10B)a^3 \cos^5(fx + e) - 49(3A - 10B)a^3 \cos^4(fx + e) + 7(30A + 329B)a^3 \cos^3(fx + e) - 63(27A + 53B)a^3 \cos^2(fx + e) + 2772(A + B)a^3 \cos(fx + e) + 5544(A + B)a^3)}{9009(c^7 f \cos(fx + e)^7 + 7c^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="fricas")

[Out] -1/9009*(2*(3*A - 10*B)*a^3*cos(f*x + e)^7 - 12*(3*A - 10*B)*a^3*cos(f*x + e)^6 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^5 + 7*(30*A + 329*B)*a^3*cos(f*x + e)^4 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^3 - 252*(19*A + 32*B)*a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3 + (2*(3*A - 10*B)*a^3*cos(f*x + e)^7 + 14*(3*A - 10*B)*a^3*cos(f*x + e)^6 - 35*(3*A - 10*B)*a^3*cos(f*x + e)^5 - 49*(3*A - 10*B)*a^3*cos(f*x + e)^4 + 7*(30*A + 329*B)*a^3*cos(f*x + e)^3 - 63*(27*A + 53*B)*a^3*cos(f*x + e)^2 + 2772*(A + B)*a^3*cos(f*x + e) + 5544*(A + B)*a^3)

$$a^3 \cos(fx + e)^4 - 63(5A + 31B)a^3 \cos(fx + e)^3 - 252(8A + 21B)a^3 \cos(fx + e)^2 + 2772(A + B)a^3 \cos(fx + e) + 5544(A + B)a^3 \sin(fx + e) / (c^7 f \cos(fx + e)^7 + 7c^7 f \cos(fx + e)^6 - 18c^7 f \cos(fx + e)^5 - 56c^7 f \cos(fx + e)^4 + 48c^7 f \cos(fx + e)^3 + 112c^7 f \cos(fx + e)^2 - 32c^7 f \cos(fx + e) - 64c^7 f - (c^7 f \cos(fx + e)^6 - 6c^7 f \cos(fx + e)^5 - 24c^7 f \cos(fx + e)^4 + 32c^7 f \cos(fx + e)^3 + 80c^7 f \cos(fx + e)^2 - 32c^7 f \cos(fx + e) - 64c^7 f) \sin(fx + e))$$

giac [B] time = 0.36, size = 445, normalized size = 2.85

$$2 \left(9009 A a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{12} - 27027 A a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{11} + 9009 B a^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^{11} + 153153 A a^3 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="giac")

[Out]
$$-2/9009*(9009*A*a^3*\tan(1/2*f*x + 1/2*e)^{12} - 27027*A*a^3*\tan(1/2*f*x + 1/2*e)^{11} + 9009*B*a^3*\tan(1/2*f*x + 1/2*e)^{11} + 153153*A*a^3*\tan(1/2*f*x + 1/2*e)^{10} + 3003*B*a^3*\tan(1/2*f*x + 1/2*e)^{10} - 297297*A*a^3*\tan(1/2*f*x + 1/2*e)^9 + 69069*B*a^3*\tan(1/2*f*x + 1/2*e)^9 + 648648*A*a^3*\tan(1/2*f*x + 1/2*e)^8 - 9009*B*a^3*\tan(1/2*f*x + 1/2*e)^8 - 738738*A*a^3*\tan(1/2*f*x + 1/2*e)^7 + 150150*B*a^3*\tan(1/2*f*x + 1/2*e)^7 + 857142*A*a^3*\tan(1/2*f*x + 1/2*e)^6 - 16302*B*a^3*\tan(1/2*f*x + 1/2*e)^6 - 548262*A*a^3*\tan(1/2*f*x + 1/2*e)^5 + 115830*B*a^3*\tan(1/2*f*x + 1/2*e)^5 + 367653*A*a^3*\tan(1/2*f*x + 1/2*e)^4 - 286*B*a^3*\tan(1/2*f*x + 1/2*e)^4 - 112827*A*a^3*\tan(1/2*f*x + 1/2*e)^3 + 30745*B*a^3*\tan(1/2*f*x + 1/2*e)^3 + 45513*A*a^3*\tan(1/2*f*x + 1/2*e)^2 + 1443*B*a^3*\tan(1/2*f*x + 1/2*e)^2 - 3081*A*a^3*\tan(1/2*f*x + 1/2*e) + 1261*B*a^3*\tan(1/2*f*x + 1/2*e) + 930*A*a^3 - 97*B*a^3)/(c^7*f*(\tan(1/2*f*x + 1/2*e) - 1)^{13})$$

maple [A] time = 0.57, size = 293, normalized size = 1.88

$$2a^3 \left(-\frac{16000A+14720B}{10\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}} - \frac{512A+512B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}} - \frac{6888A+3928B}{6\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{150A+34B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{13112A+8840B}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)

[Out]
$$2/f*a^3/c^7*(-1/10*(16000*A+14720*B)/(\tan(1/2*f*x+1/2*e)-1)^{10}-1/13*(512*A+512*B)/(\tan(1/2*f*x+1/2*e)-1)^{13}-1/6*(6888*A+3928*B)/(\tan(1/2*f*x+1/2*e)-1)^6-A/(\tan(1/2*f*x+1/2*e)-1)-1/3*(150*A+34*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/7*(13112*A+8840*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/12*(3072*A+3072*B)/(\tan(1/2*f*x+1/2*e)-1)^{12}-1/5*(2700*A+1240*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/8*(18816*A+14464*B)/(\tan(1/2*f*x+1/2*e)-1)^8-1/4*(768*A+264*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/11*(8832*A+8576*B)/(\tan(1/2*f*x+1/2*e)-1)^{11}-1/2*(18*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/9*(20256*A+17248*B)/(\tan(1/2*f*x+1/2*e)-1)^9)$$

maxima [B] time = 0.57, size = 4078, normalized size = 26.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x, algorithm="maxima")

$$\begin{aligned}
& e) + 1) - 2379*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 8723*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 18590*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 33462*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 40326*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 40326*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 27027*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 15015*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 4719*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 1287*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 - 47)/(c^7 - 13*c^7*\sin(f*x + e)/(\cos(f*x + e) + 1) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13) + 8*B*a^3*(559*\sin(f*x + e)/(\cos(f*x + e) + 1) - 3354*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 12298*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 30745*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 37323*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 49764*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24024*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 18018*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 43)/(c^7 - 13*c^7*\sin(f*x + e)/(\cos(f*x + e) + 1) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13) - 462*A*a^3*(13*\sin(f*x + e)/(\cos(f*x + e) + 1) - 78*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 286*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 520*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 936*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 858*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 858*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 351*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 195*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*\sin(f*x + e)/(\cos(f*x + e) + 1) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13) - 1386*B*a^3*(13*\sin(f*x + e)/(\cos(f*x + e) + 1) - 78*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 286*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 520*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 936*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 858*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 858*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 351*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 195*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 1)/(c^7 - 13*c^7*\sin(f*x + e)/(\cos(f*x + e) + 1) + 78*c^7*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 286*c^7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 715*c^7*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1287*c^7*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 1716*c^7*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 1716*c^7*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1287*c^7*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 715*c^7*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 286*c^7*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 78*c^7*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 13*c^7*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - c^7*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13))/f
\end{aligned}$$

mupad [B] time = 13.79, size = 500, normalized size = 3.21

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{2363 B a^3}{32} - \frac{279183 A a^3}{16} + \frac{220269 A a^3 \cos(2e+2fx)}{16} - \frac{46095 A a^3 \cos(3e+3fx)}{16} - \frac{20829 A a^3 \cos(4e+4fx)}{16} + \frac{2811}{16} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^7,x)
[Out] (2*cos(e/2 + (f*x)/2)*((2363*B*a^3)/32 - (279183*A*a^3)/16 + (220269*A*a^3*cos(2*e + 2*f*x))/16 - (46095*A*a^3*cos(3*e + 3*f*x))/16 - (20829*A*a^3*cos(4*e + 4*f*x))/16 + (2811*A*a^3*cos(5*e + 5*f*x))/16 + (231*A*a^3*cos(6*e + 6*f*x))/16 - (8995*B*a^3*cos(2*e + 2*f*x))/64 + (497*B*a^3*cos(3*e + 3*f*x))/16 + (3725*B*a^3*cos(4*e + 4*f*x))/32 - (361*B*a^3*cos(5*e + 5*f*x))/16 - (77*B*a^3*cos(6*e + 6*f*x))/64 - (19305*A*a^3*sin(2*e + 2*f*x))/4 - (81081*A*a^3*sin(3*e + 3*f*x))/16 + (15015*A*a^3*sin(4*e + 4*f*x))/16 + (3237*A*a^3*sin(5*e + 5*f*x))/16 - (117*A*a^3*sin(6*e + 6*f*x))/8 + (77649*B*a^3*sin(2*e + 2*f*x))/64 + (27027*B*a^3*sin(3*e + 3*f*x))/32 - (1001*B*a^3*sin(4*e + 4*f*x))/8 - (559*B*a^3*sin(5*e + 5*f*x))/32 + (117*B*a^3*sin(6*e + 6*f*x))/64 + (26979*A*a^3*cos(e + f*x))/4 + 40*B*a^3*cos(e + f*x) + (173745*A*a^3*sin(e + f*x))/8 - (80223*B*a^3*sin(e + f*x))/16)/(9009*c^7*f*((1287*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/64 - (429*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/16 + (715*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/64 - (143*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/32 - (39*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/32 + (13*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/64 + (2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/64))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^7,x)
[Out] Timed out
```

$$3.51 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^8} dx$$

Optimal. Leaf size=197

$$\frac{a^3 c^3 (A+B) \cos^7(e+fx)}{15f(c-c \sin(e+fx))^{11}} + \frac{a^3 c^2 (4A-11B) \cos^7(e+fx)}{195f(c-c \sin(e+fx))^{10}} + \frac{2a^3 (4A-11B) \cos^7(e+fx)}{45045cf(c-c \sin(e+fx))^7} + \frac{2a^3 (4A-11B) \cos^7(e+fx)}{6435f(c-c \sin(e+fx))^{11}}$$

[Out] 1/15*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^11+1/195*a^3*(4*A-11*B)*c^2*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^10+1/715*a^3*(4*A-11*B)*c*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^9+2/6435*a^3*(4*A-11*B)*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^8+2/45045*a^3*(4*A-11*B)*cos(f*x+e)^7/c/f/(c-c*sin(f*x+e))^7

Rubi [A] time = 0.44, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 2671}

$$\frac{a^3 c^2 (4A-11B) \cos^7(e+fx)}{195f(c-c \sin(e+fx))^{10}} + \frac{a^3 c^3 (A+B) \cos^7(e+fx)}{15f(c-c \sin(e+fx))^{11}} + \frac{2a^3 (4A-11B) \cos^7(e+fx)}{45045cf(c-c \sin(e+fx))^7} + \frac{2a^3 (4A-11B) \cos^7(e+fx)}{6435f(c-c \sin(e+fx))^{11}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^8,x]

[Out] (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(15*f*(c - c*Sin[e + f*x])^11) + (a^3*(4*A - 11*B)*c^2*Cos[e + f*x]^7)/(195*f*(c - c*Sin[e + f*x])^10) + (a^3*(4*A - 11*B)*c*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^9) + (2*a^3*(4*A - 11*B)*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^8) + (2*a^3*(4*A - 11*B)*Cos[e + f*x]^7)/(45045*c*f*(c - c*Sin[e + f*x])^7)

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1], x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]


```
[Out] 1/45045*(2*(4*A - 11*B)*a^3*cos(f*x + e)^8 + 16*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 49*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 168*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(7*A + 88*B)*a^3*cos(f*x + e)^4 - 231*(31*A + 61*B)*a^3*cos(f*x + e)^3 - 924*(22*A + 37*B)*a^3*cos(f*x + e)^2 + 12012*(A + B)*a^3*cos(f*x + e) + 24024*(A + B)*a^3 - (2*(4*A - 11*B)*a^3*cos(f*x + e)^7 - 14*(4*A - 11*B)*a^3*cos(f*x + e)^6 - 63*(4*A - 11*B)*a^3*cos(f*x + e)^5 + 105*(4*A - 11*B)*a^3*cos(f*x + e)^4 + 1155*(A + 7*B)*a^3*cos(f*x + e)^3 + 2772*(3*A + 8*B)*a^3*cos(f*x + e)^2 - 12012*(A + B)*a^3*cos(f*x + e) - 24024*(A + B)*a^3)*sin(f*x + e))/(c^8*f*cos(f*x + e)^8 - 7*c^8*f*cos(f*x + e)^7 - 32*c^8*f*cos(f*x + e)^6 + 56*c^8*f*cos(f*x + e)^5 + 160*c^8*f*cos(f*x + e)^4 - 112*c^8*f*cos(f*x + e)^3 - 256*c^8*f*cos(f*x + e)^2 + 64*c^8*f*cos(f*x + e) + 128*c^8*f + (c^8*f*cos(f*x + e)^7 + 8*c^8*f*cos(f*x + e)^6 - 24*c^8*f*cos(f*x + e)^5 - 80*c^8*f*cos(f*x + e)^4 + 80*c^8*f*cos(f*x + e)^3 + 192*c^8*f*cos(f*x + e)^2 - 64*c^8*f*cos(f*x + e) - 128*c^8*f)*sin(f*x + e))
```

giac [B] time = 0.39, size = 517, normalized size = 2.62

$$2 \left(45045 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{14} - 180180 A a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{13} + 45045 B a^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^{13} + 1066065 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorithm="giac")
```

```
[Out] -2/45045*(45045*A*a^3*tan(1/2*f*x + 1/2*e)^14 - 180180*A*a^3*tan(1/2*f*x + 1/2*e)^13 + 45045*B*a^3*tan(1/2*f*x + 1/2*e)^13 + 1066065*A*a^3*tan(1/2*f*x + 1/2*e)^12 - 15015*B*a^3*tan(1/2*f*x + 1/2*e)^12 - 2702700*A*a^3*tan(1/2*f*x + 1/2*e)^11 + 450450*B*a^3*tan(1/2*f*x + 1/2*e)^11 + 6675669*A*a^3*tan(1/2*f*x + 1/2*e)^10 - 306306*B*a^3*tan(1/2*f*x + 1/2*e)^10 - 10210200*A*a^3*tan(1/2*f*x + 1/2*e)^9 + 1456455*B*a^3*tan(1/2*f*x + 1/2*e)^9 + 14124825*A*a^3*tan(1/2*f*x + 1/2*e)^8 - 791505*B*a^3*tan(1/2*f*x + 1/2*e)^8 - 13178880*A*a^3*tan(1/2*f*x + 1/2*e)^7 + 1827540*B*a^3*tan(1/2*f*x + 1/2*e)^7 + 11026015*A*a^3*tan(1/2*f*x + 1/2*e)^6 - 580580*B*a^3*tan(1/2*f*x + 1/2*e)^6 - 6066060*A*a^3*tan(1/2*f*x + 1/2*e)^5 + 915915*B*a^3*tan(1/2*f*x + 1/2*e)^5 + 3088995*A*a^3*tan(1/2*f*x + 1/2*e)^4 - 105105*B*a^3*tan(1/2*f*x + 1/2*e)^4 - 864500*A*a^3*tan(1/2*f*x + 1/2*e)^3 + 170170*B*a^3*tan(1/2*f*x + 1/2*e)^3 + 265335*A*a^3*tan(1/2*f*x + 1/2*e)^2 + 2310*B*a^3*tan(1/2*f*x + 1/2*e)^2 - 18600*A*a^3*tan(1/2*f*x + 1/2*e) + 6105*B*a^3*tan(1/2*f*x + 1/2*e) + 4243*A*a^3 - 407*B*a^3)/(c^8*f*(tan(1/2*f*x + 1/2*e) - 1)^15)
```

maple [A] time = 0.58, size = 337, normalized size = 1.71

$$2a^3 \left(-\frac{4536A+1836B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{20A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{81344A+72512B}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}} - \frac{24320A+23808B}{13\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{13}} - \frac{1104A+336B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x)
```

```
[Out] 2/f*a^3/c^8*(-1/5*(4536*A+1836*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(20*A+2*B)/(tan(1/2*f*x+1/2*e)-1)^2-1/11*(81344*A+72512*B)/(tan(1/2*f*x+1/2*e)-1)^11-1/13*(24320*A+23808*B)/(tan(1/2*f*x+1/2*e)-1)^13-1/4*(1104*A+336*B)/(tan(1/2*f*x+1/2*e)-1)^4-A/(tan(1/2*f*x+1/2*e)-1)-1/7*(32288*A+19176*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/15*(1024*A+1024*B)/(tan(1/2*f*x+1/2*e)-1)^15-1/6*(13824*A+6936*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/14*(7168*A+7168*B)/(tan(1/2*f*x+1/2*e)-1)^14-1/8*(58816*A+40000*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/12*(52736*A+49664*B)/(t
```

$$\frac{\tan(1/2*f*x+1/2*e)-1)^{12}-1/3*(188*A+38*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/9*(8411*2*A+63856*B)/(\tan(1/2*f*x+1/2*e)-1)^9-1/10*(94144*A+78144*B)/(\tan(1/2*f*x+1/2*e)-1)^{10}}$$

maxima [B] time = 0.68, size = 4765, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^8,x, algorithm="maxima")

[Out]
$$\frac{2/45045*(3*A*a^3*(17715*\sin(f*x + e)/(\cos(f*x + e) + 1) - 78960*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 342160*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 891345*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 1960959*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 3043040*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 3912480*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3687255*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 2867865*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 1585584*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 720720*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 195195*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} + 45045*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 1181)/(c^8 - 15*c^8*\sin(f*x + e)/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 1365*c^8*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 455*c^8*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 105*c^8*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} + 15*c^8*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - c^8*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15}) + B*a^3*(17715*\sin(f*x + e)/(\cos(f*x + e) + 1) - 78960*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 342160*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 891345*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 1960959*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 3043040*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 3912480*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3687255*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 2867865*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 1585584*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 720720*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 195195*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} + 45045*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 1181)/(c^8 - 15*c^8*\sin(f*x + e)/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 1365*c^8*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 455*c^8*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 105*c^8*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} + 15*c^8*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - c^8*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15}) - 7*A*a^3*(7845*\sin(f*x + e)/(\cos(f*x + e) + 1) - 54915*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 222950*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 668850*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 1444443*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 2407405*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 3063060*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3063060*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 2357355*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 1414413*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 630630*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 210210*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} + 45045*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 6435*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - 952)/(c^8 - 15*c^8*\sin(f*x + e)/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 1365*c^8*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 455*c^8*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 105*c^8*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} + 15*c^8*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - c^8*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15})$$

$$\begin{aligned}
& + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 1365*c^8*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 455*c^8*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 105*c^8*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 15*c^8*\sin(f*x + e)^14/(\cos(f*x + e) + 1)^14 - c^8*\sin(f*x + e)^15/(\cos(f*x + e) + 1)^15 - 12*A*a^3*(1740*\sin(f*x + e))/(\cos(f*x + e) + 1) - 12180*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 37765*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 113295*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 204204*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 340340*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 373230*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 373230*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 240240*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 144144*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 45045*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 - 15015*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*\sin(f*x + e))/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 1365*c^8*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 455*c^8*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 105*c^8*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 15*c^8*\sin(f*x + e)^14/(\cos(f*x + e) + 1)^14 - c^8*\sin(f*x + e)^15/(\cos(f*x + e) + 1)^15) - 12*B*a^3*(1740*\sin(f*x + e))/(\cos(f*x + e) + 1) - 12180*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 37765*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 113295*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 204204*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 340340*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 373230*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 373230*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 240240*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 144144*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 45045*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 - 15015*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 116)/(c^8 - 15*c^8*\sin(f*x + e))/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 1365*c^8*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 455*c^8*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 105*c^8*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 15*c^8*\sin(f*x + e)^14/(\cos(f*x + e) + 1)^14 - c^8*\sin(f*x + e)^15/(\cos(f*x + e) + 1)^15) + 6*A*a^3*(675*\sin(f*x + e))/(\cos(f*x + e) + 1) - 4725*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20475*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 46410*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 102102*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 130130*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 167310*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 122265*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 95095*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 33033*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 15015*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 - 45)/(c^8 - 15*c^8*\sin(f*x + e))/(\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 1365*c^8*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 455*c^8*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 - 105*c^8*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 15*c^8*\sin(f*x + e)^14/(\cos(f*x + e) + 1)^14 - c^8*\sin(f*x + e)^15/(\cos(f*x + e) + 1)^15) + 18*B*a^3*(675*\sin(f*x + e))/(\cos(f*x + e) + 1) - 4725*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20475*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 46410*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 102102*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 130130*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 167310*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 +
\end{aligned}$$

$$1)^7 - 122265 \sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + 95095 \sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9 - 33033 \sin(f*x + e)^{10} / (\cos(f*x + e) + 1)^{10} + 15015 \sin(f*x + e)^{11} / (\cos(f*x + e) + 1)^{11} - 45 / (c^8 - 15*c^8*\sin(f*x + e) / (\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^{10} / (\cos(f*x + e) + 1)^{10} - 1365*c^8*\sin(f*x + e)^{11} / (\cos(f*x + e) + 1)^{11} + 455*c^8*\sin(f*x + e)^{12} / (\cos(f*x + e) + 1)^{12} - 105*c^8*\sin(f*x + e)^{13} / (\cos(f*x + e) + 1)^{13} + 15*c^8*\sin(f*x + e)^{14} / (\cos(f*x + e) + 1)^{14} - c^8*\sin(f*x + e)^{15} / (\cos(f*x + e) + 1)^{15}) - 48*B*a^3*(60*\sin(f*x + e) / (\cos(f*x + e) + 1) - 420*\sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 1820*\sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 - 5460*\sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 9009*\sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 - 15015*\sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 12870*\sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 - 12870*\sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + 5005*\sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9 - 3003*\sin(f*x + e)^{10} / (\cos(f*x + e) + 1)^{10} - 4) / (c^8 - 15*c^8*\sin(f*x + e) / (\cos(f*x + e) + 1) + 105*c^8*\sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 - 455*c^8*\sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 1365*c^8*\sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 - 3003*c^8*\sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 5005*c^8*\sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 - 6435*c^8*\sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + 6435*c^8*\sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 - 5005*c^8*\sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9 + 3003*c^8*\sin(f*x + e)^{10} / (\cos(f*x + e) + 1)^{10} - 1365*c^8*\sin(f*x + e)^{11} / (\cos(f*x + e) + 1)^{11} + 455*c^8*\sin(f*x + e)^{12} / (\cos(f*x + e) + 1)^{12} - 105*c^8*\sin(f*x + e)^{13} / (\cos(f*x + e) + 1)^{13} + 15*c^8*\sin(f*x + e)^{14} / (\cos(f*x + e) + 1)^{14} - c^8*\sin(f*x + e)^{15} / (\cos(f*x + e) + 1)^{15}) / f$$

mupad [B] time = 13.96, size = 577, normalized size = 2.93

$$2 \cos\left(\frac{e}{2} + \frac{f*x}{2}\right) \left(\frac{544369 A a^3}{4} - \frac{21791 B a^3}{4} - \frac{257861 A a^3 \cos(2e+2fx)}{2} + \frac{3497111 A a^3 \cos(3e+3fx)}{128} + \frac{72047 A a^3 \cos(4e+4fx)}{4} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^8,x)

[Out] (2*cos(e/2 + (f*x)/2)*((544369*A*a^3)/4 - (21791*B*a^3)/4 - (257861*A*a^3*cos(2*e + 2*f*x))/2 + (3497111*A*a^3*cos(3*e + 3*f*x))/128 + (72047*A*a^3*cos(4*e + 4*f*x))/4 - (378579*A*a^3*cos(5*e + 5*f*x))/128 - (1059*A*a^3*cos(6*e + 6*f*x))/2 + (4251*A*a^3*cos(7*e + 7*f*x))/128 + (219769*B*a^3*cos(2*e + 2*f*x))/32 - (191389*B*a^3*cos(3*e + 3*f*x))/128 - 1672*B*a^3*cos(4*e + 4*f*x) + (38841*B*a^3*cos(5*e + 5*f*x))/128 + (1551*B*a^3*cos(6*e + 6*f*x))/32 - (429*B*a^3*cos(7*e + 7*f*x))/128 + (2633345*A*a^3*sin(2*e + 2*f*x))/64 + (7210775*A*a^3*sin(3*e + 3*f*x))/128 - (89375*A*a^3*sin(4*e + 4*f*x))/8 - (504205*A*a^3*sin(5*e + 5*f*x))/128 + (29765*A*a^3*sin(6*e + 6*f*x))/64 + (4235*A*a^3*sin(7*e + 7*f*x))/128 - (451165*B*a^3*sin(2*e + 2*f*x))/64 - (854425*B*a^3*sin(3*e + 3*f*x))/128 + (9295*B*a^3*sin(4*e + 4*f*x))/8 + (46475*B*a^3*sin(5*e + 5*f*x))/128 - (3025*B*a^3*sin(6*e + 6*f*x))/64 - (385*B*a^3*sin(7*e + 7*f*x))/128 - (5734111*A*a^3*cos(e + f*x))/128 + (126929*B*a^3*cos(e + f*x))/128 - (25501905*A*a^3*sin(e + f*x))/128 + (3970395*B*a^3*sin(e + f*x))/128)/(45045*c^8*f*((6435*2^(1/2)*cos(e/2 + pi/4 + (f*x)/2))/128 - (5005*2^(1/2)*cos((3*e)/2 - pi/4 + (3*f*x)/2))/128 - (3003*2^(1/2)*cos((5*e)/2 + pi/4 + (5*f*x)/2))/128 + (1365*2^(1/2)*cos((7*e)/2 - pi/4 + (7*f*x)/2))/128 + (455*2^(1/2)*cos((9*e)/2 + pi/4 + (9*f*x)/2))/128 - (105*2^(1/2)*cos((11*e)/2 - pi/4 + (11*f*x)/2))/128 - (15*2^(1/2)*cos((13*e)/2 + pi/4 + (13*f*x)/2))/128 + (2^(1/2)*cos((15*e)/2 - pi/4 + (15*f*x)/2))/128))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**8,x)

[Out] Timed out

$$3.52 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=190

$$\frac{a^4 c^4 (A-B) \cos^9(e+fx)}{f(a \sin(e+fx)+a)^5} - \frac{2a^2 c^4 (4A-5B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^3} - \frac{35c^4 (4A-5B) \cos^3(e+fx)}{12af} - \frac{7c^4 (4A-5B) \cos^5(e+fx)}{4f(a \sin(e+fx)+a)}$$

[Out] $-35/8*(4*A-5*B)*c^4*x/a-35/12*(4*A-5*B)*c^4*\cos(f*x+e)^3/a/f-35/8*(4*A-5*B)*c^4*\cos(f*x+e)*\sin(f*x+e)/a/f-a^4*(A-B)*c^4*\cos(f*x+e)^9/f/(a+a*\sin(f*x+e))^5-2*a^2*(4*A-5*B)*c^4*\cos(f*x+e)^7/f/(a+a*\sin(f*x+e))^3-7/4*(4*A-5*B)*c^4*\cos(f*x+e)^5/f/(a+a*\sin(f*x+e))$

Rubi [A] time = 0.36, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$\frac{a^4 c^4 (A-B) \cos^9(e+fx)}{f(a \sin(e+fx)+a)^5} - \frac{2a^2 c^4 (4A-5B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^3} - \frac{35c^4 (4A-5B) \cos^3(e+fx)}{12af} - \frac{7c^4 (4A-5B) \cos^5(e+fx)}{4f(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^4/(a + a*\text{Sin}[e + f*x]),x]$

[Out] $(-35*(4*A - 5*B)*c^4*x)/(8*a) - (35*(4*A - 5*B)*c^4*\text{Cos}[e + f*x]^3)/(12*a*f) - (35*(4*A - 5*B)*c^4*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(8*a*f) - (a^4*(A - B)*c^4*\text{Cos}[e + f*x]^9)/(f*(a + a*\text{Sin}[e + f*x])^5) - (2*a^2*(4*A - 5*B)*c^4*\text{Cos}[e + f*x]^7)/(f*(a + a*\text{Sin}[e + f*x])^3) - (7*(4*A - 5*B)*c^4*\text{Cos}[e + f*x]^5)/(4*f*(a + a*\text{Sin}[e + f*x]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2679

$\text{Int}[(\cos[(e_*) + (f_*)*(x_)]*(g_*)^{(p_)*((a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]^{(m_)}), x_Symbol] \rightarrow \text{Simp}[(g*(g*\text{Cos}[e + f*x])^{(p-1)}*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+p)), x] + \text{Dist}[(g^2*(p-1))/(a*(m+p)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p-2)}*(a + b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[p, 1] \ \&\& \ (\text{GtQ}[m, -2] \ || \ \text{EqQ}[2*m + p + 1, 0] \ || \ (\text{EqQ}[m, -2] \ \&\& \ \text{IntegerQ}[p])) \ \&\& \ \text{NeQ}[m + p, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*p]$

Rule 2680

$\text{Int}[(\cos[(e_*) + (f_*)*(x_)]*(g_*)^{(p_)*((a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]^{(m_)}), x_Symbol] \rightarrow \text{Simp}[(2*g*(g*\text{Cos}[e + f*x])^{(p-1)}*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(2*m + p + 1)), x] + \text{Dist}[(g^2*(p-1))/(b^2*(2*m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p-2)}*(a + b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LeQ}[m, -2] \ \&\& \ \text{GtQ}[p, 1] \ \&\& \ \text{NeQ}[2*m + p + 1, 0] \ \&\& \ !\text{LtQ}[m + p + 1, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*p]$

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(g*(g*cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Dist[g^2/a, Int[(g*cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.)), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^4}{a + a \sin(e + fx)} dx &= (a^4 c^4) \int \frac{\cos^8(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^5} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{f(a + a \sin(e + fx))^5} - (a^3(4A - 5B)c^4) \int \frac{\cos^8(e + fx)}{(a + a \sin(e + fx))^5} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{f(a + a \sin(e + fx))^5} - \frac{2a^2(4A - 5B)c^4 \cos^7(e + fx)}{f(a + a \sin(e + fx))^3} - \frac{2a^2(4A - 5B)c^4 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{f(a + a \sin(e + fx))^5} - \frac{2a^2(4A - 5B)c^4 \cos^7(e + fx)}{f(a + a \sin(e + fx))^3} - \frac{2a^2(4A - 5B)c^4 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} \\ &= -\frac{35(4A - 5B)c^4 \cos^3(e + fx)}{12af} - \frac{a^4(A - B)c^4 \cos^9(e + fx)}{f(a + a \sin(e + fx))^5} - \frac{2a^2(4A - 5B)c^4 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} \\ &= -\frac{35(4A - 5B)c^4 \cos^3(e + fx)}{12af} - \frac{35(4A - 5B)c^4 \cos(e + fx) \sin^2(e + fx)}{8af} \\ &= -\frac{35(4A - 5B)c^4 x}{8a} - \frac{35(4A - 5B)c^4 \cos^3(e + fx)}{12af} - \frac{35(4A - 5B)c^4 \cos(e + fx) \sin^2(e + fx)}{8af} \end{aligned}$$

Mathematica [A] time = 2.36, size = 274, normalized size = 1.44

$$\frac{(c - c \sin(e + fx))^4 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(3072(A - B) \sin\left(\frac{1}{2}(e + fx)\right) - 420(4A - 5B)(e + fx) \right)}{8af}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x]), x]
```


$$\begin{aligned} & \frac{1}{2}e)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^6 A + 30/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^6 B - 5/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^5 A + 55/4/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^5 B - 70/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^4 A + 110/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^4 B + 5/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^3 A - 55/4/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^3 B - 214/3/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2 A + 350/3/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2 B + 5/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right) A - 47/4/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right) B - 70/3/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 A + 110/3/f*c^4/a / (1 + \tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)^2)^4 B - 35/f*c^4/a * \arctan\left(\tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)\right) A + 175/4/f*c^4/a * \arctan\left(\tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right)\right) B - 32/f*c^4/a / (\tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right) + 1) A + 32/f*c^4/a / (\tan\left(\frac{1}{2}f*x+\frac{1}{2}e\right) + 1) B \end{aligned}$$

maxima [B] time = 0.64, size = 1796, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out]
$$\begin{aligned} & \frac{1}{12} * (B * c^4 * ((19 * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 211 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 91 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 219 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 165 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 165 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 45 * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + 45 * \sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + 64) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 4 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 4 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 6 * a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 6 * a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 4 * a * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 4 * a * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + a * \sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + a * \sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9) + 45 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a - 4 * A * c^4 * ((7 * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 39 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 24 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 24 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 9 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 9 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 16) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 3 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 3 * a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 3 * a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + a * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + a * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7) + 9 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a + 16 * B * c^4 * ((7 * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 39 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 24 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 24 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 9 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 9 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 16) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 3 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 3 * a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 3 * a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + a * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + a * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7) + 9 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a - 4 * A * c^4 * ((\sin(f*x + e)) / (\cos(f*x + e) + 1) + 5 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 4) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 2 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 2 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5) + 3 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a + 72 * B * c^4 * ((\sin(f*x + e)) / (\cos(f*x + e) + 1) + 5 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 4) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 2 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 2 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5) + 3 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a - 144 * A * c^4 * ((\sin(f*x + e)) / (\cos(f*x + e) + 1) + \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 4) / (a + a * \sin(f*x + e)) / (\cos(f*x + e) + 1) + 2 * a * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 2 * a * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + a * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + a * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5) + 3 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a \end{aligned}$$

$$\begin{aligned}
& f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144* \\
& a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x \\
& /2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 2520*A*c**4*f*x*tan(e/2 + f*x/ \\
& 2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan \\
& (e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 \\
& + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/ \\
& 2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x*tan(e/2 \\
& + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96* \\
& a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f* \\
& x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f \\
& *tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1680*A*c**4*f*x* \\
& tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)** \\
& 8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e \\
& /2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + \\
& 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A*c** \\
& 4*f*x*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2 \\
&)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*ta \\
& n(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)** \\
& 3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 420*A* \\
& c**4*f*x/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f* \\
& tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2) \\
& **5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan \\
& (e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 888*A*c**4*tan(e/2 + \\
& f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a* \\
& f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/ \\
& 2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*t \\
& an(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 648*A*c**4*tan(e/2 \\
& + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96* \\
& a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f* \\
& x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f \\
& *tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 3720*A*c**4*tan(\\
& e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + \\
& 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + \\
& f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96* \\
& a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1800*A*c**4*t \\
& an(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 \\
& + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/ \\
& 2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + \\
& 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 6168*A*c** \\
& 4*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2) \\
& **8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan \\
& (e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 \\
& + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 1592*A* \\
& c**4*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x \\
& /2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f* \\
& tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2) \\
& **3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4664 \\
& *A*c**4*tan(e/2 + f*x/2)**2/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + \\
& f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a \\
& *f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x \\
& /2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 4 \\
& 40*A*c**4*tan(e/2 + f*x/2)/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f \\
& *x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a* \\
& f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/ \\
& 2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) - 13 \\
& 28*A*c**4/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f \\
& *tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2 \\
&)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*ta \\
& n(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(
\end{aligned}$$

$$\begin{aligned}
& e/2 + f*x/2)**9/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + \\
& 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + \\
& f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96* \\
& a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f* \\
& x*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2) \\
& **8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan \\
& (e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 \\
& + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2100*B* \\
& c**4*f*x*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + \\
& f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144* \\
& a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f* \\
& x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + \\
& 2100*B*c**4*f*x*tan(e/2 + f*x/2)**6/(24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*ta \\
& n(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 \\
& + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e \\
& /2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24* \\
& a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**5/(24*a*f*tan(e/2 + f*x/2)**9 + 24 \\
& *a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e/2 + f* \\
& x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 + 96*a* \\
& f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 + f*x/2) \\
&) + 24*a*f) + 3150*B*c**4*f*x*tan(e/2 + f*x/2)**4/(24*a*f*tan(e/2 + f*x/2)* \\
& **9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*tan(e \\
& /2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)**4 \\
& + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*tan(e/2 \\
& + f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**3/(24*a*f*tan(e/2 + \\
& f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a* \\
& f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x \\
& /2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*t \\
& an(e/2 + f*x/2) + 24*a*f) + 2100*B*c**4*f*x*tan(e/2 + f*x/2)**2/(24*a*f*tan \\
& (e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 \\
& + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/ \\
& 2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 2 \\
& 4*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x*tan(e/2 + f*x/2)/(24*a*f* \\
& tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)* \\
& **7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*ta \\
& n(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 \\
& + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 525*B*c**4*f*x/(24*a*f*tan(e/2 + f*x/ \\
& 2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f*ta \\
& n(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/2)* \\
& **4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta \\
& n(e/2 + f*x/2) + 24*a*f) + 1050*B*c**4*tan(e/2 + f*x/2)**8/(24*a*f*tan(e/2 + f \\
& *x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f \\
& *tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/ \\
& 2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta \\
& n(e/2 + f*x/2) + 24*a*f) + 1002*B*c**4*tan(e/2 + f*x/2)**7/(24*a*f*tan(e/2 \\
& + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96* \\
& a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f \\
& *x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f \\
& *tan(e/2 + f*x/2) + 24*a*f) + 4122*B*c**4*tan(e/2 + f*x/2)**6/(24*a*f*tan(e \\
& /2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + \\
& 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 \\
& + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24* \\
& a*f*tan(e/2 + f*x/2) + 24*a*f) + 2970*B*c**4*tan(e/2 + f*x/2)**5/(24*a*f*ta \\
& n(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 \\
& + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e \\
& /2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + \\
& 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 6918*B*c**4*tan(e/2 + f*x/2)**4/(24*a*f \\
& *tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2) \\
& **7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*ta
\end{aligned}$$


```

n(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2
+ 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 2470*B*c**4*tan(e/2 + f*x/2)**3/(24*
a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x
/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f
*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)
**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 5590*B*c**4*tan(e/2 + f*x/2)**2/(
24*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 +
f*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*
a*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x
/2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 598*B*c**4*tan(e/2 + f*x/2)/(2
4*a*f*tan(e/2 + f*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f
*x/2)**7 + 96*a*f*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a
*f*tan(e/2 + f*x/2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/
2)**2 + 24*a*f*tan(e/2 + f*x/2) + 24*a*f) + 1648*B*c**4/(24*a*f*tan(e/2 + f
*x/2)**9 + 24*a*f*tan(e/2 + f*x/2)**8 + 96*a*f*tan(e/2 + f*x/2)**7 + 96*a*f
*tan(e/2 + f*x/2)**6 + 144*a*f*tan(e/2 + f*x/2)**5 + 144*a*f*tan(e/2 + f*x/
2)**4 + 96*a*f*tan(e/2 + f*x/2)**3 + 96*a*f*tan(e/2 + f*x/2)**2 + 24*a*f*ta
n(e/2 + f*x/2) + 24*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**4/(
a*sin(e) + a), True))

```

$$3.53 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=157

$$\frac{a^3 c^3 (A-B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^4} - \frac{2a^3 c^3 (3A-4B) \cos^5(e+fx)}{f(a^2 \sin(e+fx)+a^2)^2} - \frac{5c^3 (3A-4B) \cos^3(e+fx)}{3af} - \frac{5c^3 (3A-4B) \sin(e+fx)}{2af}$$

[Out] $-5/2*(3*A-4*B)*c^3*x/a-5/3*(3*A-4*B)*c^3*\cos(f*x+e)^3/a/f-5/2*(3*A-4*B)*c^3*\cos(f*x+e)*\sin(f*x+e)/a/f-a^3*(A-B)*c^3*\cos(f*x+e)^7/f/(a+a*\sin(f*x+e))^4-2*a^3*(3*A-4*B)*c^3*\cos(f*x+e)^5/f/(a^2+a^2*\sin(f*x+e))^2$

Rubi [A] time = 0.32, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{a^3 c^3 (A-B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^4} - \frac{2a^3 c^3 (3A-4B) \cos^5(e+fx)}{f(a^2 \sin(e+fx)+a^2)^2} - \frac{5c^3 (3A-4B) \cos^3(e+fx)}{3af} - \frac{5c^3 (3A-4B) \sin(e+fx)}{2af}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]

[Out] $(-5*(3*A - 4*B)*c^3*x)/(2*a) - (5*(3*A - 4*B)*c^3*\text{Cos}[e + f*x]^3)/(3*a*f) - (5*(3*A - 4*B)*c^3*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(2*a*f) - (a^3*(A - B)*c^3*\text{Cos}[e + f*x]^7)/(f*(a + a*\text{Sin}[e + f*x])^4) - (2*a^3*(3*A - 4*B)*c^3*\text{Cos}[e + f*x]^5)/(f*(a^2 + a^2*\text{Sin}[e + f*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m+p+1)

```

)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]

```

Rule 2967

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^3}{a + a \sin(e + fx)} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^4} dx \\
&= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} - (a^2(3A - 4B)c^3) \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^4} dx \\
&= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} - \frac{2a(3A - 4B)c^3 \cos^5(e + fx)}{f(a + a \sin(e + fx))^2} - \frac{5(3A - 4B)c^3 \cos^3(e + fx)}{3af} \\
&= -\frac{5(3A - 4B)c^3 \cos^3(e + fx)}{3af} - \frac{a^3(A - B)c^3 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} - \frac{2a(3A - 4B)c^3 \cos^5(e + fx)}{f(a + a \sin(e + fx))^2} \\
&= -\frac{5(3A - 4B)c^3 \cos^3(e + fx)}{3af} - \frac{5(3A - 4B)c^3 \cos(e + fx) \sin(e + fx)}{2af} \\
&= -\frac{5(3A - 4B)c^3 x}{2a} - \frac{5(3A - 4B)c^3 \cos^3(e + fx)}{3af} - \frac{5(3A - 4B)c^3 \cos(e + fx) \sin(e + fx)}{2af}
\end{aligned}$$

Mathematica [A] time = 1.40, size = 220, normalized size = 1.40

$$\frac{c^3(\sin(e + fx) - 1)^3 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) (30(3A - 4B)(e + fx) - 3(A - 4B) \sin(e + fx)) \right)}{12af^2}$$

Antiderivative was successfully verified.

```

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]
),x]

```

```

[Out] (c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*(Cos[(e +
f*x)/2]*(30*(3*A - 4*B)*(e + f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos[3*(e
+ f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(-24*B*(-8 + 5*
e + 5*f*x) + 6*A*(-32 + 15*e + 15*f*x) + (48*A - 93*B)*Cos[e + f*x] + B*Cos
[3*(e + f*x)] - 3*(A - 4*B)*Sin[2*(e + f*x)])))/(12*a*f*(Cos[(e + f*x)/2] -
Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x]))

```

fricas [A] time = 0.46, size = 218, normalized size = 1.39

$$\frac{2Bc^3 \cos^4(fx + e) + (3A - 10B)c^3 \cos^3(fx + e) + 15(3A - 4B)c^3 fx + 24(A - 2B)c^3 \cos^2(fx + e) + 48(A - 2B)c^3 \cos(fx + e) + 48(A - 2B)c^3}{12af^2}$$

Verification of antiderivative is not currently implemented for this CAS.


```

*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*
x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 180*B*c**3*f*x*tan(e/2 + f*x/2)
**2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18
*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 60*B*c**3*f*x*
tan(e/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 1
8*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f
*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 6
0*B*c**3*f*x/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*
f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 120*B
*c**3*tan(e/2 + f*x/2)**6/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/
2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*ta
n(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6
*a*f) + 108*B*c**3*tan(e/2 + f*x/2)**5/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*t
an(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**
4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2
+ f*x/2) + 6*a*f) + 372*B*c**3*tan(e/2 + f*x/2)**4/(6*a*f*tan(e/2 + f*x/2)
**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e
/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 + f*x/2)**2 +
6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 192*B*c**3*tan(e/2 + f*x/2)**3/(6*a*f*tan
(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2 + f*x/2)**5 +
18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*a*f*tan(e/2 +
f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 456*B*c**3*tan(e/2 + f*x/2)*
*2/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/2
+ f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 18*
a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 68*B*c**3*tan(e
/2 + f*x/2)/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f
*tan(e/2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)
)**3 + 18*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f) + 188*B*
c**3/(6*a*f*tan(e/2 + f*x/2)**7 + 6*a*f*tan(e/2 + f*x/2)**6 + 18*a*f*tan(e/
2 + f*x/2)**5 + 18*a*f*tan(e/2 + f*x/2)**4 + 18*a*f*tan(e/2 + f*x/2)**3 + 1
8*a*f*tan(e/2 + f*x/2)**2 + 6*a*f*tan(e/2 + f*x/2) + 6*a*f), Ne(f, 0)), (x*
(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a), True))

```

$$3.54 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=118

$$\frac{a^2 c^2 (A-B) \cos^5(e+fx)}{f(a \sin(e+fx)+a)^3} - \frac{3c^2(2A-3B) \cos(e+fx)}{2af} - \frac{c^2(2A-3B) \cos^3(e+fx)}{2f(a \sin(e+fx)+a)} - \frac{3c^2 x(2A-3B)}{2a}$$

[Out] $-3/2*(2*A-3*B)*c^2*x/a-3/2*(2*A-3*B)*c^2*\cos(f*x+e)/a/f-a^2*(A-B)*c^2*\cos(f*x+e)^5/f/(a+a*\sin(f*x+e))^3-1/2*(2*A-3*B)*c^2*\cos(f*x+e)^3/f/(a+a*\sin(f*x+e))$

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2679, 2682, 8}

$$\frac{a^2 c^2 (A-B) \cos^5(e+fx)}{f(a \sin(e+fx)+a)^3} - \frac{3c^2(2A-3B) \cos(e+fx)}{2af} - \frac{c^2(2A-3B) \cos^3(e+fx)}{2f(a \sin(e+fx)+a)} - \frac{3c^2 x(2A-3B)}{2a}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]

[Out] $(-3*(2*A - 3*B)*c^2*x)/(2*a) - (3*(2*A - 3*B)*c^2*\text{Cos}[e + f*x])/(2*a*f) - (a^2*(A - B)*c^2*\text{Cos}[e + f*x]^5)/(f*(a + a*\text{Sin}[e + f*x])^3) - ((2*A - 3*B)*c^2*\text{Cos}[e + f*x]^3)/(2*f*(a + a*\text{Sin}[e + f*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2679

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)/((a_) + (b_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967


```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^2}{a + a \sin(e + fx)} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} - (a(2A - 3B)c^2) \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} - \frac{(2A - 3B)c^2 \cos^3(e + fx)}{2f(a + a \sin(e + fx))} - \frac{1}{2} \left(\frac{3(2A - 3B)c^2 \cos(e + fx)}{2af} - \frac{a^2(A - B)c^2 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} - \frac{(2A - 3B)c^2 \cos^3(e + fx)}{2f(a + a \sin(e + fx))} \right) \\ &= -\frac{3(2A - 3B)c^2 x}{2a} - \frac{3(2A - 3B)c^2 \cos(e + fx)}{2af} - \frac{a^2(A - B)c^2 \cos^5(e + fx)}{f(a + a \sin(e + fx))^3} \end{aligned}$$

Mathematica [A] time = 1.33, size = 188, normalized size = 1.59

$$\frac{c^2(\sin(e + fx) - 1)^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(\cos\left(\frac{1}{2}(e + fx)\right) (6(2A - 3B)(e + fx) + 4(A - 3B)c) \right)}{4af(\sin(e + fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]), x]
```

```
[Out] -1/4*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*(Cos[(e + f*x)/2]*(6*(2*A - 3*B)*(e + f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)]) + Sin[(e + f*x)/2]*(4*A*(-8 + 3*e + 3*f*x) - 2*B*(-16 + 9*e + 9*f*x) + 4*(A - 3*B)*Cos[e + f*x] + B*Sin[2*(e + f*x)])))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x]))
```

fricas [A] time = 0.44, size = 179, normalized size = 1.52

$$\frac{Bc^2 \cos^3(fx + e) - 3(2A - 3B)c^2 fx - 2(A - 3B)c^2 \cos^2(fx + e) - 8(A - B)c^2 - (3(2A - 3B)c^2 fx + (10A - 13B)c^2) \cos(fx + e)}{2(af \cos(fx + e) + a^2 \sin^2(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(B*c^2*cos(f*x + e)^3 - 3*(2*A - 3*B)*c^2*f*x - 2*(A - 3*B)*c^2*cos(f*x + e)^2 - 8*(A - B)*c^2 - (3*(2*A - 3*B)*c^2*f*x + (10*A - 13*B)*c^2)*cos(f*x + e) - (3*(2*A - 3*B)*c^2*f*x + B*c^2*cos(f*x + e)^2 + (2*A - 5*B)*c^2*cos(f*x + e) - 8*(A - B)*c^2)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [A] time = 0.16, size = 164, normalized size = 1.39

$$\frac{3(2Ac^2-3Bc^2)(fx+e)}{a} + \frac{16(Ac^2-Bc^2)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} - \frac{2\left(Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3-2Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Bc^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-2Ac^2\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^2} \cdot \frac{1}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/2*(3*(2*A*c^2 - 3*B*c^2)*(f*x + e)/a + 16*(A*c^2 - B*c^2)/(a*(tan(1/2*f*x + 1/2*e) + 1)) - 2*(B*c^2*tan(1/2*f*x + 1/2*e)^3 - 2*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 6*B*c^2*tan(1/2*f*x + 1/2*e) - B*c^2*tan(1/2*f*x + 1/2*e) - 2*A*c^2 + 6*B*c^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f
```

maple [B] time = 0.40, size = 299, normalized size = 2.53

$$\frac{c^2\left(\tan^3\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} - \frac{2c^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} + \frac{6c^2\left(\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)B}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} - \frac{c^2B\tan\left(\frac{fx}{2}+\frac{e}{2}\right)}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2} - \frac{c^2A}{fa\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)
```

```
[Out] 1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^3*B-2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*A+6/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*tan(1/2*f*x+1/2*e)^2*B-1/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)-2/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*A+6/f*c^2/a/(1+tan(1/2*f*x+1/2*e)^2)^2*B+9/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))*B-6/f*c^2/a*arctan(tan(1/2*f*x+1/2*e))*A-8/f*c^2/a/(tan(1/2*f*x+1/2*e)+1)*A+8/f*c^2/a/(tan(1/2*f*x+1/2*e)+1)*B
```

maxima [B] time = 0.43, size = 608, normalized size = 5.15

$$Bc^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 4}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1} + \frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right) - 2Ac^2\left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{5\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 4}{a + \frac{a\sin(fx+e)}{\cos(fx+e)+1} + \frac{2a\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2a\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a\sin(fx+e)^5}{(\cos(fx+e)+1)^5}} + \frac{3\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] (B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - 2*A*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) + 4*B*c^2*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2*a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a)
```



```

+ 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*
a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2
)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*f*x*tan(e/2 + f*x/2)**3/
(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*
x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*
B*c**2*f*x*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*
tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 +
f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*t
an(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 9*B*c**2*f*x/(2*a*f*
tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3
+ 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 18*B*c**2*
tan(e/2 + f*x/2)**4/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4
+ 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f
*x/2) + 2*a*f) + 14*B*c**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 +
2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*
x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 42*B*c**2*tan(e/2 + f*x/2)**2/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 10*B
*c**2*tan(e/2 + f*x/2)/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)*
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 28*B*c**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 +
f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*t
an(e/2 + f*x/2) + 2*a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**2/(
a*sin(e) + a), True))

```

$$3.55 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=57

$$-\frac{2c(A-B) \cos(e+fx)}{f(a \sin(e+fx)+a)} - \frac{cx(A-2B)}{a} + \frac{Bc \cos(e+fx)}{af}$$

[Out] $-(A-2*B)*c*x/a+B*c*cos(f*x+e)/a/f-2*(A-B)*c*cos(f*x+e)/f/(a+a*\sin(f*x+e))$

Rubi [A] time = 0.16, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {2967, 2857, 2638}

$$-\frac{2c(A-B) \cos(e+fx)}{f(a \sin(e+fx)+a)} - \frac{cx(A-2B)}{a} + \frac{Bc \cos(e+fx)}{af}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])]/(a + a*\text{Sin}[e + f*x]), x]$

[Out] $-\left(\frac{(A - 2*B)*c*x}{a} + \frac{B*c*\text{Cos}[e + f*x]}{a*f} - \frac{2*(A - B)*c*\text{Cos}[e + f*x]}{f*(a + a*\text{Sin}[e + f*x])}\right)$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2857

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{2*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])}, x_Symbol] \rightarrow \text{Simp}[(2*(b*c - a*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b^2*f*(2*m+3)), x] + \text{Dist}[1/(b^3*(2*m+3)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+2)}*(b*c + 2*a*d*(m+1) - b*d*(2*m+3)*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -3/2]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{a+a \sin(e+fx)} dx &= (ac) \int \frac{\cos^2(e+fx)(A+B \sin(e+fx))}{(a+a \sin(e+fx))^2} dx \\ &= -\frac{2(A-B)c \cos(e+fx)}{f(a+a \sin(e+fx))} - \frac{c \int (aA-2aB+aB \sin(e+fx)) dx}{a^2} \\ &= -\frac{(A-2B)cx}{a} - \frac{2(A-B)c \cos(e+fx)}{f(a+a \sin(e+fx))} - \frac{(Bc) \int \sin(e+fx) dx}{a} \\ &= -\frac{(A-2B)cx}{a} + \frac{Bc \cos(e+fx)}{af} - \frac{2(A-B)c \cos(e+fx)}{f(a+a \sin(e+fx))} \end{aligned}$$

Mathematica [B] time = 0.59, size = 127, normalized size = 2.23

$$\frac{(c - c \sin(e + fx)) \left(\frac{4(A-B) \sin\left(\frac{fx}{2}\right)}{f \left(\sin\left(\frac{e}{2}\right) + \cos\left(\frac{e}{2}\right) \right) \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)} - (x(A-2B)) - \frac{B \sin(e) \sin(fx)}{f} + \frac{B \cos(e) \cos(fx)}{f} \right)}{a \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x]), x]

[Out] (((-(A - 2*B)*x) + (B*Cos[e]*Cos[f*x])/f - (B*Sin[e]*Sin[f*x])/f + (4*(A - B)*Sin[(f*x)/2])/(f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])))*(c - c*Sin[e + f*x]))/(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2)

fricas [B] time = 0.43, size = 117, normalized size = 2.05

$$\frac{(A - 2B)cfx - Bc \cos(fx + e)^2 + 2(A - B)c + ((A - 2B)cfx + (2A - 3B)c) \cos(fx + e) + ((A - 2B)cfx - Bc) \sin(fx + e)}{af \cos(fx + e) + af \sin(fx + e) + af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] -((A - 2*B)*c*f*x - B*c*cos(f*x + e)^2 + 2*(A - B)*c + ((A - 2*B)*c*f*x + (2*A - 3*B)*c)*cos(f*x + e) + ((A - 2*B)*c*f*x - B*c*cos(f*x + e) - 2*(A - B)*c)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

giac [B] time = 0.14, size = 122, normalized size = 2.14

$$\frac{\frac{(Ac-2Bc)(fx+e)}{a} + \frac{2 \left(2Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 2Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2Ac - 3Bc \right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] -((A*c - 2*B*c)*(f*x + e)/a + 2*(2*A*c*tan(1/2*f*x + 1/2*e)^2 - 2*B*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e) + 2*A*c - 3*B*c)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f

maple [A] time = 0.38, size = 113, normalized size = 1.98

$$\frac{2cB}{fa \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)} - \frac{2c \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)A}{fa} + \frac{4c \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)B}{fa} - \frac{4cA}{fa \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1 \right)} + \frac{4cB}{fa \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] 2/f*c/a*B/(1+tan(1/2*f*x+1/2*e)^2)-2/f*c/a*arctan(tan(1/2*f*x+1/2*e))*A+4/f*c/a*arctan(tan(1/2*f*x+1/2*e))*B-4/f*c/a/(tan(1/2*f*x+1/2*e)+1)*A+4/f*c/a/(tan(1/2*f*x+1/2*e)+1)*B

maxima [B] time = 0.55, size = 256, normalized size = 4.49

$$2 \left(Bc \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} + \frac{a \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} \right) - Ac \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1}} \right) + Bc \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1}} \right) \right) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] 2*(B*c*((sin(f*x + e)/(cos(f*x + e) + 1) + sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a + a*sin(f*x + e)/(cos(f*x + e) + 1) + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a) - A*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) + B*c*(arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A*c/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))/f

mupad [B] time = 12.94, size = 110, normalized size = 1.93

$$\frac{(4Ac - 4Bc) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 2Bc \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 4Ac - 6Bc}{f \left(a \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + a \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + a \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + a \right)} - \frac{Acfx - 2Bcfx}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x)),x)

[Out] - (4*A*c - 6*B*c + tan(e/2 + (f*x)/2)^2*(4*A*c - 4*B*c) - 2*B*c*tan(e/2 + (f*x)/2))/(f*(a + a*tan(e/2 + (f*x)/2) + a*tan(e/2 + (f*x)/2)^2 + a*tan(e/2 + (f*x)/2)^3)) - (A*c*f*x - 2*B*c*f*x)/(a*f)

sympy [A] time = 3.94, size = 828, normalized size = 14.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] Piecewise((-A*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - A*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*A*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 4*B*c*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +

```
6*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*
x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a), T
rue))
```


$$3.56 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))} dx$$

Optimal. Leaf size=35

$$\frac{A \tan(e+fx)}{acf} + \frac{B \sec(e+fx)}{acf}$$

[Out] B*sec(f*x+e)/a/c/f+A*tan(f*x+e)/a/c/f

Rubi [A] time = 0.14, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2669, 3767, 8}

$$\frac{A \tan(e+fx)}{acf} + \frac{B \sec(e+fx)}{acf}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])),x]

[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2669

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))} dx &= \frac{\int \sec^2(e + fx)(A + B \sin(e + fx)) dx}{ac} \\ &= \frac{B \sec(e + fx)}{acf} + \frac{A \int \sec^2(e + fx) dx}{ac} \\ &= \frac{B \sec(e + fx)}{acf} - \frac{A \text{Subst}(\int 1 dx, x, -\tan(e + fx))}{acf} \\ &= \frac{B \sec(e + fx)}{acf} + \frac{A \tan(e + fx)}{acf} \end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 1.00

$$\frac{A \tan(e + fx)}{acf} + \frac{B \sec(e + fx)}{acf}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])), x]

[Out] (B*Sec[e + f*x])/(a*c*f) + (A*Tan[e + f*x])/(a*c*f)

fricas [A] time = 0.41, size = 28, normalized size = 0.80

$$\frac{A \sin(fx + e) + B}{acf \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] (A*sin(f*x + e) + B)/(a*c*f*cos(f*x + e))

giac [A] time = 0.18, size = 41, normalized size = 1.17

$$\frac{2 \left(A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + B \right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1 \right) acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] -2*(A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)*a*c*f)

maple [A] time = 0.37, size = 57, normalized size = 1.63

$$\frac{\frac{2\left(\frac{A}{2} + \frac{B}{2}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{2\left(\frac{A}{2} - \frac{B}{2}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1}}{acf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] 2/f/c/a*(-(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)-(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1))

maxima [A] time = 0.32, size = 35, normalized size = 1.00

$$\frac{\frac{A \tan(fx+e)}{ac} + \frac{B}{ac \cos(fx+e)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] $(A \cdot \tan(f \cdot x + e) / (a \cdot c) + B / (a \cdot c \cdot \cos(f \cdot x + e))) / f$

mupad [B] time = 12.55, size = 39, normalized size = 1.11

$$\frac{2 \left(B + A \tan \left(\frac{e}{2} + \frac{f x}{2} \right) \right)}{a c f \left(\tan \left(\frac{e}{2} + \frac{f x}{2} \right)^2 - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))),x)`

[Out] $-(2 \cdot (B + A \cdot \tan(e/2 + (f \cdot x)/2))) / (a \cdot c \cdot f \cdot (\tan(e/2 + (f \cdot x)/2)^2 - 1))$

sympy [A] time = 2.53, size = 83, normalized size = 2.37

$$\begin{cases} -\frac{2A \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{acf \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) - acf} - \frac{2B}{acf \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) - acf} & \text{for } f \neq 0 \\ \frac{x(A+B \sin(e))}{(a \sin(e)+a)(-c \sin(e)+c)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e)),x)`

[Out] `Piecewise((-2*A*tan(e/2 + f*x/2)/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f) - 2*B/(a*c*f*tan(e/2 + f*x/2)**2 - a*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)), True))`

$$3.57 \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=63

$$\frac{(2A-B) \tan(e+fx)}{3ac^2f} + \frac{(A+B) \sec(e+fx)}{3af(c^2 - c^2 \sin(e+fx))}$$

[Out] 1/3*(A+B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))+1/3*(2*A-B)*tan(f*x+e)/a/c^2/f

Rubi [A] time = 0.20, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A-B) \tan(e+fx)}{3ac^2f} + \frac{(A+B) \sec(e+fx)}{3af(c^2 - c^2 \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]

[Out] ((A + B)*Sec[e + f*x]/(3*a*f*(c^2 - c^2*Sin[e + f*x])) + ((2*A - B)*Tan[e + f*x])/(3*a*c^2*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^n), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^2} dx = \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx}{ac}$$

$$= \frac{(A + B) \sec(e + fx)}{3af (c^2 - c^2 \sin(e + fx))} + \frac{(2A - B) \int \sec^2(e + fx) dx}{3ac^2}$$

$$= \frac{(A + B) \sec(e + fx)}{3af (c^2 - c^2 \sin(e + fx))} - \frac{(2A - B) \text{Subst}(\int 1 dx, x, -\tan(e + fx))}{3ac^2 f}$$

$$= \frac{(A + B) \sec(e + fx)}{3af (c^2 - c^2 \sin(e + fx))} + \frac{(2A - B) \tan(e + fx)}{3ac^2 f}$$

Mathematica [A] time = 0.59, size = 108, normalized size = 1.71

$$\frac{\cos(e + fx)(-2(A + B) \cos(e + fx) + (4A - 2B) \cos(2(e + fx)) + 8A \sin(e + fx) + A \sin(2(e + fx)) - 4B \sin(2(e + fx)))}{12ac^2 f (\sin(e + fx) - 1)^2 (\sin(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^2),x]

[Out] (Cos[e + f*x]*(6*B - 2*(A + B)*Cos[e + f*x] + (4*A - 2*B)*Cos[2*(e + f*x)] + 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] + A*Sin[2*(e + f*x)] + B*Sin[2*(e + f*x)]))/(12*a*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x]))

fricas [A] time = 0.42, size = 73, normalized size = 1.16

$$\frac{(2A - B) \cos(fx + e)^2 + (2A - B) \sin(fx + e) - A + 2B}{3(ac^2 f \cos(fx + e) \sin(fx + e) - ac^2 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*((2*A - B)*cos(f*x + e)^2 + (2*A - B)*sin(f*x + e) - A + 2*B)/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))

giac [A] time = 0.29, size = 102, normalized size = 1.62

$$\frac{\frac{3(A-B)}{ac^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)} + \frac{9A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 12A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 7A + B}{ac^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] -1/6*(3*(A - B)/(a*c^2*(tan(1/2*f*x + 1/2*e) + 1)) + (9*A*tan(1/2*f*x + 1/2*e)^2 + 3*B*tan(1/2*f*x + 1/2*e)^2 - 12*A*tan(1/2*f*x + 1/2*e) + 7*A + B)/(a*c^2*(tan(1/2*f*x + 1/2*e) - 1)^3))/f

maple [A] time = 0.45, size = 93, normalized size = 1.48

$$\frac{\frac{2(A+B)}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^3} - \frac{A+B}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^2} - \frac{2\left(\frac{3A}{4} + \frac{B}{4}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{2\left(\frac{A}{4} - \frac{B}{4}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1}}{fa c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] 2/f/a/c^2*(-1/3*(A+B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(A+B)/(tan(1/2*f*x+1/2*e)-1)^2-(3/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-(1/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.63, size = 266, normalized size = 4.22

$$2 \frac{\left(\frac{B \left(\frac{2 \sin(fx+e)}{\cos(fx+e)+1} - \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - 1 \right)}{ac^2 - \frac{2ac^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{2ac^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{ac^2 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} - \frac{A \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + 1 \right)}{ac^2 - \frac{2ac^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{2ac^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{ac^2 \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} \right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) - A*(sin(f*x + e)/(cos(f*x + e) + 1) - 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a*c^2 - 2*a*c^2*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a*c^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a*c^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 12.32, size = 118, normalized size = 1.87

$$\frac{2 \left(\frac{3B}{2} + A \cos(e + fx) + B \cos(e + fx) + 2A \sin(e + fx) - B \sin(e + fx) + A \cos(2e + 2fx) - \frac{B \cos(2e + 2fx)}{2} \right)}{3a^2c^2f(2 \cos(e + fx) - \sin(2e + 2fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^2),x)
```

```
[Out] (2*((3*B)/2 + A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) - B*sin(e + f*x) + A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))/2 - (B*sin(2*e + 2*f*x))/2))/(3*a*c^2*f*(2*cos(e + f*x) - sin(2*e + 2*f*x)))
```

sympy [A] time = 7.81, size = 578, normalized size = 9.17

$$\left\{ \begin{array}{l} \frac{6A \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{3ac^2f \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) - 6ac^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 6ac^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3ac^2f} + \frac{6A \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3ac^2f \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) - 6ac^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 6ac^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3ac^2f} - \frac{x(A+B \sin(e))}{(a \sin(e)+a)(-c \sin(e)+c)^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 6*A*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A*tan(e/2 + f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c
```

```

**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*A/(3*a*c**2*f*tan(e/2 + f*x/2)**4
- 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*ta
n(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) + 4*B*tan(e/2
+ f*x/2)/(3*a*c**2*f*tan(e/2 + f*x/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3
+ 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a*c**2*f) - 2*B/(3*a*c**2*f*tan(e/2 + f*x
/2)**4 - 6*a*c**2*f*tan(e/2 + f*x/2)**3 + 6*a*c**2*f*tan(e/2 + f*x/2) - 3*a
*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)*(-c*sin(e) + c)**2),
True))

```

$$3.58 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=102

$$\frac{2(3A-2B) \tan(e+fx)}{15ac^3f} + \frac{(3A-2B) \sec(e+fx)}{15af(c^3-c^3 \sin(e+fx))} + \frac{(A+B) \sec(e+fx)}{5acf(c-c \sin(e+fx))^2}$$

[Out] 1/5*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^2+1/15*(3*A-2*B)*sec(f*x+e)/a/f/(c^3-c^3*sin(f*x+e))+2/15*(3*A-2*B)*tan(f*x+e)/a/c^3/f

Rubi [A] time = 0.26, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A-2B) \tan(e+fx)}{15ac^3f} + \frac{(3A-2B) \sec(e+fx)}{15af(c^3-c^3 \sin(e+fx))} + \frac{(A+B) \sec(e+fx)}{5acf(c-c \sin(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3),x]

[Out] ((A + B)*Sec[e + f*x])/((5*a*c*f*(c - c*Sin[e + f*x])^2) + ((3*A - 2*B)*Sec[e + f*x]))/(15*a*f*(c^3 - c^3*Sin[e + f*x])) + (2*(3*A - 2*B)*Tan[e + f*x])/((15*a*c^3*f))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]))^m, x_Symbol] := Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]))^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]))^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]))^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n] || LtQ[m, n], 0))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^3} dx &= \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx}{ac} \\ &= \frac{(A + B) \sec(e + fx)}{5acf(c - c \sin(e + fx))^2} + \frac{(3A - 2B) \int \frac{\sec^2(e+fx)}{c-c \sin(e+fx)} dx}{5ac^2} \\ &= \frac{(A + B) \sec(e + fx)}{5acf(c - c \sin(e + fx))^2} + \frac{(3A - 2B) \sec(e + fx)}{15af(c^3 - c^3 \sin(e + fx))} + \frac{(2(3A - 2B) \sec(e + fx))}{15af(c^3 - c^3 \sin(e + fx))} \\ &= \frac{(A + B) \sec(e + fx)}{5acf(c - c \sin(e + fx))^2} + \frac{(3A - 2B) \sec(e + fx)}{15af(c^3 - c^3 \sin(e + fx))} - \frac{(2(3A - 2B) \sec(e + fx))}{15af(c^3 - c^3 \sin(e + fx))} \\ &= \frac{(A + B) \sec(e + fx)}{5acf(c - c \sin(e + fx))^2} + \frac{(3A - 2B) \sec(e + fx)}{15af(c^3 - c^3 \sin(e + fx))} + \frac{2(3A - 2B) \sec(e + fx)}{15af(c^3 - c^3 \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.89, size = 157, normalized size = 1.54

$$\frac{\cos(e + fx)(5(B - 9A) \cos(e + fx) + 32(3A - 2B) \cos(2(e + fx)) + 120A \sin(e + fx) + 36A \sin(2(e + fx))) - 240ac^3 f \sin(e + fx)}{240ac^3 f \sin(e + fx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^3), x]

[Out] -1/240*(Cos[e + f*x]*(80*B + 5*(-9*A + B)*Cos[e + f*x] + 32*(3*A - 2*B)*Cos[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] - B*Cos[3*(e + f*x)] + 120*A*Sin[e + f*x] - 80*B*Sin[e + f*x] + 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] - 24*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)])/(a*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x]))

fricas [A] time = 0.42, size = 107, normalized size = 1.05

$$\frac{4(3A - 2B) \cos(fx + e)^2 - (2(3A - 2B) \cos(fx + e)^2 - 9A + 6B) \sin(fx + e) - 6A + 9B}{15(ac^3 f \cos(fx + e))^3 + 2ac^3 f \cos(fx + e) \sin(fx + e) - 2ac^3 f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*(4*(3*A - 2*B)*cos(f*x + e)^2 - (2*(3*A - 2*B)*cos(f*x + e)^2 - 9*A + 6*B)*sin(f*x + e) - 6*A + 9*B)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e))

giac [A] time = 0.26, size = 177, normalized size = 1.74

$$\frac{15(A-B)}{ac^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)} + \frac{105A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 15B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 270A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 30B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 360A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 360B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 360A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 360B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1}{ac^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out]
$$-1/60*(15*(A - B)/(a*c^3*(\tan(1/2*f*x + 1/2*e) + 1)) + (105*A*\tan(1/2*f*x + 1/2*e)^4 + 15*B*\tan(1/2*f*x + 1/2*e)^4 - 270*A*\tan(1/2*f*x + 1/2*e)^3 + 30*B*\tan(1/2*f*x + 1/2*e)^3 + 360*A*\tan(1/2*f*x + 1/2*e)^2 - 40*B*\tan(1/2*f*x + 1/2*e)^2 - 210*A*\tan(1/2*f*x + 1/2*e) + 50*B*\tan(1/2*f*x + 1/2*e) + 63*A - 7*B)/(a*c^3*(\tan(1/2*f*x + 1/2*e) - 1)^5))/f$$

maple [A] time = 0.55, size = 145, normalized size = 1.42

$$\frac{\frac{2(2A+2B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{4A+4B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{\frac{5A}{2}+\frac{3B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2\left(\frac{7A}{8}+\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{2\left(\frac{9A}{2}+\frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{2\left(\frac{A}{8}-\frac{B}{8}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{fac^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)

[Out]
$$2/f/a/c^3*(-1/5*(2*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/4*(4*A+4*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/2*(5/2*A+3/2*B)/(\tan(1/2*f*x+1/2*e)-1)^2-(7/8*A+1/8*B)/(\tan(1/2*f*x+1/2*e)-1)-1/3*(9/2*A+7/2*B)/(\tan(1/2*f*x+1/2*e)-1)^3-(1/8*A-1/8*B)/(\tan(1/2*f*x+1/2*e)+1))$$

maxima [B] time = 0.64, size = 423, normalized size = 4.15

$$\frac{2 \left(\frac{B \left(\frac{4 \sin(fx+e)}{\cos(fx+e)+1} - \frac{20 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - 1 \right)}{ac^3 - \frac{4ac^3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5ac^3 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4ac^3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{ac^3 \sin(fx+e)^6}{(\cos(fx+e)+1)^6}} + \frac{3A \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{10 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} \right)}{ac^3 - \frac{4ac^3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{5ac^3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5ac^3 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4ac^3 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{ac^3 \sin(fx+e)^6}{(\cos(fx+e)+1)^6}} \right)}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out]
$$-2/15*(B*(4*\sin(f*x + e)/(\cos(f*x + e) + 1) - 20*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 1)/(a*c^3 - 4*a*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 5*a*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4*a*c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - a*c^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6) + 3*A*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 10*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 2)/(a*c^3 - 4*a*c^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a*c^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 5*a*c^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4*a*c^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - a*c^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6))/f$$

mupad [B] time = 12.47, size = 178, normalized size = 1.75

$$2 \left(\frac{5B \sin(e+fx)}{2} - \frac{15A \cos(e+fx)}{4} - \frac{5B \cos(e+fx)}{8} - \frac{15A \sin(e+fx)}{4} - \frac{5B}{2} - 3A \cos(2e + 2fx) + \frac{3A \cos(3e+3fx)}{4} + 2B \cos(2e + 2fx) \right) / (15ac^3f \left(\frac{\cos(3e+3fx)}{4} - \frac{5 \cos(e+fx)}{4} \right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^3),x)
```

```
[Out] (2*((5*B*sin(e + f*x))/2 - (15*A*cos(e + f*x))/4 - (5*B*cos(e + f*x))/8 - (15*A*sin(e + f*x))/4 - (5*B)/2 - 3*A*cos(2*e + 2*f*x) + (3*A*cos(3*e + 3*f*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*x) + (3*A*sin(3*e + 3*f*x))/4 + (B*sin(2*e + 2*f*x))/2 - (B*sin(3*e + 3*f*x))/2))/((15*a*c^3*f*(cos(3*e + 3*f*x))/4 - (5*cos(e + f*x))/4 + sin(2*e + 2*f*x)))
```

```
sympy [A] time = 15.15, size = 1236, normalized size = 12.12
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 60*A*tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 60*A*tan(e/2 + f*x/2)**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 18*A*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 12*A/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 30*B*tan(e/2 + f*x/2)**4/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 40*B*tan(e/2 + f*x/2)**3/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) + 8*B*tan(e/2 + f*x/2)/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f) - 2*B/(15*a*c**3*f*tan(e/2 + f*x/2)**6 - 60*a*c**3*f*tan(e/2 + f*x/2)**5 + 75*a*c**3*f*tan(e/2 + f*x/2)**4 - 75*a*c**3*f*tan(e/2 + f*x/2)**2 + 60*a*c**3*f*tan(e/2 + f*x/2) - 15*a*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)*(-c*sin(e) + c)**3), True))
```

$$3.59 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=142

$$\frac{2(4A-3B) \tan(e+fx)}{35ac^4f} + \frac{(4A-3B) \sec(e+fx)}{35af(c^4-c^4 \sin(e+fx))} + \frac{(4A-3B) \sec(e+fx)}{35af(c^2-c^2 \sin(e+fx))^2} + \frac{(A+B) \sec(e+fx)}{7acf(c-c \sin(e+fx))^3}$$

[Out] 1/7*(A+B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^3+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^2-c^2*sin(f*x+e))^2+1/35*(4*A-3*B)*sec(f*x+e)/a/f/(c^4-c^4*sin(f*x+e))+2/35*(4*A-3*B)*tan(f*x+e)/a/c^4/f

Rubi [A] time = 0.31, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(4A-3B) \tan(e+fx)}{35ac^4f} + \frac{(4A-3B) \sec(e+fx)}{35af(c^4-c^4 \sin(e+fx))} + \frac{(4A-3B) \sec(e+fx)}{35af(c^2-c^2 \sin(e+fx))^2} + \frac{(A+B) \sec(e+fx)}{7acf(c-c \sin(e+fx))^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4), x]
 [Out] ((A + B)*Sec[e + f*x])/(7*a*c*f*(c - c*Sin[e + f*x])^3) + ((4*A - 3*B)*Sec[e + f*x])/(35*a*f*(c^2 - c^2*Sin[e + f*x])^2) + ((4*A - 3*B)*Sec[e + f*x])/(35*a*f*(c^4 - c^4*Sin[e + f*x])) + (2*(4*A - 3*B)*Tan[e + f*x])/(35*a*c^4*f)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^4} dx &= \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx}{ac} \\ &= \frac{(A + B) \sec(e + fx)}{7acf(c - c \sin(e + fx))^3} + \frac{(4A - 3B) \int \frac{\sec^2(e+fx)}{(c-c \sin(e+fx))^2} dx}{7ac^2} \\ &= \frac{(A + B) \sec(e + fx)}{7acf(c - c \sin(e + fx))^3} + \frac{(4A - 3B) \sec(e + fx)}{35af(c^2 - c^2 \sin(e + fx))^2} + \frac{(3(4A - 3B) \sec(e + fx))}{35af} \\ &= \frac{(A + B) \sec(e + fx)}{7acf(c - c \sin(e + fx))^3} + \frac{(4A - 3B) \sec(e + fx)}{35af(c^2 - c^2 \sin(e + fx))^2} + \frac{(4A - 3B) \sec(e + fx)}{35af} \\ &= \frac{(A + B) \sec(e + fx)}{7acf(c - c \sin(e + fx))^3} + \frac{(4A - 3B) \sec(e + fx)}{35af(c^2 - c^2 \sin(e + fx))^2} + \frac{(4A - 3B) \sec(e + fx)}{35af} \\ &= \frac{(A + B) \sec(e + fx)}{7acf(c - c \sin(e + fx))^3} + \frac{(4A - 3B) \sec(e + fx)}{35af(c^2 - c^2 \sin(e + fx))^2} + \frac{(4A - 3B) \sec(e + fx)}{35af} \end{aligned}$$

Mathematica [A] time = 1.14, size = 240, normalized size = 1.69

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left((182B - 406A) \cos(e + fx) + 224(4A - 3B) \sin(e + fx)\right)}{(a + a \sin(e + fx))^4 (c - c \sin(e + fx))^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^4), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(560*B + (-406*A + 182*B)*Cos[e + f*x] + 224*(4*A - 3*B)*Cos[2*(e + f*x)] + 174*A*Cos[3*(e + f*x)] - 78*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 48*B*Cos[4*(e + f*x)] + 896*A*Sin[e + f*x] - 672*B*Sin[e + f*x] + 406*A*Sin[2*(e + f*x)] - 182*B*Sin[2*(e + f*x)] - 384*A*Sin[3*(e + f*x)] + 288*B*Sin[3*(e + f*x)] - 29*A*Sin[4*(e + f*x)] + 13*B*Sin[4*(e + f*x)]))/(2240*a*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x]))

fricas [A] time = 0.43, size = 141, normalized size = 0.99

$$\frac{2(4A - 3B) \cos^4(fx + e) - 9(4A - 3B) \cos^2(fx + e) + (6(4A - 3B) \cos^2(fx + e) - 20A + 15B) \sin(fx + e)}{35 \left(3ac^4f \cos^3(fx + e) - 4ac^4f \cos(fx + e) - (ac^4f \cos^3(fx + e) - 4ac^4f \cos(fx + e)) \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] $1/35*(2*(4*A - 3*B)*\cos(f*x + e)^4 - 9*(4*A - 3*B)*\cos(f*x + e)^2 + (6*(4*A - 3*B)*\cos(f*x + e)^2 - 20*A + 15*B)*\sin(f*x + e) + 15*A - 20*B)/(3*a*c^4*f*\cos(f*x + e)^3 - 4*a*c^4*f*\cos(f*x + e) - (a*c^4*f*\cos(f*x + e)^3 - 4*a*c^4*f*\cos(f*x + e))*\sin(f*x + e))$

giac [A] time = 0.20, size = 237, normalized size = 1.67

$$\frac{35(A-B)}{ac^4\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} + \frac{525A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6 + 35B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^6 - 1960A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 + 280B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5 + 4025A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 665B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 4480A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 1120B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 3143A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 791B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 1176A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 392B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 243A - 51B}{ac^4(\tan(\frac{1}{2}fx+\frac{1}{2}e)-1)^7} / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out] $-1/280*(35*(A - B)/(a*c^4*(\tan(1/2*f*x + 1/2*e) + 1)) + (525*A*\tan(1/2*f*x + 1/2*e)^6 + 35*B*\tan(1/2*f*x + 1/2*e)^6 - 1960*A*\tan(1/2*f*x + 1/2*e)^5 + 280*B*\tan(1/2*f*x + 1/2*e)^5 + 4025*A*\tan(1/2*f*x + 1/2*e)^4 - 665*B*\tan(1/2*f*x + 1/2*e)^4 - 4480*A*\tan(1/2*f*x + 1/2*e)^3 + 1120*B*\tan(1/2*f*x + 1/2*e)^3 + 3143*A*\tan(1/2*f*x + 1/2*e)^2 - 791*B*\tan(1/2*f*x + 1/2*e)^2 - 1176*A*\tan(1/2*f*x + 1/2*e) + 392*B*\tan(1/2*f*x + 1/2*e) + 243*A - 51*B)/(a*c^4*(\tan(1/2*f*x + 1/2*e) - 1)^7))/f$

maple [A] time = 0.49, size = 189, normalized size = 1.33

$$\frac{\frac{2(4A+4B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{12A+12B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{18A+14B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{2(19A+17B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{2\left(\frac{15A}{16}+\frac{B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{\frac{17A}{4}+\frac{7B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2\left(\frac{45A}{4}+\frac{27B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}}{fa^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x)

[Out] $2/f/a/c^4*(-1/7*(4*A+4*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/6*(12*A+12*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/4*(18*A+14*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/5*(19*A+17*B)/(\tan(1/2*f*x+1/2*e)-1)^5-(15/16*A+1/16*B)/(\tan(1/2*f*x+1/2*e)-1)-1/2*(17/4*A+7/4*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/3*(45/4*A+27/4*B)/(\tan(1/2*f*x+1/2*e)-1)^3-(1/16*A-1/16*B)/(\tan(1/2*f*x+1/2*e)+1))$

maxima [B] time = 0.35, size = 619, normalized size = 4.36

$$\frac{2 \left(\frac{A \left(\frac{43 \sin(fx+e)}{\cos(fx+e)+1} - \frac{77 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{7 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{105 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{175 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{105 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{35 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - 13 \right)}{ac^4 \frac{6 ac^4 \sin(fx+e)}{\cos(fx+e)+1} + \frac{14 ac^4 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{14 ac^4 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{14 ac^4 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{14 ac^4 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{6 ac^4 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{ac^4 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}} - \frac{B \left(\frac{6 \sin(fx+e)}{\cos(fx+e)+1} \right)}{ac^4 \frac{6 ac^4 \sin(fx+e)}{\cos(fx+e)+1} + \frac{14 ac^4 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{14 ac^4 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{14 ac^4 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} - \frac{14 ac^4 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{6 ac^4 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{ac^4 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}}}{35 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out] $-2/35*(A*(43*\sin(f*x + e)/(\cos(f*x + e) + 1) - 77*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 7*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 105*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 175*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 105*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 35*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 13)/(a*c^4 - 6*a*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 14*a*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 14*a*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 14*a*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 14*a*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 6*a*c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - a*c^4*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8))$


```

*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 86*A*tan(e/2 + f*
x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 +
490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a
*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4
*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 26*A/(35*a*c**4*f*tan(e/2 + f*x/2)**8
- 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490
*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c
**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 7
0*B*tan(e/2 + f*x/2)**6/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan
(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2
+ f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x
/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 140*B*tan(e/2 + f*x
/2)**5/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7
+ 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490
*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c
**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 210*B*tan(e/2 + f*x/2)**4/(35*a*c**4
*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*ta
n(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2
+ f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*
x/2) - 35*a*c**4*f) + 112*B*tan(e/2 + f*x/2)**3/(35*a*c**4*f*tan(e/2 + f*x/
2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6
- 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 49
0*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*
f) - 42*B*tan(e/2 + f*x/2)**2/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4
*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*ta
n(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2
+ f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) - 12*B*tan(e/2
+ f*x/2)/(35*a*c**4*f*tan(e/2 + f*x/2)**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**
7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 - 490*a*c**4*f*tan(e/2 + f*x/2)**5 + 4
90*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*
c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f) + 2*B/(35*a*c**4*f*tan(e/2 + f*x/2)*
**8 - 210*a*c**4*f*tan(e/2 + f*x/2)**7 + 490*a*c**4*f*tan(e/2 + f*x/2)**6 -
490*a*c**4*f*tan(e/2 + f*x/2)**5 + 490*a*c**4*f*tan(e/2 + f*x/2)**3 - 490*a
*c**4*f*tan(e/2 + f*x/2)**2 + 210*a*c**4*f*tan(e/2 + f*x/2) - 35*a*c**4*f),
Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)*(-c*sin(e) + c)**4), True))

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$$3.60 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^5}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=240

$$\frac{a^5 c^5 (A-B) \cos^{11}(e+fx)}{3f(a \sin(e+fx)+a)^7} + \frac{2a^3 c^5 (4A-7B) \cos^9(e+fx)}{3f(a \sin(e+fx)+a)^5} + \frac{35c^5 (4A-7B) \cos^3(e+fx)}{4a^2 f} + \frac{21c^5 (4A-7B) \cos^5(e+fx)}{4f(a^2 \sin(e+fx)+a)}$$

[Out] 105/8*(4*A-7*B)*c^5*x/a^2+35/4*(4*A-7*B)*c^5*cos(f*x+e)^3/a^2/f+105/8*(4*A-7*B)*c^5*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^5*(A-B)*c^5*cos(f*x+e)^11/f/(a+a*sin(f*x+e))^7+2/3*a^3*(4*A-7*B)*c^5*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^5+6*a^4*(4*A-7*B)*c^5*cos(f*x+e)^7/f/(a^2+a^2*sin(f*x+e))^3+21/4*(4*A-7*B)*c^5*cos(f*x+e)^5/f/(a^2+a^2*sin(f*x+e))

Rubi [A] time = 0.41, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {2967, 2859, 2680, 2679, 2682, 2635, 8}

$$\frac{35c^5(4A-7B) \cos^3(e+fx)}{4a^2 f} - \frac{a^5 c^5 (A-B) \cos^{11}(e+fx)}{3f(a \sin(e+fx)+a)^7} + \frac{2a^3 c^5 (4A-7B) \cos^9(e+fx)}{3f(a \sin(e+fx)+a)^5} + \frac{6a^4 c^5 (4A-7B) \cos^7(e+fx)}{f(a^2 \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]

[Out] (105*(4*A - 7*B)*c^5*x)/(8*a^2) + (35*(4*A - 7*B)*c^5*Cos[e + f*x]^3)/(4*a^2*f) + (105*(4*A - 7*B)*c^5*Cos[e + f*x]*Sin[e + f*x])/(8*a^2*f) - (a^5*(A - B)*c^5*Cos[e + f*x]^11)/(3*f*(a + a*Sin[e + f*x])^7) + (2*a^3*(4*A - 7*B)*c^5*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^5) + (6*a^4*(4*A - 7*B)*c^5*Cos[e + f*x]^7)/(f*(a^2 + a^2*Sin[e + f*x])^3) + (21*(4*A - 7*B)*c^5*Cos[e + f*x]^5)/(4*f*(a^2 + a^2*Sin[e + f*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&

NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(g*(g*cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Dist[g^2/a, Int[(g*cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^5}{(a + a \sin(e + fx))^2} dx &= (a^5 c^5) \int \frac{\cos^{10}(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^7} dx \\
 &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{3f(a + a \sin(e + fx))^7} - \frac{1}{3} (a^4(4A - 7B)c^5) \int \frac{\cos^{10}(e + fx)}{(a + a \sin(e + fx))^7} dx \\
 &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{3f(a + a \sin(e + fx))^7} + \frac{2a^3(4A - 7B)c^5 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{6a^3}{3} \int \frac{\cos^8(e + fx)}{(a + a \sin(e + fx))^5} dx \\
 &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{3f(a + a \sin(e + fx))^7} + \frac{2a^3(4A - 7B)c^5 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{6a^3}{3} \int \frac{\cos^7(e + fx)}{(a + a \sin(e + fx))^5} dx \\
 &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{3f(a + a \sin(e + fx))^7} + \frac{2a^3(4A - 7B)c^5 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{6a^3}{3} \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^5} dx \\
 &= \frac{35(4A - 7B)c^5 \cos^3(e + fx)}{4a^2 f} - \frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{3f(a + a \sin(e + fx))^7} + \frac{2a^3(4A - 7B)c^5 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^5} \\
 &= \frac{35(4A - 7B)c^5 \cos^3(e + fx)}{4a^2 f} + \frac{105(4A - 7B)c^5 \cos(e + fx) \sin(e + fx)}{8a^2 f} \\
 &= \frac{105(4A - 7B)c^5 x}{8a^2} + \frac{35(4A - 7B)c^5 \cos^3(e + fx)}{4a^2 f} + \frac{105(4A - 7B)c^5 \cos(e + fx) \sin(e + fx)}{8a^2 f}
 \end{aligned}$$

Mathematica [A] time = 2.07, size = 354, normalized size = 1.48

$$(c - c \sin(e + fx))^5 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(2048(A - B) \sin\left(\frac{1}{2}(e + fx)\right) + 1260(4A - 7B)(e + fx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^2,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(2048*(A - B)*Sin[(e + f*x)/2] - 1024*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 1024*(13*A - 19*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 1260*(4*A - 7*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(95*A - 217*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 8*(A - 7*B)*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 24*(7*A - 24*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)] - 3*B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[4*(e + f*x)])/(96*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^10*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.46, size = 370, normalized size = 1.54

$$6Bc^5 \cos(fx + e)^6 + 4(2A - 11B)c^5 \cos(fx + e)^5 + (76A - 241B)c^5 \cos(fx + e)^4 - 2(212A - 431B)c^5 \cos(fx + e)^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/24*(6*B*c^5*cos(f*x + e)^6 + 4*(2*A - 11*B)*c^5*cos(f*x + e)^5 + (76*A - 241*B)*c^5*cos(f*x + e)^4 - 2*(212*A - 431*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x - 256*(A - B)*c^5 - (315*(4*A - 7*B)*c^5*f*x - (2156*A - 3485*B)*c^5)*cos(f*x + e)^2 + (315*(4*A - 7*B)*c^5*f*x + 2*(1196*A - 2141*B)*c^5)*cos(f*x + e) + (6*B*c^5*cos(f*x + e)^5 - 2*(4*A - 25*B)*c^5*cos(f*x + e)^4 + (68*A - 191*B)*c^5*cos(f*x + e)^3 + 630*(4*A - 7*B)*c^5*f*x + 3*(164*A - 351*B)*c^5*cos(f*x + e)^2 + 256*(A - B)*c^5 + (315*(4*A - 7*B)*c^5*f*x + 2*(1324*A - 2269*B)*c^5)*cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

giac [A] time = 0.26, size = 412, normalized size = 1.72

$$\frac{315(4Ac^5 - 7Bc^5)(fx+e)}{a^2} + \frac{256\left(9Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 15Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 24Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 36Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 11Ac^5 - 17Bc^5\right)}{a^2\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/24*(315*(4*A*c^5 - 7*B*c^5)*(f*x + e)/a^2 + 256*(9*A*c^5*tan(1/2*f*x + 1/2*e)^2 - 15*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 24*A*c^5*tan(1/2*f*x + 1/2*e) - 36*B*c^5*tan(1/2*f*x + 1/2*e) + 11*A*c^5 - 17*B*c^5)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3) + 2*(84*A*c^5*tan(1/2*f*x + 1/2*e)^7 - 285*B*c^5*tan(1/2*f*x + 1/2*e)^6 + 285*B*c^5*tan(1/2*f*x + 1/2*e)^5 - 285*B*c^5*tan(1/2*f*x + 1/2*e)^4 + 285*B*c^5*tan(1/2*f*x + 1/2*e)^3 - 285*B*c^5*tan(1/2*f*x + 1/2*e)^2 + 285*B*c^5*tan(1/2*f*x + 1/2*e) - 285*B*c^5)/a^2

$$\begin{aligned} & 1/2*e)^7 + 552*A*c^5*\tan(1/2*f*x + 1/2*e)^6 - 1176*B*c^5*\tan(1/2*f*x + 1/2 \\ & *e)^6 + 84*A*c^5*\tan(1/2*f*x + 1/2*e)^5 - 309*B*c^5*\tan(1/2*f*x + 1/2*e)^5 \\ & + 1704*A*c^5*\tan(1/2*f*x + 1/2*e)^4 - 3864*B*c^5*\tan(1/2*f*x + 1/2*e)^4 - 8 \\ & 4*A*c^5*\tan(1/2*f*x + 1/2*e)^3 + 309*B*c^5*\tan(1/2*f*x + 1/2*e)^3 + 1720*A* \\ & c^5*\tan(1/2*f*x + 1/2*e)^2 - 3976*B*c^5*\tan(1/2*f*x + 1/2*e)^2 - 84*A*c^5*t \\ & \tan(1/2*f*x + 1/2*e) + 285*B*c^5*\tan(1/2*f*x + 1/2*e) + 568*A*c^5 - 1288*B*c \\ & ^5)/((\tan(1/2*f*x + 1/2*e)^2 + 1)^4*a^2))/f \end{aligned}$$

maple [B] time = 0.52, size = 778, normalized size = 3.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x)

[Out] $7*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^7*A-95/4*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^7*B+46*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^6*A-98*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^6*B+7*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^5*A-103/4*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^5*B+142*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^4*A-322*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^4*B-7*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^3*A+103/4*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^3*B+430/3*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^2*A-994/3*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)^2*B-7*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)*A+95/4*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*\tan(1/2*f*x+1/2*e)*B+142/3*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*A-322/3*c^5/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2)^4*B+105*c^5/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*A-735/4*c^5/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*B+64*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A-64*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B+96*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*A-160*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B-128/3*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A+128/3*c^5/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B$

maxima [B] time = 0.76, size = 2982, normalized size = 12.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] $-1/12*(B*c^5*((603*\sin(f*x + e))/(\cos(f*x + e) + 1) + 1297*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2228*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 2628*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 3014*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2618*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 1980*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 1100*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 495*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 165*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 256)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 7*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 13*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 18*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22*a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 22*a^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 18*a^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 13*a^2*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 7*a^2*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 3*a^2*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + a^2*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11) + 165*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^2) - 20*A*c^5*((75*\sin(f*x + e))/(\cos(f*x + e) + 1) + 97*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 126*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 98*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 63*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 21*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 7*a^2*\sin$

$$\begin{aligned}
& *2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 + 312*a**2*f*\tan \\
& (e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a**2*f*\tan(e/2 + f* \\
& x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e/2 + f*x/2)**4 + \\
& 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/2)**2 + 72*a**2* \\
& f*\tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c**5*f*x*\tan(e/2 + f*x/2)**7/(24* \\
& a**2*f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan \\
& (e/2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + \\
& f*x/2)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 \\
& + 432*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a* \\
& **2*f*\tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 48510* \\
& B*c**5*f*x*\tan(e/2 + f*x/2)**6/(24*a**2*f*\tan(e/2 + f*x/2)**11 + 72*a**2*f* \\
& \tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 \\
& + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a**2*f*\tan(e/2 + f*x/2)* \\
& **6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e/2 + f*x/2)**4 + 312* \\
& a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan \\
& (e/2 + f*x/2) + 24*a**2*f) - 48510*B*c**5*f*x*\tan(e/2 + f*x/2)**5/(24*a**2* \\
& f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/ \\
& 2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2) \\
&)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 43 \\
& 2*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f* \\
& \tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 39690*B*c** \\
& 5*f*x*\tan(e/2 + f*x/2)**4/(24*a**2*f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e \\
& /2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x \\
& /2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + \\
& 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2* \\
& f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 \\
& + f*x/2) + 24*a**2*f) - 28665*B*c**5*f*x*\tan(e/2 + f*x/2)**3/(24*a**2*f*\tan \\
& (e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f \\
& *x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 \\
& + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a** \\
& 2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e \\
& /2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 15435*B*c**5*f*x \\
& *\tan(e/2 + f*x/2)**2/(24*a**2*f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + \\
& f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)** \\
& 8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528*a \\
& **2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan \\
& (e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f*x \\
& /2) + 24*a**2*f) - 6615*B*c**5*f*x*\tan(e/2 + f*x/2)/(24*a**2*f*\tan(e/2 + f* \\
& x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 \\
& + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a** \\
& 2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e \\
& /2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/ \\
& 2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 2205*B*c**5*f*x/(24*a**2* \\
& f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/ \\
& 2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2) \\
&)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 43 \\
& 2*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f* \\
& \tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 4410*B*c**5 \\
& *\tan(e/2 + f*x/2)**10/(24*a**2*f*\tan(e/2 + f*x/2)**11 + 72*a**2*f*\tan(e/2 + \\
& f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 + 312*a**2*f*\tan(e/2 + f*x/2)* \\
& **8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a**2*f*\tan(e/2 + f*x/2)**6 + 528* \\
& a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(e/2 + f*x/2)**4 + 312*a**2*f*\tan \\
& (e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x/2)**2 + 72*a**2*f*\tan(e/2 + f* \\
& x/2) + 24*a**2*f) - 13278*B*c**5*\tan(e/2 + f*x/2)**9/(24*a**2*f*\tan(e/2 + f \\
& *x/2)**11 + 72*a**2*f*\tan(e/2 + f*x/2)**10 + 168*a**2*f*\tan(e/2 + f*x/2)**9 \\
& + 312*a**2*f*\tan(e/2 + f*x/2)**8 + 432*a**2*f*\tan(e/2 + f*x/2)**7 + 528*a* \\
& **2*f*\tan(e/2 + f*x/2)**6 + 528*a**2*f*\tan(e/2 + f*x/2)**5 + 432*a**2*f*\tan(\\
& e/2 + f*x/2)**4 + 312*a**2*f*\tan(e/2 + f*x/2)**3 + 168*a**2*f*\tan(e/2 + f*x \\
& /2)**2 + 72*a**2*f*\tan(e/2 + f*x/2) + 24*a**2*f) - 29096*B*c**5*\tan(e/2 + f
\end{aligned}$$

$x/2)^{**8}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} +$
 $168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}$
 $*f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/$
 $2 + f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2$
 $)^{**3} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**$
 $2*f) - 54072*B*c^{**5}*f*\tan(e/2 + f*x/2)^{**7}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 7$
 $2*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f$
 $*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2$
 $+ f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*x/2)*$
 $*4 + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**3} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a$
 $**2*f*\tan(e/2 + f*x/2) + 24*a^{**2}*f) - 67220*B*c^{**5}*f*\tan(e/2 + f*x/2)^{**6}/(24*$
 $a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} + 168*a^{**2}*f*t$
 $an(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*f*\tan(e/2 +$
 $f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**5}$
 $+ 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**3} + 168*a*$
 $**2*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**2}*f) - 85196*$
 $B*c^{**5}*f*\tan(e/2 + f*x/2)^{**5}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan($
 $e/2 + f*x/2)^{**10} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*$
 $x/2)^{**8} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} +$
 $528*a^{**2}*f*\tan(e/2 + f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}$
 $*f*\tan(e/2 + f*x/2)^{**3} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2$
 $+ f*x/2) + 24*a^{**2}*f) - 70632*B*c^{**5}*f*\tan(e/2 + f*x/2)^{**4}/(24*a^{**2}*f*\tan(e/$
 $2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} + 168*a^{**2}*f*\tan(e/2 + f*x/$
 $2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**7} + 5$
 $28*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**5} + 432*a^{**2}*f$
 $*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**3} + 168*a^{**2}*f*\tan(e/2$
 $+ f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**2}*f) - 60968*B*c^{**5}*f*\tan(e/$
 $2 + f*x/2)^{**3}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)*$
 $*10 + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432$
 $*a^{**2}*f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*t$
 $an(e/2 + f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 +$
 $f*x/2)^{**3} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 2$
 $4*a^{**2}*f) - 35218*B*c^{**5}*f*\tan(e/2 + f*x/2)^{**2}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**1$
 $1 + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a$
 $**2*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan$
 $(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*$
 $x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**3} + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} +$
 $72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**2}*f) - 16374*B*c^{**5}*f*\tan(e/2 + f*x/2)/(2$
 $4*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} + 168*a^{**2}*f$
 $*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*f*\tan(e/2$
 $+ f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2 + f*x/2)*$
 $*5 + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**3} + 168*$
 $a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**2}*f) - 6928$
 $*B*c^{**5}/(24*a^{**2}*f*\tan(e/2 + f*x/2)^{**11} + 72*a^{**2}*f*\tan(e/2 + f*x/2)^{**10} +$
 $168*a^{**2}*f*\tan(e/2 + f*x/2)^{**9} + 312*a^{**2}*f*\tan(e/2 + f*x/2)^{**8} + 432*a^{**2}*$
 $f*\tan(e/2 + f*x/2)^{**7} + 528*a^{**2}*f*\tan(e/2 + f*x/2)^{**6} + 528*a^{**2}*f*\tan(e/2$
 $+ f*x/2)^{**5} + 432*a^{**2}*f*\tan(e/2 + f*x/2)^{**4} + 312*a^{**2}*f*\tan(e/2 + f*x/2)$
 $**3 + 168*a^{**2}*f*\tan(e/2 + f*x/2)^{**2} + 72*a^{**2}*f*\tan(e/2 + f*x/2) + 24*a^{**2}$
 $*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**2, Tru$
 $e))$

$$3.61 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=180

$$\frac{a^4 c^4 (A-B) \cos^9(e+fx)}{3f(a \sin(e+fx)+a)^6} + \frac{35c^4(A-2B) \cos^3(e+fx)}{3a^2 f} + \frac{2a^2 c^4(A-2B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^4} + \frac{35c^4(A-2B) \sin(e+fx)}{2a^2 f}$$

[Out] 35/2*(A-2*B)*c^4*x/a^2+35/3*(A-2*B)*c^4*cos(f*x+e)^3/a^2/f+35/2*(A-2*B)*c^4*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*a^4*(A-B)*c^4*cos(f*x+e)^9/f/(a+a*sin(f*x+e))^6+2*a^2*(A-2*B)*c^4*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^4+14*(A-2*B)*c^4*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^2

Rubi [A] time = 0.36, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$\frac{35c^4(A-2B) \cos^3(e+fx)}{3a^2 f} - \frac{a^4 c^4(A-B) \cos^9(e+fx)}{3f(a \sin(e+fx)+a)^6} + \frac{2a^2 c^4(A-2B) \cos^7(e+fx)}{f(a \sin(e+fx)+a)^4} + \frac{35c^4(A-2B) \sin(e+fx)}{2a^2 f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]

[Out] (35*(A - 2*B)*c^4*x)/(2*a^2) + (35*(A - 2*B)*c^4*Cos[e + f*x]^3)/(3*a^2*f) + (35*(A - 2*B)*c^4*Cos[e + f*x]*Sin[e + f*x])/(2*a^2*f) - (a^4*(A - B)*c^4*Cos[e + f*x]^9)/(3*f*(a + a*Sin[e + f*x])^6) + (2*a^2*(A - 2*B)*c^4*Cos[e + f*x]^7)/(f*(a + a*Sin[e + f*x])^4) + (14*(A - 2*B)*c^4*Cos[e + f*x]^5)/(f*(a + a*Sin[e + f*x])^2)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !LtQ[m+p+1, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*c

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^4}{(a + a \sin(e + fx))^2} dx &= (a^4 c^4) \int \frac{\cos^8(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^6} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^6} - (a^3(A - 2B)c^4) \int \frac{\cos^8(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^6} + \frac{2a^2(A - 2B)c^4 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} + (7a^4(A - 2B)c^4) \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^6} + \frac{2a^2(A - 2B)c^4 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} + \frac{14a^4(A - 2B)c^4}{f(a + a \sin(e + fx))^4} \int \frac{\cos^5(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= \frac{35(A - 2B)c^4 \cos^3(e + fx)}{3a^2 f} - \frac{a^4(A - B)c^4 \cos^9(e + fx)}{3f(a + a \sin(e + fx))^6} + \frac{2a^2(A - 2B)c^4 \cos^7(e + fx)}{f(a + a \sin(e + fx))^4} + \frac{14a^4(A - 2B)c^4}{f(a + a \sin(e + fx))^4} \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= \frac{35(A - 2B)c^4 \cos^3(e + fx)}{3a^2 f} + \frac{35(A - 2B)c^4 \cos(e + fx) \sin(e + fx)}{2a^2 f} + \frac{14a^4(A - 2B)c^4}{f(a + a \sin(e + fx))^4} \int \frac{\cos^3(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= \frac{35(A - 2B)c^4 x}{2a^2} + \frac{35(A - 2B)c^4 \cos^3(e + fx)}{3a^2 f} + \frac{35(A - 2B)c^4 \cos(e + fx) \sin(e + fx)}{2a^2 f} + \frac{14a^4(A - 2B)c^4}{f(a + a \sin(e + fx))^4} \int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^6} dx \end{aligned}$$

Mathematica [A] time = 1.32, size = 311, normalized size = 1.73

$$(c - c \sin(e + fx))^4 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(128(A - B) \sin\left(\frac{1}{2}(e + fx)\right) + 210(A - 2B)(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(128*(A - B)*Sin[(e + f*x)/2] - 64*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(5*A - 8*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 210*(A - 2*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*(24*A - 71*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + B*Cos[3*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*(A - 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)])/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.44, size = 322, normalized size = 1.79

$$2 B c^4 \cos(f x + e)^5 - (3 A - 16 B) c^4 \cos(f x + e)^4 + 2 (15 A - 38 B) c^4 \cos(f x + e)^3 - 210 (A - 2 B) c^4 f x + 32$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*B*c^4*\cos(f*x + e)^5 - (3*A - 16*B)*c^4*\cos(f*x + e)^4 + 2*(15*A - 38*B)*c^4*\cos(f*x + e)^3 - 210*(A - 2*B)*c^4*f*x + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x - (193*A - 346*B)*c^4)*\cos(f*x + e)^2 - (105*(A - 2*B)*c^4*f*x + 2*(97*A - 202*B)*c^4)*\cos(f*x + e) - (2*B*c^4*\cos(f*x + e)^4 + (3*A - 14*B)*c^4*\cos(f*x + e)^3 + 210*(A - 2*B)*c^4*f*x + 3*(11*A - 30*B)*c^4*\cos(f*x + e)^2 + 32*(A - B)*c^4 + (105*(A - 2*B)*c^4*f*x + 2*(113*A - 218*B)*c^4)*\cos(f*x + e)*\sin(f*x + e))/(a^2*f*\cos(f*x + e)^2 - a^2*f*\cos(f*x + e) - 2*a^2*f - (a^2*f*\cos(f*x + e) + 2*a^2*f)*\sin(f*x + e))$

giac [B] time = 0.22, size = 369, normalized size = 2.05

$$\frac{105(Ac^4 - 2Bc^4)(fx+e)}{a^2} + \frac{2\left(99Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 - 210Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^8 + 333Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 - 636Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^7 + 533Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 - 1160Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 1047Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 1980Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 921Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 1980Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 1107Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 2140Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 651Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 1344Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 393Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 780Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 164Ac^4 - 330Bc^4\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3 a^2} / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{6}*(105*(A*c^4 - 2*B*c^4)*(f*x + e)/a^2 + 2*(99*A*c^4*\tan(1/2*f*x + 1/2*e)^8 - 210*B*c^4*\tan(1/2*f*x + 1/2*e)^8 + 333*A*c^4*\tan(1/2*f*x + 1/2*e)^7 - 636*B*c^4*\tan(1/2*f*x + 1/2*e)^7 + 533*A*c^4*\tan(1/2*f*x + 1/2*e)^6 - 1160*B*c^4*\tan(1/2*f*x + 1/2*e)^6 + 1047*A*c^4*\tan(1/2*f*x + 1/2*e)^5 - 1980*B*c^4*\tan(1/2*f*x + 1/2*e)^5 + 921*A*c^4*\tan(1/2*f*x + 1/2*e)^4 - 1980*B*c^4*\tan(1/2*f*x + 1/2*e)^4 + 1107*A*c^4*\tan(1/2*f*x + 1/2*e)^3 - 2140*B*c^4*\tan(1/2*f*x + 1/2*e)^3 + 651*A*c^4*\tan(1/2*f*x + 1/2*e)^2 - 1344*B*c^4*\tan(1/2*f*x + 1/2*e)^2 + 393*A*c^4*\tan(1/2*f*x + 1/2*e) - 780*B*c^4*\tan(1/2*f*x + 1/2*e) + 164*A*c^4 - 330*B*c^4)/((\tan(1/2*f*x + 1/2*e)^3 + \tan(1/2*f*x + 1/2*e) + 1)^3*a^2))/f$

maple [B] time = 0.46, size = 549, normalized size = 3.05

$$\frac{c^4 \left(\tan^5 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{6c^4 \left(\tan^5 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{12c^4 \left(\tan^4 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{34c^4 \left(\tan^4 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{2c^4 \left(\tan^3 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{10c^4 \left(\tan^3 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{12c^4 \left(\tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{36c^4 \left(\tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{12c^4 \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{36c^4 \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^2 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{2c^4 A}{a^2 f} - \frac{10c^4 B}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x)

[Out] $c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^5*A-6*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^5*B+12*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^4*A-34*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^4*B+24*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^2*A-72*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)^2*B-c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)*A+6*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)*B$

$$\begin{aligned} & (1/2*f*x+1/2*e)^2)^3*\tan(1/2*f*x+1/2*e)*B+12*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e) \\ &)^2)^3*A-106/3*c^4/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^3*B+35*c^4/a^2/f*\arctan(\tan(1/2*f*x+1/2*e)) \\ &)*A-70*c^4/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*B+32*c^4/a^2/f/ \\ & /(\tan(1/2*f*x+1/2*e)+1)^2*A-32*c^4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B+32*c^4/a^2/f/ \\ & /(\tan(1/2*f*x+1/2*e)+1)*A-64*c^4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B-64/3*c^4/a^2/f/ \\ & /(\tan(1/2*f*x+1/2*e)+1)^3*A+64/3*c^4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3 \\ & *B \end{aligned}$$

maxima [B] time = 0.52, size = 2094, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/3*(A*c^4*((75*\sin(f*x + e)/(\cos(f*x + e) + 1) + 97*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 126*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 98*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 63*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 21*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 7*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 7*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5*a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 3*a^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 21*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) - 4*B*c^4*((75*\sin(f*x + e)/(\cos(f*x + e) + 1) + 97*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 126*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 98*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 63*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 21*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 7*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 7*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5*a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 3*a^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 21*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) - 2*B*c^4*((57*\sin(f*x + e)/(\cos(f*x + e) + 1) + 99*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 155*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 153*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 135*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 85*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 45*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 15*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 24)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 6*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 12*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 12*a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 10*a^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 6*a^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 3*a^2*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + a^2*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9) + 15*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) + 16*A*c^4*((12*\sin(f*x + e)/(\cos(f*x + e) + 1) + 11*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 4*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) - 24*B*c^4*((12*\sin(f*x + e)/(\cos(f*x + e) + 1) + 11*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 4*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) + 12*A*c^4*((9*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) - 8*B*c^4*((9*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1)))/a^2) \end{aligned}$$

$$\frac{e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^2) - 2*A*c^4*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 8*A*c^4*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) - 2*B*c^4*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3))/f$$

mapad [B] time = 14.95, size = 414, normalized size = 2.30

$$\frac{\tan\left(\frac{e}{2} + \frac{f*x}{2}\right) \left(131 A c^4 - 260 B c^4\right) + \frac{164 A c^4}{3} - 110 B c^4 + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 \left(33 A c^4 - 70 B c^4\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^7 \left(111 A c^4 - 212 B c^4\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 \left(217 A c^4 - 448 B c^4\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 \left(349 A c^4 - 660 B c^4\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 \left(307 A c^4 - 660 B c^4\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 \left(369 A c^4 - \frac{2140 B c^4}{3}\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 \left(\frac{533 A c^4}{3} - \frac{1160 B c^4}{3}\right) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) \left(\frac{35 c^4 \operatorname{atan}\left(\frac{35 c^4 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) (A - 2 B)}{35 A c^4 - 70 B c^4}\right) (A - 2 B)}{a^2 f}\right) + \frac{1260 A c^4}{a^2}}{f \left(a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^9 + 3 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^8 + 6 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^7 + 10 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 + 12 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 + 10 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 + 12 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 + 10 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 + 6 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^2,x)

[Out] (tan(e/2 + (f*x)/2)*(131*A*c^4 - 260*B*c^4) + (164*A*c^4)/3 - 110*B*c^4 + tan(e/2 + (f*x)/2)^8*(33*A*c^4 - 70*B*c^4) + tan(e/2 + (f*x)/2)^7*(111*A*c^4 - 212*B*c^4) + tan(e/2 + (f*x)/2)^6*(217*A*c^4 - 448*B*c^4) + tan(e/2 + (f*x)/2)^5*(349*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^4*(307*A*c^4 - 660*B*c^4) + tan(e/2 + (f*x)/2)^3*(369*A*c^4 - (2140*B*c^4)/3) + tan(e/2 + (f*x)/2)^2*(533*A*c^4/3 - (1160*B*c^4)/3) + tan(e/2 + (f*x)/2)*(35*c^4*atan((35*c^4*tan(e/2 + (f*x)/2)*(A - 2*B))/(35*A*c^4 - 70*B*c^4))*(A - 2*B))/(a^2*f)

sympy [A] time = 44.15, size = 7337, normalized size = 40.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**4/(a+a*sin(f*x+e))**2,x)

[Out] Piecewise(((105*A*c**4*f*x*tan(e/2 + f*x/2))**9/(6*a**2*f*tan(e/2 + f*x/2))**9 + 18*a**2*f*tan(e/2 + f*x/2))**8 + 36*a**2*f*tan(e/2 + f*x/2))**7 + 60*a**2*f*tan(e/2 + f*x/2))**6 + 72*a**2*f*tan(e/2 + f*x/2))**5 + 72*a**2*f*tan(e/2 + f*x/2))**4 + 60*a**2*f*tan(e/2 + f*x/2))**3 + 36*a**2*f*tan(e/2 + f*x/2))**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 315*A*c**4*f*x*tan(e/2 + f*x/2))**8/(6*a**2*f*tan(e/2 + f*x/2))**9 + 18*a**2*f*tan(e/2 + f*x/2))**8 + 36*a**2*f*tan(e/2 + f*x/2))**7 + 60*a**2*f*tan(e/2 + f*x/2))**6 + 72*a**2*f*tan(e/2 + f*x/2))**5 + 72*a**2*f*tan(e/2 + f*x/2))**4 + 60*a**2*f*tan(e/2 + f*x/2))**3 + 36*a**2*f*tan(e/2 + f*x/2))**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 630*A*c**4*f*x*tan(e/2 + f*x/2))**7/(6*a**2*f*tan(e/2 + f*x/2))**9 + 18*a**2*f*tan(e/2 + f*x/2))**8 + 36*a**2*f*tan(e/2 + f*x/2))**7 + 60*a**2*f*tan(e/2 + f*x/2))**6 + 72*a**2*f*tan(e/2 + f*x/2))**5 + 72*a**2*f*tan(e/2 + f*x/2))**4 + 60*a**2*f*tan(e/2 + f*x/2))**3 + 36*a**2*f*tan(e/2 + f*x/2))**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1050*A*c**4*f*x*tan(e/2 + f*x/2))**6/(6*a**2*f*tan(e/2 + f*x/2))**9 + 18*a**2*f*tan(e/2 + f*x/2))**8 + 36*a**2*f*tan(e/2 + f*x/2))**7 + 60*a**2*f*tan(e/2 + f*x/2))**6 + 72*a**2*f*tan(e/2 + f*x/2))**5 + 72*a**2*f*tan(e/2 + f*x/2))**4 + 60*a**2*f*tan(e/2 + f*x/2))**3 + 36*a**2*f*tan(e/2 + f*x/2))**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 1260*A*c**4


```

*f) - 2320*B*c**4*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
*2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4*tan(e/2 + f*x/2)**5/(6*a**2*
f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 +
f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5
+ 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f
*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 3960*B*c**4
*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*
x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 7
2*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*ta
n(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 4280*B*c**4*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2
)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a
**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e
/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)
**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 2688*B*c**4*tan(e/2 + f*x/2)
**2/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan(e/2 + f*x/2)**8 + 36*a**2
*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/2)**6 + 72*a**2*f*tan(e/2
+ f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*a**2*f*tan(e/2 + f*x/2)**3
+ 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
1560*B*c**4*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a**2*f*tan
(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/2 + f*x/
2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)**4 + 60*
a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(
e/2 + f*x/2) + 6*a**2*f) - 660*B*c**4/(6*a**2*f*tan(e/2 + f*x/2)**9 + 18*a*
**2*f*tan(e/2 + f*x/2)**8 + 36*a**2*f*tan(e/2 + f*x/2)**7 + 60*a**2*f*tan(e/
2 + f*x/2)**6 + 72*a**2*f*tan(e/2 + f*x/2)**5 + 72*a**2*f*tan(e/2 + f*x/2)*
*4 + 60*a**2*f*tan(e/2 + f*x/2)**3 + 36*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**
2*f*tan(e/2 + f*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) +
c)**4/(a*sin(e) + a)**2, True))

```

$$3.62 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=162

$$\frac{a^3 c^3 (A-B) \cos^7(e+fx)}{3f(a \sin(e+fx)+a)^5} + \frac{5c^3(2A-5B) \cos(e+fx)}{2a^2 f} + \frac{5c^3(2A-5B) \cos^3(e+fx)}{6f(a^2 \sin(e+fx)+a^2)} + \frac{5c^3 x(2A-5B)}{2a^2} + \frac{2ac^3(2A-5B)}{3f(a \sin(e+fx)+a)}$$

[Out] 5/2*(2*A-5*B)*c^3*x/a^2+5/2*(2*A-5*B)*c^3*cos(f*x+e)/a^2/f-1/3*a^3*(A-B)*c^3*cos(f*x+e)^7/f/(a+a*sin(f*x+e))^5+2/3*a*(2*A-5*B)*c^3*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^3+5/6*(2*A-5*B)*c^3*cos(f*x+e)^3/f/(a^2+a^2*sin(f*x+e))

Rubi [A] time = 0.33, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$\frac{5c^3(2A-5B) \cos(e+fx)}{2a^2 f} - \frac{a^3 c^3 (A-B) \cos^7(e+fx)}{3f(a \sin(e+fx)+a)^5} + \frac{5c^3(2A-5B) \cos^3(e+fx)}{6f(a^2 \sin(e+fx)+a^2)} + \frac{5c^3 x(2A-5B)}{2a^2} + \frac{2ac^3(2A-5B)}{3f(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]

[Out] (5*(2*A - 5*B)*c^3*x)/(2*a^2) + (5*(2*A - 5*B)*c^3*Cos[e + f*x])/(2*a^2*f) - (a^3*(A - B)*c^3*Cos[e + f*x]^7)/(3*f*(a + a*Sin[e + f*x])^5) + (2*a*(2*A - 5*B)*c^3*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^3) + (5*(2*A - 5*B)*c^3*Cos[e + f*x]^3)/(6*f*(a^2 + a^2*Sin[e + f*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p_*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_, x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegersQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p_*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_, x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !LtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p_/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^3}{(a + a \sin(e + fx))^2} dx = (a^3 c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^5} dx$$

$$= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{3f(a + a \sin(e + fx))^5} - \frac{1}{3} (a^2(2A - 5B)c^3) \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^3} dx$$

$$= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{2a(2A - 5B)c^3 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{1}{3} \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^2} dx$$

$$= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{2a(2A - 5B)c^3 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{5(2A - 5B)c^3 \cos^3(e + fx)}{6f(a + a \sin(e + fx))} + \frac{5(2A - 5B)c^3 \cos(e + fx)}{2a^2 f}$$

$$= \frac{5(2A - 5B)c^3 \cos(e + fx)}{2a^2 f} - \frac{a^3(A - B)c^3 \cos^7(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{2a(2A - 5B)c^3 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{5(2A - 5B)c^3 \cos^3(e + fx)}{6f(a + a \sin(e + fx))}$$

$$= \frac{5(2A - 5B)c^3 x}{2a^2} + \frac{5(2A - 5B)c^3 \cos(e + fx)}{2a^2 f} - \frac{a^3(A - B)c^3 \cos^7(e + fx)}{3f(a + a \sin(e + fx))^5} + \frac{2a(2A - 5B)c^3 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{5(2A - 5B)c^3 \cos^3(e + fx)}{6f(a + a \sin(e + fx))}$$

Mathematica [A] time = 0.88, size = 274, normalized size = 1.69

$$(c - c \sin(e + fx))^3 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(64(A - B) \sin\left(\frac{1}{2}(e + fx)\right) + 30(2A - 5B)(e + fx) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2, x]
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^3*(64*(A - B)*Sin[(e + f*x)/2] - 32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 32*(7*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 30*(2*A - 5*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 12*(A - 5*B)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 3*B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sin[2*(e + f*x)]))/(12*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x])^2)
```

fricas [A] time = 0.45, size = 291, normalized size = 1.80

$$3 B c^3 \cos(fx + e)^4 + 6(A - 4B)c^3 \cos(fx + e)^3 - 30(2A - 5B)c^3 fx + 16(A - B)c^3 + (15(2A - 5B)c^3 fx -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] $\frac{1}{6} * (3 * B * c^3 * \cos(f * x + e)^4 + 6 * (A - 4 * B) * c^3 * \cos(f * x + e)^3 - 30 * (2 * A - 5 * B) * c^3 * f * x + 16 * (A - B) * c^3 + (15 * (2 * A - 5 * B) * c^3 * f * x - (62 * A - 131 * B) * c^3) * \cos(f * x + e)^2 - (15 * (2 * A - 5 * B) * c^3 * f * x + 2 * (26 * A - 71 * B) * c^3) * \cos(f * x + e) + (3 * B * c^3 * \cos(f * x + e)^3 - 30 * (2 * A - 5 * B) * c^3 * f * x - 3 * (2 * A - 9 * B) * c^3 * \cos(f * x + e)^2 - 16 * (A - B) * c^3 - (15 * (2 * A - 5 * B) * c^3 * f * x + 2 * (34 * A - 79 * B) * c^3) * \cos(f * x + e)) * \sin(f * x + e)) / (a^2 * f * \cos(f * x + e)^2 - a^2 * f * \cos(f * x + e) - 2 * a^2 * f - (a^2 * f * \cos(f * x + e) + 2 * a^2 * f) * \sin(f * x + e))$

giac [A] time = 0.20, size = 233, normalized size = 1.44

$$\frac{15(2Ac^3 - 5Bc^3)(fx+e)}{a^2} - \frac{6\left(Bc^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 2Ac^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 10Bc^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - Bc^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 2Ac^3 + 10Bc^3\right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2 a^2} + \frac{16(3A - 5B)c^3}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{6} * (15 * (2 * A * c^3 - 5 * B * c^3) * (f * x + e) / a^2 - 6 * (B * c^3 * \tan(1/2 * f * x + 1/2 * e)^3 - 2 * A * c^3 * \tan(1/2 * f * x + 1/2 * e)^2 + 10 * B * c^3 * \tan(1/2 * f * x + 1/2 * e)^2 - B * c^3 * \tan(1/2 * f * x + 1/2 * e) - 2 * A * c^3 + 10 * B * c^3) / ((\tan(1/2 * f * x + 1/2 * e)^2 + 1)^2 * a^2) + 16 * (3 * A * c^3 * \tan(1/2 * f * x + 1/2 * e)^2 - 9 * B * c^3 * \tan(1/2 * f * x + 1/2 * e)^2 + 12 * A * c^3 * \tan(1/2 * f * x + 1/2 * e) - 24 * B * c^3 * \tan(1/2 * f * x + 1/2 * e) + 5 * A * c^3 - 11 * B * c^3) / (a^2 * (\tan(1/2 * f * x + 1/2 * e) + 1)^3) / f$

maple [B] time = 0.45, size = 399, normalized size = 2.46

$$\frac{c^3 \left(\tan^3\left(\frac{fx}{2} + \frac{e}{2}\right) \right) B}{a^2 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} + \frac{2c^3 \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) A}{a^2 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} - \frac{10c^3 \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) B}{a^2 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} + \frac{c^3 B \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{a^2 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)

[Out] $-c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^3*B+2*c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^2*A-10*c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*\tan(1/2*f*x+1/2*e)^2*B+c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*B*\tan(1/2*f*x+1/2*e)+2*c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*A-10*c^3/a^2/f/(1+\tan(1/2*f*x+1/2*e)^2)^2*B-25*c^3/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*B+10*c^3/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*A+16*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A-16*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B+8*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*A-24*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B-32/3*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A+32/3*c^3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B$

maxima [B] time = 0.48, size = 1378, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out]
$$-1/3*(B*c^3*((75*\sin(f*x + e))/(\cos(f*x + e) + 1) + 97*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 126*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 98*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 63*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 21*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 32)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 5*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 7*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 7*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5*a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 3*a^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 21*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^2 - 4*A*c^3*((12*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 4*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^2 + 12*B*c^3*((12*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 4*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4*a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*a^2*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^2 - 6*A*c^3*((9*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 3*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^2 + 6*B*c^3*((9*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 3*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^2 + 2*A*c^3*(3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) - 6*A*c^3*(3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + 2*B*c^3*(3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*\sin(f*x + e))/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3))/f$$

mupad [B] time = 14.34, size = 336, normalized size = 2.07

$$\frac{\tan\left(\frac{e}{2} + \frac{f*x}{2}\right) (38 A c^3 - 93 B c^3) + \frac{46 A c^3}{3} - \frac{118 B c^3}{3} + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 (8 A c^3 - 25 B c^3) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 (34 A c^3 - 77 B c^3) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 (106 A c^3 - 328 B c^3) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 (128 A c^3 - 359 B c^3) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 (128 A c^3 - 359 B c^3) + \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) (128 A c^3 - 359 B c^3) + 1}{f \left(a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^7 + 3 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^6 + 5 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^5 + 7 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^4 + 7 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^3 + 5 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right)^2 + 3 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) + a^2 + 3 a^2 \tan\left(\frac{e}{2} + \frac{f*x}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)

[Out]
$$(\tan(e/2 + (f*x)/2)*(38*A*c^3 - 93*B*c^3) + (46*A*c^3)/3 - (118*B*c^3)/3 + \tan(e/2 + (f*x)/2)^6*(8*A*c^3 - 25*B*c^3) + \tan(e/2 + (f*x)/2)^5*(34*A*c^3 - 77*B*c^3) + \tan(e/2 + (f*x)/2)^4*((106*A*c^3)/3 - (328*B*c^3)/3) + \tan(e/2 + (f*x)/2)^3*((128*A*c^3)/3 - (359*B*c^3)/3))/((f*(5*a^2*\tan(e/2 + (f*x)/2)^2 + 7*a^2*\tan(e/2 + (f*x)/2)^3 + 7*a^2*\tan(e/2 + (f*x)/2)^4 + 5*a^2*\tan(e/2 + (f*x)/2)^5 + 3*a^2*\tan(e/2 + (f*x)/2)^6 + a^2*\tan(e/2 + (f*x)/2)^7 + a^2 + 3*a^2*\tan(e/2 + (f*x)/2))$$

)) + (5*c^3*atan((5*c^3*tan(e/2 + (f*x)/2)*(2*A - 5*B))/(10*A*c^3 - 25*B*c^3))*(2*A - 5*B))/(a^2*f)

sympy [A] time = 25.78, size = 4665, normalized size = 28.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)

[Out] Piecewise((30*A*c**3*f*x*tan(e/2 + f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 210*A*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 150*A*c**3*f*x*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 90*A*c**3*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 30*A*c**3*f*x/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 48*A*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 204*A*c**3*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 212*A*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 432*A*c**3*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 256*A*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 228*A*c**3*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) + 92*A*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 +

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18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*t
an(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*
x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x*tan(e/2 +
f*x/2)**7/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 3
0*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*ta
n(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x
/2) + 6*a**2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*
x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 4
2*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*ta
n(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x
*tan(e/2 + f*x/2)**5/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*
x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 4
2*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*ta
n(e/2 + f*x/2) + 6*a**2*f) - 525*B*c**3*f*x*tan(e/2 + f*x/2)**4/(6*a**2*f*ta
n(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*
x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 3
0*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 525
*B*c**3*f*x*tan(e/2 + f*x/2)**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*ta
n(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 375*B*c**3*f*x*tan(e/2 + f*x/2)**2/
(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*ta
n(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*
x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a*
**2*f) - 225*B*c**3*f*x*tan(e/2 + f*x/2)/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*
a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(
e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2
)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 75*B*c**3*f*x/(6*a**2*f*ta
n(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/
2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*
a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 150*B
*c**3*tan(e/2 + f*x/2)**6/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2
+ f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**
4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2
*f*tan(e/2 + f*x/2) + 6*a**2*f) - 462*B*c**3*tan(e/2 + f*x/2)**5/(6*a**2*f*
tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f
*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 +
30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 65
6*B*c**3*tan(e/2 + f*x/2)**4/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(
e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2
)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a
**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 996*B*c**3*tan(e/2 + f*x/2)**3/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
+ 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
718*B*c**3*tan(e/2 + f*x/2)**2/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*ta
n(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*
x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 1
8*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) - 558*B*c**3*tan(e/2 + f*x/2)/(6*a**2
*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 + 30*a**2*f*tan(e/2
+ f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*tan(e/2 + f*x/2)**3
+ 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f*x/2) + 6*a**2*f) -
236*B*c**3/(6*a**2*f*tan(e/2 + f*x/2)**7 + 18*a**2*f*tan(e/2 + f*x/2)**6 +
30*a**2*f*tan(e/2 + f*x/2)**5 + 42*a**2*f*tan(e/2 + f*x/2)**4 + 42*a**2*f*
tan(e/2 + f*x/2)**3 + 30*a**2*f*tan(e/2 + f*x/2)**2 + 18*a**2*f*tan(e/2 + f
*x/2) + 6*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e
) + a)**2, True))

```


$$3.63 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=108

$$\frac{c^2(A-4B) \cos(e+fx)}{a^2 f} - \frac{a^2 c^2(A-B) \cos^5(e+fx)}{3f(a \sin(e+fx)+a)^4} + \frac{c^2 x(A-4B)}{a^2} + \frac{2c^2(A-4B) \cos^3(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

[Out] (A-4*B)*c^2*x/a^2+(A-4*B)*c^2*cos(f*x+e)/a^2/f-1/3*a^2*(A-B)*c^2*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^4+2/3*(A-4*B)*c^2*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^2

Rubi [A] time = 0.28, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{c^2(A-4B) \cos(e+fx)}{a^2 f} - \frac{a^2 c^2(A-B) \cos^5(e+fx)}{3f(a \sin(e+fx)+a)^4} + \frac{c^2 x(A-4B)}{a^2} + \frac{2c^2(A-4B) \cos^3(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]

[Out] ((A - 4*B)*c^2*x)/a^2 + ((A - 4*B)*c^2*Cos[e + f*x])/(a^2*f) - (a^2*(A - B)*c^2*Cos[e + f*x]^5)/(3*f*(a + a*Sin[e + f*x])^4) + (2*(A - 4*B)*c^2*Cos[e + f*x]^3)/(3*f*(a + a*Sin[e + f*x])^2)

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegerQ[2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[

$e + f*x$), x], x] /; FreeQ[{ $a, b, c, d, e, f, A, B, n$ }, x] && EqQ[$b*c + a*d$, 0] && EqQ[$a^2 - b^2$, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[$m, 0$] & GtQ[$n, 0$]) || LtQ[0, n, m] || LtQ[$m, n, 0$]))

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^2}{(a + a \sin(e + fx))^2} dx = (a^2 c^2) \int \frac{\cos^4(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^4} dx$$

$$= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^4} - \frac{1}{3} (a(A - 4B)c^2) \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^2} dx$$

$$= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^4} + \frac{2(A - 4B)c^2 \cos^3(e + fx)}{3f(a + a \sin(e + fx))^2} + \frac{(A - 4B)c^2 \cos(e + fx)}{a^2 f}$$

$$= \frac{(A - 4B)c^2 \cos(e + fx)}{a^2 f} - \frac{a^2(A - B)c^2 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^4} + \frac{2(A - 4B)c^2 \cos^3(e + fx)}{3f(a + a \sin(e + fx))^2}$$

$$= \frac{(A - 4B)c^2 x}{a^2} + \frac{(A - 4B)c^2 \cos(e + fx)}{a^2 f} - \frac{a^2(A - B)c^2 \cos^5(e + fx)}{3f(a + a \sin(e + fx))^4}$$

Mathematica [B] time = 0.60, size = 234, normalized size = 2.17

$$\frac{(c - c \sin(e + fx))^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(8(A - B) \sin\left(\frac{1}{2}(e + fx)\right) + 3(A - 4B)(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)}{3(a^2 f \cos^4\left(\frac{1}{2}(e + fx)\right) + 2(A - 4B)c^2 \cos^2\left(\frac{1}{2}(e + fx)\right) + (A - 4B)c^2)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] - 4*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(2*A - 5*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*(A - 4*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)*(c - c*Sin[e + f*x])^2/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x])^2)

fricas [B] time = 0.45, size = 242, normalized size = 2.24

$$\frac{3 B c^2 \cos^3(fx + e) + 6(A - 4B)c^2 fx - 4(A - B)c^2 - (3(A - 4B)c^2 fx - (8A - 23B)c^2) \cos(fx + e)^2 + (3(A - 4B)c^2 \cos^2(fx + e) - 2(A - 4B)c^2) \sin(fx + e)}{3(a^2 f \cos^4\left(\frac{1}{2}(e + fx)\right) + 2(A - 4B)c^2 \cos^2\left(\frac{1}{2}(e + fx)\right) + (A - 4B)c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(3*B*c^2*cos(f*x + e)^3 + 6*(A - 4*B)*c^2*f*x - 4*(A - B)*c^2 - (3*(A - 4*B)*c^2*f*x - (8*A - 23*B)*c^2)*cos(f*x + e)^2 + (3*(A - 4*B)*c^2*f*x + 2*(2*A - 11*B)*c^2)*cos(f*x + e) + (6*(A - 4*B)*c^2*f*x - 3*B*c^2*cos(f*x + e)^2 + 4*(A - B)*c^2 + (3*(A - 4*B)*c^2*f*x + 2*(4*A - 13*B)*c^2)*cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f*cos(f*x + e) + 2*a^2*f*sin(f*x + e))

giac [A] time = 0.19, size = 136, normalized size = 1.26

$$\frac{3(Ac^2 - 4Bc^2)(fx+e)}{a^2} - \frac{6Bc^2}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)a^2} - \frac{8\left(3Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 3Ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 9Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - Ac^2 + 4Bc^2\right)}{a^2\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}$$

$$3f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] 1/3*(3*(A*c^2 - 4*B*c^2)*(f*x + e)/a^2 - 6*B*c^2/((tan(1/2*f*x + 1/2*e)^2 + 1)*a^2) - 8*(3*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 3*A*c^2*tan(1/2*f*x + 1/2*e) + 9*B*c^2*tan(1/2*f*x + 1/2*e) - A*c^2 + 4*B*c^2)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3))/f

maple [A] time = 0.43, size = 198, normalized size = 1.83

$$-\frac{2c^2B}{a^2f\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)} + \frac{2c^2 \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)A}{a^2f} - \frac{8c^2 \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)B}{a^2f} + \frac{8c^2A}{a^2f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)

[Out] -2*c^2/a^2/f*B/(1+tan(1/2*f*x+1/2*e)^2)+2*c^2/a^2/f*arctan(tan(1/2*f*x+1/2*e))*A-8*c^2/a^2/f*arctan(tan(1/2*f*x+1/2*e))*B+8*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A-8*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B-16/3*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A+16/3*c^2/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B-8*c^2/a^2/f*B/(tan(1/2*f*x+1/2*e)+1)

maxima [B] time = 0.46, size = 833, normalized size = 7.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -2/3*(2*B*c^2*((12*sin(f*x + e)/(cos(f*x + e) + 1) + 11*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 9*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 4*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*a^2*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^2*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)))/a^2) - A*c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)))/a^2) + 2*B*c^2*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1)))/a^2) + A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - 2*A*c^2*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a


```

12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*t
an(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x
/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2
*f) - 48*B*c**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a
**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e
/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*c**2*f*x*tan(
e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*t
an(e/2 + f*x/2) + 3*a**2*f) - 12*B*c**2*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*t
an(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c**2*tan(
e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**
4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*
f*tan(e/2 + f*x/2) + 3*a**2*f) - 78*B*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*ta
n(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/
2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*
f) - 74*B*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f
*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 +
f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 90*B*c**2*tan(e/2 + f*x
/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*
f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 +
f*x/2) + 3*a**2*f) - 38*B*c**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan
(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/
2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))
*(-c*sin(e) + c)**2/(a*sin(e) + a)**2, True))

```

$$3.64 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=72

$$\frac{c(A-7B) \cos(e+fx)}{3a^2 f(\sin(e+fx)+1)} - \frac{Bcx}{a^2} - \frac{2c(A-B) \cos(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

[Out] $-B*c*x/a^2+1/3*(A-7*B)*c*\cos(f*x+e)/a^2/f/(1+\sin(f*x+e))-2/3*(A-B)*c*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^2$

Rubi [A] time = 0.21, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2967, 2857, 2735, 2648}

$$\frac{c(A-7B) \cos(e+fx)}{3a^2 f(\sin(e+fx)+1)} - \frac{Bcx}{a^2} - \frac{2c(A-B) \cos(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]

[Out] $-((B*c*x)/a^2) + ((A - 7*B)*c*\text{Cos}[e + f*x])/(3*a^2*f*(1 + \text{Sin}[e + f*x])) - (2*(A - B)*c*\text{Cos}[e + f*x])/(3*f*(a + a*\text{Sin}[e + f*x])^2)$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))}{(a + a \sin(e + fx))^2} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^3} dx \\
&= -\frac{2(A - B)c \cos(e + fx)}{3f(a + a \sin(e + fx))^2} - \frac{c \int \frac{aA - 4aB + 3aB \sin(e + fx)}{a + a \sin(e + fx)} dx}{3a^2} \\
&= -\frac{Bcx}{a^2} - \frac{2(A - B)c \cos(e + fx)}{3f(a + a \sin(e + fx))^2} - \frac{((A - 7B)c) \int \frac{1}{a + a \sin(e + fx)} dx}{3a} \\
&= -\frac{Bcx}{a^2} - \frac{2(A - B)c \cos(e + fx)}{3f(a + a \sin(e + fx))^2} + \frac{(A - 7B)c \cos(e + fx)}{3f(a^2 + a^2 \sin(e + fx))}
\end{aligned}$$

Mathematica [B] time = 0.58, size = 156, normalized size = 2.17

$$\frac{c \left(-6(A - 3B) \cos\left(e + \frac{fx}{2}\right) + 2A \cos\left(e + \frac{3fx}{2}\right) - 9Bfx \sin\left(e + \frac{fx}{2}\right) - 3Bfx \sin\left(e + \frac{3fx}{2}\right) - 14B \cos\left(e + \frac{3fx}{2}\right) \right)}{6a^2 f \left(\sin\left(\frac{e}{2}\right) + \cos\left(\frac{e}{2}\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]

[Out] (c*(-9*B*f*x*Cos[(f*x)/2] - 6*(A - 3*B)*Cos[e + (f*x)/2] + 2*A*Cos[e + (3*f*x)/2] - 14*B*Cos[e + (3*f*x)/2] + 3*B*f*x*Cos[2*e + (3*f*x)/2] + 24*B*Sin[(f*x)/2] - 9*B*f*x*Sin[e + (f*x)/2] - 3*B*f*x*Sin[e + (3*f*x)/2]))/(6*a^2*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

fricas [B] time = 0.44, size = 166, normalized size = 2.31

$$\frac{6Bcfx - (3Bcfx + (A - 7B)c) \cos^2(fx + e) + 2(A - B)c + (3Bcfx + (A + 5B)c) \cos(fx + e) + (6Bcfx - (3Bcfx + (A + 5B)c) \cos(fx + e)) \sin(fx + e)}{3(a^2 f \cos^2(fx + e) - a^2 f \cos(fx + e) - 2a^2 f - (a^2 f \cos(fx + e) + 2a^2 f \sin(fx + e)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*(6*B*c*f*x - (3*B*c*f*x + (A - 7*B)*c)*cos(f*x + e)^2 + 2*(A - B)*c + (3*B*c*f*x + (A + 5*B)*c)*cos(f*x + e) + (6*B*c*f*x - 2*(A - B)*c + (3*B*c*f*x - (A - 7*B)*c)*cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f*sin(f*x + e)))

giac [A] time = 0.16, size = 92, normalized size = 1.28

$$\frac{\frac{3(fx+e)Bc}{a^2} + \frac{2\left(3Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 12Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + Ac + 5Bc\right)}{a^2\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] -1/3*(3*(f*x + e)*B*c/a^2 + 2*(3*A*c*tan(1/2*f*x + 1/2*e)^2 + 3*B*c*tan(1/2*f*x + 1/2*e)^2 + 12*B*c*tan(1/2*f*x + 1/2*e) + A*c + 5*B*c)/(a^2*(tan(1/2*f*x + 1/2*e) + 1)^3)/f

maple [B] time = 0.42, size = 160, normalized size = 2.22

$$\frac{2cB \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{a^2 f} + \frac{4cA}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} - \frac{4cB}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} - \frac{2cA}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} - \frac{2cA}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)

[Out] -2*c/a^2/f*B*arctan(tan(1/2*f*x+1/2*e))+4*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*A-4*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^2*B-2*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)*A-2*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)*B-8/3*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*A+8/3*c/a^2/f/(tan(1/2*f*x+1/2*e)+1)^3*B

maxima [B] time = 0.46, size = 452, normalized size = 6.28

$$\frac{2 \left(Bc \left(\frac{9 \sin(fx+e) + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 4}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2} \right) + \frac{Ac \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2 \right)}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} - \frac{3c}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -2/3*(B*c*((9*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + 3*arctan(sin(f*x + e)/(cos(f*x + e) + 1))/a^2) + A*c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) - A*c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*c*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f

mupad [B] time = 12.83, size = 133, normalized size = 1.85

$$\frac{Bcx \left(\frac{c(6A+6B+9B(e+fx))}{3} - 3Bc(e+fx) \right) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + \left(\frac{c(24B+9B(e+fx))}{3} - 3Bc(e+fx) \right) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{a^2} - \frac{2cA}{a^2 f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)

[Out] - (B*c*x)/a^2 - (tan(e/2 + (f*x)/2)^2*((c*(6*A + 6*B + 9*B*(e + f*x)))/3 - 3*B*c*(e + f*x)) + tan(e/2 + (f*x)/2)*((c*(24*B + 9*B*(e + f*x)))/3 - 3*B*c*(e + f*x)) + (c*(2*A + 10*B + 3*B*(e + f*x)))/3 - B*c*(e + f*x)/(a^2*f*(tan(e/2 + (f*x)/2) + 1)^3)

sympy [A] time = 7.39, size = 702, normalized size = 9.75

$$\left\{ \begin{array}{l} \frac{6Ac \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 3a^2f} - \frac{2Ac}{3a^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 3a^2f} - \frac{1}{3a^2f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 3a^2f} \\ \frac{x(A+B \sin(e))(-c \sin(e)+c)}{(a \sin(e)+a)^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)

[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 9*B*c*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 3*B*c*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*B*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**2, True))

$$3.65 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))} dx$$

Optimal. Leaf size=62

$$\frac{(2A+B) \tan(e+fx)}{3a^2cf} - \frac{(A-B) \sec(e+fx)}{3cf(a^2 \sin(e+fx) + a^2)}$$

[Out] $-1/3*(A-B)*\sec(f*x+e)/c/f/(a^2+a^2*\sin(f*x+e))+1/3*(2*A+B)*\tan(f*x+e)/a^2/c/f$

Rubi [A] time = 0.20, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 3767, 8}

$$\frac{(2A+B) \tan(e+fx)}{3a^2cf} - \frac{(A-B) \sec(e+fx)}{3cf(a^2 \sin(e+fx) + a^2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x])),x]$

[Out] $-\frac{(A - B)*\text{Sec}[e + f*x]}{3*c*f*(a^2 + a^2*\text{Sin}[e + f*x])} + \frac{(2*A + B)*\text{Tan}[e + f*x]}{3*a^2*c*f}$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2859

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(g*\cos[e + f*x])^{p+1}*(a + b*\sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), \text{Int}[(g*\cos[e + f*x])^p*(a + b*\sin[e + f*x])^{m+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{LtQ}[m, -1] || \text{ILtQ}[\text{Simplify}[m + p], 0]) \&\& \text{NeQ}[2*m + p + 1, 0]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\cos[e + f*x]^{2*m}*(c + d*\sin[e + f*x])^{n-m}*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^n, x_Symbol] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))} dx = \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{a+a \sin(e+fx)} dx}{ac}$$

$$= -\frac{(A - B) \sec(e + fx)}{3cf (a^2 + a^2 \sin(e + fx))} + \frac{(2A + B) \int \sec^2(e + fx) dx}{3a^2c}$$

$$= -\frac{(A - B) \sec(e + fx)}{3cf (a^2 + a^2 \sin(e + fx))} - \frac{(2A + B) \text{Subst}(\int 1 dx, x, -\tan(e + fx))}{3a^2cf}$$

$$= -\frac{(A - B) \sec(e + fx)}{3cf (a^2 + a^2 \sin(e + fx))} + \frac{(2A + B) \tan(e + fx)}{3a^2cf}$$

Mathematica [A] time = 0.51, size = 110, normalized size = 1.77

$$\frac{\cos(e + fx)(-2(A - B) \cos(e + fx) + 2(2A + B) \cos(2(e + fx)) - 8A \sin(e + fx) - A \sin(2(e + fx)) - 4B \sin(e + fx))}{12a^2cf(\sin(e + fx) - 1)(\sin(e + fx) + 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])),x]

[Out] (Cos[e + f*x]*(-6*B - 2*(A - B)*Cos[e + f*x] + 2*(2*A + B)*Cos[2*(e + f*x)] - 8*A*Sin[e + f*x] - 4*B*Sin[e + f*x] - A*Sin[2*(e + f*x)] + B*Sin[2*(e + f*x)]))/(12*a^2*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.41, size = 69, normalized size = 1.11

$$\frac{(2A + B) \cos^2(fx + e) - (2A + B) \sin(fx + e) - A - 2B}{3(a^2cf \cos(fx + e) \sin(fx + e) + a^2cf \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] -1/3*((2*A + B)*cos(f*x + e)^2 - (2*A + B)*sin(f*x + e) - A - 2*B)/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))

giac [A] time = 0.18, size = 102, normalized size = 1.65

$$\frac{\frac{3(A+B)}{a^2c \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1 \right)} + \frac{9A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 3B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 12A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 7A - B}{a^2c \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^3}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] -1/6*(3*(A + B)/(a^2*c*(tan(1/2*f*x + 1/2*e) - 1)) + (9*A*tan(1/2*f*x + 1/2*e)^2 - 3*B*tan(1/2*f*x + 1/2*e)^2 + 12*A*tan(1/2*f*x + 1/2*e) + 7*A - B)/(a^2*c*(tan(1/2*f*x + 1/2*e) + 1)^3))/f

maple [A] time = 0.44, size = 97, normalized size = 1.56

$$\frac{\frac{2\left(\frac{A}{4} + \frac{B}{4}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{-A+B}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} - \frac{2(A-B)}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^3} - \frac{2\left(\frac{3A}{4} - \frac{B}{4}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1}}{a^2cf}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2/c*(-(1/4*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-A+B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(A-B)/(tan(1/2*f*x+1/2*e)+1)^3-(3/4*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.35, size = 265, normalized size = 4.27

$$2 \left(\frac{B \left(\frac{2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1 \right)}{a^2c + \frac{2a^2c \sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^2c \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{a^2c \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} + \frac{A \left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - 1 \right)}{a^2c + \frac{2a^2c \sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^2c \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{a^2c \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} \right) \frac{1}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2/3*(B*(2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4) + A*(sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 1)/(a^2*c + 2*a^2*c*sin(f*x + e)/(cos(f*x + e) + 1) - 2*a^2*c*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - a^2*c*sin(f*x + e)^4/(cos(f*x + e) + 1)^4))/f
```

mupad [B] time = 12.29, size = 117, normalized size = 1.89

$$\frac{2 \left(\frac{3B}{2} - A \cos(e + fx) + B \cos(e + fx) + 2A \sin(e + fx) + B \sin(e + fx) - A \cos(2e + 2fx) - \frac{B \cos(2e + 2fx)}{2} \right)}{3a^2cf (2 \cos(e + fx) + \sin(2e + 2fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))),x)
```

```
[Out] (2*((3*B)/2 - A*cos(e + f*x) + B*cos(e + f*x) + 2*A*sin(e + f*x) + B*sin(e + f*x) - A*cos(2*e + 2*f*x) - (B*cos(2*e + 2*f*x))/2 - (A*sin(2*e + 2*f*x))/2 + (B*sin(2*e + 2*f*x))/2))/(3*a^2*c*f*(2*cos(e + f*x) + sin(2*e + 2*f*x)))
```

sympy [A] time = 7.23, size = 578, normalized size = 9.32

$$\left\{ \begin{array}{l} \frac{6A \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2cf \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) + 6a^2cf \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) - 6a^2cf \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3a^2cf} - \frac{6A \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2cf \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) + 6a^2cf \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) - 6a^2cf \tan\left(\frac{e}{2} + \frac{fx}{2}\right) - 3a^2cf} - \frac{x(A+B \sin(e))}{(a \sin(e)+a)^2(-c \sin(e)+c)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**3/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 6*A*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*A*tan(e/2 + f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f))
```

```

2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) + 2*A/(3*a**2*c*f*tan(e/2 + f*x/2)**4
+ 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f
) - 6*B*tan(e/2 + f*x/2)**2/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*ta
n(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 4*B*tan(e/2
+ f*x/2)/(3*a**2*c*f*tan(e/2 + f*x/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3
- 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a**2*c*f) - 2*B/(3*a**2*c*f*tan(e/2 + f*x
/2)**4 + 6*a**2*c*f*tan(e/2 + f*x/2)**3 - 6*a**2*c*f*tan(e/2 + f*x/2) - 3*a
**2*c*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)),
True))

```

$$3.66 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=62

$$\frac{A \tan^3(e+fx)}{3a^2c^2f} + \frac{A \tan(e+fx)}{a^2c^2f} + \frac{B \sec^3(e+fx)}{3a^2c^2f}$$

[Out] $1/3*B*\sec(f*x+e)^3/a^2/c^2/f+A*\tan(f*x+e)/a^2/c^2/f+1/3*A*\tan(f*x+e)^3/a^2/c^2/f$

Rubi [A] time = 0.14, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2669, 3767}

$$\frac{A \tan^3(e+fx)}{3a^2c^2f} + \frac{A \tan(e+fx)}{a^2c^2f} + \frac{B \sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^2*(c - c*\text{Sin}[e + f*x])^2), x]$

[Out] $(B*\text{Sec}[e + f*x]^3)/(3*a^2*c^2*f) + (A*\text{Tan}[e + f*x])/(a^2*c^2*f) + (A*\text{Tan}[e + f*x]^3)/(3*a^2*c^2*f)$

Rule 2669

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^p)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] \rightarrow -\text{Simp}[(b*(g*\cos[e + f*x])^{p+1})/(f*g^{p+1}), x] + \text{Dist}[a, \text{Int}[(g*\cos[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x] \&\& (\text{IntegerQ}[2*p] \parallel \text{NeQ}[a^2 - b^2, 0])$

Rule 2967

$\text{Int}(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{m_})*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])^{n_}), x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\cos[e + f*x]^{2*m}*(c + d*\sin[e + f*x])^{n-m}*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) \parallel \text{LtQ}[0, n, m] \parallel \text{LtQ}[m, n, 0]))$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_)]^{n_}), x_Symbol] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^2} dx &= \frac{\int \sec^4(e+fx)(A+B \sin(e+fx)) dx}{a^2c^2} \\ &= \frac{B \sec^3(e+fx)}{3a^2c^2f} + \frac{A \int \sec^4(e+fx) dx}{a^2c^2} \\ &= \frac{B \sec^3(e+fx)}{3a^2c^2f} - \frac{A \text{Subst}\left(\int (1+x^2) dx, x, -\tan(e+fx)\right)}{a^2c^2f} \\ &= \frac{B \sec^3(e+fx)}{3a^2c^2f} + \frac{A \tan(e+fx)}{a^2c^2f} + \frac{A \tan^3(e+fx)}{3a^2c^2f} \end{aligned}$$

Mathematica [A] time = 0.12, size = 53, normalized size = 0.85

$$\frac{A\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{a^2c^2f}+\frac{B\sec^3(e+fx)}{3a^2c^2f}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^2), x]

[Out] (B*Sec[e + f*x]^3)/(3*a^2*c^2*f) + (A*(Tan[e + f*x] + Tan[e + f*x]^3/3))/(a^2*c^2*f)

fricas [A] time = 0.42, size = 41, normalized size = 0.66

$$\frac{\left(2A\cos(fx+e)^2+A\right)\sin(fx+e)+B}{3a^2c^2f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*((2*A*cos(f*x + e)^2 + A)*sin(f*x + e) + B)/(a^2*c^2*f*cos(f*x + e)^3)

giac [A] time = 0.20, size = 87, normalized size = 1.40

$$\frac{2\left(3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^5+3B\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4-2A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+3A\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+B\right)}{3\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^2c^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^5 + 3*B*tan(1/2*f*x + 1/2*e)^4 - 2*A*tan(1/2*f*x + 1/2*e)^3 + 3*A*tan(1/2*f*x + 1/2*e) + B)/((tan(1/2*f*x + 1/2*e)^2 - 1)^3*a^2*c^2*f)

maple [B] time = 0.35, size = 145, normalized size = 2.34

$$\frac{-\frac{2\left(\frac{A}{2}+\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}-\frac{\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2}-\frac{2\left(\frac{A}{2}+\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1}-\frac{-\frac{A}{2}+\frac{B}{2}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2}-\frac{2\left(\frac{A}{2}-\frac{B}{2}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}-\frac{2\left(\frac{A}{2}-\frac{B}{4}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{fa^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)

[Out] 2/f/a^2/c^2*(-1/3*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+1/4*B)/(tan(1/2*f*x+1/2*e)-1)-1/2*(-1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^3-(1/2*A-1/4*B)/(tan(1/2*f*x+1/2*e)+1))

maxima [A] time = 0.39, size = 47, normalized size = 0.76

$$\frac{\frac{\left(\tan(fx+e)^3+3\tan(fx+e)\right)A}{a^2c^2}+\frac{B}{a^2c^2\cos(fx+e)^3}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/3*((tan(f*x + e)^3 + 3*tan(f*x + e))*A/(a^2*c^2) + B/(a^2*c^2*cos(f*x + e)^3))/f

mupad [B] time = 12.38, size = 82, normalized size = 1.32

$$\frac{2 \left(3 A \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + 3 B \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 - 2 A \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 3 A \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + B \right)}{3 a^2 c^2 f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^2),x)

[Out] -(2*(B + 3*A*tan(e/2 + (f*x)/2) - 2*A*tan(e/2 + (f*x)/2)^3 + 3*A*tan(e/2 + (f*x)/2)^5 + 3*B*tan(e/2 + (f*x)/2)^4))/(3*a^2*c^2*f*(tan(e/2 + (f*x)/2)^2 - 1)^3)

sympy [A] time = 7.63, size = 469, normalized size = 7.56

$$\left\{ \begin{array}{l} \frac{6A \tan^5\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2c^2f \tan^6\left(\frac{e}{2} + \frac{fx}{2}\right) - 9a^2c^2f \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2c^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) - 3a^2c^2f} + \frac{4A \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2c^2f \tan^6\left(\frac{e}{2} + \frac{fx}{2}\right) - 9a^2c^2f \tan^4\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2c^2f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) - 3a^2c^2f} \\ \frac{x(A+B \sin(e))}{(a \sin(e)+a)^2(-c \sin(e)+c)^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^2,x)

[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**5/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f) + 4*A*tan(e/2 + f*x/2)**3/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f) - 6*B*tan(e/2 + f*x/2)**4/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f) - 2*B/(3*a**2*c**2*f*tan(e/2 + f*x/2)**6 - 9*a**2*c**2*f*tan(e/2 + f*x/2)**4 + 9*a**2*c**2*f*tan(e/2 + f*x/2)**2 - 3*a**2*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**2), True))

$$3.67 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=93

$$\frac{(4A-B) \tan^3(e+fx)}{15a^2c^3f} + \frac{(4A-B) \tan(e+fx)}{5a^2c^3f} + \frac{(A+B) \sec^3(e+fx)}{5a^2f(c^3-c^3 \sin(e+fx))}$$

[Out] 1/5*(A+B)*sec(f*x+e)^3/a^2/f/(c^3-c^3*sin(f*x+e))+1/5*(4*A-B)*tan(f*x+e)/a^2/c^3/f+1/15*(4*A-B)*tan(f*x+e)^3/a^2/c^3/f

Rubi [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2859, 3767}

$$\frac{(4A-B) \tan^3(e+fx)}{15a^2c^3f} + \frac{(4A-B) \tan(e+fx)}{5a^2c^3f} + \frac{(A+B) \sec^3(e+fx)}{5a^2f(c^3-c^3 \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3), x]

[Out] ((A + B)*Sec[e + f*x]^3)/(5*a^2*f*(c^3 - c^3*Sin[e + f*x])) + ((4*A - B)*Tan[e + f*x])/((5*a^2*c^3*f) + ((4*A - B)*Tan[e + f*x]^3)/(15*a^2*c^3*f)

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^3} dx = \frac{\int \frac{\sec^4(e+fx)(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx}{a^2 c^2}$$

$$= \frac{(A + B) \sec^3(e + fx)}{5a^2 f (c^3 - c^3 \sin(e + fx))} + \frac{(4A - B) \int \sec^4(e + fx) dx}{5a^2 c^3}$$

$$= \frac{(A + B) \sec^3(e + fx)}{5a^2 f (c^3 - c^3 \sin(e + fx))} - \frac{(4A - B) \text{Subst} \left(\int (1 + x^2) dx, x, -\frac{1}{c \sin(e + fx)} \right)}{5a^2 c^3 f}$$

$$= \frac{(A + B) \sec^3(e + fx)}{5a^2 f (c^3 - c^3 \sin(e + fx))} + \frac{(4A - B) \tan(e + fx)}{5a^2 c^3 f} + \frac{(4A - B) \tan(e + fx)}{15a^2 c^3}$$

Mathematica [B] time = 1.02, size = 237, normalized size = 2.55

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (54(A + B) \cos(e + fx) - 32(4A - B) \cos(e + fx))}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^3),x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-240*B + 54*(A + B)*Cos[e + f*x] - 32*(4*A - B)*Cos[2*(e + f*x)] + 18*A*Cos[3*(e + f*x)] + 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] + 16*B*Cos[4*(e + f*x)] - 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] - 18*A*Sin[2*(e + f*x)] - 18*B*Sin[2*(e + f*x)] - 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)] - 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^2*c^3*f*(-1 + Sin[e + f*x])^3*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.42, size = 116, normalized size = 1.25

$$\frac{2(4A - B) \cos^4(fx + e) - (4A - B) \cos^2(fx + e) + (2(4A - B) \cos^2(fx + e) + 4A - B) \sin(fx + e) - A + 4B}{15(a^2 c^3 f \cos(fx + e)^3 \sin(fx + e) - a^2 c^3 f \cos(fx + e)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*(2*(4*A - B)*cos(f*x + e)^4 - (4*A - B)*cos(f*x + e)^2 + (2*(4*A - B)*cos(f*x + e)^2 + 4*A - B)*sin(f*x + e) - A + 4*B)/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)

giac [B] time = 0.20, size = 235, normalized size = 2.53

$$\frac{5 \left(15A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 9B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 24A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 12B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 13A - 7B \right)}{a^2 c^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^3} + \frac{165A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 45B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4}{a^2 c^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x, algorithm="giac")

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^3),x)
[Out] ((2*A)/15 - (8*B)/15 - (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 - (2*B)/15 - (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x)
))/15)/(a^2*c^3*f*(2*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^3)) - ((
2*A)/5 + (2*B)/5 - (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^2*c^3*f*
(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((16*A)/15 - (4*B)/15))/(a^2*c^3*f*(2
*sin(e + f*x) - 2))
```

sympy [A] time = 27.73, size = 2674, normalized size = 28.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^3,x)
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 30*A*tan(e/2 + f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 50*A*tan(e/2 + f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) + 42*A*tan(e/2 + f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 -
30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 9
0*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30
*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**
2*c**3*f) - 18*A*tan(e/2 + f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*
a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a
**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a*
**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c
**3*f) - 6*A/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 30*B*tan(e/2
+ f*x/2)**6/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 20*B*tan(e/2
+ f*x/2)**5/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 10*B*tan(e/2
+ f*x/2)**4/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
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f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 16*B*tan(e/2
+ f*x/2)**3/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 18*B*tan(e/2
+ f*x/2)**2/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 +
f*x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 +
f*x/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f
*x/2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) + 12*B*tan(e/2
+ f*x/2)/(15*a**2*c**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*
x/2)**7 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x
/2)**5 - 90*a**2*c**3*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/
2)**2 + 30*a**2*c**3*f*tan(e/2 + f*x/2) - 15*a**2*c**3*f) - 6*B/(15*a**2*c*
**3*f*tan(e/2 + f*x/2)**8 - 30*a**2*c**3*f*tan(e/2 + f*x/2)**7 - 30*a**2*c**
3*f*tan(e/2 + f*x/2)**6 + 90*a**2*c**3*f*tan(e/2 + f*x/2)**5 - 90*a**2*c**3
*f*tan(e/2 + f*x/2)**3 + 30*a**2*c**3*f*tan(e/2 + f*x/2)**2 + 30*a**2*c**3*
f*tan(e/2 + f*x/2) - 15*a**2*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**2*(-c*sin(e) + c)**3), True))

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$$3.68 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=135

$$\frac{4(5A-2B) \tan^3(e+fx)}{105a^2c^4f} + \frac{4(5A-2B) \tan(e+fx)}{35a^2c^4f} + \frac{(5A-2B) \sec^3(e+fx)}{35a^2f(c^4-c^4 \sin(e+fx))} + \frac{(A+B) \sec^3(e+fx)}{7a^2f(c^2-c^2 \sin(e+fx))^2}$$

[Out] 1/7*(A+B)*sec(f*x+e)^3/a^2/f/(c^2-c^2*sin(f*x+e))^2+1/35*(5*A-2*B)*sec(f*x+e)^3/a^2/f/(c^4-c^4*sin(f*x+e))+4/35*(5*A-2*B)*tan(f*x+e)/a^2/c^4/f+4/105*(5*A-2*B)*tan(f*x+e)^3/a^2/c^4/f

Rubi [A] time = 0.27, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(5A-2B) \tan^3(e+fx)}{105a^2c^4f} + \frac{4(5A-2B) \tan(e+fx)}{35a^2c^4f} + \frac{(5A-2B) \sec^3(e+fx)}{35a^2f(c^4-c^4 \sin(e+fx))} + \frac{(A+B) \sec^3(e+fx)}{7a^2f(c^2-c^2 \sin(e+fx))^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4),x]

[Out] ((A + B)*Sec[e + f*x]^3)/(7*a^2*f*(c^2 - c^2*Sin[e + f*x])^2) + ((5*A - 2*B)*Sec[e + f*x]^3)/(35*a^2*f*(c^4 - c^4*Sin[e + f*x])) + (4*(5*A - 2*B)*Tan[e + f*x])/((35*a^2*c^4*f) + (4*(5*A - 2*B)*Tan[e + f*x]^3)/(105*a^2*c^4*f)

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^n, x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^4} dx &= \frac{\int \frac{\sec^4(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx}{a^2 c^2} \\ &= \frac{(A + B) \sec^3(e + fx)}{7a^2 f (c^2 - c^2 \sin(e + fx))^2} + \frac{(5A - 2B) \int \frac{\sec^4(e+fx)}{c-c \sin(e+fx)} dx}{7a^2 c^3} \\ &= \frac{(A + B) \sec^3(e + fx)}{7a^2 f (c^2 - c^2 \sin(e + fx))^2} + \frac{(5A - 2B) \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} + \frac{(4A - 2B) \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} \\ &= \frac{(A + B) \sec^3(e + fx)}{7a^2 f (c^2 - c^2 \sin(e + fx))^2} + \frac{(5A - 2B) \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} - \frac{(4A - 2B) \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} \\ &= \frac{(A + B) \sec^3(e + fx)}{7a^2 f (c^2 - c^2 \sin(e + fx))^2} + \frac{(5A - 2B) \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} + \frac{4B \sec^3(e + fx)}{35a^2 f (c^4 - c^4 \sin(e + fx))} \end{aligned}$$

Mathematica [B] time = 0.96, size = 285, normalized size = 2.11

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(42(25A + 4B) \cos(e + fx) - 512(5A - 2B)\right)}{105\left(a^2 c^4 f \cos^5(fx + e) + 2a^2 c^4 f \cos^3(fx + e) \sin(fx + e) - 2a^2 c^4 f \cos(fx + e) \sin^3(fx + e) - 2a^2 c^4 f \cos^3(fx + e) \sin^3(fx + e)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^4), x]

[Out] -1/13440*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2688*B + 42*(25*A + 4*B)*Cos[e + f*x] - 512*(5*A - 2*B)*Cos[2*(e + f*x)] + 225*A*Cos[3*(e + f*x)] + 36*B*Cos[3*(e + f*x)] - 1280*A*Cos[4*(e + f*x)] + 512*B*Cos[4*(e + f*x)] - 75*A*Cos[5*(e + f*x)] - 12*B*Cos[5*(e + f*x)] - 4480*A*Sin[e + f*x] + 1792*B*Sin[e + f*x] - 600*A*Sin[2*(e + f*x)] - 96*B*Sin[2*(e + f*x)] - 960*A*Sin[3*(e + f*x)] + 384*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*x)] - 48*B*Sin[4*(e + f*x)] + 320*A*Sin[5*(e + f*x)] - 128*B*Sin[5*(e + f*x)]))/(a^2*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.43, size = 151, normalized size = 1.12

$$\frac{16(5A - 2B) \cos^4(fx + e) - 8(5A - 2B) \cos^2(fx + e) - \left(8(5A - 2B) \cos^4(fx + e) - 12(5A - 2B) \cos^2(fx + e) - 25A + 10B\right) \sin(fx + e) - 10A + 25B}{105\left(a^2 c^4 f \cos^5(fx + e) + 2a^2 c^4 f \cos^3(fx + e) \sin(fx + e) - 2a^2 c^4 f \cos(fx + e) \sin^3(fx + e) - 2a^2 c^4 f \cos^3(fx + e) \sin^3(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] -1/105*(16*(5*A - 2*B)*cos(f*x + e)^4 - 8*(5*A - 2*B)*cos(f*x + e)^2 - (8*(5*A - 2*B)*cos(f*x + e)^4 - 12*(5*A - 2*B)*cos(f*x + e)^2 - 25*A + 10*B)*sin(f*x + e) - 10*A + 25*B)/(a^2*c^4*f*cos(f*x + e)^5 + 2*a^2*c^4*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2*c^4*f*cos(f*x + e)^3)

giac [B] time = 0.23, size = 295, normalized size = 2.19

$$\frac{35 \left(9A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 6B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 15A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 9B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 8A - 5B \right)}{a^2 c^4 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^3} + \frac{1365A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6 + 210B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^6}{f a^2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out]
$$\frac{-1/840*(35*(9*A*\tan(1/2*f*x + 1/2*e)^2 - 6*B*\tan(1/2*f*x + 1/2*e)^2 + 15*A*\tan(1/2*f*x + 1/2*e) - 9*B*\tan(1/2*f*x + 1/2*e) + 8*A - 5*B)/(a^2*c^4*(\tan(1/2*f*x + 1/2*e) + 1)^3) + (1365*A*\tan(1/2*f*x + 1/2*e)^6 + 210*B*\tan(1/2*f*x + 1/2*e)^6 - 5775*A*\tan(1/2*f*x + 1/2*e)^5 - 105*B*\tan(1/2*f*x + 1/2*e)^5 + 12250*A*\tan(1/2*f*x + 1/2*e)^4 - 175*B*\tan(1/2*f*x + 1/2*e)^4 - 14350*A*\tan(1/2*f*x + 1/2*e)^3 + 910*B*\tan(1/2*f*x + 1/2*e)^3 + 10185*A*\tan(1/2*f*x + 1/2*e)^2 - 756*B*\tan(1/2*f*x + 1/2*e)^2 - 3955*A*\tan(1/2*f*x + 1/2*e) + 427*B*\tan(1/2*f*x + 1/2*e) + 760*A - 31*B)/(a^2*c^4*(\tan(1/2*f*x + 1/2*e) - 1)^7))/f$$

maple [A] time = 0.50, size = 233, normalized size = 1.73

$$\frac{\frac{2(2A+2B)}{7\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^7} - \frac{6A+6B}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^6} - \frac{10A+8B}{2\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^4} - \frac{2(10A+9B)}{5\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^5} - \frac{2\left(\frac{13A}{16} + \frac{B}{8}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{\frac{23A}{8} + \frac{11B}{8}}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)^2} - \frac{2\left(\frac{55A}{8} + \frac{35B}{8}\right)}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1\right)}}{f a^2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x)

[Out]
$$\frac{2/f/a^2/c^4*(-1/7*(2*A+2*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/6*(6*A+6*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/4*(10*A+8*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/5*(10*A+9*B)/(\tan(1/2*f*x+1/2*e)-1)^5-(13/16*A+1/8*B)/(\tan(1/2*f*x+1/2*e)-1)-1/2*(23/8*A+11/8*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/3*(55/8*A+35/8*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/2*(-1/8*A+1/8*B)/(\tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/8*A-1/8*B)/(\tan(1/2*f*x+1/2*e)+1)^3-(3/16*A-1/8*B)/(\tan(1/2*f*x+1/2*e)+1))$$

maxima [B] time = 0.37, size = 835, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out]
$$\frac{-2/105*(B*(36*\sin(f*x + e)/(\cos(f*x + e) + 1) - 132*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 68*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 14*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 84*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 140*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 140*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 105*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 9)/(a^2*c^4 - 4*a^2*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 8*a^2*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 14*a^2*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 14*a^2*c^4*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 8*a^2*c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3*a^2*c^4*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 4*a^2*c^4*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - a^2*c^4*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10) + 5*A*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 24*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 76*\sin(f*x + e)^3/(\cos(f*x + e) +$$

$$\begin{aligned}
& (e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f* \\
& tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f \\
& *tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) \\
& + 280*A*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a** \\
& 2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a* \\
& **2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470 \\
& *a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 31 \\
& 5*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105* \\
& a**2*c**4*f) - 760*A*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f*tan(e/2 + f*x/2)* \\
& **10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2) \\
&)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x \\
& /2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f \\
& *x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + \\
& f*x/2) - 105*a**2*c**4*f) + 240*A*tan(e/2 + f*x/2)**2/(105*a**2*c**4*f*tan(\\
& e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*ta \\
& n(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f* \\
& tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4* \\
& f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4 \\
& *f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 30*A*tan(e/2 + f*x/2)/(105*a**2*c* \\
& **4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2* \\
& c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a** \\
& 2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a \\
& **2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420* \\
& a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 60*A/(105*a**2*c**4*f*tan \\
& (e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f* \\
& tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f \\
& *tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4 \\
& *f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c** \\
& 4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 210*B*tan(e/2 + f*x/2)**8/(105*a* \\
& **2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315* \\
& a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 147 \\
& 0*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - \\
& 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + \\
& 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 280*B*tan(e/2 + f*x/ \\
& 2)**7/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x \\
& /2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f* \\
& x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + \\
& f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 \\
& + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 280*B*t \\
& an(e/2 + f*x/2)**6/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f* \\
& tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f \\
& *tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c** \\
& 4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c* \\
& **4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4 \\
& *f) - 168*B*tan(e/2 + f*x/2)**5/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420 \\
& *a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 84 \\
& 0*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + \\
& 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 \\
& - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - \\
& 105*a**2*c**4*f) - 28*B*tan(e/2 + f*x/2)**4/(105*a**2*c**4*f*tan(e/2 + f*x/ \\
& 2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f*tan(e/2 + f* \\
& x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4*f*tan(e/2 + \\
& f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c**4*f*tan(e/2 \\
& + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 \\
& + f*x/2) - 105*a**2*c**4*f) + 136*B*tan(e/2 + f*x/2)**3/(105*a**2*c**4*f* \\
& tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 315*a**2*c**4*f \\
& *tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1470*a**2*c**4 \\
& *f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4 - 840*a**2*c* \\
& **4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2 + 420*a**2*c
\end{aligned}$$

```

**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 264*B*tan(e/2 + f*x/2)**2/(105*
a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 31
5*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 - 1
470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
- 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**2
+ 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) + 72*B*tan(e/2 + f*x
/2)/(105*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2
)**9 + 315*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/
2)**7 - 1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f
*x/2)**4 - 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 +
f*x/2)**2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f) - 18*B/(105
*a**2*c**4*f*tan(e/2 + f*x/2)**10 - 420*a**2*c**4*f*tan(e/2 + f*x/2)**9 + 3
15*a**2*c**4*f*tan(e/2 + f*x/2)**8 + 840*a**2*c**4*f*tan(e/2 + f*x/2)**7 -
1470*a**2*c**4*f*tan(e/2 + f*x/2)**6 + 1470*a**2*c**4*f*tan(e/2 + f*x/2)**4
- 840*a**2*c**4*f*tan(e/2 + f*x/2)**3 - 315*a**2*c**4*f*tan(e/2 + f*x/2)**
2 + 420*a**2*c**4*f*tan(e/2 + f*x/2) - 105*a**2*c**4*f), Ne(f, 0)), (x*(A +
B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c)**4), True))

```

$$3.69 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^5} dx$$

Optimal. Leaf size=175

$$\frac{4(2A-B) \tan^3(e+fx)}{63a^2c^5f} + \frac{4(2A-B) \tan(e+fx)}{21a^2c^5f} + \frac{(2A-B) \sec^3(e+fx)}{21a^2f(c^5-c^5 \sin(e+fx))} + \frac{(2A-B) \sec^3(e+fx)}{21a^2c^3f(c-c \sin(e+fx))^2} + \frac{(A-B) \sec^3(e+fx)}{9a^2c^3f}$$

[Out] 1/9*(A+B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^3+1/21*(2*A-B)*sec(f*x+e)^3/a^2/c^3/f/(c-c*sin(f*x+e))^2+1/21*(2*A-B)*sec(f*x+e)^3/a^2/f/(c^5-c^5*sin(f*x+e))+4/21*(2*A-B)*tan(f*x+e)/a^2/c^5/f+4/63*(2*A-B)*tan(f*x+e)^3/a^2/c^5/f

Rubi [A] time = 0.32, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 3767}

$$\frac{4(2A-B) \tan^3(e+fx)}{63a^2c^5f} + \frac{4(2A-B) \tan(e+fx)}{21a^2c^5f} + \frac{(2A-B) \sec^3(e+fx)}{21a^2f(c^5-c^5 \sin(e+fx))} + \frac{(2A-B) \sec^3(e+fx)}{21a^2c^3f(c-c \sin(e+fx))^2} + \frac{(A-B) \sec^3(e+fx)}{9a^2c^3f}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5),x]
 [Out] ((A + B)*Sec[e + f*x]^3)/(9*a^2*c^2*f*(c - c*Sin[e + f*x])^3) + ((2*A - B)*Sec[e + f*x]^3)/(21*a^2*c^3*f*(c - c*Sin[e + f*x])^2) + ((2*A - B)*Sec[e + f*x]^3)/(21*a^2*f*(c^5 - c^5*Sin[e + f*x])) + (4*(2*A - B)*Tan[e + f*x])/(21*a^2*c^5*f) + (4*(2*A - B)*Tan[e + f*x]^3)/(63*a^2*c^5*f)

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^n, x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^5} dx &= \frac{\int \frac{\sec^4(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx}{a^2 c^2} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \int \frac{\sec^4(e+fx)}{(c-c \sin(e+fx))^2} dx}{3a^2 c^3} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \sec^3(e + fx)}{21a^2 c^3 f (c - c \sin(e + fx))^2} + \frac{(5)}{21} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \sec^3(e + fx)}{21a^2 c^3 f (c - c \sin(e + fx))^2} + \frac{(5)}{21} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \sec^3(e + fx)}{21a^2 c^3 f (c - c \sin(e + fx))^2} + \frac{(5)}{21} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \sec^3(e + fx)}{21a^2 c^3 f (c - c \sin(e + fx))^2} + \frac{(5)}{21} \\ &= \frac{(A + B) \sec^3(e + fx)}{9a^2 c^2 f (c - c \sin(e + fx))^3} + \frac{(2A - B) \sec^3(e + fx)}{21a^2 c^3 f (c - c \sin(e + fx))^2} + \frac{(5)}{21} \end{aligned}$$

Mathematica [A] time = 1.16, size = 329, normalized size = 1.88

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)(180(31A - 5B)\cos(e + fx) - 6912(2A - B)\cos^2(e + fx) + 310A\cos(3(e + fx)) - 50B\cos(3(e + fx)) - 6144A\cos(4(e + fx)) + 3072B\cos(4(e + fx)) - 930A\cos(5(e + fx)) + 150B\cos(5(e + fx)) + 512A\cos(6(e + fx)) - 256B\cos(6(e + fx)) - 18432A\sin(e + fx) + 9216B\sin(e + fx) - 4185A\sin(2(e + fx)) + 675B\sin(2(e + fx)) - 1024A\sin(3(e + fx)) + 512B\sin(3(e + fx)) - 1860A\sin(4(e + fx)) + 300B\sin(4(e + fx)) + 3072A\sin(5(e + fx)) - 1536B\sin(5(e + fx)) + 155A\sin(6(e + fx)) - 25B\sin(6(e + fx)))}{(64512a^2c^5f(-1 + \sin(e + fx))^5(1 + \sin(e + fx))^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^5), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-10752*B + 180*(31*A - 5*B)*Cos[e + f*x] - 6912*(2*A - B)*Cos[2*(e + f*x)] + 310*A*Cos[3*(e + f*x)] - 50*B*Cos[3*(e + f*x)] - 6144*A*Cos[4*(e + f*x)] + 3072*B*Cos[4*(e + f*x)] - 930*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*x)] + 512*A*Cos[6*(e + f*x)] - 256*B*Cos[6*(e + f*x)] - 18432*A*Sin[e + f*x] + 9216*B*Sin[e + f*x] - 4185*A*Sin[2*(e + f*x)] + 675*B*Sin[2*(e + f*x)] - 1024*A*Sin[3*(e + f*x)] + 512*B*Sin[3*(e + f*x)] - 1860*A*Sin[4*(e + f*x)] + 300*B*Sin[4*(e + f*x)] + 3072*A*Sin[5*(e + f*x)] - 1536*B*Sin[5*(e + f*x)] + 155*A*Sin[6*(e + f*x)] - 25*B*Sin[6*(e + f*x)]))/(64512*a^2*c^5*f*(-1 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.43, size = 187, normalized size = 1.07

$$\frac{8(2A - B)\cos^6(fx + e) - 36(2A - B)\cos^4(fx + e) + 15(2A - B)\cos^2(fx + e) + (24(2A - B)\cos(fx + e) - 20(2A - B))\cos(fx + e)}{63\left(3a^2c^5f\cos^5(fx + e) - 4a^2c^5f\cos^3(fx + e) - \left(a^2c^5f\cos(fx + e)\right)^5 - \left(a^2c^5f\cos(fx + e)\right)^3 - \left(a^2c^5f\cos(fx + e)\right)^5 - \left(a^2c^5f\cos(fx + e)\right)^3 - \left(a^2c^5f\cos(fx + e)\right)^5 - \left(a^2c^5f\cos(fx + e)\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] 1/63*(8*(2*A - B)*cos(f*x + e)^6 - 36*(2*A - B)*cos(f*x + e)^4 + 15*(2*A - B)*cos(f*x + e)^2 + (24*(2*A - B)*cos(f*x + e)^4 - 20*(2*A - B)*cos(f*x + e)

)² - 14*A + 7*B)*sin(f*x + e) + 7*A - 14*B)/(3*a²*c⁵*f*cos(f*x + e)⁵ - 4*a²*c⁵*f*cos(f*x + e)³ - (a²*c⁵*f*cos(f*x + e)⁵ - 4*a²*c⁵*f*cos(f*x + e)³)*sin(f*x + e))

giac [B] time = 0.25, size = 355, normalized size = 2.03

$$\frac{21 \left(21 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 15 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 36 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 24 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 19 A - 13 B \right)}{a^2 c^5 \left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1 \right)^3} + \frac{3591 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8 + 315 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^8}{f a^2 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="giac")

[Out] -1/2016*(21*(21*A*tan(1/2*f*x + 1/2*e)^2 - 15*B*tan(1/2*f*x + 1/2*e)^2 + 36*A*tan(1/2*f*x + 1/2*e) - 24*B*tan(1/2*f*x + 1/2*e) + 19*A - 13*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) + 1)^3) + (3591*A*tan(1/2*f*x + 1/2*e)^8 + 315*B*tan(1/2*f*x + 1/2*e)^8 - 19656*A*tan(1/2*f*x + 1/2*e)^7 + 756*B*tan(1/2*f*x + 1/2*e)^7 + 56196*A*tan(1/2*f*x + 1/2*e)^6 - 4200*B*tan(1/2*f*x + 1/2*e)^6 - 95760*A*tan(1/2*f*x + 1/2*e)^5 + 11340*B*tan(1/2*f*x + 1/2*e)^5 + 107730*A*tan(1/2*f*x + 1/2*e)^4 - 14994*B*tan(1/2*f*x + 1/2*e)^4 - 79464*A*tan(1/2*f*x + 1/2*e)^3 + 13356*B*tan(1/2*f*x + 1/2*e)^3 + 38484*A*tan(1/2*f*x + 1/2*e)^2 - 6768*B*tan(1/2*f*x + 1/2*e)^2 - 10944*A*tan(1/2*f*x + 1/2*e) + 2196*B*tan(1/2*f*x + 1/2*e) + 1615*A - 209*B)/(a^2*c^5*(tan(1/2*f*x + 1/2*e) - 1)^9))/f

maple [A] time = 0.53, size = 277, normalized size = 1.58

$$\frac{\frac{2(4A+4B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8} - \frac{2(34A+32B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{46A+40B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{\frac{9A}{2}+\frac{13B}{8}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2\left(\frac{57A}{64}+\frac{5B}{64}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{\frac{59A}{2}+\frac{39B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)}}{f a^2 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)

[Out] 2/f/a^2/c^5*(-1/9*(4*A+4*B)/(tan(1/2*f*x+1/2*e)-1)^9-1/8*(16*A+16*B)/(tan(1/2*f*x+1/2*e)-1)^8-1/7*(34*A+32*B)/(tan(1/2*f*x+1/2*e)-1)^7-1/6*(46*A+40*B)/(tan(1/2*f*x+1/2*e)-1)^6-1/2*(9/2*A+13/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(57/64*A+5/64*B)/(tan(1/2*f*x+1/2*e)-1)-1/4*(59/2*A+39/2*B)/(tan(1/2*f*x+1/2*e)-1)^4-1/3*(57/4*A+59/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/5*(175/4*A+135/4*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(-1/16*A+1/16*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/3*(1/16*A-1/16*B)/(tan(1/2*f*x+1/2*e)+1)^3-(7/64*A-5/64*B)/(tan(1/2*f*x+1/2*e)+1))

maxima [B] time = 0.57, size = 998, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x, algorithm="maxima")

[Out] -2/63*(A*(51*sin(f*x + e)/(cos(f*x + e) + 1) - 39*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 235*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 450*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 306*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 - 294*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 378*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 - 63*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 - 273*sin(f*x + e)^9/(cos(f*x + e) + 1

$$\begin{aligned} &)^9 + 189\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 63\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 19/(a^2*c^5 - 6*a^2*c^5*\sin(f*x + e))/(\cos(f*x + e) + 1) \\ & + 12*a^2*c^5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 2*a^2*c^5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 27*a^2*c^5*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 36 \\ & *a^2*c^5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 36*a^2*c^5*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 27*a^2*c^5*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 2*a^2 \\ & *c^5*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 12*a^2*c^5*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 6*a^2*c^5*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - a^2*c^5 \\ & *5*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} + B*(6*\sin(f*x + e)/(\cos(f*x + e) + 1) - 75*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 128*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 162*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 36*\sin(f*x + e)^5/ \\ & (\cos(f*x + e) + 1)^5 + 42*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 189*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 126*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 63 \\ & *\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 1)/(a^2*c^5 - 6*a^2*c^5*\sin(f*x + e))/(\cos(f*x + e) + 1) + 12*a^2*c^5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 2*a \\ & a^2*c^5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 27*a^2*c^5*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 36*a^2*c^5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 36*a^2 \\ & *c^5*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 27*a^2*c^5*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 2*a^2*c^5*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 12*a^2*c^5 \\ & *\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 6*a^2*c^5*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - a^2*c^5*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12})/f \end{aligned}$$

mupad [B] time = 12.88, size = 337, normalized size = 1.93

$$2 \left(7A - 14B - 14A \sin(e + fx) + 7B \sin(e + fx) + 30A \cos(e + fx)^2 - 76A \cos(e + fx)^3 - 72A \cos(e + fx)^4 + 57A \cos(e + fx)^5 + 16A \cos(e + fx)^6 - 15B \cos(e + fx)^2 - 4B \cos(e + fx)^3 + 36B \cos(e + fx)^4 + 3B \cos(e + fx)^5 - 8B \cos(e + fx)^6 - 40A \cos(e + fx)^2 \sin(e + fx) + 76A \cos(e + fx)^3 \sin(e + fx) + 48A \cos(e + fx)^4 \sin(e + fx) - 19A \cos(e + fx)^5 \sin(e + fx) + 20B \cos(e + fx)^2 \sin(e + fx) + 4B \cos(e + fx)^3 \sin(e + fx) - 24B \cos(e + fx)^4 \sin(e + fx) - B \cos(e + fx)^5 \sin(e + fx) \right) / (63a^2c^5f(8\cos(e + fx)^3\sin(e + fx) - 2\cos(e + fx)^5\sin(e + fx) - 8\cos(e + fx)^3 + 6\cos(e + fx)^5))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^5),x)

[Out] (2*(7*A - 14*B - 14*A*sin(e + f*x) + 7*B*sin(e + f*x) + 30*A*cos(e + f*x)^2 - 76*A*cos(e + f*x)^3 - 72*A*cos(e + f*x)^4 + 57*A*cos(e + f*x)^5 + 16*A*cos(e + f*x)^6 - 15*B*cos(e + f*x)^2 - 4*B*cos(e + f*x)^3 + 36*B*cos(e + f*x)^4 + 3*B*cos(e + f*x)^5 - 8*B*cos(e + f*x)^6 - 40*A*cos(e + f*x)^2*sin(e + f*x) + 76*A*cos(e + f*x)^3*sin(e + f*x) + 48*A*cos(e + f*x)^4*sin(e + f*x) - 19*A*cos(e + f*x)^5*sin(e + f*x) + 20*B*cos(e + f*x)^2*sin(e + f*x) + 4*B*cos(e + f*x)^3*sin(e + f*x) - 24*B*cos(e + f*x)^4*sin(e + f*x) - B*cos(e + f*x)^5*sin(e + f*x)))/(63*a^2*c^5*f*(8*cos(e + f*x)^3*sin(e + f*x) - 2*cos(e + f*x)^5*sin(e + f*x) - 8*cos(e + f*x)^3 + 6*cos(e + f*x)^5))

sympy [A] time = 90.73, size = 5868, normalized size = 33.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^5,x)

[Out] Piecewise((-126*A*tan(e/2 + f*x/2)**11/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a**2*c**5*f) + 378*A*tan(e/2 + f*x/2)**10/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2

$$\begin{aligned}
& c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2} \\
& *c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) - 546*A*\tan(e/2 + f*x/2)**9/(63 \\
& *a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + \\
& 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 \\
& - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)* \\
& *7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/ \\
& /2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x \\
& /2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) - 126*A*\tan(e/2 \\
& + f*x/2)**8/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 \\
& + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(\\
& e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*t \\
& an(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**5 + 1701*a^{**2}*c^{**5} \\
& f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5} \\
& *f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) \\
& + 756*A*\tan(e/2 + f*x/2)**7/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378*a^{**2} \\
& *c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - 126* \\
& a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 + 22 \\
& 68*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**5 + \\
& 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 \\
& - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - \\
& 63*a^{**2}*c^{**5}f) - 588*A*\tan(e/2 + f*x/2)**6/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/ \\
& /2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f \\
& *x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 \\
& + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e \\
& /2 + f*x/2)**5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan \\
& (e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*ta \\
& n(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) - 612*A*\tan(e/2 + f*x/2)**5/(63*a^{**2}*c^{**5} \\
& f*\tan(e/2 + f*x/2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c \\
& *5*f*\tan(e/2 + f*x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 - 1701*a^{**2} \\
& *c^{**5}f*\tan(e/2 + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**7 - 2268*a \\
& **2*c^{**5}f*\tan(e/2 + f*x/2)**5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**4 + 126 \\
& *a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 37 \\
& 8*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) + 900*A*\tan(e/2 + f*x/2)** \\
& 4/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)** \\
& 11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2 \\
&)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f \\
& x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + \\
& f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 \\
& + f*x/2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) - 470*A*ta \\
& n(e/2 + f*x/2)**3/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378*a^{**2}*c^{**5}f*ta \\
& n(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - 126*a^{**2}*c^{**5}f \\
& *tan(e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 + 2268*a^{**2}*c^{** \\
& 5*f*\tan(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**5 + 1701*a^{**2}* \\
& c^{**5}f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**3 - 756*a^{**2} \\
& *c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2) - 63*a^{**2}*c \\
& *5*f) - 78*A*\tan(e/2 + f*x/2)**2/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**12 - 378 \\
& *a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**10 - \\
& 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**8 \\
& + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)* \\
& *5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2 \\
&)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/ \\
& /2) - 63*a^{**2}*c^{**5}f) + 102*A*\tan(e/2 + f*x/2)/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x \\
& /2)**12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + \\
& f*x/2)**10 - 126*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**9 - 1701*a^{**2}*c^{**5}f*\tan(e/2 \\
& + f*x/2)**8 + 2268*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**7 - 2268*a^{**2}*c^{**5}f*\tan(\\
& e/2 + f*x/2)**5 + 1701*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**4 + 126*a^{**2}*c^{**5}f*ta \\
& n(e/2 + f*x/2)**3 - 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**2 + 378*a^{**2}*c^{**5}f*t \\
& an(e/2 + f*x/2) - 63*a^{**2}*c^{**5}f) - 38*A/(63*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)** \\
& 12 - 378*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2)**11 + 756*a^{**2}*c^{**5}f*\tan(e/2 + f*x/2
\end{aligned}$$


```

3 - 756*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2)
- 63*a**2*c**5*f) - 2*B/(63*a**2*c**5*f*tan(e/2 + f*x/2)**12 - 378*a**2*c**
5*f*tan(e/2 + f*x/2)**11 + 756*a**2*c**5*f*tan(e/2 + f*x/2)**10 - 126*a**2*
c**5*f*tan(e/2 + f*x/2)**9 - 1701*a**2*c**5*f*tan(e/2 + f*x/2)**8 + 2268*a*
**2*c**5*f*tan(e/2 + f*x/2)**7 - 2268*a**2*c**5*f*tan(e/2 + f*x/2)**5 + 1701
*a**2*c**5*f*tan(e/2 + f*x/2)**4 + 126*a**2*c**5*f*tan(e/2 + f*x/2)**3 - 75
6*a**2*c**5*f*tan(e/2 + f*x/2)**2 + 378*a**2*c**5*f*tan(e/2 + f*x/2) - 63*a
**2*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**2*(-c*sin(e) + c
)**5), True))

```

$$3.70 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^5}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=243

$$\frac{a^5 c^5 (A-B) \cos^{11}(e+fx)}{5f(a \sin(e+fx)+a)^8} - \frac{7c^5 (3A-8B) \cos^3(e+fx)}{a^3 f} + \frac{2a^3 c^5 (3A-8B) \cos^9(e+fx)}{15f(a \sin(e+fx)+a)^6} - \frac{21c^5 (3A-8B) \sin(e+fx)}{2a^3 f}$$

[Out] $-21/2*(3*A-8*B)*c^5*x/a^3-7*(3*A-8*B)*c^5*\cos(f*x+e)^3/a^3/f-21/2*(3*A-8*B)*c^5*\cos(f*x+e)*\sin(f*x+e)/a^3/f-1/5*a^5*(A-B)*c^5*\cos(f*x+e)^{11}/f/(a+a*\sin(f*x+e))^8+2/15*a^3*(3*A-8*B)*c^5*\cos(f*x+e)^9/f/(a+a*\sin(f*x+e))^6-6/5*a^5*(3*A-8*B)*c^5*\cos(f*x+e)^7/f/(a^2+a^2*\sin(f*x+e))^4-42/5*a^5*(3*A-8*B)*c^5*\cos(f*x+e)^5/f/(a^4+a^4*\sin(f*x+e))^2$

Rubi [A] time = 0.41, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2682, 2635, 8}

$$-\frac{7c^5(3A-8B)\cos^3(e+fx)}{a^3f} - \frac{a^5c^5(A-B)\cos^{11}(e+fx)}{5f(a\sin(e+fx)+a)^8} + \frac{2a^3c^5(3A-8B)\cos^9(e+fx)}{15f(a\sin(e+fx)+a)^6} - \frac{6a^5c^5(3A-8B)\cos^7(e+fx)}{5f(a^2\sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3,x]

[Out] $(-21*(3*A-8*B)*c^5*x)/(2*a^3) - (7*(3*A-8*B)*c^5*\cos[e+f*x]^3)/(a^3*f) - (21*(3*A-8*B)*c^5*\cos[e+f*x]*\sin[e+f*x])/(2*a^3*f) - (a^5*(A-B)*c^5*\cos[e+f*x]^{11})/(5*f*(a+a*\sin[e+f*x])^8) + (2*a^3*(3*A-8*B)*c^5*\cos[e+f*x]^9)/(15*f*(a+a*\sin[e+f*x])^6) - (6*a^5*(3*A-8*B)*c^5*\cos[e+f*x]^7)/(5*f*(a^2+a^2*\sin[e+f*x])^4) - (42*a^5*(3*A-8*B)*c^5*\cos[e+f*x]^5)/(5*f*(a^4+a^4*\sin[e+f*x])^2)$

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !LtQ[m+p+1, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[((b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^5}{(a + a \sin(e + fx))^3} dx &= (a^5 c^5) \int \frac{\cos^{10}(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{5f(a + a \sin(e + fx))^8} - \frac{1}{5} (a^4(3A - 8B)c^5) \int \frac{\cos^{10}(e + fx)}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{5f(a + a \sin(e + fx))^8} + \frac{2a^3(3A - 8B)c^5 \cos^9(e + fx)}{15f(a + a \sin(e + fx))^6} + \frac{1}{5} \int \frac{\cos^8(e + fx)}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{5f(a + a \sin(e + fx))^8} + \frac{2a^3(3A - 8B)c^5 \cos^9(e + fx)}{15f(a + a \sin(e + fx))^6} - \frac{6a^3}{15} \int \frac{\cos^7(e + fx)}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{5f(a + a \sin(e + fx))^8} + \frac{2a^3(3A - 8B)c^5 \cos^9(e + fx)}{15f(a + a \sin(e + fx))^6} - \frac{6a^3}{15} \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{7(3A - 8B)c^5 \cos^3(e + fx)}{a^3 f} - \frac{a^5(A - B)c^5 \cos^{11}(e + fx)}{5f(a + a \sin(e + fx))^8} + \frac{2a^3}{15} \int \frac{\cos^5(e + fx)}{(a + a \sin(e + fx))^8} dx \\ &= -\frac{7(3A - 8B)c^5 \cos^3(e + fx)}{a^3 f} - \frac{21(3A - 8B)c^5 \cos(e + fx) \sin(e + fx)}{2a^3 f} \\ &= -\frac{21(3A - 8B)c^5 x}{2a^3} - \frac{7(3A - 8B)c^5 \cos^3(e + fx)}{a^3 f} - \frac{21(3A - 8B)c^5}{a^3 f} \end{aligned}$$

Mathematica [A] time = 2.50, size = 388, normalized size = 1.60

$$(c - c \sin(e + fx))^5 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(768(A - B) \sin\left(\frac{1}{2}(e + fx)\right) - 630(3A - 8B)(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^5)/(a + a*Sin[e + f*x])^3, x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^5*(768*(A - B)*Sin[(e + f*x)/2] - 384*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(21*A - 31*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64
```

$(21A - 31B) \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^3 + 128(54A - 119B) \cdot \sin[(e + fx)/2] \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^4 - 630(3A - 8B) \cdot (e + fx) \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^5 - 15(32A - 127B) \cdot \cos[e + fx] \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^5 - 5B \cdot \cos[3(e + fx)] \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^5 + 15(A - 8B) \cdot (\cos[(e + fx)/2] + \sin[(e + fx)/2])^5 \cdot \sin[2(e + fx)] / (60a^3 \cdot f \cdot (\cos[(e + fx)/2] - \sin[(e + fx)/2])^{10} \cdot (1 + \sin[e + fx])^3)$

fricas [A] time = 0.46, size = 429, normalized size = 1.77

$$\frac{10Bc^5 \cos(fx + e)^6 + 15(A - 6B)c^5 \cos(fx + e)^5 + 10(21A - 74B)c^5 \cos(fx + e)^4 - 1260(3A - 8B)c^5 f \cos(fx + e)^3 + \dots}{(60a^3 f (\cos((e + fx)/2) - \sin((e + fx)/2))^{10} (1 + \sin(e + fx))^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] $-1/30(10Bc^5 \cos(fx + e)^6 + 15(A - 6B)c^5 \cos(fx + e)^5 + 10(21A - 74B)c^5 \cos(fx + e)^4 - 1260(3A - 8B)c^5 f \cos(fx + e)^3 - 192(A - B)c^5 + (315(3A - 8B)c^5 f \cos(fx + e) + (2373A - 6128B)c^5) \cos(fx + e)^2 - 6(105(3A - 8B)c^5 f \cos(fx + e) + 2(323A - 848B)c^5) \cos(fx + e) + (10Bc^5 \cos(fx + e)^5 - 5(3A - 20B)c^5 \cos(fx + e)^4 + 5(39A - 128B)c^5 \cos(fx + e)^3 - 1260(3A - 8B)c^5 f \cos(fx + e) + 192(A - B)c^5 + (315(3A - 8B)c^5 f \cos(fx + e) - 2(1089A - 2744B)c^5) \cos(fx + e)^2 - 6(105(3A - 8B)c^5 f \cos(fx + e) + 2(307A - 832B)c^5) \cos(fx + e)) \sin(fx + e) / (a^3 f \cos(fx + e)^3 + 3a^3 f \cos(fx + e)^2 - 2a^3 f \cos(fx + e) - 4a^3 f + (a^3 f \cos(fx + e)^2 - 2a^3 f \cos(fx + e) - 4a^3 f) \sin(fx + e))$

giac [A] time = 0.28, size = 376, normalized size = 1.55

$$\frac{315(3Ac^5 - 8Bc^5)(fx + e)}{a^3} + \frac{10 \left(3Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 - 24Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^5 + 48Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 186Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 96Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 396Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 255Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 595Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 165Ac^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 395Bc^5 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 39Ac^5 - 94Bc^5 \right)}{a^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^5} / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^5/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] $-1/30(315(3A \cdot c^5 - 8B \cdot c^5) \cdot (fx + e) / a^3 + 10(3A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^5 - 24B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^5 + 48A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^4 - 186B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^4 + 96A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^3 - 384B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^3 + 255A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^2 - 595B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^2 + 165A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e) - 395B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e) + 39A \cdot c^5 - 94B \cdot c^5) / ((\tan(1/2 \cdot fx + 1/2 \cdot e)^2 + 1)^3 \cdot a^3) + 64(30A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^4 - 75B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^4 + 135A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^3 - 345B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^3 + 255A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^2 - 595B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e)^2 + 165A \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e) - 395B \cdot c^5 \cdot \tan(1/2 \cdot fx + 1/2 \cdot e) + 39A \cdot c^5 - 94B \cdot c^5) / (a^3 \cdot (\tan(1/2 \cdot fx + 1/2 \cdot e) + 1)^5) / f$

maple [B] time = 0.52, size = 649, normalized size = 2.67

$$\frac{c^5 \left(\tan^5 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^3 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{8c^5 \left(\tan^5 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^3 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \frac{16c^5 \left(\tan^4 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) A}{a^3 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} + \frac{62c^5 \left(\tan^4 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) B}{a^3 f \left(1 + \tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right)^3} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sin(f*x+e))*(c-c*\sin(f*x+e))^5/(a+a*\sin(f*x+e))^3,x)$

[Out] $-c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^5*A+8*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^5*B-16*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^4*A+62*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^4*B-32*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^2*A+128*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)^2*B+c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)*A-8*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*\tan(1/2*f*x+1/2*e)*B-16*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*A+190/3*c^5/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^3*B-63*c^5/a^3/f*\arctan(\tan(1/2*f*x+1/2*e))*A+168*c^5/a^3/f*\arctan(\tan(1/2*f*x+1/2*e))*B+128*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A-128*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B-32*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*A+96*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*B-64*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*A+160*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*B-64*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A+64/3*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*B-256/5*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A+256/5*c^5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B$

maxima [B] time = 0.57, size = 3282, normalized size = 13.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\sin(f*x+e))*(c-c*\sin(f*x+e))^5/(a+a*\sin(f*x+e))^3,x, \text{algorithm}="maxima")$

[Out] $1/15*(B*c^5*((2375*\sin(f*x + e))/(\cos(f*x + e) + 1) + 5347*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9230*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 12622*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 13340*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 11684*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 8050*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 4370*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 1725*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 345*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 544)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 13*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 25*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 38*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 46*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 46*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 38*a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 25*a^3*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 13*a^3*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 5*a^3*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + a^3*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11) + 345*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^3 - A*c^5*((1325*\sin(f*x + e))/(\cos(f*x + e) + 1) + 2673*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3805*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 4329*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 3575*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2275*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 975*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 195*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 12*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 26*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 26*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 20*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 12*a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 5*a^3*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + a^3*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9) + 195*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^3) + 5*B*c^5*((1325*\sin(f*x + e))/(\cos(f*x + e) + 1) + 2673*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3805*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 4329*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 3575*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2275*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 975*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 195*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 304)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 12*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 26*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 26*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 20*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 12*a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 5*a^3*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + a^3*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9) + 195*\arctan(\sin(f*x + e))/(\cos(f*x + e) + 1))/a^3)$

$$\begin{aligned}
& e)^6/(\cos(f*x + e) + 1)^6 + 12*a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 5 \\
& *a^3*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + a^3*\sin(f*x + e)^9/(\cos(f*x + e) \\
& + 1)^9) + 195*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^3) - 30*A*c^5*((10 \\
& 5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 189*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 \\
& + 200*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 160*\sin(f*x + e)^4/(\cos(f*x + \\
& e) + 1)^4 + 75*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\sin(f*x + e)^6/(\cos \\
& (f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 11*a^ \\
& 3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*a^3*\sin(f*x + e)^3/(\cos(f*x + e) \\
& + 1)^3 + 15*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 11*a^3*\sin(f*x + e)^ \\
& 5/(\cos(f*x + e) + 1)^5 + 5*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a^3*si \\
& n(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 15*\arctan(\sin(f*x + e)/(\cos(f*x + e) + \\
& 1))/a^3) + 60*B*c^5*((105*\sin(f*x + e)/(\cos(f*x + e) + 1) + 189*\sin(f*x + \\
& e)^2/(\cos(f*x + e) + 1)^2 + 200*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 160*s \\
& in(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 75*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 \\
& + 15*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e)/ \\
& \cos(f*x + e) + 1) + 11*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*a^3*\sin \\
& (f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1) \\
& ^4 + 11*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5*a^3*\sin(f*x + e)^6/(\cos \\
& (f*x + e) + 1)^6 + a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 15*\arctan(\sin \\
& (f*x + e)/(\cos(f*x + e) + 1))/a^3) - 20*A*c^5*((95*\sin(f*x + e)/(\cos(f*x + \\
& e) + 1) + 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos \\
& f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3 \\
& *\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1) \\
& ^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos \\
& (f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin \\
& (f*x + e)/(\cos(f*x + e) + 1))/a^3) + 20*B*c^5*((95*\sin(f*x + e)/(\cos(f*x + \\
& e) + 1) + 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos \\
& f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3 \\
& *\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1) \\
& ^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos \\
& (f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin \\
& (f*x + e)/(\cos(f*x + e) + 1))/a^3) - 2*A*c^5*(20*\sin(f*x + e)/(\cos(f*x + e) \\
& + 1) + 40*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x \\
& + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*\sin \\
& (f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + \\
& 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x \\
& + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 40*A*c^5*(5*\sin(f \\
& *x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/(a \\
& ^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x \\
& + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + \\
& e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 20* \\
& B*c^5*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) \\
& + 1)^2 + 1)/(a^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + \\
& e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5 \\
& *a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) \\
& + 1)^5) + 30*A*c^5*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/ \\
& \cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a \\
& ^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + \\
& 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/ \\
& \cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 6*B*c^5*(5* \\
& \sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5 \\
& *\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*\sin(f*x + e)/(\cos(f* \\
& x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + \\
& e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^ \\
& 3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5))/f
\end{aligned}$$

$$\begin{aligned}
& 40a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f \\
& * \tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 43470A*c \\
& **5f*x\tan(e/2 + f*x/2)^{*6}/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan \\
& (e/2 + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + \\
& f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)* \\
& *6 + 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 75 \\
& 0a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan \\
& (e/2 + f*x/2) + 30a^{*3}f) - 43470A*c**5f*x\tan(e/2 + f*x/2)^{*5}/(30a* \\
& *3f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390a^{*3}f\tan \\
& (e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + \\
& f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + f*x/2)* \\
& *5 + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390 \\
& a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 35 \\
& 910A*c**5f*x\tan(e/2 + f*x/2)^{*4}/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a* \\
& *3f\tan(e/2 + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan \\
& (e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + \\
& f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)* \\
& *4 + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150* \\
& a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 23625A*c**5f*x\tan(e/2 + f*x/2)^{*3} \\
& /(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390a* \\
& *3f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan \\
& (e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + \\
& f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{* \\
& 3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}* \\
& f) - 12285A*c**5f*x\tan(e/2 + f*x/2)^{*2}/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + \\
& 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a** \\
& 3f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan \\
& (e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + \\
& f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} \\
& + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 4725A*c**5f*x\tan(e/2 + f*x \\
& /2)/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390 \\
& a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f* \\
& \tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 \\
& + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2 \\
&)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a* \\
& *3f) - 945A*c**5f*x/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 \\
& + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2 \\
&)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + \\
& 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a** \\
& 3f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e \\
& /2 + f*x/2) + 30a^{*3}f) - 1950A*c**5\tan(e/2 + f*x/2)^{*10}/(30a^{*3}f\tan(\\
& e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f \\
& *x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} \\
& + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140 \\
& a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan \\
& (e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 9270A*c**5 \\
& * \tan(e/2 + f*x/2)^{*9}/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + \\
& f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)* \\
& *8 + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 13 \\
& 80a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}* \\
& f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan(e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 \\
& + f*x/2) + 30a^{*3}f) - 24780A*c**5\tan(e/2 + f*x/2)^{*8}/(30a^{*3}f\tan(e/ \\
& 2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x \\
& /2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8} + 1140a^{*3}f\tan(e/2 + f*x/2)^{*7} + \\
& 1380a^{*3}f\tan(e/2 + f*x/2)^{*6} + 1380a^{*3}f\tan(e/2 + f*x/2)^{*5} + 1140a* \\
& **3f\tan(e/2 + f*x/2)^{*4} + 750a^{*3}f\tan(e/2 + f*x/2)^{*3} + 390a^{*3}f\tan \\
& (e/2 + f*x/2)^{*2} + 150a^{*3}f\tan(e/2 + f*x/2) + 30a^{*3}f) - 42540A*c**5* \\
& \tan(e/2 + f*x/2)^{*7}/(30a^{*3}f\tan(e/2 + f*x/2)^{*11} + 150a^{*3}f\tan(e/2 + \\
& f*x/2)^{*10} + 390a^{*3}f\tan(e/2 + f*x/2)^{*9} + 750a^{*3}f\tan(e/2 + f*x/2)^{*8}
\end{aligned}$$


```

*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 64180*B*c**5*tan(e/2
+ f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**
10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140
*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f
*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2
+ f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2)
+ 30*a**3*f) + 116360*B*c**5*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/
2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 +
750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a*
**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 173416*B*c**5*tan(e/2
+ f*x/2)**6/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)*
*10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 114
0*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*
f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/
2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2
) + 30*a**3*f) + 190720*B*c**5*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x
/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a
**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*ta
n(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 184448*B*c**5*tan(e/
2 + f*x/2)**4/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 133720*B*c**5*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*
x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9
+ 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*
a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*
tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 +
f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 77768*B*c**5*tan(e/
2 + f*x/2)**2/(30*a**3*f*tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)
**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 11
40*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3
*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e
/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/
2) + 30*a**3*f) + 34540*B*c**5*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)
**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2 + f*x/2)**9 + 7
50*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2)**7 + 1380*a**3
*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 + 1140*a**3*f*tan(
e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3*f*tan(e/2 + f*x
/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7916*B*c**5/(30*a**3*f*
tan(e/2 + f*x/2)**11 + 150*a**3*f*tan(e/2 + f*x/2)**10 + 390*a**3*f*tan(e/2
+ f*x/2)**9 + 750*a**3*f*tan(e/2 + f*x/2)**8 + 1140*a**3*f*tan(e/2 + f*x/2
)**7 + 1380*a**3*f*tan(e/2 + f*x/2)**6 + 1380*a**3*f*tan(e/2 + f*x/2)**5 +
1140*a**3*f*tan(e/2 + f*x/2)**4 + 750*a**3*f*tan(e/2 + f*x/2)**3 + 390*a**3
*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)
), (x*(A + B*sin(e))*(-c*sin(e) + c)**5/(a*sin(e) + a)**3, True))

```

$$3.71 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^4}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=201

$$\frac{a^4 c^4 (A-B) \cos^9(e+fx)}{5f(a \sin(e+fx)+a)^7} - \frac{7c^4(2A-7B) \cos(e+fx)}{2a^3 f} - \frac{7c^4(2A-7B) \cos^3(e+fx)}{6f(a^3 \sin(e+fx)+a^3)} - \frac{7c^4 x(2A-7B)}{2a^3} + \frac{2a^2 c^4(2A-7B)}{15f(a \sin(e+fx)+a)^5}$$

[Out] $-7/2*(2*A-7*B)*c^4*x/a^3-7/2*(2*A-7*B)*c^4*\cos(f*x+e)/a^3/f-1/5*a^4*(A-B)*c^4*\cos(f*x+e)^9/f/(a+a*\sin(f*x+e))^7+2/15*a^2*(2*A-7*B)*c^4*\cos(f*x+e)^7/f/(a+a*\sin(f*x+e))^5-14/15*(2*A-7*B)*c^4*\cos(f*x+e)^5/f/(a+a*\sin(f*x+e))^3-7/6*(2*A-7*B)*c^4*\cos(f*x+e)^3/f/(a^3+a^3*\sin(f*x+e))$

Rubi [A] time = 0.39, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2859, 2680, 2679, 2682, 8}

$$-\frac{7c^4(2A-7B) \cos(e+fx)}{2a^3 f} - \frac{a^4 c^4 (A-B) \cos^9(e+fx)}{5f(a \sin(e+fx)+a)^7} + \frac{2a^2 c^4(2A-7B) \cos^7(e+fx)}{15f(a \sin(e+fx)+a)^5} - \frac{7c^4(2A-7B) \cos^3(e+fx)}{6f(a^3 \sin(e+fx)+a^3)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*SIN[e + f*x])*(c - c*SIN[e + f*x])^4)/(a + a*SIN[e + f*x])^3,x]

[Out] $(-7*(2*A - 7*B)*c^4*x)/(2*a^3) - (7*(2*A - 7*B)*c^4*\cos[e + f*x])/(2*a^3*f) - (a^4*(A - B)*c^4*\cos[e + f*x]^9)/(5*f*(a + a*\sin[e + f*x])^7) + (2*a^2*(2*A - 7*B)*c^4*\cos[e + f*x]^7)/(15*f*(a + a*\sin[e + f*x])^5) - (14*(2*A - 7*B)*c^4*\cos[e + f*x]^5)/(15*f*(a + a*\sin[e + f*x])^3) - (7*(2*A - 7*B)*c^4*\cos[e + f*x]^3)/(6*f*(a^3 + a^3*\sin[e + f*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(g*(g*cos[e + f*x])^(p-1)*(a + b*sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*cos[e + f*x])^(p-2)*(a + b*sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(2*g*(g*cos[e + f*x])^(p-1)*(a + b*sin[e + f*x])^(m+1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m + p + 1)), Int[(g*cos[e + f*x])^(p-2)*(a + b*sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !LtQ[m + p + 1, 0] && IntegerQ[2*m, 2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^4}{(a + a \sin(e + fx))^3} dx &= (a^4 c^4) \int \frac{\cos^8(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^7} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} - \frac{1}{5} (a^3(2A - 7B)c^4) \int \frac{\cos^8(e + fx)}{(a + a \sin(e + fx))^6} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} + \frac{2a^2(2A - 7B)c^4 \cos^7(e + fx)}{15f(a + a \sin(e + fx))^5} + \frac{1}{15} \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^4} dx \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} + \frac{2a^2(2A - 7B)c^4 \cos^7(e + fx)}{15f(a + a \sin(e + fx))^5} - \frac{14a^2(2A - 7B)c^4 \cos^5(e + fx)}{15f(a + a \sin(e + fx))^3} \\ &= -\frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} + \frac{2a^2(2A - 7B)c^4 \cos^7(e + fx)}{15f(a + a \sin(e + fx))^5} - \frac{14a^2(2A - 7B)c^4 \cos^5(e + fx)}{15f(a + a \sin(e + fx))^3} \\ &= -\frac{7(2A - 7B)c^4 \cos(e + fx)}{2a^3 f} - \frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} + \frac{2a^2(2A - 7B)c^4 \cos^7(e + fx)}{15f(a + a \sin(e + fx))^5} \\ &= -\frac{7(2A - 7B)c^4 x}{2a^3} - \frac{7(2A - 7B)c^4 \cos(e + fx)}{2a^3 f} - \frac{a^4(A - B)c^4 \cos^9(e + fx)}{5f(a + a \sin(e + fx))^7} + \frac{2a^2(2A - 7B)c^4 \cos^7(e + fx)}{15f(a + a \sin(e + fx))^5} \end{aligned}$$

Mathematica [A] time = 1.61, size = 348, normalized size = 1.73

$$(c - c \sin(e + fx))^4 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(384(A - B) \sin\left(\frac{1}{2}(e + fx)\right) - 210(2A - 7B)(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^4)/(a + a*Sin[e + f*x])^3, x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^4*(384*(A - B)*Sin[(e + f*x)/2] - 192*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 128*(8*A - 13*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 64*(8*A - 13*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 64*(29*A - 79*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(a + a*Sin[e + f*x])^3
```

$$\frac{((e + fx)/2) * (\cos((e + fx)/2) + \sin((e + fx)/2))^4 - 210 * (2A - 7B) * (e + fx) * (\cos((e + fx)/2) + \sin((e + fx)/2))^5 - 60 * (A - 7B) * \cos[e + fx] * (\cos((e + fx)/2) + \sin((e + fx)/2))^5 - 15 * B * (\cos((e + fx)/2) + \sin((e + fx)/2))^5 * \sin[2 * (e + fx)]}{(60 * a^3 * f * (\cos((e + fx)/2) - \sin((e + fx)/2))^8 * (1 + \sin[e + fx])^3}$$

fricas [B] time = 0.46, size = 392, normalized size = 1.95

$$\frac{15 B c^4 \cos(fx + e)^5 - 30(A - 6B)c^4 \cos(fx + e)^4 + 420(2A - 7B)c^4 fx + 96(A - B)c^4 - (105(2A - 7B)c^4}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1/30 * (15 * B * c^4 * \cos(f * x + e)^5 - 30 * (A - 6 * B) * c^4 * \cos(f * x + e)^4 + 420 * (2 * A - 7 * B) * c^4 * f * x + 96 * (A - B) * c^4 - (105 * (2 * A - 7 * B) * c^4 * f * x + (554 * A - 1819 * B) * c^4) * \cos(f * x + e)^3 - (315 * (2 * A - 7 * B) * c^4 * f * x - 2 * (134 * A - 619 * B) * c^4) * \cos(f * x + e)^2 + 6 * (35 * (2 * A - 7 * B) * c^4 * f * x + 2 * (74 * A - 249 * B) * c^4) * \cos(f * x + e) - (15 * B * c^4 * \cos(f * x + e)^4 + 15 * (2 * A - 11 * B) * c^4 * \cos(f * x + e)^3 - 420 * (2 * A - 7 * B) * c^4 * f * x + 96 * (A - B) * c^4 + (105 * (2 * A - 7 * B) * c^4 * f * x - 2 * (262 * A - 827 * B) * c^4) * \cos(f * x + e)^2 - 6 * (35 * (2 * A - 7 * B) * c^4 * f * x + 2 * (66 * A - 241 * B) * c^4) * \cos(f * x + e)) * \sin(f * x + e)}{(a^3 * f * \cos(f * x + e)^3 + 3 * a^3 * f * \cos(f * x + e)^2 - 2 * a^3 * f * \cos(f * x + e) - 4 * a^3 * f + (a^3 * f * \cos(f * x + e)^2 - 2 * a^3 * f * \cos(f * x + e) - 4 * a^3 * f) * \sin(f * x + e))}$

giac [A] time = 0.26, size = 305, normalized size = 1.52

$$\frac{105(2Ac^4 - 7Bc^4)(fx + e)}{a^3} - \frac{30 \left(Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 2Ac^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 14Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - Bc^4 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 2Ac^4 + 14Bc^4 \right)}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^2 a^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] $\frac{-1/30 * (105 * (2 * A * c^4 - 7 * B * c^4) * (f * x + e) / a^3 - 30 * (B * c^4 * \tan(1/2 * f * x + 1/2 * e))^3 - 2 * A * c^4 * \tan(1/2 * f * x + 1/2 * e)^2 + 14 * B * c^4 * \tan(1/2 * f * x + 1/2 * e)^2 - B * c^4 * \tan(1/2 * f * x + 1/2 * e) - 2 * A * c^4 + 14 * B * c^4) / ((\tan(1/2 * f * x + 1/2 * e))^2 + 1)^2 * a^3 + 32 * (15 * A * c^4 * \tan(1/2 * f * x + 1/2 * e)^4 - 45 * B * c^4 * \tan(1/2 * f * x + 1/2 * e)^4 + 60 * A * c^4 * \tan(1/2 * f * x + 1/2 * e)^3 - 210 * B * c^4 * \tan(1/2 * f * x + 1/2 * e)^3 + 130 * A * c^4 * \tan(1/2 * f * x + 1/2 * e)^2 - 380 * B * c^4 * \tan(1/2 * f * x + 1/2 * e)^2 + 80 * A * c^4 * \tan(1/2 * f * x + 1/2 * e) - 250 * B * c^4 * \tan(1/2 * f * x + 1/2 * e) + 19 * A * c^4 - 59 * B * c^4) / (a^3 * (\tan(1/2 * f * x + 1/2 * e) + 1)^5) / f}$

maple [B] time = 0.50, size = 474, normalized size = 2.36

$$\frac{c^4 \left(\tan^3\left(\frac{fx}{2} + \frac{e}{2}\right) \right) B}{a^3 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} - \frac{2c^4 \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) A}{a^3 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} + \frac{14c^4 \left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) B}{a^3 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} - \frac{c^4 B \tan\left(\frac{fx}{2} + \frac{e}{2}\right)}{a^3 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right)^2} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)

[Out] $\frac{c^4/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^2 * \tan(1/2*f*x+1/2*e)^3 * B - 2*c^4/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^2 * \tan(1/2*f*x+1/2*e)^2 * A + 14*c^4/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^2 * \tan(1/2*f*x+1/2*e) * B - c^4/a^3/f/(1+\tan(1/2*f*x+1/2*e))^2)^2 * \tan(1/2*f*x+1/2*e) * B}{a^3 * f * (1 + \tan^2(1/2 * f * x + 1/2 * e))^5}$

$$\begin{aligned} & \frac{1}{2}e)^2)^2 \tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)^2 B - c^4/a^3/f / \left(1 + \tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)^2\right)^2 B * \tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) - 2*c^4/a^3/f / \left(1 + \tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)^2\right)^2 A + 14*c^4/a^3/f / \left(1 + \tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)^2\right)^2 B + 49*c^4/a^3/f * \arctan\left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)\right) * B - 14*c^4/a^3/f * \arctan\left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right)\right) * A + 64*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^4 * A - 64*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^4 * B - 16*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right) * A + 48*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right) * B - 128/5*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^5 * A + 128/5*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^5 * B - 128/3*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^3 * A + 64/3*c^4/a^3/f / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^3 * B + 32*c^4/a^3/f * B / \left(\tan\left(\frac{1}{2}f*x + \frac{1}{2}e\right) + 1\right)^2 \end{aligned}$$

maxima [B] time = 0.52, size = 2394, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{1}{15} * (B * c^4 * ((1325 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 2673 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 3805 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 4329 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 3575 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 2275 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 975 * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + 195 * \sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + 304) / (a^3 + 5 * a^3 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 12 * a^3 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 20 * a^3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 26 * a^3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 26 * a^3 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 20 * a^3 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 12 * a^3 * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7 + 5 * a^3 * \sin(f*x + e)^8 / (\cos(f*x + e) + 1)^8 + a^3 * \sin(f*x + e)^9 / (\cos(f*x + e) + 1)^9) + 195 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a^3) - 6 * A * c^4 * ((105 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 189 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 200 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 160 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 75 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 15 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 24) / (a^3 + 5 * a^3 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 11 * a^3 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 15 * a^3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 15 * a^3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 11 * a^3 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 5 * a^3 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + a^3 * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7) + 15 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a^3) + 24 * B * c^4 * ((105 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 189 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 200 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 160 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 75 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 15 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + 24) / (a^3 + 5 * a^3 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 11 * a^3 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 15 * a^3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 15 * a^3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 11 * a^3 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5 + 5 * a^3 * \sin(f*x + e)^6 / (\cos(f*x + e) + 1)^6 + a^3 * \sin(f*x + e)^7 / (\cos(f*x + e) + 1)^7) + 15 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a^3) - 8 * A * c^4 * ((95 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 145 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 75 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 15 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 22) / (a^3 + 5 * a^3 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 10 * a^3 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 10 * a^3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 5 * a^3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + a^3 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5) + 15 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a^3) + 12 * B * c^4 * ((95 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 145 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 75 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 15 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 22) / (a^3 + 5 * a^3 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 10 * a^3 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 10 * a^3 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 5 * a^3 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + a^3 * \sin(f*x + e)^5 / (\cos(f*x + e) + 1)^5) + 15 * \arctan(\sin(f*x + e) / (\cos(f*x + e) + 1)) / a^3) - 2 * A * c^4 * (20 * \sin(f*x + e) / (\cos(f*x + e) + 1) + 40 * \sin(f*x + e)^2 / (\cos(f*x + e) + 1)^2 + 30 * \sin(f*x + e)^3 / (\cos(f*x + e) + 1)^3 + 15 * \sin(f*x + e)^4 / (\cos(f*x + e) + 1)^4 + 7) / (a^3 + 5 * a^3 * \sin(f$


```
*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1
0*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x +
e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 24*A*c^4*(5*sin(f*x
+ e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3
+ 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e
)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 16*B*
c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) +
1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a
^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) +
1)^5) + 24*A*c^4*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3
*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)
^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) - 6*B*c^4*(5*si
n(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*s
in(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x
+ e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e
)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.71, size = 419, normalized size = 2.08

$$\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{286Ac^4}{3} - \frac{1007Bc^4}{3}\right) + \frac{334Ac^4}{15} - \frac{1154Bc^4}{15} + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 (16Ac^4 - 49Bc^4) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7 (6f \left(a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 12a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + a^3\right)}{f \left(a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^9 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^8 + 12a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 10a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^4)/(a + a*sin(e + f*x))^3,x)
[Out] - (tan(e/2 + (f*x)/2)*((286*A*c^4)/3 - (1007*B*c^4)/3) + (334*A*c^4)/15 - (
1154*B*c^4)/15 + tan(e/2 + (f*x)/2)^8*(16*A*c^4 - 49*B*c^4) + tan(e/2 + (f*
x)/2)^7*(66*A*c^4 - 243*B*c^4) + tan(e/2 + (f*x)/2)^6*((542*A*c^4)/3 - (174
1*B*c^4)/3) + tan(e/2 + (f*x)/2)^5*((706*A*c^4)/3 - (2621*B*c^4)/3) + tan(e
/2 + (f*x)/2)^3*((794*A*c^4)/3 - (2875*B*c^4)/3) + tan(e/2 + (f*x)/2)^2*((1
006*A*c^4)/5 - (3401*B*c^4)/5) + tan(e/2 + (f*x)/2)^4*((1718*A*c^4)/5 - (56
33*B*c^4)/5))/(f*(12*a^3*tan(e/2 + (f*x)/2)^2 + 20*a^3*tan(e/2 + (f*x)/2)^3
+ 26*a^3*tan(e/2 + (f*x)/2)^4 + 26*a^3*tan(e/2 + (f*x)/2)^5 + 20*a^3*tan(e
/2 + (f*x)/2)^6 + 12*a^3*tan(e/2 + (f*x)/2)^7 + 5*a^3*tan(e/2 + (f*x)/2)^8
+ a^3*tan(e/2 + (f*x)/2)^9 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (7*c^4*atan
((7*c^4*tan(e/2 + (f*x)/2)*(2*A - 7*B))/(14*A*c^4 - 49*B*c^4))*(2*A - 7*B)
)/(a^3*f)
```

sympy [A] time = 74.02, size = 7337, normalized size = 36.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^4/(a+a*sin(f*x+e))^3,x)
[Out] Piecewise((-210*A*c**4*f*x*tan(e/2 + f*x/2)**9/(30*a**3*f*tan(e/2 + f*x/2)*
*9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*
a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*ta
n(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f
*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) - 1050*A*c**4*f*x*tan(e
/2 + f*x/2)**8/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)
```



```

*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
*3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 735*B*c**4*f*x/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 +
f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6
+ 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a*
**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(
e/2 + f*x/2) + 30*a**3*f) + 1470*B*c**4*tan(e/2 + f*x/2)**8/(30*a**3*f*tan(
e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x
/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 +
780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*
f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 7290*B*c
**4*tan(e/2 + f*x/2)**7/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2
+ f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)
**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600
*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*t
an(e/2 + f*x/2) + 30*a**3*f) + 17410*B*c**4*tan(e/2 + f*x/2)**6/(30*a**3*f*
tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 +
f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**
5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a
**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2621
0*B*c**4*tan(e/2 + f*x/2)**5/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*ta
n(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f
*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4
+ 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**
3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 33798*B*c**4*tan(e/2 + f*x/2)**4/(30*a*
**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(
e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x
/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 +
360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) +
28750*B*c**4*tan(e/2 + f*x/2)**3/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3
*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/
2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)
)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 15
0*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 20406*B*c**4*tan(e/2 + f*x/2)**2/(
30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f
*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2
+ f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)*
**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3
*f) + 10070*B*c**4*tan(e/2 + f*x/2)/(30*a**3*f*tan(e/2 + f*x/2)**9 + 150*a*
**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**7 + 600*a**3*f*tan(
e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a**3*f*tan(e/2 + f*x
/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan(e/2 + f*x/2)**2 +
150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f) + 2308*B*c**4/(30*a**3*f*tan(e/2 +
f*x/2)**9 + 150*a**3*f*tan(e/2 + f*x/2)**8 + 360*a**3*f*tan(e/2 + f*x/2)**
7 + 600*a**3*f*tan(e/2 + f*x/2)**6 + 780*a**3*f*tan(e/2 + f*x/2)**5 + 780*a
**3*f*tan(e/2 + f*x/2)**4 + 600*a**3*f*tan(e/2 + f*x/2)**3 + 360*a**3*f*tan
(e/2 + f*x/2)**2 + 150*a**3*f*tan(e/2 + f*x/2) + 30*a**3*f), Ne(f, 0)), (x
(A + B*sin(e))*(-c*sin(e) + c)**4/(a*sin(e) + a)**3, True))

```

$$3.72 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^3}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=153

$$\frac{c^3(A-6B) \cos(e+fx)}{a^3 f} - \frac{a^3 c^3(A-B) \cos^7(e+fx)}{5f(a \sin(e+fx)+a)^6} - \frac{2a^3 c^3(A-6B) \cos^3(e+fx)}{3f(a^3 \sin(e+fx)+a^3)^2} - \frac{c^3 x(A-6B)}{a^3} + \frac{2ac^3(A-6B)}{15f(a \sin(e+fx)+a)^4}$$

[Out] $-(A-6*B)*c^3*x/a^3-(A-6*B)*c^3*\cos(f*x+e)/a^3/f-1/5*a^3*(A-B)*c^3*\cos(f*x+e)^7/f/(a+a*\sin(f*x+e))^6+2/15*a*(A-6*B)*c^3*\cos(f*x+e)^5/f/(a+a*\sin(f*x+e))^4-2/3*a^3*(A-6*B)*c^3*\cos(f*x+e)^3/f/(a^3+a^3*\sin(f*x+e))^2$

Rubi [A] time = 0.33, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2680, 2682, 8}

$$\frac{c^3(A-6B) \cos(e+fx)}{a^3 f} - \frac{a^3 c^3(A-B) \cos^7(e+fx)}{5f(a \sin(e+fx)+a)^6} - \frac{2a^3 c^3(A-6B) \cos^3(e+fx)}{3f(a^3 \sin(e+fx)+a^3)^2} - \frac{c^3 x(A-6B)}{a^3} + \frac{2ac^3(A-6B)}{15f(a \sin(e+fx)+a)^4}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]

[Out] $-(((A-6*B)*c^3*x)/a^3)-((A-6*B)*c^3*\cos[e+f*x])/(a^3*f)-(a^3*(A-B)*c^3*\cos[e+f*x]^7)/(5*f*(a+a*\sin[e+f*x])^6)+(2*a*(A-6*B)*c^3*\cos[e+f*x]^5)/(15*f*(a+a*\sin[e+f*x])^4)-(2*a^3*(A-6*B)*c^3*\cos[e+f*x]^3)/(3*f*(a^3+a^3*\sin[e+f*x])^2)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m+p+1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m+p+1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m+p+1, 0] && !ILtQ[m+p+1, 0] && IntegerQ[2*p]

Rule 2682

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1))/(b*f*(p-1)), x] + Dist[g^2/a, Int[(g*Cos[e + f*x])^(p-2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(a*b*(2*m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m+p], 0]) && NeQ[2*m+p+1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^3}{(a + a \sin(e + fx))^3} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^6} dx \\ &= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{5f(a + a \sin(e + fx))^6} - \frac{1}{5} (a^2(A - 6B)c^3) \int \frac{\cos^6(e + fx)}{(a + a \sin(e + fx))^5} dx \\ &= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{5f(a + a \sin(e + fx))^6} + \frac{2a(A - 6B)c^3 \cos^5(e + fx)}{15f(a + a \sin(e + fx))^4} + \frac{1}{3} (A - 6B)c^3 \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{a^3(A - B)c^3 \cos^7(e + fx)}{5f(a + a \sin(e + fx))^6} + \frac{2a(A - 6B)c^3 \cos^5(e + fx)}{15f(a + a \sin(e + fx))^4} - \frac{2(A - 6B)c^3}{3af} \int \frac{\cos^3(e + fx)}{(a + a \sin(e + fx))^2} dx \\ &= -\frac{(A - 6B)c^3 \cos(e + fx)}{a^3 f} - \frac{a^3(A - B)c^3 \cos^7(e + fx)}{5f(a + a \sin(e + fx))^6} + \frac{2a(A - 6B)c^3}{15f(a + a \sin(e + fx))^4} \\ &= -\frac{(A - 6B)c^3 x}{a^3} - \frac{(A - 6B)c^3 \cos(e + fx)}{a^3 f} - \frac{a^3(A - B)c^3 \cos^7(e + fx)}{5f(a + a \sin(e + fx))^6} \end{aligned}$$

Mathematica [B] time = 1.09, size = 308, normalized size = 2.01

$$(c - c \sin(e + fx))^3 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(48(A - B) \sin\left(\frac{1}{2}(e + fx)\right) - 15(A - 6B)(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(48*(A - B)*Sin[(e + f*x)/2] - 24*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(11*A - 21*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(11*A - 21*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(23*A - 93*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 15*(A - 6*B)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*B*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)*(c - c*Sin[e + f*x])^3/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(1 + Sin[e + f*x])^3)
```

fricas [B] time = 0.44, size = 338, normalized size = 2.21

$$15 Bc^3 \cos^4(fx + e) + 60(A - 6B)c^3 fx + 24(A - B)c^3 - (15(A - 6B)c^3 fx + (46A - 231B)c^3) \cos^3(fx + e) - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

[Out] $1/15*(15*B*c^3*\cos(f*x + e)^4 + 60*(A - 6*B)*c^3*f*x + 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x + (46*A - 231*B)*c^3)*\cos(f*x + e)^3 - (45*(A - 6*B)*c^3*f*x - 2*(A - 66*B)*c^3)*\cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(6*A - 31*B)*c^3)*\cos(f*x + e) + (15*B*c^3*\cos(f*x + e)^3 + 60*(A - 6*B)*c^3*f*x - 24*(A - B)*c^3 - (15*(A - 6*B)*c^3*f*x - 2*(23*A - 108*B)*c^3)*\cos(f*x + e)^2 + 6*(5*(A - 6*B)*c^3*f*x + 2*(4*A - 29*B)*c^3)*\cos(f*x + e))*\sin(f*x + e)/(a^3*f*\cos(f*x + e)^3 + 3*a^3*f*\cos(f*x + e)^2 - 2*a^3*f*\cos(f*x + e) - 4*a^3*f + (a^3*f*\cos(f*x + e)^2 - 2*a^3*f*\cos(f*x + e) - 4*a^3*f)*\sin(f*x + e))$

giac [A] time = 0.21, size = 226, normalized size = 1.48

$$\frac{30 B c^3}{\left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 1\right) a^3} - \frac{15 (A c^3 - 6 B c^3) (f x + e)}{a^3} - \frac{4 \left(15 A c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 45 B c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 30 A c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 210 B c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 100 A c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 420 B c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 + 50 A c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) - 270 B c^3 \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 13 A c^3 - 63 B c^3\right)}{a^3 (\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1)^5}}{15 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] $1/15*(30*B*c^3/((\tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(A*c^3 - 6*B*c^3)*(f*x + e)/a^3 - 4*(15*A*c^3*\tan(1/2*f*x + 1/2*e)^4 - 45*B*c^3*\tan(1/2*f*x + 1/2*e)^4 + 30*A*c^3*\tan(1/2*f*x + 1/2*e)^3 - 210*B*c^3*\tan(1/2*f*x + 1/2*e)^3 + 100*A*c^3*\tan(1/2*f*x + 1/2*e)^2 - 420*B*c^3*\tan(1/2*f*x + 1/2*e)^2 + 50*A*c^3*\tan(1/2*f*x + 1/2*e) - 270*B*c^3*\tan(1/2*f*x + 1/2*e) + 13*A*c^3 - 63*B*c^3)/(a^3*(\tan(1/2*f*x + 1/2*e) + 1)^5))/f$

maple [B] time = 0.48, size = 323, normalized size = 2.11

$$\frac{2c^3 B}{a^3 f \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)} - \frac{2c^3 \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right) A}{a^3 f} + \frac{12c^3 \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right) B}{a^3 f} + \frac{32c^3 A}{a^3 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)

[Out] $2*c^3/a^3/f*B/(1+\tan(1/2*f*x+1/2*e)^2)-2*c^3/a^3/f*\arctan(\tan(1/2*f*x+1/2*e))*A+12*c^3/a^3/f*\arctan(\tan(1/2*f*x+1/2*e))*B+32*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A-32*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B+8*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*A+8*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*B-4*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*A+12*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*B-64/5*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A+64/5*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B-80/3*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A+16*c^3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*B$

maxima [B] time = 0.52, size = 1679, normalized size = 10.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $2/15*(3*B*c^3*((105*\sin(f*x + e))/(\cos(f*x + e) + 1) + 189*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 200*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 160*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 75*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 11*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 15*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 11*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 15*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1)$

$$\begin{aligned} & e^3/(\cos(f*x + e) + 1)^3 + 15*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 11 \\ & *a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5*a^3*\sin(f*x + e)^6/(\cos(f*x + \\ & e) + 1)^6 + a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 15*\arctan(\sin(f*x + \\ & e)/(\cos(f*x + e) + 1))/a^3 - A*c^3*((95*\sin(f*x + e)/(\cos(f*x + e) + 1) + \\ & 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + \\ & 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*\sin(f*x + \\ & e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^ \\ & 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) \\ & + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin(f*x + e)/ \\ & (\cos(f*x + e) + 1))/a^3 + 3*B*c^3*((95*\sin(f*x + e)/(\cos(f*x + e) + 1) + 1 \\ & 45*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + \\ & 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*\sin(f*x + \\ & e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3 \\ & *\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + \\ & 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*\arctan(\sin(f*x + e)/ \\ & (\cos(f*x + e) + 1))/a^3 - A*c^3*(20*\sin(f*x + e)/(\cos(f*x + e) + 1) + 40*si \\ & n(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 \\ & + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*\sin(f*x + e)/(co \\ & s(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f \\ & *x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 \\ & + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 6*A*c^3*(5*\sin(f*x + e)/(\cos(f \\ & *x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*\sin \\ & (f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + \\ & 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x \\ & + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 6*B*c^3*(5*\sin(f* \\ & x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/(a^ \\ & 3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x \\ & + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + \\ & e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 9*A \\ & c^3*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1 \\ &)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*\sin(f*x + e)/ \\ & (\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*si \\ & n(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1) \\ & ^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 3*B*c^3*(5*\sin(f*x + e)/(co \\ & s(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/ \\ & (\cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10 \\ & *a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + \\ & e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5 \\ & /(\cos(f*x + e) + 1)^5))/f \end{aligned}$$

mupad [B] time = 14.36, size = 333, normalized size = 2.18

$$\frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{40Ac^3}{3} - 82Bc^3\right) + \frac{52Ac^3}{15} - \frac{94Bc^3}{5} + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 (4Ac^3 - 12Bc^3) + \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 (8Ac^3 - 52Bc^3)}{f \left(a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^7 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^6 + 11a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 + 15a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 11a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 5a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + a^3 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + a^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)

[Out] - (tan(e/2 + (f*x)/2)*((40*A*c^3)/3 - 82*B*c^3) + (52*A*c^3)/15 - (94*B*c^3)/5 + tan(e/2 + (f*x)/2)^6*(4*A*c^3 - 12*B*c^3) + tan(e/2 + (f*x)/2)^5*(8*A*c^3 - 52*B*c^3) + tan(e/2 + (f*x)/2)^3*((64*A*c^3)/3 - 148*B*c^3) + tan(e/2 + (f*x)/2)^4*((92*A*c^3)/3 - 134*B*c^3) + tan(e/2 + (f*x)/2)^2*((452*A*c^3)/15 - (744*B*c^3)/5))/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2 + (f*x)/2))) - (2*c^3*atan((2*c^3*tan(e/2 + (f*x)/2)*(A - 6*B))/(2*A*c^3 - 12*B*c^3))*(A - 6*B))/(a^3*f)

sympy [A] time = 46.58, size = 4665, normalized size = 30.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**3/(a+a*sin(f*x+e))**3,x)

[Out] Piecewise((-15*A*c**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f*x*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 225*A*c**3*f*x*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 165*A*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 75*A*c**3*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 120*A*c**3*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 460*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 320*A*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 452*A*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 200*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 52*A*c**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +

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225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**
3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/
2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2
)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*x*tan(e/2 + f*x/2)**6/
(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f
*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2
+ f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225
*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t
an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*
f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2
+ f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)*
**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1350*B*c**3*f*x*tan(e/2 + f*x/2)**3/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) + 990*B*c**3*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)
)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*
a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 450*B*c**3*f*
x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x
/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 +
225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 90*B*c**3*f*x/(15*a**3*f*tan(e/2 + f*x/2)*
**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan
(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 180*B*c**3*tan
(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2
)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 22
5*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*t
an(e/2 + f*x/2) + 15*a**3*f) + 870*B*c**3*tan(e/2 + f*x/2)**5/(15*a**3*f*ta
n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*
x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) +
2010*B*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f
*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2
+ f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)*
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 2220*B*c**3*tan(e/2 + f*x/2)
**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a*
**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(
e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/
2) + 15*a**3*f) + 2232*B*c**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/
2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 22
5*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 1230*B*c**3
*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/
2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 2
25*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 282*B*c**3/(15*a**3*f*tan(e/2 + f*x/2)**7 +
75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*
f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2
+ f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A +
B*sin(e))*(-c*sin(e) + c)**3/(a*sin(e) + a)**3, True))

```

$$3.73 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^2}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=110

$$\frac{2Bc^2 \cos(e+fx)}{f(a^3 \sin(e+fx) + a^3)} + \frac{Bc^2 x}{a^3} - \frac{a^2 c^2 (A-B) \cos^5(e+fx)}{5f(a \sin(e+fx) + a)^5} - \frac{2Bc^2 \cos^3(e+fx)}{3f(a \sin(e+fx) + a)^3}$$

[Out] $B*c^2*x/a^3-1/5*a^2*(A-B)*c^2*\cos(f*x+e)^5/f/(a+a*\sin(f*x+e))^5-2/3*B*c^2*\cos(f*x+e)^3/f/(a+a*\sin(f*x+e))^3+2*B*c^2*\cos(f*x+e)/f/(a^3+a^3*\sin(f*x+e))$

Rubi [A] time = 0.26, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2680, 8}

$$-\frac{a^2 c^2 (A-B) \cos^5(e+fx)}{5f(a \sin(e+fx) + a)^5} + \frac{2Bc^2 \cos(e+fx)}{f(a^3 \sin(e+fx) + a^3)} + \frac{Bc^2 x}{a^3} - \frac{2Bc^2 \cos^3(e+fx)}{3f(a \sin(e+fx) + a)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^2]/(a + a*\text{Sin}[e + f*x])^3, x]$

[Out] $(B*c^2*x)/a^3 - (a^2*(A - B)*c^2*\text{Cos}[e + f*x]^5)/(5*f*(a + a*\text{Sin}[e + f*x])^5) - (2*B*c^2*\text{Cos}[e + f*x]^3)/(3*f*(a + a*\text{Sin}[e + f*x])^3) + (2*B*c^2*\text{Cos}[e + f*x])/(f*(a^3 + a^3*\text{Sin}[e + f*x]))$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2680

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] \rightarrow \text{Simp}[(2*g*(g*\text{Cos}[e + f*x])^{p-1}*(a + b*\text{Sin}[e + f*x])^{m+1})/(b*f*(2*m + p + 1)), x] + \text{Dist}[(g^2*(p-1))/(b^2*(2*m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^{p-2}*(a + b*\text{Sin}[e + f*x])^{m+2}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LeQ}[m, -2] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& !\text{ILtQ}[m + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 2859

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(g*\text{Cos}[e + f*x])^{p+1}*(a + b*\text{Sin}[e + f*x])^m]/(a*f*g*(2*m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^{m+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{LtQ}[m, -1] || \text{ILtQ}[\text{Simplify}[m + p], 0]) \&\& \text{NeQ}[2*m + p + 1, 0]$

Rule 2967

$\text{Int}[(a_. + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{2*m}*(c + d*\text{Sin}[e + f*x])^{n-m}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^2}{(a + a \sin(e + fx))^3} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^5} dx \\
&= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{5f(a + a \sin(e + fx))^5} + (aBc^2) \int \frac{\cos^4(e + fx)}{(a + a \sin(e + fx))^4} dx \\
&= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{5f(a + a \sin(e + fx))^5} - \frac{2Bc^2 \cos^3(e + fx)}{3f(a + a \sin(e + fx))^3} - \frac{(Bc^2) \int \frac{\cos^2(e + fx)}{(a + a \sin(e + fx))^2} dx}{f(a + a \sin(e + fx))} \\
&= -\frac{a^2(A - B)c^2 \cos^5(e + fx)}{5f(a + a \sin(e + fx))^5} - \frac{2Bc^2 \cos^3(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{2Bc^2 \cos(e + fx)}{f(a + a \sin(e + fx))} \\
&= \frac{Bc^2 x}{a^3} - \frac{a^2(A - B)c^2 \cos^5(e + fx)}{5f(a + a \sin(e + fx))^5} - \frac{2Bc^2 \cos^3(e + fx)}{3f(a + a \sin(e + fx))^3} + \frac{2Bc^2 \cos(e + fx)}{f(a + a \sin(e + fx))}
\end{aligned}$$

Mathematica [B] time = 0.70, size = 272, normalized size = 2.47

$$(c - c \sin(e + fx))^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(24(A - B) \sin\left(\frac{1}{2}(e + fx)\right) + 2(3A - 43B) \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*Sin[(e + f*x)/2] - 12*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 8*(3*A - 8*B)*Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*(3*A - 8*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(3*A - 43*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 15*B*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^2/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(1 + Sin[e + f*x])^3)

fricas [B] time = 0.42, size = 279, normalized size = 2.54

$$\frac{60 Bc^2 fx - (15 Bc^2 fx - (3A - 43B)c^2) \cos(fx + e)^3 - 12(A - B)c^2 - (45 Bc^2 fx - (9A + 11B)c^2) \cos(fx + e)^2}{15 \left(a^3 f \cos(fx + e) \right)^3 + 3 a^3 f \cos(fx + e)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*(60*B*c^2*f*x - (15*B*c^2*f*x - (3*A - 43*B)*c^2)*cos(f*x + e)^3 - 12*(A - B)*c^2 - (45*B*c^2*f*x - (9*A + 11*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x - (A - 11*B)*c^2)*cos(f*x + e) + (60*B*c^2*f*x + 12*(A - B)*c^2 - (15*B*c^2*f*x + (3*A - 43*B)*c^2)*cos(f*x + e)^2 + 6*(5*B*c^2*f*x + (A + 9*B)*c^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e))^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)

giac [A] time = 0.21, size = 159, normalized size = 1.45

$$\frac{15(fx+e)Bc^2}{a^3} - \frac{2 \left(15Ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 60Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 30Ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 170Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 100Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 10Ac^2 \right)}{a^3 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^5}$$

15 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] $\frac{1}{15} \cdot (15 \cdot (f \cdot x + e) \cdot B \cdot c^2 / a^3 - 2 \cdot (15 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 - 15 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 - 60 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 30 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 170 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 100 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 3 \cdot A \cdot c^2 - 23 \cdot B \cdot c^2) / (a^3 \cdot (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^5) / f$

maple [B] time = 0.47, size = 249, normalized size = 2.26

$$\frac{2c^2B \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{a^3f} + \frac{16c^2A}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^4} - \frac{16c^2B}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^4} + \frac{2c^2A}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \frac{2c^2A}{a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)

[Out] $2 \cdot c^2 / a^3 / f \cdot B \cdot \arctan(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) + 16 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^4 \cdot A - 16 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^4 \cdot B - 2 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \cdot A + 2 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \cdot B - 32 / 5 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^5 \cdot A + 32 / 5 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^5 \cdot B - 16 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot A + 32 / 3 \cdot c^2 / a^3 / f / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot B + 8 \cdot c^2 / a^3 / f \cdot A / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2$

maxima [B] time = 0.47, size = 1134, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $\frac{2}{15} \cdot (B \cdot c^2 \cdot ((95 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 145 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 75 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 15 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 22) / (a^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) + 15 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1)) / a^3) - A \cdot c^2 \cdot (20 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 40 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 30 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 15 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 7) / (a^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) - 2 \cdot A \cdot c^2 \cdot (5 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 1) / (a^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) + 4 \cdot B \cdot c^2 \cdot (5 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 1) / (a^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) + 6 \cdot A \cdot c^2 \cdot (5 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 5 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 5 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 1) / (a^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 10 \cdot a^3 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 5 \cdot a^3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^3 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) - 3 \cdot B \cdot c^2 \cdot (5 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 5 \cdot \sin$


```

*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 340*B*c**2*tan(e/2 + f
*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 1
50*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*
tan(e/2 + f*x/2) + 15*a**3*f) + 200*B*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(
e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/
2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a*
**3*f) + 46*B*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*si
n(e) + c)**2/(a*sin(e) + a)**3, True))

```

$$3.74 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=103

$$\frac{c(A+4B) \cos(e+fx)}{15f(a^3 \sin(e+fx)+a^3)} + \frac{ac(A-11B) \cos(e+fx)}{15f(a^2 \sin(e+fx)+a^2)^2} - \frac{2c(A-B) \cos(e+fx)}{5f(a \sin(e+fx)+a)^3}$$

[Out] $-2/5*(A-B)*c*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^3+1/15*a*(A-11*B)*c*\cos(f*x+e)/f/(a^2+a^2*\sin(f*x+e))^2+1/15*(A+4*B)*c*\cos(f*x+e)/f/(a^3+a^3*\sin(f*x+e))$

Rubi [A] time = 0.23, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2967, 2857, 2750, 2648}

$$\frac{c(A+4B) \cos(e+fx)}{15f(a^3 \sin(e+fx)+a^3)} + \frac{ac(A-11B) \cos(e+fx)}{15f(a^2 \sin(e+fx)+a^2)^2} - \frac{2c(A-B) \cos(e+fx)}{5f(a \sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]

[Out] $(-2*(A - B)*c*\text{Cos}[e + f*x])/(5*f*(a + a*\text{Sin}[e + f*x])^3) + (a*(A - 11*B)*c*\text{Cos}[e + f*x])/(15*f*(a^2 + a^2*\text{Sin}[e + f*x])^2) + ((A + 4*B)*c*\text{Cos}[e + f*x])/(15*f*(a^3 + a^3*\text{Sin}[e + f*x]))$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_) + (f_)*(x_)]^2*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))}{(a + a \sin(e + fx))^3} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(a + a \sin(e + fx))^4} dx \\
&= \frac{2(A - B)c \cos(e + fx)}{5f(a + a \sin(e + fx))^3} - \frac{c \int \frac{aA - 6aB + 5aB \sin(e + fx)}{(a + a \sin(e + fx))^2} dx}{5a^2} \\
&= \frac{2(A - B)c \cos(e + fx)}{5f(a + a \sin(e + fx))^3} + \frac{(A - 11B)c \cos(e + fx)}{15af(a + a \sin(e + fx))^2} - \frac{((A + 4B)c \cos(e + fx) + 5A \sin(e + fx) - 15B \sin(2e + 3fx))}{15f(a^3 \cos^3(e + fx) + 3a^3 f \cos(e + fx) \sin(e + fx) - 4a^3 f \sin^3(e + fx) + 3a^3 \cos^2(e + fx) \sin(e + fx) - 2a^3 \cos(e + fx) \sin^2(e + fx) + a^3 \sin^2(e + fx))} \\
&= \frac{2(A - B)c \cos(e + fx)}{5f(a + a \sin(e + fx))^3} + \frac{(A - 11B)c \cos(e + fx)}{15af(a + a \sin(e + fx))^2} + \frac{(A + 4B)c \cos(e + fx) + 5A \sin(e + fx) - 15B \sin(2e + 3fx)}{15f(a^3 \cos^3(e + fx) + 3a^3 f \cos(e + fx) \sin(e + fx) - 4a^3 f \sin^3(e + fx) + 3a^3 \cos^2(e + fx) \sin(e + fx) - 2a^3 \cos(e + fx) \sin^2(e + fx) + a^3 \sin^2(e + fx))}
\end{aligned}$$

Mathematica [A] time = 0.82, size = 139, normalized size = 1.35

$$\frac{c \left(-15(A + B) \cos\left(e + \frac{fx}{2}\right) + 5(A + B) \cos\left(e + \frac{3fx}{2}\right) + A \sin\left(2e + \frac{5fx}{2}\right) + 5A \sin\left(\frac{fx}{2}\right) - 15B \sin\left(2e + \frac{3fx}{2}\right) + 4B \sin\left(2e + \frac{5fx}{2}\right) \right)}{30a^3 f \left(\sin\left(\frac{e}{2}\right) + \cos\left(\frac{e}{2}\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]

[Out] (c*(-15*(A + B)*Cos[e + (f*x)/2] + 5*(A + B)*Cos[e + (3*f*x)/2] + 5*A*Sin[(f*x)/2] - 25*B*Sin[(f*x)/2] - 15*B*Sin[2*e + (3*f*x)/2] + A*Sin[2*e + (5*f*x)/2] + 4*B*Sin[2*e + (5*f*x)/2]))/(30*a^3*f*(Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.42, size = 191, normalized size = 1.85

$$\frac{(A + 4B)c \cos^3(fx + e) - (2A - 7B)c \cos^2(fx + e) + 3(A - B)c \cos(fx + e) + 6(A - B)c - ((A + 4B)c \cos^3(fx + e) + 3a^3 f \cos^2(fx + e) - 2a^3 f \cos(fx + e) - 4a^3 f + (a^3 f \cos(fx + e) \sin(fx + e) - 2a^3 f \cos^2(fx + e) \sin(fx + e) + a^3 f \sin^2(fx + e)))}{15a^3 f \left(\cos^3(fx + e) + 3 \cos^2(fx + e) \sin(fx + e) - 2 \cos(fx + e) \sin^2(fx + e) + \sin^3(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*((A + 4*B)*c*cos(f*x + e)^3 - (2*A - 7*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*cos(f*x + e) + 6*(A - B)*c - ((A + 4*B)*c*cos(f*x + e)^2 + 3*(A - B)*c*cos(f*x + e) + 6*(A - B)*c*sin(f*x + e))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))

giac [A] time = 0.17, size = 138, normalized size = 1.34

$$\frac{2 \left(15 A c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 15 A c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 15 B c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 25 A c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 5 A c \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 5 A c \right)}{15 a^3 f \left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] $-2/15*(15*A*c*\tan(1/2*f*x + 1/2*e)^4 + 15*A*c*\tan(1/2*f*x + 1/2*e)^3 + 15*B*c*\tan(1/2*f*x + 1/2*e)^3 + 25*A*c*\tan(1/2*f*x + 1/2*e)^2 - 5*B*c*\tan(1/2*f*x + 1/2*e)^2 + 5*A*c*\tan(1/2*f*x + 1/2*e) + 5*B*c*\tan(1/2*f*x + 1/2*e) + 4*A*c + B*c)/(a^3*f*(\tan(1/2*f*x + 1/2*e) + 1)^5)$

maple [A] time = 0.45, size = 115, normalized size = 1.12

$$2c \left(\frac{-16A+16B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4} - \frac{8A-8B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5} - \frac{14A-10B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3} - \frac{-6A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} - \frac{A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1} \right) \frac{1}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)`

[Out] $2/f*c/a^3*(-1/4*(-16*A+16*B)/(\tan(1/2*f*x+1/2*e)+1)^4-1/5*(8*A-8*B)/(\tan(1/2*f*x+1/2*e)+1)^5-1/3*(14*A-10*B)/(\tan(1/2*f*x+1/2*e)+1)^3-1/2*(-6*A+2*B)/(\tan(1/2*f*x+1/2*e)+1)^2-A/(\tan(1/2*f*x+1/2*e)+1))$

maxima [B] time = 0.35, size = 733, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")`

[Out] $-2/15*(A*c*(20*\sin(f*x + e)/(\cos(f*x + e) + 1) + 40*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 2*B*c*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/(\tan(1/2*f*x + 1/2*e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) - 3*A*c*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1)/(\tan(1/2*f*x + 1/2*e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*B*c*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 1)/(\tan(1/2*f*x + 1/2*e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5))/f$

mupad [B] time = 13.03, size = 172, normalized size = 1.67

$$2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{41Ac}{4} - \frac{Bc}{4} - \frac{11Ac \cos(e+fx)}{2} + \frac{Bc \cos(e+fx)}{2} + 5Ac \sin(e+fx) + 5Bc \sin(e+fx) - \frac{3Ac \cos(2e+fx)}{4} \right) \frac{1}{15a^3 f \left(\frac{5\sqrt{2} \cos\left(\frac{3e}{2} + \frac{\pi}{4} + \frac{3fx}{2}\right)}{4} - \frac{5\sqrt{2} \cos\left(\frac{e}{2} - \frac{\pi}{4} + \frac{fx}{2}\right)}{2} + \frac{\sqrt{2} \cos\left(\frac{5e}{2} - \frac{\pi}{4} + \frac{5fx}{2}\right)}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)`

[Out] $(2*\cos(e/2 + (f*x)/2)*((41*A*c)/4 - (B*c)/4 - (11*A*c*\cos(e + f*x))/2 + (B*c*\cos(e + f*x))/2 + 5*A*c*\sin(e + f*x) + 5*B*c*\sin(e + f*x) - (3*A*c*\cos(2e + f*x))/4) + 10*a^3*\sin(e + f*x)^2/(\cos(e + f*x) + 1)^2 + 10*a^3*\sin(e + f*x)^3/(\cos(e + f*x) + 1)^3 + 5*a^3*\sin(e + f*x)^4/(\cos(e + f*x) + 1)^4 + a^3*\sin(e + f*x)^5/(\cos(e + f*x) + 1)^5) - 2*B*c*(5*\sin(e + f*x)/(\cos(e + f*x) + 1) + 10*\sin(e + f*x)^2/(\cos(e + f*x) + 1)^2 + 1)/(\tan(1/2*f*x + 1/2*e) + 1) + 10*a^3*\sin(e + f*x)^2/(\cos(e + f*x) + 1)^2 + 10*a^3*\sin(e + f*x)^3/(\cos(e + f*x) + 1)^3 + 5*a^3*\sin(e + f*x)^4/(\cos(e + f*x) + 1)^4 + a^3*\sin(e + f*x)^5/(\cos(e + f*x) + 1)^5) - 3*A*c*(5*\sin(e + f*x)/(\cos(e + f*x) + 1) + 5*\sin(e + f*x)^2/(\cos(e + f*x) + 1)^2 + 5*\sin(e + f*x)^3/(\cos(e + f*x) + 1)^3 + 1)/(\tan(1/2*f*x + 1/2*e) + 1) + 10*a^3*\sin(e + f*x)^2/(\cos(e + f*x) + 1)^2 + 10*a^3*\sin(e + f*x)^3/(\cos(e + f*x) + 1)^3 + 5*a^3*\sin(e + f*x)^4/(\cos(e + f*x) + 1)^4 + a^3*\sin(e + f*x)^5/(\cos(e + f*x) + 1)^5))/f$

$$\frac{e + 2fx}{4} + \frac{(3Bc \cos(2e + 2fx))}{4} - \frac{(5A \sin(2e + 2fx))}{4} - \frac{(5B \sin(2e + 2fx))}{4} / (15a^3 f ((5^{1/2} \cos((3e)/2 + \pi/4 + (3fx)/2))) / 4 - (5^{1/2} \cos(e/2 - \pi/4 + (fx)/2)) / 2 + (2^{1/2} \cos((5e)/2 - \pi/4 + (5fx)/2)) / 4)$$

sympy [A] time = 13.79, size = 1035, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)

[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 50*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*A*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 10*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 10*B*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 2*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(-c*sin(e) + c)/(a*sin(e) + a)**3, True))

$$3.75 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))} dx$$

Optimal. Leaf size=102

$$\frac{2(3A+2B) \tan(e+fx)}{15a^3cf} - \frac{(3A+2B) \sec(e+fx)}{15cf(a^3 \sin(e+fx)+a^3)} - \frac{(A-B) \sec(e+fx)}{5acf(a \sin(e+fx)+a)^2}$$

[Out] $-1/5*(A-B)*\sec(f*x+e)/a/c/f/(a+a*\sin(f*x+e))^2-1/15*(3*A+2*B)*\sec(f*x+e)/c/f/(a^3+a^3*\sin(f*x+e))+2/15*(3*A+2*B)*\tan(f*x+e)/a^3/c/f$

Rubi [A] time = 0.25, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2859, 2672, 3767, 8}

$$\frac{2(3A+2B) \tan(e+fx)}{15a^3cf} - \frac{(3A+2B) \sec(e+fx)}{15cf(a^3 \sin(e+fx)+a^3)} - \frac{(A-B) \sec(e+fx)}{5acf(a \sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]

[Out] $-((A - B)*\text{Sec}[e + f*x])/((5*a*c*f*(a + a*\text{Sin}[e + f*x])^2) - ((3*A + 2*B)*\text{Sec}[e + f*x]))/(15*c*f*(a^3 + a^3*\text{Sin}[e + f*x])) + (2*(3*A + 2*B)*\text{Tan}[e + f*x])/(15*a^3*c*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2672

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))} dx &= \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{(a+a \sin(e+fx))^2} dx}{ac} \\ &= -\frac{(A - B) \sec(e + fx)}{5acf(a + a \sin(e + fx))^2} + \frac{(3A + 2B) \int \frac{\sec^2(e+fx)}{a+a \sin(e+fx)} dx}{5a^2c} \\ &= -\frac{(A - B) \sec(e + fx)}{5acf(a + a \sin(e + fx))^2} - \frac{(3A + 2B) \sec(e + fx)}{15cf(a^3 + a^3 \sin(e + fx))} + \frac{(2(3A + 2B) \sec(e + fx))}{15cf(a^3 + a^3 \sin(e + fx))} \\ &= -\frac{(A - B) \sec(e + fx)}{5acf(a + a \sin(e + fx))^2} - \frac{(3A + 2B) \sec(e + fx)}{15cf(a^3 + a^3 \sin(e + fx))} - \frac{(2(3A + 2B) \sec(e + fx))}{15cf(a^3 + a^3 \sin(e + fx))} \\ &= -\frac{(A - B) \sec(e + fx)}{5acf(a + a \sin(e + fx))^2} - \frac{(3A + 2B) \sec(e + fx)}{15cf(a^3 + a^3 \sin(e + fx))} + \frac{2(3A + 2B) \sec(e + fx)}{15cf(a^3 + a^3 \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.84, size = 156, normalized size = 1.53

$$\frac{\cos(e + fx)(-5(9A + B) \cos(e + fx) + 32(3A + 2B) \cos(2(e + fx)) - 120A \sin(e + fx) - 36A \sin(2(e + fx)) - 240a^3cf(\sin(e + fx)))}{240a^3cf(\sin(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])),x]

[Out] (Cos[e + f*x]*(-80*B - 5*(9*A + B)*Cos[e + f*x] + 32*(3*A + 2*B)*Cos[2*(e + f*x)] + 9*A*Cos[3*(e + f*x)] + B*Cos[3*(e + f*x)] - 120*A*Sin[e + f*x] - 80*B*Sin[e + f*x] - 36*A*Sin[2*(e + f*x)] - 4*B*Sin[2*(e + f*x)] + 24*A*Sin[3*(e + f*x)] + 16*B*Sin[3*(e + f*x)])/(240*a^3*c*f*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.41, size = 106, normalized size = 1.04

$$\frac{4(3A + 2B) \cos^2(fx + e) + (2(3A + 2B) \cos^2(fx + e) - 9A - 6B) \sin(fx + e) - 6A - 9B}{15(a^3cf \cos^3(fx + e) - 2a^3cf \cos(fx + e) \sin(fx + e) - 2a^3cf \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/15*(4*(3*A + 2*B)*cos(f*x + e)^2 + (2*(3*A + 2*B)*cos(f*x + e)^2 - 9*A - 6*B)*sin(f*x + e) - 6*A - 9*B)/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))

giac [A] time = 0.18, size = 175, normalized size = 1.72

$$\frac{15(A+B)}{a^3c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-1\right)} + \frac{105A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 - 15B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4 + 270A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 30B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 + 360A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 360B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 + 360A \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 360B \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 360A + 360B}{a^3c\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out]
$$-1/60*(15*(A + B)/(a^3*c*(\tan(1/2*f*x + 1/2*e) - 1)) + (105*A*\tan(1/2*f*x + 1/2*e)^4 - 15*B*\tan(1/2*f*x + 1/2*e)^4 + 270*A*\tan(1/2*f*x + 1/2*e)^3 + 30*B*\tan(1/2*f*x + 1/2*e)^3 + 360*A*\tan(1/2*f*x + 1/2*e)^2 + 40*B*\tan(1/2*f*x + 1/2*e)^2 + 210*A*\tan(1/2*f*x + 1/2*e) + 50*B*\tan(1/2*f*x + 1/2*e) + 63*A + 7*B)/(a^3*c*(\tan(1/2*f*x + 1/2*e) + 1)^5))/f$$

maple [A] time = 0.52, size = 145, normalized size = 1.42

$$\frac{\frac{2\left(\frac{A}{8} + \frac{B}{8}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{-4A + 4B}{2\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^4} - \frac{2(2A - 2B)}{5\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^5} - \frac{-\frac{5A}{2} + \frac{3B}{2}}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} - \frac{2\left(\frac{7A}{8} - \frac{B}{8}\right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1} - \frac{2\left(\frac{9A}{2} - \frac{7B}{2}\right)}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^3}}{a^3 c f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)

[Out]
$$2/f/a^3/c*(-(1/8*A+1/8*B)/(\tan(1/2*f*x+1/2*e)-1)-1/4*(-4*A+4*B)/(\tan(1/2*f*x+1/2*e)+1)^4-1/5*(2*A-2*B)/(\tan(1/2*f*x+1/2*e)+1)^5-1/2*(-5/2*A+3/2*B)/(\tan(1/2*f*x+1/2*e)+1)^2-(7/8*A-1/8*B)/(\tan(1/2*f*x+1/2*e)+1)-1/3*(9/2*A-7/2*B)/(\tan(1/2*f*x+1/2*e)+1)^3)$$

maxima [B] time = 0.42, size = 423, normalized size = 4.15

$$2 \left(\frac{B \left(\frac{4 \sin(fx+e)}{\cos(fx+e)+1} + \frac{20 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{20 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{15 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 1 \right)}{a^3 c + \frac{4 a^3 c \sin(fx+e)}{\cos(fx+e)+1} + \frac{5 a^3 c \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{5 a^3 c \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4 a^3 c \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{a^3 c \sin(fx+e)^6}{(\cos(fx+e)+1)^6}} - \frac{3 A \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} - \frac{10 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{10 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right)}{a^3 c + \frac{4 a^3 c \sin(fx+e)}{\cos(fx+e)+1} + \frac{5 a^3 c \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{5 a^3 c \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{4 a^3 c \sin(fx+e)^5}{(\cos(fx+e)+1)^5}} \right) / 15 f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out]
$$2/15*(B*(4*\sin(f*x + e)/(\cos(f*x + e) + 1) + 20*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 20*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 1)/(a^3*c + 4*a^3*c*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a^3*c*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 5*a^3*c*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 4*a^3*c*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - a^3*c*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6) - 3*A*(3*\sin(f*x + e)/(\cos(f*x + e) + 1) - 10*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 10*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2)/(a^3*c + 4*a^3*c*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*a^3*c*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 5*a^3*c*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 4*a^3*c*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - a^3*c*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6))/f$$

mupad [B] time = 12.43, size = 178, normalized size = 1.75

$$\frac{2 \left(\frac{15 A \cos(e+f x)}{4} - \frac{5 B}{2} - \frac{5 B \cos(e+f x)}{8} - \frac{15 A \sin(e+f x)}{4} - \frac{5 B \sin(e+f x)}{2} + 3 A \cos(2 e+2 f x) - \frac{3 A \cos(3 e+3 f x)}{4} + 2 B \right)}{15 a^3 c f \left(\frac{5 \cos(e+f x)}{4} - \frac{\cos(3 e+3 f x)}{4} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))),x)
```

```
[Out] -(2*((15*A*cos(e + f*x))/4 - (5*B)/2 - (5*B*cos(e + f*x))/8 - (15*A*sin(e +
f*x))/4 - (5*B*sin(e + f*x))/2 + 3*A*cos(2*e + 2*f*x) - (3*A*cos(3*e + 3*f
*x))/4 + 2*B*cos(2*e + 2*f*x) + (B*cos(3*e + 3*f*x))/8 + 3*A*sin(2*e + 2*f*
x) + (3*A*sin(3*e + 3*f*x))/4 - (B*sin(2*e + 2*f*x))/2 + (B*sin(3*e + 3*f*x
))/2))/(15*a^3*c*f*((5*cos(e + f*x))/4 - cos(3*e + 3*f*x)/4 + sin(2*e + 2*f
*x)))
```

```
sympy [A] time = 14.44, size = 1236, normalized size = 12.12
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e)),x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**5/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*
a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*
f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*
tan(e/2 + f*x/2)**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
+ f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 60*A*tan(e/2 + f*x/2)*
**3/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*
a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*
f*tan(e/2 + f*x/2) - 15*a**3*c*f) + 18*A*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(
e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f
*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) -
15*a**3*c*f) + 12*A/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2
+ f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2
)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 30*B*tan(e/2 + f*x/2)
**4/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75
*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c
*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**3/(15*a**3*c*f*
tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
+ f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2)
- 15*a**3*c*f) - 40*B*tan(e/2 + f*x/2)**2/(15*a**3*c*f*tan(e/2 + f*x/2)*
**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75
*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f)
- 8*B*tan(e/2 + f*x/2)/(15*a**3*c*f*tan(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(
e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2 + f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f
*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2) - 15*a**3*c*f) - 2*B/(15*a**3*c*f*t
an(e/2 + f*x/2)**6 + 60*a**3*c*f*tan(e/2 + f*x/2)**5 + 75*a**3*c*f*tan(e/2
+ f*x/2)**4 - 75*a**3*c*f*tan(e/2 + f*x/2)**2 - 60*a**3*c*f*tan(e/2 + f*x/2
) - 15*a**3*c*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**3*(-c*sin(e
) + c)), True))
```

$$3.76 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=90

$$\frac{(4A+B) \tan^3(e+fx)}{15a^3c^2f} + \frac{(4A+B) \tan(e+fx)}{5a^3c^2f} - \frac{(A-B) \sec^3(e+fx)}{5c^2f(a^3 \sin(e+fx) + a^3)}$$

[Out] $-1/5*(A-B)*\sec(f*x+e)^3/c^2/f/(a^3+a^3*\sin(f*x+e))+1/5*(4*A+B)*\tan(f*x+e)/a^3/c^2/f+1/15*(4*A+B)*\tan(f*x+e)^3/a^3/c^2/f$

Rubi [A] time = 0.20, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2859, 3767}

$$\frac{(4A+B) \tan^3(e+fx)}{15a^3c^2f} + \frac{(4A+B) \tan(e+fx)}{5a^3c^2f} - \frac{(A-B) \sec^3(e+fx)}{5c^2f(a^3 \sin(e+fx) + a^3)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^3*(c - c*\text{Sin}[e + f*x])^2), x]$

[Out] $-((A - B)*\text{Sec}[e + f*x]^3)/(5*c^2*f*(a^3 + a^3*\text{Sin}[e + f*x])) + ((4*A + B)*\text{Tan}[e + f*x])/((5*a^3*c^2*f) + ((4*A + B)*\text{Tan}[e + f*x]^3)/(15*a^3*c^2*f))$

Rule 2859

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)*(g*\text{Cos}[e + f*x])^{p+1}*(a + b*\text{Sin}[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^{m+1}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{LtQ}[m, -1] || \text{ILtQ}[\text{Simplify}[m + p], 0]) \&\& \text{NeQ}[2*m + p + 1, 0]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.))]^m*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{2*m}*(c + d*\text{Sin}[e + f*x])^{n-m}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^n, x_Symbol] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{n/2 - 1}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^2} dx = \frac{\int \frac{\sec^4(e+fx)(A+B \sin(e+fx))}{a+a \sin(e+fx)} dx}{a^2 c^2}$$

$$= -\frac{(A - B) \sec^3(e + fx)}{5c^2 f (a^3 + a^3 \sin(e + fx))} + \frac{(4A + B) \int \sec^4(e + fx) dx}{5a^3 c^2}$$

$$= -\frac{(A - B) \sec^3(e + fx)}{5c^2 f (a^3 + a^3 \sin(e + fx))} - \frac{(4A + B) \text{Subst} \left(\int (1 + x^2) dx \right)}{5a^3 c^2 f}$$

$$= -\frac{(A - B) \sec^3(e + fx)}{5c^2 f (a^3 + a^3 \sin(e + fx))} + \frac{(4A + B) \tan(e + fx)}{5a^3 c^2 f} + \frac{(4A + B)}{5a^3 c^2 f}$$

Mathematica [B] time = 1.05, size = 237, normalized size = 2.63

$$\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (54(A - B) \cos(e + fx) - 32(4A + B))$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^2), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(240*B + 54*(A - B)*Cos[e + f*x] - 32*(4*A + B)*Cos[2*(e + f*x)] + 18*A*Cos[3*(e + f*x)] - 18*B*Cos[3*(e + f*x)] - 64*A*Cos[4*(e + f*x)] - 16*B*Cos[4*(e + f*x)] + 384*A*Sin[e + f*x] + 96*B*Sin[e + f*x] + 18*A*Sin[2*(e + f*x)] - 18*B*Sin[2*(e + f*x)] + 128*A*Sin[3*(e + f*x)] + 32*B*Sin[3*(e + f*x)] + 9*A*Sin[4*(e + f*x)] - 9*B*Sin[4*(e + f*x)]))/(960*a^3*c^2*f*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.42, size = 108, normalized size = 1.20

$$\frac{2(4A + B) \cos(fx + e)^4 - (4A + B) \cos(fx + e)^2 - \left(2(4A + B) \cos(fx + e)^2 + 4A + B\right) \sin(fx + e) - A}{15 \left(a^3 c^2 f \cos(fx + e)^3 \sin(fx + e) + a^3 c^2 f \cos(fx + e)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/15*(2*(4*A + B)*cos(f*x + e)^4 - (4*A + B)*cos(f*x + e)^2 - (2*(4*A + B)*cos(f*x + e)^2 + 4*A + B)*sin(f*x + e) - A - 4*B)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)

giac [B] time = 0.20, size = 235, normalized size = 2.61

$$\frac{5 \left(15A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 9B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 24A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 12B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 13A + 7B\right)}{a^3 c^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^3} + \frac{165A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 45B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4}{a^3 c^2 \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] $-1/120*(5*(15*A*\tan(1/2*f*x + 1/2*e)^2 + 9*B*\tan(1/2*f*x + 1/2*e)^2 - 24*A*\tan(1/2*f*x + 1/2*e) - 12*B*\tan(1/2*f*x + 1/2*e) + 13*A + 7*B)/(a^3*c^2*(\tan(1/2*f*x + 1/2*e) - 1)^3) + (165*A*\tan(1/2*f*x + 1/2*e)^4 - 45*B*\tan(1/2*f*x + 1/2*e)^4 + 480*A*\tan(1/2*f*x + 1/2*e)^3 - 60*B*\tan(1/2*f*x + 1/2*e)^3 + 650*A*\tan(1/2*f*x + 1/2*e)^2 - 70*B*\tan(1/2*f*x + 1/2*e)^2 + 400*A*\tan(1/2*f*x + 1/2*e) - 20*B*\tan(1/2*f*x + 1/2*e) + 113*A - 13*B)/(a^3*c^2*(\tan(1/2*f*x + 1/2*e) + 1)^5))/f$

maple [B] time = 0.46, size = 185, normalized size = 2.06

$$\frac{2\left(\frac{A+B}{4}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3} - \frac{\frac{A}{4}+\frac{B}{4}}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2\left(\frac{5A}{16}+\frac{3B}{16}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{-2A+2B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4} - \frac{2(A-B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5} - \frac{-\frac{3A}{2}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} - \frac{2\left(\frac{5A}{2}-2B\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3}$$

$f a^3 c^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)`

[Out] $2/f/a^3/c^2*(-1/3*(1/4*A+1/4*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/2*(1/4*A+1/4*B)/(\tan(1/2*f*x+1/2*e)-1)^2-(5/16*A+3/16*B)/(\tan(1/2*f*x+1/2*e)-1)-1/4*(-2*A+2*B)/(\tan(1/2*f*x+1/2*e)+1)^4-1/5*(A-B)/(\tan(1/2*f*x+1/2*e)+1)^5-1/2*(-3/2*A+B)/(\tan(1/2*f*x+1/2*e)+1)^2-1/3*(5/2*A-2*B)/(\tan(1/2*f*x+1/2*e)+1)^3-(11/16*A-3/16*B)/(\tan(1/2*f*x+1/2*e)+1))$

maxima [B] time = 0.37, size = 650, normalized size = 7.22

$$2 \left(\frac{A \left(\frac{9 \sin(fx+e)}{\cos(fx+e)+1} + \frac{21 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{13 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} - \frac{25 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{15 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} + \frac{15 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - 3 \right)}{a^3 c^2 + \frac{2 a^3 c^2 \sin(fx+e)}{\cos(fx+e)+1} - \frac{2 a^3 c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{6 a^3 c^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{6 a^3 c^2 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{2 a^3 c^2 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{2 a^3 c^2 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{a^3 c^2 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}} + \frac{B \left(\frac{6 \sin(fx+e)}{\cos(fx+e)+1} + \frac{15 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{5 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{5 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{5 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{5 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} + \frac{5 \sin(fx+e)^8}{(\cos(fx+e)+1)^8} \right)}{a^3 c^2 + \frac{2 a^3 c^2 \sin(fx+e)}{\cos(fx+e)+1} - \frac{2 a^3 c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{6 a^3 c^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{6 a^3 c^2 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{2 a^3 c^2 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} - \frac{2 a^3 c^2 \sin(fx+e)^7}{(\cos(fx+e)+1)^7} - \frac{a^3 c^2 \sin(fx+e)^8}{(\cos(fx+e)+1)^8}} \right)$$

$15 f$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x, algorithm="maxima")`

[Out] $2/15*(A*(9*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 13*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 25*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 5*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 15*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 3)/(a^3*c^2 + 2*a^3*c^2*\sin(f*x + e)/(\cos(f*x + e) + 1) - 2*a^3*c^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 6*a^3*c^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 6*a^3*c^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2*a^3*c^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 2*a^3*c^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - a^3*c^2*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8) + B*(6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 9*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 8*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 10*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 3)/(a^3*c^2 + 2*a^3*c^2*\sin(f*x + e)/(\cos(f*x + e) + 1) - 2*a^3*c^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 6*a^3*c^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 6*a^3*c^2*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 2*a^3*c^2*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 2*a^3*c^2*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - a^3*c^2*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8))/f$

mupad [B] time = 12.47, size = 183, normalized size = 2.03

$$\frac{\left(\frac{8A}{15} + \frac{2B}{15} + \frac{16A \sin(e+fx)}{15} + \frac{4B \sin(e+fx)}{15}\right) \cos(e+fx)^2 + \frac{2A}{15} + \frac{8B}{15} + \frac{8A \sin(e+fx)}{15} + \frac{2B \sin(e+fx)}{15}}{a^3 c^2 f \left(2 \cos(e+fx)^3 \sin(e+fx) + 2 \cos(e+fx)^3 \right)} - \frac{\frac{2A}{5} - \frac{2B}{5} + \frac{2A \sin(e+fx)}{5}}{a^3 c^2 f \left(2 \cos(e+fx)^3 \sin(e+fx) + 2 \cos(e+fx)^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^2),x)
```

```
[Out] ((2*A)/15 + (8*B)/15 + (8*A*sin(e + f*x))/15 + (2*B*sin(e + f*x))/15 + cos(
e + f*x)^2*((8*A)/15 + (2*B)/15 + (16*A*sin(e + f*x))/15 + (4*B*sin(e + f*x)
))/15)/(a^3*c^2*f*(2*cos(e + f*x)^3*sin(e + f*x) + 2*cos(e + f*x)^3)) - ((
2*A)/5 - (2*B)/5 + (2*A*sin(e + f*x))/5 - (2*B*sin(e + f*x))/5)/(a^3*c^2*f*
(2*sin(e + f*x) + 2)) - (cos(e + f*x)*((16*A)/15 + (4*B)/15))/(a^3*c^2*f*(2
*sin(e + f*x) + 2))
```

sympy [A] time = 27.39, size = 2674, normalized size = 29.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^2,x)
```

```
[Out] Piecewise((-30*A*tan(e/2 + f*x/2)**7/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 30*A*tan(e/2 + f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 10*A*tan(e/2 + f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) + 50*A*tan(e/2 + f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 26*A*tan(e/2 + f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 42*A*tan(e/2 + f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 +
30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 9
0*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30
*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**
3*c**2*f) - 18*A*tan(e/2 + f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*
a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a
**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a*
**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c
**2*f) + 6*A/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 30*B*tan(e/2
+ f*x/2)**6/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 20*B*tan(e/2
+ f*x/2)**5/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 10*B*tan(e/2
+ f*x/2)**4/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
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f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) + 16*B*tan(e/2
+ f*x/2)**3/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 18*B*tan(e/2
+ f*x/2)**2/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 +
f*x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 +
f*x/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f
*x/2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 12*B*tan(e/2
+ f*x/2)/(15*a**3*c**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*
x/2)**7 - 30*a**3*c**2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x
/2)**5 + 90*a**3*c**2*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/
2)**2 - 30*a**3*c**2*f*tan(e/2 + f*x/2) - 15*a**3*c**2*f) - 6*B/(15*a**3*c*
**2*f*tan(e/2 + f*x/2)**8 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**7 - 30*a**3*c**
2*f*tan(e/2 + f*x/2)**6 - 90*a**3*c**2*f*tan(e/2 + f*x/2)**5 + 90*a**3*c**2
*f*tan(e/2 + f*x/2)**3 + 30*a**3*c**2*f*tan(e/2 + f*x/2)**2 - 30*a**3*c**2*
f*tan(e/2 + f*x/2) - 15*a**3*c**2*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(
e) + a)**3*(-c*sin(e) + c)**2), True))

```

$$3.77 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=84

$$\frac{A \tan^5(e+fx)}{5a^3c^3f} + \frac{2A \tan^3(e+fx)}{3a^3c^3f} + \frac{A \tan(e+fx)}{a^3c^3f} + \frac{B \sec^5(e+fx)}{5a^3c^3f}$$

[Out] 1/5*B*sec(f*x+e)^5/a^3/c^3/f+A*tan(f*x+e)/a^3/c^3/f+2/3*A*tan(f*x+e)^3/a^3/c^3/f+1/5*A*tan(f*x+e)^5/a^3/c^3/f

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2669, 3767}

$$\frac{A \tan^5(e+fx)}{5a^3c^3f} + \frac{2A \tan^3(e+fx)}{3a^3c^3f} + \frac{A \tan(e+fx)}{a^3c^3f} + \frac{B \sec^5(e+fx)}{5a^3c^3f}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3), x]

[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*Tan[e + f*x])/(a^3*c^3*f) + (2*A*Tan[e + f*x]^3)/(3*a^3*c^3*f) + (A*Tan[e + f*x]^5)/(5*a^3*c^3*f)

Rule 2669

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^3} dx &= \frac{\int \sec^6(e+fx)(A+B \sin(e+fx)) dx}{a^3c^3} \\ &= \frac{B \sec^5(e+fx)}{5a^3c^3f} + \frac{A \int \sec^6(e+fx) dx}{a^3c^3} \\ &= \frac{B \sec^5(e+fx)}{5a^3c^3f} - \frac{A \text{Subst}\left(\int (1+2x^2+x^4) dx, x, -\tan(e+fx)\right)}{a^3c^3f} \\ &= \frac{B \sec^5(e+fx)}{5a^3c^3f} + \frac{A \tan(e+fx)}{a^3c^3f} + \frac{2A \tan^3(e+fx)}{3a^3c^3f} + \frac{A \tan^5(e+fx)}{5a^3c^3f} \end{aligned}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 0.77

$$\frac{A \left(\frac{1}{5} \tan^5(e + fx) + \frac{2}{3} \tan^3(e + fx) + \tan(e + fx) \right)}{a^3 c^3 f} + \frac{B \sec^5(e + fx)}{5 a^3 c^3 f}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^3), x]

[Out] (B*Sec[e + f*x]^5)/(5*a^3*c^3*f) + (A*(Tan[e + f*x] + (2*Tan[e + f*x]^3)/3 + Tan[e + f*x]^5/5))/(a^3*c^3*f)

fricas [A] time = 0.41, size = 56, normalized size = 0.67

$$\frac{\left(8 A \cos(fx + e)^4 + 4 A \cos(fx + e)^2 + 3 A \right) \sin(fx + e) + 3 B}{15 a^3 c^3 f \cos(fx + e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 1/15*((8*A*cos(f*x + e)^4 + 4*A*cos(f*x + e)^2 + 3*A)*sin(f*x + e) + 3*B)/(a^3*c^3*f*cos(f*x + e)^5)

giac [A] time = 0.35, size = 134, normalized size = 1.60

$$\frac{2 \left(15 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^9 + 15 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^8 - 20 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^7 + 58 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^5 + 30 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 20 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + 3 B \right)}{15 \left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 1 \right)^5 a^3 c^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^9 + 15*B*tan(1/2*f*x + 1/2*e)^8 - 20*A*tan(1/2*f*x + 1/2*e)^7 + 58*A*tan(1/2*f*x + 1/2*e)^5 + 30*B*tan(1/2*f*x + 1/2*e)^3 - 20*A*tan(1/2*f*x + 1/2*e) + 3*B)/((tan(1/2*f*x + 1/2*e)^2 - 1)^5*a^3*c^3*f)

maple [B] time = 0.42, size = 227, normalized size = 2.70

$$\frac{\frac{A+B}{2 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^4} - \frac{2 \left(\frac{A}{2} + \frac{B}{2} \right)}{5 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^5} - \frac{\frac{7A}{8} + \frac{5B}{8}}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^2} - \frac{2 \left(\frac{A}{2} + \frac{3B}{16} \right)}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1} - \frac{2 \left(\frac{11A}{8} + \frac{9B}{8} \right)}{3 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) - 1 \right)^3} - \frac{-A+B}{2 \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1 \right)^4} - \frac{-\frac{7A}{8} + \frac{5B}{8}}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1 \right)^5}}{f a^3 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^3,x)

[Out] 2/f/a^3/c^3*(-1/4*(A+B)/(tan(1/2*f*x+1/2*e)-1)^4-1/5*(1/2*A+1/2*B)/(tan(1/2*f*x+1/2*e)-1)^5-1/2*(7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)-1)^2-(1/2*A+3/16*B)/(tan(1/2*f*x+1/2*e)-1)-1/3*(11/8*A+9/8*B)/(tan(1/2*f*x+1/2*e)-1)^3-1/4*(-A+B)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-7/8*A+5/8*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/5*(1/2*A-1/2*B)/(tan(1/2*f*x+1/2*e)+1)^5-(1/2*A-3/16*B)/(tan(1/2*f*x+1/2*e)+1)-1/3*(11/8*A-9/8*B)/(tan(1/2*f*x+1/2*e)+1)^3)


```
3*f*tan(e/2 + f*x/2)**10 - 75*a**3*c**3*f*tan(e/2 + f*x/2)**8 + 150*a**3*c*  
*3*f*tan(e/2 + f*x/2)**6 - 150*a**3*c**3*f*tan(e/2 + f*x/2)**4 + 75*a**3*c*  
*3*f*tan(e/2 + f*x/2)**2 - 15*a**3*c**3*f), Ne(f, 0)), (x*(A + B*sin(e))/((  
a*sin(e) + a)**3*(-c*sin(e) + c)**3), True))
```


$$3.78 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^4} dx$$

Optimal. Leaf size=121

$$\frac{(6A-B) \tan^5(e+fx)}{35a^3c^4f} + \frac{2(6A-B) \tan^3(e+fx)}{21a^3c^4f} + \frac{(6A-B) \tan(e+fx)}{7a^3c^4f} + \frac{(A+B) \sec^5(e+fx)}{7a^3f(c^4-c^4 \sin(e+fx))}$$

[Out] 1/7*(A+B)*sec(f*x+e)^5/a^3/f/(c^4-c^4*sin(f*x+e))+1/7*(6*A-B)*tan(f*x+e)/a^3/c^4/f+2/21*(6*A-B)*tan(f*x+e)^3/a^3/c^4/f+1/35*(6*A-B)*tan(f*x+e)^5/a^3/c^4/f

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2859, 3767}

$$\frac{(6A-B) \tan^5(e+fx)}{35a^3c^4f} + \frac{2(6A-B) \tan^3(e+fx)}{21a^3c^4f} + \frac{(6A-B) \tan(e+fx)}{7a^3c^4f} + \frac{(A+B) \sec^5(e+fx)}{7a^3f(c^4-c^4 \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4), x]

[Out] ((A + B)*Sec[e + f*x]^5)/(7*a^3*f*(c^4 - c^4*Sin[e + f*x])) + ((6*A - B)*Tan[e + f*x])/(7*a^3*c^4*f) + (2*(6*A - B)*Tan[e + f*x]^3)/(21*a^3*c^4*f) + ((6*A - B)*Tan[e + f*x]^5)/(35*a^3*c^4*f)

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^4} dx = \frac{\int \frac{\sec^6(e+fx)(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx}{a^3 c^3}$$

$$= \frac{(A + B) \sec^5(e + fx)}{7a^3 f (c^4 - c^4 \sin(e + fx))} + \frac{(6A - B) \int \sec^6(e + fx) dx}{7a^3 c^4}$$

$$= \frac{(A + B) \sec^5(e + fx)}{7a^3 f (c^4 - c^4 \sin(e + fx))} - \frac{(6A - B) \text{Subst} \left(\int (1 + 2x^2 + x^4) dx \right)}{7a^3 c^4 f}$$

$$= \frac{(A + B) \sec^5(e + fx)}{7a^3 f (c^4 - c^4 \sin(e + fx))} + \frac{(6A - B) \tan(e + fx)}{7a^3 c^4 f} + \frac{2(6A - B) \tan(e + fx)}{21a^3 c^4 f}$$

Mathematica [B] time = 1.14, size = 325, normalized size = 2.69

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (1500(A + B) \cos(e + fx) - 640(6A - B))}{105 \left(a^3 c^4 f \cos^5(e + fx) \sin(e + fx) - a^3 c^4 f \cos^5(e + fx) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^4), x]

[Out] -1/53760*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))*(-8960*B + 1500*(A + B)*Cos[e + f*x] - 640*(6*A - B)*Cos[2*(e + f*x)] + 750*A*Cos[3*(e + f*x)] + 750*B*Cos[3*(e + f*x)] - 3072*A*Cos[4*(e + f*x)] + 512*B*Cos[4*(e + f*x)] + 150*A*Cos[5*(e + f*x)] + 150*B*Cos[5*(e + f*x)] - 768*A*Cos[6*(e + f*x)] + 128*B*Cos[6*(e + f*x)] - 15360*A*Sin[e + f*x] + 2560*B*Sin[e + f*x] - 375*A*Sin[2*(e + f*x)] - 375*B*Sin[2*(e + f*x)] - 7680*A*Sin[3*(e + f*x)] + 1280*B*Sin[3*(e + f*x)] - 300*A*Sin[4*(e + f*x)] - 300*B*Sin[4*(e + f*x)] - 1536*A*Sin[5*(e + f*x)] + 256*B*Sin[5*(e + f*x)] - 75*A*Sin[6*(e + f*x)] - 75*B*Sin[6*(e + f*x)])))/(a^3*c^4*f*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.44, size = 150, normalized size = 1.24

$$\frac{8(6A - B) \cos^6(fx + e) - 4(6A - B) \cos^4(fx + e) - (6A - B) \cos^2(fx + e) + (8(6A - B) \cos^4(fx + e) + 4(6A - B) \cos^2(fx + e) - a^3 c^4 f \cos^5(fx + e) \sin(fx + e) - a^3 c^4 f \cos^5(fx + e))}{105 \left(a^3 c^4 f \cos^5(fx + e) \sin(fx + e) - a^3 c^4 f \cos^5(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="fricas")

[Out] -1/105*(8*(6*A - B)*cos(f*x + e)^6 - 4*(6*A - B)*cos(f*x + e)^4 - (6*A - B)*cos(f*x + e)^2 + (8*(6*A - B)*cos(f*x + e)^4 + 4*(6*A - B)*cos(f*x + e)^2 + 18*A - 3*B)*sin(f*x + e) - 3*A + 18*B)/(a^3*c^4*f*cos(f*x + e)^5*sin(f*x + e) - a^3*c^4*f*cos(f*x + e)^5)

giac [B] time = 0.27, size = 355, normalized size = 2.93

$$\frac{7 \left(165 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 - 75 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^4 + 540 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 - 210 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^3 + 750 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 - 280 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right)^2 + 480 A \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) - 240 B \tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) \right)}{a^3 c^4 \left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e\right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="giac")

[Out]
$$\frac{-1/1680*(7*(165*A*\tan(1/2*f*x + 1/2*e)^4 - 75*B*\tan(1/2*f*x + 1/2*e)^4 + 540*A*\tan(1/2*f*x + 1/2*e)^3 - 210*B*\tan(1/2*f*x + 1/2*e)^3 + 750*A*\tan(1/2*f*x + 1/2*e)^2 - 280*B*\tan(1/2*f*x + 1/2*e)^2 + 480*A*\tan(1/2*f*x + 1/2*e) - 170*B*\tan(1/2*f*x + 1/2*e) + 129*A - 49*B)/(a^3*c^4*(\tan(1/2*f*x + 1/2*e) + 1)^5) + (2205*A*\tan(1/2*f*x + 1/2*e)^6 + 525*B*\tan(1/2*f*x + 1/2*e)^6 - 10080*A*\tan(1/2*f*x + 1/2*e)^5 - 1470*B*\tan(1/2*f*x + 1/2*e)^5 + 21945*A*\tan(1/2*f*x + 1/2*e)^4 + 2555*B*\tan(1/2*f*x + 1/2*e)^4 - 26460*A*\tan(1/2*f*x + 1/2*e)^3 - 2240*B*\tan(1/2*f*x + 1/2*e)^3 + 18963*A*\tan(1/2*f*x + 1/2*e)^2 + 1407*B*\tan(1/2*f*x + 1/2*e)^2 - 7476*A*\tan(1/2*f*x + 1/2*e) - 434*B*\tan(1/2*f*x + 1/2*e) + 1383*A + 137*B)/(a^3*c^4*(\tan(1/2*f*x + 1/2*e) - 1)^7))/f$$

maple [B] time = 0.51, size = 271, normalized size = 2.24

$$\frac{\frac{2(A+B)}{7\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^7} - \frac{3A+3B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{\frac{11A}{2}+\frac{9B}{2}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{\frac{15A}{8}+B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^2} - \frac{2\left(\frac{21A}{32}+\frac{5B}{32}\right)}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1} - \frac{2\left(\frac{21A}{4}+\frac{19B}{4}\right)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^5} - \frac{2\left(\frac{33A}{8}+\frac{19B}{8}\right)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^3}}{f a^3 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)

[Out]
$$\frac{2/f/a^3/c^4*(-1/7*(A+B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/6*(3*A+3*B)/(\tan(1/2*f*x+1/2*e)-1)^6-1/4*(11/2*A+9/2*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/2*(15/8*A+B)/(\tan(1/2*f*x+1/2*e)-1)^2-(21/32*A+5/32*B)/(\tan(1/2*f*x+1/2*e)-1)-1/5*(21/4*A+9/4*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/3*(33/8*A+11/4*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/2*(-1/2*A+3/8*B)/(\tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/2*A+1/2*B)/(\tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/4*A-1/4*B)/(\tan(1/2*f*x+1/2*e)+1)^5-1/3*(3/4*A-5/8*B)/(\tan(1/2*f*x+1/2*e)+1)^3-(11/32*A-5/32*B)/(\tan(1/2*f*x+1/2*e)+1))$$

maxima [B] time = 0.41, size = 1019, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2/105*(B*(30*\sin(f*x + e)/(\cos(f*x + e) + 1) - 45*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 80*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 110*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 188*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 266*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 112*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 35*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 70*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 105*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 - 15)/(a^3*c^4 - 2*a^3*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) - 4*a^3*c^4*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*c^4*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*c^4*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 20*a^3*c^4*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 20*a^3*c^4*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 5*a^3*c^4*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 10*a^3*c^4*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 4*a^3*c^4*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 2*a^3*c^4*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 - a^3*c^4*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12) - 3*A*(25*\sin(f*x + e)/(\cos(f*x + e) + 1) - 55*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 130*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 26*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 182*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 126*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 105*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 35*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 35*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 35*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 5)/(a^3*c^4 - 2*a^3*c^4*\sin(f*x + e)/(\cos(f*x + e) + 1) - 4*a^3*c^4* \end{aligned}$$

$$\frac{\sin(fx + e)^2/(\cos(fx + e) + 1)^2 + 10a^3c^4\sin(fx + e)^3/(\cos(fx + e) + 1)^3 + 5a^3c^4\sin(fx + e)^4/(\cos(fx + e) + 1)^4 - 20a^3c^4\sin(fx + e)^5/(\cos(fx + e) + 1)^5 + 20a^3c^4\sin(fx + e)^7/(\cos(fx + e) + 1)^7 - 5a^3c^4\sin(fx + e)^8/(\cos(fx + e) + 1)^8 - 10a^3c^4\sin(fx + e)^9/(\cos(fx + e) + 1)^9 + 4a^3c^4\sin(fx + e)^{10}/(\cos(fx + e) + 1)^{10} + 2a^3c^4\sin(fx + e)^{11}/(\cos(fx + e) + 1)^{11} - a^3c^4\sin(fx + e)^{12}/(\cos(fx + e) + 1)^{12}}{f}$$

mupad [B] time = 13.16, size = 217, normalized size = 1.79

$$\frac{\left(\frac{16A}{35} - \frac{8B}{105} - \frac{32A \sin(e+fx)}{35} + \frac{16B \sin(e+fx)}{105}\right) \cos(e+fx)^4 + \left(\frac{4A}{35} - \frac{2B}{105} - \frac{16A \sin(e+fx)}{35} + \frac{8B \sin(e+fx)}{105}\right) \cos(e+fx)}{a^3 c^4 f \left(2 \cos(e+fx)^5 \sin(e+fx) - 2 \cos(e+fx)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^4),x)

[Out] ((2*A)/35 - (12*B)/35 - (12*A*sin(e + f*x))/35 + (2*B*sin(e + f*x))/35 + cos(e + f*x)^2*((4*A)/35 - (2*B)/105 - (16*A*sin(e + f*x))/35 + (8*B*sin(e + f*x))/105) + cos(e + f*x)^4*((16*A)/35 - (8*B)/105 - (32*A*sin(e + f*x))/35 + (16*B*sin(e + f*x))/105))/(a^3*c^4*f*(2*cos(e + f*x)^5*sin(e + f*x) - 2*cos(e + f*x)^5)) - ((2*A)/7 + (2*B)/7 - (2*A*sin(e + f*x))/7 - (2*B*sin(e + f*x))/7)/(a^3*c^4*f*(2*sin(e + f*x) - 2)) - (cos(e + f*x)*((32*A)/35 - (16*B)/105))/(a^3*c^4*f*(2*sin(e + f*x) - 2))

sympy [A] time = 92.70, size = 6135, normalized size = 50.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^4,x)

[Out] Piecewise((-210*A*tan(e/2 + f*x/2)**11/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**10/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) + 210*A*tan(e/2 + f*x/2)**9/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 630*A*tan(e/2 + f*x/2)**8/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12 - 210*a**3*c**4*f*tan(e/2 + f*x/2)**11 - 420*a**3*c**4*f*tan(e/2 + f*x/2)**10 + 1050*a**3*c**4*f*tan(e/2 + f*x/2)**9 + 525*a**3*c**4*f*tan(e/2 + f*x/2)**8 - 2100*a**3*c**4*f*tan(e/2 + f*x/2)**7 + 2100*a**3*c**4*f*tan(e/2 + f*x/2)**5 - 525*a**3*c**4*f*tan(e/2 + f*x/2)**4 - 1050*a**3*c**4*f*tan(e/2 + f*x/2)**3 + 420*a**3*c**4*f*tan(e/2 + f*x/2)**2 + 210*a**3*c**4*f*tan(e/2 + f*x/2) - 105*a**3*c**4*f) - 756*A*tan(e/2 + f*x/2)**7/(105*a**3*c**4*f*tan(e/2 + f*x/2)**12

))

$$3.79 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^5} dx$$

Optimal. Leaf size=162

$$\frac{2(7A-2B) \tan^5(e+fx)}{105a^3c^5f} + \frac{4(7A-2B) \tan^3(e+fx)}{63a^3c^5f} + \frac{2(7A-2B) \tan(e+fx)}{21a^3c^5f} + \frac{(7A-2B) \sec^5(e+fx)}{63a^3f(c^5-c^5 \sin(e+fx))} + \frac{(A+B) \sec^5(e+fx)}{9a^3c^5f}$$

[Out] 1/9*(A+B)*sec(f*x+e)^5/a^3/c^3/f/(c-c*sin(f*x+e))^2+1/63*(7*A-2*B)*sec(f*x+e)^5/a^3/f/(c^5-c^5*sin(f*x+e))+2/21*(7*A-2*B)*tan(f*x+e)/a^3/c^5/f+4/63*(7*A-2*B)*tan(f*x+e)^3/a^3/c^5/f+2/105*(7*A-2*B)*tan(f*x+e)^5/a^3/c^5/f

Rubi [A] time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(7A-2B) \tan^5(e+fx)}{105a^3c^5f} + \frac{4(7A-2B) \tan^3(e+fx)}{63a^3c^5f} + \frac{2(7A-2B) \tan(e+fx)}{21a^3c^5f} + \frac{(7A-2B) \sec^5(e+fx)}{63a^3f(c^5-c^5 \sin(e+fx))} + \frac{(A+B) \sec^5(e+fx)}{9a^3c^5f}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5),x]

[Out] ((A + B)*Sec[e + f*x]^5)/(9*a^3*c^3*f*(c - c*Sin[e + f*x])^2) + ((7*A - 2*B)*Sec[e + f*x]^5)/(63*a^3*f*(c^5 - c^5*Sin[e + f*x])) + (2*(7*A - 2*B)*Tan[e + f*x])/((21*a^3*c^5*f) + (4*(7*A - 2*B)*Tan[e + f*x]^3)/(63*a^3*c^5*f) + (2*(7*A - 2*B)*Tan[e + f*x]^5)/(105*a^3*c^5*f))

Rule 2672

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^5} dx &= \frac{\int \frac{\sec^6(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx}{a^3 c^3} \\ &= \frac{(A + B) \sec^5(e + fx)}{9a^3 c^3 f (c - c \sin(e + fx))^2} + \frac{(7A - 2B) \int \frac{\sec^6(e+fx)}{c-c \sin(e+fx)} dx}{9a^3 c^4} \\ &= \frac{(A + B) \sec^5(e + fx)}{9a^3 c^3 f (c - c \sin(e + fx))^2} + \frac{(7A - 2B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} + \frac{(2A - B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} \\ &= \frac{(A + B) \sec^5(e + fx)}{9a^3 c^3 f (c - c \sin(e + fx))^2} + \frac{(7A - 2B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} - \frac{(2A - B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} \\ &= \frac{(A + B) \sec^5(e + fx)}{9a^3 c^3 f (c - c \sin(e + fx))^2} + \frac{(7A - 2B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} + \frac{(2A - B) \sec^5(e + fx)}{63a^3 f (c^5 - c^5 \sin(e + fx))} \end{aligned}$$

Mathematica [B] time = 1.41, size = 373, normalized size = 2.30

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)(1125(49A + 13B) \cos(e + fx) - 20480 \sin(e + fx))}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^5), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-184320*B + 1125*(49*A + 13*B)*Cos[e + f*x] - 20480*(7*A - 2*B)*Cos[2*(e + f*x)] + 23275*A*Cos[3*(e + f*x)] + 6175*B*Cos[3*(e + f*x)] - 114688*A*Cos[4*(e + f*x)] + 32768*B*Cos[4*(e + f*x)] + 1225*A*Cos[5*(e + f*x)] + 325*B*Cos[5*(e + f*x)] - 28672*A*Cos[6*(e + f*x)] + 8192*B*Cos[6*(e + f*x)] - 1225*A*Cos[7*(e + f*x)] - 325*B*Cos[7*(e + f*x)] - 322560*A*Sin[e + f*x] + 92160*B*Sin[e + f*x] - 24500*A*Sin[2*(e + f*x)] - 6500*B*Sin[2*(e + f*x)] - 136192*A*Sin[3*(e + f*x)] + 38912*B*Sin[3*(e + f*x)] - 19600*A*Sin[4*(e + f*x)] - 5200*B*Sin[4*(e + f*x)] - 7168*A*Sin[5*(e + f*x)] + 2048*B*Sin[5*(e + f*x)] - 4900*A*Sin[6*(e + f*x)] - 1300*B*Sin[6*(e + f*x)] + 7168*A*Sin[7*(e + f*x)] - 2048*B*Sin[7*(e + f*x)])))/(1290240*a^3*c^5*f*(-1 + Sin[e + f*x])^5*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.44, size = 185, normalized size = 1.14

$$\frac{32(7A - 2B) \cos^6(fx + e) - 16(7A - 2B) \cos^4(fx + e) - 4(7A - 2B) \cos^2(fx + e) - (16(7A - 2B) \cos^2(fx + e) - 10(7A - 2B) \cos(fx + e) + 49A + 14B) \sin(fx + e)}{315(a^3 c^5 f \cos(fx + e))^7 + 2a^3 c^5 f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x, algorithm="fricas")

[Out] -1/315*(32*(7*A - 2*B)*cos(f*x + e)^6 - 16*(7*A - 2*B)*cos(f*x + e)^4 - 4*(7*A - 2*B)*cos(f*x + e)^2 - (16*(7*A - 2*B)*cos(f*x + e)^2 - 10*(7*A - 2*B)*cos(f*x + e) + 49*A + 14*B)*sin(f*x + e)

$$\begin{aligned} & x + e)^6/(\cos(f*x + e) + 1)^6 + 1032*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - \\ & 483*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 588*\sin(f*x + e)^9/(\cos(f*x + e) \\ & + 1)^9 - 420*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 420*\sin(f*x + e)^{11}/(\cos \\ & (f*x + e) + 1)^{11} - 315*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 25)/(a^3*c^5 \\ & c^5 - 4*a^3*c^5*\sin(f*x + e)/(\cos(f*x + e) + 1) + a^3*c^5*\sin(f*x + e)^2/(\cos \\ & (f*x + e) + 1)^2 + 16*a^3*c^5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 19*a^3*c^5 \\ & 3*c^5*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 20*a^3*c^5*\sin(f*x + e)^5/(\cos \\ & (f*x + e) + 1)^5 + 45*a^3*c^5*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 45*a^3*c^5 \\ & ^5*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 20*a^3*c^5*\sin(f*x + e)^9/(\cos(f*x \\ & + e) + 1)^9 + 19*a^3*c^5*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 16*a^3*c^5 \\ & 5*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - a^3*c^5*\sin(f*x + e)^{12}/(\cos(f*x \\ & + e) + 1)^{12} + 4*a^3*c^5*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - a^3*c^5*\sin \\ & (f*x + e)^{14}/(\cos(f*x + e) + 1)^{14}) - 7*A*(5*\sin(f*x + e)/(\cos(f*x + e) + \\ & 1) - 80*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 190*\sin(f*x + e)^3/(\cos(f*x + \\ & e) + 1)^3 + 50*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 269*\sin(f*x + e)^5/(\cos \\ & (f*x + e) + 1)^5 + 96*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 516*\sin(f*x + \\ & e)^7/(\cos(f*x + e) + 1)^7 - 354*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 69*\sin \\ & (f*x + e)^9/(\cos(f*x + e) + 1)^9 + 240*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} \\ & + 30*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 90*\sin(f*x + e)^{12}/(\cos(f*x \\ & + e) + 1)^{12} + 45*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} + 10)/(a^3*c^5 - \\ & 4*a^3*c^5*\sin(f*x + e)/(\cos(f*x + e) + 1) + a^3*c^5*\sin(f*x + e)^2/(\cos(f*x \\ & + e) + 1)^2 + 16*a^3*c^5*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 19*a^3*c^5 \\ & \sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 20*a^3*c^5*\sin(f*x + e)^5/(\cos(f*x + \\ & e) + 1)^5 + 45*a^3*c^5*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 45*a^3*c^5*\sin \\ & (f*x + e)^8/(\cos(f*x + e) + 1)^8 + 20*a^3*c^5*\sin(f*x + e)^9/(\cos(f*x + e) \\ & + 1)^9 + 19*a^3*c^5*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 16*a^3*c^5*\sin \\ & (f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - a^3*c^5*\sin(f*x + e)^{12}/(\cos(f*x + e) \\ & + 1)^{12} + 4*a^3*c^5*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - a^3*c^5*\sin(f*x \\ & + e)^{14}/(\cos(f*x + e) + 1)^{14}))/f \end{aligned}$$

mupad [B] time = 13.14, size = 231, normalized size = 1.43

$$\left(\frac{128B}{315} - \frac{64A}{45} + \frac{32A \sin(e+fx)}{45} - \frac{64B \sin(e+fx)}{315} \right) \cos(e+fx)^6 + \left(\frac{8A \sin(e+fx)}{9} - \frac{20B}{63} - \frac{8A}{9} + \frac{20B \sin(e+fx)}{63} - \frac{4 \sin(e+fx)}{9} \right) \cos(e+fx)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^5),x)
[Out] ((4*A)/45 - (14*B)/45 - (14*A*sin(e + f*x))/45 + (4*B*sin(e + f*x))/45 - co
s(e + f*x)^5*((8*A)/9 + (20*B)/63 - (8*A*sin(e + f*x))/9 - (20*B*sin(e + f*
x))/63 + ((4*sin(e + f*x) - 4)*((4*A)/9 + (10*B)/63))/2) + cos(e + f*x)^2*(
(8*A)/45 - (16*B)/315 - (4*A*sin(e + f*x))/9 + (8*B*sin(e + f*x))/63) + cos
(e + f*x)^4*((32*A)/45 - (64*B)/315 - (16*A*sin(e + f*x))/15 + (32*B*sin(e
+ f*x))/105) - cos(e + f*x)^6*((64*A)/45 - (128*B)/315 - (32*A*sin(e + f*x)
)/45 + (64*B*sin(e + f*x))/315))/(a^3*c^5*f*(4*cos(e + f*x)^5*sin(e + f*x)
- 4*cos(e + f*x)^5 + 2*cos(e + f*x)^7))
```

sympy [A] time = 155.47, size = 8396, normalized size = 51.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^5,x)
[Out] Piecewise((-630*A*tan(e/2 + f*x/2)**13/(315*a**3*c**5*f*tan(e/2 + f*x/2)**1
4 - 1260*a**3*c**5*f*tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2
)**12 + 5040*a**3*c**5*f*tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 +
```


$$\begin{aligned}
& **5*f*\tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**9 + 14175*a \\
& **3*c**5*f*\tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**6 + 63 \\
& 00*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**4 - \\
& 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*\tan(e/2 + f*x/2)**2 \\
& + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 700*A*\tan(e/2 + f \\
& *x/2)**4/(315*a**3*c**5*f*\tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*\tan(e/2 + \\
& f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*\tan(e \\
& /2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f* \\
& \tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**8 - 14175*a**3*c* \\
& *5*f*\tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + 5985*a**3 \\
& *c**5*f*\tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**3 - 315*a* \\
& **3*c**5*f*\tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) - 315*a** \\
& 3*c**5*f) - 2660*A*\tan(e/2 + f*x/2)**3/(315*a**3*c**5*f*\tan(e/2 + f*x/2)**1 \\
& 4 - 1260*a**3*c**5*f*\tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f*x/2) \\
&)**12 + 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/2 + \\
& f*x/2)**10 - 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*\tan(e \\
& /2 + f*x/2)**8 - 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*t \\
& \tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f \\
& *\tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*\tan(e/2 + f*x/2)**2 + 1260*a**3*c** \\
& 5*f*\tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 1120*A*\tan(e/2 + f*x/2)**2/(315*a \\
& **3*c**5*f*\tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*\tan(e/2 + f*x/2)**13 + 3 \\
& 15*a**3*c**5*f*\tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**11 \\
& - 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*\tan(e/2 + f*x/2) \\
&)**9 + 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*\tan(e/2 + \\
& f*x/2)**6 + 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*\tan(e/2 \\
& + f*x/2)**4 - 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*\tan(e \\
& /2 + f*x/2)**2 + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) - 315*a**3*c**5*f) - 70* \\
& A*\tan(e/2 + f*x/2)/(315*a**3*c**5*f*\tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f \\
& *\tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f*x/2)**12 + 5040*a**3*c* \\
& *5*f*\tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**10 - 6300*a* \\
& **3*c**5*f*\tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**8 - 141 \\
& 75*a**3*c**5*f*\tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + \\
& 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*\tan(e/2 + f*x/2)** \\
& 3 - 315*a**3*c**5*f*\tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) \\
& - 315*a**3*c**5*f) - 140*A/(315*a**3*c**5*f*\tan(e/2 + f*x/2)**14 - 1260*a* \\
& **3*c**5*f*\tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f*x/2)**12 + 504 \\
& 0*a**3*c**5*f*\tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**10 \\
& - 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*\tan(e/2 + f*x/2) \\
& **8 - 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*\tan(e/2 + f* \\
& x/2)**5 + 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*\tan(e/2 + \\
& f*x/2)**3 - 315*a**3*c**5*f*\tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*\tan(e/2 \\
& + f*x/2) - 315*a**3*c**5*f) - 630*B*\tan(e/2 + f*x/2)**12/(315*a**3*c**5*f* \\
& \tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*\tan(e/2 + f*x/2)**13 + 315*a**3*c** \\
& 5*f*\tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**11 - 5985*a** \\
& 3*c**5*f*\tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**9 + 1417 \\
& 5*a**3*c**5*f*\tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**6 + \\
& 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*\tan(e/2 + f*x/2)** \\
& 4 - 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*\tan(e/2 + f*x/2) \\
& **2 + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 840*B*\tan(e/2 \\
& + f*x/2)**11/(315*a**3*c**5*f*\tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*\tan(e \\
& /2 + f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*t \\
& \tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/2 + f*x/2)**10 - 6300*a**3*c** \\
& 5*f*\tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*\tan(e/2 + f*x/2)**8 - 14175*a** \\
& 3*c**5*f*\tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*\tan(e/2 + f*x/2)**5 + 5985* \\
& a**3*c**5*f*\tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**3 - 31 \\
& 5*a**3*c**5*f*\tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*\tan(e/2 + f*x/2) - 315 \\
& *a**3*c**5*f) - 840*B*\tan(e/2 + f*x/2)**10/(315*a**3*c**5*f*\tan(e/2 + f*x/2) \\
&)**14 - 1260*a**3*c**5*f*\tan(e/2 + f*x/2)**13 + 315*a**3*c**5*f*\tan(e/2 + f \\
& *x/2)**12 + 5040*a**3*c**5*f*\tan(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*\tan(e/
\end{aligned}$$


```

*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/2 + f*x/2)**9 + 14175*a**3
*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*tan(e/2 + f*x/2)**6 + 6300*
a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*f*tan(e/2 + f*x/2)**4 - 50
40*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**5*f*tan(e/2 + f*x/2)**2 +
1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5*f) + 200*B*tan(e/2 + f*x/
2)/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2 + f*x/2
)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan(e/2 + f
*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*f*tan(e/
2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*c**5*f*t
an(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a**3*c**5*
f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*a**3*c**
5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a**3*c**5
*f) - 50*B/(315*a**3*c**5*f*tan(e/2 + f*x/2)**14 - 1260*a**3*c**5*f*tan(e/2
+ f*x/2)**13 + 315*a**3*c**5*f*tan(e/2 + f*x/2)**12 + 5040*a**3*c**5*f*tan
(e/2 + f*x/2)**11 - 5985*a**3*c**5*f*tan(e/2 + f*x/2)**10 - 6300*a**3*c**5*
f*tan(e/2 + f*x/2)**9 + 14175*a**3*c**5*f*tan(e/2 + f*x/2)**8 - 14175*a**3*
c**5*f*tan(e/2 + f*x/2)**6 + 6300*a**3*c**5*f*tan(e/2 + f*x/2)**5 + 5985*a*
**3*c**5*f*tan(e/2 + f*x/2)**4 - 5040*a**3*c**5*f*tan(e/2 + f*x/2)**3 - 315*
a**3*c**5*f*tan(e/2 + f*x/2)**2 + 1260*a**3*c**5*f*tan(e/2 + f*x/2) - 315*a
**3*c**5*f), Ne(f, 0)), (x*(A + B*sin(e))/((a*sin(e) + a)**3*(-c*sin(e) + c
)**5), True))

```

$$3.80 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^6} dx$$

Optimal. Leaf size=205

$$\frac{2(8A-3B) \tan^5(e+fx)}{165a^3c^6f} + \frac{4(8A-3B) \tan^3(e+fx)}{99a^3c^6f} + \frac{2(8A-3B) \tan(e+fx)}{33a^3c^6f} + \frac{(8A-3B) \sec^5(e+fx)}{99a^3f(c^6-c^6 \sin(e+fx))} + \frac{(8A-3B) \sec^5(e+fx)}{99a^3}$$

[Out] 1/11*(A+B)*sec(f*x+e)^5/a^3/f/(c^2-c^2*sin(f*x+e))^3+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^3-c^3*sin(f*x+e))^2+1/99*(8*A-3*B)*sec(f*x+e)^5/a^3/f/(c^6-c^6*sin(f*x+e))+2/33*(8*A-3*B)*tan(f*x+e)/a^3/c^6/f+4/99*(8*A-3*B)*tan(f*x+e)^3/a^3/c^6/f+2/165*(8*A-3*B)*tan(f*x+e)^5/a^3/c^6/f

Rubi [A] time = 0.35, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2859, 2672, 3767}

$$\frac{2(8A-3B) \tan^5(e+fx)}{165a^3c^6f} + \frac{4(8A-3B) \tan^3(e+fx)}{99a^3c^6f} + \frac{2(8A-3B) \tan(e+fx)}{33a^3c^6f} + \frac{(8A-3B) \sec^5(e+fx)}{99a^3f(c^6-c^6 \sin(e+fx))} + \frac{(8A-3B) \sec^5(e+fx)}{99a^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6),x]

[Out] ((A + B)*Sec[e + f*x]^5)/(11*a^3*f*(c^2 - c^2*Sin[e + f*x])^3) + ((8*A - 3*B)*Sec[e + f*x]^5)/(99*a^3*f*(c^6 - c^6*Sin[e + f*x])) + (2*(8*A - 3*B)*Tan[e + f*x])/(33*a^3*c^6*f) + (4*(8*A - 3*B)*Tan[e + f*x]^3)/(99*a^3*c^6*f) + (2*(8*A - 3*B)*Tan[e + f*x]^5)/(165*a^3*c^6*f)

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^6} dx &= \frac{\int \frac{\sec^6(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx}{a^3 c^3} \\ &= \frac{(A + B) \sec^5(e + fx)}{11a^3 f (c^2 - c^2 \sin(e + fx))^3} + \frac{(8A - 3B) \int \frac{\sec^6(e+fx)}{(c-c \sin(e+fx))^2} dx}{11a^3 c^4} \\ &= \frac{(A + B) \sec^5(e + fx)}{11a^3 f (c^2 - c^2 \sin(e + fx))^3} + \frac{(8A - 3B) \sec^5(e + fx)}{99a^3 f (c^3 - c^3 \sin(e + fx))^2} + \\ &= \frac{(A + B) \sec^5(e + fx)}{11a^3 f (c^2 - c^2 \sin(e + fx))^3} + \frac{(8A - 3B) \sec^5(e + fx)}{99a^3 f (c^3 - c^3 \sin(e + fx))^2} + \\ &= \frac{(A + B) \sec^5(e + fx)}{11a^3 f (c^2 - c^2 \sin(e + fx))^3} + \frac{(8A - 3B) \sec^5(e + fx)}{99a^3 f (c^3 - c^3 \sin(e + fx))^2} + \\ &= \frac{(A + B) \sec^5(e + fx)}{11a^3 f (c^2 - c^2 \sin(e + fx))^3} + \frac{(8A - 3B) \sec^5(e + fx)}{99a^3 f (c^3 - c^3 \sin(e + fx))^2} + \end{aligned}$$

Mathematica [A] time = 3.55, size = 401, normalized size = 1.96

$$\frac{-3850(107A - 3B) \cos(e + fx) + 135168(8A - 3B) \cos(2(e + fx)) + 1802240A \sin(e + fx) + 247170A \sin(2(e + fx))}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^6}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^6), x]

[Out] (1013760*B - 3850*(107*A - 3*B)*Cos[e + f*x] + 135168*(8*A - 3*B)*Cos[2*(e + f*x)] - 127330*A*Cos[3*(e + f*x)] + 3570*B*Cos[3*(e + f*x)] + 819200*A*Cos[4*(e + f*x)] - 307200*B*Cos[4*(e + f*x)] + 37450*A*Cos[5*(e + f*x)] - 10500*B*Cos[5*(e + f*x)] + 163840*A*Cos[6*(e + f*x)] - 61440*B*Cos[6*(e + f*x)] + 22470*A*Cos[7*(e + f*x)] - 630*B*Cos[7*(e + f*x)] - 16384*A*Cos[8*(e + f*x)] + 6144*B*Cos[8*(e + f*x)] + 1802240*A*Sin[e + f*x] - 675840*B*Sin[e + f*x] + 247170*A*Sin[2*(e + f*x)] - 6930*B*Sin[2*(e + f*x)] + 557056*A*Sin[3*(e + f*x)] - 208896*B*Sin[3*(e + f*x)] + 187250*A*Sin[4*(e + f*x)] - 5250*B*Sin[4*(e + f*x)] - 163840*A*Sin[5*(e + f*x)] + 61440*B*Sin[5*(e + f*x)] + 37450*A*Sin[6*(e + f*x)] - 1050*B*Sin[6*(e + f*x)] - 98304*A*Sin[7*(e + f*x)] + 36864*B*Sin[7*(e + f*x)] - 3745*A*Sin[8*(e + f*x)] + 105*B*Sin[8*(e + f*x)])/(8110080*a^3*c^6*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.46, size = 221, normalized size = 1.08

$$\frac{16(8A - 3B) \cos(fx + e)^8 - 72(8A - 3B) \cos(fx + e)^6 + 30(8A - 3B) \cos(fx + e)^4 + 7(8A - 3B) \cos(fx + e)^2 + 495(3a^3c^6f \cos(fx + e)^7 - 4a^3c^6f \cos(fx + e)^5)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="fricas")

[Out] $\frac{1}{495}*(16*(8*A - 3*B)*\cos(f*x + e)^8 - 72*(8*A - 3*B)*\cos(f*x + e)^6 + 30*(8*A - 3*B)*\cos(f*x + e)^4 + 7*(8*A - 3*B)*\cos(f*x + e)^2 + (48*(8*A - 3*B)*\cos(f*x + e)^6 - 40*(8*A - 3*B)*\cos(f*x + e)^4 - 14*(8*A - 3*B)*\cos(f*x + e)^2 - 72*A + 27*B)*\sin(f*x + e) + 27*A - 72*B)/(3*a^3*c^6*f*\cos(f*x + e)^7 - 4*a^3*c^6*f*\cos(f*x + e)^5 - (a^3*c^6*f*\cos(f*x + e)^7 - 4*a^3*c^6*f*\cos(f*x + e)^5)*\sin(f*x + e))$

giac [B] time = 0.30, size = 475, normalized size = 2.32

$$\frac{33 \left(555 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 - 315 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^4 + 1920 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 - 1020 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^3 + 2710 A \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 - 1410 B \tan\left(\frac{1}{2} f x + \frac{1}{2} e\right)^2 \right)}{a^3 c^6 \left(\tan\left(\frac{1}{2} f x + \frac{1}{2} e\right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="giac")

[Out] $-\frac{1}{63360}*(33*(555*A*\tan(1/2*f*x + 1/2*e)^4 - 315*B*\tan(1/2*f*x + 1/2*e)^4 + 1920*A*\tan(1/2*f*x + 1/2*e)^3 - 1020*B*\tan(1/2*f*x + 1/2*e)^3 + 2710*A*\tan(1/2*f*x + 1/2*e)^2 - 1410*B*\tan(1/2*f*x + 1/2*e)^2 + 1760*A*\tan(1/2*f*x + 1/2*e) - 900*B*\tan(1/2*f*x + 1/2*e) + 463*A - 243*B)/(a^3*c^6*(\tan(1/2*f*x + 1/2*e) + 1)^5) + (108405*A*\tan(1/2*f*x + 1/2*e)^{10} + 10395*B*\tan(1/2*f*x + 1/2*e)^{10} - 784080*A*\tan(1/2*f*x + 1/2*e)^9 - 5940*B*\tan(1/2*f*x + 1/2*e)^9 + 2901195*A*\tan(1/2*f*x + 1/2*e)^8 - 79695*B*\tan(1/2*f*x + 1/2*e)^8 - 6652800*A*\tan(1/2*f*x + 1/2*e)^7 + 388080*B*\tan(1/2*f*x + 1/2*e)^7 + 10407474*A*\tan(1/2*f*x + 1/2*e)^6 - 816354*B*\tan(1/2*f*x + 1/2*e)^6 - 11435424*A*\tan(1/2*f*x + 1/2*e)^5 + 1114344*B*\tan(1/2*f*x + 1/2*e)^5 + 8949270*A*\tan(1/2*f*x + 1/2*e)^4 - 990990*B*\tan(1/2*f*x + 1/2*e)^4 - 4899840*A*\tan(1/2*f*x + 1/2*e)^3 + 609840*B*\tan(1/2*f*x + 1/2*e)^3 + 1816265*A*\tan(1/2*f*x + 1/2*e)^2 - 235785*B*\tan(1/2*f*x + 1/2*e)^2 - 411664*A*\tan(1/2*f*x + 1/2*e) + 56364*B*\tan(1/2*f*x + 1/2*e) + 47279*A - 4179*B)/(a^3*c^6*(\tan(1/2*f*x + 1/2*e) - 1)^{11}))/f$

maple [A] time = 0.51, size = 365, normalized size = 1.78

$$\frac{2(4A+4B)}{11\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{11}} - \frac{20A+20B}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^{10}} - \frac{2(53A+51B)}{9\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^9} - \frac{92A+84B}{4\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^8} - \frac{\frac{169A}{4}+\frac{99B}{4}}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^4} - \frac{\frac{217A}{2}+84B}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)-1\right)^6} - \frac{2\left(\frac{219A}{256}\right)}{\tan\left(\frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x)

[Out] $\frac{2}{f/a^3/c^6}*(-1/11*(4*A+4*B)/(\tan(1/2*f*x+1/2*e)-1)^{11}-1/10*(20*A+20*B)/(\tan(1/2*f*x+1/2*e)-1)^{10}-1/9*(53*A+51*B)/(\tan(1/2*f*x+1/2*e)-1)^9-1/8*(92*A+84*B)/(\tan(1/2*f*x+1/2*e)-1)^8-1/4*(169/4*A+99/4*B)/(\tan(1/2*f*x+1/2*e)-1)^4-1/6*(217/2*A+84*B)/(\tan(1/2*f*x+1/2*e)-1)^6-(219/256*A+21/256*B)/(\tan(1/2*f*x+1/2*e)-1)-1/7*(231/2*A+98*B)/(\tan(1/2*f*x+1/2*e)-1)^7-1/2*(303/64*A+99/64*B)/(\tan(1/2*f*x+1/2*e)-1)^2-1/5*(623/8*A+427/8*B)/(\tan(1/2*f*x+1/2*e)-1)^5-1/3*(1095/64*A+507/64*B)/(\tan(1/2*f*x+1/2*e)-1)^3-1/2*(-5/32*A+1/8*B)/(\tan(1/2*f*x+1/2*e)+1)^2-1/4*(-1/8*A+1/8*B)/(\tan(1/2*f*x+1/2*e)+1)^4-1/5*(1/16*A-1/16*B)/(\tan(1/2*f*x+1/2*e)+1)^5-1/3*(7/32*A-3/16*B)/(\tan(1/2*f*x+1/2*e)+1)^3-(37/256*A-21/256*B)/(\tan(1/2*f*x+1/2*e)+1))$

maxima [B] time = 0.58, size = 1387, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^6,x, algorithm="maxima")

[Out]
$$-2/495*(A*(255*\sin(f*x + e)/(\cos(f*x + e) + 1) + 235*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 3065*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3775*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 667*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 8217*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 2035*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 8745*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 - 11715*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 33*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 4917*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 2475*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 1815*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} + 1485*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - 495*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15} - 125)/(a^3*c^6 - 6*a^3*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 50*a^3*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 34*a^3*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 66*a^3*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 110*a^3*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 110*a^3*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 66*a^3*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 34*a^3*c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 50*a^3*c^6*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 10*a^3*c^6*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 10*a^3*c^6*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} + 6*a^3*c^6*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15} - a^3*c^6*\sin(f*x + e)^{16}/(\cos(f*x + e) + 1)^{16}) + 3*B*(30*\sin(f*x + e)/(\cos(f*x + e) + 1) - 215*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 280*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 245*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 434*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 - 231*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 880*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 - 1815*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 330*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 99*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 264*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} - 495*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} + 330*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 165*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} - 5)/(a^3*c^6 - 6*a^3*c^6*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*c^6*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*c^6*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 50*a^3*c^6*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 34*a^3*c^6*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 66*a^3*c^6*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 110*a^3*c^6*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 110*a^3*c^6*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 - 66*a^3*c^6*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} - 34*a^3*c^6*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 50*a^3*c^6*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} - 10*a^3*c^6*\sin(f*x + e)^{13}/(\cos(f*x + e) + 1)^{13} - 10*a^3*c^6*\sin(f*x + e)^{14}/(\cos(f*x + e) + 1)^{14} + 6*a^3*c^6*\sin(f*x + e)^{15}/(\cos(f*x + e) + 1)^{15} - a^3*c^6*\sin(f*x + e)^{16}/(\cos(f*x + e) + 1)^{16}))/f$$

mupad [B] time = 14.52, size = 474, normalized size = 2.31

$$2 \left(\frac{165 B \sin(e+fx)}{4} - \frac{6875 A \cos(e+fx)}{64} - \frac{825 B \cos(e+fx)}{64} - 110 A \sin(e+fx) - \frac{495 B}{8} - 66 A \cos(2e+2fx) - \frac{2125}{64} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^6),x)

[Out]
$$(2*((165*B*\sin(e + f*x))/4 - (6875*A*\cos(e + f*x))/64 - (825*B*\cos(e + f*x))/64 - 110*A*\sin(e + f*x) - (495*B)/8 - 66*A*\cos(2*e + 2*f*x) - (2125*A*\cos(3*e + 3*f*x))/64 - 50*A*\cos(4*e + 4*f*x) + (625*A*\cos(5*e + 5*f*x))/64 - 1$$

```

0*A*cos(6*e + 6*f*x) + (375*A*cos(7*e + 7*f*x))/64 + A*cos(8*e + 8*f*x) + (
99*B*cos(2*e + 2*f*x))/4 - (255*B*cos(3*e + 3*f*x))/64 + (75*B*cos(4*e + 4*
f*x))/4 + (75*B*cos(5*e + 5*f*x))/64 + (15*B*cos(6*e + 6*f*x))/4 + (45*B*co
s(7*e + 7*f*x))/64 - (3*B*cos(8*e + 8*f*x))/8 + (4125*A*sin(2*e + 2*f*x))/6
4 - 34*A*sin(3*e + 3*f*x) + (3125*A*sin(4*e + 4*f*x))/64 + 10*A*sin(5*e + 5
*f*x) + (625*A*sin(6*e + 6*f*x))/64 + 6*A*sin(7*e + 7*f*x) - (125*A*sin(8*e
+ 8*f*x))/128 + (495*B*sin(2*e + 2*f*x))/64 + (51*B*sin(3*e + 3*f*x))/4 +
(375*B*sin(4*e + 4*f*x))/64 - (15*B*sin(5*e + 5*f*x))/4 + (75*B*sin(6*e + 6
*f*x))/64 - (9*B*sin(7*e + 7*f*x))/4 - (15*B*sin(8*e + 8*f*x))/128)/(495*a
^3*c^6*f*((5*cos(5*e + 5*f*x))/32 - (17*cos(3*e + 3*f*x))/32 - (55*cos(e +
f*x))/32 + (3*cos(7*e + 7*f*x))/32 + (33*sin(2*e + 2*f*x))/32 + (25*sin(4*e
+ 4*f*x))/32 + (5*sin(6*e + 6*f*x))/32 - sin(8*e + 8*f*x)/64))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**6,x)

[Out] Timed out

$$3.81 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=198

$$\frac{256ac^5(11A - 5B) \cos^3(e + fx)}{3465f(c - c \sin(e + fx))^{3/2}} + \frac{64ac^4(11A - 5B) \cos^3(e + fx)}{1155f\sqrt{c - c \sin(e + fx)}} + \frac{8ac^3(11A - 5B) \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{231f}$$

[Out] 256/3465*a*(11*A-5*B)*c^5*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/99*a*(11*A-5*B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f-2/11*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/f+64/1155*a*(11*A-5*B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)+8/231*a*(11*A-5*B)*c^3*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.49, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256ac^5(11A - 5B) \cos^3(e + fx)}{3465f(c - c \sin(e + fx))^{3/2}} + \frac{64ac^4(11A - 5B) \cos^3(e + fx)}{1155f\sqrt{c - c \sin(e + fx)}} + \frac{8ac^3(11A - 5B) \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{231f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2), x]

[Out] (256*a*(11*A - 5*B)*c^5*Cos[e + f*x]^3)/(3465*f*(c - c*Sin[e + f*x])^(3/2)) + (64*a*(11*A - 5*B)*c^4*Cos[e + f*x]^3)/(1155*f*Sqrt[c - c*Sin[e + f*x]]) + (8*a*(11*A - 5*B)*c^3*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(231*f) + (2*a*(11*A - 5*B)*c^2*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(99*f) - (2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(11*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] := Di

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx = (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

$$= -\frac{2aBc \cos^3(e + fx)(c - c \sin(e + fx))^{5/2}}{11f} + \frac{1}{11} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

$$= \frac{2a(11A - 5B)c^2 \cos^3(e + fx)(c - c \sin(e + fx))^{3/2}}{99f}$$

$$= \frac{8a(11A - 5B)c^3 \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{231f}$$

$$= \frac{64a(11A - 5B)c^4 \cos^3(e + fx)}{1155f\sqrt{c - c \sin(e + fx)}} + \frac{8a(11A - 5B)c^3 \cos^3(e + fx)}{1155f\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{256a(11A - 5B)c^5 \cos^3(e + fx)}{3465f(c - c \sin(e + fx))^{3/2}} + \frac{64a(11A - 5B)c^3 \cos^3(e + fx)}{1155f\sqrt{c - c \sin(e + fx)}}$$

Mathematica [A] time = 2.99, size = 149, normalized size = 0.75

$$\frac{ac^3 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^3 (60(121A - 202B) \cos(2(e + fx)) + 30558A \sin(e + fx))}{13860f \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2), x]
```

```
[Out] -1/13860*(a*c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x])*(-35332*A + 27085*B + 60*(121*A - 202*B)*Cos[2*(e + f*x)] + 315*B*Cos[4*(e + f*x)] + 30558*A*Sin[e + f*x] - 31530*B*Sin[e + f*x] - 770*A*Sin[3*(e + f*x)] + 2870*B*Sin[3*(e + f*x)])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

fricas [A] time = 0.44, size = 287, normalized size = 1.45

$$\frac{2 \left(315 Bac^3 \cos(fx + e)^6 - 35(11A - 32B)ac^3 \cos(fx + e)^5 + 5(209A - 221B)ac^3 \cos(fx + e)^4 + 2(1243A - 1195B)ac^3 \cos(fx + e)^3 - 32(11A - 5B)a^2c^3 \cos(fx + e)^2 + 128(11A - 5B)a^2c^3 \cos(fx + e) + 256(11A - 5B)a^2c^3 - (315B^2a^2c^3 \cos(fx + e)^5 + 35(11A - 23B)a^2c^3 \cos(fx + e)^4 + 10(143A - 191B)a^2c^3 \cos(fx + e)^3 - 96(11A - 5B)a^2c^3 \cos(fx + e)^2 - 128(11A - 5B)a^2c^3 \cos(fx + e) + 64(11A - 5B)a^2c^3 \right)}{13860f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2), x, algorithm="fricas")
```

```
[Out] 2/3465*(315*B*a*c^3*cos(f*x + e)^6 - 35*(11*A - 32*B)*a*c^3*cos(f*x + e)^5 + 5*(209*A - 221*B)*a*c^3*cos(f*x + e)^4 + 2*(1243*A - 1195*B)*a*c^3*cos(f*x + e)^3 - 32*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 + 128*(11*A - 5*B)*a*c^3*cos(f*x + e) + 256*(11*A - 5*B)*a*c^3 - (315*B^2*a*c^3*cos(f*x + e)^5 + 35*(11*A - 23*B)*a*c^3*cos(f*x + e)^4 + 10*(143*A - 191*B)*a*c^3*cos(f*x + e)^3 - 96*(11*A - 5*B)*a*c^3*cos(f*x + e)^2 - 128*(11*A - 5*B)*a*c^3*cos(f*x + e) + 64*(11*A - 5*B)*a*c^3)
```


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x)) (c - c \sin(e + f x))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] Timed out
```


$$3.82 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=157

$$\frac{64ac^4(3A - B) \cos^3(e + fx)}{315f(c - c \sin(e + fx))^{3/2}} + \frac{16ac^3(3A - B) \cos^3(e + fx)}{105f\sqrt{c - c \sin(e + fx)}} + \frac{2ac^2(3A - B) \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{21f} - \frac{2a}{f}$$

[Out] 64/315*a*(3*A-B)*c^4*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)-2/9*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/f+16/105*a*(3*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)+2/21*a*(3*A-B)*c^2*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.41, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{64ac^4(3A - B) \cos^3(e + fx)}{315f(c - c \sin(e + fx))^{3/2}} + \frac{16ac^3(3A - B) \cos^3(e + fx)}{105f\sqrt{c - c \sin(e + fx)}} + \frac{2ac^2(3A - B) \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{21f} - \frac{2a}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] (64*a*(3*A - B)*c^4*Cos[e + f*x]^3)/(315*f*(c - c*Sin[e + f*x])^(3/2)) + (16*a*(3*A - B)*c^3*Cos[e + f*x]^3)/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*(3*A - B)*c^2*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(21*f) - (2*a*B*c*Cos[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(9*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx \\ &= -\frac{2aBc \cos^3(e + fx)(c - c \sin(e + fx))^{3/2}}{9f} + \frac{1}{3} \int (c - c \sin(e + fx))^{3/2} dx \\ &= \frac{2a(3A - B)c^2 \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{21f} \\ &= \frac{16a(3A - B)c^3 \cos^3(e + fx)}{105f\sqrt{c - c \sin(e + fx)}} + \frac{2a(3A - B)c^2}{105f} \int (c - c \sin(e + fx))^{3/2} dx \\ &= \frac{64a(3A - B)c^4 \cos^3(e + fx)}{315f(c - c \sin(e + fx))^{3/2}} + \frac{16a(3A - B)c^3}{105f\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.52, size = 123, normalized size = 0.78

$$\frac{ac^2\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^3 ((648A - 741B) \sin(e + fx) + 30(3A - 8B) \cos(2e + 2fx))}{630f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] -1/630*(a*c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqrt[c - c*Sin[e + f*x])*(-942*A + 664*B + 30*(3*A - 8*B)*Cos[2*(e + f*x)] + (648*A - 741*B)*Sin[e + f*x] + 35*B*Sin[3*(e + f*x)])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [A] time = 0.43, size = 243, normalized size = 1.55

$$2 \left(35 Bac^2 \cos(fx + e)^5 + 5(9A - 10B)ac^2 \cos(fx + e)^4 + (117A - 109B)ac^2 \cos(fx + e)^3 - 8(3A - B)ac^2 \cos(fx + e)^2 + 32(3A - B)ac^2 \cos(fx + e) + 64(3A - B)ac^2 + (35Bac^2 \cos(fx + e))^4 - 5(9A - 17B)ac^2 \cos(fx + e)^3 + 24(3A - B)ac^2 \cos(fx + e)^2 + 32(3A - B)ac^2 \cos(fx + e) + 64(3A - B)ac^2 \sin(fx + e) \right) \sqrt{-c \sin(fx + e) + c} / (f \cos(fx + e) - f \sin(fx + e) + f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] 2/315*(35*B*a*c^2*cos(f*x + e)^5 + 5*(9*A - 10*B)*a*c^2*cos(f*x + e)^4 + (117*A - 109*B)*a*c^2*cos(f*x + e)^3 - 8*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2 + (35*B*a*c^2*cos(f*x + e))^4 - 5*(9*A - 17*B)*a*c^2*cos(f*x + e)^3 + 24*(3*A - B)*a*c^2*cos(f*x + e)^2 + 32*(3*A - B)*a*c^2*cos(f*x + e) + 64*(3*A - B)*a*c^2)*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.


```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

$$3.83 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=116

$$\frac{8ac^3(7A - B) \cos^3(e + fx)}{105f(c - c \sin(e + fx))^{3/2}} + \frac{2ac^2(7A - B) \cos^3(e + fx)}{35f\sqrt{c - c \sin(e + fx)}} - \frac{2aBc \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{7f}$$

[Out] 8/105*a*(7*A-B)*c^3*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(3/2)+2/35*a*(7*A-B)*c^2*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(1/2)-2/7*a*B*c*cos(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.32, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2ac^2(7A - B) \cos^3(e + fx)}{35f\sqrt{c - c \sin(e + fx)}} + \frac{8ac^3(7A - B) \cos^3(e + fx)}{105f(c - c \sin(e + fx))^{3/2}} - \frac{2aBc \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{7f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2), x]

[Out] (8*a*(7*A - B)*c^3*Cos[e + f*x]^3)/(105*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*(7*A - B)*c^2*Cos[e + f*x]^3)/(35*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a*B*c*Cos[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(7*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && (LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx &= (ac) \int \cos^2(e + fx)(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx \\ &= -\frac{2aBc \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{7f} + \frac{1}{7}(a^2 \cos^2(e + fx)\sqrt{c - c \sin(e + fx)}) \\ &= \frac{2a(7A - B)c^2 \cos^3(e + fx)}{35f\sqrt{c - c \sin(e + fx)}} - \frac{2aBc \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{7f} \\ &= \frac{8a(7A - B)c^3 \cos^3(e + fx)}{105f(c - c \sin(e + fx))^{3/2}} + \frac{2a(7A - B)c^2 \cos^3(e + fx)\sqrt{c - c \sin(e + fx)}}{35f\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.96, size = 104, normalized size = 0.90

$$\frac{ac\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^3 ((66B - 42A) \sin(e + fx) + 98A + 15B \cos(2(e + fx)))}{105f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]

[Out] (a*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(98*A - 59*B + 15*B*Cos[2*(e + f*x)]) + (-42*A + 66*B)*Sin[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [A] time = 0.42, size = 184, normalized size = 1.59

$$\frac{2 \left(15 Bac \cos(fx + e)^4 - 3(7A - 6B)ac \cos(fx + e)^3 + (7A - B)ac \cos(fx + e)^2 - 4(7A - B)ac \cos(fx + e) \right)}{105f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -2/105*(15*B*a*c*cos(f*x + e)^4 - 3*(7*A - 6*B)*a*c*cos(f*x + e)^3 + (7*A - B)*a*c*cos(f*x + e)^2 - 4*(7*A - B)*a*c*cos(f*x + e) - 8*(7*A - B)*a*c*cos(f*x + e) + 3*(7*A - B)*a*c*cos(f*x + e)^2 + 4*(7*A - B)*a*c*cos(f*x + e) + 8*(7*A - B)*a*c*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4

3.84 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx$

Optimal. Leaf size=73

$$\frac{2ac^2(5A + B) \cos^3(e + fx)}{15f(c - c \sin(e + fx))^{3/2}} - \frac{2aBc \cos^3(e + fx)}{5f\sqrt{c - c \sin(e + fx)}}$$

[Out] $2/15*a*(5*A+B)*c^2*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^(3/2)-2/5*a*B*c*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^(1/2)$

Rubi [A] time = 0.24, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2967, 2856, 2673}

$$\frac{2ac^2(5A + B) \cos^3(e + fx)}{15f(c - c \sin(e + fx))^{3/2}} - \frac{2aBc \cos^3(e + fx)}{5f\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]

[Out] $(2*a*(5*A + B)*c^2*\cos[e + f*x]^3)/(15*f*(c - c*\sin[e + f*x])^(3/2)) - (2*a*B*c*\cos[e + f*x]^3)/(5*f*Sqrt[c - c*\sin[e + f*x]])$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx \\ &= -\frac{2aBc \cos^3(e + fx)}{5f\sqrt{c - c \sin(e + fx)}} + \frac{1}{5}(a(5A + B)c) \int \frac{1}{\sqrt{c - c \sin(e + fx)}} dx \\ &= \frac{2a(5A + B)c^2 \cos^3(e + fx)}{15f(c - c \sin(e + fx))^{3/2}} - \frac{2aBc \cos^3(e + fx)}{5f\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

[In] `int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)`

[Out] `-2/15*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^2*a*(3*B*sin(f*x+e)+5*A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a) \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2),x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int A \sqrt{-c \sin(e + fx) + c} dx + \int A \sqrt{-c \sin(e + fx) + c} \sin(e + fx) dx + \int B \sqrt{-c \sin(e + fx) + c} \sin(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)`

[Out] `a*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x))`

$$3.85 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=122

$$-\frac{2a(3A+5B) \cos(e+fx)}{3f\sqrt{c-c \sin(e+fx)}} + \frac{2\sqrt{2} a(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{c} f} + \frac{2aB \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{3cf}$$

[Out] 2*a*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))*2^(1/2)/f/c^(1/2)-2/3*a*(3*A+5*B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(1/2)+2/3*a*B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/c/f

Rubi [A] time = 0.34, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2858, 2751, 2649, 206}

$$-\frac{2a(3A+5B) \cos(e+fx)}{3f\sqrt{c-c \sin(e+fx)}} + \frac{2\sqrt{2} a(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{c} f} + \frac{2aB \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{3cf}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] (2*Sqrt[2]*a*(A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(Sqrt[c]*f) - (2*a*(3*A + 5*B)*Cos[e + f*x])/(3*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a*B*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*c*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2858

Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2))/(b^2*f*(m + 3)), x] - Dist[1/(b^2*(m + 3)), Int[(a + b*Sin[e + f*x])^(m + 1)*(b*d*(m + 2) - a*c*(m + 3) + (b*c*(m + 3) - a*d*(m + 4))*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GeQ[m, -3/2] && LtQ[m, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &&
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx \\ &= \frac{2aB \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{3cf} - \frac{(2a) \int \frac{-\frac{3Ac}{2} - \frac{Bc}{2} + \left(-\frac{3Ac}{2} - \frac{5Bc}{2}\right) \sin(e + fx)}{\sqrt{c - c \sin(e + fx)}} dx}{3c} \\ &= -\frac{2a(3A + 5B) \cos(e + fx)}{3f\sqrt{c - c \sin(e + fx)}} + \frac{2aB \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{3cf} \\ &= -\frac{2a(3A + 5B) \cos(e + fx)}{3f\sqrt{c - c \sin(e + fx)}} + \frac{2aB \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{3cf} \\ &= \frac{2\sqrt{2} a(A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{\sqrt{c} f} - \frac{2a(3A + 5B) \cos(e + fx)}{3f\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.26, size = 166, normalized size = 1.36

$$\frac{a \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(6\sqrt{2}(A + B)\sqrt{-c(\sin(e + fx) + 1)} \tan^{-1}\left(\frac{\sqrt{-c(\sin(e + fx) + 1)}}{\sqrt{2}\sqrt{c}}\right) + \sqrt{c}(2(3A + 5B)\sin(e + fx) + 3A) \right)}{3\sqrt{c} f \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]
```

```
[Out] -1/3*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))*(6*Sqrt[2]*(A + B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])]*Sqrt[-(c*(1 + Sin[e + f*x]))] + Sqrt[c]*(6*A + 9*B - B*Cos[2*(e + f*x)] + 2*(3*A + 5*B)*Sin[e + f*x]))/(Sqrt[c]*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.44, size = 254, normalized size = 2.08

$$\frac{3\sqrt{2}((A+B)ac \cos(fx+e) - (A+B)ac \sin(fx+e) + (A+B)ac) \log\left(\frac{\cos(fx+e)^2 + (\cos(fx+e)-2)\sin(fx+e) + \frac{2\sqrt{2}\sqrt{-c \sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}} + 3\cos(fx+e)}{\cos(fx+e)^2 + (\cos(fx+e)+2)\sin(fx+e) - \cos(fx+e) - 2}\right)}{\sqrt{c}}$$

$3(cf \cos(e + fx) + 3A)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*((A + B)*a*c*cos(f*x + e) - (A + B)*a*c*sin(f*x + e) + (A + B)*a*c)*log(-(cos(f*x + e))^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*
```


[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{A \sin(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{B \sin(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{B \sin^2(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] a*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x))

$$3.86 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=115

$$-\frac{a(A+5B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{2} c^{3/2} f} + \frac{a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))^{3/2}} + \frac{2aB \cos(e+fx)}{cf \sqrt{c-c \sin(e+fx)}}$$

[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)-1/2*a*(A+5*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(3/2)/f*2^(1/2)+2*a*B*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.32, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2857, 2751, 2649, 206}

$$-\frac{a(A+5B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{2} c^{3/2} f} + \frac{a(A+B) \cos(e+fx)}{f(c-c \sin(e+fx))^{3/2}} + \frac{2aB \cos(e+fx)}{cf \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] -((a*(A + 5*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(Sqrt[2]*c^(3/2)*f) + (a*(A + B)*Cos[e + f*x])/(f*(c - c*Sin[e + f*x])^(3/2)) + (2*a*B*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx \\ &= \frac{a(A + B) \cos(e + fx)}{f(c - c \sin(e + fx))^{3/2}} + \frac{a \int \frac{-Ac - 3Bc - 2Bc \sin(e + fx)}{\sqrt{c - c \sin(e + fx)}} dx}{2c^2} \\ &= \frac{a(A + B) \cos(e + fx)}{f(c - c \sin(e + fx))^{3/2}} + \frac{2aB \cos(e + fx)}{cf\sqrt{c - c \sin(e + fx)}} - \frac{(a(A + 5B)) \int \frac{1}{\sqrt{c - c \sin(e + fx)}} dx}{2c^2} \\ &= \frac{a(A + B) \cos(e + fx)}{f(c - c \sin(e + fx))^{3/2}} + \frac{2aB \cos(e + fx)}{cf\sqrt{c - c \sin(e + fx)}} + \frac{(a(A + 5B)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{c - c \sin(e + fx)}} dx, x, \frac{c - c \sin(e + fx)}{2}\right)}{2c^2} \\ &= -\frac{a(A + 5B) \operatorname{tanh}^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{\sqrt{2} c^{3/2} f} + \frac{a(A + B) \cos(e + fx)}{f(c - c \sin(e + fx))^{3/2}} + \end{aligned}$$

Mathematica [A] time = 1.61, size = 157, normalized size = 1.37

$$\frac{a \sec(e + fx) \left(2\sqrt{c} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2 (A - 2B \sin(e + fx) + 3B) + \sqrt{2}(A + 5B)\sqrt{-c(\sin(e + fx) + 1)} \right)}{2c^{3/2} f \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]
```

```
[Out] (a*Sec[e + f*x]*(Sqrt[2]*(A + 5*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]]/(Sqrt[2]*Sqrt[c]))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sqrt[-(c*(1 + Sin[e + f*x]))] + 2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*(A + 3*B - 2*B*Sin[e + f*x]))/(2*c^(3/2)*f*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.45, size = 318, normalized size = 2.77

$$\frac{\sqrt{2} \left((A+5B)ac \cos(fx+e)^2 - (A+5B)ac \cos(fx+e) - 2(A+5B)ac + ((A+5B)ac \cos(fx+e) + 2(A+5B)ac) \sin(fx+e) \right) \log \left(-\frac{\cos(fx+e)^2 + (\cos(fx+e)-2) \sin(fx+e)}{\cos(fx+e)} \right)}{\sqrt{c}}$$

$$4 \left(c^2 f \cos(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x, algor
ithm="fricas")
```

```
[Out] 1/4*(sqrt(2)*((A + 5*B)*a*c*cos(f*x + e)^2 - (A + 5*B)*a*c*cos(f*x + e) - 2
*(A + 5*B)*a*c + ((A + 5*B)*a*c*cos(f*x + e) + 2*(A + 5*B)*a*c)*sin(f*x + e
```



```
) * log(-(cos(f*x + e)^2 + (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(c) - 4*(2*B*a*cos(f*x + e)^2 + (A + 3*B)*a*cos(f*x + e) + (A + B)*a - (2*B*a*cos(f*x + e) - (A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^2*f*cos(f*x + e)^2 - c^2*f*cos(f*x + e) - 2*c^2*f + (c^2*f*cos(f*x + e) + 2*c^2*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorith="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(-1/2*B*a/c/sign(tan((f*x+exp(1))/2)-1)-1/2*B*a*tan((f*x+exp(1))/2)/c/sign(tan((f*x+exp(1))/2)-1)))/(c*tan((f*x+exp(1))/2)^2+c)+2*(1/2*(-3*A*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*a*sqrt(c)*c-B*a*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)+1/2*(-A*a-5*B*a)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1)))

maple [B] time = 1.24, size = 227, normalized size = 1.97

$$a \left(A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) \sin(fx+e) c + 5B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) \sin(fx+e) c - A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) \sin(fx+e) c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] 1/2*a/c^(5/2)*(A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c+5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c-A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c-4*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*sin(f*x+e)-5*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c+2*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*A+6*(c*(1+sin(f*x+e)))^(1/2)*c^(1/2)*B*(c*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(a \sin(fx+e) + a)}{(-c \sin(fx+e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorith="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a \left(\int \frac{A}{-c\sqrt{-c \sin(e + fx) + c} \sin(e + fx) + c\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{A \sin(e + fx)}{-c\sqrt{-c \sin(e + fx) + c} \sin(e + fx) + c\sqrt{-c \sin(e + fx) + c}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] a*(Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(A*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)**2/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + c*sqrt(-c*sin(e + f*x) + c)), x))

$$3.87 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=126

$$-\frac{a(A-7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2} c^{5/2} f} - \frac{a(A+9B) \cos(e+fx)}{8cf(c-c \sin(e+fx))^{3/2}} + \frac{a(A+B) \cos(e+fx)}{2f(c-c \sin(e+fx))^{5/2}}$$

[Out] 1/2*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)-1/8*a*(A+9*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)-1/16*a*(A-7*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)

Rubi [A] time = 0.33, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2857, 2750, 2649, 206}

$$-\frac{a(A-7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2} c^{5/2} f} - \frac{a(A+9B) \cos(e+fx)}{8cf(c-c \sin(e+fx))^{3/2}} + \frac{a(A+B) \cos(e+fx)}{2f(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] -(a*(A - 7*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(8*Sqrt[2]*c^(5/2)*f) + (a*(A + B)*Cos[e + f*x])/(2*f*(c - c*Sin[e + f*x])^(5/2)) - (a*(A + 9*B)*Cos[e + f*x])/(8*c*f*(c - c*Sin[e + f*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

Int[cos[(e_.) + (f_.)*(x_)]^2*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(2*(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -3/2]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx \\ &= \frac{a(A + B) \cos(e + fx)}{2f(c - c \sin(e + fx))^{5/2}} + \frac{a \int \frac{-Ac - 5Bc - 4Bc \sin(e + fx)}{(c - c \sin(e + fx))^{3/2}} dx}{4c^2} \\ &= \frac{a(A + B) \cos(e + fx)}{2f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 9B) \cos(e + fx)}{8cf(c - c \sin(e + fx))^{3/2}} - \frac{(a(A - 7B))}{16c^2} \\ &= \frac{a(A + B) \cos(e + fx)}{2f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 9B) \cos(e + fx)}{8cf(c - c \sin(e + fx))^{3/2}} + \frac{(a(A - 7B))}{16c^2} \\ &= -\frac{a(A - 7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{8\sqrt{2} c^{5/2} f} + \frac{a(A + B) \cos(e + fx)}{2f(c - c \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 2.24, size = 199, normalized size = 1.58

$$\frac{a(\sin(e + fx) - 1)(\sin(e + fx) + 1) \left(\frac{2\sqrt{c} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) ((A + 9B) \sin(e + fx) + 3A - 5B)}{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^5} + \sqrt{2}(A - 7B) \sec(e + fx) \right)}{16c^{5/2} f \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]
```

```
[Out] -1/16*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(Sqrt[2]*(A - 7*B)*ArcTan[Sqrt[-(c*(1 + Sin[e + f*x]))]/(Sqrt[2]*Sqrt[c])])*Sec[e + f*x]*Sqrt[-(c*(1 + Sin[e + f*x]))] + (2*Sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*A - 5*B + (A + 9*B)*Sin[e + f*x]))/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)/(c^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[c - c*Sin[e + f*x]])
```

fricas [B] time = 0.46, size = 394, normalized size = 3.13

$$\sqrt{2} \left((A - 7B)a \cos^3(fx + e) + 3(A - 7B)a \cos^2(fx + e) - 2(A - 7B)a \cos(fx + e) - 4(A - 7B)a - (A - 7B) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, algorith="fricas")
```

```
[Out] -1/32*(sqrt(2))*((A - 7*B)*a*cos(f*x + e)^3 + 3*(A - 7*B)*a*cos(f*x + e)^2 -
2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a - ((A - 7*B)*a*cos(f*x + e)^2 -
2*(A - 7*B)*a*cos(f*x + e) - 4*(A - 7*B)*a)*sin(f*x + e))*sqrt(c)*log(-(c*
cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e)
+ sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e)
) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) -
2)) - 4*((A + 9*B)*a*cos(f*x + e)^2 - (3*A - 5*B)*a*cos(f*x + e) - 4*(A +
B)*a - ((A + 9*B)*a*cos(f*x + e) + 4*(A + B)*a)*sin(f*x + e))*sqrt(-c*sin(f
*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(
f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f
)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/16*(-17*A*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^7+7*B*a*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))^7-23*A*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x
+exp(1))/2)^2+c))^6+81*B*a*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^6-19*A*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^5+53*B*a*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan(
(f*x+exp(1))/2)^2+c))^5+39*A*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))^4-65*B*a*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+5*A*a*c^2*(-sqrt(c)*tan((f*x+exp(1))/2
)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+13*B*a*c^2*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+7*A*a*c^3*(-sqrt(c)*tan((f*x+exp(1))/
2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-37*A*a*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+e
xp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-33*B*a*c^3*(-sqrt(c)*tan((f*x+
exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+19*B*a*sqrt(c)*c^2*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-3*A*a*sqrt(c)*c^3+5*B*
a*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/
2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/16*(-A*a+7*B*a)*atan((-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-
c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.61, size = 268, normalized size = 2.13

$$a \left(-2 \sin(fx + e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^2 (A - 7B) - \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^2 (A - 7B) \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/16*a*(-2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(
1/2))*c^2*(A-7*B)-2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/
2))*c^2*(A-7*B)*cos(f*x+e)^2+2*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)
)*2^(1/2)/c^(1/2))*c^2-2*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)-4*A*(c+c*sin(f*x+e)
))^(1/2)*c^(3/2)-14*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(
1/2))*c^2-18*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+28*B*(c+c*sin(f*x+e))^(1/2)*
```

$c^{(3/2)} * (c * (1 + \sin(f * x + e)))^{(1/2)} / c^{(9/2)} / (\sin(f * x + e) - 1) / \cos(f * x + e) / (c - c * \sin(f * x + e))^{(1/2)} / f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] Timed out

$$3.88 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=163

$$\frac{a(A-3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} - \frac{a(A-3B) \cos(e+fx)}{32c^2f(c-c \sin(e+fx))^{3/2}} - \frac{a(A+13B) \cos(e+fx)}{24cf(c-c \sin(e+fx))^{5/2}} + \frac{a(A+B) \cos(e+fx)}{3f(c-c \sin(e+fx))^{7/2}}$$

[Out] 1/3*a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(7/2)-1/24*a*(A+13*B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(5/2)-1/32*a*(A-3*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)-1/64*a*(A-3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)

Rubi [A] time = 0.37, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2967, 2857, 2750, 2650, 2649, 206}

$$\frac{a(A-3B) \cos(e+fx)}{32c^2f(c-c \sin(e+fx))^{3/2}} - \frac{a(A-3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{32\sqrt{2}c^{7/2}f} - \frac{a(A+13B) \cos(e+fx)}{24cf(c-c \sin(e+fx))^{5/2}} + \frac{a(A+B) \cos(e+fx)}{3f(c-c \sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] -(a*(A - 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(32*Sqrt[2]*c^(7/2)*f) + (a*(A + B)*Cos[e + f*x])/(3*f*(c - c*Sin[e + f*x])^(7/2)) - (a*(A + 13*B)*Cos[e + f*x])/(24*c*f*(c - c*Sin[e + f*x])^(5/2)) - (a*(A - 3*B)*Cos[e + f*x])/(32*c^2*f*(c - c*Sin[e + f*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2750

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2857

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*
(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(2*(b*c - a*d)*Co
s[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(2*m + 3)), x] + Dist[1/(b^
3*(2*m + 3)), Int[(a + b*Sin[e + f*x])^(m + 2)*(b*c + 2*a*d*(m + 1) - b*d*(
2*m + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 -
b^2, 0] && LtQ[m, -3/2]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx &= (ac) \int \frac{\cos^2(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx \\ &= \frac{a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^{7/2}} + \frac{a \int \frac{-Ac - 7Bc - 6Bc \sin(e + fx)}{(c - c \sin(e + fx))^{5/2}} dx}{6c^2} \\ &= \frac{a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 13B) \cos(e + fx)}{24cf(c - c \sin(e + fx))^{5/2}} - \frac{a(A - 3B)}{32c^2 f(c - c \sin(e + fx))^{3/2}} \\ &= \frac{a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 13B) \cos(e + fx)}{24cf(c - c \sin(e + fx))^{5/2}} - \frac{a(A - 3B)}{32c^2 f(c - c \sin(e + fx))^{3/2}} \\ &= \frac{a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 13B) \cos(e + fx)}{24cf(c - c \sin(e + fx))^{5/2}} - \frac{a(A - 3B)}{32c^2 f(c - c \sin(e + fx))^{3/2}} \\ &= -\frac{a(A - 3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{32\sqrt{2} c^{7/2} f} + \frac{a(A + B) \cos(e + fx)}{3f(c - c \sin(e + fx))^{7/2}} \end{aligned}$$

Mathematica [A] time = 3.45, size = 217, normalized size = 1.33

$$\frac{a(\sin(e + fx) - 1)(\sin(e + fx) + 1) \left(\frac{\sqrt{c} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (4(5A + 17B) \sin(e + fx) + 3(A - 3B) \cos(2(e + fx)) + 47A - 13B)}{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7} + 3 \right)}{192c^{7/2} f \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(
(7/2), x]
```

```
[Out] -1/192*(a*(-1 + Sin[e + f*x])*(1 + Sin[e + f*x])*(3*sqrt[2]*(A - 3*B)*ArcTa
n[sqrt[-(c*(1 + Sin[e + f*x]))]/(sqrt[2]*sqrt[c])]*Sec[e + f*x]*sqrt[-(c*(1
+ Sin[e + f*x]))] + (sqrt[c]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(47*A -
13*B + 3*(A - 3*B)*Cos[2*(e + f*x)] + 4*(5*A + 17*B)*Sin[e + f*x]))/(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^(7/2))/(c^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^2*sqrt[c - c*Sin[e + f*x]])
```


fricas [B] time = 0.45, size = 490, normalized size = 3.01

$$3\sqrt{2}\left((A-3B)a\cos(fx+e)^4 - 3(A-3B)a\cos(fx+e)^3 - 8(A-3B)a\cos(fx+e)^2 + 4(A-3B)a\cos(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out]
$$\frac{-1/384*(3*\sqrt{2})*((A-3*B)*a*\cos(f*x+e)^4 - 3*(A-3*B)*a*\cos(f*x+e)^3 - 8*(A-3*B)*a*\cos(f*x+e)^2 + 4*(A-3*B)*a*\cos(f*x+e) + ((A-3*B)*a*\cos(f*x+e)^3 + 4*(A-3*B)*a*\cos(f*x+e)^2 - 4*(A-3*B)*a*\cos(f*x+e) - 8*(A-3*B)*a*\sin(f*x+e))*\sqrt{c}*\log(-(c*\cos(f*x+e))^2 + 2*\sqrt{2}*\sqrt{-c*\sin(f*x+e)+c}*\sqrt{c}*(\cos(f*x+e)+\sin(f*x+e)+1) + 3*c*\cos(f*x+e) + (c*\cos(f*x+e) - 2*c)*\sin(f*x+e) + 2*c)/(\cos(f*x+e)^2 + (\cos(f*x+e)+2)*\sin(f*x+e) - \cos(f*x+e) - 2)) - 4*(3*(A-3*B)*a*\cos(f*x+e)^3 - (7*A+43*B)*a*\cos(f*x+e)^2 + 2*(11*A-B)*a*\cos(f*x+e) + 32*(A+B)*a + (3*(A-3*B)*a*\cos(f*x+e)^2 + 2*(5*A+17*B)*a*\cos(f*x+e) + 32*(A+B)*a)*\sin(f*x+e))*\sqrt{-c*\sin(f*x+e)+c}}{(c^4*f*\cos(f*x+e)^4 - 3*c^4*f*\cos(f*x+e)^3 - 8*c^4*f*\cos(f*x+e)^2 + 4*c^4*f*\cos(f*x+e) + 8*c^4*f + (c^4*f*\cos(f*x+e)^3 + 4*c^4*f*\cos(f*x+e)^2 - 4*c^4*f*\cos(f*x+e) - 8*c^4*f)*\sin(f*x+e))}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: $(4*\pi/x/2)>(-4*\pi/x/2)$ Unable to check sign: $(4*\pi/x/2)>(-4*\pi/x/2)$
$$\frac{2/f*2*(1/192*(-195*A*a*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c)})^{11}+9*B*a*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^{11}-609*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^{10}+483*B*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^{10}-1347*A*a*c*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^9+393*B*a*c*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^9+3*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^8+183*B*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^8+1842*A*a*c^2*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^7-534*B*a*c^2*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^7+950*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^6-418*B*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^6-2358*A*a*c^3*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^5+1314*B*a*c^3*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^5-378*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^4-18*B*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^4+1153*A*a*c^4*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^3-707*B*a*c^4*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^3+105*A*a*c^5*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2-741*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2-123*B*a*c^5*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2+111*B*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2-25*A*a*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2+11*B*a*s$$

```

qrt(c)*c^5)/c^3/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^
2+c))+c)^6/sign(tan((f*x+exp(1))/2)-1)+1/64*(-A*a+3*B*a)*atan((-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c)
/sqrt(2)/c^3/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))

```

maple [B] time = 1.73, size = 352, normalized size = 2.16

$$a \left(-3\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^4 (A-3B) \sin(fx+e) (\cos^2(fx+e)) + 12\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/192*a*(-3*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4
*(A-3*B)*sin(f*x+e)*cos(f*x+e)^2+12*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1
/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*sin(f*x+e)+9*2^(1/2)*arctanh(1/2*(c+c*sin(
f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(A-3*B)*cos(f*x+e)^2+24*A*(c+c*sin(f*x+e
))^(1/2)*c^(7/2)+32*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-6*A*(c+c*sin(f*x+e))^(
5/2)*c^(3/2)-72*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+32*B*(c+c*sin(f*x+e))^(3/2
)*c^(5/2)+18*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-12*A*2^(1/2)*arctanh(1/2*(c+c
*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4+36*B*2^(1/2)*arctanh(1/2*(c+c*sin(f
*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(c*(1+sin(f*x+e)))^(1/2)/c^(15/2)/(sin(f
*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)}{(-c \sin(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(
7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))}{(c - c \sin(e + fx))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c - c*sin(e + f*x))^(7/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] Timed out
```

$$3.89 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=210

$$\frac{256a^2c^6(13A - 3B) \cos^5(e + fx)}{15015f(c - c \sin(e + fx))^{5/2}} + \frac{64a^2c^5(13A - 3B) \cos^5(e + fx)}{3003f(c - c \sin(e + fx))^{3/2}} + \frac{8a^2c^4(13A - 3B) \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}} + \frac{2a^2c^3(13A - 3B) \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}}$$

[Out] 256/15015*a^2*(13*A-3*B)*c^6*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+64/3003*a^2*(13*A-3*B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/f+8/429*a^2*(13*A-3*B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)+2/143*a^2*(13*A-3*B)*c^3*cos(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.55, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{256a^2c^6(13A - 3B) \cos^5(e + fx)}{15015f(c - c \sin(e + fx))^{5/2}} + \frac{64a^2c^5(13A - 3B) \cos^5(e + fx)}{3003f(c - c \sin(e + fx))^{3/2}} + \frac{8a^2c^4(13A - 3B) \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}} + \frac{2a^2c^3(13A - 3B) \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2), x]

[Out] (256*a^2*(13*A - 3*B)*c^6*Cos[e + f*x]^5)/(15015*f*(c - c*Sin[e + f*x])^(5/2)) + (64*a^2*(13*A - 3*B)*c^5*Cos[e + f*x]^5)/(3003*f*(c - c*Sin[e + f*x])^(3/2)) + (8*a^2*(13*A - 3*B)*c^4*Cos[e + f*x]^5)/(429*f*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(13*A - 3*B)*c^3*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(143*f) - (2*a^2*B*c^2*Cos[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(13*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx &= (a^2 c^2) \int \cos^4(e + fx)(A + B \sin(e + fx))(c - \\ &= -\frac{2a^2 B c^2 \cos^5(e + fx)(c - c \sin(e + fx))^{3/2}}{13f} + \\ &= \frac{2a^2(13A - 3B)c^3 \cos^5(e + fx)\sqrt{c - c \sin(e + fx)}}{143f} \\ &= \frac{8a^2(13A - 3B)c^4 \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}} + \frac{2a^2(13A - 3B)c^5 \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}} \\ &= \frac{64a^2(13A - 3B)c^5 \cos^5(e + fx)}{3003f(c - c \sin(e + fx))^{3/2}} + \frac{8a^2(13A - 3B)c^6 \cos^5(e + fx)}{429f\sqrt{c - c \sin(e + fx)}} \\ &= \frac{256a^2(13A - 3B)c^6 \cos^5(e + fx)}{15015f(c - c \sin(e + fx))^{5/2}} + \frac{64a^2(13A - 3B)c^7 \cos^5(e + fx)}{3003f\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [B] time = 6.64, size = 1355, normalized size = 6.45

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] ((7*A - 2*B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(160*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((A - 4*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (A*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(48*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - 3*B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(416*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((7*A - 2*B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((22*A - 7*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(7/2)*Sin[(5*(e + f*x))/2])/(160*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((A - 4*B)*
```



```

2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-6*B*a^2*c^3*sign(sin(1/2*(f*x+exp(
1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-16*f*(12*A*a^2*c^3*s
ign(sin(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4
*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(16*f)^2-320*f*(32*A*a^2*c^3*sign(sin
(1/2*(f*x+exp(1))-1/4*pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*
cos(1/4*(10*f*x+10*exp(1)-pi))/(320*f)^2+16*f*(-2*A*a^2*c^3*sign(sin(1/2*(f
*x+exp(1))-1/4*pi))+2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4
*(2*f*x+2*exp(1)+pi))/(16*f)^2+288*f*(-2*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1)
)-1/4*pi))+2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+
18*exp(1)+pi))/(288*f)^2-224*f*(-4*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*
pi))-2*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp
(1)+pi))/(224*f)^2+160*f*(-6*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+6
*B*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi
))/(160*f)^2-192*f*(-32*A*a^2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))+2*B*a^
2*c^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)+pi))/(192
*f)^2+1408*B*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+2
2*exp(1)+pi))/(704*f)^2-1664*B*a^2*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
*cos(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2

```

maple [A] time = 1.16, size = 121, normalized size = 0.58

$$\frac{2(\sin(fx + e) - 1)c^4(1 + \sin(fx + e))^3 a^2((-1365A + 4935B)\sin(fx + e)(\cos^2(fx + e)) + (11180A - 11820B)\cos(fx + e)\sqrt{c - c\sin(fx + e)})}{15015 \cos(fx + e) \sqrt{c - c\sin(fx + e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 2/15015*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^3*a^2*((-1365*A+4935*B)*sin(f*x+e)
)*cos(f*x+e)^2+(11180*A-11820*B)*sin(f*x+e)+1155*B*cos(f*x+e)^4+(5915*A-106
05*B)*cos(f*x+e)^2-12844*A+12204*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (-c \sin(fx + e) + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)
^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(7/2),
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

$$3.90 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=167

$$\frac{64a^2c^5(11A - B) \cos^5(e + fx)}{3465f(c - c \sin(e + fx))^{5/2}} + \frac{16a^2c^4(11A - B) \cos^5(e + fx)}{693f(c - c \sin(e + fx))^{3/2}} + \frac{2a^2c^3(11A - B) \cos^5(e + fx)}{99f\sqrt{c - c \sin(e + fx)}} - \frac{2a^2Bc^2 \cos^5(e + fx)}{11f}$$

[Out] 64/3465*a^2*(11*A-B)*c^5*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(5/2)+16/693*a^2*(11*A-B)*c^4*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(3/2)+2/99*a^2*(11*A-B)*c^3*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(1/2)-2/11*a^2*B*c^2*cos(f*x+e)^5*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.45, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(11A - B) \cos^5(e + fx)}{99f\sqrt{c - c \sin(e + fx)}} + \frac{16a^2c^4(11A - B) \cos^5(e + fx)}{693f(c - c \sin(e + fx))^{3/2}} + \frac{64a^2c^5(11A - B) \cos^5(e + fx)}{3465f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2Bc^2 \cos^5(e + fx)}{11f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] (64*a^2*(11*A - B)*c^5*Cos[e + f*x]^5)/(3465*f*(c - c*Sin[e + f*x])^(5/2)) + (16*a^2*(11*A - B)*c^4*Cos[e + f*x]^5)/(693*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^2*(11*A - B)*c^3*Cos[e + f*x]^5)/(99*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^2*B*c^2*Cos[e + f*x]^5*Sqrt[c - c*Sin[e + f*x]])/(11*f)

Rule 2673

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di


```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx &= (a^2 c^2) \int \cos^4(e + fx) (A + B \sin(e + fx)) \\ &= -\frac{2a^2 B c^2 \cos^5(e + fx) \sqrt{c - c \sin(e + fx)}}{11f} \\ &= \frac{2a^2 (11A - B) c^3 \cos^5(e + fx)}{99f \sqrt{c - c \sin(e + fx)}} - \frac{2a^2 B c^2 \cos^5(e + fx)}{99f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{16a^2 (11A - B) c^4 \cos^5(e + fx)}{693f (c - c \sin(e + fx))^{3/2}} + \frac{2a^2 (11A - B) c^3 \cos^5(e + fx)}{99f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{64a^2 (11A - B) c^5 \cos^5(e + fx)}{3465f (c - c \sin(e + fx))^{5/2}} + \frac{16a^2 (11A - B) c^3 \cos^5(e + fx)}{693f \sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [B] time = 6.51, size = 1173, normalized size = 7.02

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]
```

```
[Out] ((6*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - ((2*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) - (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((6*A - B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((4*A + B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(3*(e + f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((8*A - 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(5*(e + f*x))/2])/(80*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A + 3*B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + ((2*A - B)*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4) + (B*(a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)
```


[Out]
$$\frac{-2/3465*(\sin(f*x+e)-1)*c^3*(1+\sin(f*x+e))^3*a^2*(-315*B*\cos(f*x+e)^2*\sin(f*x+e)+(-1210*A+1370*B)*\sin(f*x+e)+(-385*A+980*B)*\cos(f*x+e)^2+1562*A-1402*B)}{\cos(f*x+e)/(c-c*\sin(f*x+e))^{1/2}/f}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (-c \sin(fx + e) + c)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(-c*sin(f*x + e) + c)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2), x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)`

[Out] Timed out

3.91 $\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=120

$$\frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} + \frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

[Out] $8/315*a^2*(9*A+B)*c^4*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(5/2)}+2/63*a^2*(9*A+B)*c^3*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(3/2)}-2/9*a^2*B*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.39, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^2c^3(9A+B)\cos^5(e+fx)}{63f(c-c\sin(e+fx))^{3/2}} + \frac{8a^2c^4(9A+B)\cos^5(e+fx)}{315f(c-c\sin(e+fx))^{5/2}} - \frac{2a^2Bc^2\cos^5(e+fx)}{9f\sqrt{c-c\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^2*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(3/2)}, x]$

[Out] $(8*a^2*(9*A + B)*c^4*\text{Cos}[e + f*x]^5)/(315*f*(c - c*\text{Sin}[e + f*x])^{(5/2)}) + (2*a^2*(9*A + B)*c^3*\text{Cos}[e + f*x]^5)/(63*f*(c - c*\text{Sin}[e + f*x])^{(3/2)}) - (2*a^2*B*c^2*\text{Cos}[e + f*x]^5)/(9*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 2673

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_)}, x_Symbol] :> \text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p + 1)}*(a + b*\text{Sin}[e + f*x])^{(m - 1)})/(f*g*(m - 1)), x] /; \text{FreeQ}[\{a, b, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[2*m + p - 1, 0] \&\& \text{NeQ}[m, 1]$

Rule 2674

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_)}, x_Symbol] :> -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p + 1)}*(a + b*\text{Sin}[e + f*x])^{(m - 1)})/(f*g*(m + p)), x] + \text{Dist}[(a*(2*m + p - 1))/(m + p), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^{(m - 1)}, x], x] /; \text{FreeQ}[\{a, b, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[\text{Simplify}[(2*m + p - 1)/2], 0] \&\& \text{NeQ}[m + p, 0]$

Rule 2856

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -\text{Simp}[(d*(g*\text{Cos}[e + f*x])^{(p + 1)}*(a + b*\text{Sin}[e + f*x])^m)/(f*g*(m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[\text{Simplify}[(2*m + p + 1)/2], 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}), x_Symbol] :> \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n - m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d$

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx \\ &= -\frac{2a^2 B c^2 \cos^5(e + fx)}{9f \sqrt{c - c \sin(e + fx)}} + \frac{1}{9} (a^2 (9A + B) c^3) \\ &= \frac{2a^2 (9A + B) c^3 \cos^5(e + fx)}{63f (c - c \sin(e + fx))^{3/2}} - \frac{2a^2 B c^2 \cos^5(e + fx)}{9f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{8a^2 (9A + B) c^4 \cos^5(e + fx)}{315f (c - c \sin(e + fx))^{5/2}} + \frac{2a^2 (9A + B) c^3 \cos^5(e + fx)}{63f (c - c \sin(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 4.59, size = 106, normalized size = 0.88

$$\frac{a^2 c \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^5 \left((130B - 90A) \sin(e + fx) + 162A + 35B \cos(2(e + fx)) \right)}{315f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]

[Out] (a^2*c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(162*A - 87*B + 35*B*Cos[2*(e + f*x)] + (-90*A + 130*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]/(315*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [B] time = 0.43, size = 228, normalized size = 1.90

$$\frac{2 \left(35 B a^2 c \cos(fx + e)^5 + 5 (9 A + 8 B) a^2 c \cos(fx + e)^4 - (9 A + B) a^2 c \cos(fx + e)^3 + 2 (9 A + B) a^2 c \cos(fx + e)^2 - (9 A + B) a^2 c \cos(fx + e) \right)}{(f \cos(fx + e) - f \sin(fx + e) + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] -2/315*(35*B*a^2*c*cos(f*x + e)^5 + 5*(9*A + 8*B)*a^2*c*cos(f*x + e)^4 - (9*A + B)*a^2*c*cos(f*x + e)^3 + 2*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c + (35*B*a^2*c*cos(f*x + e)^4 - 5*(9*A + B)*a^2*c*cos(f*x + e)^3 - 6*(9*A + B)*a^2*c*cos(f*x + e)^2 - 8*(9*A + B)*a^2*c*cos(f*x + e) - 16*(9*A + B)*a^2*c)*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e) - f*sin(f*x + e) + f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int Ac\sqrt{-c\sin(e+fx)+c} dx + \int Ac\sqrt{-c\sin(e+fx)+c} \sin(e+fx) dx + \int (-Ac\sqrt{-c\sin(e+fx)+c} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)

[Out] a**2*(Integral(A*c*sqrt(-c*sin(e + f*x) + c), x) + Integral(A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-A*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x) + Integral(-B*c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**4, x))

3.92 $\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx$

Optimal. Leaf size=81

$$\frac{2a^2c^3(7A + 3B) \cos^5(e + fx)}{35f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2Bc^2 \cos^5(e + fx)}{7f(c - c \sin(e + fx))^{3/2}}$$

[Out] $2/35*a^2*(7*A+3*B)*c^3*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(5/2)}-2/7*a^2*B*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(3/2)}$

Rubi [A] time = 0.33, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2967, 2856, 2673}

$$\frac{2a^2c^3(7A + 3B) \cos^5(e + fx)}{35f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2Bc^2 \cos^5(e + fx)}{7f(c - c \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^2*(A + B*\text{Sin}[e + f*x])*Sqrt[c - c*\text{Sin}[e + f*x]],x]$

[Out] $(2*a^2*(7*A + 3*B)*c^3*\text{Cos}[e + f*x]^5)/(35*f*(c - c*\text{Sin}[e + f*x])^{(5/2)}) - (2*a^2*B*c^2*\text{Cos}[e + f*x]^5)/(7*f*(c - c*\text{Sin}[e + f*x])^{(3/2)})$

Rule 2673

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_)}, x_Symbol] :> \text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p + 1)}*(a + b*\text{Sin}[e + f*x])^{(m - 1)})/(f*g*(m - 1)), x] /; \text{FreeQ}\{a, b, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[2*m + p - 1, 0] \&\& \text{NeQ}[m, 1]$

Rule 2856

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -\text{Simp}[(d*(g*\text{Cos}[e + f*x])^{(p + 1)}*(a + b*\text{Sin}[e + f*x])^m)/(f*g*(m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[\text{Simplify}[(2*m + p + 1)/2], 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}), x_Symbol] :> \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n - m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx &= (a^2c^2) \int \frac{\cos^4(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx \\ &= -\frac{2a^2Bc^2 \cos^5(e + fx)}{7f(c - c \sin(e + fx))^{3/2}} + \frac{1}{7} (a^2(7A + 3B)c^2) \\ &= \frac{2a^2(7A + 3B)c^3 \cos^5(e + fx)}{35f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2Bc^2 \cos^5(e + fx)}{7f(c - c \sin(e + fx))^{3/2}} \end{aligned}$$

maple [A] time = 1.30, size = 65, normalized size = 0.80

$$\frac{2(\sin(fx + e) - 1)c(1 + \sin(fx + e))^3 a^2 (5B \sin(fx + e) + 7A - 2B)}{35 \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)

[Out] -2/35*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^3*a^2*(5*B*sin(f*x+e)+7*A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^2 \sqrt{c - c \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2), x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int A \sqrt{-c \sin(e + fx) + c} dx + \int 2A \sqrt{-c \sin(e + fx) + c} \sin(e + fx) dx + \int A \sqrt{-c \sin(e + fx) + c} \sin^2(e + fx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),x)

[Out] a**2*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(2*B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x))

$$3.93 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=161

$$\frac{2a^2c(A+B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^{3/2}} - \frac{4a^2(A+B) \cos(e+fx)}{f\sqrt{c-c \sin(e+fx)}} + \frac{4\sqrt{2}a^2(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2 \cos^5(e+fx)}{5f(c-c \sin(e+fx))^{5/2}}$$

[Out] $-2/5*a^2*B*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(5/2)}-2/3*a^2*(A+B)*c*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^{(3/2)}+4*a^2*(A+B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/f/c^{(1/2)}-4*a^2*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.44, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2860, 2679, 2649, 206}

$$\frac{2a^2c(A+B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^{3/2}} - \frac{4a^2(A+B) \cos(e+fx)}{f\sqrt{c-c \sin(e+fx)}} + \frac{4\sqrt{2}a^2(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{c}f} - \frac{2a^2Bc^2 \cos^5(e+fx)}{5f(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] $(4*\sqrt{2}*a^2*(A+B)*\operatorname{ArcTanh}[(\sqrt{c}*\cos[e+f*x])/(\sqrt{2}*\sqrt{c-c*\sin[e+f*x]})])/(\sqrt{c}*f) - (2*a^2*B*c^2*\cos[e+f*x]^5)/(5*f*(c-c*\sin[e+f*x])^{(5/2)}) - (2*a^2*(A+B)*c*\cos[e+f*x]^3)/(3*f*(c-c*\sin[e+f*x])^{(3/2)}) - (4*a^2*(A+B)*\cos[e+f*x])/f*\sqrt{c-c*\sin[e+f*x]}$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegersQ[2*m, 2*p]

Rule 2860

Int[(cos[(e_) + (f_)*(x_)])*(g_)^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(f*g*(m+p+1)), x] + Dist[(a*d*m + b*c*(m+p+1))/(b*(m+p+1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x]

$\text{Sin}[e + f*x]^m, x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * (A + B*\text{sin}[e + f*x]) * (c + d*\text{sin}[e + f*x])^n, x_Symbol] \text{ :> Dist}[a^m * c^m, \text{Int}[\text{Cos}[e + f*x]^{2*m} * (c + d*\text{Sin}[e + f*x])^{n-m} * (A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \& \ \text{GtQ}[n, 0]) \ || \ \text{LtQ}[0, n, m] \ || \ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx = (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx$$

$$= -\frac{2a^2 Bc^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} + (a^2(A + B)c^2) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{5/2}} dx$$

$$= -\frac{2a^2 Bc^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2(A + B)c \cos^3(e + fx)}{3f(c - c \sin(e + fx))^{3/2}} + (2a^2(A + B)c^2) \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{5/2}} dx$$

$$= -\frac{2a^2 Bc^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} - \frac{2a^2(A + B)c \cos^3(e + fx)}{3f(c - c \sin(e + fx))^{3/2}} - \frac{4a^2(A + B)c^2}{f\sqrt{c - c \sin(e + fx)}} + \frac{4\sqrt{2} a^2(A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2}\sqrt{c - c \sin(e + fx)}}\right)}{\sqrt{c} f} - \frac{2a^2 Bc^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}}$$

Mathematica [C] time = 1.18, size = 175, normalized size = 1.09

$$\frac{a^2(\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (2(5A + 11B) \sin(e + fx) + 1) \right)}{15f\sqrt{c - c \sin(e + fx)} \left(\sin(e + fx) + 1 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] -1/15*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*((120 + 120*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] + (Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(70*A + 79*B - 3*B*Cos[2*(e + f*x)] + 2*(5*A + 11*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sqrt[c - c*Sin[e + f*x]])

fricas [B] time = 0.45, size = 310, normalized size = 1.93

$$2 \left[\frac{15\sqrt{2}((A+B)a^2c \cos(fx+e) - (A+B)a^2c \sin(fx+e) + (A+B)a^2c) \log\left(\frac{\cos(fx+e)^2 + (\cos(fx+e)-2)\sin(fx+e) + \frac{2\sqrt{2}\sqrt{-c \sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}}}{\cos(fx+e)^2 + (\cos(fx+e)+2)\sin(fx+e) - \cos(fx+e) - 2}\right)}{\sqrt{c}} \right]$$

$$\frac{nh(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*B+3*B*(c*(1+\sin(f*x+e)))^{(5/2)}+5*A*(c*(1+\sin(f*x+e)))^{(3/2)}*c+5*B*(c*(1+\sin(f*x+e)))^{(3/2)}*c+30*A*c^2*(c*(1+\sin(f*x+e)))^{(1/2)}+30*B*c^2*(c*(1+\sin(f*x+e)))^{(1/2)})/c^3/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^2}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^2}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int \frac{A}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{2A \sin(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{A \sin^2(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{B \sin(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)

[Out] a**2*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*A*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(A*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(2*B*sin(e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x))

$$3.94 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=176

$$-\frac{\sqrt{2} a^2(3A+7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{c^{3/2} f} + \frac{a^2 c^2(A+B) \cos^5(e+fx)}{2f(c-c \sin(e+fx))^{7/2}} + \frac{a^2(3A+7B) \cos^3(e+fx)}{6f(c-c \sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)}{cf \sqrt{c-c \sin(e+fx)}}$$

[Out] $1/2*a^2*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(7/2)}+1/6*a^2*(3*A+7*B)*c*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^{(3/2)}-a^2*(3*A+7*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/c^{(3/2)}/f+a^2*(3*A+7*B)*\cos(f*x+e)/c/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^2 c^2(A+B) \cos^5(e+fx)}{2f(c-c \sin(e+fx))^{7/2}} - \frac{\sqrt{2} a^2(3A+7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{c^{3/2} f} + \frac{a^2(3A+7B) \cos^3(e+fx)}{6f(c-c \sin(e+fx))^{3/2}} + \frac{a^2(3A+7B)}{cf \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + aSin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] $-((\operatorname{Sqrt}[2]*a^2*(3*A+7*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c-c*\sin[e+f*x]])])/(c^{(3/2)}*f)) + (a^2*(A+B)*c^2*\operatorname{Cos}[e+f*x]^5)/(2*f*(c-c*\sin[e+f*x])^{(7/2)}) + (a^2*(3*A+7*B)*\operatorname{Cos}[e+f*x]^3)/(6*f*(c-c*\sin[e+f*x])^{(3/2)}) + (a^2*(3*A+7*B)*\operatorname{Cos}[e+f*x])/(c*f*\operatorname{Sqrt}[c-c*\sin[e+f*x]])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1) * ((c + d*Sin[e + f*x])^(m+1) - (c - d*Sin[e + f*x])^(m+1))), x]

```

)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]

```

Rule 2967

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sin(e + fx))^2(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= \frac{a^2(A + B)c^2 \cos^5(e + fx)}{2f(c - c \sin(e + fx))^{7/2}} - \frac{1}{4} (a^2(3A + 7B)c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= \frac{a^2(A + B)c^2 \cos^5(e + fx)}{2f(c - c \sin(e + fx))^{7/2}} + \frac{a^2(3A + 7B) \cos^3(e + fx)}{6f(c - c \sin(e + fx))^{3/2}} - \frac{1}{2} (a^2(3A + 7B)c) \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= \frac{a^2(A + B)c^2 \cos^5(e + fx)}{2f(c - c \sin(e + fx))^{7/2}} + \frac{a^2(3A + 7B) \cos^3(e + fx)}{6f(c - c \sin(e + fx))^{3/2}} + \frac{a^2(3A + 7B)c}{cf\sqrt{c - c \sin(e + fx)}} \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= \frac{a^2(A + B)c^2 \cos^5(e + fx)}{2f(c - c \sin(e + fx))^{7/2}} + \frac{a^2(3A + 7B) \cos^3(e + fx)}{6f(c - c \sin(e + fx))^{3/2}} + \frac{a^2(3A + 7B)c}{cf\sqrt{c - c \sin(e + fx)}} \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= -\frac{\sqrt{2} a^2(3A + 7B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{c^{3/2} f} + \frac{a^2(A + B)c^2 \cos^5(e + fx)}{2f(c - c \sin(e + fx))^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 0.90, size = 355, normalized size = 2.02

$$a^2(\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(12(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 3(2A + 7B) \cos\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

```

```

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(6*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (6 + 6*I)*(-1)^(1/4)*(3*A + 7*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(2*A + 7*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - B*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 12*(A + B)*Sin[(e + f*x)/2] + 3*(2*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(3*(e + f*x))/2]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(3/2))

```


maple [A] time = 1.46, size = 282, normalized size = 1.60

$$a^2 \left(\sin(fx + e) \left(9A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^2 - 6A\sqrt{c+c\sin(fx+e)} c^{\frac{3}{2}} + 21B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)}}{2\sqrt{c}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)`

[Out] $\frac{1}{3}a^2(\sin(fx+e)(9A2^{1/2}\operatorname{arctanh}(1/2(c+c\sin(fx+e))^{1/2})2^{1/2}/c^{1/2})c^2-6A(c+c\sin(fx+e))^{1/2}c^{3/2}+21B2^{1/2}\operatorname{arctanh}(1/2(c+c\sin(fx+e))^{1/2})2^{1/2}/c^{1/2})c^2-18B(c+c\sin(fx+e))^{1/2}c^{3/2}-2B(c+c\sin(fx+e))^{3/2}c^{1/2})-9A2^{1/2}\operatorname{arctanh}(1/2(c+c\sin(fx+e))^{1/2})2^{1/2}/c^{1/2})c^2+12A(c+c\sin(fx+e))^{1/2}c^{3/2}-21B2^{1/2}\operatorname{arctanh}(1/2(c+c\sin(fx+e))^{1/2})2^{1/2}/c^{1/2})c^2+24B(c+c\sin(fx+e))^{1/2}c^{3/2}+2B(c+c\sin(fx+e))^{3/2}c^{1/2})(c(1+\sin(fx+e)))^{1/2}/c^{7/2}/\cos(fx+e)/(c-c\sin(fx+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^2}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx))(a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2),x)`

[Out] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)`

[Out] Timed out

$$3.95 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=175

$$\frac{3a^2(A+9B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{4\sqrt{2} c^{5/2} f} + \frac{a^2 c^2 (A+B) \cos^5(e+fx)}{4f(c-c \sin(e+fx))^{9/2}} - \frac{3a^2(A+9B) \cos(e+fx)}{8c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{a^2(A+9B) \cos^3(e+fx)}{8f(c-c \sin(e+fx))^{5/2}}$$

[Out] $1/4*a^2*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(9/2)}-1/8*a^2*(A+9*B)*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^{(5/2)}+3/8*a^2*(A+9*B)*\arctanh(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)/(c-c*\sin(f*x+e))^{(1/2)})/c^{(5/2)}/f*2^{(1/2)}-3/8*a^2*(A+9*B)*\cos(f*x+e)/c^2/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{4f(c-c \sin(e+fx))^{9/2}} - \frac{3a^2(A+9B) \cos(e+fx)}{8c^2 f \sqrt{c-c \sin(e+fx)}} + \frac{3a^2(A+9B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{4\sqrt{2} c^{5/2} f} - \frac{a^2(A+9B) \cos^3(e+fx)}{8f(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + aSin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] $(3*a^2*(A+9*B)*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Cos}[e+f*x])/(\text{Sqrt}[2]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])])/(4*\text{Sqrt}[2]*c^{(5/2)}*f) + (a^2*(A+B)*c^2*\text{Cos}[e+f*x]^5)/(4*f*(c-c*\text{Sin}[e+f*x])^{(9/2)}) - (a^2*(A+9*B)*\text{Cos}[e+f*x]^3)/(8*f*(c-c*\text{Sin}[e+f*x])^{(5/2)}) - (3*a^2*(A+9*B)*\text{Cos}[e+f*x])/(8*c^2*f*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

1)), Int[(g*cos[e + f*x])^(p - 2)*(a + b*sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2859

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= (a^2 c^2) \int \frac{\cos^4(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{4f(c - c \sin(e + fx))^{9/2}} - \frac{1}{8} (a^2 (A + 9B) c) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{4f(c - c \sin(e + fx))^{9/2}} - \frac{a^2 (A + 9B) \cos^3(e + fx)}{8f(c - c \sin(e + fx))^{5/2}} + \frac{(3a^2 (A + 9B) \cos^2(e + fx))}{8f(c - c \sin(e + fx))^{3/2}} \\
 &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{4f(c - c \sin(e + fx))^{9/2}} - \frac{a^2 (A + 9B) \cos^3(e + fx)}{8f(c - c \sin(e + fx))^{5/2}} - \frac{3a^2 (A + 9B) \cos^2(e + fx)}{8c^2 f \sqrt{c - c \sin(e + fx)}} \\
 &= \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{4f(c - c \sin(e + fx))^{9/2}} - \frac{a^2 (A + 9B) \cos^3(e + fx)}{8f(c - c \sin(e + fx))^{5/2}} - \frac{3a^2 (A + 9B) \cos^2(e + fx)}{8c^2 f \sqrt{c - c \sin(e + fx)}} \\
 &= \frac{3a^2 (A + 9B) \tanh^{-1} \left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}} \right)}{4\sqrt{2} c^{5/2} f} + \frac{a^2 (A + B) c^2 \cos^5(e + fx)}{4f(c - c \sin(e + fx))^{9/2}}
 \end{aligned}$$

Mathematica [C] time = 1.17, size = 344, normalized size = 1.97

$$\frac{a^2 (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(8(A + B) \sin\left(\frac{1}{2}(e + fx)\right) - (5A + 13B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{(c - c \sin(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*sin[e + f*x])^2*(A + B*sin[e + f*x]))/(c - c*sin[e + f*x])^(5/2), x]

```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - (5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(A + 9*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 8*B*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 8*(A + B)*Sin[(e + f*x)/2] - 2*(5*A + 13*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Ssin[(e + f*x)/2] - 8*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Ssin[(e + f*x)/2]*(1 + Sin[e + f*x])^2)/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Ssin[e + f*x])^(5/2))
```

fricas [B] time = 0.45, size = 449, normalized size = 2.57

$$3\sqrt{2}\left((A+9B)a^2\cos(fx+e)^3+3(A+9B)a^2\cos(fx+e)^2-2(A+9B)a^2\cos(fx+e)-4(A+9B)a^2-\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*((A + 9*B)*a^2*cos(f*x + e)^3 + 3*(A + 9*B)*a^2*cos(f*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2 - ((A + 9*B)*a^2*cos(f*x + e)^2 - 2*(A + 9*B)*a^2*cos(f*x + e) - 4*(A + 9*B)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(8*B*a^2*cos(f*x + e)^3 - (5*A + 21*B)*a^2*cos(f*x + e)^2 - (A + 25*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2 + (8*B*a^2*cos(f*x + e)^2 + (5*A + 29*B)*a^2*cos(f*x + e) + 4*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^3*f*cos(f*x + e)^3 + 3*c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f - (c^3*f*cos(f*x + e)^2 - 2*c^3*f*cos(f*x + e) - 4*c^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(c*tan((f*x+exp(1))/2)^2+c)*(1/2*B*a^2/c^2/sign(tan((f*x+exp(1))/2)-1)+1/2*B*a^2*tan((f*x+exp(1))/2)/c^2/sign(tan((f*x+exp(1))/2)-1)))/(c*tan((f*x+exp(1))/2)^2+c)+2*(1/8*(-5*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+19*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+29*A*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+133*B*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+17*A*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+89*B*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-13*A*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-117*B*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+9*A*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+17*B*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-13*A*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-9*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
```

```

an((f*x+exp(1))/2)^2+c))^2-53*B*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(
c*tan((f*x+exp(1))/2)^2+c))+47*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1)
)/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+A*a^2*sqrt(c)*c^3+9*B*a^2*sqrt(c)*c^
3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2
*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^
4/sign(tan((f*x+exp(1))/2)-1)+1/8*(3*A*a^2+27*B*a^2)*atan((-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqr
t(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)))

```

maple [B] time = 1.60, size = 386, normalized size = 2.21

$$a^2 \left(3A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) (\sin^2(fx+e)) c^2 + 27B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) (\sin^2(fx+e)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out]
$$-1/8/c^{(9/2)}*a^2*(3*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)^2*c^2+27*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)^2*c^2-6*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c^2-16*B*(c*(1+\sin(f*x+e)))^{(1/2)}*c^{(3/2)}*\sin(f*x+e)^2-54*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c^2+10*A*(c*(1+\sin(f*x+e)))^{(3/2)}*c^{(1/2)}+3*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e))^2+26*B*(c*(1+\sin(f*x+e)))^{(3/2)}*c^{(1/2)}+32*B*c^{(3/2)}*(c*(1+\sin(f*x+e)))^{(1/2)}*\sin(f*x+e)+27*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)-12*A*(c*(1+\sin(f*x+e)))^{(1/2)}*c^{(3/2)}-60*B*(c*(1+\sin(f*x+e)))^{(1/2)}*c^{(3/2)}*(c*(1+\sin(f*x+e)))^{(1/2)})/(\sin(f*x+e)-1)/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(a \sin(fx+e) + a)^2}{(-c \sin(fx+e) + c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorith="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

$$3.96 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=175

$$\frac{a^2(A-11B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{16\sqrt{2} c^{7/2} f} + \frac{a^2 c^2 (A+B) \cos^5(e+fx)}{6f(c-c \sin(e+fx))^{11/2}} - \frac{a^2(A-11B) \cos(e+fx)}{16c^2 f(c-c \sin(e+fx))^{3/2}} + \frac{a^2(A-11B) \cos^3(e+fx)}{24f(c-c \sin(e+fx))^{5/2}}$$

[Out] 1/6*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(11/2)+1/24*a^2*(A-11*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(7/2)-1/16*a^2*(A-11*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(3/2)+1/32*a^2*(A-11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)

Rubi [A] time = 0.49, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^2 c^2 (A+B) \cos^5(e+fx)}{6f(c-c \sin(e+fx))^{11/2}} - \frac{a^2(A-11B) \cos(e+fx)}{16c^2 f(c-c \sin(e+fx))^{3/2}} + \frac{a^2(A-11B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{16\sqrt{2} c^{7/2} f} + \frac{a^2(A-11B) \cos^3(e+fx)}{24f(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] (a^2*(A - 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(16*Sqrt[2]*c^(7/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(6*f*(c - c*Sin[e + f*x])^(11/2)) + (a^2*(A - 11*B)*Cos[e + f*x]^3)/(24*f*(c - c*Sin[e + f*x])^(7/2)) - (a^2*(A - 11*B)*Cos[e + f*x])/(16*c^2*f*(c - c*Sin[e + f*x])^(3/2))

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2680

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p-1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2859

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x]

fricas [B] time = 0.46, size = 521, normalized size = 2.98

$$3\sqrt{2}\left((A-11B)a^2\cos(fx+e)^4 - 3(A-11B)a^2\cos(fx+e)^3 - 8(A-11B)a^2\cos(fx+e)^2 + 4(A-11B)a^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out] -1/192*(3*sqrt(2)*((A - 11*B)*a^2*cos(f*x + e)^4 - 3*(A - 11*B)*a^2*cos(f*x + e)^3 - 8*(A - 11*B)*a^2*cos(f*x + e)^2 + 4*(A - 11*B)*a^2*cos(f*x + e) + 8*(A - 11*B)*a^2 + ((A - 11*B)*a^2*cos(f*x + e)^3 + 4*(A - 11*B)*a^2*cos(f*x + e)^2 - 4*(A - 11*B)*a^2*cos(f*x + e) - 8*(A - 11*B)*a^2)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(A + 21*B)*a^2*cos(f*x + e)^3 + (25*A + 13*B)*a^2*cos(f*x + e)^2 - 2*(5*A + 41*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2 + (3*(A + 21*B)*a^2*cos(f*x + e)^2 - 2*(11*A - 25*B)*a^2*cos(f*x + e) - 32*(A + B)*a^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/96*(-93*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11-33*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11-63*A*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10-171*B*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10-477*A*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-1953*B*a^2*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+93*A*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-1503*B*a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+654*A*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+2214*B*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+266*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+722*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-522*A*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-978*B*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-198*A*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+1218*B*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+223*A*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-53*B*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-9*A*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-315*A*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+195*B*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))

$-855*B*a^2*\sqrt{c}*c^4*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2-7*A*a^2*\sqrt{c}*c^5-19*B*a^2*\sqrt{c}*c^5)/c^3/(-(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})^2-2*\sqrt{c}*(-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})+c)^6/\text{sign}(\tan((f*x+\exp(1))/2)-1)+1/32*(A*a^2-11*B*a^2)*\text{atan}((-\sqrt{c})*\tan((f*x+\exp(1))/2)+\sqrt{c})*\sqrt{c*\tan((f*x+\exp(1))/2)^2+c})/\sqrt{2}/\sqrt{-c})/\sqrt{2}/c^3/\sqrt{-c}/\text{sign}(\tan((f*x+\exp(1))/2)-1))$

maple [B] time = 1.69, size = 354, normalized size = 2.02

$$a^2 \left(-3\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^3 (A-11B) \sin(fx+e) (\cos^2(fx+e)) + 12\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)`

[Out] $-1/96*a^2*(-3*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))*c^3*(A-11*B)*\sin(f*x+e)*\cos(f*x+e)^2+12*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))*c^3*(A-11*B)*\sin(f*x+e)+9*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))*c^3*(A-11*B)*\cos(f*x+e)^2+24*A*(c+c*\sin(f*x+e))^{(1/2)}*c^{(5/2)}-32*A*(c+c*\sin(f*x+e))^{(3/2)}*c^{(3/2)}-6*A*(c+c*\sin(f*x+e))^{(5/2)}*c^{(1/2)}-264*B*(c+c*\sin(f*x+e))^{(1/2)}*c^{(5/2)}+352*B*(c+c*\sin(f*x+e))^{(3/2)}*c^{(3/2)}-126*B*(c+c*\sin(f*x+e))^{(5/2)}*c^{(1/2)}-12*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))*c^3+132*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)}))*c^3*(c*(1+\sin(f*x+e)))^{(1/2)}/c^{(13/2)}/(\sin(f*x+e)-1)^2/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^2}{(-c \sin(fx + e) + c)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(7/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2),x)`

[Out] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(7/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

$$3.97 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$$

Optimal. Leaf size=222

$$\frac{a^2(3A-13B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f} + \frac{a^2(3A-13B) \cos(e+fx)}{256c^3f(c-c \sin(e+fx))^{3/2}} + \frac{a^2c^2(A+B) \cos^5(e+fx)}{8f(c-c \sin(e+fx))^{13/2}} - \frac{a^2(3A-13B)}{64c^2f(c-c \sin(e+fx))^{5/2}}$$

[Out] 1/8*a^2*(A+B)*c^2*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)+1/48*a^2*(3*A-13*B)*cos(f*x+e)^3/f/(c-c*sin(f*x+e))^(9/2)-1/64*a^2*(3*A-13*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(5/2)+1/256*a^2*(3*A-13*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(3/2)+1/512*a^2*(3*A-13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(9/2)/f*2^(1/2)

Rubi [A] time = 0.51, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^2c^2(A+B) \cos^5(e+fx)}{8f(c-c \sin(e+fx))^{13/2}} + \frac{a^2(3A-13B) \cos(e+fx)}{256c^3f(c-c \sin(e+fx))^{3/2}} - \frac{a^2(3A-13B) \cos(e+fx)}{64c^2f(c-c \sin(e+fx))^{5/2}} + \frac{a^2(3A-13B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{256\sqrt{2}c^{9/2}f}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] (a^2*(3*A - 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(256*Sqrt[2]*c^(9/2)*f) + (a^2*(A + B)*c^2*Cos[e + f*x]^5)/(8*f*(c - c*Sin[e + f*x])^(13/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x]^3)/(48*f*(c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 13*B)*Cos[e + f*x])/(64*c^2*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*(3*A - 13*B)*Cos[e + f*x])/(256*c^3*f*(c - c*Sin[e + f*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F


```
[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2*(2013*A*Cos
[(e + f*x)/2] + 1517*B*Cos[(e + f*x)/2] - 999*A*Cos[(3*(e + f*x))/2] - 791*
B*Cos[(3*(e + f*x))/2] - 69*A*Cos[(5*(e + f*x))/2] - 725*B*Cos[(5*(e + f*x)
)/2] - 9*A*Cos[(7*(e + f*x))/2] + 39*B*Cos[(7*(e + f*x))/2] - (24 + 24*I)*(-
-1)^(1/4)*(3*A - 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])
]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 2013*A*Sin[(e + f*x)/2] + 1517*
B*Sin[(e + f*x)/2] + 999*A*Sin[(3*(e + f*x))/2] + 791*B*Sin[(3*(e + f*x))/2
] - 69*A*Sin[(5*(e + f*x))/2] - 725*B*Sin[(5*(e + f*x))/2] + 9*A*Sin[(7*(e
+ f*x))/2] - 39*B*Sin[(7*(e + f*x))/2]))/(6144*f*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))
```

fricas [B] time = 0.49, size = 654, normalized size = 2.95

$$3\sqrt{2}\left((3A - 13B)a^2 \cos(fx + e)^5 + 5(3A - 13B)a^2 \cos(fx + e)^4 - 8(3A - 13B)a^2 \cos(fx + e)^3 - 20(3A - 13B)a^2 \cos(fx + e)^2 + 8(3A - 13B)a^2 \cos(fx + e) + 16(3A - 13B)a^2 - ((3A - 13B)a^2 \cos(fx + e)^4 - 4(3A - 13B)a^2 \cos(fx + e)^3 - 12(3A - 13B)a^2 \cos(fx + e)^2 + 8(3A - 13B)a^2 \cos(fx + e) + 16(3A - 13B)a^2) \sin(fx + e) \sqrt{c} \log(-c \cos(fx + e)^2 - 2\sqrt{2} \sqrt{-c \sin(fx + e) + c}) \sqrt{c} (\cos(fx + e) + \sin(fx + e) + 1) + 3c \cos(fx + e) + (c \cos(fx + e) - 2c) \sin(fx + e) + 2c) / (\cos(fx + e)^2 + (\cos(fx + e) + 2) \sin(fx + e) - \cos(fx + e) - 2) + 4(3(3A - 13B)a^2 \cos(fx + e)^4 + (39A + 343B)a^2 \cos(fx + e)^3 + 2(129A + 209B)a^2 \cos(fx + e)^2 - 12(13A + 29B)a^2 \cos(fx + e) - 384(A + B)a^2 - (3(3A - 13B)a^2 \cos(fx + e)^3 - 2(15A + 191B)a^2 \cos(fx + e)^2 + 12(19A + 3B)a^2 \cos(fx + e) + 384(A + B)a^2) \sin(fx + e) \sqrt{-c \sin(fx + e) + c}) / (c^5 f \cos(fx + e)^5 + 5c^5 f \cos(fx + e)^4 - 8c^5 f \cos(fx + e)^3 - 20c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f - (c^5 f \cos(fx + e)^4 - 4c^5 f \cos(fx + e)^3 - 12c^5 f \cos(fx + e)^2 + 8c^5 f \cos(fx + e) + 16c^5 f) \sin(fx + e))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="fricas")
```

```
[Out] -1/3072*(3*sqrt(2))*((3*A - 13*B)*a^2*cos(f*x + e)^5 + 5*(3*A - 13*B)*a^2*co
s(f*x + e)^4 - 8*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 20*(3*A - 13*B)*a^2*cos(
f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B)*a^2 - ((3*A
- 13*B)*a^2*cos(f*x + e)^4 - 4*(3*A - 13*B)*a^2*cos(f*x + e)^3 - 12*(3*A -
13*B)*a^2*cos(f*x + e)^2 + 8*(3*A - 13*B)*a^2*cos(f*x + e) + 16*(3*A - 13*B
)*a^2)*sin(f*x + e)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin
(f*x + e) + c))*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e)
+ (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x +
e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(3*(3*A - 13*B)*a^2*cos(f*x +
e)^4 + (39*A + 343*B)*a^2*cos(f*x + e)^3 + 2*(129*A + 209*B)*a^2*cos(f*x +
e)^2 - 12*(13*A + 29*B)*a^2*cos(f*x + e) - 384*(A + B)*a^2 - (3*(3*A - 13*
B)*a^2*cos(f*x + e)^3 - 2*(15*A + 191*B)*a^2*cos(f*x + e)^2 + 12*(19*A + 3*
B)*a^2*cos(f*x + e) + 384*(A + B)*a^2)*sin(f*x + e)*sqrt(-c*sin(f*x + e) +
c))/(c^5*f*cos(f*x + e)^5 + 5*c^5*f*cos(f*x + e)^4 - 8*c^5*f*cos(f*x + e)^
3 - 20*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos(f*x + e) + 16*c^5*f - (c^5*f*cos(
f*x + e)^4 - 4*c^5*f*cos(f*x + e)^3 - 12*c^5*f*cos(f*x + e)^2 + 8*c^5*f*cos
(f*x + e) + 16*c^5*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/1536*(-1527*A*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^15-39*B*a^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan
((f*x+exp(1))/2)^2+c))^15-4473*A*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+
sqrt(c*tan((f*x+exp(1))/2)^2+c))^14+2487*B*a^2*sqrt(c)*(-sqrt(c)*tan((f*x+e
xp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^14-22233*A*a^2*c*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-7593*B*a^2*c*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^13-23811*A*a^2*sqrt(c)*
c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^12+1293*B*
a^2*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))
```

```

)^12+2133*A*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11-1563*B*a^2*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^11+68019*A*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10-10589*B*a^2*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^10+25371*A*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+9355*B*a^2*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9-71487*A*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-3055*B*a^2*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-25173*A*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+7195*B*a^2*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+56469*A*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+15909*B*a^2*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-10971*A*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-2123*B*a^2*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-31881*A*a^2*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-3673*B*a^2*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+17079*A*a^2*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-5913*B*a^2*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+345*A*a^2*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-7695*A*a^2*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+41*B*a^2*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-3007*B*a^2*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-117*A*a^2*sqrt(c)*c^7-5*B*a^2*sqrt(c)*c^7/c^4/(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8/sign(tan((f*x+exp(1))/2)-1)+1/512*(3*A*a^2-13*B*a^2)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))

```

maple [B] time = 1.70, size = 440, normalized size = 1.98

$$a^2 \left(-12 \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2} c^5 (3A - 13B) \sin(fx + e) (\cos^2(fx + e)) + 24 \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)

```

[Out] 1/1536/c^(19/2)*a^2*(-12*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)*cos(f*x+e)^2+24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*sin(f*x+e)-3*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^4+24*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^5*(3*A-13*B)*cos(f*x+e)^2+144*A*(c+c*sin(f*x+e))^(1/2)*c^(9/2)-264*A*(c+c*sin(f*x+e))^(3/2)*c^(7/2)-132*A*(c+c*sin(f*x+e))^(5/2)*c^(5/2)+18*A*(c+c*sin(f*x+e))^(7/2)*c^(3/2)-624*B*(c+c*sin(f*x+e))^(1/2)*c^(9/2)+1144*B*(c+c*sin(f*x+e))^(3/2)*c^(7/2)-452*B*(c+c*sin(f*x+e))^(5/2)*c^(5/2)-78*B*(c+c*sin(f*x+e))^(7/2)*c^(3/2)-72*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5+312*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^5*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^2}{(-c \sin(fx + e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2/(-c*sin(f*x + e) + c)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^2}{(c - c \sin(e + fx))^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c - c*sin(e + f*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)
```

```
[Out] Timed out
```

$$3.98 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=210

$$\frac{256a^3c^7(15A - B)\cos^7(e + fx)}{45045f(c - c\sin(e + fx))^{7/2}} + \frac{64a^3c^6(15A - B)\cos^7(e + fx)}{6435f(c - c\sin(e + fx))^{5/2}} + \frac{8a^3c^5(15A - B)\cos^7(e + fx)}{715f(c - c\sin(e + fx))^{3/2}} + \frac{2a^3c^4(15A - B)\cos^7(e + fx)}{195f\sqrt{c - c\sin(e + fx)}}$$

[Out] 256/45045*a^3*(15*A-B)*c^7*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+64/6435*a^3*(15*A-B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+8/715*a^3*(15*A-B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)+2/195*a^3*(15*A-B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(1/2)-2/15*a^3*B*c^3*cos(f*x+e)^7*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.53, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(15A - B)\cos^7(e + fx)}{195f\sqrt{c - c\sin(e + fx)}} + \frac{8a^3c^5(15A - B)\cos^7(e + fx)}{715f(c - c\sin(e + fx))^{3/2}} + \frac{64a^3c^6(15A - B)\cos^7(e + fx)}{6435f(c - c\sin(e + fx))^{5/2}} + \frac{256a^3c^7(15A - B)\cos^7(e + fx)}{45045f(c - c\sin(e + fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2), x]

[Out] (256*a^3*(15*A - B)*c^7*Cos[e + f*x]^7)/(45045*f*(c - c*Sin[e + f*x])^(7/2)) + (64*a^3*(15*A - B)*c^6*Cos[e + f*x]^7)/(6435*f*(c - c*Sin[e + f*x])^(5/2)) + (8*a^3*(15*A - B)*c^5*Cos[e + f*x]^7)/(715*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*(15*A - B)*c^4*Cos[e + f*x]^7)/(195*f*Sqrt[c - c*Sin[e + f*x]]) - (2*a^3*B*c^3*Cos[e + f*x]^7*Sqrt[c - c*Sin[e + f*x]])/(15*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx &= (a^3 c^3) \int \cos^6(e + fx) (A + B \sin(e + fx)) \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx) \sqrt{c - c \sin(e + fx)}}{15f} \\ &= \frac{2a^3 (15A - B) c^4 \cos^7(e + fx)}{195f \sqrt{c - c \sin(e + fx)}} - \frac{2a^3 B c^3 \cos^7(e + fx)}{195f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{8a^3 (15A - B) c^5 \cos^7(e + fx)}{715f (c - c \sin(e + fx))^{3/2}} + \frac{2a^3 (15A - B) c^4 \cos^7(e + fx)}{195f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{64a^3 (15A - B) c^6 \cos^7(e + fx)}{6435f (c - c \sin(e + fx))^{5/2}} + \frac{8a^3 (15A - B) c^5 \cos^7(e + fx)}{715f (c - c \sin(e + fx))^{3/2}} \\ &= \frac{256a^3 (15A - B) c^7 \cos^7(e + fx)}{45045f (c - c \sin(e + fx))^{7/2}} + \frac{64a^3 (15A - B) c^6 \cos^7(e + fx)}{6435f (c - c \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [B] time = 6.83, size = 1569, normalized size = 7.47

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
7/2),x]
```

```
[Out] (5*(8*A - B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(
7/2))/(64*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^6) - (5*(6*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^
3*(c - c*Sin[e + f*x])^(7/2))/(192*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^
7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (3*(10*A - 3*B)*Cos[(5*(e + f*
x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(320*f*(Cos[(e +
f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (3
*(4*A + 3*B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x
])^(7/2))/(448*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) + ((12*A - 5*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e +
f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(576*f*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 5*B)*Cos[(11*(e
+ f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(704*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((2*A - B)*Cos[(13*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*
x])^(7/2))/(832*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2])^6) - (B*Cos[(15*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(
c - c*Sin[e + f*x])^(7/2))/(960*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(8*A - B)*Sin[(e + f*x)/2]*(a
+ a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2))/(64*f*(Cos[(e + f*x)/2] - S
in[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*(6*A + B)*
(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(7/2)*Sin[(3*(e + f*x))/2])/(19
```


) * cos(1/4*(14*f*x+14*exp(1)+pi))/(224*f)^2-1728*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)-pi))/(288*f)^2+1408*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+22*exp(1)+pi))/(704*f)^2-1664*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2-160*A*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x-pi)+1/2*exp(1))/(16*f)^2+1280*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x+2*exp(1)+pi))/(128*f)^2-3840*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(384*f)^2+11520*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)+pi))/(640*f)^2-16128*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)-pi))/(896*f)^2+11520*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)+pi))/(1152*f)^2-14080*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*f*x+22*exp(1)-pi))/(1408*f)^2+3328*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(26*f*x+26*exp(1)+pi))/(1664*f)^2-3840*B*a^3*c^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(30*f*x+30*exp(1)-pi))/(1920*f)^2)

maple [A] time = 1.37, size = 121, normalized size = 0.58

$$\frac{2 \left(\sin (f x+e)-1 \right) c^4 \left(1+\sin (f x+e) \right)^4 a^3 \left((-3465 A+12243 B) \sin (f x+e) \left(\cos ^2 (f x+e) \right)+\left(24780 A-24969 B \right) \cos (f x+e) \right) \sqrt{c}}{45045 \cos (f x+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)

[Out] 2/45045*(sin(f*x+e)-1)*c^4*(1+sin(f*x+e))^4*a^3*((-3465*A+12243*B)*sin(f*x+e)*cos(f*x+e)^2+(24780*A-25676*B)*sin(f*x+e)+3003*B*cos(f*x+e)^4+(14175*A-24969*B)*cos(f*x+e)^2-26700*A+25804*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (f x+e)+A)(a \sin (f x+e)+a)^3(-c \sin (f x+e)+c)^{\frac{7}{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A+B \sin (e+f x))(a+a \sin (e+f x))^3(c-c \sin (e+f x))^{\frac{7}{2}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2), x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x)

[Out] Timed out

$$3.99 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=161

$$\frac{64a^3c^6(13A + B) \cos^7(e + fx)}{9009f(c - c \sin(e + fx))^{7/2}} + \frac{16a^3c^5(13A + B) \cos^7(e + fx)}{1287f(c - c \sin(e + fx))^{5/2}} + \frac{2a^3c^4(13A + B) \cos^7(e + fx)}{143f(c - c \sin(e + fx))^{3/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{13f\sqrt{c - c \sin(e + fx)}}$$

[Out] 64/9009*a^3*(13*A+B)*c^6*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+16/1287*a^3*(13*A+B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)+2/143*a^3*(13*A+B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)-2/13*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.47, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(13A + B) \cos^7(e + fx)}{143f(c - c \sin(e + fx))^{3/2}} + \frac{16a^3c^5(13A + B) \cos^7(e + fx)}{1287f(c - c \sin(e + fx))^{5/2}} + \frac{64a^3c^6(13A + B) \cos^7(e + fx)}{9009f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{13f\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] (64*a^3*(13*A + B)*c^6*Cos[e + f*x]^7)/(9009*f*(c - c*Sin[e + f*x])^(7/2)) + (16*a^3*(13*A + B)*c^5*Cos[e + f*x]^7)/(1287*f*(c - c*Sin[e + f*x])^(5/2)) + (2*a^3*(13*A + B)*c^4*Cos[e + f*x]^7)/(143*f*(c - c*Sin[e + f*x])^(3/2)) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(13*f*Sqrt[c - c*Sin[e + f*x]])

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Di

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{13f \sqrt{c - c \sin(e + fx)}} + \frac{1}{13} (a^3 (13A + B) \cos^6(e + fx)) \\ &= \frac{2a^3 (13A + B) c^4 \cos^7(e + fx)}{143f (c - c \sin(e + fx))^{3/2}} - \frac{2a^3 B c^3 \cos^6(e + fx)}{13f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{16a^3 (13A + B) c^5 \cos^7(e + fx)}{1287f (c - c \sin(e + fx))^{5/2}} + \frac{2a^3 (13A + B) c^4 \cos^6(e + fx)}{143f \sqrt{c - c \sin(e + fx)}} \\ &= \frac{64a^3 (13A + B) c^6 \cos^7(e + fx)}{9009f (c - c \sin(e + fx))^{7/2}} + \frac{16a^3 (13A + B) c^5 \cos^6(e + fx)}{1287f \sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [B] time = 6.71, size = 1351, normalized size = 8.39

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
(5/2),x]
```

```
[Out] (5*A*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8
*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x
)/2])^6) - (5*(4*A + B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*
Sin[e + f*x])^(5/2))/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*Cos[(5*(e + f*x))/2]*(a + a*S
in[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(32*f*(Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((5*A + 2*B)*Cos[(
7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(112*f*(
Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^6) + ((A - 2*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e
+ f*x])^(5/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) - ((2*A + B)*Cos[(11*(e + f*x))/2]*(a + a*Sin[e
+ f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(352*f*(Cos[(e + f*x)/2] - Sin[(e +
f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(13*(e + f*x)
)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(416*f*(Cos[(e + f*x
)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (5*A*
Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))/(8*f*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^6) + (5*(4*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(3
*(e + f*x))/2])/(96*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x
)/2] + Sin[(e + f*x)/2])^6) + ((2*A - B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[
e + f*x])^(5/2)*Sin[(5*(e + f*x))/2])/(32*f*(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((5*A + 2*B)*(a + a*Sin
[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(
e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
+ ((A - 2*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)*Sin[(9*(e +
f*x))/2])/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2]
```


$$\begin{aligned} & ((1/2*(f*x+\exp(1))-1/4*\pi))+2*B*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))) * \\ & \cos(1/4*(10*f*x+10*\exp(1)-\pi))/(320*f)^2+48*f*(-2*A*a^3*c^2*\text{sign}(\sin(1/2*(f \\ & *x+\exp(1))-1/4*\pi))-2*B*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)))*\sin(1/4 \\ & *(6*f*x+6*\exp(1)-\pi))/(48*f)^2+352*f*(-2*A*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1) \\ &)-1/4*\pi))-2*B*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)))*\sin(1/4*(22*f*x+ \\ & 22*\exp(1)-\pi))/(352*f)^2-224*f*(-4*A*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4* \\ & \pi))+2*B*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)))*\cos(1/4*(14*f*x+14*\exp \\ & (1)+\pi))/(224*f)^2+224*f*(-6*A*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))-6 \\ & *B*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)))*\sin(1/4*(14*f*x+14*\exp(1)-\pi \\ &))/(224*f)^2-192*f*(-32*A*a^3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))-2*B*a^ \\ & 3*c^2*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)))*\cos(1/4*(6*f*x+6*\exp(1)+\pi))/(192 \\ & *f)^2-1408*B*a^3*c^2*f*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\cos(1/4*(22*f*x+2 \\ & 2*\exp(1)+\pi))/(704*f)^2+1664*B*a^3*c^2*f*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) \\ & *\cos(1/4*(26*f*x+26*\exp(1)-\pi))/(832*f)^2 \end{aligned}$$

maple [A] time = 1.34, size = 105, normalized size = 0.65

$$\frac{2(\sin(fx+e)-1)c^3(1+\sin(fx+e))^4 a^3(-693B(\cos^2(fx+e))\sin(fx+e)+(-2366A+2590B)\sin(fx+e))}{9009 \cos(fx+e) \sqrt{c-c\sin(fx+e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)

[Out] $-2/9009*(\sin(f*x+e)-1)*c^3*(1+\sin(f*x+e))^4*a^3*(-693*B*\cos(f*x+e)^2*\sin(f*x+e)+(-2366*A+2590*B)*\sin(f*x+e)+(-819*A+2016*B)*\cos(f*x+e)^2+2782*A-2558*B)/\cos(f*x+e)/(c-c*\sin(f*x+e))^(1/2)/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx+e) + A)(a \sin(fx+e) + a)^3 (-c \sin(fx+e) + c)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2), x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)

[Out] Timed out

$$3.100 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=124

$$\frac{8a^3c^5(11A + 3B) \cos^7(e + fx)}{693f(c - c \sin(e + fx))^{7/2}} + \frac{2a^3c^4(11A + 3B) \cos^7(e + fx)}{99f(c - c \sin(e + fx))^{5/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{11f(c - c \sin(e + fx))^{3/2}}$$

[Out] 8/693*a^3*(11*A+3*B)*c^5*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(7/2)+2/99*a^3*(11*A+3*B)*c^4*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(5/2)-2/11*a^3*B*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(3/2)

Rubi [A] time = 0.41, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2856, 2674, 2673}

$$\frac{2a^3c^4(11A + 3B) \cos^7(e + fx)}{99f(c - c \sin(e + fx))^{5/2}} + \frac{8a^3c^5(11A + 3B) \cos^7(e + fx)}{693f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{11f(c - c \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2), x]

[Out] (8*a^3*(11*A + 3*B)*c^5*Cos[e + f*x]^7)/(693*f*(c - c*Sin[e + f*x])^(7/2)) + (2*a^3*(11*A + 3*B)*c^4*Cos[e + f*x]^7)/(99*f*(c - c*Sin[e + f*x])^(5/2)) - (2*a^3*B*c^3*Cos[e + f*x]^7)/(11*f*(c - c*Sin[e + f*x])^(3/2))

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])^n*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{11f(c - c \sin(e + fx))^{3/2}} + \frac{1}{11} (a^3(11A + \\ &= \frac{2a^3(11A + 3B)c^4 \cos^7(e + fx)}{99f(c - c \sin(e + fx))^{5/2}} - \frac{2a^3 B c^3}{11f(c - c \sin(e + fx))^{3/2}} \\ &= \frac{8a^3(11A + 3B)c^5 \cos^7(e + fx)}{693f(c - c \sin(e + fx))^{7/2}} + \frac{2a^3(11A + 3B)c^4 \cos^7(e + fx)}{99f(c - c \sin(e + fx))^{3/2}} \end{aligned}$$

Mathematica [B] time = 6.50, size = 1157, normalized size = 9.33

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]

[Out] ((6*A + B)*Cos[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((8*A + 3*B)*Cos[(3*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*Cos[(5*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((6*A + B)*Cos[(7*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*Cos[(9*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (B*Cos[(11*(e + f*x))/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2))/(8*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((8*A + 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(3*(e + f*x))/2])/(24*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(5*(e + f*x))/2])/(16*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + ((6*A + B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(7*(e + f*x))/2])/(112*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - ((2*A + 3*B)*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(9*(e + f*x))/2])/(144*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) - (B*(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)*Sin[(11*(e + f*x))/2])/(176*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)

fricas [B] time = 0.45, size = 287, normalized size = 2.31

$$2 \left(63 B a^3 c \cos(fx + e)^6 - 7(11 A + 12 B) a^3 c \cos(fx + e)^5 - (187 A + 177 B) a^3 c \cos(fx + e)^4 + 2(11 A + 3 B) \right)$$

[In] `int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)`

[Out] `2/693*(sin(f*x+e)-1)*c^2*(1+sin(f*x+e))^4*a^3*(sin(f*x+e)*(77*A-105*B)-63*B*cos(f*x+e)^2-121*A+93*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^3 (-c \sin(fx + e) + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*(-c*sin(f*x + e) + c)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2), x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)`

[Out] Timed out

3.101 $\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$

Optimal. Leaf size=81

$$\frac{2a^3c^4(9A + 5B) \cos^7(e + fx)}{63f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{9f(c - c \sin(e + fx))^{5/2}}$$

[Out] $2/63*a^3*(9*A+5*B)*c^4*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^{(7/2)}-2/9*a^3*B*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^{(5/2)}$

Rubi [A] time = 0.30, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2967, 2856, 2673}

$$\frac{2a^3c^4(9A + 5B) \cos^7(e + fx)}{63f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{9f(c - c \sin(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]

[Out] $(2*a^3*(9*A + 5*B)*c^4*\cos[e + f*x]^7)/(63*f*(c - c*\sin[e + f*x])^{(7/2)}) - (2*a^3*B*c^3*\cos[e + f*x]^7)/(9*f*(c - c*\sin[e + f*x])^{(5/2)})$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2856

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p + 1)/2], 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx &= (a^3c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx \\ &= -\frac{2a^3Bc^3 \cos^7(e + fx)}{9f(c - c \sin(e + fx))^{5/2}} + \frac{1}{9} (a^3(9A + 5B)c^3) \\ &= \frac{2a^3(9A + 5B)c^4 \cos^7(e + fx)}{63f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3Bc^3 \cos^7(e + fx)}{9f(c - c \sin(e + fx))^{5/2}} \end{aligned}$$


```
*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)-pi))/(20
*f)^2+24*f*(-30*A*a^3*sign(sin(1/2*(f*x+exp(1))-1/4*pi))-26*B*a^3*sign(sin(
1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+224*B*a^3*
f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)+pi))/(112*f)
^2-288*B*a^3*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)
-pi))/(144*f)^2)
```

maple [A] time = 1.24, size = 65, normalized size = 0.80

$$\frac{2(\sin(fx + e) - 1)c(1 + \sin(fx + e))^4 a^3 (7B \sin(fx + e) + 9A - 2B)}{63 \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -2/63*(sin(f*x+e)-1)*c*(1+sin(f*x+e))^4*a^3*(7*B*sin(f*x+e)+9*A-2*B)/cos(f*
x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^3 \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3*sqrt(-c*sin(f*x + e)
+ c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^3 \sqrt{c - c \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int A \sqrt{-c \sin(e + fx) + c} dx + \int 3A \sqrt{-c \sin(e + fx) + c} \sin(e + fx) dx + \int 3A \sqrt{-c \sin(e + fx) + c} \sin^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] a**3*(Integral(A*sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sqrt(-c*sin(e
+ f*x) + c)*sin(e + f*x), x) + Integral(3*A*sqrt(-c*sin(e + f*x) + c)*sin(
e + f*x)**2, x) + Integral(A*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**3, x)
+ Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x), x) + Integral(3*B*sqrt
(-c*sin(e + f*x) + c)*sin(e + f*x)**2, x) + Integral(3*B*sqrt(-c*sin(e + f*
x) + c)*sin(e + f*x)**3, x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e +
f*x)**4, x))
```


$$3.102 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=200

$$\frac{2a^3c^2(A+B) \cos^5(e+fx)}{5f(c-c \sin(e+fx))^{5/2}} - \frac{4a^3c(A+B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^{3/2}} - \frac{8a^3(A+B) \cos(e+fx)}{f\sqrt{c-c \sin(e+fx)}} + \frac{8\sqrt{2}a^3(A+B) \tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{c}f}$$

[Out] $-2/7*a^3*B*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^{(7/2)}-2/5*a^3*(A+B)*c^2*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(5/2)}-4/3*a^3*(A+B)*c*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^{(3/2)}+8*a^3*(A+B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)})/(c-c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/f/c^{(1/2)}-8*a^3*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.52, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2860, 2679, 2649, 206}

$$\frac{2a^3c^2(A+B) \cos^5(e+fx)}{5f(c-c \sin(e+fx))^{5/2}} - \frac{4a^3c(A+B) \cos^3(e+fx)}{3f(c-c \sin(e+fx))^{3/2}} - \frac{8a^3(A+B) \cos(e+fx)}{f\sqrt{c-c \sin(e+fx)}} + \frac{8\sqrt{2}a^3(A+B) \tanh^{-1}\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] $(8*\sqrt{2}*a^3*(A+B)*\operatorname{ArcTanh}[(\sqrt{c}*\cos[e+f*x])/(\sqrt{2}*\sqrt{c-c*\sin[e+f*x]})])/(\sqrt{c}*f) - (2*a^3*B*c^3*\cos[e+f*x]^7)/(7*f*(c-c*\sin[e+f*x])^{(7/2)}) - (2*a^3*(A+B)*c^2*\cos[e+f*x]^5)/(5*f*(c-c*\sin[e+f*x])^{(5/2)}) - (4*a^3*(A+B)*c*\cos[e+f*x]^3)/(3*f*(c-c*\sin[e+f*x])^{(3/2)}) - (8*a^3*(A+B)*\cos[e+f*x])/(f*\sqrt{c-c*\sin[e+f*x]})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegersQ[2*m, 2*p]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(

$g \cos[e + f*x]^{(p+1)}(a + b \sin[e + f*x]^m) / (f*g*(m+p+1)), x] + \text{Dist}[(a*d*m + b*c*(m+p+1)) / (b*(m+p+1)), \text{Int}[(g \cos[e + f*x])^p (a + b \sin[e + f*x])^m, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

$\text{Int}[(a + b \sin[e + f*x])^m ((c + d \sin[e + f*x])^n (A + B \sin[e + f*x])), x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && ! (IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^{7/2}} + (a^3 (A + B) c^3) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} + (2a^3 (A + B) c^3) \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} - \frac{4a^3 (A + B) c^3}{3f(c - c \sin(e + fx))^{3/2}} + (2a^3 (A + B) c^3) \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\ &= -\frac{2a^3 B c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^{7/2}} - \frac{2a^3 (A + B) c^2 \cos^5(e + fx)}{5f(c - c \sin(e + fx))^{5/2}} - \frac{4a^3 (A + B) c^3}{3f(c - c \sin(e + fx))^{3/2}} - \frac{8\sqrt{2} a^3 (A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{\sqrt{c} f} - \frac{2a^3 B c^3 \cos^7(e + fx)}{7f(c - c \sin(e + fx))^{7/2}} \end{aligned}$$

Mathematica [C] time = 1.35, size = 193, normalized size = 0.96

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right) (-448A + 6720B)}{420f}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] -1/420*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*((6720 + 6720*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])] - 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2086*A - 2236*B + 6*(7*A + 22*B)*Cos[2*(e + f*x)] - (448*A + 673*B)*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)])))/((f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Sqrt[c - c*Sin[e + f*x]]))


```

xp(1))/2)-1)-7069440*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1))+sqrt(2)*(8*A*a
^3+8*B*a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(
1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1)+(-840*A
*a^3*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-644*A*a^3*sqrt(-c)*sqrt(2)*sqrt(c)-84
0*B*a^3*c*sqrt(2)*atan(sqrt(c)/sqrt(-c))-764*B*a^3*sqrt(-c)*sqrt(2)*sqrt(c)
)/105/c/sqrt(-c)*sign(tan((f*x+exp(1))/2)-1))

```

maple [A] time = 1.56, size = 233, normalized size = 1.16

$$2(\sin(fx + e) - 1) \sqrt{c(1 + \sin(fx + e))} a^3 \left(420c^{\frac{7}{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) A + 420c^{\frac{7}{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) B - 15B(c(1 + \sin(fx + e)))^{\frac{7}{2}} - 21A(c(1 + \sin(fx + e)))^{\frac{5}{2}} - 21B(c(1 + \sin(fx + e)))^{\frac{5}{2}} - 70A(c(1 + \sin(fx + e)))^{\frac{3}{2}} - 70B(c(1 + \sin(fx + e)))^{\frac{3}{2}} - 420A(c(1 + \sin(fx + e)))^{\frac{1}{2}} - 420B(c(1 + \sin(fx + e)))^{\frac{1}{2}} \right) / c^4 / \cos(fx + e) / (c - c \sin(fx + e))^{\frac{1}{2}} / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] -2/105*(sin(f*x+e)-1)*(c*(1+sin(f*x+e)))^(1/2)*a^3*(420*c^(7/2)*2^(1/2)*arc
tanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*A+420*c^(7/2)*2^(1/2)*ar
ctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*B-15*B*(c*(1+sin(f*x+e)
))^7/2-21*A*(c*(1+sin(f*x+e)))^5/2-21*B*(c*(1+sin(f*x+e)))^5/2*c-70
A(c*(1+sin(f*x+e)))^3/2*c^2-70*B*(c*(1+sin(f*x+e)))^3/2*c^2-420*A*c^3
(c(1+sin(f*x+e)))^1/2-420*B*c^3*(c*(1+sin(f*x+e)))^1/2)/c^4/cos(f*x+e
)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^3}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/sqrt(-c*sin(f*x + e)
+ c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(a + a \sin(e + fx))^3}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(1/2
), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int \frac{A}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{3A \sin(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{3A \sin^2(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx + \int \frac{A \sin^3(e + fx)}{\sqrt{-c \sin(e + fx) + c}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] a**3*(Integral(A/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)/  
sqrt(-c*sin(e + f*x) + c), x) + Integral(3*A*sin(e + f*x)**2/sqrt(-c*sin(e  
+ f*x) + c), x) + Integral(A*sin(e + f*x)**3/sqrt(-c*sin(e + f*x) + c), x)  
+ Integral(B*sin(e + f*x)/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(  
e + f*x)**2/sqrt(-c*sin(e + f*x) + c), x) + Integral(3*B*sin(e + f*x)**3/sq  
rt(-c*sin(e + f*x) + c), x) + Integral(B*sin(e + f*x)**4/sqrt(-c*sin(e + f*  
x) + c), x))
```

$$3.103 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=218

$$-\frac{2\sqrt{2}a^3(5A+9B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c^3(A+B) \cos^7(e+fx)}{2f(c-c \sin(e+fx))^{9/2}} + \frac{a^3c(5A+9B) \cos^5(e+fx)}{10f(c-c \sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)}{3f(c-c \sin(e+fx))^{3/2}}$$

[Out] $1/2*a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^{(9/2)}+1/10*a^3*(5*A+9*B)*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(5/2)}+1/3*a^3*(5*A+9*B)*\cos(f*x+e)^3/f/(c-c*\sin(f*x+e))^{(3/2)}-2*a^3*(5*A+9*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)})/(c-c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(3/2)}/f+2*a^3*(5*A+9*B)*\cos(f*x+e)/c/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.55, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2859, 2679, 2649, 206}

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{2f(c-c \sin(e+fx))^{9/2}} - \frac{2\sqrt{2}a^3(5A+9B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{c^{3/2}f} + \frac{a^3c(5A+9B) \cos^5(e+fx)}{10f(c-c \sin(e+fx))^{5/2}} + \frac{a^3(5A+9B)}{3f(c-c \sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]`

[Out] `(-2*Sqrt[2]*a^3*(5*A + 9*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(c^(3/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(2*f*(c - c*Sin[e + f*x])^(9/2)) + (a^3*(5*A + 9*B)*c*Cos[e + f*x]^5)/(10*f*(c - c*Sin[e + f*x])^(5/2)) + (a^3*(5*A + 9*B)*Cos[e + f*x]^3)/(3*f*(c - c*Sin[e + f*x])^(3/2)) + (2*a^3*(5*A + 9*B)*Cos[e + f*x])/(c*f*Sqrt[c - c*Sin[e + f*x]])`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 2649

`Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]`

Rule 2679

`Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]`

Rule 2859

`Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*c`

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1
)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e +
f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g
, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]
) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^n, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &&
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx = (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{2f (c - c \sin(e + fx))^{9/2}} - \frac{1}{4} (a^3 (5A + 9B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{2f (c - c \sin(e + fx))^{9/2}} + \frac{a^3 (5A + 9B) c \cos^5(e + fx)}{10f (c - c \sin(e + fx))^{5/2}} - \frac{1}{2} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{2f (c - c \sin(e + fx))^{9/2}} + \frac{a^3 (5A + 9B) c \cos^5(e + fx)}{10f (c - c \sin(e + fx))^{5/2}} + \frac{a^3}{3} \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{2f (c - c \sin(e + fx))^{9/2}} + \frac{a^3 (5A + 9B) c \cos^5(e + fx)}{10f (c - c \sin(e + fx))^{5/2}} + \frac{a^3}{3} \int \frac{1}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{2f (c - c \sin(e + fx))^{9/2}} + \frac{a^3 (5A + 9B) c \cos^5(e + fx)}{10f (c - c \sin(e + fx))^{5/2}} + \frac{a^3}{3} \int \frac{1}{(c - c \sin(e + fx))^{9/2}} dx$$

$$= -\frac{2\sqrt{2} a^3 (5A + 9B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{c^{3/2} f} + \frac{a^3 (A + B) c^3}{2f (c - c \sin(e + fx))^{9/2}}$$

Mathematica [C] time = 1.69, size = 444, normalized size = 2.04

$$a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(240(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 30(9A + 20B) \cos\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(120*(A + B)
)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) + (120 + 120*I)*(-1)^(1/4)*(5*A + 9
*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^2 + 30*(9*A + 20*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])^2 - 5*(2*A + 9*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^2 - 3*B*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^2 + 240*(A + B)*Sin[(e + f*x)/2] + 30*(9*A + 20*B)*(Cos[(e +
```


$$+2Aa^3c*(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})-2$$

$$*Aa^3*\sqrt{c}*(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})$$

$$)^2+2B*a^3c*(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})$$

$$)-2B*a^3*\sqrt{c}*(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})$$

$$)^2-2Aa^3*\sqrt{c}*c-2B*a^3*\sqrt{c}*c)/(-(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})^2-2*\sqrt{c}*(-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})+c)^2/c/\text{sign}(\tan((fx+\exp(1))/2)-1)+1/2*(-20Aa^3-36B*a^3)*\text{atan}((-\sqrt{c}*\tan((fx+\exp(1))/2)+\sqrt{c}+\sqrt{c*\tan((fx+\exp(1))/2)^2+c})/\sqrt{2}/\sqrt{-c})/\sqrt{2}/\sqrt{-c}/c/\text{sign}(\tan((fx+\exp(1))/2)-1)))$$

maple [A] time = 1.47, size = 354, normalized size = 1.62

$$2a^3 \left(\sin(fx + e) \left(-60A\sqrt{c + c \sin(fx + e)} c^{\frac{5}{2}} - 5A(c + c \sin(fx + e))^{\frac{3}{2}} c^{\frac{3}{2}} - 120B\sqrt{c + c \sin(fx + e)} c^{\frac{5}{2}} - \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] $\frac{2}{15}a^3(\sin(fx+e)*(-60A*(c+c*\sin(fx+e))^{1/2}*c^{5/2}-5A*(c+c*\sin(fx+e))^{3/2}*c^{3/2}-120B*(c+c*\sin(fx+e))^{1/2}*c^{5/2}-15B*(c+c*\sin(fx+e))^{3/2}*c^{3/2}-3B*(c+c*\sin(fx+e))^{5/2}*c^{1/2}+75A*2^{1/2}*\text{arctanh}(1/2*(c+c*\sin(fx+e))^{1/2}*2^{1/2}/c^{1/2})*c^3+135B*2^{1/2}*\text{arctanh}(1/2*(c+c*\sin(fx+e))^{1/2}*2^{1/2}/c^{1/2})*c^3)+90A*(c+c*\sin(fx+e))^{1/2}*c^{5/2}+5A*(c+c*\sin(fx+e))^{3/2}*c^{3/2}+150B*(c+c*\sin(fx+e))^{1/2}*c^{5/2}+15B*(c+c*\sin(fx+e))^{3/2}*c^{3/2}+3B*(c+c*\sin(fx+e))^{5/2}*c^{1/2}-75A*2^{1/2}*\text{arctanh}(1/2*(c+c*\sin(fx+e))^{1/2}*2^{1/2}/c^{1/2})*c^3-135B*2^{1/2}*\text{arctanh}(1/2*(c+c*\sin(fx+e))^{1/2}*2^{1/2}/c^{1/2})*c^3)*(c*(1+\sin(fx+e)))^{1/2}/c^{9/2}/\cos(fx+e)/(c-c*\sin(fx+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^3}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)

[Out] Timed out

$$3.104 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=225

$$\frac{5a^3(3A+11B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} + \frac{a^3c^3(A+B) \cos^7(e+fx)}{4f(c-c \sin(e+fx))^{11/2}} - \frac{5a^3(3A+11B) \cos(e+fx)}{4c^2f\sqrt{c-c \sin(e+fx)}} - \frac{a^3c(3A+11B)}{8f(c-c \sin(e+fx))^{5/2}}$$

[Out] 1/4*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(11/2)-1/8*a^3*(3*A+11*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(7/2)-5/24*a^3*(3*A+11*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(3/2)+5/4*a^3*(3*A+11*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(5/2)/f*2^(1/2)-5/4*a^3*(3*A+11*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.55, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3c^3(A+B) \cos^7(e+fx)}{4f(c-c \sin(e+fx))^{11/2}} - \frac{5a^3(3A+11B) \cos(e+fx)}{4c^2f\sqrt{c-c \sin(e+fx)}} + \frac{5a^3(3A+11B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{2\sqrt{2}c^{5/2}f} - \frac{a^3c(3A+11B)}{8f(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] (5*a^3*(3*A + 11*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(2*Sqrt[2]*c^(5/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(4*f*(c - c*Sin[e + f*x])^(11/2)) - (a^3*(3*A + 11*B)*c*Cos[e + f*x]^5)/(8*f*(c - c*Sin[e + f*x])^(7/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x]^3)/(24*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*a^3*(3*A + 11*B)*Cos[e + f*x])/(4*c^2*f*Sqrt[c - c*Sin[e + f*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}} - \frac{1}{8} (a^3 (3A + 11B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}} - \frac{a^3 (3A + 11B) c \cos^5(e + fx)}{8f(c - c \sin(e + fx))^{7/2}} + \frac{1}{16} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{3/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}} - \frac{a^3 (3A + 11B) c \cos^5(e + fx)}{8f(c - c \sin(e + fx))^{7/2}} - \frac{5a^3 (3A + 11B)}{24c} \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{1/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}} - \frac{a^3 (3A + 11B) c \cos^5(e + fx)}{8f(c - c \sin(e + fx))^{7/2}} - \frac{5a^3 (3A + 11B)}{24c} \int \frac{1}{(c - c \sin(e + fx))^{1/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}} - \frac{a^3 (3A + 11B) c \cos^5(e + fx)}{8f(c - c \sin(e + fx))^{7/2}} - \frac{5a^3 (3A + 11B)}{24c} \int \frac{1}{(c - c \sin(e + fx))^{1/2}} dx \\
 &= \frac{5a^3 (3A + 11B) \tanh^{-1} \left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}} \right)}{2\sqrt{2} c^{5/2} f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{4f(c - c \sin(e + fx))^{11/2}}
 \end{aligned}$$

Mathematica [C] time = 2.21, size = 434, normalized size = 1.93

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(24(A + B) \sin\left(\frac{1}{2}(e + fx)\right) - 6(2A + 11B) \cos\left(\frac{1}{2}(e + fx)\right) \right)}{2\sqrt{2} c^{5/2} f}$$

Antiderivative was successfully verified.


```

n(tan((f*x+exp(1))/2)-1))/c^4)-1/144*(-72*A*a^3*c^3*sign(tan((f*x+exp(1))/2)-1)-384*B*a^3*c^3*sign(tan((f*x+exp(1))/2)-1))/c^4)+2*(1/4*(7*A*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+31*B*a^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+81*A*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+185*B*a^3*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+53*A*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+125*B*a^3*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-65*A*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-169*B*a^3*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+13*A*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+21*B*a^3*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-33*A*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+19*A*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-73*B*a^3*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+75*B*a^3*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+5*A*a^3*sqrt(c)*c^3+13*B*a^3*sqrt(c)*c^3)/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/4*(15*A*a^3+55*B*a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))

```

maple [B] time = 1.99, size = 434, normalized size = 1.93

$$a^3 \left(\sin(fx + e) \left(-90A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}} \right) c^2 + 48A\sqrt{c+c\sin(fx+e)} c^{\frac{3}{2}} - 330B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c\sin(fx+e)}}{\sqrt{2}} \right) c^{\frac{3}{2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x)
```

```
[Out] -1/12/c^(9/2)*a^3*(sin(f*x+e)*(-90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+48*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+240*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+16*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))+(-45*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+24*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)-165*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2+120*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+8*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))*cos(f*x+e)^2+90*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-132*A*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+54*A*(c+c*sin(f*x+e))^(3/2)*c^(1/2)+330*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^2-420*B*(c+c*sin(f*x+e))^(1/2)*c^(3/2)+86*B*(c+c*sin(f*x+e))^(3/2)*c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^3}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2), x)

[Out] Timed out

$$3.105 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=217

$$-\frac{5a^3(A+13B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2} c^{7/2} f} + \frac{a^3 c^3 (A+B) \cos^7(e+fx)}{6f(c-c \sin(e+fx))^{13/2}} + \frac{5a^3(A+13B) \cos(e+fx)}{16c^3 f \sqrt{c-c \sin(e+fx)}} - \frac{a^3 c(A+13B) \cos(e+fx)}{24f(c-c \sin(e+fx))^{7/2}}$$

[Out] 1/6*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(13/2)-1/24*a^3*(A+13*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(9/2)+5/48*a^3*(A+13*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(5/2)-5/16*a^3*(A+13*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(7/2)/f*2^(1/2)+5/16*a^3*(A+13*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.55, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2680, 2679, 2649, 206}

$$\frac{a^3 c^3 (A+B) \cos^7(e+fx)}{6f(c-c \sin(e+fx))^{13/2}} + \frac{5a^3(A+13B) \cos(e+fx)}{16c^3 f \sqrt{c-c \sin(e+fx)}} - \frac{5a^3(A+13B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2} c^{7/2} f} - \frac{a^3 c(A+13B) \cos(e+fx)}{24f(c-c \sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] (-5*a^3*(A + 13*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(8*Sqrt[2]*c^(7/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(6*f*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(A + 13*B)*c*Cos[e + f*x]^5)/(24*f*(c - c*Sin[e + f*x])^(9/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x]^3)/(48*c*f*(c - c*Sin[e + f*x])^(5/2)) + (5*a^3*(A + 13*B)*Cos[e + f*x])/(16*c^3*f*Sqrt[c - c*Sin[e + f*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2679

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Simp[(g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p-1)*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+p)), x] + Dist[(g^2*(p-1))/(a*(m+p)), Int[(g*Cos[e + f*x])^(p-2)*(a + b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && (GtQ[m, -2] || EqQ[2*m + p + 1, 0] || (EqQ[m, -2] && IntegerQ[p])) && NeQ[m + p, 0] && IntegerQ[2*m, 2*p]


```
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p + 1)), Int[(g*cos[e + f*x])^(p - 2)*(a + b*sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx &= (a^3 c^3) \int \frac{\cos^6(e + fx) (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{13/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}} - \frac{1}{12} (a^3 (A + 13B) c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{13/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}} - \frac{a^3 (A + 13B) c \cos^5(e + fx)}{24f(c - c \sin(e + fx))^{9/2}} + \frac{1}{48} \int \frac{\cos^4(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}} - \frac{a^3 (A + 13B) c \cos^5(e + fx)}{24f(c - c \sin(e + fx))^{9/2}} + \frac{5a^3}{48} \int \frac{\cos^2(e + fx)}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}} - \frac{a^3 (A + 13B) c \cos^5(e + fx)}{24f(c - c \sin(e + fx))^{9/2}} + \frac{5a^3}{48} \int \frac{1}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}} - \frac{a^3 (A + 13B) c \cos^5(e + fx)}{24f(c - c \sin(e + fx))^{9/2}} + \frac{5a^3}{48} \int \frac{1}{(c - c \sin(e + fx))^{9/2}} dx \\
 &= -\frac{5a^3 (A + 13B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{8\sqrt{2} c^{7/2} f} + \frac{a^3 (A + B) c^3 \cos^7(e + fx)}{6f(c - c \sin(e + fx))^{13/2}}
 \end{aligned}$$

Mathematica [C] time = 3.09, size = 422, normalized size = 1.94

$$\frac{a^3 (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(64(A + B) \sin\left(\frac{1}{2}(e + fx)\right) + 3(11A + 47B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{8\sqrt{2} c^{7/2} f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2),x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(32*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]) - 4*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 + 3*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5 + (15 + 15*I)*(-1)^(1/4)*(A + 13*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 48*B*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 64*(A + B)*Sin[(e + f*x)/2] - 8*(13*A + 25*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2] + 6*(11*A + 47*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[(e + f*x)/2] + 48*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)/(24*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(7/2))
```

fricas [B] time = 0.47, size = 554, normalized size = 2.55

$$15\sqrt{2}\left((A+13B)a^3\cos(fx+e)^4 - 3(A+13B)a^3\cos(fx+e)^3 - 8(A+13B)a^3\cos(fx+e)^2 + 4(A+13B)a^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/96*(15*sqrt(2))*((A + 13*B)*a^3*cos(f*x + e)^4 - 3*(A + 13*B)*a^3*cos(f*x + e)^3 - 8*(A + 13*B)*a^3*cos(f*x + e)^2 + 4*(A + 13*B)*a^3*cos(f*x + e) + 8*(A + 13*B)*a^3 + ((A + 13*B)*a^3*cos(f*x + e)^3 + 4*(A + 13*B)*a^3*cos(f*x + e)^2 - 4*(A + 13*B)*a^3*cos(f*x + e) - 8*(A + 13*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(48*B*a^3*cos(f*x + e)^4 + 3*(11*A + 95*B)*a^3*cos(f*x + e)^3 + (19*A - 137*B)*a^3*cos(f*x + e)^2 - 2*(23*A + 203*B)*a^3*cos(f*x + e) - 32*(A + B)*a^3 - (48*B*a^3*cos(f*x + e)^3 - 3*(11*A + 79*B)*a^3*cos(f*x + e)^2 - 2*(7*A + 187*B)*a^3*cos(f*x + e) + 32*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^4*f*cos(f*x + e)^4 - 3*c^4*f*cos(f*x + e)^3 - 8*c^4*f*cos(f*x + e)^2 + 4*c^4*f*cos(f*x + e) + 8*c^4*f + (c^4*f*cos(f*x + e)^3 + 4*c^4*f*cos(f*x + e)^2 - 4*c^4*f*cos(f*x + e) - 8*c^4*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
```

ion of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep-1)]Evaluation time: 2.23Not invertible Error: Bad Argument Value

maple [B] time = 1.98, size = 524, normalized size = 2.41

$$a^3 \left(15A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) (\sin^3(fx+e)) c^3 + 195B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) (\sin^3(fx+e)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)

[Out] 1/48/c^(13/2)*a^3*(15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3+195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^3*c^3-45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-96*B*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3-585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)^2*c^3+66*A*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+45*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3+282*B*(c*(1+sin(f*x+e)))^(5/2)*c^(1/2)+288*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)*sin(f*x+e)^2+585*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*sin(f*x+e)*c^3-160*A*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-15*A*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3-928*B*(c*(1+sin(f*x+e)))^(3/2)*c^(3/2)-288*B*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)*sin(f*x+e)-195*B*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*c^3+120*A*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)+888*B*(c*(1+sin(f*x+e)))^(1/2)*c^(5/2)*(c*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)^2/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(a \sin(fx+e) + a)^3}{(-c \sin(fx+e) + c)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(7/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

$$3.106 \quad \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx$$

Optimal. Leaf size=217

$$-\frac{5a^3(A - 15B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{128\sqrt{2} c^{9/2} f} + \frac{a^3 c^3 (A + B) \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{5a^3(A - 15B) \cos(e + fx)}{128c^3 f(c - c \sin(e + fx))^{3/2}} + \frac{a^3 c(A - 15B)}{48f(c - c \sin(e + fx))^{3/2}}$$

[Out] $1/8*a^3*(A+B)*c^3*\cos(f*x+e)^7/f/(c-c*\sin(f*x+e))^{(15/2)}+1/48*a^3*(A-15*B)*c*\cos(f*x+e)^5/f/(c-c*\sin(f*x+e))^{(11/2)}-5/192*a^3*(A-15*B)*\cos(f*x+e)^3/c/f/(c-c*\sin(f*x+e))^{(7/2)}+5/128*a^3*(A-15*B)*\cos(f*x+e)/c^3/f/(c-c*\sin(f*x+e))^{(3/2)}-5/256*a^3*(A-15*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)/(c-c*\sin(f*x+e))^{(1/2)})/c^{(9/2)}/f*2^{(1/2)}$

Rubi [A] time = 0.56, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2859, 2680, 2649, 206}

$$\frac{a^3 c^3 (A + B) \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{5a^3(A - 15B) \cos(e + fx)}{128c^3 f(c - c \sin(e + fx))^{3/2}} - \frac{5a^3(A - 15B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{128\sqrt{2} c^{9/2} f} + \frac{a^3 c(A - 15B)}{48f(c - c \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a \sin[e + f*x])^3 * (A + B \sin[e + f*x]) / (c - c \sin[e + f*x])^{(9/2)}, x]$

[Out] $(-5*a^3*(A - 15*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Cos}[e + f*x]) / (\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c - c*\sin[e + f*x]])]) / (128*\operatorname{Sqrt}[2]*c^{(9/2)}*f) + (a^3*(A + B)*c^3*\operatorname{Cos}[e + f*x]^7) / (8*f*(c - c*\sin[e + f*x])^{(15/2)}) + (a^3*(A - 15*B)*c*\operatorname{Cos}[e + f*x]^5) / (48*f*(c - c*\sin[e + f*x])^{(11/2)}) - (5*a^3*(A - 15*B)*\operatorname{Cos}[e + f*x]^3) / (192*c*f*(c - c*\sin[e + f*x])^{(7/2)}) + (5*a^3*(A - 15*B)*\operatorname{Cos}[e + f*x]) / (128*c^3*f*(c - c*\sin[e + f*x])^{(3/2)})$

Rule 206

$\operatorname{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a + b*x)*\sin[(c + d*x)]]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x]) / \operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2680

$\operatorname{Int}[(\cos[(e + f*x)]*(g + h*x))^p * (a + b*\sin[(e + f*x)]*(g + h*x))^m], x_Symbol] \rightarrow \operatorname{Simp}[(2*g*(g*\operatorname{Cos}[e + f*x])^{(p-1)}*(a + b*\sin[e + f*x])^{(m+1)}) / (b*f*(2*m + p + 1)), x] + \operatorname{Dist}[(g^2*(p-1)) / (b^2*(2*m + p + 1)), \operatorname{Int}[(g*\operatorname{Cos}[e + f*x])^{(p-2)}*(a + b*\sin[e + f*x])^{(m+2)}, x], x] /;$ FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] && NeQ[2*m + p + 1, 0] && !LtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2859

$\operatorname{Int}[(\cos[(e + f*x)]*(g + h*x))^p * (a + b*\sin[(e + f*x)]*(g + h*x))^m * (c + d*\sin[(e + f*x)]*(g + h*x))], x_Symbol] \rightarrow \operatorname{Simp}[(b*c$

```
- a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^3(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx = (a^3 c^3) \int \frac{\cos^6(e + fx)(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{15/2}} dx$$

$$= \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{1}{16} (a^3(A - 15B)c^2) \int \frac{\cos^6(e + fx)}{(c - c \sin(e + fx))^{11/2}} dx$$

$$= \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{a^3(A - 15B)c \cos^5(e + fx)}{48f(c - c \sin(e + fx))^{11/2}} - \frac{1}{96} (5a^3(A - 15B)c^2) \int \frac{\cos^5(e + fx)}{(c - c \sin(e + fx))^{7/2}} dx$$

$$= \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{a^3(A - 15B)c \cos^5(e + fx)}{48f(c - c \sin(e + fx))^{11/2}} - \frac{5a^3(A - 15B)c^2 \cos^3(e + fx)}{192cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{a^3(A - 15B)c \cos^5(e + fx)}{48f(c - c \sin(e + fx))^{11/2}} - \frac{5a^3(A - 15B)c^2 \cos^3(e + fx)}{192cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}} + \frac{a^3(A - 15B)c \cos^5(e + fx)}{48f(c - c \sin(e + fx))^{11/2}} - \frac{5a^3(A - 15B)c^2 \cos^3(e + fx)}{192cf(c - c \sin(e + fx))^{7/2}}$$

$$= -\frac{5a^3(A - 15B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{128\sqrt{2} c^{9/2} f} + \frac{a^3(A + B)c^3 \cos^7(e + fx)}{8f(c - c \sin(e + fx))^{15/2}}$$

Mathematica [C] time = 4.38, size = 355, normalized size = 1.64

$$a^3(\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left((120 + 120i)\sqrt[4]{-1} (A - 15B) \tan^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt[4]{-1} \left(\tan\left(\frac{1}{2}(e + fx)\right) + 1\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*(1765*A*Cos[(e + f*x)/2] + 405*B*Cos[(e + f*x)/2] - 895*A*Cos[(3*(e + f*x))/2] - 2703*B*Cos[(3*(e + f*x))/2] - 397*A*Cos[(5*(e + f*x))/2] + 579*B*Cos[(5*(e + f*x))/2] + 15*A*Cos[(7*(e + f*x))/2] + 543*B*Cos[(7*(e + f*x))/2] + (120 + 120*I)*(-1)^(1/4)*(A - 15*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8 + 1765*A*Sin[(e + f*x)/2] + 405*B*Sin[(e + f*x)/2] + 895*A*Sin[(3*(e + f*x))/2] + 2703*B*Sin[(3*(e + f*x))/2] - 397*A*Sin[(5*(e + f*x))/2] - 579*B*Sin[(5*(e + f*x))/2] - 15*A*Sin[(7*(e + f*x))/2] - 543*B*Sin[(7*(e + f*x))/2])
```



```
t(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8+
9817*B*a^3*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-8989*A*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-10317*B*a^3*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+36189*A*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-32115*B*a^3*sqrt(c)*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-6547*A*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+71325*B*a^3*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-17777*A*a^3*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-7521*B*a^3*sqrt(c)*c^5*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+5583*A*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-35361*B*a^3*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+193*A*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-5351*A*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2127*B*a^3*c^7*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+10377*B*a^3*sqrt(c)*c^6*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-61*A*a^3*sqrt(c)*c^7+147*B*a^3*sqrt(c)*c^7/c^4/(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c^8/sign(tan((f*x+exp(1))/2)-1)+1/256*(-5*A*a^3+75*B*a^3)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/c^4/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))
```

maple [B] time = 1.88, size = 432, normalized size = 1.99

$$a^3 \left(60 \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{2} c^4 (A-15B) \sin(fx+e) (\cos^2(fx+e)) - 120 \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)
```

```
[Out] 1/768*a^3*(60*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)*cos(f*x+e)^2-120*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*sin(f*x+e)+15*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)^4-120*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*2^(1/2)*c^4*(A-15*B)*cos(f*x+e)^2-30*A*(c+c*sin(f*x+e))^(7/2)*c^(1/2)-292*A*(c+c*sin(f*x+e))^(5/2)*c^(3/2)+440*A*(c+c*sin(f*x+e))^(3/2)*c^(5/2)-240*A*(c+c*sin(f*x+e))^(1/2)*c^(7/2)-1086*B*(c+c*sin(f*x+e))^(7/2)*c^(1/2)+4380*B*(c+c*sin(f*x+e))^(5/2)*c^(3/2)-6600*B*(c+c*sin(f*x+e))^(3/2)*c^(5/2)+3600*B*(c+c*sin(f*x+e))^(1/2)*c^(7/2)+120*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4-1800*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^4*(c*(1+sin(f*x+e)))^(1/2)/c^(17/2)/(sin(f*x+e)-1)^3/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(a \sin(fx+e) + a)^3}{(-c \sin(fx+e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")
```


[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^3}{(c - c \sin(e + f x))^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2), x)

[Out] Timed out

$$3.107 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$$

Optimal. Leaf size=266

$$\frac{a^3(3A-17B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{512\sqrt{2} c^{11/2} f} - \frac{a^3(3A-17B) \cos(e+fx)}{512c^4 f(c-c \sin(e+fx))^{3/2}} + \frac{a^3 c^3(A+B) \cos^7(e+fx)}{10f(c-c \sin(e+fx))^{17/2}} + \frac{a^3(3A-17B)}{128c^3 f(c-c \sin(e+fx))^{11/2}}$$

[Out] 1/10*a^3*(A+B)*c^3*cos(f*x+e)^7/f/(c-c*sin(f*x+e))^(17/2)+1/80*a^3*(3*A-17*B)*c*cos(f*x+e)^5/f/(c-c*sin(f*x+e))^(13/2)-1/96*a^3*(3*A-17*B)*cos(f*x+e)^3/c/f/(c-c*sin(f*x+e))^(9/2)+1/128*a^3*(3*A-17*B)*cos(f*x+e)/c^3/f/(c-c*sin(f*x+e))^(5/2)-1/512*a^3*(3*A-17*B)*cos(f*x+e)/c^4/f/(c-c*sin(f*x+e))^(3/2)-1/1024*a^3*(3*A-17*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/c^(11/2)/f*2^(1/2)

Rubi [A] time = 0.59, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2680, 2650, 2649, 206}

$$\frac{a^3 c^3(A+B) \cos^7(e+fx)}{10f(c-c \sin(e+fx))^{17/2}} - \frac{a^3(3A-17B) \cos(e+fx)}{512c^4 f(c-c \sin(e+fx))^{3/2}} + \frac{a^3(3A-17B) \cos(e+fx)}{128c^3 f(c-c \sin(e+fx))^{5/2}} - \frac{a^3(3A-17B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{512\sqrt{2} c^{11/2} f}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] -(a^3*(3*A - 17*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(512*Sqrt[2]*c^(11/2)*f) + (a^3*(A + B)*c^3*Cos[e + f*x]^7)/(10*f*(c - c*Sin[e + f*x])^(17/2)) + (a^3*(3*A - 17*B)*c*Cos[e + f*x]^5)/(80*f*(c - c*Sin[e + f*x])^(13/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x]^3)/(96*c*f*(c - c*Sin[e + f*x])^(9/2)) + (a^3*(3*A - 17*B)*Cos[e + f*x])/(128*c^3*f*(c - c*Sin[e + f*x])^(5/2)) - (a^3*(3*A - 17*B)*Cos[e + f*x])/(512*c^4*f*(c - c*Sin[e + f*x])^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2680

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^m), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2),x]
```

```
[Out] ((1/512 + I/512)*(-1)^(1/4)*(3*A - 17*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] + Sin[(e + f*x)/4])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^11*(a + a*Sin[e + f*x])^3)/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2)) + ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a + a*Sin[e + f*x])^3*(56370*A*Cos[(e + f*x)/2] + 38970*B*Cos[(e + f*x)/2] - 31140*A*Cos[(3*(e + f*x))/2] - 38580*B*Cos[(3*(e + f*x))/2] - 10404*A*Cos[(5*(e + f*x))/2] - 12724*B*Cos[(5*(e + f*x))/2] + 435*A*Cos[(7*(e + f*x))/2] + 7775*B*Cos[(7*(e + f*x))/2] - 45*A*Cos[(9*(e + f*x))/2] + 255*B*Cos[(9*(e + f*x))/2] + 56370*A*Sin[(e + f*x)/2] + 38970*B*Sin[(e + f*x)/2] + 31140*A*Sin[(3*(e + f*x))/2] + 38580*B*Sin[(3*(e + f*x))/2] - 10404*A*Sin[(5*(e + f*x))/2] - 12724*B*Sin[(5*(e + f*x))/2] - 435*A*Sin[(7*(e + f*x))/2] - 7775*B*Sin[(7*(e + f*x))/2] - 45*A*Sin[(9*(e + f*x))/2] + 255*B*Sin[(9*(e + f*x))/2]))/(122880*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*(c - c*Sin[e + f*x])^(11/2))
```

fricas [B] time = 0.49, size = 760, normalized size = 2.86

$$15\sqrt{2}\left((3A - 17B)a^3 \cos(fx + e)^6 - 5(3A - 17B)a^3 \cos(fx + e)^5 - 18(3A - 17B)a^3 \cos(fx + e)^4 + 20(3A - 17B)a^3 \cos(fx + e)^3 + 48(3A - 17B)a^3 \cos(fx + e)^2 - 16(3A - 17B)a^3 \cos(fx + e) - 32(3A - 17B)a^3 + ((3A - 17B)a^3 \cos(fx + e)^5 + 6(3A - 17B)a^3 \cos(fx + e)^4 - 12(3A - 17B)a^3 \cos(fx + e)^3 - 32(3A - 17B)a^3 \cos(fx + e)^2 + 16(3A - 17B)a^3 \cos(fx + e) + 32(3A - 17B)a^3) \sin(fx + e) \sqrt{c} \log(-c \cos(fx + e)^2 + 2\sqrt{2} \sqrt{-c \sin(fx + e) + c}) \sqrt{c} (\cos(fx + e) + \sin(fx + e) + 1) + 3c \cos(fx + e) + (c \cos(fx + e) - 2c) \sin(fx + e) + 2c) / (\cos(fx + e)^2 + (\cos(fx + e) + 2) \sin(fx + e) - \cos(fx + e) - 2) - 4(15(3A - 17B)a^3 \cos(fx + e)^5 - 5(39A + 803B)a^3 \cos(fx + e)^4 + 4(609A + 389B)a^3 \cos(fx + e)^3 + 12(449A + 869B)a^3 \cos(fx + e)^2 - 24(143A + 43B)a^3 \cos(fx + e) - 6144(A + B)a^3 + (15(3A - 17B)a^3 \cos(fx + e)^4 + 80(3A + 47B)a^3 \cos(fx + e)^3 + 12(223A + 443B)a^3 \cos(fx + e)^2 - 24(113A + 213B)a^3 \cos(fx + e) - 6144(A + B)a^3) \sin(fx + e) \sqrt{-c \sin(fx + e) + c} / (c^6 f \cos(fx + e)^6 - 5c^6 f \cos(fx + e)^5 - 18c^6 f \cos(fx + e)^4 + 20c^6 f \cos(fx + e)^3 + 48c^6 f \cos(fx + e)^2 - 16c^6 f \cos(fx + e) - 32c^6 f + (c^6 f \cos(fx + e)^5 + 6c^6 f \cos(fx + e)^4 - 12c^6 f \cos(fx + e)^3 - 32c^6 f \cos(fx + e)^2 + 16c^6 f \cos(fx + e) + 32c^6 f) \sin(fx + e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="fricas")
```

```
[Out] -1/30720*(15*sqrt(2)*((3*A - 17*B)*a^3*cos(f*x + e)^6 - 5*(3*A - 17*B)*a^3*cos(f*x + e)^5 - 18*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 20*(3*A - 17*B)*a^3*cos(f*x + e)^3 + 48*(3*A - 17*B)*a^3*cos(f*x + e)^2 - 16*(3*A - 17*B)*a^3*cos(f*x + e) - 32*(3*A - 17*B)*a^3 + ((3*A - 17*B)*a^3*cos(f*x + e)^5 + 6*(3*A - 17*B)*a^3*cos(f*x + e)^4 - 12*(3*A - 17*B)*a^3*cos(f*x + e)^3 - 32*(3*A - 17*B)*a^3*cos(f*x + e)^2 + 16*(3*A - 17*B)*a^3*cos(f*x + e) + 32*(3*A - 17*B)*a^3)*sin(f*x + e))*sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c))*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(15*(3*A - 17*B)*a^3*cos(f*x + e)^5 - 5*(39*A + 803*B)*a^3*cos(f*x + e)^4 + 4*(609*A + 389*B)*a^3*cos(f*x + e)^3 + 12*(449*A + 869*B)*a^3*cos(f*x + e)^2 - 24*(143*A + 43*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3 + (15*(3*A - 17*B)*a^3*cos(f*x + e)^4 + 80*(3*A + 47*B)*a^3*cos(f*x + e)^3 + 12*(223*A + 443*B)*a^3*cos(f*x + e)^2 - 24*(113*A + 213*B)*a^3*cos(f*x + e) - 6144*(A + B)*a^3)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c))/(c^6*f*cos(f*x + e)^6 - 5*c^6*f*cos(f*x + e)^5 - 18*c^6*f*cos(f*x + e)^4 + 20*c^6*f*cos(f*x + e)^3 + 48*c^6*f*cos(f*x + e)^2 - 16*c^6*f*cos(f*x + e) - 32*c^6*f + (c^6*f*cos(f*x + e)^5 + 6*c^6*f*cos(f*x + e)^4 - 12*c^6*f*cos(f*x + e)^3 - 32*c^6*f*cos(f*x + e)^2 + 16*c^6*f*cos(f*x + e) + 32*c^6*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="giac")
```

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: $(4\pi/x/2) > (-4\pi/x/2)$ Unable to check sign: $(4\pi/x/2) > (-4\pi/x/2)$

$$\frac{2}{f^2} \left(\frac{1}{15360} (-15405 A^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{19} + 255 B A^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{19} - 46935 A^3 \sqrt{c} (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{18} + 35565 B A^3 \sqrt{c} (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{18} - 452265 A^3 c (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{17} - 43245 B A^3 c (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{17} - 812175 A^3 \sqrt{c} c (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{16} + 583125 B A^3 \sqrt{c} c (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{16} - 1157268 A^3 c^2 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{15} + 270492 B A^3 c^2 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{15} + 999780 A^3 \sqrt{c} c^2 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{14} - 320140 B A^3 \sqrt{c} c^2 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{14} + 2949420 A^3 c^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{13} - 308900 B A^3 c^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{13} + 775380 A^3 \sqrt{c} c^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{12} - 768860 B A^3 \sqrt{c} c^3 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{12} - 3937830 A^3 c^4 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{11} + 820610 B A^3 c^4 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{11} - 1793586 A^3 \sqrt{c} c^4 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{10} + 507334 B A^3 \sqrt{c} c^4 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^{10} + 4240770 A^3 c^5 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^9 - 1492790 B A^3 c^5 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^9 + 850830 A^3 \sqrt{c} c^5 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^8 + 308870 B A^3 \sqrt{c} c^5 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^8 - 3213060 A^3 c^6 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^7 + 1434220 B A^3 c^6 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^7 + 712020 A^3 \sqrt{c} c^6 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^6 - 164060 B A^3 \sqrt{c} c^6 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^6 + 805596 A^3 c^7 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^5 - 284724 B A^3 c^7 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^5 - 1056540 A^3 \sqrt{c} c^7 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^4 - 54540 B A^3 \sqrt{c} c^7 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^4 + 289515 A^3 c^8 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^3 - 253065 B A^3 c^8 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^3 + 3615 A^3 c^9 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^2 - 114735 A^3 \sqrt{c} c^8 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^2 - 5125 B A^3 c^9 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c}) + 10165 B A^3 \sqrt{c} c^8 (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^2 - 951 A^3 \sqrt{c} c^9 + 269 B A^3 \sqrt{c} c^9 / c^5 / (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c})^2 - 2 \sqrt{c} (-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c}) + c)^{10} / \text{sign}(\tan((fx+\exp(1))/2) - 1) + 1/1024 (-3 A^3 + 17 B A^3) \text{atanh}((-\sqrt{c} \tan((fx+\exp(1))/2) + \sqrt{c \tan((fx+\exp(1))/2)^2 + c}) / \sqrt{2} / \sqrt{-c}) / \sqrt{2} / c^5 / \sqrt{-c} / \text{sign}(\tan((fx+\exp(1))/2) - 1))$$

maple [B] time = 2.05, size = 526, normalized size = 1.98

$$a^3 \left(15\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)} \sqrt{2}}{2\sqrt{c}} \right) c^6 (3A - 17B) \sin(fx + e) (\cos^4(fx + e)) - 180\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c+c \sin(fx+e)}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)

[Out] 1/15360*a^3*(15*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin(f*x+e)*cos(f*x+e)^4-180*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2*sin(f*x+e)+240*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*sin(f*x+e)-75*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^4+300*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(3*A-17*B)*cos(f*x+e)^2-90*A*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+840*A*(c+c*sin(f*x+e))^(7/2)*c^(5/2)+3072*A*(c+c*sin(f*x+e))^(5/2)*c^(7/2)-3360*A*(c+c*sin(f*x+e))^(3/2)*c^(9/2)+1440*A*(c+c*sin(f*x+e))^(1/2)*c^(11/2)+510*B*(c+c*sin(f*x+e))^(9/2)*c^(3/2)+5480*B*(c+c*sin(f*x+e))^(7/2)*c^(5/2)-17408*B*(c+c*sin(f*x+e))^(5/2)*c^(7/2)+19040*B*(c+c*sin(f*x+e))^(3/2)*c^(9/2)-8160*B*(c+c*sin(f*x+e))^(1/2)*c^(11/2)-720*A*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6+4080*B*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c^6*(c*(1+sin(f*x+e)))^(1/2)/c^(23/2)/(sin(f*x+e)-1)^4/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^3}{(-c \sin(fx + e) + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^3/(-c*sin(f*x + e) + c)^(11/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^3}{(c - c \sin(e + fx))^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c - c*sin(e + f*x))^(11/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)

[Out] Timed out

$$3.108 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=200

$$\frac{128c^4(7A-9B) \cos(e+fx)}{35af\sqrt{c-c \sin(e+fx)}} - \frac{32c^3(7A-9B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{35af} - \frac{12c^2(7A-9B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{35af}$$

[Out] $-12/35*(7*A-9*B)*c^2*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a/f-1/7*(7*A-9*B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/a/f-(A-B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(9/2)}/a/c/f-128/35*(7*A-9*B)*c^4*\cos(f*x+e)/a/f/(c-c*\sin(f*x+e))^{(1/2)}-32/35*(7*A-9*B)*c^3*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f$

Rubi [A] time = 0.38, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2647, 2646}

$$\frac{12c^2(7A-9B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{35af} - \frac{32c^3(7A-9B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{35af} - \frac{128c^4(7A-9B) \cos(e+fx)}{35af\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]), x]

[Out] $(-128*(7*A - 9*B)*c^4*\cos[e + f*x])/(35*a*f*\text{Sqrt}[c - c*\sin[e + f*x]]) - (32*(7*A - 9*B)*c^3*\cos[e + f*x]*\text{Sqrt}[c - c*\sin[e + f*x]])/(35*a*f) - (12*(7*A - 9*B)*c^2*\cos[e + f*x]*(c - c*\sin[e + f*x])^{(3/2)})/(35*a*f) - ((7*A - 9*B)*c*\cos[e + f*x]*(c - c*\sin[e + f*x])^{(5/2)})/(7*a*f) - ((A - B)*\text{Sec}[e + f*x]*(c - c*\sin[e + f*x])^{(9/2)})/(a*c*f)$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2647

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(a*(2*n-1))/n, Int[(a + b*Sin[c + d*x])^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*Cos[c + a*d]*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p+1)), x] + Dist[(b*(a*d*m + b*c*(m+p+1)))/(a*g^2*(p+1)), Int[(g*Cos[e + f*x])^(p+2)*(a + b*Sin[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{a + a \sin(e + fx)} dx &= \frac{\int \sec^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2} dx}{ac} \\ &= -\frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{9/2}}{acf} - \frac{(7A - 9B) \int (c - c \sin(e + fx))^{7/2} dx}{2ac} \\ &= -\frac{(7A - 9B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{7af} - \frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{9/2}}{2ac} \\ &= -\frac{12(7A - 9B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{35af} - \frac{(7A - 9B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{7af} \\ &= -\frac{32(7A - 9B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{35af} - \frac{12(7A - 9B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{35af} \\ &= -\frac{128(7A - 9B)c^4 \cos(e + fx)}{35af\sqrt{c - c \sin(e + fx)}} - \frac{32(7A - 9B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{35af} \end{aligned}$$

Mathematica [A] time = 5.59, size = 157, normalized size = 0.78

$$\frac{c^3 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (196(A - 2B) \cos(2(e + fx)) + 2450A \sin(e + fx) - 140af(\sin(e + fx) + 1) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right))}{140af(\sin(e + fx) + 1) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x]), x]

[Out] -1/140*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4900*A - 6125*B + 196*(A - 2*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] + 2450*A*Sin[e + f*x] - 3430*B*Sin[e + f*x] - 14*A*Sin[3*(e + f*x)] + 58*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))

fricas [A] time = 0.44, size = 115, normalized size = 0.58

$$\frac{2 \left(5 B c^3 \cos(fx + e)^4 + (49 A - 103 B) c^3 \cos(fx + e)^2 + 4 (147 A - 179 B) c^3 - \left((7 A - 29 B) c^3 \cos(fx + e)^2 - 4 (147 A - 179 B) c^3 \right) \right)}{35 a f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)), x, algorithm="fricas")

[Out] -2/35*(5*B*c^3*cos(f*x + e)^4 + (49*A - 103*B)*c^3*cos(f*x + e)^2 + 4*(147*A - 179*B)*c^3 - ((7*A - 29*B)*c^3*cos(f*x + e)^2 - 4*(77*A - 109*B)*c^3)*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorith="giac")

[Out] Timed out

maple [A] time = 1.17, size = 111, normalized size = 0.56

$$\frac{2c^4 \left(\sin(fx + e) - 1 \right) \left((-7A + 29B) \sin(fx + e) \left(\cos^2(fx + e) \right) + (308A - 436B) \sin(fx + e) + 5B \left(\cos^4(fx + e) \right) \right)}{35a \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x)

[Out] 2/35*c^4/a*(sin(f*x+e)-1)*((-7*A+29*B)*sin(f*x+e)*cos(f*x+e)^2+(308*A-436*B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(49*A-103*B)*cos(f*x+e)^2+588*A-716*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.50, size = 478, normalized size = 2.39

$$2 \frac{7 \left(91c^{\frac{7}{2}} + \frac{86c^{\frac{7}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{336c^{\frac{7}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{266c^{\frac{7}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{490c^{\frac{7}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{266c^{\frac{7}{2}} \sin^5(fx+e)}{(\cos(fx+e)+1)^5} + \frac{336c^{\frac{7}{2}} \sin^6(fx+e)}{(\cos(fx+e)+1)^6} + \frac{86c^{\frac{7}{2}} \sin^7(fx+e)}{(\cos(fx+e)+1)^7} + \frac{91c^{\frac{7}{2}} \sin^8(fx+e)}{(\cos(fx+e)+1)^8} \right)}{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e)),x, algorith="maxima")

[Out] 2/35*(7*(91*c^(7/2) + 86*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 336*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 266*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 490*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 266*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 336*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 86*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 91*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*A/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(407*c^(7/2) + 407*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1442*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1337*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 2030*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 1337*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 1442*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 407*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 407*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)*B/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2}}{a + a \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x)),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e)),x)

[Out] Timed out

$$3.109 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=159

$$\frac{32c^3(5A-7B) \cos(e+fx)}{15af\sqrt{c-c \sin(e+fx)}} - \frac{8c^2(5A-7B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{15af} - \frac{c(5A-7B) \cos(e+fx)(c-c \sin(e+fx))^{5/2}}{5af}$$

[Out] $-1/5*(5*A-7*B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a/f-(A-B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/a/c/f-32/15*(5*A-7*B)*c^3*\cos(f*x+e)/a/f/(c-c*\sin(f*x+e))^{(1/2)}-8/15*(5*A-7*B)*c^2*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f$

Rubi [A] time = 0.35, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2647, 2646}

$$\frac{8c^2(5A-7B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{15af} - \frac{32c^3(5A-7B) \cos(e+fx)}{15af\sqrt{c-c \sin(e+fx)}} - \frac{c(5A-7B) \cos(e+fx)(c-c \sin(e+fx))^{5/2}}{5af}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]), x]

[Out] $(-32*(5*A - 7*B)*c^3*\cos[e + f*x])/(15*a*f*\sqrt{c - c*\sin[e + f*x]}) - (8*(5*A - 7*B)*c^2*\cos[e + f*x]*\sqrt{c - c*\sin[e + f*x]})/(15*a*f) - ((5*A - 7*B)*c*\cos[e + f*x]*(c - c*\sin[e + f*x])^{(3/2)})/(5*a*f) - ((A - B)*\sec[e + f*x]*(c - c*\sin[e + f*x])^{(7/2)})/(a*c*f)$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2647

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(a*(2*n-1))/n, Int[(a + b*Sin[c + d*x])^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p+1)), x] + Dist[(b*(a*d*m + b*c*(m+p+1)))/(a*g^2*(p+1)), Int[(g*Cos[e + f*x])^(p+2)*(a + b*Sin[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{a + a \sin(e + fx)} dx &= \frac{\int \sec^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx}{ac} \\
&= -\frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{7/2}}{acf} - \frac{(5A - 7B) \int (c - c \sin(e + fx))^{5/2} dx}{2ac} \\
&= -\frac{(5A - 7B)c \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{5af} - \frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{5/2}}{2ac} \\
&= -\frac{8(5A - 7B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{15af} - \frac{(5A - 7B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2ac} \\
&= -\frac{32(5A - 7B)c^3 \cos(e + fx)}{15af\sqrt{c - c \sin(e + fx)}} - \frac{8(5A - 7B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{15af}
\end{aligned}$$

Mathematica [A] time = 1.76, size = 134, normalized size = 0.84

$$\frac{c^2 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (25(8A - 13B) \sin(e + fx) + 2(5A - 16B) \cos(2(e + fx)))}{30af(\sin(e + fx) + 1) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x]),x]

[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(450*A - 600*B + 2*(5*A - 16*B)*Cos[2*(e + f*x)] + 25*(8*A - 13*B)*Sin[e + f*x] + 3*B*Sin[3*(e + f*x)]))/(a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))

fricas [A] time = 0.43, size = 95, normalized size = 0.60

$$\frac{2 \left((5A - 16B)c^2 \cos^2(fx + e) + 2(55A - 71B)c^2 + (3Bc^2 \cos^2(fx + e) + 2(25A - 41B)c^2) \sin(fx + e) \right) \sqrt{-c \sin(fx + e)}}{15af \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorith="fricas")

[Out] -2/15*((5*A - 16*B)*c^2*cos(f*x + e)^2 + 2*(55*A - 71*B)*c^2 + (3*B*c^2*cos(f*x + e)^2 + 2*(25*A - 41*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorith="giac")

[Out] Timed out

maple [A] time = 1.10, size = 95, normalized size = 0.60

$$\frac{2c^3 (\sin(fx + e) - 1) (-3B (\cos^2(fx + e)) \sin(fx + e) + (-50A + 82B) \sin(fx + e) + (-5A + 16B) (\cos^2(fx + e) - 1))}{15a \cos(fx + e) \sqrt{c - c \sin(fx + e)}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)

[Out]
$$-2/15*c^3/a*(\sin(f*x+e)-1)*(-3*B*\cos(f*x+e)^2*\sin(f*x+e)+(-50*A+82*B)*\sin(f*x+e)+(-5*A+16*B)*\cos(f*x+e)^2-110*A+142*B)/\cos(f*x+e)/(c-c*\sin(f*x+e))^(1/2)/f$$

maxima [B] time = 0.46, size = 386, normalized size = 2.43

$$2 \frac{\left(5 \left(23c^{\frac{5}{2}} + \frac{20c^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{65c^{\frac{5}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{40c^{\frac{5}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{65c^{\frac{5}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{20c^{\frac{5}{2}} \sin^5(fx+e)}{(\cos(fx+e)+1)^5} + \frac{23c^{\frac{5}{2}} \sin^6(fx+e)}{(\cos(fx+e)+1)^6} \right) A - 2 \left(79c^{\frac{5}{2}} + \frac{79c^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1} \right)}{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{5}{2}}}$$

15 f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out]
$$2/15*(5*(23*c^{(5/2)} + 20*c^{(5/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 65*c^{(5/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 40*c^{(5/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 65*c^{(5/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 20*c^{(5/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 23*c^{(5/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6)*A/((a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(5/2)}) - 2*(79*c^{(5/2)} + 79*c^{(5/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 205*c^{(5/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 170*c^{(5/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 205*c^{(5/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 79*c^{(5/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 79*c^{(5/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6)*B/((a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(5/2}))/f$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2}}{a + a \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x)),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e)),x)

[Out] Timed out

$$3.110 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=118

$$\frac{4c^2(3A-5B) \cos(e+fx)}{3af\sqrt{c-c \sin(e+fx)}} - \frac{c(3A-5B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{3af} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{5/2}}{acf}$$

[Out] -(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a/c/f-4/3*(3*A-5*B)*c^2*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(1/2)-1/3*(3*A-5*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/f

Rubi [A] time = 0.32, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2647, 2646}

$$\frac{4c^2(3A-5B) \cos(e+fx)}{3af\sqrt{c-c \sin(e+fx)}} - \frac{c(3A-5B) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{3af} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{5/2}}{acf}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]), x]

[Out] (-4*(3*A - 5*B)*c^2*Cos[e + f*x])/(3*a*f*Sqrt[c - c*Sin[e + f*x]]) - ((3*A - 5*B)*c*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a*f) - ((A - B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(a*c*f)

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2647

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(a*(2*n-1))/n, Int[(a + b*Sin[c + d*x])^(n-1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n - 1/2, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p+1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p+1)), x] + Dist[(b*(a*d*m + b*c*(m+p+1)))/(a*g^2*(p+1)), Int[(g*Cos[e + f*x])^(p+2)*(a + b*Sin[e + f*x])^(m-1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n-m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{a + a \sin(e + fx)} dx = \frac{\int \sec^2(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx}{ac}$$

$$= -\frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{5/2}}{acf} - \frac{(3A - 5B) \int (c - c \sin(e + fx))^{3/2} dx}{3af}$$

$$= -\frac{(3A - 5B)c \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{3af} - \frac{(A - B) \sec(e + fx)(c - c \sin(e + fx))^{3/2}}{3af}$$

$$= -\frac{4(3A - 5B)c^2 \cos(e + fx)}{3af \sqrt{c - c \sin(e + fx)}} - \frac{(3A - 5B)c \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{3af}$$

Mathematica [A] time = 0.60, size = 113, normalized size = 0.96

$$\frac{c \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((14B - 6A) \sin(e + fx) - 18A + B \cos(2(e + fx)) + 2B \right)}{3af(\sin(e + fx) + 1) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x]),x]

[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-18*A + 27*B + B*Cos[2*(e + f*x)] + (-6*A + 14*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x]))

fricas [A] time = 0.43, size = 67, normalized size = 0.57

$$\frac{2 \left(Bc \cos^2(fx + e) - (3A - 7B)c \sin(fx + e) - (9A - 13B)c \right) \sqrt{-c \sin(fx + e) + c}}{3af \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorith="fricas")

[Out] 2/3*(B*c*cos(f*x + e)^2 - (3*A - 7*B)*c*sin(f*x + e) - (9*A - 13*B)*c)*sqrt(-c*sin(f*x + e) + c)/(a*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorith="giac")

[Out] Timed out

maple [A] time = 1.12, size = 73, normalized size = 0.62

$$\frac{2c^2 \left(\sin(fx + e) - 1 \right) \left(\sin(fx + e) (3A - 7B) - B \left(\cos^2(fx + e) \right) + 9A - 13B \right)}{3a \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)

[Out] $\frac{2}{3} \frac{c^2}{a} \frac{(\sin(fx+e)-1) \cdot (\sin(fx+e) \cdot (3A-7B) - B \cos(fx+e)^2 + 9A - 13B)}{\cos(fx+e) \cdot (c-c \sin(fx+e))^{1/2}} / f$

maxima [B] time = 0.46, size = 294, normalized size = 2.49

$$2 \frac{\left(3 \left(3c^{\frac{3}{2}} + \frac{2c^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{6c^{\frac{3}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{2c^{\frac{3}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{3c^{\frac{3}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} \right) A - 2 \left(7c^{\frac{3}{2}} + \frac{7c^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{12c^{\frac{3}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{7c^{\frac{3}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{7c^{\frac{3}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} \right) B}{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{3}{2}}} - \frac{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{3}{2}}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] $\frac{2}{3} \frac{c^2}{a} \frac{(\sin(fx+e)-1) \cdot (\sin(fx+e) \cdot (3A-7B) - B \cos(fx+e)^2 + 9A - 13B)}{\cos(fx+e) \cdot (c-c \sin(fx+e))^{1/2}} / f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2}}{a + a \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x)),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{Ac \sqrt{-c \sin(e+fx)+c}}{\sin(e+fx)+1} dx + \int \left(-\frac{Ac \sqrt{-c \sin(e+fx)+c} \sin(e+fx)}{\sin(e+fx)+1} \right) dx + \int \frac{Bc \sqrt{-c \sin(e+fx)+c} \sin(e+fx)}{\sin(e+fx)+1} dx + \int \left(-\frac{Bc \sqrt{-c \sin(e+fx)+c} \sin^2(e+fx)}{\sin(e+fx)+1} \right) dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e)),x)

[Out] $\frac{(\text{Integral}(A \cdot c \cdot \sqrt{-c \sin(e + fx) + c}) / (\sin(e + fx) + 1), x) + \text{Integral}(-A \cdot c \cdot \sqrt{-c \sin(e + fx) + c}) \cdot \sin(e + fx) / (\sin(e + fx) + 1), x) + \text{Integral}(B \cdot c \cdot \sqrt{-c \sin(e + fx) + c}) \cdot \sin(e + fx) / (\sin(e + fx) + 1), x) + \text{Integral}(-B \cdot c \cdot \sqrt{-c \sin(e + fx) + c}) \cdot \sin^2(e + fx) / (\sin(e + fx) + 1), x)}{a}$

$$3.111 \quad \int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=73

$$\frac{c(A-3B) \cos(e+fx)}{af\sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{acf}$$

[Out] $-(A-B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a/c/f-(A-3*B)*c*\cos(f*x+e)/a/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2967, 2855, 2646}

$$\frac{c(A-3B) \cos(e+fx)}{af\sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{acf}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x]),x]

[Out] $-(((A - 3*B)*c*\cos[e + f*x])/(a*f*\sqrt{c - c*\sin[e + f*x]})) - ((A - B)*\sec[e + f*x]*(c - c*\sin[e + f*x])^{(3/2)})/(a*c*f)$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A+B \sin(e+fx))\sqrt{c-c \sin(e+fx)}}{a+a \sin(e+fx)} dx &= \frac{\int \sec^2(e+fx)(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2} dx}{ac} \\ &= -\frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{acf} - \frac{(A-3B) \int \sqrt{c-c \sin(e+fx)}}{2af} \\ &= -\frac{(A-3B)c \cos(e+fx)}{af\sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{acf} \end{aligned}$$

$$\begin{aligned}
& \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^2 * \\
& \tan(1/4 * \exp(1))^2 + 1055531162664960 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \\
& \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^2 * \tan(1/4 * \exp(1))^4 - 70368744177664 * \sqrt{2} \\
& * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^2 * \tan(1/4 \\
& * \exp(1))^6 - 211106232532992 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan \\
& (1/2 * (1/2 * f * x + 2 * \exp(1)))^2 * \tan(1/2 * \exp(1)) - 422212465065984 * \sqrt{2} * A * \text{sign}(\sin \\
& (1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^3 * \tan(1/2 * \exp(1))^2 \\
& - 703687441776640 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 \\
& * f * x + 2 * \exp(1)))^3 * \tan(1/2 * \exp(1))^3 + 633318697598976 * \sqrt{2} * A * \text{sign}(\sin(1/2 * \\
& (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^3 * \tan(1/2 * \exp(1))^4 + 21110 \\
& 6232532992 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \\
& \exp(1)))^3 * \tan(1/2 * \exp(1))^5 - 1055531162664960 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + e \\
& xp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^3 * \tan(1/4 * \exp(1))^2 + 10555311626 \\
& 64960 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1 \\
&)))^3 * \tan(1/4 * \exp(1))^4 - 70368744177664 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - \\
& 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^3 * \tan(1/4 * \exp(1))^6 + 211106232532992 * \sqrt{2} \\
& * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1)))^3 * \tan \\
& (1/2 * \exp(1)) + 6333186975989760 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) \\
& * \tan(1/2 * \exp(1))^2 * \tan(1/4 * \exp(1))^2 + 12666373951979520 * \sqrt{2} * A * \text{sign}(\sin(1 \\
& /2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^2 * \tan(1/4 * \exp(1))^3 - 63331869759897 \\
& 60 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^2 * \tan(1/4 * e \\
& xp(1))^4 - 3799912185593856 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(\\
& 1/2 * \exp(1))^2 * \tan(1/4 * \exp(1))^5 + 422212465065984 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x \\
& + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^2 * \tan(1/4 * \exp(1))^6 - 3799912185593856 * \sqrt{2} \\
& (2) * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^2 * \tan(1/4 * \exp(1)) - \\
& 10555311626649600 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(\\
& 1))^3 * \tan(1/4 * \exp(1))^2 + 14073748835532800 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1 \\
&)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^3 * \tan(1/4 * \exp(1))^3 + 10555311626649600 * \sqrt{2} * A \\
& * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^3 * \tan(1/4 * \exp(1))^4 - 422 \\
& 2124650659840 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^3 \\
& * \tan(1/4 * \exp(1))^5 - 703687441776640 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 \\
& * \pi)) * \tan(1/2 * \exp(1))^3 * \tan(1/4 * \exp(1))^6 - 4222124650659840 * \sqrt{2} * A * \text{sign}(\sin \\
& (1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^3 * \tan(1/4 * \exp(1)) - 949978046398 \\
& 4640 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^4 * \tan(1/4 \\
& * \exp(1))^2 - 8444249301319680 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan \\
& (1/2 * \exp(1))^4 * \tan(1/4 * \exp(1))^3 + 9499780463984640 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (\\
& f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^4 * \tan(1/4 * \exp(1))^4 + 2533274790395904 * \sqrt{2} \\
& * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^4 * \tan(1/4 * \exp(1 \\
&))^5 - 633318697598976 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * e \\
& xp(1))^4 * \tan(1/4 * \exp(1))^6 + 2533274790395904 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp \\
& (1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^4 * \tan(1/4 * \exp(1)) + 3166593487994880 * \sqrt{2} * A * \\
& \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^5 * \tan(1/4 * \exp(1))^2 - 4222 \\
& 124650659840 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^5 \\
& * \tan(1/4 * \exp(1))^3 - 3166593487994880 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 \\
& * \pi)) * \tan(1/2 * \exp(1))^5 * \tan(1/4 * \exp(1))^4 + 1266637395197952 * \sqrt{2} * A * \text{sign}(\sin \\
& (1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^5 * \tan(1/4 * \exp(1))^5 + 2111062325 \\
& 32992 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^5 * \tan(1/ \\
& 4 * \exp(1))^6 + 1266637395197952 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan \\
& (1/2 * \exp(1))^5 * \tan(1/4 * \exp(1)) + 1407374883553280 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f \\
& * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^6 * \tan(1/4 * \exp(1))^3 - 422212465065984 * \sqrt{2} \\
& * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp(1))^6 * \tan(1/4 * \exp(1)) \\
& ^5 - 422212465065984 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * \exp \\
& (1))^6 * \tan(1/4 * \exp(1)) - 422212465065984 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - \\
& 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1))) * \tan(1/2 * \exp(1))^2 - 703687441776640 * \sqrt{2} \\
& (2) * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1))) * \tan(1/ \\
& 2 * \exp(1))^3 + 633318697598976 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan \\
& (1/2 * (1/2 * f * x + 2 * \exp(1))) * \tan(1/2 * \exp(1))^4 + 211106232532992 * \sqrt{2} * A * \text{sign}(\sin \\
& (1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2 * f * x + 2 * \exp(1))) * \tan(1/2 * \exp(1))^5 \\
& - 1055531162664960 * \sqrt{2} * A * \text{sign}(\sin(1/2 * (f * x + \exp(1)) - 1/4 * \pi)) * \tan(1/2 * (1/2
\end{aligned}$$

$$\begin{aligned}
 & *f*x+2*\exp(1)) * \tan(1/4*\exp(1))^2 + 1055531162664960 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1))) * \tan(1/4*\exp(1))^4 - 70368744 \\
 & 177664 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1))) * \tan(1/4*\exp(1))^6 + 211106232532992 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))- \\
 & 1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1))) * \tan(1/2*\exp(1)) + 3166593487994880 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1)) * \tan(1/4*\exp(1))^2 - 4 \\
 & 222124650659840 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1)) * \tan(1/4*\exp(1))^3 - 3166593487994880 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/ \\
 & 4*\pi)) * \tan(1/2*\exp(1)) * \tan(1/4*\exp(1))^4 + 1266637395197952 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1)) * \tan(1/4*\exp(1))^5 + 2111062325329 \\
 & 92 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1)) * \tan(1/4*\exp(1))^6 + 1266637395197952 * \sqrt{2} * A * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/ \\
 & 2*\exp(1)) * \tan(1/4*\exp(1)) + 2744381022928896 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/2*\exp(1))^2 + 70368744177664 \\
 & 0 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/2*\exp(1))^3 - 2533274790395904 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/ \\
 & 4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/2*\exp(1))^4 - 211106232532992 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(\\
 & 1/2*\exp(1))^5 + 211106232532992 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/2*\exp(1))^6 + 2111062325329920 * \sqrt{2} * B * \\
 & \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1))^2 + 4222124650659840 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1 \\
 & /2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1))^3 - 2111062325329920 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1)) \\
 & ^4 - 1266637395197952 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1))^5 + 140737488355328 * \sqrt{2} * B * \text{sign}(\sin(1/ \\
 & 2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1))^6 - 211 \\
 & 106232532992 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+ \\
 & 2*\exp(1)))^2 * \tan(1/2*\exp(1)) - 1266637395197952 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^2 * \tan(1/4*\exp(1)) + 6333186975989 \\
 & 76 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1))) \\
 & ^3 * \tan(1/2*\exp(1))^2 + 2111062325329920 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1 \\
 & /4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/2*\exp(1))^3 - 422212465065984 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan \\
 & (1/2*\exp(1))^4 - 633318697598976 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) \\
 & * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/2*\exp(1))^5 + 70368744177664 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/2*\exp \\
 & (1))^6 - 1407374883553280 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/ \\
 & 2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/4*\exp(1))^3 + 422212465065984 * \sqrt{2} * B * \text{sign}(\sin \\
 & (1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/4*\exp(1))^5 \\
 & - 633318697598976 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2* \\
 & f*x+2*\exp(1)))^3 * \tan(1/2*\exp(1)) + 422212465065984 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f* \\
 & x+\exp(1))-1/4*\pi)) * \tan(1/2*(1/2*f*x+2*\exp(1)))^3 * \tan(1/4*\exp(1)) - 3166593487 \\
 & 994880 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^2 * \tan(1 \\
 & /4*\exp(1))^2 - 16888498602639360 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) \\
 & * \tan(1/2*\exp(1))^2 * \tan(1/4*\exp(1))^3 + 3166593487994880 * \sqrt{2} * B * \text{sign}(\sin(1/ \\
 & 2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^2 * \tan(1/4*\exp(1))^4 + 506654958079180 \\
 & 8 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^2 * \tan(1/4*\exp \\
 & (1))^5 - 211106232532992 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/ \\
 & 2*\exp(1))^2 * \tan(1/4*\exp(1))^6 + 5066549580791808 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+ \\
 & \exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^2 * \tan(1/4*\exp(1)) + 31665934879948800 * \sqrt{2} \\
 &) * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^3 * \tan(1/4*\exp(1))^2 - \\
 & 42221246506598400 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(\\
 & 1))^3 * \tan(1/4*\exp(1))^3 - 31665934879948800 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1 \\
 &))-1/4*\pi)) * \tan(1/2*\exp(1))^3 * \tan(1/4*\exp(1))^4 + 12666373951979520 * \sqrt{2} * B \\
 & * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^3 * \tan(1/4*\exp(1))^5 + 211 \\
 & 1062325329920 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \tan(1/2*\exp(1))^ \\
 & 3 * \tan(1/4*\exp(1))^6 + 12666373951979520 * \sqrt{2} * B * \text{sign}(\sin(1/2*(f*x+\exp(1))-1 \\
 & /4*\pi)) * \tan(1/2*\exp(1))^3 * \tan(1/4*\exp(1)) + 12666373951979520 * \sqrt{2} * B * \text{sign}
 \end{aligned}$$

$+2*\exp(1))^{2*\tan(1/2*\exp(1))^{3*\tan(1/4*\exp(1))^{2+14073748835532800*\sqrt{2}}}$
 $*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2}$
 $*\exp(1))^{3*\tan(1/4*\exp(1))^{3+10555311626649600*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+$
 $\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{3*\tan(1/4*\exp}$
 $\exp(1))^{4-4222124650659840*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1$
 $/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{3*\tan(1/4*\exp(1))^{5-70368744177664$
 $0*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{$
 $2*\tan(1/2*\exp(1))^{3*\tan(1/4*\exp(1))^{6-4222124650659840*\sqrt{2}}*A*\text{sign}(\sin(1$
 $/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{3*ta$
 $n(1/4*\exp(1))-9499780463984640*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))$
 $*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{4*\tan(1/4*\exp(1))^{2-84442493$
 $01319680*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp$
 $\exp(1)))^{2*\tan(1/2*\exp(1))^{4*\tan(1/4*\exp(1))^{3+9499780463984640*\sqrt{2}}*A*\text{sig}$
 $n}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1$
 $))^{4*\tan(1/4*\exp(1))^{4+2533274790395904*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))$
 $-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{4*\tan(1/4*\exp(1))^{5$
 $-633318697598976*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*$
 $f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{4*\tan(1/4*\exp(1))^{6+2533274790395904*\sqrt{2}}$
 $(2)*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1$
 $/2*\exp(1))^{4*\tan(1/4*\exp(1))+3166593487994880*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+e$
 $xp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{5*\tan(1/4*\exp$
 $(1))^{2-4222124650659840*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/$
 $2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{5*\tan(1/4*\exp(1))^{3-316659348799488$
 $0*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{$
 $2*\tan(1/2*\exp(1))^{5*\tan(1/4*\exp(1))^{4+1266637395197952*\sqrt{2}}*A*\text{sign}(\sin(1$
 $/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{5*ta$
 $n(1/4*\exp(1))^{5+211106232532992*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi)$
 $)*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{5*\tan(1/4*\exp(1))^{6+1266637$
 $395197952*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*e$
 $xp(1)))^{2*\tan(1/2*\exp(1))^{5*\tan(1/4*\exp(1))+1407374883553280*\sqrt{2}}*A*\text{sign}$
 $(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1$
 $))^{6*\tan(1/4*\exp(1))^{3-422212465065984*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1$
 $/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{6*\tan(1/4*\exp(1))^{5-4$
 $22212465065984*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*$
 $x+2*\exp(1)))^{2*\tan(1/2*\exp(1))^{6*\tan(1/4*\exp(1))+3166593487994880*\sqrt{2}}*A$
 $*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*e$
 $xp(1))*\tan(1/4*\exp(1))^{2-4222124650659840*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1$
 $))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))*\tan(1/4*\exp(1))^{3$
 $-3166593487994880*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2$
 $*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))*\tan(1/4*\exp(1))^{4+1266637395197952*\sqrt{2}}$
 $(2)*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/$
 $2*\exp(1))*\tan(1/4*\exp(1))^{5+211106232532992*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp$
 $(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))*\tan(1/4*\exp(1))$
 $^{6+1266637395197952*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1$
 $/2*f*x+2*\exp(1)))^{2*\tan(1/2*\exp(1))*\tan(1/4*\exp(1))+6333186975989760*\sqrt{2}}$
 $(2)*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/$
 $2*\exp(1))^{2*\tan(1/4*\exp(1))^{2-12666373951979520*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x$
 $+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/2*\exp(1))^{2*\tan(1/4*e$
 $xp(1))^{3-6333186975989760*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan($
 $1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/2*\exp(1))^{2*\tan(1/4*\exp(1))^{4+3799912185593$
 $856*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1))$
 $)^{3*\tan(1/2*\exp(1))^{2*\tan(1/4*\exp(1))^{5+422212465065984*\sqrt{2}}*A*\text{sign}(\sin($
 $1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/2*\exp(1))^{2*ta$
 $n(1/4*\exp(1))^{6+3799912185593856*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*p$
 $i))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/2*\exp(1))^{2*\tan(1/4*\exp(1))+1055531$
 $1626649600*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*$
 $\exp(1)))^{3*\tan(1/2*\exp(1))^{3*\tan(1/4*\exp(1))^{2+14073748835532800*\sqrt{2}}*A*$
 $\text{sign}(\sin(1/2*(f*x+\exp(1))-1/4*\pi))*\tan(1/2*(1/2*f*x+2*\exp(1)))^{3*\tan(1/2*\exp$
 $\exp(1))^{3*\tan(1/4*\exp(1))^{3-10555311626649600*\sqrt{2}}*A*\text{sign}(\sin(1/2*(f*x+\exp$


```
(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5-237684487542793012780631851008*sqrt
(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))+475368975085586025
561263702016*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))-7
9228162514264337593543950336*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/2*exp(1))^3+1584563250285286751870879006720*sqrt(2)*B*sign(sin(1/2*(f*
x+exp(1))-1/4*pi))*tan(1/4*exp(1))^3-475368975085586025561263702016*sqrt(2)
*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/4*exp(1))^5+2376844875427930127
80631851008*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))-47
5368975085586025561263702016*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*t
an(1/4*exp(1))+4753689750855860255612637020160*sqrt(2)*A*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^3-1426106925256758076683
791106048*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan
(1/4*exp(1))^5-1426106925256758076683791106048*sqrt(2)*A*sign(sin(1/2*(f*x
+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))-11884224377139650639031
59255040*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan
(1/4*exp(1))^2+1188422437713965063903159255040*sqrt(2)*A*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^4-7922816251426433759354
3950336*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(
1/4*exp(1))^6+3565267313141895191709477765120*sqrt(2)*A*sign(sin(1/2*(f*x+e
xp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^2-3565267313141895191709477
765120*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4
*exp(1))^4+237684487542793012780631851008*sqrt(2)*A*sign(sin(1/2*(f*x+exp(1)
))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^6-47536897508558602556126370201
60*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp
(1))^3+1426106925256758076683791106048*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*exp(1))^5+1426106925256758076683791106
048*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^2*tan(1/4*
exp(1))+1188422437713965063903159255040*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1)
))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^2-11884224377139650639031592550
40*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp
(1))^4+79228162514264337593543950336*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*tan(1/2*exp(1))^3*tan(1/4*exp(1))^6-356526731314189519170947776512
0*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(
1))^2+3565267313141895191709477765120*sqrt(2)*B*sign(sin(1/2*(f*x+exp(1))-1
/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^4-237684487542793012780631851008*sq
rt(2)*B*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*tan(1/2*exp(1))*tan(1/4*exp(1))^
6)*ln(abs(2*tan(1/2*exp(1))^3-2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*
exp(1))^2+1)+6*tan(1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1))))*tan(1/2*exp(
1))^2+2*tan(1/2*(1/2*f*x+2*exp(1))))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*
exp(1))))*tan(1/2*exp(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2)
/abs(2*tan(1/2*exp(1))^3+2*sqrt(2)*sqrt(tan(1/2*exp(1))^2+1)*(tan(1/2*exp(1)
))^2+1)+6*tan(1/2*exp(1))^2-6*tan(1/2*(1/2*f*x+2*exp(1))))*tan(1/2*exp(1))^2
+2*tan(1/2*(1/2*f*x+2*exp(1))))*tan(1/2*exp(1))^3-6*tan(1/2*(1/2*f*x+2*exp(1)
)))*tan(1/2*exp(1))+2*tan(1/2*(1/2*f*x+2*exp(1)))-6*tan(1/2*exp(1))-2)/sqrt
(2)/sqrt(tan(1/2*exp(1))^2+1)/(tan(1/2*exp(1))^2+1)/(158456325028528675187
087900672*a*tan(1/4*exp(1))^6+475368975085586025561263702016*a*tan(1/4*exp(
1))^4+475368975085586025561263702016*a*tan(1/4*exp(1))^2+158456325028528675
187087900672*a))/f
```

maple [A] time = 1.02, size = 53, normalized size = 0.73

$$\frac{2c(\sin(fx+e)-1)(-B\sin(fx+e)+A-2B)}{a\cos(fx+e)\sqrt{c-c\sin(fx+e)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)

[Out] 2*c/a*(sin(f*x+e)-1)*(-B*sin(f*x+e)+A-2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.45, size = 174, normalized size = 2.38

$$\frac{2 \left(\frac{2B \left(\sqrt{c} + \frac{\sqrt{c} \sin(fx+e)}{\cos(fx+e)+1} + \frac{\sqrt{c} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} \right)}{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \sqrt{\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1}} - \frac{A \left(\sqrt{c} + \frac{\sqrt{c} \sin(fx+e)}{\cos(fx+e)+1} \right)}{\left(a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} \right) \sqrt{\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1}} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x, algorith="maxima")

[Out] -2*(2*B*(sqrt(c) + sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) - A*(sqrt(c) + sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)/((a + a*sin(f*x + e)/(cos(f*x + e) + 1))*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/f

mupad [B] time = 13.15, size = 128, normalized size = 1.75

$$\frac{2 \sqrt{-c (\sin(e + fx) - 1)} \left(2B \sin(2e + 2fx) - 2A \sin(2e + 2fx) - 4A \left(2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right) + 7B \left(2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right) \right)}{af \left(4 \sin^2(e + fx) + \sin(e + fx) + \sin(3e + 3fx) - 4 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x)), x)

[Out] (2*(-c*(sin(e + f*x) - 1))^(1/2)*(2*B*sin(2*e + 2*f*x) - 2*A*sin(2*e + 2*f*x) - 4*A*(2*sin(e/2 + (f*x)/2)^2 - 1) + 7*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a*f*(sin(e + f*x) + sin(3*e + 3*f*x) + 4*sin(e + f*x)^2 - 4))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \sqrt{-c \sin(e+fx)+c}}{\sin(e+fx)+1} dx + \int \frac{B \sqrt{-c \sin(e+fx)+c} \sin(e+fx)}{\sin(e+fx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e)),x)

[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x) + 1), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x) + 1), x))/a

$$3.112 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=91

$$\frac{(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{acf}$$

[Out] 1/2*(A+B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/f*2^(1/2)/c^(1/2)-(A-B)*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a/c/f

Rubi [A] time = 0.28, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2649, 206}

$$\frac{(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{\sqrt{2}a\sqrt{c}f} - \frac{(A-B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{acf}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]),x]

[Out] ((A + B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(Sqrt[2]*a*Sqrt[c]*f) - ((A - B)*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(a*c*f)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m]/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))\sqrt{c - c \sin(e + fx)}} dx = \frac{\int \sec^2(e + fx)(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx}{ac}$$

$$= -\frac{(A - B) \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{acf} + \frac{(A + B) \int \frac{1}{\sqrt{c - c \sin(e + fx)}} dx}{2a}$$

$$= -\frac{(A - B) \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{acf} - \frac{(A + B) \operatorname{Subst}\left(\int \frac{1}{\sqrt{c - c \sin(e + fx)}} dx\right)}{2a}$$

$$= \frac{(A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{\sqrt{2} a \sqrt{c} f} - \frac{(A - B) \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{acf}$$

Mathematica [C] time = 0.45, size = 140, normalized size = 1.54

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(-1 + i\sqrt[4]{-1}\right)(A + B) \tan^{-1}\left(\frac{1}{2} + \frac{i}{2}\right)}{af(\sin(e + fx) + 1)\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-A + B - (1 + I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.43, size = 162, normalized size = 1.78

$$\sqrt{2}(A + B)\sqrt{c} \cos(fx + e) \log\left(-\frac{\cos(fx+e)^2 + (\cos(fx+e)-2) \sin(fx+e) + \frac{2\sqrt{2}\sqrt{-c\sin(fx+e)+c}(\cos(fx+e)+\sin(fx+e)+1)}{\sqrt{c}} + 3\cos(fx+e)+2}{\cos(fx+e)^2 + (\cos(fx+e)+2) \sin(fx+e) - \cos(fx+e) - 2}\right)$$

$$4 acf \cos(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x, algorithm="fricas")

[Out] 1/4*(sqrt(2)*(A + B)*sqrt(c)*cos(f*x + e)*log(-(cos(f*x + e))^2 + (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*(cos(f*x + e) + sin(f*x + e) + 1)/sqrt(c) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(-c*sin(f*x + e) + c)*(A - B)/(a*c*f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)

Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(sin((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nostep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep-1)]index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [A] time = 1.29, size = 130, normalized size = 1.43

$$\frac{(\sin(fx + e) - 1) \left(\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) \sqrt{c(1 + \sin(fx + e))} A + \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) \right)}{2a\sqrt{c} \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x)

[Out] -1/2/a*(sin(f*x+e)-1)*(2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e))))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*A+2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e))))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(1/2)*B-2*c^(1/2)*A+2*c^(1/2)*B/c^(1/2)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a) \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx)) \sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)), x)

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2)),
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{-c \sin(e+fx)+c} \sin(e+fx)+\sqrt{-c \sin(e+fx)+c}} dx + \int \frac{B \sin(e+fx)}{\sqrt{-c \sin(e+fx)+c} \sin(e+fx)+\sqrt{-c \sin(e+fx)+c}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x) + sqrt(-c*sin(e + f*x)
+ c)), x) + Integral(B*sin(e + f*x)/(sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)
+ sqrt(-c*sin(e + f*x) + c)), x))/a
```

$$3.113 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=136

$$\frac{(3A - B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{4\sqrt{2} ac^{3/2} f} + \frac{(3A - B) \cos(e + fx)}{4af(c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec(e + fx)}{acf \sqrt{c - c \sin(e + fx)}}$$

[Out] 1/4*(3*A-B)*cos(f*x+e)/a/f/(c-c*sin(f*x+e))^(3/2)+1/8*(3*A-B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a/c^(3/2)/f*2^(1/2)-(A-B)*sec(f*x+e)/a/c/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.33, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2855, 2650, 2649, 206}

$$\frac{(3A - B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{4\sqrt{2} ac^{3/2} f} + \frac{(3A - B) \cos(e + fx)}{4af(c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec(e + fx)}{acf \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2)), x]

[Out] ((3*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])]/(4*Sqrt[2]*a*c^(3/2)*f) + ((3*A - B)*Cos[e + f*x])/(4*a*f*(c - c*Sin[e + f*x])^(3/2)) - ((A - B)*Sec[e + f*x])/(a*c*f*Sqrt[c - c*Sin[e + f*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967


```
[Out] -1/16*(sqrt(2)*((3*A - B)*cos(f*x + e)*sin(f*x + e) - (3*A - B)*cos(f*x + e)))*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*((3*A - B)*sin(f*x + e) - A + 3*B)*sqrt(-c*sin(f*x + e) + c))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/4*(-A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)+B*sqrt(c))/a/c/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f*x+exp(1))/2)-1)+1/8*(-3*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*sqrt(c)*c-B*sqrt(c)*c)/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/a/c/sign(tan((f*x+exp(1))/2)-1)+1/8*(3*A-B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/sqrt(-c)/a/c/sign(tan((f*x+exp(1))/2)-1))
```

maple [A] time = 1.28, size = 225, normalized size = 1.65

$$\sin(fx + e) \left(3A \sqrt{c + c \sin(fx + e)} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c + c \sin(fx + e)} \sqrt{2}}{2\sqrt{c}} \right) c - B \sqrt{c + c \sin(fx + e)} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c + c \sin(fx + e)} \sqrt{2}}{2\sqrt{c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/8/c^(5/2)/a*(sin(f*x+e)*(3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c-6*A*c^(3/2)+2*B*c^(3/2))-3*A*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c+B*(c+c*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(c+c*sin(f*x+e))^(1/2)*2^(1/2)/c^(1/2))*c+2*A*c^(3/2)-6*B*c^(3/2))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorith="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x)) (c - c \sin(e + f x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)), x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{-c\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+c\sqrt{-c\sin(e+fx)+c}} dx + \int \frac{B\sin(e+fx)}{-c\sqrt{-c\sin(e+fx)+c}\sin^2(e+fx)+c\sqrt{-c\sin(e+fx)+c}} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] (Integral(A/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x) + Integral(B*sin(e + f*x)/(-c*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)**2 + c*sqrt(-c*sin(e + f*x) + c)), x))/a

$$3.114 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=180

$$\frac{3(5A-3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{32\sqrt{2} ac^{5/2} f} - \frac{(5A-3B) \sec(e+fx)}{8ac^2 f \sqrt{c-c \sin(e+fx)}} + \frac{3(5A-3B) \cos(e+fx)}{32acf(c-c \sin(e+fx))^{3/2}} + \frac{(A+B) \sec(e+fx)}{4acf(c-c \sin(e+fx))^{3/2}}$$

[Out] $3/32*(5*A-3*B)*\cos(f*x+e)/a/c/f/(c-c*\sin(f*x+e))^{(3/2)}+1/4*(A+B)*\sec(f*x+e)/a/c/f/(c-c*\sin(f*x+e))^{(3/2)}+3/64*(5*A-3*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)}*2^{(1/2)/(c-c*\sin(f*x+e))^{(1/2)})}/a/c^{(5/2)}/f*2^{(1/2)}-1/8*(5*A-3*B)*\sec(f*x+e)/a/c^2/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.42, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2859, 2687, 2650, 2649, 206}

$$-\frac{(5A-3B) \sec(e+fx)}{8ac^2 f \sqrt{c-c \sin(e+fx)}} + \frac{3(5A-3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{32\sqrt{2} ac^{5/2} f} + \frac{3(5A-3B) \cos(e+fx)}{32acf(c-c \sin(e+fx))^{3/2}} + \frac{(A+B) \sec(e+fx)}{4acf(c-c \sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A+B*\sin[e+fx])/((a+a*\sin[e+fx])*(c-c*\sin[e+fx])^{(5/2)})]$, x]

[Out] $(3*(5*A-3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Cos}[e+fx])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c-c*\sin[e+fx]])])/(32*\operatorname{Sqrt}[2]*a*c^{(5/2)}*f) + (3*(5*A-3*B)*\operatorname{Cos}[e+fx])/(32*a*c*f*(c-c*\sin[e+fx])^{(3/2)}) + ((A+B)*\operatorname{Sec}[e+fx])/(4*a*c*f*(c-c*\sin[e+fx])^{(3/2)}) - ((5*A-3*B)*\operatorname{Sec}[e+fx])/(8*a*c^2*f*\operatorname{Sqrt}[c-c*\sin[e+fx]])]$

Rule 206

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(c_+ + (d_+)*(x_+)])], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/ \operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2650

$\operatorname{Int}[(a_+ + (b_+)*\sin[(c_+ + (d_+)*(x_+)])^{(n_+)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Cos}[c + d*x]*(a + b*\sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + \operatorname{Dist}[(n + 1)/(a*(2*n + 1)), \operatorname{Int}[(a + b*\sin[c + d*x])^{(n + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{LtQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 2687

$\operatorname{Int}[(\cos[(e_+ + (f_+)*(x_+)])*(g_+))^{(p_+)}/\operatorname{Sqrt}[(a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)])], x_Symbol] \rightarrow -\operatorname{Simp}[(b*(g*\operatorname{Cos}[e + f*x])^{(p + 1)})/(a*f*g*(p + 1)*\operatorname{Sqrt}[a + b*\sin[e + f*x]], x] + \operatorname{Dist}[(a*(2*p + 1))/(2*g^2*(p + 1)), \operatorname{Int}[(g*\operatorname{Cos}[e + f*x])^{(p + 2)}]/(a + b*\sin[e + f*x])^{(3/2)}, x], x] /; \operatorname{FreeQ}\{a, b, e, f, g\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{LtQ}[p, -1] \ \&\& \operatorname{IntegerQ}[2*p]$

Rule 2859

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Simp[((b*c - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0]) && NeQ[2*m + p + 1, 0]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{5/2}} dx = \frac{\int \frac{\sec^2(e+fx)(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx}{ac}$$

$$= \frac{(A + B) \sec(e + fx)}{4acf(c - c \sin(e + fx))^{3/2}} + \frac{(5A - 3B) \int \frac{\sec^2(e+fx)}{\sqrt{c-c \sin(e+fx)}} dx}{8ac^2}$$

$$= \frac{(A + B) \sec(e + fx)}{4acf(c - c \sin(e + fx))^{3/2}} - \frac{(5A - 3B) \sec(e + fx)}{8ac^2 f \sqrt{c - c \sin(e + fx)}} + \frac{(3(5A - 3B) \cos(e + fx))}{32acf(c - c \sin(e + fx))^{3/2}} + \frac{(A + B) \sec(e + fx)}{4acf(c - c \sin(e + fx))^{3/2}} - \frac{(5A - 3B) \sec(e + fx)}{8ac^2 f \sqrt{c - c \sin(e + fx)}} + \frac{(3(5A - 3B) \cos(e + fx))}{32acf(c - c \sin(e + fx))^{3/2}} + \frac{(A + B) \sec(e + fx)}{4acf(c - c \sin(e + fx))^{3/2}} - \frac{(5A - 3B) \sec(e + fx)}{8ac^2 f \sqrt{c - c \sin(e + fx)}} + \frac{3(5A - 3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{32\sqrt{2} ac^{5/2} f} + \frac{3(5A - 3B) \cos(e + fx)}{32acf(c - c \sin(e + fx))^{3/2}}$$

Mathematica [C] time = 0.85, size = 404, normalized size = 2.24

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(8(B - A)\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) - (3(5A - 3B) \cos(e + fx))\right)}{32\sqrt{2} ac^{5/2} f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2)), x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 4*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (7*A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - (3 + 3*I)*(-1)^(1/4)*(5*A - 3*B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1
```

$$+ \tan\left(\frac{e + fx}{4}\right) \cdot \left(\cos\left(\frac{e + fx}{2}\right) - \sin\left(\frac{e + fx}{2}\right)\right)^4 \cdot \left(\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right)\right) + 8 \cdot (A + B) \cdot \sin\left(\frac{e + fx}{2}\right) \cdot \left(\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right)\right) + 2 \cdot (7A - B) \cdot \left(\cos\left(\frac{e + fx}{2}\right) - \sin\left(\frac{e + fx}{2}\right)\right)^2 \cdot \sin\left(\frac{e + fx}{2}\right) \cdot \left(\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right)\right) \Big/ \left(32 \cdot a \cdot f \cdot (1 + \sin[e + fx])\right) \cdot (c - c \cdot \sin[e + fx])^{5/2}$$

fricas [A] time = 0.44, size = 282, normalized size = 1.57

$$3\sqrt{2} \left((5A - 3B) \cos^3(fx + e) + 2(5A - 3B) \cos(fx + e) \sin(fx + e) - 2(5A - 3B) \cos(fx + e) \right) \sqrt{c} \log\left(\frac{\dots}{\dots}\right)$$

12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out]
$$-1/128 \cdot (3 \cdot \sqrt{2}) \cdot ((5A - 3B) \cdot \cos(fx + e)^3 + 2 \cdot (5A - 3B) \cdot \cos(fx + e) \cdot \sin(fx + e) - 2 \cdot (5A - 3B) \cdot \cos(fx + e)) \cdot \sqrt{c} \cdot \log\left(\frac{-c \cdot \cos(fx + e)^2 - 2 \cdot \sqrt{2} \cdot \sqrt{-c \cdot \sin(fx + e) + c} \cdot \sqrt{c} \cdot (\cos(fx + e) + \sin(fx + e) + 1) + 3 \cdot c \cdot \cos(fx + e) + (c \cdot \cos(fx + e) - 2 \cdot c) \cdot \sin(fx + e) + 2 \cdot c}{(\cos(fx + e))^2 + (\cos(fx + e) + 2) \cdot \sin(fx + e) - \cos(fx + e) - 2}\right) + 4 \cdot (3 \cdot (5A - 3B) \cdot \cos(fx + e)^2 + 4 \cdot (5A - 3B) \cdot \sin(fx + e) - 12A + 20B) \cdot \sqrt{-c \cdot \sin(fx + e) + c} \Big/ (a \cdot c^3 \cdot f \cdot \cos(fx + e)^3 + 2 \cdot a \cdot c^3 \cdot f \cdot \cos(fx + e) \cdot \sin(fx + e) - 2 \cdot a \cdot c^3 \cdot f \cdot \cos(fx + e))$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/8*(-A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)+B*sqrt(c))/c^2/a/((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)/sign(tan((f*x+exp(1))/2)-1)+1/64*(-41*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-17*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-127*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-23*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-91*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-19*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+143*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+39*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-3*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+5*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+47*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-93*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+7*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-37*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-11*A*sqrt(c)*c^3-3*B*sqrt(c)*c^3)/c^2/((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/a/sign(tan((f*x+exp(1))/2)-1)+1/64*(15*A-9*B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c

$c + \sqrt{c \tan((f \cdot x + \exp(1))/2)^2 + c} / \sqrt{2} / \sqrt{-c} / \sqrt{2} / c^{5/2} / \sqrt{-c} / a / \text{sign}(\tan((f \cdot x + \exp(1))/2) - 1)$

maple [B] time = 1.78, size = 350, normalized size = 1.94

$$\sin(fx + e) \left(-30A \sqrt{c + c \sin(fx + e)} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c + c \sin(fx + e)} \sqrt{2}}{2\sqrt{c}} \right) c^2 + 40A c^{5/2} + 18B \sqrt{c + c \sin(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] $-1/64/c^{9/2}/a*(\sin(f*x+e)*(-30*A*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2})*c^2+40*A*c^{5/2}+18*B*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))*c^2-24*B*c^{5/2})+(-15*A*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))*c^2+30*A*c^{5/2}+9*B*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))*c^2-18*B*c^{5/2})*\cos(f*x+e)^2+30*A*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))*c^2-24*A*c^{5/2}-18*B*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{1/2}*2^{1/2}/c^{1/2}))*c^2+40*B*c^{5/2})/(\sin(f*x+e)-1)/\cos(f*x+e)/(c-c*\sin(f*x+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)(-c \sin(fx + e) + c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorith="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2)),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] Timed out

$$3.115 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=242

$$\frac{2048c^4(7A-13B) \sec(e+fx) \sqrt{c-c \sin(e+fx)}}{105a^2f} - \frac{512c^3(7A-13B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{105a^2f} - \frac{(A-B) \sec(e+fx)(c-c \sin(e+fx))^{9/2}}{105a^2f}$$

[Out] -512/105*(7*A-13*B)*c^3*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-64/105*(7*A-13*B)*c^2*sec(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f-16/105*(7*A-13*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a^2/f-1/21*(7*A-13*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(9/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(13/2)/a^2/c^2/f+2048/105*(7*A-13*B)*c^4*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f

Rubi [A] time = 0.65, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{13/2}}{3a^2c^2f} - \frac{64c^2(7A-13B) \sec(e+fx)(c-c \sin(e+fx))^{5/2}}{105a^2f} - \frac{512c^3(7A-13B) \sec(e+fx)(c-c \sin(e+fx))^{9/2}}{105a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))^(9/2)/(a + a*Sin[e + f*x])^2, x]

[Out] (2048*(7*A - 13*B)*c^4*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(105*a^2*f) - (512*(7*A - 13*B)*c^3*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(105*a^2*f) - (64*(7*A - 13*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(105*a^2*f) - (16*(7*A - 13*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(105*a^2*f) - ((7*A - 13*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(21*a^2*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(13/2))/(3*a^2*c^2*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^2} dx &= \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{13/2} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{13/2}}{3a^2 c^2 f} - \frac{(7A - 13B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{13/2} dx}{21a^2 f} \\ &= -\frac{(7A - 13B) \sec(e + fx)(c - c \sin(e + fx))^{9/2}}{21a^2 f} - \frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{13/2}}{105a^2 f} \\ &= -\frac{16(7A - 13B)c \sec(e + fx)(c - c \sin(e + fx))^{7/2}}{105a^2 f} - \frac{(7A - 13B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{13/2} dx}{105a^2 f} \\ &= -\frac{64(7A - 13B)c^2 \sec(e + fx)(c - c \sin(e + fx))^{5/2}}{105a^2 f} - \frac{16(7A - 13B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{13/2} dx}{105a^2 f} \\ &= -\frac{512(7A - 13B)c^3 \sec(e + fx)(c - c \sin(e + fx))^{3/2}}{105a^2 f} - \frac{64(7A - 13B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{13/2} dx}{105a^2 f} \\ &= -\frac{2048(7A - 13B)c^4 \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{105a^2 f} - \frac{512(7A - 13B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{13/2} dx}{105a^2 f} \end{aligned}$$

Mathematica [B] time = 6.82, size = 953, normalized size = 3.94

$$\frac{(26A - 83B) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)^4 \sin\left(\frac{3}{2}(e + fx)\right) (c - c \sin(e + fx))^{9/2}}{12f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^9 (\sin(e + fx)a + a)^2} - \frac{(2A - 13B) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)^4 \sin\left(\frac{3}{2}(e + fx)\right) (c - c \sin(e + fx))^{9/2}}{20f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^9 (\sin(e + fx)a + a)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (-32*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2))/(3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) + (32*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) + ((164*A - 351*B)*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) + ((26*A - 83*B)*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((2*A - 13*B)*Cos[(5*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(20*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) + (B*Cos[(7*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(28*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) + ((164*A - 351*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a + a*Sin[e + f*x])^2) - ((26*A - 83*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*(c - c*Sin[e + f*x])^(9/2)*Sin[(3*(e + f*x))/2])/(12*f*(
```

$$\frac{\cos\left(\frac{e+fx}{2}\right) - \sin\left(\frac{e+fx}{2}\right)}{20f\left(\cos\left(\frac{e+fx}{2}\right) - \sin\left(\frac{e+fx}{2}\right)\right)^9} \cdot \frac{\left(a + a\sin\left[\frac{e+fx}{2}\right]\right)^2 - \left((2A - 13B)\left(\cos\left(\frac{e+fx}{2}\right) + \sin\left(\frac{e+fx}{2}\right)\right)^4\right) \cdot \left(c - c\sin\left[\frac{e+fx}{2}\right]\right)^{9/2} \cdot \sin\left[\frac{e+fx}{2}\right]}{\left(B\left(\cos\left(\frac{e+fx}{2}\right) + \sin\left(\frac{e+fx}{2}\right)\right)^4\right) \cdot \left(c - c\sin\left[\frac{e+fx}{2}\right]\right)^{9/2} \cdot \sin\left[\frac{7(e+fx)}{2}\right]} \cdot \frac{1}{\left(28f\left(\cos\left(\frac{e+fx}{2}\right) - \sin\left(\frac{e+fx}{2}\right)\right)\right)^9} \cdot \left(a + a\sin\left[\frac{e+fx}{2}\right]\right)^2$$

fricas [A] time = 0.44, size = 153, normalized size = 0.63

$$\frac{2\left(3(7A - 38B)c^4 \cos^4(fx + e) - 12(154A - 311B)c^4 \cos^2(fx + e) + 24(287A - 543B)c^4 + 15Bc^4 \cos(fx + e)\right)}{105\left(a^2 f \cos(fx + e) \sin(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 2/105*(3*(7*A - 38*B)*c^4*cos(f*x + e)^4 - 12*(154*A - 311*B)*c^4*cos(f*x + e)^2 + 24*(287*A - 543*B)*c^4 + (15*B*c^4*cos(f*x + e)^4 + 4*(49*A - 136*B)*c^4*cos(f*x + e)^2 + 8*(931*A - 1699*B)*c^4)*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.36, size = 143, normalized size = 0.59

$$\frac{2c^5\left(\sin(fx + e) - 1\right)\left(15B \sin(fx + e)\left(\cos^4(fx + e)\right) + (196A - 544B)\left(\cos^2(fx + e)\right) \sin(fx + e) + (7448A - 13592B)\right)}{105a^2\left(1 + \sin(fx + e)\right) \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x)

[Out] -2/105*c^5/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(15*B*sin(f*x+e)*cos(f*x+e)^4+(196*A-544*B)*cos(f*x+e)^2*sin(f*x+e)+(7448*A-13592*B)*sin(f*x+e)+(21*A-114*B)*cos(f*x+e)^4+(-1848*A+3732*B)*cos(f*x+e)^2+6888*A-13032*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.46, size = 762, normalized size = 3.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -2/105*(7*(723*c^(9/2) + 2184*c^(9/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 5370*c^(9/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10696*c^(9/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15021*c^(9/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 21168*c^(9/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 20748*c^(9/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21168*c^(9/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))

$$\begin{aligned}
& 1)^7 + 15021*c^{(9/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 10696*c^{(9/2)}*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 5370*c^{(9/2)}*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 2184*c^{(9/2)}*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 723*c^{(9/2)}*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} * A / ((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) * (\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(9/2)}) - 2*(4707*c^{(9/2)} + 14121*c^{(9/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 35250*c^{(9/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 68549*c^{(9/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 99549*c^{(9/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 134802*c^{(9/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 138012*c^{(9/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 134802*c^{(9/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 99549*c^{(9/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 68549*c^{(9/2)}*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 35250*c^{(9/2)}*\sin(f*x + e)^{10}/(\cos(f*x + e) + 1)^{10} + 14121*c^{(9/2)}*\sin(f*x + e)^{11}/(\cos(f*x + e) + 1)^{11} + 4707*c^{(9/2)}*\sin(f*x + e)^{12}/(\cos(f*x + e) + 1)^{12} * B / ((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) * (\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(9/2)})) / f
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

$$3.116 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=201

$$\frac{128c^3(5A-11B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{15a^2f} - \frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{32c^2(5A-11B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^2f} + \frac{128c^3(5A-11B)}{15a^2f}$$

[Out] $-32/15*(5*A-11*B)*c^2*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a^2/f-4/15*(5*A-11*B)*c*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/a^2/f-1/15*(5*A-11*B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/a^2/f-1/3*(A-B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(11/2)}/a^2/c^2/f+128/15*(5*A-11*B)*c^3*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a^2/f$

Rubi [A] time = 0.56, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{11/2}}{3a^2c^2f} - \frac{32c^2(5A-11B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^2f} + \frac{128c^3(5A-11B)}{15a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))^(7/2)/(a + a*Sin[e + f*x])^2,x]

[Out] $(128*(5*A - 11*B)*c^3*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(15*a^2*f) - (32*(5*A - 11*B)*c^2*Sec[e + f*x]*(c - c*Sin[e + f*x])^{(3/2)})/(15*a^2*f) - (4*(5*A - 11*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^{(5/2)})/(15*a^2*f) - ((5*A - 11*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^{(7/2)})/(15*a^2*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(11/2)})/(3*a^2*c^2*f)$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Di

```
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^2} dx &= \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{11/2} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{11/2}}{3a^2 c^2 f} - \frac{(5A - 11B) \int}{15a^2 f} \\ &= -\frac{(5A - 11B) \sec(e + fx)(c - c \sin(e + fx))^{7/2}}{15a^2 f} - \frac{(A - B) \sec^3}{15a^2 f} \\ &= -\frac{4(5A - 11B)c \sec(e + fx)(c - c \sin(e + fx))^{5/2}}{15a^2 f} - \frac{(5A - 11B) \int}{15a^2 f} \\ &= -\frac{32(5A - 11B)c^2 \sec(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^2 f} - \frac{4(5A - 11B) \int}{15a^2 f} \\ &= \frac{128(5A - 11B)c^3 \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{15a^2 f} - \frac{32(5A - 11B) \int}{15a^2 f} \end{aligned}$$

Mathematica [A] time = 2.83, size = 159, normalized size = 0.79

$$\frac{c^3 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (12(25A - 62B) \cos(2(e + fx)) - 2730A \sin(e + fx) - 60a^2 f (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right))}{60a^2 f (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e +
f*x])^2, x]
```

```
[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(
-2100*A + 4725*B + 12*(25*A - 62*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)]
- 2730*A*Sin[e + f*x] + 5838*B*Sin[e + f*x] - 10*A*Sin[3*(e + f*x)] + 46*B
*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e
+ f*x])^2)
```

fricas [A] time = 0.43, size = 133, normalized size = 0.66

$$\frac{2 \left(3Bc^3 \cos(fx + e)^4 + 3(25A - 63B)c^3 \cos(fx + e)^2 - 12(25A - 57B)c^3 - \left((5A - 23B)c^3 \cos(fx + e) \right)^2 \right)}{15 \left(a^2 f \cos(fx + e) \sin(fx + e) + a^2 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] -2/15*(3*B*c^3*cos(f*x + e)^4 + 3*(25*A - 63*B)*c^3*cos(f*x + e)^2 - 12*(25
*A - 57*B)*c^3 - ((5*A - 23*B)*c^3*cos(f*x + e)^2 + 4*(85*A - 181*B)*c^3)*s
in(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a
^2*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.51, size = 121, normalized size = 0.60

$$\frac{2c^4 (\sin (fx + e) - 1) ((-5A + 23B) \sin (fx + e) (\cos^2 (fx + e)) + (-340A + 724B) \sin (fx + e) + 3B (\cos^4 (fx + e) - 1))}{15a^2 (1 + \sin (fx + e)) \cos (fx + e) \sqrt{c - c \sin (fx + e)}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x)

[Out] 2/15*c^4/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*((-5*A+23*B)*sin(f*x+e)*cos(f*x+e)^2+(-340*A+724*B)*sin(f*x+e)+3*B*cos(f*x+e)^4+(75*A-189*B)*cos(f*x+e)^2-300*A+684*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.53, size = 670, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -2/15*(5*(45*c^(7/2) + 138*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 285*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 544*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 630*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 812*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 630*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 544*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 285*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 138*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 45*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*A/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2)) - 2*(249*c^(7/2) + 747*c^(7/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 1611*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2896*c^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3612*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4298*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 3612*c^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 2896*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 1611*c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8 + 747*c^(7/2)*sin(f*x + e)^9/(cos(f*x + e) + 1)^9 + 249*c^(7/2)*sin(f*x + e)^10/(cos(f*x + e) + 1)^10)*B/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(7/2))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

$$3.117 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=154

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{9/2}}{3a^2c^2f} + \frac{32c^2(A-3B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2f} - \frac{(A-3B) \sec(e+fx)(c-c \sin(e+fx))^{5/2}}{3a^2}$$

[Out] $-8/3*(A-3*B)*c*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a^2/f-1/3*(A-3*B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/a^2/f-1/3*(A-B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(9/2)}/a^2/c^2/f+32/3*(A-3*B)*c^2*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a^2/f$

Rubi [A] time = 0.48, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{9/2}}{3a^2c^2f} + \frac{32c^2(A-3B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2f} - \frac{(A-3B) \sec(e+fx)(c-c \sin(e+fx))^{5/2}}{3a^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))^(5/2))/(a + a*Sin[e + f*x])^2, x]

[Out] $(32*(A - 3*B)*c^2*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - (8*(A - 3*B)*c*Sec[e + f*x]*(c - c*Sin[e + f*x])^{(3/2)})/(3*a^2*f) - ((A - 3*B)*Sec[e + f*x]*c[e + f*x]*(c - c*Sin[e + f*x])^{(5/2)})/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(9/2)})/(3*a^2*c^2*f)$

Rule 2673

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(b*(c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^2} dx &= \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{9/2}}{3a^2 c^2 f} - \frac{(A - 3B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{9/2} dx}{3a^2 c^2 f} \\ &= -\frac{(A - 3B) \sec(e + fx)(c - c \sin(e + fx))^{5/2}}{3a^2 f} - \frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{3a^2 f} \\ &= -\frac{8(A - 3B)c \sec(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 f} - \frac{(A - 3B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 f} \\ &= \frac{32(A - 3B)c^2 \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{3a^2 f} - \frac{8(A - 3B)c \sec^3(e + fx)\sqrt{c - c \sin(e + fx)}}{3a^2 f} \end{aligned}$$

Mathematica [A] time = 1.16, size = 130, normalized size = 0.84

$$\frac{c^2 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((201B - 72A) \sin(e + fx) + 6(A - 4B) \cos(2(e + fx)) + B \sin(3(e + fx)) \right)}{6a^2 f (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^2, x]

[Out] -1/6*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-50*A + 160*B + 6*(A - 4*B)*Cos[2*(e + f*x)] + (-72*A + 201*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.44, size = 110, normalized size = 0.71

$$\frac{2 \left(3(A - 4B)c^2 \cos^2(fx + e) - 2(7A - 23B)c^2 + (Bc^2 \cos^2(fx + e) - 2(9A - 25B)c^2) \sin(fx + e) \right) \sqrt{-c \sin(fx + e) + c}}{3(a^2 f \cos(fx + e) \sin(fx + e) + a^2 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2, x, algorithm="fricas")

[Out] -2/3*(3*(A - 4*B)*c^2*cos(f*x + e)^2 - 2*(7*A - 23*B)*c^2 + (B*c^2*cos(f*x + e)^2 - 2*(9*A - 25*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2, x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.27, size = 105, normalized size = 0.68

$$\frac{2c^3 (\sin(fx + e) - 1) (-B (\cos^2(fx + e)) \sin(fx + e) + (18A - 50B) \sin(fx + e) + (-3A + 12B) (\cos^2(fx + e) - 1))}{3a^2 (1 + \sin(fx + e)) \cos(fx + e) \sqrt{c - c \sin(fx + e)}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x)

[Out] $-2/3*c^3/a^2*(\sin(f*x+e)-1)/(1+\sin(f*x+e))*(-B*\cos(f*x+e)^2*\sin(f*x+e)+(18*A-50*B)*\sin(f*x+e)+(-3*A+12*B)*\cos(f*x+e)^2+14*A-46*B)/\cos(f*x+e)/(c-c*\sin(f*x+e))^(1/2)/f$

maxima [B] time = 0.46, size = 577, normalized size = 3.75

$$2 \left(\frac{11c^{\frac{5}{2}} + \frac{36c^{\frac{5}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{56c^{\frac{5}{2}} \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{108c^{\frac{5}{2}} \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{90c^{\frac{5}{2}} \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{108c^{\frac{5}{2}} \sin^5(fx+e)}{(\cos(fx+e)+1)^5} + \frac{56c^{\frac{5}{2}} \sin^6(fx+e)}{(\cos(fx+e)+1)^6} + \frac{36c^{\frac{5}{2}} \sin^7(fx+e)}{(\cos(fx+e)+1)^7} + \frac{11c^{\frac{5}{2}} \sin^8(fx+e)}{(\cos(fx+e)+1)^8}}{\left(a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin^3(fx+e)}{(\cos(fx+e)+1)^3} \right) \left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] $-2/3*((11*c^{(5/2)} + 36*c^{(5/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 56*c^{(5/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 108*c^{(5/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 90*c^{(5/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 108*c^{(5/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 56*c^{(5/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 36*c^{(5/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 11*c^{(5/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8)*A/((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*(17*c^{(5/2)} + 51*c^{(5/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 92*c^{(5/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 149*c^{(5/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 150*c^{(5/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 149*c^{(5/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 92*c^{(5/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 51*c^{(5/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 17*c^{(5/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8)*B/((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)*(sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^(5/2))/f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

$$3.118 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=115

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(A-7B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{3a^2f} + \frac{4c(A-7B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2f}$$

[Out] -1/3*(A-7*B)*sec(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f-1/3*(A-B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^2/c^2/f+4/3*(A-7*B)*c*sec(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f

Rubi [A] time = 0.41, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{7/2}}{3a^2c^2f} - \frac{(A-7B) \sec(e+fx)(c-c \sin(e+fx))^{3/2}}{3a^2f} + \frac{4c(A-7B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]))^(3/2))/(a + a*Sin[e + f*x])^2,x]

[Out] (4*(A - 7*B)*c*Sec[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f) - ((A - 7*B)*Sec[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(3*a^2*f) - ((A - B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(3*a^2*c^2*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*(c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^2} dx &= \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{7/2}}{3a^2 c^2 f} - \frac{(A - 7B) \int \sec^3(e + fx)(c - c \sin(e + fx))^{7/2} dx}{3a^2 c^2 f} \\ &= -\frac{(A - 7B) \sec(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 f} - \frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{7/2}}{3a^2 c^2 f} \\ &= \frac{4(A - 7B)c \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{3a^2 f} - \frac{(A - 7B) \sec(e + fx)(c - c \sin(e + fx))^{7/2}}{3a^2 c^2 f} \end{aligned}$$

Mathematica [A] time = 0.66, size = 113, normalized size = 0.98

$$\frac{c \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (6(A - 5B) \sin(e + fx) + 2A + 3B \cos(2(e + fx)) - 2)}{3a^2 f (\sin(e + fx) + 1)^2 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^2,x]

[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A - 23*B + 3*B*Cos[2*(e + f*x)] + 6*(A - 5*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.43, size = 80, normalized size = 0.70

$$\frac{2 \left(3 B c \cos(fx + e)^2 + 3(A - 5B)c \sin(fx + e) + (A - 13B)c \right) \sqrt{-c \sin(fx + e) + c}}{3(a^2 f \cos(fx + e) \sin(fx + e) + a^2 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 2/3*(3*B*c*cos(f*x + e)^2 + 3*(A - 5*B)*c*sin(f*x + e) + (A - 13*B)*c)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.42, size = 81, normalized size = 0.70

$$\frac{2c^2 (\sin(fx + e) - 1) (\sin(fx + e) (3A - 15B) + 3B (\cos^2(fx + e)) + A - 13B)}{3a^2 (1 + \sin(fx + e)) \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x)`

[Out] $-2/3*c^2/a^2*(\sin(f*x+e)-1)/(1+\sin(f*x+e))*(\sin(f*x+e)*(3*A-15*B)+3*B*\cos(f*x+e)^2+A-13*B)/\cos(f*x+e)/(c-c*\sin(f*x+e))^(1/2)/f$

maxima [B] time = 0.58, size = 482, normalized size = 4.19

$$2 \frac{\left(\frac{c^3}{c^2} + \frac{6c^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{12c^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3c^2 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{6c^2 \sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{c^2 \sin(fx+e)^6}{(\cos(fx+e)+1)^6} \right) A - 2 \left(5c^3 + \frac{15c^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{21c^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} \right)}{\left(a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right) \left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{3}{2}} \left(a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} \right)}$$

3 f

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^2,x, algorithm="maxima")`

[Out] $-2/3*((c^(3/2) + 6*c^(3/2)*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*c^(3/2)*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 12*c^(3/2)*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*c^(3/2)*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 6*c^(3/2)*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + c^(3/2)*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6)*A /((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^(3/2)) - 2*(5*c^(3/2) + 15*c^(3/2)*\sin(f*x + e)/(\cos(f*x + e) + 1) + 21*c^(3/2)*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*c^(3/2)*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 21*c^(3/2)*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 15*c^(3/2)*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5*c^(3/2)*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6)*B/((a^2 + 3*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^2*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^(3/2)))/f$

mupad [B] time = 17.18, size = 492, normalized size = 4.28

$$-\frac{\sqrt{c-c\left(\frac{e^{-e1i-fx1i}1i}{2}-\frac{e^{e1i+fx1i}1i}{2}\right)}\left(\frac{2Bc}{a^2f}-\frac{Bce^{e1i+fx1i}2i}{a^2f}\right)e^{e1i+fx1i}}{e^{e1i+fx1i}-i}+\frac{\sqrt{c-c\left(\frac{e^{-e1i-fx1i}1i}{2}-\frac{e^{e1i+fx1i}1i}{2}\right)}\left(\frac{2Bc}{3a^2f}-\frac{c(2i)}{3}\right)}{(e^{e1i+fx1i}-i)(e^{e1i+fx1i}+i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^2,x)`

[Out] $(\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((2*B*c)/(3*a^2*f) - (c*(2*A - 3*B))/(3*a^2*f) - (2*c*(3*A - 2*B))/(3*a^2*f) + (c*(A*2i - B*3i)*1i)/(3*a^2*f) + (c*(A*3i - B*2i)*2i)/(3*a^2*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^3) - ((c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((2*B*c)/(a^2*f) - (B*c*\exp(e*1i + f*x*1i)*2i)/(a^2*f)))/(\exp(e*1i + f*x*1i) - 1i) - (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((c*(A - B)*4i)/(a^2*f) + (c*(A*1i - B*2i))/(a^2*f) + (c*(A*1i + B*2i))/(3*a^2*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^2) - (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((4*B*c)/(a^2*f) + (c*(A*1i - B*2i)*4i)/(a^2*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

$$3.119 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=78

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{5/2}}{3a^2c^2f} - \frac{(A+5B) \sec(e+fx) \sqrt{c-c \sin(e+fx)}}{3a^2f}$$

[Out] $-1/3*(A-B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(5/2)}/a^2/c^2/f-1/3*(A+5*B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a^2/f$

Rubi [A] time = 0.31, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{5/2}}{3a^2c^2f} - \frac{(A+5B) \sec(e+fx) \sqrt{c-c \sin(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2, x]

[Out] $-((A + 5*B)*\text{Sec}[e + f*x]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(3*a^2*f) - ((A - B)*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(3*a^2*c^2*f)$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^2} dx = \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx}{a^2 c^2}$$

$$= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{3a^2 c^2 f} + \frac{(A + 5B) \int \sec^2}{3a^2 c^2 f}$$

$$= -\frac{(A + 5B) \sec(e + fx)\sqrt{c - c \sin(e + fx)}}{3a^2 f} - \frac{(A - B) \sec^3(e + fx)}{3a^2 c^2 f}$$

Mathematica [A] time = 0.28, size = 87, normalized size = 1.12

$$\frac{2\sqrt{c - c \sin(e + fx)}(A + 3B \sin(e + fx) + 2B)}{3a^2 f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^2,x]

[Out] (-2*(A + 2*B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(3*a^2*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)

fricas [A] time = 0.43, size = 60, normalized size = 0.77

$$\frac{2(3B \sin(fx + e) + A + 2B)\sqrt{-c \sin(fx + e) + c}}{3(a^2 f \cos(fx + e) \sin(fx + e) + a^2 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -2/3*(3*B*sin(f*x + e) + A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^2*f*cos(f*x + e)*sin(f*x + e) + a^2*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.31, size = 63, normalized size = 0.81

$$\frac{2c(\sin(fx + e) - 1)(3B \sin(fx + e) + A + 2B)}{3a^2(1 + \sin(fx + e))\cos(fx + e)\sqrt{c - c \sin(fx + e)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)

[Out] 2/3*c/a^2*(sin(f*x+e)-1)/(1+sin(f*x+e))*(3*B*sin(f*x+e)+A+2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.46, size = 343, normalized size = 4.40

$$2 \frac{\left(2B \left(\sqrt{c} + \frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1} + \frac{2\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} \right) \right.}{\left(a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right) \sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1}} + \frac{A \left(\sqrt{c} + \frac{2\sqrt{c}\sin(fx+e)}{(\cos(fx+e)+1)^2} + \frac{\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} \right)}{\left(a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right) \sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1}}$$

$3f$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x, alg orithm="maxima")

[Out] 2/3*(2*B*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + A*(sqrt(c) + 2*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/((a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/f

mupad [B] time = 17.51, size = 137, normalized size = 1.76

$$\frac{4e^{e^{1i+fx1i}} \sqrt{c - c \left(\frac{e^{-e^{1i-fx1i}1i}}{2} - \frac{e^{e^{1i+fx1i}1i}}{2} \right)} (B3i + 2Ae^{e^{1i+fx1i}} + 4Be^{e^{1i+fx1i}} - Be^{e^{2i+fx2i}3i})}{3a^2 f (e^{e^{1i+fx1i}} + 1i)^3 (1 + e^{e^{1i+fx1i}1i})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^2,x)

[Out] (4*exp(e*1i + f*x*1i)*(c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(B*3i + 2*A*exp(e*1i + f*x*1i) + 4*B*exp(e*1i + f*x*1i) - B*exp(e*2i + f*x*2i)*3i))/(3*a^2*f*(exp(e*1i + f*x*1i) + 1i)^3*(exp(e*1i + f*x*1i)*1i + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{-c\sin(e+fx)+c}}{\sin^2(e+fx)+2\sin(e+fx)+1} dx + \int \frac{B\sqrt{-c\sin(e+fx)+c}\sin(e+fx)}{\sin^2(e+fx)+2\sin(e+fx)+1} dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^2,x)

[Out] (Integral(A*sqrt(-c*sin(e + f*x) + c)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x) + Integral(B*sqrt(-c*sin(e + f*x) + c)*sin(e + f*x)/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x))/a**2

$$3.120 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2 \sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=135

$$\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{3a^2c^2f} - \frac{(A+B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{2a^2cf} + \frac{(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

[Out] $-1/3*(A-B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(3/2)}/a^2/c^2/f+1/4*(A+B)*\operatorname{arctanh}\left(\frac{1/2*\cos(f*x+e)*c^{(1/2)*2^{(1/2)}}/(c-c*\sin(f*x+e))^{(1/2)}}{a^2/f*2^{(1/2)}/c^{(1/2)}}\right)-1/2*(A+B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a^2/c/f$

Rubi [A] time = 0.35, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2855, 2675, 2649, 206}

$$\frac{(A-B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{3a^2c^2f} - \frac{(A+B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{2a^2cf} + \frac{(A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{2\sqrt{2}a^2\sqrt{c}f}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] $((A+B)*\operatorname{ArcTanh}[\frac{\sqrt{c}*\cos[e+f*x]}{\sqrt{2}*\sqrt{c-c*\sin[e+f*x]}}])/(2*\sqrt{2}*a^2*\sqrt{c}*f) - ((A+B)*\sec[e+f*x]*\sqrt{c-c*\sin[e+f*x]})/(2*a^2*c*f) - ((A-B)*\sec[e+f*x]^3*(c-c*\sin[e+f*x])^{(3/2)})/(3*a^2*c^2*f)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2675

Int[(cos[(e_) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_), x_Symbol] := -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m + 1/2, 2*p]

Rule 2855

Int[(cos[(e_) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)]), x_Symbol] := -Simp[(b*(c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f,

g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 \sqrt{c - c \sin(e + fx)}} dx &= \frac{\int \sec^4(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 c^2 f} + \frac{(A + B) \int \sec^2(e + fx) dx}{3a^2 c^2 f} \\ &= -\frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{2a^2 c f} - \frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 c^2 f} \\ &= -\frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{2a^2 c f} - \frac{(A - B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{3a^2 c^2 f} \\ &= \frac{(A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{2\sqrt{2} a^2 \sqrt{c} f} - \frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{2a^2 c f} \end{aligned}$$

Mathematica [C] time = 0.51, size = 176, normalized size = 1.30

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(-3(A + B)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{6a^2 f(\sin(e + fx) + \cos(e + fx))}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(-A + B) - 3*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3 + 3*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(6*a^2*f*(1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.45, size = 217, normalized size = 1.61

$$\frac{3\sqrt{2}\left((A + B)\cos(fx + e)\sin(fx + e) + (A + B)\cos(fx + e)\right)\sqrt{c} \log\left(-\frac{c\cos(fx + e)^2 + 2\sqrt{2}\sqrt{-c\sin(fx + e) + c}\sqrt{c}(\cos(fx + e) + \sin(fx + e))}{\cos(fx + e)^2 + (\cos(fx + e) + \sin(fx + e))^2}\right)}{24(a^2 c f \cos(fx + e) \sin(fx + e) + (A + B) \cos(fx + e)) \sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2), x, algorithm="fricas")

```
[Out] 1/24*(3*sqrt(2)*((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*
sqrt(c)*log(-(c*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c
)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) -
2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e
- cos(f*x + e) - 2)) - 4*(3*(A + B)*sin(f*x + e) + 5*A + B)*sqrt(-c*sin(f*x
+ e) + c))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_noste
p/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to che
ck sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_
nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(sin((f
*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t
_nostep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were
not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unabl
e to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign:
(4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi
/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to che
ck sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep-1)]index.cc index_m i_lex_is_greater Error: Bad Argument Value
```

maple [A] time = 1.78, size = 168, normalized size = 1.24

$$\frac{(\sin(fx + e) - 1) \left(-3\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) (c(1 + \sin(fx + e)))^{\frac{3}{2}} cA - 3\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1 + \sin(fx + e))} \sqrt{2}}{2\sqrt{c}} \right) \right)}{12a^2c^{\frac{5}{2}} (1 + \sin(fx + e)) \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/12/a^2*(sin(f*x+e)-1)/c^(5/2)/(1+sin(f*x+e))*(-3*2^(1/2)*arctanh(1/2*(c*(
1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(3/2)*c*A-3*2^(1/2
)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^
(3/2)*c*B+10*A*c^(5/2)+6*A*c^(5/2)*sin(f*x+e)+2*B*c^(5/2)+6*B*c^(5/2)*sin(f
*x+e))/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^2 \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^2*sqrt(-c*sin(f*x + e) + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^2 \sqrt{c - c \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(1/2),x)
```

```
[Out] Timed out
```

$$3.121 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=175

$$\frac{(5A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} - \frac{(A-B) \sec^3(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B) \cos(e+fx)}{8a^2f(c-c \sin(e+fx))^{3/2}} - \frac{(5A+B) \cos(e+fx)}{6a^2c}$$

[Out] $1/8*(5*A+B)*\cos(f*x+e)/a^2/f/(c-c*\sin(f*x+e))^{3/2}+1/16*(5*A+B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{1/2}*2^{1/2}/(c-c*\sin(f*x+e))^{1/2})/a^2/c^{3/2}/f*2^{1/2}-1/6*(5*A+B)*\sec(f*x+e)/a^2/c/f/(c-c*\sin(f*x+e))^{1/2}-1/3*(A-B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{1/2}/a^2/c^2/f$

Rubi [A] time = 0.39, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2967, 2855, 2687, 2650, 2649, 206}

$$-\frac{(A-B) \sec^3(e+fx)\sqrt{c-c \sin(e+fx)}}{3a^2c^2f} + \frac{(5A+B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2}\sqrt{c-c \sin(e+fx)}}\right)}{8\sqrt{2}a^2c^{3/2}f} + \frac{(5A+B) \cos(e+fx)}{8a^2f(c-c \sin(e+fx))^{3/2}} - \frac{(5A+B) \cos(e+fx)}{6a^2c}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)), x]

[Out] $((5*A + B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[c - c*\sin[e + f*x]])])/(8*\operatorname{Sqrt}[2]*a^2*c^{3/2}*f) + ((5*A + B)*\operatorname{Cos}[e + f*x])/(8*a^2*f*(c - c*\sin[e + f*x])^{3/2}) - ((5*A + B)*\operatorname{Sec}[e + f*x])/(6*a^2*c*f*\operatorname{Sqrt}[c - c*\sin[e + f*x]]) - ((A - B)*\operatorname{Sec}[e + f*x]^3*\operatorname{Sqrt}[c - c*\sin[e + f*x]])/(3*a^2*c^2*f)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2687

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2855

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[((b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

Rule 2967

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{3/2}} dx &= \frac{\int \sec^4(e + fx) (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx}{a^2 c^2} \\ &= -\frac{(A - B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{3a^2 c^2 f} + \frac{(5A + B) \int \frac{\sec^2(e + fx)}{\sqrt{c - c \sin(e + fx)}} dx}{6a^2 c} \\ &= -\frac{(5A + B) \sec(e + fx)}{6a^2 c f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{3a^2 c^2 f} \\ &= \frac{(5A + B) \cos(e + fx)}{8a^2 f (c - c \sin(e + fx))^{3/2}} - \frac{(5A + B) \sec(e + fx)}{6a^2 c f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B)}{3a^2 c^2 f} \\ &= \frac{(5A + B) \cos(e + fx)}{8a^2 f (c - c \sin(e + fx))^{3/2}} - \frac{(5A + B) \sec(e + fx)}{6a^2 c f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B)}{3a^2 c^2 f} \\ &= \frac{(5A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{8\sqrt{2} a^2 c^{3/2} f} + \frac{(5A + B) \cos(e + fx)}{8a^2 f (c - c \sin(e + fx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.86, size = 300, normalized size = 1.71

$$\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(3(A + B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2)), x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-12*A*Cos[e + f*x]^2 + 4*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (3 + 3*I)*(-1)^(1/4)*(5*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(24*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(3/2))
```

fricas [A] time = 0.45, size = 206, normalized size = 1.18

$$3\sqrt{2}(5A+B)\sqrt{c}\cos(fx+e)^3\log\left(\frac{-c\cos(fx+e)^2+2\sqrt{2}\sqrt{-c\sin(fx+e)+c}\sqrt{c}(\cos(fx+e)+\sin(fx+e)+1)+3c\cos(fx+e)+(c\cos(fx+e))^2}{\cos(fx+e)^2+(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right)$$

96 a²c²f c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 1/96*(3*sqrt(2)*(5*A + B)*sqrt(c)*cos(f*x + e)^3*log(-(c*cos(f*x + e))^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(3*(5*A + B)*cos(f*x + e)^2 - 2*(5*A + B)*sin(f*x + e) - 2*A - 10*B)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/16*(-3*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-A*sqrt(c)*c-B*sqrt(c)*c)/a^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^2/c/sign(tan((f*x+exp(1))/2)-1)-1/12*(-6*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+3*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+12*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-3*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+8*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-2*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-18*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-24*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+3*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+6*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-4*A*sqrt(c)*c^2+B*sqrt(c)*c^2)/a^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^3/c/sign(tan((f*x+exp(1))/2)-1)+1/16*(5*A+B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/a^2/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1))

maple [A] time = 1.56, size = 258, normalized size = 1.47

$$\sin(fx+e)\left(15\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)(c+c\sin(fx+e))^{\frac{3}{2}}cA+3\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x)`

[Out]
$$-1/48/c^{(7/2)}/a^2*(\sin(f*x+e)*(15*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(3/2)}*c*A+3*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(3/2)}*c*B-20*A*c^{(5/2)}-4*B*c^{(5/2)})+(30*A*c^{(5/2)}+6*B*c^{(5/2)})*\cos(f*x+e)^2-15*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(3/2)}*c*A-3*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)})*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(3/2)}*c*B-4*A*c^{(5/2)}-20*B*c^{(5/2)})/(1+\sin(f*x+e))/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^2 (c - c \sin(e + f x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)),x)`

[Out] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(3/2)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(3/2),x)`

[Out] Timed out

$$3.122 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=225

$$\frac{5(7A - B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{64\sqrt{2} a^2 c^{5/2} f} - \frac{(A - B) \sec^3(e + fx)}{3a^2 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{5(7A - B) \sec(e + fx)}{48a^2 c^2 f \sqrt{c - c \sin(e + fx)}} + \frac{5(7A - B) \cos(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{5/2}}$$

[Out] 5/64*(7*A-B)*cos(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+1/24*(7*A-B)*sec(f*x+e)/a^2/c/f/(c-c*sin(f*x+e))^(3/2)+5/128*(7*A-B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^2/c^(5/2)/f*2^(1/2)-5/48*(7*A-B)*sec(f*x+e)/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)-1/3*(A-B)*sec(f*x+e)^3/a^2/c^2/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.48, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2967, 2855, 2681, 2687, 2650, 2649, 206}

$$-\frac{(A - B) \sec^3(e + fx)}{3a^2 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{5(7A - B) \sec(e + fx)}{48a^2 c^2 f \sqrt{c - c \sin(e + fx)}} + \frac{5(7A - B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{64\sqrt{2} a^2 c^{5/2} f} + \frac{5(7A - B) \cos(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)), x]

[Out] (5*(7*A - B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(64*Sqrt[2]*a^2*c^(5/2)*f) + (5*(7*A - B)*Cos[e + f*x])/(64*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) + ((7*A - B)*Sec[e + f*x])/(24*a^2*c*f*(c - c*Sin[e + f*x])^(3/2)) - (5*(7*A - B)*Sec[e + f*x])/(48*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]]) - ((A - B)*Sec[e + f*x]^3)/(3*a^2*c^2*f*Sqrt[c - c*Sin[e + f*x]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2681

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(2*m + p + 1)), x] + Dist[(m + p + 1)/(a*(2*m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f}

, g, m, p}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -1] && NeQ[2*m + p + 1, 0] && IntegersQ[2*m, 2*p]

Rule 2687

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]], x_Symbol] :> -Simp[(b*(g*cos[e + f*x])^(p + 1))/(a*f*g*(p + 1)*Sqrt[a + b*sin[e + f*x]]), x] + Dist[(a*(2*p + 1))/(2*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)/(a + b*sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[((b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*sin[e + f*x])^(n - m)*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c - c \sin(e + fx))^{5/2}} dx &= \frac{\int \frac{\sec^4(e+fx)(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx}{a^2 c^2} \\
 &= -\frac{(A - B) \sec^3(e + fx)}{3a^2 c^2 f \sqrt{c - c \sin(e + fx)}} + \frac{(7A - B) \int \frac{\sec^2(e+fx)}{(c-c \sin(e+fx))^{3/2}} dx}{6a^2 c} \\
 &= \frac{(7A - B) \sec(e + fx)}{24a^2 c f (c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec^3(e + fx)}{3a^2 c^2 f \sqrt{c - c \sin(e + fx)}} + \frac{(5(7A - B) \cos(e + fx))}{64a^2 c f (c - c \sin(e + fx))^{3/2}} \\
 &= \frac{(7A - B) \sec(e + fx)}{24a^2 c f (c - c \sin(e + fx))^{3/2}} - \frac{5(7A - B) \sec(e + fx)}{48a^2 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{(7A - B) \sec(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{3/2}} + \frac{(7A - B) \sec(e + fx)}{24a^2 c f (c - c \sin(e + fx))^{3/2}} - \frac{(7A - B) \sec(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{3/2}} \\
 &= \frac{5(7A - B) \cos(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{3/2}} + \frac{(7A - B) \sec(e + fx)}{24a^2 c f (c - c \sin(e + fx))^{3/2}} - \frac{(7A - B) \sec^3(e + fx)}{3a^2 c^2 f \sqrt{c - c \sin(e + fx)}} \\
 &= \frac{5(7A - B) \cos(e + fx)}{64\sqrt{2} a^2 c^{5/2} f} + \frac{5(7A - B) \cos(e + fx)}{64a^2 c f (c - c \sin(e + fx))^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 1.41, size = 430, normalized size = 1.91

$$\left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\left(3(11A+3B)\cos^3(e+fx) + 24(B - \dots)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2)), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(11*A + 3*B)*Cos[e + f*x]^3 + 16*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 24*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 12*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - (15 + 15*I)*(-1)^(1/4)*(7*A - B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 24*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 6*(11*A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(192*a^2*f*(1 + Sin[e + f*x])^2*(c - c*Sin[e + f*x])^(5/2))

fricas [A] time = 0.46, size = 279, normalized size = 1.24

$$15\sqrt{2}\left((7A - B)\cos^3(fx + e)\sin(fx + e) - (7A - B)\cos^3(fx + e)\right)\sqrt{c}\log\left(-\frac{c\cos^2(fx+e) - 2\sqrt{2}\sqrt{-c\sin(fx+e)} + \dots}{\cos(\dots)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] -1/768*(15*sqrt(2))*((7*A - B)*cos(f*x + e)^3*sin(f*x + e) - (7*A - B)*cos(f*x + e)^3)*sqrt(c)*log(-(c*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(5*(7*A - B)*cos(f*x + e)^2 - (15*(7*A - B)*cos(f*x + e)^2 + 56*A - 8*B)*sin(f*x + e) + 8*A - 56*B)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^3*f*cos(f*x + e)^3*sin(f*x + e) - a^2*c^3*f*cos(f*x + e)^3)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(-1/48*(-15*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+9*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+33*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-15*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+22*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-10*B*c*(-sqrt(c)

```

*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-51*A*c^2*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-66*A*sqrt(c)*c*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+21*B*c^2*(-sqrt(
c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+30*B*sqrt(c)*c*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-11*A*sqrt(c)*c^
2+5*B*sqrt(c)*c^2)/a^2/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+
exp(1))/2)^2+c))+c)^3/sign(tan((f*x+exp(1))/2)-1)+1/128*(-53*A*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-29*B*(-sqrt(c)*tan((f*
x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-179*A*sqrt(c)*(-sqrt(c)*tan
((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-75*B*sqrt(c)*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-127*A*c*(-sqrt(c)*ta
n((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-55*B*c*(-sqrt(c)*tan((
f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+195*A*sqrt(c)*c*(-sqrt(c)
*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+91*B*sqrt(c)*c*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-7*A*c^2*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+B*c^2*(-sqrt(c)*
tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+67*A*c^3*(-sqrt(c)*t
an((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-121*A*sqrt(c)*c^2*(-sqr
t(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+27*B*c^3*(-sqrt
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-65*B*sqrt(c)*c^2*(
-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-15*A*sqrt(c
)*c^3-7*B*sqrt(c)*c^3)/a^2/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((
f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)+1/128*(35*A-5*B)*atan
((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqr
t(2)/sqrt(-c))/sqrt(2)/a^2/c^2/sqrt(-c)/sign(tan((f*x+exp(1))/2)-1))

```

maple [B] time = 1.74, size = 426, normalized size = 1.89

$$-86A c^{\frac{7}{2}} + 122B c^{\frac{7}{2}} + 70A c^{\frac{7}{2}} (\sin^2(fx + e)) - 10B c^{\frac{7}{2}} (\sin^2(fx + e)) + 322A c^{\frac{7}{2}} \sin(fx + e) - 46B c^{\frac{7}{2}} \sin(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2), x)

[Out]
$$-1/384/c^{(11/2)}/a^2*(-86*A*c^{(7/2)}+122*B*c^{(7/2)}+70*A*c^{(7/2)}*\sin(f*x+e)^2-10*B*c^{(7/2)}*\sin(f*x+e)^2+322*A*c^{(7/2)}*\sin(f*x+e)-46*B*c^{(7/2)}*\sin(f*x+e)-210*A*c^{(7/2)}*\sin(f*x+e)^3+30*B*c^{(7/2)}*\sin(f*x+e)^3+105*A*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)^2*c^2-15*B*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)^2*c^2-210*A*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c^2+30*B*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c^2+105*A*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*c^2-15*B*(c*(1+\sin(f*x+e)))^{(3/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e))))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*c^2)/(1+\sin(f*x+e))/(\sin(f*x+e)-1)/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c-c*sin(f*x+e))^(5/2), x, alg orithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^2 (c - c \sin(e + f x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)), x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c - c*sin(e + f*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c-c*sin(f*x+e))**(5/2), x)

[Out] Timed out

$$3.123 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=242

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{15/2}}{5a^3c^3f} - \frac{2048c^3(A-3B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3f} + \frac{512c^2(A-3B) \sec^5(e+fx)(c-c \sin(e+fx))^{1/2}}{5a^3c^3f}$$

[Out] -2048/15*(A-3*B)*c^3*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f+512/5*(A-3*B)*c^2*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/f-64/5*(A-3*B)*c*sec(f*x+e)^3*(c-c*sin(f*x+e))^(7/2)/a^3/f-16/15*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(9/2)/a^3/f-1/5*(A-3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(11/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(15/2)/a^3/c^3/f

Rubi [A] time = 0.65, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{15/2}}{5a^3c^3f} + \frac{512c^2(A-3B) \sec^3(e+fx)(c-c \sin(e+fx))^{5/2}}{5a^3f} - \frac{2048c^3(A-3B) \sec^5(e+fx)(c-c \sin(e+fx))^{1/2}}{5a^3c^3f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^3,x]

[Out] (-2048*(A - 3*B)*c^3*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) + (512*(A - 3*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*f) - (64*(A - 3*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(7/2))/(5*a^3*f) - (16*(A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(9/2))/(15*a^3*f) - ((A - 3*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(11/2))/(5*a^3*c*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(15/2))/(5*a^3*c^3*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^3} dx &= \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{15/2} dx}{a^3 c^3} \\ &= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{15/2}}{5a^3 c^3 f} - \frac{(A - 3B) \int \sec^5(e + fx)(c - c \sin(e + fx))^{15/2} dx}{5a^3 c^3 f} \\ &= -\frac{(A - 3B) \sec^3(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c f} \\ &= -\frac{16(A - 3B) \sec^3(e + fx)(c - c \sin(e + fx))^{9/2}}{15a^3 f} - \frac{(A - 3B) \sec^5(e + fx)(c - c \sin(e + fx))^{9/2}}{15a^3 f} \\ &= -\frac{64(A - 3B)c \sec^3(e + fx)(c - c \sin(e + fx))^{7/2}}{5a^3 f} - \frac{16(A - 3B) \sec^5(e + fx)(c - c \sin(e + fx))^{7/2}}{5a^3 f} \\ &= \frac{512(A - 3B)c^2 \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 f} - \frac{64(A - 3B) \sec^5(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 f} \\ &= -\frac{2048(A - 3B)c^3 \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 f} + \frac{512(A - 3B)c^2 \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 f} \end{aligned}$$

Mathematica [A] time = 4.19, size = 176, normalized size = 0.73

$$\frac{c^4(\sin(e + fx) - 1)^4 \sqrt{c - c \sin(e + fx)} (-40(137A - 402B) \cos(2(e + fx)) - 10(A - 6B) \cos(4(e + fx)) + 15600A \sin(e + fx) - 47430B \sin(e + fx) - 400A \sin(3(e + fx)) + 1335B \sin(3(e + fx)) - 3B \sin(5(e + fx)))}{120a^3 f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e +
f*x])^3, x]
```

```
[Out] -1/120*(c^4*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]]*(11298*A - 33516
*B - 40*(137*A - 402*B)*Cos[2*(e + f*x)] - 10*(A - 6*B)*Cos[4*(e + f*x)] +
15600*A*Sin[e + f*x] - 47430*B*Sin[e + f*x] - 400*A*Sin[3*(e + f*x)] + 1335
*B*Sin[3*(e + f*x)] - 3*B*Sin[5*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [A] time = 0.45, size = 166, normalized size = 0.69

$$\frac{2 \left(5(A - 6B)c^4 \cos(fx + e)^4 + 20(34A - 99B)c^4 \cos(fx + e)^2 - 8(131A - 387B)c^4 + (3Bc^4 \cos(fx + e) - 15(a^3 f \cos(fx + e)^3 - 2a^3 f \cos(fx + e) \sin(fx + e)) \right)}{15 \left(a^3 f \cos(fx + e)^3 - 2a^3 f \cos(fx + e) \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, alg
orithm="fricas")
```

```
[Out] -2/15*(5*(A - 6*B)*c^4*cos(f*x + e)^4 + 20*(34*A - 99*B)*c^4*cos(f*x + e)^2
- 8*(131*A - 387*B)*c^4 + (3*B*c^4*cos(f*x + e)^4 + 4*(25*A - 84*B)*c^4*co
```

$$\frac{s(f*x + e)^2 - 8*(125*A - 381*B)*c^4*\sin(f*x + e)*\sqrt{-c*\sin(f*x + e) + c}}{(a^3*f*\cos(f*x + e)^3 - 2*a^3*f*\cos(f*x + e)*\sin(f*x + e) - 2*a^3*f*\cos(f*x + e))}$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.47, size = 143, normalized size = 0.59

$$\frac{2c^5(\sin(fx+e)-1)(3B\sin(fx+e)(\cos^4(fx+e))+(100A-336B)(\cos^2(fx+e))\sin(fx+e)+(-1000A+3048B)\cos(fx+e))}{15a^3(1+\sin(fx+e))^2\cos(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x)

[Out]
$$\frac{-2/15*c^5/a^3*(\sin(f*x+e)-1)/(1+\sin(f*x+e))^2*(3*B*\sin(f*x+e)*\cos(f*x+e)^4+(100*A-336*B)*\cos(f*x+e)^2*\sin(f*x+e)+(-1000*A+3048*B)*\sin(f*x+e)+(5*A-30*B)*\cos(f*x+e)^4+(680*A-1980*B)*\cos(f*x+e)^2-1048*A+3096*B)/\cos(f*x+e)}{(c-c*\sin(f*x+e))^{1/2}/f}$$

maxima [B] time = 0.49, size = 945, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out]
$$\frac{2/15*((363*c^{(9/2)} + 1800*c^{(9/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 5301*c^{(9/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 11600*c^{(9/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 21343*c^{(9/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 30200*c^{(9/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 40065*c^{(9/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 40800*c^{(9/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 40065*c^{(9/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 30200*c^{(9/2)}*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 21343*c^{(9/2)}*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 11600*c^{(9/2)}*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 5301*c^{(9/2)}*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 + 1800*c^{(9/2)}*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 363*c^{(9/2)}*\sin(f*x + e)^14/(\cos(f*x + e) + 1)^14)*A/((a^3 + 5*a^3*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(9/2)} - 6*(181*c^{(9/2)} + 905*c^{(9/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + 2627*c^{(9/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 5870*c^{(9/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 10521*c^{(9/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 15351*c^{(9/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 19695*c^{(9/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 20772*c^{(9/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + 19695*c^{(9/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8 + 15351*c^{(9/2)}*\sin(f*x + e)^9/(\cos(f*x + e) + 1)^9 + 10521*c^{(9/2)}*\sin(f*x + e)^10/(\cos(f*x + e) + 1)^10 + 5870*c^{(9/2)}*\sin(f*x + e)^11/(\cos(f*x + e) + 1)^11 + 2627*c^{(9/2)}*\sin(f*x + e)^12/(\cos(f*x + e) + 1)^12 + 905*c^{(9/2)}*\sin(f*x + e)^13/(\cos(f*x + e) + 1)^13 + 181*c^{(9/2)}*$$


```
sin(f*x + e)^14/(cos(f*x + e) + 1)^14)*B/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*
x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x +
e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^
3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2
+ 1)^(9/2)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.124 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=209

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{128c^2(3A-13B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3f} - \frac{(3A-13B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3f}$$

[Out] $-128/15*(3*A-13*B)*c^2*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(3/2)}/a^3/f+32/5*(3*A-13*B)*c*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(5/2)}/a^3/f-4/5*(3*A-13*B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(7/2)}/a^3/f-1/15*(3*A-13*B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(9/2)}/a^3/c/f-1/5*(A-B)*\sec(f*x+e)^5*(c-c*\sin(f*x+e))^{(13/2)}/a^3/c^3/f$

Rubi [A] time = 0.57, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{13/2}}{5a^3c^3f} - \frac{128c^2(3A-13B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3f} - \frac{(3A-13B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^3, x]

[Out] $(-128*(3*A - 13*B)*c^2*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(3/2)})/(15*a^3*f) + (32*(3*A - 13*B)*c*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(5/2)})/(5*a^3*f) - (4*(3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(7/2)})/(5*a^3*f) - ((3*A - 13*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^{(9/2)})/(15*a^3*c*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^{(13/2)})/(5*a^3*c^3*f)$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m, n] && NeQ[m, 0]

$e + f*x]), x], x] /; FreeQ[\{a, b, c, d, e, f, A, B, n\}, x] \&\& EqQ[b*c + a*d, 0] \&\& EqQ[a^2 - b^2, 0] \&\& IntegerQ[m] \&\& !(IntegerQ[n] \&\& ((LtQ[m, 0] \& \& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^3} dx &= \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{13/2} dx}{a^3 c^3} \\ &= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{13/2}}{5a^3 c^3 f} - \frac{(3A - 13B) \int \sec^4(e + fx)(c - c \sin(e + fx))^{13/2} dx}{15a^3 c f} \\ &= -\frac{(3A - 13B) \sec^3(e + fx)(c - c \sin(e + fx))^{9/2}}{15a^3 c f} - \frac{(A - B) \int \sec^2(e + fx)(c - c \sin(e + fx))^{13/2} dx}{5a^3 f} \\ &= -\frac{4(3A - 13B) \sec^3(e + fx)(c - c \sin(e + fx))^{7/2}}{5a^3 f} - \frac{(3A - 13B) \int \sec(e + fx)(c - c \sin(e + fx))^{13/2} dx}{5a^3 f} \\ &= \frac{32(3A - 13B)c \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 f} - \frac{4(3A - 13B) \int \sec(e + fx)(c - c \sin(e + fx))^{13/2} dx}{15a^3 f} \\ &= -\frac{128(3A - 13B)c^2 \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 f} + \frac{32(3A - 13B) \int \sec(e + fx)(c - c \sin(e + fx))^{13/2} dx}{15a^3 f} \end{aligned}$$

Mathematica [A] time = 2.63, size = 158, normalized size = 0.76

$$\frac{c^3 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((2200B - 540A) \cos(2(e + fx)) + 1410A \sin(e + fx) \right)}{60a^3 f (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^3,x]

[Out] -1/60*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(1092*A - 4557*B + (-540*A + 2200*B)*Cos[2*(e + f*x)] + 5*B*Cos[4*(e + f*x)] + 1410*A*Sin[e + f*x] - 6390*B*Sin[e + f*x] - 30*A*Sin[3*(e + f*x)] + 170*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.44, size = 148, normalized size = 0.71

$$\frac{2 \left(5 B c^3 \cos(fx + e)^4 - 5 (27 A - 109 B) c^3 \cos(fx + e)^2 + 4 (51 A - 211 B) c^3 - 5 \left((3 A - 17 B) c^3 \cos(fx + e) \right) \right)}{15 \left(a^3 f \cos(fx + e)^3 - 2 a^3 f \cos(fx + e) \sin(fx + e) - 2 a^3 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 2/15*(5*B*c^3*cos(f*x + e)^4 - 5*(27*A - 109*B)*c^3*cos(f*x + e)^2 + 4*(51*A - 211*B)*c^3 - 5*((3*A - 17*B)*c^3*cos(f*x + e)^2 - 4*(9*A - 41*B)*c^3)*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.60, size = 121, normalized size = 0.58

$$\frac{2c^4(\sin(fx+e)-1)((-15A+85B)\sin(fx+e)(\cos^2(fx+e))+(180A-820B)\sin(fx+e)+5B(\cos^4(fx+e)-1))}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x)

[Out] 2/15*c^4/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*((-15*A+85*B)*sin(f*x+e)*cos(f*x+e)^2+(180*A-820*B)*sin(f*x+e)+5*B*cos(f*x+e)^4+(-135*A+545*B)*cos(f*x+e)^2+204*A-844*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 1.25, size = 854, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 2/15*(3*(23*c^(7/2)+110*c^(7/2)*sin(f*x+e)/(cos(f*x+e)+1)+318*c^(7/2)*sin(f*x+e)^2/(cos(f*x+e)+1)^2+590*c^(7/2)*sin(f*x+e)^3/(cos(f*x+e)+1)^3+1065*c^(7/2)*sin(f*x+e)^4/(cos(f*x+e)+1)^4+1220*c^(7/2)*sin(f*x+e)^5/(cos(f*x+e)+1)^5+1540*c^(7/2)*sin(f*x+e)^6/(cos(f*x+e)+1)^6+1220*c^(7/2)*sin(f*x+e)^7/(cos(f*x+e)+1)^7+1065*c^(7/2)*sin(f*x+e)^8/(cos(f*x+e)+1)^8+590*c^(7/2)*sin(f*x+e)^9/(cos(f*x+e)+1)^9+318*c^(7/2)*sin(f*x+e)^10/(cos(f*x+e)+1)^10+110*c^(7/2)*sin(f*x+e)^11/(cos(f*x+e)+1)^11+23*c^(7/2)*sin(f*x+e)^12/(cos(f*x+e)+1)^12)*A/((a^3+5*a^3*sin(f*x+e)/(cos(f*x+e)+1)+10*a^3*sin(f*x+e)^2/(cos(f*x+e)+1)^2+10*a^3*sin(f*x+e)^3/(cos(f*x+e)+1)^3+5*a^3*sin(f*x+e)^4/(cos(f*x+e)+1)^4+a^3*sin(f*x+e)^5/(cos(f*x+e)+1)^5)*(sin(f*x+e)^2/(cos(f*x+e)+1)^2+1)^(7/2))-2*(147*c^(7/2)+735*c^(7/2)*sin(f*x+e)/(cos(f*x+e)+1)+1992*c^(7/2)*sin(f*x+e)^2/(cos(f*x+e)+1)^2+4015*c^(7/2)*sin(f*x+e)^3/(cos(f*x+e)+1)^3+6605*c^(7/2)*sin(f*x+e)^4/(cos(f*x+e)+1)^4+8370*c^(7/2)*sin(f*x+e)^5/(cos(f*x+e)+1)^5+9520*c^(7/2)*sin(f*x+e)^6/(cos(f*x+e)+1)^6+8370*c^(7/2)*sin(f*x+e)^7/(cos(f*x+e)+1)^7+6605*c^(7/2)*sin(f*x+e)^8/(cos(f*x+e)+1)^8+4015*c^(7/2)*sin(f*x+e)^9/(cos(f*x+e)+1)^9+1992*c^(7/2)*sin(f*x+e)^10/(cos(f*x+e)+1)^10+735*c^(7/2)*sin(f*x+e)^11/(cos(f*x+e)+1)^11+147*c^(7/2)*sin(f*x+e)^12/(cos(f*x+e)+1)^12)*B/((a^3+5*a^3*sin(f*x+e)/(cos(f*x+e)+1)+10*a^3*sin(f*x+e)^2/(cos(f*x+e)+1)^2+10*a^3*sin(f*x+e)^3/(cos(f*x+e)+1)^3+5*a^3*sin(f*x+e)^4/(cos(f*x+e)+1)^4+a^3*sin(f*x+e)^5/(cos(f*x+e)+1)^5)*(sin(f*x+e)^2/(cos(f*x+e)+1)^2+1)^(7/2)))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.125 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=160

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11B) \sec^3(e+fx)(c-c \sin(e+fx))^{7/2}}{5a^3cf} + \frac{8(A-11B) \sec^3(e+fx)}{5a^3cf}$$

[Out] $-32/15*(A-11*B)*c*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(3/2)}/a^3/f+8/5*(A-11*B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(5/2)}/a^3/f-1/5*(A-11*B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(7/2)}/a^3/c/f-1/5*(A-B)*\sec(f*x+e)^5*(c-c*\sin(f*x+e))^{(11/2)}/a^3/c^3/f$

Rubi [A] time = 0.48, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{11/2}}{5a^3c^3f} - \frac{(A-11B) \sec^3(e+fx)(c-c \sin(e+fx))^{7/2}}{5a^3cf} + \frac{8(A-11B) \sec^3(e+fx)}{5a^3cf}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^3, x]

[Out] $(-32*(A - 11*B)*c*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(15*a^3*f) + (8*(A - 11*B)*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(5*a^3*f) - ((A - 11*B)*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(5*a^3*c*f) - ((A - B)*\text{Sec}[e + f*x]^5*(c - c*\text{Sin}[e + f*x])^{(11/2)})/(5*a^3*c^3*f)$

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*c + a*d)*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d

, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^3} dx &= \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{11/2} dx}{a^3 c^3} \\ &= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c^3 f} - \frac{(A - 11B) \int \sec^5(e + fx)(c - c \sin(e + fx))^{11/2} dx}{5a^3 c^3 f} \\ &= -\frac{(A - 11B) \sec^3(e + fx)(c - c \sin(e + fx))^{7/2}}{5a^3 c f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c f} \\ &= \frac{8(A - 11B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 f} - \frac{(A - 11B) \sec^5(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c f} \\ &= -\frac{32(A - 11B)c \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 f} + \frac{8(A - 11B) \sec^5(e + fx)(c - c \sin(e + fx))^{11/2}}{5a^3 c f} \end{aligned}$$

Mathematica [A] time = 1.21, size = 132, normalized size = 0.82

$$\frac{c^2 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (5(8A - 133B) \sin(e + fx) - 30(A - 8B) \cos(2(e + fx)))}{30a^3 f (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^3, x]

[Out] -1/30*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(58*A - 488*B - 30*(A - 8*B)*Cos[2*(e + f*x)] + 5*(8*A - 133*B)*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)]))/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.44, size = 125, normalized size = 0.78

$$\frac{2 \left(15(A - 8B)c^2 \cos^2(fx + e) - 2(11A - 91B)c^2 - 5 \left(3Bc^2 \cos^2(fx + e) + 2(A - 17B)c^2 \right) \sin(fx + e) \right) \sqrt{c - c \sin(fx + e)}}{15 \left(a^3 f \cos^3(fx + e) - 2a^3 f \cos(fx + e) \sin(fx + e) - 2a^3 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -2/15*(15*(A - 8*B)*c^2*cos(f*x + e)^2 - 2*(11*A - 91*B)*c^2 - 5*(3*B*c^2*cos(f*x + e)^2 + 2*(A - 17*B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.60, size = 105, normalized size = 0.66

$$\frac{2c^3(\sin(fx+e)-1)(15B(\cos^2(fx+e))\sin(fx+e)+(10A-170B)\sin(fx+e)+(-15A+120B)(\cos^2(fx+e)-1))}{15a^3(1+\sin(fx+e))^2\cos(fx+e)\sqrt{c-c\sin(fx+e)}}f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x)

[Out] 2/15*c^3/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(15*B*cos(f*x+e)^2*sin(f*x+e)+(10*A-170*B)*sin(f*x+e)+(-15*A+120*B)*cos(f*x+e)^2+22*A-182*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.48, size = 761, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 2/15*((7*c^(5/2)+20*c^(5/2)*sin(f*x+e)/(cos(f*x+e)+1)+95*c^(5/2)*sin(f*x+e)^2/(cos(f*x+e)+1)^2+80*c^(5/2)*sin(f*x+e)^3/(cos(f*x+e)+1)^3+250*c^(5/2)*sin(f*x+e)^4/(cos(f*x+e)+1)^4+120*c^(5/2)*sin(f*x+e)^5/(cos(f*x+e)+1)^5+250*c^(5/2)*sin(f*x+e)^6/(cos(f*x+e)+1)^6+80*c^(5/2)*sin(f*x+e)^7/(cos(f*x+e)+1)^7+95*c^(5/2)*sin(f*x+e)^8/(cos(f*x+e)+1)^8+20*c^(5/2)*sin(f*x+e)^9/(cos(f*x+e)+1)^9+7*c^(5/2)*sin(f*x+e)^10/(cos(f*x+e)+1)^10)*A/((a^3+5*a^3*sin(f*x+e)/(cos(f*x+e)+1)+10*a^3*sin(f*x+e)^2/(cos(f*x+e)+1)^2+10*a^3*sin(f*x+e)^3/(cos(f*x+e)+1)^3+5*a^3*sin(f*x+e)^4/(cos(f*x+e)+1)^4+a^3*sin(f*x+e)^5/(cos(f*x+e)+1)^5)*(sin(f*x+e)^2/(cos(f*x+e)+1)^2+1)^(5/2))-2*(31*c^(5/2)+155*c^(5/2)*sin(f*x+e)/(cos(f*x+e)+1)+395*c^(5/2)*sin(f*x+e)^2/(cos(f*x+e)+1)^2+680*c^(5/2)*sin(f*x+e)^3/(cos(f*x+e)+1)^3+1030*c^(5/2)*sin(f*x+e)^4/(cos(f*x+e)+1)^4+1050*c^(5/2)*sin(f*x+e)^5/(cos(f*x+e)+1)^5+1030*c^(5/2)*sin(f*x+e)^6/(cos(f*x+e)+1)^6+680*c^(5/2)*sin(f*x+e)^7/(cos(f*x+e)+1)^7+395*c^(5/2)*sin(f*x+e)^8/(cos(f*x+e)+1)^8+155*c^(5/2)*sin(f*x+e)^9/(cos(f*x+e)+1)^9+31*c^(5/2)*sin(f*x+e)^10/(cos(f*x+e)+1)^10)*B/((a^3+5*a^3*sin(f*x+e)/(cos(f*x+e)+1)+10*a^3*sin(f*x+e)^2/(cos(f*x+e)+1)^2+10*a^3*sin(f*x+e)^3/(cos(f*x+e)+1)^3+5*a^3*sin(f*x+e)^4/(cos(f*x+e)+1)^4+a^3*sin(f*x+e)^5/(cos(f*x+e)+1)^5)*(sin(f*x+e)^2/(cos(f*x+e)+1)^2+1)^(5/2))/f

mupad [B] time = 22.79, size = 904, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A+B*sin(e+f*x))*(c-c*sin(e+f*x))^(5/2))/(a+a*sin(e+f*x))^3,x)

[Out] ((c-c*(exp(-e*1i-f*x*1i)*1i)/2-(exp(e*1i+f*x*1i)*1i)/2))^(1/2)*((2*B*c^2)/(a^3*f)-(B*c^2*exp(e*1i+f*x*1i)*2i)/(a^3*f))/(exp(e*1i+f*x*1i)-1i)-(exp(e*1i+f*x*1i)*(c-c*(exp(-e*1i-f*x*1i)*1i)/2-(exp(e*1i+f*x*1i)*1i)/2))^(1/2)*((c^2*(A*2i-B*7i)*1i)/(3*a^3*f)-(2*c^2*(7*A-12*B))/(3*a^3*f)+(c^2*(A*23i-B*28i)*2i)/(3*a^3*f)-(c^2*(42*A-67*B))/(15*a^3*f)+(2*B*c^2)/(3*a^3*f)))/((exp(e*1i+f*x*1i)-1i)*(exp(e*1i+f*x*1i)+1i))

$$\begin{aligned}
& i + f*x*1i) + 1i)^3) + (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i) \\
&)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)*((c^2*(A*1i - B*4i)*4i)/(a^3*f) + (\\
& 4*B*c^2)/(a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)) - \\
& (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i) \\
& i)*1i)/2))^{(1/2)*((8*c^2*(A*1i - B*1i))/(a^3*f) + (c^2*(A*1i - B*3i))/(2*a^ \\
& 3*f) + (c^2*(A*11i - B*1i))/(10*a^3*f) + (c^2*(12*A - 17*B)*1i)/(4*a^3*f) + \\
& (c^2*(52*A - 47*B)*1i)/(4*a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + \\
& f*x*1i) + 1i)^4) + (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 \\
& - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)*((c^2*(A*1i - B*4i))/(a^3*f) + (c^2*(A* \\
& 5i - B*4i))/(3*a^3*f) + (c^2*(A - 2*B)*8i)/(a^3*f)))/((\exp(e*1i + f*x*1i) - \\
& 1i)*(\exp(e*1i + f*x*1i) + 1i)^2) + (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i \\
& - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)*((c^2*(A*2i - B*5i)*1i) \\
&)/(5*a^3*f) - (c^2*(4*A - 3*B))/(a^3*f) - (c^2*(2*A - 5*B))/(5*a^3*f) + (c^ \\
& 2*(A*4i - B*3i)*1i)/(a^3*f) - (c^2*(10*A - 11*B))/(5*a^3*f) + (c^2*(A*10i - \\
& B*11i)*1i)/(5*a^3*f) + (2*B*c^2)/(5*a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(e \\
& xp(e*1i + f*x*1i) + 1i)^5)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**3,x)

[Out] Timed out

$$3.126 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=121

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{9/2}}{5a^3c^3f} - \frac{(A+9B) \sec^3(e+fx)(c-c \sin(e+fx))^{5/2}}{5a^3cf} + \frac{4(A+9B) \sec^3(e+fx)}{15a^3}$$

[Out] 4/15*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(3/2)/a^3/f-1/5*(A+9*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(5/2)/a^3/c/f-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(9/2)/a^3/c^3/f

Rubi [A] time = 0.41, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2967, 2855, 2674, 2673}

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{9/2}}{5a^3c^3f} - \frac{(A+9B) \sec^3(e+fx)(c-c \sin(e+fx))^{5/2}}{5a^3cf} + \frac{4(A+9B) \sec^3(e+fx)}{15a^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^3,x]

[Out] (4*(A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(3/2))/(15*a^3*f) - ((A + 9*B)*Sec[e + f*x]^3*(c - c*Sin[e + f*x])^(5/2))/(5*a^3*c*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(9/2))/(5*a^3*c^3*f)

Rule 2673

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m - 1)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[2*m + p - 1, 0] && NeQ[m, 1]

Rule 2674

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m - 1))/(f*g*(m + p)), x] + Dist[(a*(2*m + p - 1))/(m + p), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[Simplify[(2*m + p - 1)/2], 0] && NeQ[m + p, 0]

Rule 2855

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> -Simp[(b*(c + a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &

& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^3} dx &= \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2} dx}{a^3 c^3} \\ &= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{9/2}}{5a^3 c^3 f} + \frac{(A + 9B) \int \sec^5(e + fx)(c - c \sin(e + fx))^{9/2} dx}{5a^3 c^3 f} \\ &= -\frac{(A + 9B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 c f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{9/2}}{5a^3 c^3 f} \\ &= \frac{4(A + 9B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 f} - \frac{(A + 9B) \sec^5(e + fx)(c - c \sin(e + fx))^{9/2}}{15a^3 c^3 f} \end{aligned}$$

Mathematica [A] time = 0.70, size = 113, normalized size = 0.93

$$\frac{c\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (10(A + 3B) \sin(e + fx) - 2A - 15B \cos(2(e + fx)) + 15a^3 f (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right))}{15a^3 f (\sin(e + fx) + 1)^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^3, x]

[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A + 27*B - 15*B*Cos[2*(e + f*x)]) + 10*(A + 3*B)*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.43, size = 95, normalized size = 0.79

$$\frac{2 \left(15 B c \cos(fx + e)^2 - 5(A + 3B)c \sin(fx + e) + (A - 21B)c \right) \sqrt{-c \sin(fx + e) + c}}{15 \left(a^3 f \cos(fx + e)^3 - 2a^3 f \cos(fx + e) \sin(fx + e) - 2a^3 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 2/15*(15*B*c*cos(f*x + e)^2 - 5*(A + 3*B)*c*sin(f*x + e) + (A - 21*B)*c)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.46, size = 83, normalized size = 0.69

$$\frac{2c^2 \left(\sin(fx + e) - 1 \right) \left(\sin(fx + e) (5A + 15B) - 15B \left(\cos^2(fx + e) \right) - A + 21B \right)}{15a^3 \left(1 + \sin(fx + e) \right)^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sin(f*x+e))*(c-c*\sin(f*x+e))^{(3/2)}/(a+a*\sin(f*x+e))^3,x)$

[Out] $-2/15*c^2/a^3*(\sin(f*x+e)-1)/(1+\sin(f*x+e))^2*(\sin(f*x+e)*(5*A+15*B)-15*B*\cos(f*x+e)^2-A+21*B)/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$

maxima [B] time = 0.48, size = 663, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\sin(f*x+e))*(c-c*\sin(f*x+e))^{(3/2)}/(a+a*\sin(f*x+e))^3,x, \text{algorithm}="maxima")$

[Out] $2/15*((c^{(3/2)} - 10*c^{(3/2)}*\sin(f*x + e))/(\cos(f*x + e) + 1) + 4*c^{(3/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 - 30*c^{(3/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 6*c^{(3/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 - 30*c^{(3/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 4*c^{(3/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 - 10*c^{(3/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + c^{(3/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8)*A/((a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(3/2)} - 6*(c^{(3/2)} + 5*c^{(3/2)}*\sin(f*x + e))/(\cos(f*x + e) + 1) + 14*c^{(3/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*c^{(3/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 26*c^{(3/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 15*c^{(3/2)}*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 14*c^{(3/2)}*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 5*c^{(3/2)}*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7 + c^{(3/2)}*\sin(f*x + e)^8/(\cos(f*x + e) + 1)^8)*B/((a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(3/2)})/f$

mupad [B] time = 19.16, size = 683, normalized size = 5.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(((A + B*\sin(e + f*x))*(c - c*\sin(e + f*x))^{(3/2)})/(a + a*\sin(e + f*x))^3,x)$

[Out] $(\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)}*((4*B*c)/(5*a^3*f) - (2*c*(2*A - 3*B))/(5*a^3*f) - (4*c*(3*A - 2*B))/(5*a^3*f) + (c*(A*2i - B*3i)*2i)/(5*a^3*f) + (c*(A*3i - B*2i)*4i)/(5*a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^5) - (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)}*((2*B*c)/(3*a^3*f) + (c*(A*2i - B*5i)*2i)/(3*a^3*f) - (2*c*(10*A - 13*B))/(3*a^3*f) + (c*(A*8i - B*13i)*2i)/(15*a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^3) + (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)}*((c*(2*A - 3*B)*1i)/(3*a^3*f) - (B*c*1i)/(a^3*f) + (2*c*(A*1i - B*3i))/(a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^2) - (\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)}*((c*(A - B)*4i)/(a^3*f) - (B*c*1i)/(2*a^3*f) + (c*(8*A - 3*B)*1i)/(10*a^3*f) + (c*(A*1i - B*2i))/(a^3*f) + (c*(A*7i - B*6i))/(a^3*f)))/((\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i)^4) + (4*B*c*\exp(e*1i + f*x*1i)*(c - c*((\exp(- e*1i - f*x*1i)*1i)/2 - (\exp(e*1i + f*x*1i)*1i)/2))^{(1/2)})/(a^3*f*(\exp(e*1i + f*x*1i) - 1i)*(\exp(e*1i + f*x*1i) + 1i))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**3,x)

[Out] Timed out

$$3.127 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=85

$$-\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{7/2}}{5a^3c^3f} - \frac{(3A+7B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3cf}$$

[Out] $-1/15*(3*A+7*B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(3/2)}/a^3/c/f-1/5*(A-B)*\sec(f*x+e)^5*(c-c*\sin(f*x+e))^{(7/2)}/a^3/c^3/f$

Rubi [A] time = 0.32, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2967, 2855, 2673}

$$-\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{7/2}}{5a^3c^3f} - \frac{(3A+7B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{15a^3cf}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])* \text{Sqrt}[c - c*\text{Sin}[e + f*x]]/(a + a*\text{Sin}[e + f*x])^3, x]$

[Out] $-((3*A + 7*B)*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(15*a^3*c*f) - ((A - B)*\text{Sec}[e + f*x]^5*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(5*a^3*c^3*f)$

Rule 2673

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_.)}, x_Symbol] :> \text{Simp}[(b*(g*\cos[e + f*x])^{(p + 1)}*(a + b*\sin[e + f*x])^{(m - 1)})/(f*g*(m - 1)), x] /; \text{FreeQ}\{a, b, e, f, g, m, p\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{EqQ}[2*m + p - 1, 0] \ \&\& \ \text{NeQ}[m, 1]$

Rule 2855

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]))^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -\text{Simp}[(b*c + a*d)*(g*\cos[e + f*x])^{(p + 1)}*(a + b*\sin[e + f*x])^m/(a*f*g*(p + 1)), x] + \text{Dist}[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), \text{Int}[(g*\cos[e + f*x])^{(p + 2)}*(a + b*\sin[e + f*x])^{(m - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, -1] \ \&\& \ \text{LtQ}[p, -1]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]*(c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}), x_Symbol] :> \text{Dist}[a^m*c^m, \text{Int}[\cos[e + f*x]^{(2*m)}*(c + d*\sin[e + f*x])^{(n - m)}*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]) \ || \ \text{LtQ}[0, n] \ || \ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\int \frac{(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^3} dx = \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx}{a^3 c^3}$$

$$= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{7/2}}{5a^3 c^3 f} + \frac{(3A + 7B) \int \sec^5(e + fx)(c - c \sin(e + fx))^{7/2} dx}{15a^3 c^3 f}$$

$$= -\frac{(3A + 7B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{15a^3 c f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{7/2}}{15a^3 c^3 f}$$

Mathematica [A] time = 0.30, size = 89, normalized size = 1.05

$$-\frac{2\sqrt{c - c \sin(e + fx)}(3A + 5B \sin(e + fx) + 2B)}{15a^3 f \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^3,x]

[Out] (-2*(3*A + 2*B + 5*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(15*a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.44, size = 77, normalized size = 0.91

$$\frac{2(5B \sin(fx + e) + 3A + 2B)\sqrt{-c \sin(fx + e) + c}}{15(a^3 f \cos(fx + e)^3 - 2a^3 f \cos(fx + e) \sin(fx + e) - 2a^3 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 2/15*(5*B*sin(f*x + e) + 3*A + 2*B)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.41, size = 65, normalized size = 0.76

$$\frac{2c(\sin(fx + e) - 1)(5B \sin(fx + e) + 3A + 2B)}{15a^3(1 + \sin(fx + e))^2 \cos(fx + e) \sqrt{c - c \sin(fx + e)}} f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)

[Out] 2/15*c/a^3*(sin(f*x+e)-1)/(1+sin(f*x+e))^2*(5*B*sin(f*x+e)+3*A+2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f

maxima [B] time = 0.51, size = 505, normalized size = 5.94

$$2 \frac{\left(2B \left(\sqrt{c} + \frac{5\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{5\sqrt{c}\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{\sqrt{c}\sin(fx+e)^6}{(\cos(fx+e)+1)^6} \right) + \frac{3A \left(\sqrt{c} + \frac{3\sqrt{c}\sin(fx+e)}{\cos(fx+e)+1} + \frac{5\sqrt{c}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10\sqrt{c}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5\sqrt{c}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{\sqrt{c}\sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right)}{\left(a^3 + \frac{5a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right) \sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1} + \left(a^3 + \frac{5a^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{10a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{5a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a^3\sin(fx+e)^5}{(\cos(fx+e)+1)^5} \right) \sqrt{\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1}}{15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 2/15*(2*B*(sqrt(c) + 5*sqrt(c)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*sqrt(c)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)) + 3*A*(sqrt(c) + 3*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3*sqrt(c)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + sqrt(c)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6)/((a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)*sqrt(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)))/f

mupad [B] time = 17.54, size = 479, normalized size = 5.64

$$\frac{e^{e li + f x li} \sqrt{c - c \left(\frac{e^{-e li - f x li} li}{2} - \frac{e^{e li + f x li} li}{2} \right) \left(\frac{8B}{5a^3 f} - \frac{16A - 8B}{10a^3 f} + \frac{(A16i - B8i) li}{10a^3 f} \right)}{(e^{e li + f x li} - i) (e^{e li + f x li} + i)^5} - \frac{e^{e li + f x li} \sqrt{c - c \left(\frac{e^{-e li - f x li} li}{2} - \frac{e^{e li + f x li} li}{2} \right)}}{(e^{e li + f x li} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^3,x)

[Out] (exp(e*li + f*x*li)*(c - c*((exp(- e*li - f*x*li)*li)/2 - (exp(e*li + f*x*li)*li)/2))^(1/2)*((8*B)/(5*a^3*f) - (16*A - 8*B)/(10*a^3*f) + ((A*16i - B*8i)*li)/(10*a^3*f))/((exp(e*li + f*x*li) - li)*(exp(e*li + f*x*li) + li)^5) - (exp(e*li + f*x*li)*(c - c*((exp(- e*li - f*x*li)*li)/2 - (exp(e*li + f*x*li)*li)/2))^(1/2)*((4*B)/(3*a^3*f) - (16*A - 16*B)/(30*a^3*f) + ((A*80i - B*120i)*li)/(30*a^3*f))/((exp(e*li + f*x*li) - li)*(exp(e*li + f*x*li) + li)^3) - (exp(e*li + f*x*li)*(c - c*((exp(- e*li - f*x*li)*li)/2 - (exp(e*li + f*x*li)*li)/2))^(1/2)*((A*16i - B*16i)/(40*a^3*f) - (B*li)/(a^3*f) + (A*80i - B*80i)/(40*a^3*f) + ((160*A - 120*B)*li)/(40*a^3*f))/((exp(e*li + f*x*li) - li)*(exp(e*li + f*x*li) + li)^4) - (B*exp(e*li + f*x*li)*(c - c*((exp(- e*li - f*x*li)*li)/2 - (exp(e*li + f*x*li)*li)/2))^(1/2)*8i)/(3*a^3*f*(exp(e*li + f*x*li) - li)*(exp(e*li + f*x*li) + li)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^3,x)

[Out] Timed out

$$3.128 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3 \sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=174

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{5/2}}{5a^3c^3f} - \frac{(A+B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A+B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{4a^3cf}$$

[Out] $-1/6*(A+B)*\sec(f*x+e)^3*(c-c*\sin(f*x+e))^{(3/2)}/a^3/c^2/f-1/5*(A-B)*\sec(f*x+e)^5*(c-c*\sin(f*x+e))^{(5/2)}/a^3/c^3/f+1/8*(A+B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*c^{(1/2)*2^{(1/2)}}/(c-c*\sin(f*x+e))^{(1/2)})/a^3/f*2^{(1/2)}/c^{(1/2)}-1/4*(A+B)*\sec(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a^3/c/f$

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2967, 2855, 2675, 2649, 206}

$$\frac{(A-B) \sec^5(e+fx)(c-c \sin(e+fx))^{5/2}}{5a^3c^3f} - \frac{(A+B) \sec^3(e+fx)(c-c \sin(e+fx))^{3/2}}{6a^3c^2f} - \frac{(A+B) \sec(e+fx)\sqrt{c-c \sin(e+fx)}}{4a^3cf}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^3*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])]$, x]

[Out] $((A + B)*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Cos}[e + f*x])/(\text{Sqrt}[2]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])])/(4*\text{Sqrt}[2]*a^3*\text{Sqrt}[c]*f) - ((A + B)*\text{Sec}[e + f*x]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(4*a^3*c*f) - ((A + B)*\text{Sec}[e + f*x]^3*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(6*a^3*c^2*f) - ((A - B)*\text{Sec}[e + f*x]^5*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(5*a^3*c^3*f)$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*\sin[(c_ + (d_)*(x_))]], x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, (b*\text{Cos}[c + d*x])/(\text{Sqrt}[a + b*\text{Sin}[c + d*x]])], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2675

$\text{Int}[(\cos[(e_ + (f_)*(x_)]*(g_))^{(p_)*((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}, x_Symbol] \rightarrow -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)}*(a + b*\text{Sin}[e + f*x])^m)/(a*f*g*(p+1)), x] + \text{Dist}[(a*(m+p+1))/(g^2*(p+1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p+2)}*(a + b*\text{Sin}[e + f*x])^{(m-1)}, x], x] /;$ FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[p, -2*m] && IntegersQ[m + 1/2, 2*p]

Rule 2855

$\text{Int}[(\cos[(e_ + (f_)*(x_)]*(g_))^{(p_)*((a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)*((c_ + (d_)*\sin[(e_ + (f_)*(x_))]), x_Symbol] \rightarrow -\text{Simp}[(b*c + a*d)*(g*\text{Cos}[e + f*x])^{(p+1)}*(a + b*\text{Sin}[e + f*x])^m)/(a*f*g*(p+1)), x] + \text{Dist}[(b*(a*d*m + b*c*(m+p+1)))/(a*g^2*(p+1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p+2)}*(a + b*\text{Sin}[e + f*x])^{(m-1)}, x], x] /;$ FreeQ[{a, b, c, d, e, f,

g}], x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 \sqrt{c - c \sin(e + fx)}} dx = \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx}{a^3 c^3}$$

$$= -\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 c^3 f} + \frac{(A + B) \int \sec^4(e + fx)(c - c \sin(e + fx))^{5/2} dx}{5a^3 c^3 f}$$

$$= -\frac{(A + B) \sec^3(e + fx)(c - c \sin(e + fx))^{3/2}}{6a^3 c^2 f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{5/2}}{5a^3 c^3 f}$$

$$= -\frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{4a^3 c f} - \frac{(A + B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{6a^3 c^3 f}$$

$$= -\frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{4a^3 c f} - \frac{(A + B) \sec^3(e + fx)(c - c \sin(e + fx))^{5/2}}{6a^3 c^3 f}$$

$$= \frac{(A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{4\sqrt{2} a^3 \sqrt{c} f} - \frac{(A + B) \sec(e + fx) \sqrt{c - c \sin(e + fx)}}{4a^3 c f}$$

Mathematica [C] time = 0.77, size = 204, normalized size = 1.17

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(-15(A + B)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(-A + B) - 10*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 15*(A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (15 + 15*I)*(-1)^(1/4)*(A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(60*a^3*f*(1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.44, size = 262, normalized size = 1.51

$$15 \sqrt{2} \left((A + B) \cos^3(fx + e) - 2(A + B) \cos(fx + e) \sin(fx + e) - 2(A + B) \cos(fx + e) \right) \sqrt{c} \log \left(-\frac{c \cos(fx + e)^2}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{240} \cdot (15 \sqrt{2}) \cdot ((A+B) \cos(fx+e))^3 - 2(A+B) \cos(fx+e) \sin(fx+e) - 2(A+B) \cos(fx+e) \sqrt{c} \log(-c \cos(fx+e)^2 + 2\sqrt{2} \sqrt{-c \sin(fx+e) + c} \sqrt{c} (\cos(fx+e) + \sin(fx+e) + 1) + 3c \cos(fx+e) + (c \cos(fx+e) - 2c) \sin(fx+e) + 2c) / (\cos(fx+e)^2 + (\cos(fx+e) + 2) \sin(fx+e) - \cos(fx+e) - 2) - 4(15(A+B) \cos(fx+e)^2 - 40(A+B) \sin(fx+e) - 52A - 28B) \sqrt{-c \sin(fx+e) + c} / (a^3 c f \cos(fx+e)^3 - 2a^3 c f \cos(fx+e) \sin(fx+e) - 2a^3 c f \cos(fx+e))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2)
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
 (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
 e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
 *pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
 *pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
 check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-
 4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
 Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check s
 ign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nost
 ep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes constan
 t sign by intervals (correct if the argument is real):Check [abs(sin((f*t_n
 ostep+exp(1))/2-pi/4))]Unable to check sign: (8*pi/t_nostep/2)>(-8*pi/t_nos
 tep/2)Discontinuities at zeroes of sin((f*t_nostep+exp(1))/2-pi/4) were not
 checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
 check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi
 i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2
)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/
 2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
 ign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes cons
 tant sign by intervals (correct if the argument is real):Check [abs(t_noste
 p-1)]Evaluation time: 0.43index.cc index_m i_lex_is_greater Error: Bad Argu
 ment Value

maple [A] time = 1.94, size = 200, normalized size = 1.15

$$\frac{(\sin(fx+e)-1) \left(74c^{\frac{9}{2}}A + 80Ac^{\frac{9}{2}} \sin(fx+e) + 30Ac^{\frac{9}{2}} (\sin^2(fx+e)) - 15\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{c(1+\sin(fx+e))} \sqrt{2}}{2\sqrt{c}} \right) \right)}{120a^3c^{\frac{9}{2}} (1+\sin(fx+e))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x)

[Out] $\frac{1}{120} \cdot \frac{1}{a^3} \cdot \frac{(\sin(fx+e)-1)}{c^{9/2}} / (1+\sin(fx+e))^2 \cdot (74c^{9/2}A + 80Ac^{9/2} \sin(fx+e) + 30Ac^{9/2} \sin^2(fx+e) - 15\sqrt{2} \operatorname{arctanh}(1/2 \cdot (c(1+\sin(fx+e)))^{1/2} \cdot 2^{1/2} / c^{1/2}))^{5/2} \cdot c^2 A + 26c^{9/2} B + 80$

```
*B*c^(9/2)*sin(f*x+e)+30*B*c^(9/2)*sin(f*x+e)^2-15*2^(1/2)*arctanh(1/2*(c*(1+sin(f*x+e)))^(1/2)*2^(1/2)/c^(1/2))*(c*(1+sin(f*x+e)))^(5/2)*c^2*B)/cos(f*x+e)/(c-c*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^3 \sqrt{c - c \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(1/2),x)
```

[Out] Timed out

$$3.129 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=224

$$\frac{(7A + 3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{16\sqrt{2} a^3 c^{3/2} f} - \frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{3/2}}{5a^3 c^3 f} - \frac{(7A + 3B) \sec^3(e + fx) \sqrt{c}}{30a^3 c^2 f}$$

[Out] 1/16*(7*A+3*B)*cos(f*x+e)/a^3/f/(c-c*sin(f*x+e))^(3/2)-1/5*(A-B)*sec(f*x+e)^5*(c-c*sin(f*x+e))^(3/2)/a^3/c^3/f+1/32*(7*A+3*B)*arctanh(1/2*cos(f*x+e)*c^(1/2)*2^(1/2)/(c-c*sin(f*x+e))^(1/2))/a^3/c^(3/2)/f*2^(1/2)-1/12*(7*A+3*B)*sec(f*x+e)/a^3/c/f/(c-c*sin(f*x+e))^(1/2)-1/30*(7*A+3*B)*sec(f*x+e)^3*(c-c*sin(f*x+e))^(1/2)/a^3/c^2/f

Rubi [A] time = 0.48, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.184$, Rules used = {2967, 2855, 2675, 2687, 2650, 2649, 206}

$$\frac{(A - B) \sec^5(e + fx)(c - c \sin(e + fx))^{3/2}}{5a^3 c^3 f} - \frac{(7A + 3B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{30a^3 c^2 f} + \frac{(7A + 3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e+fx)}{\sqrt{2} \sqrt{c-c \sin(e+fx)}}\right)}{16\sqrt{2} a^3 c^{3/2} f}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(3/2)), x]

[Out] ((7*A + 3*B)*ArcTanh[(Sqrt[c]*Cos[e + f*x])/(Sqrt[2]*Sqrt[c - c*Sin[e + f*x]])])/(16*Sqrt[2]*a^3*c^(3/2)*f) + ((7*A + 3*B)*Cos[e + f*x])/(16*a^3*f*(c - c*Sin[e + f*x])^(3/2)) - ((7*A + 3*B)*Sec[e + f*x])/(12*a^3*c*f*Sqrt[c - c*Sin[e + f*x]]) - ((7*A + 3*B)*Sec[e + f*x]^3*Sqrt[c - c*Sin[e + f*x]])/(30*a^3*c^2*f) - ((A - B)*Sec[e + f*x]^5*(c - c*Sin[e + f*x])^(3/2))/(5*a^3*c^3*f)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2675

Int[(cos[(e_.) + (f_.)*(x_)])*(g_.)^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*(p + 1)), x] + Dist[(a*(m + p + 1))/(g^2*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e,

$f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, 0] \&\& \text{LeQ}[p, -2*m] \&\& \text{IntegersQ}[m + 1/2, 2*p]$

Rule 2687

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.))^p / \text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]], x_Symbol] :> -\text{Simp}[(b*(g*\cos[e + f*x])^{p+1}) / (a*f*g*(p+1)*\text{Sqrt}[a + b*\sin[e + f*x]]), x] + \text{Dist}[(a*(2*p+1)) / (2*g^2*(p+1)), \text{Int}[(g*\cos[e + f*x])^{p+2} / (a + b*\sin[e + f*x])^{3/2}], x], x] /; \text{FreeQ}[\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2*p]$

Rule 2855

$\text{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.))^p * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^m * ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -\text{Simp}[(b*c + a*d)*(g*\cos[e + f*x])^{p+1}*(a + b*\sin[e + f*x])^m / (a*f*g*(p+1)), x] + \text{Dist}[(b*(a*d*m + b*c*(m+p+1))) / (a*g^2*(p+1)), \text{Int}[(g*\cos[e + f*x])^{p+2}*(a + b*\sin[e + f*x])^{m-1}], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, -1] \&\& \text{LtQ}[p, -1]$

Rule 2967

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^m * ((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_)]) * ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])^n, x_Symbol] :> \text{Dist}[a^m*c^m, \text{Int}[\cos[e + f*x]^{2*m}*(c + d*\sin[e + f*x])^{n-m}*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{3/2}} dx &= \frac{\int \sec^6(e + fx) (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx}{a^3 c^3} \\ &= -\frac{(A - B) \sec^5(e + fx) (c - c \sin(e + fx))^{3/2}}{5a^3 c^3 f} + \frac{(7A + 3B) \int \sec^4}{30a^3 c^2 f} \\ &= -\frac{(7A + 3B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{30a^3 c^2 f} - \frac{(A - B) \sec^5(e + fx)}{12a^3 c f \sqrt{c - c \sin(e + fx)}} - \frac{(7A + 3B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{30a^3 c^2 f} \\ &= -\frac{(7A + 3B) \sec(e + fx)}{12a^3 c f \sqrt{c - c \sin(e + fx)}} - \frac{(7A + 3B) \sec^3(e + fx) \sqrt{c - c \sin(e + fx)}}{30a^3 c^2 f} - \frac{(7A + 3B) \cos(e + fx)}{16a^3 f (c - c \sin(e + fx))^{3/2}} - \frac{(7A + 3B) \sec(e + fx)}{12a^3 c f \sqrt{c - c \sin(e + fx)}} - \frac{(7A + 3B) \cos(e + fx)}{16a^3 f (c - c \sin(e + fx))^{3/2}} - \frac{(7A + 3B) \sec(e + fx)}{12a^3 c f \sqrt{c - c \sin(e + fx)}} - \frac{(7A + 3B) \cos(e + fx)}{16a^3 f (c - c \sin(e + fx))^{3/2}} \\ &= \frac{(7A + 3B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{16\sqrt{2} a^3 c^{3/2} f} + \frac{(7A + 3B) \cos(e + fx)}{16a^3 f (c - c \sin(e + fx))^{3/2}} \end{aligned}$$


```
(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2/c/sign(tan((f*x+exp(1))/2)-1)-1/240*(-165*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+45*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+795*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-75*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-1100*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+60*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-1180*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+60*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+2618*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+102*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+1130*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+150*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-1980*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-180*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-605*A*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-1900*A*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-75*B*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-180*B*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-77*A*sqrt(c)*c^4-3*B*sqrt(c)*c^4)/a^3/(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+c)^5/c/sign(tan((f*x+exp(1))/2)-1)+1/32*(7*A+3*B)*atan((-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c)+sqrt(c*tan((f*x+exp(1))/2)^2+c))/sqrt(2)/sqrt(-c))/sqrt(2)/a^3/sqrt(-c)/c/sign(tan((f*x+exp(1))/2)-1)
```

maple [A] time = 1.69, size = 308, normalized size = 1.38

$$278A c^{\frac{7}{2}} - 18B c^{\frac{7}{2}} - 350A c^{\frac{7}{2}} \left(\sin^2(fx + e)\right) - 150B c^{\frac{7}{2}} \left(\sin^2(fx + e)\right) + 42A c^{\frac{7}{2}} \sin(fx + e) + 18B c^{\frac{7}{2}} \sin(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x)

[Out] $-1/480/c^{(9/2)}/a^3*(278*A*c^{(7/2)}-18*B*c^{(7/2)}-350*A*c^{(7/2)}*\sin(f*x+e)^2-150*B*c^{(7/2)}*\sin(f*x+e)^2+42*A*c^{(7/2)}*\sin(f*x+e)+18*B*c^{(7/2)}*\sin(f*x+e)-210*A*c^{(7/2)}*\sin(f*x+e)^3-90*B*c^{(7/2)}*\sin(f*x+e)^3+45*B*(c*(1+\sin(f*x+e)))^{(5/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c-105*A*(c*(1+\sin(f*x+e)))^{(5/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*c-45*B*(c*(1+\sin(f*x+e)))^{(5/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*c+105*A*(c*(1+\sin(f*x+e)))^{(5/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(c*(1+\sin(f*x+e)))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*\sin(f*x+e)*c)/(1+\sin(f*x+e))^2/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```


$]^m)/(a*f*g*(2*m + p + 1)), x] + \text{Dist}[(m + p + 1)/(a*(2*m + p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^{m + 1}, x], x] /; \text{FreeQ}\{a, b, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[2*m + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*p]$

Rule 2687

$\text{Int}[(\text{cos}[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}/\text{Sqrt}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]], x_Symbol] \text{:>} -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p + 1)})/(a*f*g*(p + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(a*(2*p + 1))/(2*g^2*(p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p + 2)}(a + b*\text{Sin}[e + f*x])^{(3/2)}, x], x] /; \text{FreeQ}\{a, b, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IntegerQ}[2*p]$

Rule 2855

$\text{Int}[(\text{cos}[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \text{:>} -\text{Simp}[(b*c + a*d)*(g*\text{Cos}[e + f*x])^{(p + 1)}(a + b*\text{Sin}[e + f*x])^m)/(a*f*g*(p + 1)), x] + \text{Dist}[(b*(a*d*m + b*c*(m + p + 1)))/(a*g^2*(p + 1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p + 2)}(a + b*\text{Sin}[e + f*x])^{(m - 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{GtQ}[m, -1] \&\& \text{LtQ}[p, -1]$

Rule 2967

$\text{Int}[(a_. + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(n_.)*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \text{:>} \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n - m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \& \text{GtQ}[n, 0]) || \text{LtQ}[0, n] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2}} dx &= \frac{\int \sec^6(e + fx)(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)} dx}{a^3 c^3} \\
&= -\frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} + \frac{(9A + B) \int \frac{\sec^4(e + fx)}{\sqrt{c - c \sin(e + fx)}} dx}{10a^3 c^2} \\
&= -\frac{(9A + B) \sec^3(e + fx)}{30a^3 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} \\
&= \frac{7(9A + B) \sec(e + fx)}{240a^3 c f (c - c \sin(e + fx))^{3/2}} - \frac{(9A + B) \sec^3(e + fx)}{30a^3 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} \\
&= \frac{7(9A + B) \sec(e + fx)}{240a^3 c f (c - c \sin(e + fx))^{3/2}} - \frac{7(9A + B) \sec(e + fx)}{96a^3 c^2 f \sqrt{c - c \sin(e + fx)}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} \\
&= \frac{7(9A + B) \cos(e + fx)}{128a^3 c f (c - c \sin(e + fx))^{3/2}} + \frac{7(9A + B) \sec(e + fx)}{240a^3 c f (c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} \\
&= \frac{7(9A + B) \cos(e + fx)}{128a^3 c f (c - c \sin(e + fx))^{3/2}} + \frac{7(9A + B) \sec(e + fx)}{240a^3 c f (c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f} \\
&= \frac{7(9A + B) \tanh^{-1}\left(\frac{\sqrt{c} \cos(e + fx)}{\sqrt{2} \sqrt{c - c \sin(e + fx)}}\right)}{128\sqrt{2} a^3 c^{5/2} f} + \frac{7(9A + B) \cos(e + fx)}{128a^3 c f (c - c \sin(e + fx))^{3/2}} - \frac{(A - B) \sec^5(e + fx)\sqrt{c - c \sin(e + fx)}}{5a^3 c^3 f}
\end{aligned}$$

Mathematica [C] time = 2.29, size = 479, normalized size = 1.86

$$\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(15(15A + 7B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2)), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-720*A*Cos[e + f*x]^4 + 96*(-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 80*(-3*A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 60*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 15*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - (105 + 105*I)*(-1)^(1/4)*(9*A + B)*ArcTan[(1/2 + I/2)*(-1)^(1/4)*(1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 120*(A + B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 + 30*(15*A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(1920*a^3*f*(1 + Sin[e + f*x])^3*(c - c*Sin[e + f*x])^(5/2))

fricas [A] time = 0.49, size = 238, normalized size = 0.92

$$105\sqrt{2}(9A + B)\sqrt{c} \cos(fx + e)^5 \log\left(-\frac{c \cos(fx + e)^2 + 2\sqrt{2}\sqrt{-c \sin(fx + e)} + c\sqrt{c}(\cos(fx + e) + \sin(fx + e) + 1) + 3c \cos(fx + e) + (c \cos(fx + e))^2}{\cos(fx + e)^2 + (\cos(fx + e) + 2)\sin(fx + e) - \cos(fx + e) - 2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/7680*(105*sqrt(2)*(9*A + B)*sqrt(c)*cos(f*x + e)^5*log(-(c*cos(f*x + e))^2 + 2*sqrt(2)*sqrt(-c*sin(f*x + e) + c)*sqrt(c)*(cos(f*x + e) + sin(f*x + e) + 1) + 3*c*cos(f*x + e) + (c*cos(f*x + e) - 2*c)*sin(f*x + e) + 2*c)/(cos(f*x + e)^2 + (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(105*(9*A + B)*cos(f*x + e)^4 - 14*(9*A + B)*cos(f*x + e)^2 - 2*(35*(9*A + B)*cos(f*x + e)^2 + 216*A + 24*B)*sin(f*x + e) - 48*A - 432*B)*sqrt(-c*sin(f*x + e) + c))/(a^3*c^3*f*cos(f*x + e)^5)
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/256*(-65*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-41*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-231*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-127*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6-163*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-91*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+247*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4+143*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-11*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-3*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+87*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-149*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+47*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-93*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-19*A*sqrt(c)*c^3-11*B*sqrt(c)*c^3)/a^3/c^2/(-(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-2*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+c)^4/sign(tan((f*x+exp(1))/2)-1)-1/240*(-120*A*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+45*B*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^9+630*A*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-165*B*sqrt(c)*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^8-900*A*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7+200*B*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^7-960*A*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+220*B*sqrt(c)*c*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^6+2244*A*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5-374*B*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^5+1020*A*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-110*B*sqrt(c)*c^2*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^4-1740*A*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3+240*B*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^3-540*A*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))-1680*A*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2+65*B*c^4*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))+220*B*sqrt(c)*c^3*(-sqrt(c)*tan((f*x+exp(1))/2)+sqrt(c*tan((f*x+exp(1))/2)^2+c))^2-66*A*sqrt(c)*c^4+11*B*sqrt(c)*c^4/
```

$$a^3/c^2/(-(-\sqrt{c})\tan((f*x+\exp(1))/2)+\sqrt{c}\tan((f*x+\exp(1))/2)^2+c)^2+2*\sqrt{c}*(-\sqrt{c})\tan((f*x+\exp(1))/2)+\sqrt{c}\tan((f*x+\exp(1))/2)^2+c)^5/\text{sign}(\tan((f*x+\exp(1))/2)-1)+1/256*(63*A+7*B)*\text{atan}((-\sqrt{c})\tan((f*x+\exp(1))/2)+\sqrt{c})/\sqrt{2}/\sqrt{-c})/\sqrt{2}/a^3/c^2/\sqrt{-c}/\text{sign}(\tan((f*x+\exp(1))/2)-1))$$

maple [A] time = 2.06, size = 410, normalized size = 1.59

$$\left(1260c^{\frac{9}{2}}A + 140c^{\frac{9}{2}}B\right) \sin(fx + e) \left(\cos^2(fx + e)\right) + \left(-1890\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{c+c\sin(fx+e)}\sqrt{2}}{2\sqrt{c}}\right)\right) (c + c\sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2), x)

[Out]
$$-1/3840/c^{(13/2)}/a^3*((1260*c^{(9/2)}*A+140*c^{(9/2)}*B)*\sin(f*x+e)*\cos(f*x+e)^2+(-1890*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*A+864*c^{(9/2)}*A-210*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*B+96*c^{(9/2)}*B)*\sin(f*x+e)+(-1890*c^{(9/2)}*A-210*c^{(9/2)}*B)*\cos(f*x+e)^4+(-945*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*A+252*c^{(9/2)}*A-105*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*B+28*c^{(9/2)}*B)*\cos(f*x+e)^2+1890*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*A+96*c^{(9/2)}*A+210*2^{(1/2)}*\operatorname{arctanh}(1/2*(c+c*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/c^{(1/2)})*(c+c*\sin(f*x+e))^{(5/2)}*c^2*B+864*c^{(9/2)}*B)/(1+\sin(f*x+e))^2/(\sin(f*x+e)-1)/\cos(f*x+e)/(c-c*\sin(f*x+e))^{(1/2)}/f$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2), x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)), x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c - c*sin(e + f*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c-c*sin(f*x+e))^(5/2), x)

[Out] Timed out

$$3.131 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=94

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{9/2}}{5cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4f\sqrt{a \sin(e + fx) + a}}$$

[Out] $-1/4*a*(A+B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}+1/5*a*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(9/2)}/c/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{9/2}}{5cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4f\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2), x]`

[Out] $-(a*(A + B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(4*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (a*B*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(9/2)})/(5*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 2738

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]`

Rule 2971

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx = (A + B) \int \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2} dx \\ = -\frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 1.00, size = 118, normalized size = 1.26

$$\frac{c^3 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (4(23B - 60A) \sin(e + fx) + 4 \cos(2(e + fx)))(4(5A - 60B) \cos(e + fx) + 4 \sin(2(e + fx)))}{160f}$$

Antiderivative was successfully verified.

maple [B] time = 0.87, size = 174, normalized size = 1.85

$$\frac{(-4B(\cos^4(fx + e)) + 5A(\cos^2(fx + e))\sin(fx + e) - 15B(\cos^2(fx + e))\sin(fx + e) - 20A(\cos^2(fx + e))\sin(fx + e))}{20f((\cos^2(fx + e))\sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)-15*B*cos(f*x+e)^2*sin(f*x+e)-20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)+25*B*sin(f*x+e)+40*A-24*B)*(-c*(sin(f*x+e)-1))^(7/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)+4)/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(7/2), x)

mupad [B] time = 16.17, size = 173, normalized size = 1.84

$$\frac{c^3 \sqrt{a(\sin(e + fx) + 1)} \sqrt{-c(\sin(e + fx) - 1)} (100B \cos(e + fx) - 140A \cos(e + fx) - 135A \cos(3e + 3fx) + 5A \cos(5e + 5fx) + 85B \cos(3e + 3fx) - 15B \cos(5e + 5fx) - 240A \sin(2e + 2fx) + 40A \sin(4e + 4fx) + 90B \sin(2e + 2fx) - 48B \sin(4e + 4fx) + 2B \sin(6e + 6fx))}{(160f(\cos(2e + 2fx) + 1))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(7/2),x)

[Out] -(c^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(100*B*cos(e + f*x) - 140*A*cos(e + f*x) - 135*A*cos(3*e + 3*f*x) + 5*A*cos(5*e + 5*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x) + 40*A*sin(4*e + 4*f*x) + 90*B*sin(2*e + 2*f*x) - 48*B*sin(4*e + 4*f*x) + 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)*(a+a*sin(f*x+e))^(1/2),x)

[Out] Timed out

$$3.132 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=94

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3f\sqrt{a \sin(e + fx) + a}}$$

[Out] $-1/3*a*(A+B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/f/(a+a*\sin(f*x+e))^{(1/2)+1/4}$
 $*a*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/c/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3f\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] $-(a*(A + B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(3*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (a*B*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(4*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx = (A + B) \int \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2} dx - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.82, size = 102, normalized size = 1.09

$$\frac{c^2 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (16(7A - 2B) \sin(e + fx) - 4 \cos(2(e + fx))(4(A - 2B) \sin(e + fx) - 2))}{96f}$$

Antiderivative was successfully verified.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2-8*B*cos(f*x+e)^2+12*A*sin(f*x+e)-9*B*sin(f*x+e)-16*A+8*B)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(2*sin(f*x+e)+cos(f*x+e)^2-2)/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2), x)

mupad [B] time = 14.96, size = 149, normalized size = 1.59

$$c^2 \sqrt{a (\sin(e + fx) + 1)} \sqrt{-c (\sin(e + fx) - 1)} (48 A \cos(e + fx) - 36 B \cos(e + fx) + 48 A \cos(3e + 3fx))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2),x)

[Out] (c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(e + f*x) - 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) - 33*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) + 112*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x) - 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)*(a+a*sin(f*x+e))^(1/2),x)

[Out] Timed out

$$3.133 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=94

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a \sin(e + fx) + a}}$$

[Out] $-1/2*a*(A+B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/f/(a+a*\sin(f*x+e))^{(1/2)+1/3}$
 $*a*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/c/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3cf\sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2), x]

[Out] $-(a*(A + B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (a*B*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(3*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx = (A + B) \int \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2} dx$$

$$= -\frac{a(A + B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.56, size = 84, normalized size = 0.89

$$\frac{c \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (2(6A - B) \sin(e + fx) + \cos(2(e + fx)))(3A + 2B \sin(e + fx))}{12f}$$

Antiderivative was successfully verified.

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)*(a+a*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2), x)
```

mupad [B] time = 1.79, size = 122, normalized size = 1.30

$$\frac{c \sqrt{a (\sin(e + fx) + 1)} \sqrt{-c (\sin(e + fx) - 1)} (3A \cos(e + fx) - 3B \cos(e + fx) + 3A \cos(3e + 3fx) - 3B \cos(3e + 3fx) + 12f (\cos(2e + 2fx)))}{12f (\cos(2e + 2fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(3/2),x)
```

```
[Out] (c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
f*x) - 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) - 3*B*cos(3*e + 3*f*x) + 12*
A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(cos
(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sin(e + fx) + 1)} (-c (\sin(e + fx) - 1))^{\frac{3}{2}} (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)*(a+a*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x) - 1))**(3/2)*(A + B*s
in(e + f*x)), x)
```

3.134 $\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$

Optimal. Leaf size=92

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2cf \sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{f \sqrt{a \sin(e + fx) + a}}$$

[Out] $1/2*a*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^(3/2)/c/f/(a+a*\sin(f*x+e))^(1/2)-a*(A+B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^(1/2)/f/(a+a*\sin(f*x+e))^(1/2)$

Rubi [A] time = 0.31, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{aB \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2cf \sqrt{a \sin(e + fx) + a}} - \frac{a(A + B) \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{f \sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]], x]`

[Out] $-\left(\frac{a(A + B)\cos[e + f*x]\sqrt{c - c\sin[e + f*x]}}{f\sqrt{a + a\sin[e + f*x]}}\right) + \frac{aB\cos[e + f*x](c - c\sin[e + f*x])^{3/2}}{2c*f*\sqrt{a + a\sin[e + f*x]}}$

Rule 2738

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]`

Rule 2971

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx = (A + B) \int \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)} dx - \frac{a(A + B) \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{f \sqrt{a + a \sin(e + fx)}} + \frac{aB \cos(e + fx) (c - c \sin(e + fx))^{3/2}}{2cf \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.17, size = 63, normalized size = 0.68

$$\frac{\sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (4A \sin(e + fx) - B \cos(2(e + fx)))}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]],x]

[Out] (Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(-(B*Cos[2*(e + f*x)]) + 4*A*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(4*f)

fricas [A] time = 0.44, size = 61, normalized size = 0.66

$$\frac{(B \cos(fx + e))^2 - 2A \sin(fx + e) - B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{2f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] -1/2*(B*cos(f*x + e)^2 - 2*A*sin(f*x + e) - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-2*A*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(2*f)^2+8*B*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(8*f)^2)

maple [A] time = 0.80, size = 57, normalized size = 0.62

$$\frac{(B \sin(fx + e) + 2A) \sqrt{-c(\sin(fx + e) - 1)} \sin(fx + e) \sqrt{a(1 + \sin(fx + e))}}{2f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/2/f*(B*sin(f*x+e)+2*A)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c), x)

mupad [B] time = 0.94, size = 75, normalized size = 0.82

$$\frac{\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}(B\cos(e+fx)+B\cos(3e+3fx)-4A\sin(2e+2fx))}{4f(\cos(2e+2fx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2),x)

[Out] -((a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(B*cos(e + f*x) + B*cos(3*e + 3*f*x) - 4*A*sin(2*e + 2*f*x)))/(4*f*(cos(2*e + 2*f*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)*(a+a*sin(f*x+e))**(1/2), x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x)), x)

$$3.135 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=100

$$\frac{aB \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{cf \sqrt{a \sin(e+fx)+a}} - \frac{a(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

[Out] $-a*(A+B)*\cos(f*x+e)*\ln(1-\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}+a*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/c/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{aB \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{cf \sqrt{a \sin(e+fx)+a}} - \frac{a(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] $-((a*(A+B)*\cos[e+f*x]*\log[1-\sin[e+f*x]])/(f*\sqrt{a+a*\sin[e+f*x]}*\sqrt{c-c*\sin[e+f*x]})+(a*B*\cos[e+f*x]*\sqrt{c-c*\sin[e+f*x]})/(c*f*\sqrt{a+a*\sin[e+f*x]})$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] => Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^{(p_)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_), x_Symbol] => Dist[1/(b^p*f), Subst[Int[(a + x)^{(m + (p - 1)/2)((p - 1)/2)}, x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a² - b², 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])}

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] => Dist[(a*c*cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a² - b², 0]

Rule 2738

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] => Simp[(-2*b*cos[e + f*x]*(c + d*Sin[e + f*x])ⁿ)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a² - b², 0] && NeQ[n, -2⁽⁻¹⁾]

Rule 2971

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_) + (B_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] => Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])ⁿ, x],

x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= (A + B) \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx - \frac{B \int \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}}{c} \\ &= \frac{aB \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{cf \sqrt{a + a \sin(e + fx)}} + \frac{(a(A + B)c \cos(e + fx)) \int \frac{1}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} \\ &= \frac{aB \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{cf \sqrt{a + a \sin(e + fx)}} - \frac{(a(A + B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx, x, \frac{c - c \sin(e + fx)}{2}\right)}{f \sqrt{a + a \sin(e + fx)}} \\ &= -\frac{a(A + B) \cos(e + fx) \log(1 - \sin(e + fx))}{f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{aB \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{cf \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.13, size = 120, normalized size = 1.20

$$\frac{\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(B \sin(e + fx) + (A + B) \left(2 \log(i - e^{i(e + fx)}) - ifx \right) \right)}{f \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] -(((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*((A + B)*((-I)*f*x + 2*Log[I - E^(I*(e + f*x))]) + B*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]))

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\operatorname{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c \sin(fx + e) - c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2), x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*(A*sqrt(c*tan(1/2*exp(1)

$$\begin{aligned}
&)^2+c)*(-12582912*\tan(1/2*\exp(1))^5-37748736*\tan(1/2*\exp(1))^4+41943040*\tan \\
& (1/2*\exp(1))^3+25165824*\tan(1/2*\exp(1))^2-12582912*\tan(1/2*\exp(1))-4194304) \\
& +B*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(-12582912*\tan(1/2*\exp(1))^5-37748736*\tan(1/ \\
& 2*\exp(1))^4+41943040*\tan(1/2*\exp(1))^3+25165824*\tan(1/2*\exp(1))^2-12582912* \\
& \tan(1/2*\exp(1))-4194304)+A*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(12582912*\tan(1/2*\exp \\
& (1))^5+37748736*\tan(1/2*\exp(1))^4-41943040*\tan(1/2*\exp(1))^3-25165824*\tan(\\
& 1/2*\exp(1))^2+12582912*\tan(1/2*\exp(1))+4194304)*\tan(1/4*\exp(1))^6+A*\sqrt{c* \\
& \tan(1/2*\exp(1))^2+c)*(83886080*\tan(1/2*\exp(1))^6+251658240*\tan(1/2*\exp(1))^ \\
& 5-503316480*\tan(1/2*\exp(1))^4-838860800*\tan(1/2*\exp(1))^3+754974720*\tan(1/2 \\
& *\exp(1))^2+251658240*\tan(1/2*\exp(1)))*\tan(1/4*\exp(1))^3+A*\sqrt{c*\tan(1/2*\exp \\
& (1))^2+c)*(188743680*\tan(1/2*\exp(1))^5+566231040*\tan(1/2*\exp(1))^4-6291456 \\
& 00*\tan(1/2*\exp(1))^3-377487360*\tan(1/2*\exp(1))^2+188743680*\tan(1/2*\exp(1))+ \\
& 62914560)*\tan(1/4*\exp(1))^2+A*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(-25165824*\tan(1/ \\
& 2*\exp(1))^6-75497472*\tan(1/2*\exp(1))^5+150994944*\tan(1/2*\exp(1))^4+25165824 \\
& 0*\tan(1/2*\exp(1))^3-226492416*\tan(1/2*\exp(1))^2-75497472*\tan(1/2*\exp(1)))*\tan \\
& (1/4*\exp(1))^5+A*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(-25165824*\tan(1/2*\exp(1))^6 \\
& -75497472*\tan(1/2*\exp(1))^5+150994944*\tan(1/2*\exp(1))^4+251658240*\tan(1/2*\exp \\
& (1))^3-226492416*\tan(1/2*\exp(1))^2-75497472*\tan(1/2*\exp(1)))*\tan(1/4*\exp(\\
& 1))+A*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(-188743680*\tan(1/2*\exp(1))^5-566231040*\tan \\
& (1/2*\exp(1))^4+629145600*\tan(1/2*\exp(1))^3+377487360*\tan(1/2*\exp(1))^2-18 \\
& 8743680*\tan(1/2*\exp(1))-62914560)*\tan(1/4*\exp(1))^4+B*\sqrt{c*\tan(1/2*\exp(1) \\
&)^2+c)*(12582912*\tan(1/2*\exp(1))^5+37748736*\tan(1/2*\exp(1))^4-41943040*\tan(\\
& 1/2*\exp(1))^3-25165824*\tan(1/2*\exp(1))^2+12582912*\tan(1/2*\exp(1))+4194304)* \\
& \tan(1/4*\exp(1))^6+B*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(83886080*\tan(1/2*\exp(1))^6 \\
& +251658240*\tan(1/2*\exp(1))^5-503316480*\tan(1/2*\exp(1))^4-838860800*\tan(1/2* \\
& \exp(1))^3+754974720*\tan(1/2*\exp(1))^2+251658240*\tan(1/2*\exp(1)))*\tan(1/4*\exp \\
& (1))^3+B*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(188743680*\tan(1/2*\exp(1))^5+56623104 \\
& 0*\tan(1/2*\exp(1))^4-629145600*\tan(1/2*\exp(1))^3-377487360*\tan(1/2*\exp(1))^2 \\
& +188743680*\tan(1/2*\exp(1))+62914560)*\tan(1/4*\exp(1))^2+B*\sqrt{c*\tan(1/2*\exp \\
& (1))^2+c)*(-25165824*\tan(1/2*\exp(1))^6-75497472*\tan(1/2*\exp(1))^5+150994944 \\
& *\tan(1/2*\exp(1))^4+251658240*\tan(1/2*\exp(1))^3-226492416*\tan(1/2*\exp(1))^2- \\
& 75497472*\tan(1/2*\exp(1)))*\tan(1/4*\exp(1))^5+B*\sqrt{c*\tan(1/2*\exp(1))^2+c)* \\
& (-25165824*\tan(1/2*\exp(1))^6-75497472*\tan(1/2*\exp(1))^5+150994944*\tan(1/2*\exp \\
& (1))^4+251658240*\tan(1/2*\exp(1))^3-226492416*\tan(1/2*\exp(1))^2-75497472*\tan \\
& (1/2*\exp(1)))*\tan(1/4*\exp(1))+B*\sqrt{c*\tan(1/2*\exp(1))^2+c)*(-188743680*\tan \\
& (1/2*\exp(1))^5-566231040*\tan(1/2*\exp(1))^4+629145600*\tan(1/2*\exp(1))^3+377 \\
& 487360*\tan(1/2*\exp(1))^2-188743680*\tan(1/2*\exp(1))-62914560)*\tan(1/4*\exp(1) \\
&)^4)*\ln(\text{abs}(-2*\tan(1/2*\exp(1))^3+6*\tan(1/2*\exp(1))^2+(\tan(1/2*(1/2*f*x+2*\exp \\
& (1)))-1/\tan(1/2*(1/2*f*x+2*\exp(1))))*(\tan(1/2*\exp(1))^3+3*\tan(1/2*\exp(1))^ \\
& 2-3*\tan(1/2*\exp(1))-1)+6*\tan(1/2*\exp(1))-2)/f/(-8388608*\sqrt{2}*c*\tan(1/2* \\
& \exp(1))^7+8388608*\sqrt{2}*c+(-8388608*\sqrt{2}*c*\tan(1/2*\exp(1))^7-25165824* \\
& \sqrt{2}*c*\tan(1/2*\exp(1))^6+8388608*\sqrt{2}*c*\tan(1/2*\exp(1))^5-41943040*\sqrt{ \\
& 2}*c*\tan(1/2*\exp(1))^4+41943040*\sqrt{2}*c*\tan(1/2*\exp(1))^3-8388608*\sqrt{ \\
& 2}*c*\tan(1/2*\exp(1))^2+8388608*\sqrt{2}*c+25165824*\sqrt{2}*c*\tan(1/2*\exp(1) \\
&))*\tan(1/4*\exp(1))^6+(-25165824*\sqrt{2}*c*\tan(1/2*\exp(1))^7-75497472*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^6+25165824*\sqrt{2}*c*\tan(1/2*\exp(1))^5-125829120*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^4+125829120*\sqrt{2}*c*\tan(1/2*\exp(1))^3-25165824*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^2+25165824*\sqrt{2}*c+75497472*\sqrt{2}*c*\tan(1/2*\exp(1) \\
&))*\tan(1/4*\exp(1))^2+(-25165824*\sqrt{2}*c*\tan(1/2*\exp(1))^7-75497472*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^6+25165824*\sqrt{2}*c*\tan(1/2*\exp(1))^5-125829120*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^4+125829120*\sqrt{2}*c*\tan(1/2*\exp(1))^3-25165824*\sqrt{2} \\
&)*c*\tan(1/2*\exp(1))^2+25165824*\sqrt{2}*c+75497472*\sqrt{2}*c*\tan(1/2*\exp(1) \\
&))*\tan(1/4*\exp(1))^4-25165824*\sqrt{2}*c*\tan(1/2*\exp(1))^6+8388608*\sqrt{2}*c*\tan \\
& (1/2*\exp(1))^5-41943040*\sqrt{2}*c*\tan(1/2*\exp(1))^4+41943040*\sqrt{2}*c*\tan \\
& (1/2*\exp(1))^3-8388608*\sqrt{2}*c*\tan(1/2*\exp(1))^2+25165824*\sqrt{2}*c*\tan(\\
& 1/2*\exp(1))
\end{aligned}$$

maple [B] time = 0.73, size = 395, normalized size = 3.95

$$\left(2A \sin(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - A \sin(fx + e) \ln\left(\frac{2}{\cos(fx + e) + 1}\right) + 2A \cos(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x)

[Out] -1/f*(2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+B*sin(f*x+e)*cos(f*x+e)+2*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^2+2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln(2/(cos(f*x+e)+1))-B*sin(f*x+e)-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln(2/(cos(f*x+e)+1))+B*(a*(1+sin(f*x+e)))^(1/2)/(1-cos(f*x+e)+sin(f*x+e))/(-c*(sin(f*x+e)-1))^(1/2)

maxima [A] time = 0.55, size = 175, normalized size = 1.75

$$B \left[\frac{2\sqrt{a} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{\sqrt{c}} - \frac{\sqrt{a} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{c}} + \frac{2\sqrt{a}\sqrt{c} \sin(fx+e)}{\left(c + \frac{c \sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} \right] + A \left[\frac{2\sqrt{a} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{\sqrt{c}} - \frac{\sqrt{a} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] (B*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c) + 2*sqrt(a)*sqrt(c)*sin(f*x + e)/((c + c*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) + A*(2*sqrt(a)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/sqrt(c) - sqrt(a)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(c)))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx))}{\sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(1/2),  
x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f  
*x) - 1)), x)
```

$$3.136 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=99

$$\frac{a(A+B) \cos(e+fx)}{f \sqrt{a \sin(e+fx) + a(c-c \sin(e+fx))^{3/2}}} + \frac{aB \cos(e+fx) \log(1-\sin(e+fx))}{cf \sqrt{a \sin(e+fx) + a} \sqrt{c-c \sin(e+fx)}}$$

[Out] a*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+a*B*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.36, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{a(A+B) \cos(e+fx)}{f \sqrt{a \sin(e+fx) + a(c-c \sin(e+fx))^{3/2}}} + \frac{aB \cos(e+fx) \log(1-\sin(e+fx))}{cf \sqrt{a \sin(e+fx) + a} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] (a*(A + B)*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + (a*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2738

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_) + (B_.)*sin[(e_) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]

$x]$ /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx &= (A + B) \int \frac{\sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{3/2}} dx - \frac{B \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx}{c} \\ &= \frac{a(A + B) \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} - \frac{(aB \cos(e + fx))}{\sqrt{a + a \sin(e + fx)}} \\ &= \frac{a(A + B) \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} + \frac{(aB \cos(e + fx))}{cf \sqrt{a + a \sin(e + fx)}} \\ &= \frac{a(A + B) \cos(e + fx)}{f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} + \frac{aB \cos(e + fx)}{cf \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.16, size = 147, normalized size = 1.48

$$\frac{\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(A + 2B \log\left(i - e^{i(e + fx)}\right) + iB \left(fx + 2i \log\left(i - e^{i(e + fx)}\right) \right) \right)}{f(c - c \sin(e + fx))^{3/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + B - I*B*f*x + 2*B*Log[I - E^(I*(e + f*x))] + I*B*(f*x + (2*I)*Log[I - E^(I*(e + f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(3/2))

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.69, size = 403, normalized size = 4.07

$$\left(B \left(\cos^2(fx + e) \right) \ln \left(\frac{2}{\cos(fx+e)+1} \right) - 2B \left(\cos^2(fx + e) \right) \ln \left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)} \right) - B \ln \left(\frac{2}{\cos(fx+e)+1} \right) \sin(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2), x)

[Out] 1/f*(B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)+B*sin(f*x+e)-2*B*ln(2/(cos(f*x+e)+1))+4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A-B)*(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a}}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx))}{(-c(\sin(e + fx) - 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(3/2), x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) - 1))^(3/2), x)

$$3.137 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=92

$$\frac{a(A+B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{5/2}} - \frac{aB \cos(e+fx)}{cf\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}}$$

[Out] $1/2*a*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))^{(5/2)}/(a+a*\sin(f*x+e))^{(1/2)}-a*B*\cos(f*x+e)/c/f/(c-c*\sin(f*x+e))^{(3/2)}/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{a(A+B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{5/2}} - \frac{aB \cos(e+fx)}{cf\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(A + B*\text{Sin}[e + f*x]))/(c - c*\text{Sin}[e + f*x])^{(5/2)}, x]$

[Out] $(a*(A + B)*\text{Cos}[e + f*x])/(2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)}) - (a*B*\text{Cos}[e + f*x])/(c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(3/2)})$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] := \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] := \text{Dist}[B/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n+1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx &= (A+B) \int \frac{\sqrt{a+a \sin(e+fx)}}{(c-c \sin(e+fx))^{5/2}} dx - \frac{B \int \frac{\sqrt{a+a \sin(e+fx)}}{(c-c \sin(e+fx))^{3/2}} dx}{c} \\ &= \frac{a(A+B) \cos(e+fx)}{2f\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{5/2}} - \frac{aB \cos(e+fx)}{cf\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.54, size = 101, normalized size = 1.10

$$\frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{c-c \sin(e+fx)} (A+2B \sin(e+fx)-B)}{2c^3 f \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right)^5 \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]
```

```
[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(A - B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(2*c^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 87, normalized size = 0.95

$$\frac{(2B \sin(fx + e) + A - B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{2 \left(c^3 f \cos(fx + e)^3 + 2c^3 f \cos(fx + e) \sin(fx + e) - 2c^3 f \cos(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2), x, algorithm="fricas")
```

```
[Out] -1/2*(2*B*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^3*f*cos(f*x + e)^3 + 2*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*c^3*f*cos(f*x + e))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2), x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.71, size = 137, normalized size = 1.49

$$\frac{(A(\cos^2(fx + e)) - A \sin(fx + e) \cos(fx + e) - B(\cos^2(fx + e)) + B \sin(fx + e) \cos(fx + e) + 2A \cos(fx + e))}{2f(-1 + \cos(fx + e) - \sin(fx + e))(-c \sin(fx + e) + c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2), x)
```

```
[Out] -1/2/f*(A*cos(f*x+e)^2-A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)+3*A*sin(f*x+e)-B*sin(f*x+e)-3*A+B)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/(-1+cos(f*x+e)-sin(f*x+e))/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a}}{(-c \sin(fx + e) + c)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(5/2), x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx))}{(-c(\sin(e + fx) - 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c-c*sin(f*x+e))**(5/2), x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) - 1))**(5/2), x)

$$3.138 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=94

$$\frac{a(A+B) \cos(e+fx)}{3f\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{7/2}} - \frac{aB \cos(e+fx)}{2cf\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{5/2}}$$

[Out] $1/3*a*(A+B)*\cos(f*x+e)/f/(c-c*\sin(f*x+e))^{(7/2)}/(a+a*\sin(f*x+e))^{(1/2)}-1/2*a*B*\cos(f*x+e)/c/f/(c-c*\sin(f*x+e))^{(5/2)}/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{a(A+B) \cos(e+fx)}{3f\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{7/2}} - \frac{aB \cos(e+fx)}{2cf\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(A + B*\text{Sin}[e + f*x]))/(c - c*\text{Sin}[e + f*x])^{(7/2)}, x]$

[Out] $(a*(A + B)*\text{Cos}[e + f*x])/(3*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(7/2)}) - (a*B*\text{Cos}[e + f*x])/(2*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)})$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[n, -2^{(-1)}]$

Rule 2971

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Dist}[B/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx &= (A+B) \int \frac{\sqrt{a+a \sin(e+fx)}}{(c-c \sin(e+fx))^{7/2}} dx - \frac{B \int \frac{\sqrt{a+a \sin(e+fx)}}{(c-c \sin(e+fx))^{5/2}} dx}{c} \\ &= \frac{a(A+B) \cos(e+fx)}{3f\sqrt{a+a \sin(e+fx)}(c-c \sin(e+fx))^{7/2}} - \frac{aB \cos(e+fx)}{2cf\sqrt{a+a \sin(e+fx)}(c-c \sin(e+fx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.60, size = 103, normalized size = 1.10

$$\frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{c-c \sin(e+fx)} (2A+3B \sin(e+fx)-B)}{6c^4 f \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right)^7 \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] (Sqrt[a*(1 + Sin[e + f*x])]*(2*A - B + 3*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(6*c^4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))

fricas [A] time = 0.44, size = 106, normalized size = 1.13

$$\frac{(3B \sin(fx + e) + 2A - B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{6 \left(3c^4 f \cos(fx + e)^3 - 4c^4 f \cos(fx + e) - \left(c^4 f \cos(fx + e)^3 - 4c^4 f \cos(fx + e) \right) \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2), x, algorithm="fricas")

[Out] -1/6*(3*B*sin(f*x + e) + 2*A - B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.72, size = 205, normalized size = 2.18

$$\frac{\sin(fx + e) \sqrt{a(1 + \sin(fx + e))} (2A (\cos^2(fx + e)) \sin(fx + e) + 2A (\cos^3(fx + e)) - B (\cos^2(fx + e)))}{(-c \sin(fx + e) + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2), x)

[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)*(2*A*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^3-B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*x+e)^3+6*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)+4*B*cos(f*x+e)^2-14*A*sin(f*x+e)-8*A*cos(f*x+e)+4*B*sin(f*x+e)+B*cos(f*x+e)+14*A-4*B)/(-c*(sin(f*x+e)-1))^(7/2)/(1-cos(f*x+e)+sin(f*x+e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a}}{(-c \sin(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(-c*sin(f*x + e) + c)^(7/2), x)

mupad [B] time = 17.63, size = 153, normalized size = 1.63

$$\frac{2A\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)} - B\sqrt{a+a\sin(e+fx)}\sqrt{c-c\sin(e+fx)} + 3B\sin(e+fx)}{\frac{9c^4f\cos(3e+3fx)}{2} + \frac{21c^4f\sin(2e+2fx)}{2} - \frac{3c^4f\sin(4e+4fx)}{4} - \frac{21c^4f\cos(e+fx)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c - c*sin(e + f*x))^(7/2), x)

[Out] -(2*A*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) - B*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2) + 3*B*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2))/((9*c^4*f*cos(3*e + 3*f*x))/2 + (21*c^4*f*sin(2*e + 2*f*x))/2 - (3*c^4*f*sin(4*e + 4*f*x))/4 - (21*c^4*f*cos(e + f*x))/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(7/2), x)

[Out] Timed out

$$3.139 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=146

$$\frac{a^2(3A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{30f\sqrt{a \sin(e + fx) + a}} - \frac{a(3A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}(c - c \sin(e + fx))^{7/2}}{15f}$$

[Out] $-1/6*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-1/30*a^2*(3*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-1/15*a*(3*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.36, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{a^2(3A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{30f\sqrt{a \sin(e + fx) + a}} - \frac{a(3A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}(c - c \sin(e + fx))^{7/2}}{15f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(7/2)}, x]$

[Out] $-(a^2*(3*A - B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(30*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (a*(3*A - B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(15*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(6*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

mupad [B] time = 17.53, size = 323, normalized size = 2.21

$$e^{-e6i-fx6i} \sqrt{c - c \sin(e + fx)} \left(\frac{Bac^3 e^{e6i+fx6i} \cos(6e+6fx) \sqrt{a+a \sin(e+fx)}}{96f} - \frac{ac^3 e^{e6i+fx6i} \sin(e+fx) (A7i-B2i) \sqrt{a+a \sin(e+fx)}}{4f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(7/2),x)

[Out] (exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((B*a*c^3*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A - B)*(a + a*sin(e + f*x))^(1/2))/(16*f) + (a*c^3*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A - 11*B)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (a*c^3*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(A*3i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) + (a*c^3*exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f))/(2*cos(e + f*x))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),x)

[Out] Timed out

$$3.140 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=146

$$\frac{a^2(5A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{30f\sqrt{a \sin(e + fx) + a}} - \frac{a(5A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}(c - c \sin(e + fx))^{5/2}}{20f}$$

[Out] $-1/5*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(5/2)}/f-1/30*a^2*(5*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-1/20*a*(5*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.36, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{a^2(5A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{30f\sqrt{a \sin(e + fx) + a}} - \frac{a(5A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}(c - c \sin(e + fx))^{5/2}}{20f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(5/2)}, x]$

[Out] $-(a^2*(5*A - B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(30*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (a*(5*A - B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(20*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(5*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps


```
x) - 75*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) + 40*A*sin(2*e + 2*f*x)
+ 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
6*B*sin(6*e + 6*f*x))/(480*f*(cos(2*e + 2*f*x) + 1))
```

```
sympy [F(-1)]    time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```


$$3.141 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=134

$$\frac{a^2 A \cos(e + fx) (c - c \sin(e + fx))^{3/2}}{3f \sqrt{a \sin(e + fx) + a}} - \frac{a A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{3/2}}{3f} - \frac{B \cos(e + fx)}{3f}$$

[Out] $-1/4*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(3/2)}/f-1/3*a^2*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-1/3*a*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.35, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{a^2 A \cos(e + fx) (c - c \sin(e + fx))^{3/2}}{3f \sqrt{a \sin(e + fx) + a}} - \frac{a A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{3/2}}{3f} - \frac{B \cos(e + fx)}{3f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(3/2)}, x]$

[Out] $-(a^2*A*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(3*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (a*A*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(3*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(4*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)])*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(64*f)^2+32*B*a*c*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-32*f)^2

maple [A] time = 0.75, size = 86, normalized size = 0.64

$$\frac{(3B(\cos^2(fx+e))\sin(fx+e)+4A(\cos^2(fx+e))+3B\sin(fx+e)+8A)(-c(\sin(fx+e)-1))^{\frac{3}{2}}\sin(fx+e)}{12f\cos(fx+e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+3*B*sin(f*x+e)+8*A)*(-c*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)/cos(f*x+e)^3

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}(-c \sin(fx + e) + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2), x)

mupad [B] time = 1.84, size = 103, normalized size = 0.77

$$\frac{ac\sqrt{a(\sin(e+fx)+1)}\sqrt{-c(\sin(e+fx)-1)}(12B\cos(e+fx)+15B\cos(3e+3fx)+3B\cos(5e+5fx)-80A\sin(2e+2fx)-8A\sin(4e+4fx))}{96f(\cos(2e+2fx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2),x)

[Out] -(a*c*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(12*B*cos(e + f*x) + 15*B*cos(3*e + 3*f*x) + 3*B*cos(5*e + 5*f*x) - 80*A*sin(2*e + 2*f*x) - 8*A*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

[Out] Timed out

$$3.142 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$$

Optimal. Leaf size=96

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{3af\sqrt{c - c \sin(e + fx)}}$$

[Out] 1/2*(A-B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(1/2)+1/3*B*c*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/a/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{2f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{3af\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]], x]

[Out] ((A - B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*Sqrt[c - c*Sin[e + f*x]]) + (B*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f*Sqrt[c - c*Sin[e + f*x]])

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx = \frac{B \int (a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)} dx}{a} = \frac{(A - B)c \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{2f\sqrt{c - c \sin(e + fx)}} +$$

Mathematica [A] time = 0.56, size = 81, normalized size = 0.84

$$\frac{a \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (\cos(2(e + fx))(3(A + B) + 2B \sin(e + fx)) - 2(6A + B) \sin(e + fx))}{12f}$$

Antiderivative was successfully verified.

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x +
e) + c), x)
```

mupad [B] time = 14.29, size = 122, normalized size = 1.27

$$\frac{a \sqrt{a (\sin(e + fx) + 1)} \sqrt{-c (\sin(e + fx) - 1)} (3A \cos(e + fx) + 3B \cos(e + fx) + 3A \cos(3e + 3fx))}{12f (\cos(2e + 2fx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1
/2),x)
```

```
[Out] -(a*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(3*A*cos(e +
f*x) + 3*B*cos(e + f*x) + 3*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12
*A*sin(2*e + 2*f*x) - 2*B*sin(2*e + 2*f*x) + B*sin(4*e + 4*f*x)))/(12*f*(co
s(2*e + 2*f*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin(e + fx) + 1))^{\frac{3}{2}} \sqrt{-c (\sin(e + fx) - 1)} (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2),
x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e + f*x) - 1))*(A + B*s
in(e + f*x)), x)
```

$$3.143 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=145

$$\frac{2a^2(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} - \frac{a(A+B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{B \cos(e+fx)(a \sin(e+fx))^{3/2}}{2f\sqrt{c-c \sin(e+fx)}}$$

[Out] $-1/2*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-2*a^2*(A+B)*\cos(f*x+e)*\ln(1-\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-a*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.38, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2a^2(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} - \frac{a(A+B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{B \cos(e+fx)(a \sin(e+fx))^{3/2}}{2f\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x])]/\text{Sqrt}[c - c*\text{Sin}[e + f*x]], x]$

[Out] $(-2*a^2*(A + B)*\text{Cos}[e + f*x]*\text{Log}[1 - \text{Sin}[e + f*x]])/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) - (a*(A + B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(2*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 31

$\text{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

Rule 2667

$\text{Int}[\cos[(e + f*x)]^{(p)}*((a + b*\sin[(e + f*x)]^{(m)})), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{(m + (p - 1)/2)}*(a - x)^{((p - 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \ \&\& \ \text{IntegerQ}[(p - 1)/2] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ (\text{GeQ}[p, -1] \ \|\ \ !\text{IntegerQ}[m + 1/2])$

Rule 2737

$\text{Int}[\text{Sqrt}[(a + b*\sin[(e + f*x)])/\text{Sqrt}[(c + d*\sin[(e + f*x)])], x_Symbol] \rightarrow \text{Dist}[(a*c*\text{Cos}[e + f*x])/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[\text{Cos}[e + f*x]/(c + d*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2740

$\text{Int}[(a + b*\sin[(e + f*x)]^{(m)}*((c + d*\sin[(e + f*x)]^{(n)})), x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m - 1/2, 0] \ \&\& \ !\text{LtQ}[n, -1] \ \&\& \ !(\text{IGtQ}[n - 1/2, 0] \ \&\& \ \text{LtQ}[n, m]) \ \&\& \ !(\text{LtQ}[m + n, 0] \ \&\& \ \text{GtQ}[2*m + n + 1, 0])$

Rule 2973

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
+ 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= -\frac{B \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f \sqrt{c - c \sin(e + fx)}} + (A + B) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx \\ &= -\frac{a(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{a(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{a(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{2a^2(A + B) \cos(e + fx) \log(1 - \sin(e + fx))}{f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} - \frac{a(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.69, size = 136, normalized size = 0.94

$$\frac{(a(\sin(e + fx) + 1))^{3/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(4(A + 2B) \sin(e + fx) + 16(A + B) \log\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) \right)}{4f \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]
```

```
[Out] -1/4*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2)*(-
(B*Cos[2*(e + f*x)]) + 16*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
+ 4*(A + 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*Sqr
t[c - c*Sin[e + f*x]])
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Ba \cos(fx + e)^2 - (A + B)a \sin(fx + e) - (A + B)a \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c \sin(fx + e) - c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2), x,
algorithm="fricas")
```

```
[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x)
```


giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable
to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*a)*(A*a*sqrt(c*tan(1/2*exp(
1))^2+c)*(-27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1/2*ex
p(1))^4+90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*exp(1)
)^2-27021597764222976*tan(1/2*exp(1))-9007199254740992)+B*a*sqrt(c*tan(1/2
*exp(1))^2+c)*(-27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1
/2*exp(1))^4+90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*
exp(1))^2-27021597764222976*tan(1/2*exp(1))-9007199254740992)+A*a*sqrt(c*ta
n(1/2*exp(1))^2+c)*(27021597764222976*tan(1/2*exp(1))^5+81064793292668928*t
an(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3-54043195528445952*tan(
1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))+9007199254740992)*tan(1/4*e
xp(1))^6+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(180143985094819840*tan(1/2*exp(1)
)^6+540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(1)
)^4-1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp(1)
)^2+540431955284459520*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+A*a*sqrt(c*tan(1
/2*exp(1))^2+c)*(405323966463344640*tan(1/2*exp(1))^5+1215971899390033920*t
an(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3-810647932926689280*t
an(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))+135107988821114880)*tan
(1/4*exp(1))^2+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*
exp(1))^6-162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*
exp(1))^4+540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*
exp(1))^2-162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+A*a*sqrt(c*ta
n(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*exp(1))^6-162129586585337856
*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4+540431955284459520*
tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2-162129586585337856*t
an(1/2*exp(1)))*tan(1/4*exp(1))+A*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-405323966
463344640*tan(1/2*exp(1))^5-1215971899390033920*tan(1/2*exp(1))^4+135107988
8211148800*tan(1/2*exp(1))^3+810647932926689280*tan(1/2*exp(1))^2-405323966
463344640*tan(1/2*exp(1))-135107988821114880)*tan(1/4*exp(1))^4+B*a*sqrt(c*
tan(1/2*exp(1))^2+c)*(27021597764222976*tan(1/2*exp(1))^5+81064793292668928
*tan(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3-54043195528445952*ta
n(1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))+9007199254740992)*tan(1/4
*exp(1))^6+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(180143985094819840*tan(1/2*exp(
1))^6+540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(
1))^4-1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp
(1))^2+540431955284459520*tan(1/2*exp(1)))*tan(1/4*exp(1))^3+B*a*sqrt(c*tan
(1/2*exp(1))^2+c)*(405323966463344640*tan(1/2*exp(1))^5+1215971899390033920
*tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3-810647932926689280
*tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))+135107988821114880)*t
an(1/4*exp(1))^2+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/
2*exp(1))^6-162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2
*exp(1))^4+540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*
exp(1))^2-162129586585337856*tan(1/2*exp(1)))*tan(1/4*exp(1))^5+B*a*sqrt(c*
tan(1/2*exp(1))^2+c)*(-54043195528445952*tan(1/2*exp(1))^6-1621295865853378
56*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4+54043195528445952
0*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2-162129586585337856
*tan(1/2*exp(1)))*tan(1/4*exp(1))+B*a*sqrt(c*tan(1/2*exp(1))^2+c)*(-4053239
66463344640*tan(1/2*exp(1))^5-1215971899390033920*tan(1/2*exp(1))^4+1351079
888211148800*tan(1/2*exp(1))^3+810647932926689280*tan(1/2*exp(1))^2-4053239
66463344640*tan(1/2*exp(1))-135107988821114880)*tan(1/4*exp(1))^4)*ln(abs(-
2*tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1))))-1/tan(
```

```

1/2*(1/2*f*x+2*exp(1)))*(tan(1/2*exp(1))^3+3*tan(1/2*exp(1))^2-3*tan(1/2*exp(1))-1)+6*tan(1/2*exp(1))-2)/f/(-9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^7+9007199254740992*sqrt(2)*c+(-9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^7-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^6+9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^5-45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^4+45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^3-9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^2+9007199254740992*sqrt(2)*c+27021597764222976*sqrt(2)*c*tan(1/2*exp(1)))
*tan(1/4*exp(1))^6+(-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^7-81064793292668928*sqrt(2)*c*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^5-135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^4+135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^3-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^2+27021597764222976*sqrt(2)*c+81064793292668928*sqrt(2)*c*tan(1/2*exp(1)))
*tan(1/4*exp(1))^2+(-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^7-81064793292668928*sqrt(2)*c*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^5-135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^4+135107988821114880*sqrt(2)*c*tan(1/2*exp(1))^3-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^2+27021597764222976*sqrt(2)*c+81064793292668928*sqrt(2)*c*tan(1/2*exp(1)))
*tan(1/4*exp(1))^4-27021597764222976*sqrt(2)*c*tan(1/2*exp(1))^6+9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^5-45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^4+45035996273704960*sqrt(2)*c*tan(1/2*exp(1))^3-9007199254740992*sqrt(2)*c*tan(1/2*exp(1))^2+27021597764222976*sqrt(2)*c*tan(1/2*exp(1)))

```

maple [B] time = 0.70, size = 494, normalized size = 3.41

$$\left(B(\cos^3(fx + e)) + B(\cos^2(fx + e))\sin(fx + e) + 2A(\cos^2(fx + e)) - 2A\sin(fx + e)\cos(fx + e) - 8A\cos(fx + e) \right) \sqrt{-c\sin(fx + e) + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] -1/2/f*(B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2-2*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+3*B*cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-8*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*sin(f*x+e)+8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)+3*B*sin(f*x+e)+8*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*ln(2/(cos(f*x+e)+1))-2*A-3*B)*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^{\frac{3}{2}} (A + B \sin(e + fx))}{\sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2), x)

[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*x) - 1)), x)

$$3.144 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=158

$$\frac{a^2(A+3B) \cos(e+fx) \log(1-\sin(e+fx))}{cf\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{a(A+3B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2cf\sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)}{2f(c-c \sin(e+fx))}$$

[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(3/2)+a^2*(A+3*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+1/2*a*(A+3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.39, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{a^2(A+3B) \cos(e+fx) \log(1-\sin(e+fx))}{cf\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{a(A+3B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2cf\sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)}{2f(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^2*(A + 3*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 3*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n]/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f(c - c \sin(e + fx))^{3/2}} - \frac{(A + 3B) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2cf \sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a(A + 3B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2cf \sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a(A + 3B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2cf \sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a(A + 3B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2cf \sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a^2(A + 3B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{3/2}}{\sqrt{c - c \sin(e + fx)}} dx}{cf \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.86, size = 210, normalized size = 1.33

$$\frac{a \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-2 \sin(e + fx) \left(2(A + 3B) \log\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) - 2cf(\sin(e + fx) - 1) \right) \right)}{2cf(\sin(e + fx) - 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2), x]
```

```
[Out] -1/2*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]))*(4
*A + 3*B + B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]
] + 12*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 2*(-B + 2*(A + 3*B)*Log
[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x))/(c*f*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])
```

fricas [F] time = 6.01, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Ba \cos(fx + e)^2 - (A + B)a \sin(fx + e) - (A + B)a \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x,
algorithm="fricas")
```

[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.67, size = 748, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x)

[Out]
$$\frac{1}{f} \left(-6B \cos(fx+e)^2 \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 2A \sin(fx+e) - 2A \sin(fx+e) \cos(fx+e) - 4A \sin(fx+e) \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 3B \cos(fx+e)^2 \ln\left(\frac{2}{\cos(fx+e)+1}\right) - B \cos(fx+e)^2 \sin(fx+e) - 2A - 4B + A \cos(fx+e)^2 \ln\left(\frac{2}{\cos(fx+e)+1}\right) - 2A \cos(fx+e)^2 \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 2A \sin(fx+e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) - 2A \cos(fx+e) \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + A \cos(fx+e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) - 3B \sin(fx+e) \cos(fx+e) - 12B \sin(fx+e) \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 6B \sin(fx+e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) - 6B \cos(fx+e) \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 3B \cos(fx+e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) - A \cos(fx+e) \sin(fx+e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) + 2A \cos(fx+e) \sin(fx+e) \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 3B \ln\left(\frac{2}{\cos(fx+e)+1}\right) \sin(fx+e) \cos(fx+e) + 6B \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) \sin(fx+e) \cos(fx+e) + 4B \cos(fx+e)^2 - B \cos(fx+e)^3 + 4B \sin(fx+e) + B \cos(fx+e) + 2A \cos(fx+e)^2 + 4A \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 2A \ln\left(\frac{2}{\cos(fx+e)+1}\right) + 12B \ln\left(\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 6B \ln\left(\frac{2}{\cos(fx+e)+1}\right) \right) \cdot \frac{(a(1+\sin(fx+e)))^{3/2}}{(\cos(fx+e)^2 + \sin(fx+e) \cos(fx+e) + \cos(fx+e) - 2 \sin(fx+e) - 2)^{3/2}} \cdot \frac{1}{(-c(\sin(fx+e) - 1))^{3/2}}$$

maxima [B] time = 0.57, size = 366, normalized size = 2.32

$$\frac{B \left(\frac{6a^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c^{\frac{3}{2}}} - \frac{3a^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2 + 1}\right)}{c^{\frac{3}{2}}} + \frac{2 \left(\frac{3a^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} - \frac{2a^{\frac{3}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3a^{\frac{3}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{c^{\frac{3}{2}} - \frac{2c^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{2c^{\frac{3}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{2c^{\frac{3}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{c^{\frac{3}{2}} \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} \right) + A \left(\frac{2a^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} - 1\right)}{c^{\frac{3}{2}}} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x, algorithm="maxima")

[Out]
$$-(B(6a^{3/2} \log(\sin(fx+e)/(\cos(fx+e)+1) - 1)/c^{3/2} - 3a^{3/2} \log(\sin(fx+e)^2/(\cos(fx+e)+1)^2 + 1)/c^{3/2} + 2(3a^{3/2} \sin(fx+e)/(\cos(fx+e)+1) - 2a^{3/2} \sin(fx+e)^2/(\cos(fx+e)+1)^2 + 3a^{3/2} \sin(fx+e)^3/(\cos(fx+e)+1)^3)/(c^{3/2} - 2c^{3/2} \sin(fx+e)/(\cos(fx+e)+1) + 2c^{3/2} \sin(fx+e)^2/(\cos(fx+e)+1)^2 - 2c^{3/2} \sin(fx+e)^3/(\cos(fx+e)+1)^3 + c^{3/2} \sin(fx+e)^4/(\cos(fx+e)+1)^4)) + A(2a^{3/2} \log(\sin(fx+e)/(\cos(fx+e)+1) - 1)$$

$/c^{3/2} - a^{3/2} \log(\sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 1) / c^{3/2} + 4$
 $*a^{3/2} \sqrt{c} \sin(fx + e) / ((c^2 - 2c^2 \sin(fx + e) / (\cos(fx + e) + 1)$
 $+ c^2 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2) * (\cos(fx + e) + 1)) / f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^{3/2} (A + B \sin(e + fx))}{(-c(\sin(e + fx) - 1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2), x)

[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) - 1))**(3/2), x)

$$3.145 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=149

$$\frac{a^2 B \cos(e+fx) \log(1-\sin(e+fx))}{c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{4f(c-c \sin(e+fx))^{5/2}} - \frac{aB \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{cf(c-c \sin(e+fx))}$$

[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(5/2)-a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^2*B*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.39, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2739, 2737, 2667, 31}

$$\frac{a^2 B \cos(e+fx) \log(1-\sin(e+fx))}{c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{4f(c-c \sin(e+fx))^{5/2}} - \frac{aB \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{cf(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*(c - c*Sin[e + f*x])^(3/2)) - (a^2*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972


```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{B \int \frac{(a + a \sin(e + fx))}{(c - c \sin(e + fx))} dx}{c} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{aB \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{cf(c - c \sin(e + fx))^{5/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{aB \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{cf(c - c \sin(e + fx))^{5/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{aB \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{cf(c - c \sin(e + fx))^{5/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{aB \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{cf(c - c \sin(e + fx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.94, size = 198, normalized size = 1.33

$$\frac{a \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin(e + fx) \left(A + 4B \log\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) \right) \right)}{c^2 f (\sin(e + fx) - 1)^2 \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

```

[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(5/2), x]

```

```

[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(B*Cos[
2*(e + f*x)]*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]])) + (A + 3*B + 4*B*Log[Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]])*Sin[e + f*x))/(c^2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]])

```

fricas [F] time = 2.95, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Ba \cos(fx + e)^2 - (A + B)a \sin(fx + e) - (A + B)a \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{3c^3 \cos(fx + e)^2 - 4c^3 - \left(c^3 \cos(fx + e)^2 - 4c^3 \right) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x,
algorithm="fricas")

```

```

[Out] integral((B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*sqrt(a*s
in(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 -
(c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
algorithm="giac")

[Out] Timed out

maple [B] time = 0.66, size = 594, normalized size = 3.99

$$\frac{\left(A \sin(fx + e) - A \sin(fx + e) \cos(fx + e) + 2B \left(\cos^2(fx + e)\right) \sin(fx + e) - A + 3B + B \sin(fx + e) \cos(fx + e)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] 1/f*(6*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)-A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)^2*sin(f*x+e)-A+3*B-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+B*sin(f*x+e)*cos(f*x+e)+8*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+2*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+e)^2+2*B*cos(f*x+e)^3-3*B*sin(f*x+e)-2*B*cos(f*x+e)+A*cos(f*x+e)^2-8*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(3/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)/(-c*(sin(f*x+e)-1))^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{3}{2}}}{(c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2),x)

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2), x)
```

```
[Out] Timed out
```

$$3.146 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=96

$$\frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{24cf(c-c \sin(e+fx))^{5/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{6f(c-c \sin(e+fx))^{7/2}}$$

[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(7/2)+1/24*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)

Rubi [A] time = 0.27, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2972, 2742}

$$\frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{24cf(c-c \sin(e+fx))^{5/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{6f(c-c \sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(6*f*(c - c*Sin[e + f*x])^(7/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(5/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{3/2}}{6f(c-c \sin(e+fx))^{7/2}} + \frac{(A-5B) \int \frac{(a+a \sin(e+fx))^{3/2}}{(c-c \sin(e+fx))^{7/2}} dx}{6c} \\ &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{3/2}}{6f(c-c \sin(e+fx))^{7/2}} + \frac{(A-5B) \cos(e+fx)(a+a \sin(e+fx))^{3/2}}{24cf(c-c \sin(e+fx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.99, size = 125, normalized size = 1.30

$$\frac{a\sqrt{a(\sin(e+fx)+1)} \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right) (3(A-B) \sin(e+fx) + A - 3B \cos(2(e+fx)) + 4B)}{6c^3 f (\sin(e+fx) - 1)^3 \sqrt{c - c \sin(e+fx)} \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] -1/6*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(A + 4*B - 3*B*Cos[2*(e + f*x)] + 3*(A - B)*Sin[e + f*x]))/(c^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^3*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.45, size = 125, normalized size = 1.30

$$\frac{(6Ba \cos(fx + e)^2 - 3(A - B)a \sin(fx + e) - (A + 7B)a) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{6(3c^4 f \cos(fx + e)^3 - 4c^4 f \cos(fx + e) - (c^4 f \cos(fx + e)^3 - 4c^4 f \cos(fx + e)) \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x, algorithm="fricas")

[Out] 1/6*(6*B*a*cos(f*x + e)^2 - 3*(A - B)*a*sin(f*x + e) - (A + 7*B)*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e) - (c^4*f*cos(f*x + e)^3 - 4*c^4*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.66, size = 223, normalized size = 2.32

$$\frac{\sin(fx + e) (a (1 + \sin(fx + e)))^{\frac{3}{2}} (A (\cos^3(fx + e)) + A (\cos^2(fx + e)) \sin(fx + e) + B (\cos^3(fx + e)))}{(-c \sin(fx + e) + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x)

[Out] 1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(3/2)*(A*cos(f*x+e)^3+A*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-4*A*cos(f*x+e)^2+3*A*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^2-3*B*sin(f*x+e)*cos(f*x+e)-7*A*cos(f*x+e)-10*A*sin(f*x+e)-B*cos(f*x+e)+2*B*sin(f*x+e)+10*A-2*B)/(-c*(sin(f*x+e)-1))^(7/2)/(cos(f*x+e)^2+sin(f*x+e)*cos(f*x+e)+cos(f*x+e)-2*sin(f*x+e)-2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{(-c \sin(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(7/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{3/2}}{(c - c \sin(e + f x))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2), x)

[Out] Timed out

$$3.147 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$$

Optimal. Leaf size=146

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{96c^2 f(c-c \sin(e+fx))^{5/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{24cf(c-c \sin(e+fx))^{7/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{8f(c-c \sin(e+fx))^{9/2}}$$

[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(9/2)+1/24*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/96*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{96c^2 f(c-c \sin(e+fx))^{5/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{24cf(c-c \sin(e+fx))^{7/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{8f(c-c \sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(8*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(24*c*f*(c - c*Sin[e + f*x])^(7/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(96*c^2*f*(c - c*Sin[e + f*x])^(5/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{8f(c - c \sin(e + fx))^{9/2}} + \frac{(A - 3B) \int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{9/2}} dx}{4c}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{8f(c - c \sin(e + fx))^{9/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{24cf(c - c \sin(e + fx))^{9/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{8f(c - c \sin(e + fx))^{9/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{24cf(c - c \sin(e + fx))^{9/2}}$$

Mathematica [A] time = 1.35, size = 123, normalized size = 0.84

$$\frac{a \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (4A \sin(e + fx) + 2A - 3B \cos(2(e + fx)) + 3B)}{12c^4 f (\sin(e + fx) - 1)^4 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] (a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(2*A + 3*B - 3*B*Cos[2*(e + f*x)] + 4*A*Sin[e + f*x]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.47, size = 134, normalized size = 0.92

$$\frac{\left(3Ba \cos^2(fx + e) - 2Aa \sin(fx + e) - (A + 3B)a \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{6 \left(c^5 f \cos^5(fx + e) - 8c^5 f \cos^3(fx + e) + 8c^5 f \cos(fx + e) + 4 \left(c^5 f \cos^3(fx + e) - 2c^5 f \cos(fx + e) \right) \sin(fx + e) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2), x, algorithm="fricas")

[Out] -1/6*(3*B*a*cos(f*x + e)^2 - 2*A*a*sin(f*x + e) - (A + 3*B)*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.68, size = 217, normalized size = 1.49

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{3/2} \left(A \left(\cos^4(fx + e) \right) - A \left(\cos^3(fx + e) \right) \sin(fx + e) + 4A \left(\cos^3(fx + e) \right) + 5 \right)}{6f(-c \sin(fx + e) + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)`

[Out] $\frac{1}{6} \frac{f \sin(fx+e) (a(1+\sin(fx+e)))^{3/2} (A \cos(fx+e)^4 - A \cos(fx+e)^3 \sin(fx+e) + 4A \cos(fx+e)^3 + 5A \cos(fx+e)^2 \sin(fx+e) - 12A \cos(fx+e)^2 + 7A \sin(fx+e) \cos(fx+e) + 3B \cos(fx+e)^2 - 3B \sin(fx+e) \cos(fx+e) - 10A \cos(fx+e) - 17A \sin(fx+e) + 3B \sin(fx+e) + 17A - 3B)}{(-c(\sin(fx+e)-1))^{9/2} (\cos(fx+e)^2 + \sin(fx+e) \cos(fx+e) + \cos(fx+e) - 2 \sin(fx+e) - 2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{(-c \sin(fx + e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(9/2), x)`

mupad [B] time = 18.99, size = 245, normalized size = 1.68

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\frac{8 a e^{5i+fx5i} (2A+3B) \sqrt{a+a \sin(e+fx)}}{3c^5 f} + \frac{32 A a e^{5i+fx5i} \sin(e+fx) \sqrt{a+a \sin(e+fx)}}{3c^5 f} - \frac{8 B a e^{5i+fx5i} \cos(e+fx)}{3c^5 f} \right)}{84 \cos(e + fx) e^{5i+fx5i} - 54 e^{5i+fx5i} \cos(3e + 3fx) + 2 e^{5i+fx5i} \cos(5e + 5fx) - 96 e^{5i+fx5i} \sin(2e + 2fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(9/2),x)`

[Out] $((c - c \sin(e + fx))^{1/2} ((8a \exp(e5i + fx5i) (2A + 3B) (a + a \sin(e + fx))^{1/2}) / (3c^5 f) + (32Aa \exp(e5i + fx5i) \sin(e + fx) (a + a \sin(e + fx))^{1/2}) / (3c^5 f) - (8B a \exp(e5i + fx5i) \cos(2e + 2fx) (a + a \sin(e + fx))^{1/2}) / (c^5 f)) / (84 \cos(e + fx) \exp(e5i + fx5i) - 54 \exp(e5i + fx5i) \cos(3e + 3fx) + 2 \exp(e5i + fx5i) \cos(5e + 5fx) - 96 \exp(e5i + fx5i) \sin(2e + 2fx) + 16 \exp(e5i + fx5i) \sin(4e + 4fx))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),x)`

[Out] Timed out

$$3.148 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$$

Optimal. Leaf size=154

$$-\frac{a^2(3A-7B) \cos(e+fx)}{120c^2 f \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{7/2}} + \frac{a(3A-7B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{40cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)}{10f(c-c \sin(e+fx))^{11/2}}$$

[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f/(c-c*sin(f*x+e))^(11/2)-1/120*a^2*(3*A-7*B)*cos(f*x+e)/c^2/f/(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(1/2)+1/40*a*(3*A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(9/2)

Rubi [A] time = 0.37, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2739, 2738}

$$-\frac{a^2(3A-7B) \cos(e+fx)}{120c^2 f \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{7/2}} + \frac{a(3A-7B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{40cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)}{10f(c-c \sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)) + (a*(3*A - 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(40*c*f*(c - c*Sin[e + f*x])^(9/2)) - (a^2*(3*A - 7*B)*Cos[e + f*x])/(120*c^2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(7/2))

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2739

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{(3A - 7B) \int \frac{(a + a \sin(e + fx))^{3/2}}{(c - c \sin(e + fx))^{11/2}} dx}{10f(c - c \sin(e + fx))^{11/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{a(3A - 7B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{40cf(c - c \sin(e + fx))^{11/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{a(3A - 7B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{40cf(c - c \sin(e + fx))^{11/2}}$$

Mathematica [A] time = 1.95, size = 126, normalized size = 0.82

$$\frac{a\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (5(3A + B) \sin(e + fx) + 9(A + B) - 10B \cos(2(e + fx)))}{60c^5 f (\sin(e + fx) - 1)^5 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] -1/60*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(9*(A + B) - 10*B*Cos[2*(e + f*x)] + 5*(3*A + B)*Sin[e + f*x]))/(c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.46, size = 155, normalized size = 1.01

$$\frac{\left(20Ba \cos^2(fx + e) - 5(3A + B)a \sin(fx + e) - (9A + 19B)a\right) \sqrt{a \sin(fx + e) + a} \sqrt{c - c \sin(fx + e)}}{60 \left(5c^6 f \cos^5(fx + e) - 20c^6 f \cos^3(fx + e) + 16c^6 f \cos(fx + e) - \left(c^6 f \cos^5(fx + e) - 12c^6 f \cos^3(fx + e) + 16c^6 f \cos(fx + e)\right) \sin(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="fricas")

[Out] -1/60*(20*B*a*cos(f*x + e)^2 - 5*(3*A + B)*a*sin(f*x + e) - (9*A + 19*B)*a)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.69, size = 339, normalized size = 2.20

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{\frac{3}{2}} \left(9A \cos^5(fx + e) + 9A \cos^4(fx + e) \right) \sin(fx + e) - B \cos^5(fx + e)}{60c^5 f (\sin(fx + e) - 1)^5 \sqrt{c - c \sin(fx + e)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)`

[Out]
$$-1/60/f*\sin(f*x+e)*(a*(1+\sin(f*x+e)))^{3/2}*(9*A*\cos(f*x+e)^5+9*A*\cos(f*x+e)^4*\sin(f*x+e)-B*\cos(f*x+e)^5-B*\sin(f*x+e)*\cos(f*x+e)^4-54*A*\cos(f*x+e)^4+45*A*\cos(f*x+e)^3*\sin(f*x+e)+6*B*\cos(f*x+e)^4-5*B*\cos(f*x+e)^3*\sin(f*x+e)-108*A*\cos(f*x+e)^3-153*A*\cos(f*x+e)^2*\sin(f*x+e)+12*B*\cos(f*x+e)^3+17*B*\cos(f*x+e)^2*\sin(f*x+e)+288*A*\cos(f*x+e)^2-135*A*\sin(f*x+e)*\cos(f*x+e)-52*B*\cos(f*x+e)^2+35*B*\sin(f*x+e)*\cos(f*x+e)+159*A*\cos(f*x+e)+294*A*\sin(f*x+e)-11*B*\cos(f*x+e)-46*B*\sin(f*x+e)-294*A+46*B)/(-c*(\sin(f*x+e)-1))^{11/2}/(\cos(f*x+e)^2+\sin(f*x+e)*\cos(f*x+e)+\cos(f*x+e)-2*\sin(f*x+e)-2)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{(-c \sin(fx + e) + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(-c*sin(f*x + e) + c)^(11/2), x)`

mupad [B] time = 20.23, size = 279, normalized size = 1.81

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\frac{a e^{e6i+fx6i} (A+B) \sqrt{a+a \sin(e+fx)} 48i}{5c^6 f} - \frac{B a e^{e6i+fx6i} \cos(2e+2fx) \sqrt{a+a \sin(e+fx)}}{3c^6 f} \right)}{\cos(e + fx) e^{e6i+fx6i} 264i - e^{e6i+fx6i} \cos(3e + 3fx) 220i + e^{e6i+fx6i} \cos(5e + 5fx) 20i - e^{e6i+fx6i} \sin(2e + 2fx) 330i + e^{e6i+fx6i} \sin(4e + 4fx) 88i - e^{e6i+fx6i} \sin(6e + 6fx) 2i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c - c*sin(e + f*x))^(11/2),x)`

[Out]
$$((c - c*\sin(e + f*x))^{1/2}*((a*\exp(e*6i + f*x*6i)*(A + B)*(a + a*\sin(e + f*x))^{1/2}*48i)/(5*c^6*f) - (B*a*\exp(e*6i + f*x*6i)*\cos(2*e + 2*f*x)*(a + a*\sin(e + f*x))^{1/2}*32i)/(3*c^6*f) + (16*a*\exp(e*6i + f*x*6i)*\sin(e + f*x)*(A*3i + B*1i)*(a + a*\sin(e + f*x))^{1/2})/(3*c^6*f)))/(\cos(e + f*x)*\exp(e*6i + f*x*6i)*264i - \exp(e*6i + f*x*6i)*\cos(3*e + 3*f*x)*220i + \exp(e*6i + f*x*6i)*\cos(5*e + 5*f*x)*20i - \exp(e*6i + f*x*6i)*\sin(2*e + 2*f*x)*330i + \exp(e*6i + f*x*6i)*\sin(4*e + 4*f*x)*88i - \exp(e*6i + f*x*6i)*\sin(6*e + 6*f*x)*2i)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)`

[Out] Timed out

$$3.149 \quad \int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=198

$$\frac{a^3(7A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{105f \sqrt{a \sin(e + fx) + a}} - \frac{2a^2(7A - B) \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{7/2}}{105f}$$

[Out] $-1/42*a*(7*A-B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-1/7*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-1/105*a^3*(7*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-2/105*a^2*(7*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.48, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{2a^2(7A - B) \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{7/2}}{105f} - \frac{a^3(7A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{105f \sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(5/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(7/2)}, x]$

[Out] $-(a^3*(7*A - B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(105*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*a^2*(7*A - B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(105*f) - (a*(7*A - B)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(42*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(7*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx &= -\frac{B \cos(e + fx) (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{7/2}}{7f} \\
&= -\frac{a(7A - B) \cos(e + fx) (a + a \sin(e + fx))^3}{42f} \\
&= -\frac{2a^2(7A - B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{105f} \\
&= -\frac{a^3(7A - B) \cos(e + fx) (c - c \sin(e + fx))^{7/2}}{105f \sqrt{a + a \sin(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 2.65, size = 223, normalized size = 1.13

$$\frac{c^3(\sin(e + fx) - 1)^3(a(\sin(e + fx) + 1))^{5/2}\sqrt{c - c \sin(e + fx)}(525(A - B) \cos(2(e + fx)) + 210(A - B) \cos(4(e + fx)))}{210f \cos(fx + e)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]

[Out] -1/6720*(c^3*(-1 + Sin[e + f*x])^3*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]]*(525*(A - B)*Cos[2*(e + f*x)] + 210*(A - B)*Cos[4*(e + f*x)] + 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] - 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] + 35*B*Sin[3*(e + f*x)] + 84*A*Sin[5*(e + f*x)] + 63*B*Sin[5*(e + f*x)] + 15*B*Sin[7*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.48, size = 160, normalized size = 0.81

$$\frac{(35(A - B)a^2c^3 \cos(fx + e)^6 - 35(A - B)a^2c^3 + 2(15Ba^2c^3 \cos(fx + e)^6 + 3(7A - B)a^2c^3 \cos(fx + e)^4 + 4(7A - B)a^2c^3 \cos(fx + e)^2 + 8(7A - B)a^2c^3 \sin(fx + e)) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c})}{210f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out] 1/210*(35*(A - B)*a^2*c^3*cos(f*x + e)^6 - 35*(A - B)*a^2*c^3 + 2*(15*B*a^2*c^3*cos(f*x + e)^6 + 3*(7*A - B)*a^2*c^3*cos(f*x + e)^4 + 4*(7*A - B)*a^2*c^3*cos(f*x + e)^2 + 8*(7*A - B)*a^2*c^3*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [B] time = 18.17, size = 383, normalized size = 1.93

$$e^{-e7i-fx7i} \sqrt{c - c \sin(e + fx)} \left(-\frac{a^2 c^3 e^{e7i+fx7i} \cos(2e+2fx)(A1i-B1i) \sqrt{a+a \sin(e+fx)} 5i}{32f} - \frac{a^2 c^3 e^{e7i+fx7i} \cos(4e+4fx)(A1i-B1i)}{16f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(7/2),x)
```

```
[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^2*c^3*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(4*A + 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(16*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) - (a^2*c^3*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (5*a^2*c^3*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A - B)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (a^2*c^3*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A + B)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (B*a^2*c^3*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2),
x)
```

```
[Out] Timed out
```


$$3.150 \quad \int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=180

$$\frac{2a^3 A \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{15f \sqrt{a \sin(e + fx) + a}} - \frac{a^2 A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{5/2}}{5f} - \frac{a A \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{5f}$$

[Out] $-1/5*a*A*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(5/2)}/f-1/6*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}*(c-c*\sin(f*x+e))^{(5/2)}/f-2/15*a^3*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-1/5*a^2*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.47, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{2a^3 A \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{15f \sqrt{a \sin(e + fx) + a}} - \frac{a^2 A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{5/2}}{5f} - \frac{a A \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{5f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(5/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(5/2)}, x]$

[Out] $(-2*a^3*A*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(15*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (a^2*A*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(5*f) - (a*A*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(5*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)})/(6*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]]*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}(((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}(((a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)])*((c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)*(-80*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))/(16*f)^2-480*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(3*f*x+3*exp(1))/(96*f)^2-160*A*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(160*f)^2+64*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(2*f*x+2*exp(1))/(64*f)^2+768*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(256*f)^2+384*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(384*f)^2+384*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-2*f*x-2*exp(1))/(-128*f)^2+256*B*a^2*c^2*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-256*f)^2)

maple [A] time = 0.77, size = 114, normalized size = 0.63

$$\frac{(5B \sin(fx + e) (\cos^4(fx + e)) + 6A (\cos^4(fx + e)) + 5B (\cos^2(fx + e)) \sin(fx + e) + 8A (\cos^2(fx + e)))}{30f \cos(fx + e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)

[Out] 1/30/f*(5*B*sin(f*x+e)*cos(f*x+e)^4+6*A*cos(f*x+e)^4+5*B*cos(f*x+e)^2*sin(f*x+e)+8*A*cos(f*x+e)^2+5*B*sin(f*x+e)+16*A)*(-c*(sin(f*x+e)-1))^(5/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/cos(f*x+e)^5

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2), x)

mupad [B] time = 16.00, size = 131, normalized size = 0.73

$$\frac{a^2 c^2 \sqrt{a (\sin(e + fx) + 1)} \sqrt{-c (\sin(e + fx) - 1)} (75 B \cos(e + fx) + 105 B \cos(3e + 3fx) + 35 B \cos(5e + 5fx) + 5 B \cos(7e + 7fx) - 700 A \sin(2e + 2fx) - 112 A \sin(4e + 4fx) - 12 A \sin(6e + 6fx))}{960 f (\cos(e + fx) + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2),x)

[Out] -(a^2*c^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(75*B*cos(e + f*x) + 105*B*cos(3*e + 3*f*x) + 35*B*cos(5*e + 5*f*x) + 5*B*cos(7*e + 7*f*x) - 700*A*sin(2*e + 2*f*x) - 112*A*sin(4*e + 4*f*x) - 12*A*sin(6*e + 6*f*x)))/(960*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),  
x)
```

```
[Out] Timed out
```

$$3.151 \quad \int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=142

$$\frac{c^2(5A + B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{30f\sqrt{c - c \sin(e + fx)}} + \frac{c(5A + B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}\sqrt{c - c \sin(e + fx)}}{20f}$$

[Out] $-1/5*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}*(c-c*\sin(f*x+e))^{(3/2)}/f+1/30*(5*A+B)*c^2*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}+1/20*(5*A+B)*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}*(c-c*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.36, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{c^2(5A + B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{30f\sqrt{c - c \sin(e + fx)}} + \frac{c(5A + B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}\sqrt{c - c \sin(e + fx)}}{20f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(5/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(3/2)}, x]$

[Out] $((5*A + B)*c^2*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)})/(30*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((5*A + B)*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)}*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(20*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(5*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps


```
*x) + 75*B*cos(3*e + 3*f*x) + 15*B*cos(5*e + 5*f*x) - 400*A*sin(2*e + 2*f*x)
) - 40*A*sin(4*e + 4*f*x) - 50*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x) +
6*B*sin(6*e + 6*f*x))/(480*f*(cos(2*e + 2*f*x) + 1))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),
x)
```

```
[Out] Timed out
```


3.152 $\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$

Optimal. Leaf size=96

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{3f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{4af\sqrt{c - c \sin(e + fx)}}$$

[Out] $1/3*(A-B)*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}/f/(c-c*\sin(f*x+e))^{(1/2)+1/4*}$
 $B*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}/a/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, number of rules / integrand size = 0.050, Rules used = {2971, 2738}

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{5/2}}{3f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{4af\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(5/2)}*(A + B*\text{Sin}[e + f*x])*Sqrt[c - c*\text{Sin}[e + f*x]], x]$

[Out] $((A - B)*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)})/(3*f*Sqrt[c - c*\text{Sin}[e + f*x]]) + (B*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)})/(4*a*f*Sqrt[c - c*\text{Sin}[e + f*x]])$

Rule 2738

$\text{Int}[Sqrt[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{(n)})/(f*(2*n + 1)*Sqrt[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

$\text{Int}[Sqrt[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Dist}[B/d, \text{Int}[Sqrt[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[Sqrt[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx = \frac{B \int (a + a \sin(e + fx))^{7/2} \sqrt{c - c \sin(e + fx)} dx}{a}$$

$$= \frac{(A - B)c \cos(e + fx)(a + a \sin(e + fx))^{5/2}}{3f\sqrt{c - c \sin(e + fx)}}$$

Mathematica [A] time = 0.83, size = 102, normalized size = 1.06

$$\frac{a^2 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (16(7A + 2B) \sin(e + fx) - 4 \cos(2(e + fx))(4(A + 2B) \sin(e + fx) - 4 \cos(2(e + fx))))}{96f}$$

Antiderivative was successfully verified.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)

[Out] 1/12/f*(3*B*cos(f*x+e)^2*sin(f*x+e)+4*A*cos(f*x+e)^2+8*B*cos(f*x+e)^2-12*A*sin(f*x+e)-9*B*sin(f*x+e)-16*A-8*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^2-2*sin(f*x+e)-2)/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) (a \sin(fx + e) + a)^{\frac{5}{2}} \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c), x)

mupad [B] time = 2.70, size = 149, normalized size = 1.55

$$\frac{a^2 \sqrt{a (\sin(e + fx) + 1)} \sqrt{-c (\sin(e + fx) - 1)} (48 A \cos(e + fx) + 36 B \cos(e + fx) + 48 A \cos(3e -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2),x)

[Out] -(a^2*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(48*A*cos(e + f*x) + 36*B*cos(e + f*x) + 48*A*cos(3*e + 3*f*x) + 33*B*cos(3*e + 3*f*x) - 3*B*cos(5*e + 5*f*x) - 112*A*sin(2*e + 2*f*x) + 8*A*sin(4*e + 4*f*x) - 32*B*sin(2*e + 2*f*x) + 16*B*sin(4*e + 4*f*x)))/(96*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)

[Out] Timed out

$$3.153 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=193

$$\frac{4a^3(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{2a^2(A+B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{a(A+B) \cos(e+fx)}{2f\sqrt{c-c \sin(e+fx)}}$$

[Out] $-1/2*a*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-1/3$
 $*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-4*a^3*(A+B)*$
 $\cos(f*x+e)*\ln(1-\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-$
 $2*a^2*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.46, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2a^2(A+B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{4a^3(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{a(A+B) \cos(e+fx)}{2f\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] $(-4*a^3*(A+B)*\cos[e+f*x]*\log[1-\sin[e+f*x]])/(f*\sqrt{a+a*\sin[e+f*x]})$
 $*\sqrt{c-c*\sin[e+f*x]} - (2*a^2*(A+B)*\cos[e+f*x]*\sqrt{a+a*\sin[e+f*x]})$
 $/(f*\sqrt{c-c*\sin[e+f*x]}) - (a*(A+B)*\cos[e+f*x]*(a+a*\sin[e+f*x])^{(3/2)})$
 $/(2*f*\sqrt{c-c*\sin[e+f*x]}) - (B*\cos[e+f*x]*(a+a*\sin[e+f*x])^{(5/2)})$
 $/(3*f*\sqrt{c-c*\sin[e+f*x]})$

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1)/2, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILTQ[n, m])

Q[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= -\frac{B \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{3f \sqrt{c - c \sin(e + fx)}} + (A + B) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx \\ &= -\frac{a(A + B) \cos(e + fx) (a + a \sin(e + fx))^{3/2}}{2f \sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{3f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{2a^2(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{a(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{2a^2(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{a(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{2a^2(A + B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{f \sqrt{c - c \sin(e + fx)}} - \frac{a(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{4a^3(A + B) \cos(e + fx) \log(1 - \sin(e + fx))}{f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} - \frac{2a^2(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f \sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.50, size = 177, normalized size = 0.92

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left((36A + 51B) \sin(e + fx) - 3(A + 3B) \cos(2(e + fx)) \right)}{12f \sqrt{c - c \sin(e + fx)} \left(\sin(e + fx) + 1 \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] -1/12*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(-3*(A + 3*B)*Cos[2*(e + f*x)] + 96*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (36*A + 51*B)*Sin[e + f*x] - B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 4.60, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A + 2B)a^2 \cos^2(fx + e) - 2(A + B)a^2 + (Ba^2 \cos^2(fx + e) - 2(A + B)a^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{c \sin(fx + e) - c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="fricas")

[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e)
)^2 - 2*(A + B)*a^2)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(c*sin(f*x + e) - c), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="giac")

[Out] Timed out

maple [B] time = 0.72, size = 590, normalized size = 3.06

$$\frac{(15A \sin(fx + e) - 18A \sin(fx + e) \cos(fx + e) + 7B (\cos^2(fx + e)) \sin(fx + e) - 48A \sin(fx + e) \ln\left(-\frac{-1+\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) + 3A \cos(fx+e)^2 \sin(fx+e) + 7B \cos(fx+e)^2 \sin(fx+e) - 2B \cos(fx+e)^4 - 15A - 17B + 3A \cos(fx+e)^3 + 24A \sin(fx+e) \ln(2/(\cos(fx+e)+1)) - 48A \cos(fx+e) \ln(-(-1+\cos(fx+e)+\sin(fx+e))/\sin(fx+e)) + 24A \cos(fx+e) \ln(2/(\cos(fx+e)+1)) - 26B \sin(fx+e) \cos(fx+e) - 48B \sin(fx+e) \ln(-(-1+\cos(fx+e)+\sin(fx+e))/\sin(fx+e)) + 24B \sin(fx+e) \ln(2/(\cos(fx+e)+1)) - 48B \cos(fx+e) \ln(-(-1+\cos(fx+e)+\sin(fx+e))/\sin(fx+e)) + 24B \cos(fx+e) \ln(2/(\cos(fx+e)+1)) + 19B \cos(fx+e)^2 + 9B \cos(fx+e)^3 + 17B \sin(fx+e) - 3A \cos(fx+e) - 9B \cos(fx+e) + 15A \cos(fx+e)^2 + 48A \ln(-(-1+\cos(fx+e)+\sin(fx+e))/\sin(fx+e)) - 24A \ln(2/(\cos(fx+e)+1)) + 48B \ln(-(-1+\cos(fx+e)+\sin(fx+e))/\sin(fx+e)) - 24B \ln(2/(\cos(fx+e)+1))) * (a*(1+\sin(fx+e)))^(5/2) / (\cos(fx+e)^3 - \cos(fx+e)^2 \sin(fx+e) - 3 \cos(fx+e)^2 - 2 \sin(fx+e) \cos(fx+e) - 2 \cos(fx+e) + 4 \sin(fx+e) + 4) / (-c*(\sin(fx+e)-1))^(1/2)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] 1/6/f*(2*B*cos(f*x+e)^3*sin(f*x+e)+15*A*sin(f*x+e)-18*A*sin(f*x+e)*cos(f*x+
e)-48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e
)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e)-2*B*cos(f*x+e)^4-15*A-17*B+3*A*c
os(f*x+e)^3+24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*cos(f*x+e)*ln(-(-1+co
s(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-26*B*
sin(f*x+e)*cos(f*x+e)-48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*
x+e))+24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*B*cos(f*x+e)*ln(-(-1+cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+19*B*cos(f*
x+e)^2+9*B*cos(f*x+e)^3+17*B*sin(f*x+e)-3*A*cos(f*x+e)-9*B*cos(f*x+e)+15*A*
cos(f*x+e)^2+48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*A*ln(2/(cos
(f*x+e)+1))+48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*ln(2/(cos(
f*x+e)+1)))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-
3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)/(-c*(si
n(f*x+e)-1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/sqrt(-c*sin(f*x +
e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{5}{2}}}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(1/2), x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2), x)
```

```
[Out] Timed out
```

$$3.154 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=210

$$\frac{4a^3(A+2B) \cos(e+fx) \log(1-\sin(e+fx))}{cf\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} + \frac{2a^2(A+2B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{cf\sqrt{c-c \sin(e+fx)}} + \frac{a(A+2B) \cos(e+fx)}{2cf\sqrt{c-c \sin(e+fx)}}$$

[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^3*(A+2*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a^2*(A+2*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^2(A+2B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{cf\sqrt{c-c \sin(e+fx)}} + \frac{4a^3(A+2B) \cos(e+fx) \log(1-\sin(e+fx))}{cf\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} + \frac{a(A+2B) \cos(e+fx)}{2cf\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(2*f*(c - c*Sin[e + f*x])^(3/2)) + (4*a^3*(A + 2*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^2*(A + 2*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(A + 2*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILTQ[n, m])

Q[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} - \frac{(A + 2B) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{c} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a(A + 2B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2cf\sqrt{c - c \sin(e + fx)}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^2(A + 2B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{cf\sqrt{c - c \sin(e + fx)}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^2(A + 2B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{cf\sqrt{c - c \sin(e + fx)}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^2(A + 2B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{cf\sqrt{c - c \sin(e + fx)}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{4a^3(A + 2B) \cos(e + fx) \int \frac{(a + a \sin(e + fx))^{5/2}}{\sqrt{c - c \sin(e + fx)}} dx}{cf\sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.64, size = 231, normalized size = 1.10

$$\frac{a^2 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(2(2A + 7B) \cos(2(e + fx)) + \sin(e + fx) \right) \left(-64(A + B) \cos(e + fx) \sqrt{c - c \sin(e + fx)} \right)}{c^2 \cos^2(e + fx) + 2c^2 \sin(e + fx) - 2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] -1/8*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(28*A + 16*B + 2*(2*A + 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 128*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (8*A + 31*B - 64*(A + 2*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])*Sin[e + f*x] + B*Sin[3*(e + f*x)])/(c*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 10.49, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A + 2B)a^2 \cos^2(fx + e) - 2(A + B)a^2 + (Ba^2 \cos^2(fx + e) - 2(A + B)a^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{c^2 \cos^2(fx + e) + 2c^2 \sin(fx + e) - 2c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e)
)^2 - 2*(A + B)*a^2)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 846, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(32*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(
f*x+e)^3*sin(f*x+e)-12*A*sin(f*x+e)+10*A*sin(f*x+e)*cos(f*x+e)+32*A*sin(f*x
+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*B*cos(f*x+e)^2*ln(2/(cos(
f*x+e)+1))+2*A*cos(f*x+e)^2*sin(f*x+e)+6*B*cos(f*x+e)^2*sin(f*x+e)-B*cos(f*
x+e)^4+12*A+22*B+2*A*cos(f*x+e)^3-8*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*
A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*A*sin(f*x+e)*l
n(2/(cos(f*x+e)+1))+16*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+
e))-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*sin(f*x+e)*cos(f*x+e)+64*B*sin
(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*sin(f*x+e)*ln(2/(co
s(f*x+e)+1))+32*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-16*
B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-16*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
+16*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-32*B*ln(-(-1+cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-21*B*cos(f*x+e)^2+7*B*cos(f*x+e
)^3-22*B*sin(f*x+e)-2*A*cos(f*x+e)-7*B*cos(f*x+e)-12*A*cos(f*x+e)^2-32*A*ln
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+16*A*ln(2/(cos(f*x+e)+1))-64*B*ln(
(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+32*B*ln(2/(cos(f*x+e)+1)))*(a*(1+si
n(f*x+e)))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)-cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x
+e)+3*cos(f*x+e)^2-4*sin(f*x+e)+2*cos(f*x+e)-4)/(-c*(sin(f*x+e)-1))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2}}{(c - c \sin(e + f x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2), x)

[Out] Timed out

$$3.155 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=212

$$\frac{a^3(A+5B) \cos(e+fx) \log(1-\sin(e+fx))}{c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{a^2(A+5B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{a(A+5B) \cos(e+fx)}{4cf(c-c \sin(e+fx))}$$

[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(5/2)-1/4*a*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(3/2)-a^3*(A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-1/2*a^2*(A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.49, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{a^2(A+5B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{a^3(A+5B) \cos(e+fx) \log(1-\sin(e+fx))}{c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{a(A+5B) \cos(e+fx)}{4cf(c-c \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(4*f*(c - c*Sin[e + f*x])^(5/2)) - (a*(A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c*f*(c - c*Sin[e + f*x])^(3/2)) - (a^3*(A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (a^2*(A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^2*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n, 0])

+ 1, 0])

Rule 2740

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{(A + 5B) \int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{5/2}} dx}{4c} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 5B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4cf(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 5B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4cf(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 5B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4cf(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 5B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4cf(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 5B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{4cf(c - c \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 1.16, size = 207, normalized size = 0.98

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-4(A + 2B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)^2}{f(c - c \sin(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*SIN[e + f*x])^(5/2)*(A + B*SIN[e + f*x]))/(c - c*SIN[e + f*x])^(5/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2)*(2*(A + B) - 4*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 - 2*(A + 5*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(5/2))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A + 2B)a^2 \cos(fx + e)^2 - 2(A + B)a^2 + (Ba^2 \cos(fx + e)^2 - 2(A + B)a^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{3c^3 \cos(fx + e)^2 - 4c^3 - (c^3 \cos(fx + e)^2 - 4c^3) \sin(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e))^2 - 2*(A + B)*a^2)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 1092, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] -1/f*(30*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^3*sin(f*x+e)-2*A*sin(f*x+e)+8*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^4+2*A+14*B+2*A*cos(f*x+e)^3-10*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+5*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-3*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+6*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+4*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*sin(f*x+e)*cos(f*x+e)+40*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+20*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-10*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-4*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-10*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+5*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+10*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-20*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-15*B*cos(f*x+e)^2-2*A*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*B*cos(f*x+e)^3-14*B*sin(f*x+e)-2*A*cos(f*x+e)-8*B*cos(f*x+e)-2*A*cos(f*x+e)^2-8*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*ln(2/(cos(f*x+e)+1)))
```

$f*x+e)+1))-40*B*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))+20*B*\ln(2/(\cos(f*x+e)+1)))*(a*(1+\sin(f*x+e)))^{(5/2)}/(\cos(f*x+e)^3-\cos(f*x+e)^2*\sin(f*x+e)-3*\cos(f*x+e)^2-2*\sin(f*x+e)*\cos(f*x+e)-2*\cos(f*x+e)+4*\sin(f*x+e)+4)/(-c*(\sin(f*x+e)-1))^{(5/2)}$

maxima [B] time = 0.48, size = 506, normalized size = 2.39

$$\left(\frac{8a^{\frac{5}{2}}\sqrt{c}\sin^2(fx+e)}{\left(c^3 - \frac{4c^3\sin(fx+e)}{\cos(fx+e)+1} + \frac{6c^3\sin^2(fx+e)}{(\cos(fx+e)+1)^2} - \frac{4c^3\sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{c^3\sin^4(fx+e)}{(\cos(fx+e)+1)^4} \right) (\cos(fx+e)+1)^2} - \frac{2a^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)}{c^{\frac{5}{2}}} + \frac{a^{\frac{5}{2}}\log\left(\frac{\sin^2(fx+e)}{(\cos(fx+e)+1)^2+1}\right)}{c^{\frac{5}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] $-\left((8a^{(5/2)}\sqrt{c}\sin(fx+e)^2/((c^3-4c^3\sin(fx+e)/(\cos(fx+e)+1)+6c^3\sin^2(fx+e)/(\cos(fx+e)+1)^2-4c^3\sin^3(fx+e)/(\cos(fx+e)+1)^3+c^3\sin^4(fx+e)/(\cos(fx+e)+1)^4)*(\cos(fx+e)+1)^2) - 2a^{(5/2)}\log(\sin(fx+e)/(\cos(fx+e)+1)-1)/c^{(5/2)} + a^{(5/2)}\log(\sin^2(fx+e)/(\cos(fx+e)+1)^2+1)/c^{(5/2)} * A - B*(10a^{(5/2)}\log(\sin(fx+e)/(\cos(fx+e)+1)-1)/c^{(5/2)} - 5a^{(5/2)}\log(\sin^2(fx+e)/(\cos(fx+e)+1)^2+1)/c^{(5/2)} + 2*(5a^{(5/2)}\sin(fx+e)/(\cos(fx+e)+1)-16a^{(5/2)}\sin^2(fx+e)/(\cos(fx+e)+1)^2+14a^{(5/2)}\sin^3(fx+e)/(\cos(fx+e)+1)^3-16a^{(5/2)}\sin^4(fx+e)/(\cos(fx+e)+1)^4+5a^{(5/2)}\sin^5(fx+e)/(\cos(fx+e)+1)^5)/(c^{(5/2)}-4c^{(5/2)}\sin(fx+e)/(\cos(fx+e)+1)+7c^{(5/2)}\sin^2(fx+e)/(\cos(fx+e)+1)^2-8c^{(5/2)}\sin^3(fx+e)/(\cos(fx+e)+1)^3+7c^{(5/2)}\sin^4(fx+e)/(\cos(fx+e)+1)^4-4c^{(5/2)}\sin^5(fx+e)/(\cos(fx+e)+1)^5+c^{(5/2)}\sin^6(fx+e)/(\cos(fx+e)+1)^6)) \right) / f$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B\sin(e+fx))(a+a\sin(e+fx))^{5/2}}{(c-c\sin(e+fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A+B*sin(e+f*x))*(a+a*sin(e+f*x))^(5/2))/(c-c*sin(e+f*x))^(5/2),x)

[Out] int(((A+B*sin(e+f*x))*(a+a*sin(e+f*x))^(5/2))/(c-c*sin(e+f*x))^(5/2),x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),x)

[Out] Timed out

$$3.156 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=196

$$\frac{a^3 B \cos(e+fx) \log(1-\sin(e+fx))}{c^3 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{a^2 B \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^2 f (c-c \sin(e+fx))^{3/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx))^{7/2}}{6 f (c-c \sin(e+fx))^{7/2}}$$

[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(7/2)-1/2*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(5/2)+a^2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^3*B*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.49, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2739, 2737, 2667, 31}

$$\frac{a^2 B \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^2 f (c-c \sin(e+fx))^{3/2}} + \frac{a^3 B \cos(e+fx) \log(1-\sin(e+fx))}{c^3 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx))^{7/2}}{6 f (c-c \sin(e+fx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*f*(c - c*Sin[e + f*x])^(7/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]/(c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^3*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]]/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]))

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x, x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n

+ 1, 0])

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{B \int \frac{(a + a \sin(e + fx))}{(c - c \sin(e + fx))} dx}{c} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{aB \cos(e + fx)}{2cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{aB \cos(e + fx)}{2cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{aB \cos(e + fx)}{2cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{aB \cos(e + fx)}{2cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{aB \cos(e + fx)}{2cf(c - c \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 1.19, size = 204, normalized size = 1.04

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(3(A + 5B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)^4}{3f(c - c \sin(e + fx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(7/2), x]
```

```
[Out] ((4*(A + B) - 6*(A + 2*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 3*(A +
5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*B*Log[Cos[(e + f*x)/2] - S
in[(e + f*x)/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(3*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(7/2))
```

fricas [F] time = 6.12, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A + 2B)a^2 \cos^2(fx + e) - 2(A + B)a^2 + (Ba^2 \cos^2(fx + e) - 2(A + B)a^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{c^4 \cos^4(fx + e) - 8c^4 \cos^2(fx + e) + 8c^4 + 4(c^4 \cos^2(fx + e) - 2c^4) \sin(fx + e)}}{c^4 \cos^4(fx + e) - 8c^4 \cos^2(fx + e) + 8c^4 + 4(c^4 \cos^2(fx + e) - 2c^4) \sin(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
algorithm="fricas")
```

```
[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x +
e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*
x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos
(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.69, size = 832, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x)
```

```
[Out] 1/3/f*(48*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-7*B*cos
(f*x+e)^3*sin(f*x+e)-4*A*sin(f*x+e)+4*A*sin(f*x+e)*cos(f*x+e)-A*cos(f*x+e)^
3*sin(f*x+e)-24*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)^2*sin(f*x+
e)+13*B*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^4+4*A+16*B-18*B*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+9*B*ln(2/(cos(f*x+e)+1))*cos(f*
x+e)^3+10*B*sin(f*x+e)*cos(f*x+e)+48*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))-24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*B*cos(f*x+e)*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))
-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+12
*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+12*B*ln(2/(cos(f*x+e)+1))*s
in(f*x+e)*cos(f*x+e)-24*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*
x+e)*cos(f*x+e)+3*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-6*B*cos(f*x+e)^4*ln(-
(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-23*B*cos(f*x+e)^2+A*cos(f*x+e)^4+6*B
*cos(f*x+e)^3-16*B*sin(f*x+e)+6*B*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e
)+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-
6*B*cos(f*x+e)-5*A*cos(f*x+e)^2-48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+24*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(5/2)/(cos(f*x+e)^3-cos(
f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*s
in(f*x+e)+4)/(-c*(sin(f*x+e)-1))^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}}{(-c \sin(fx + e) + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2}}{(c - c \sin(e + f x))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(7/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2), x)

[Out] Timed out

$$3.157 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$$

Optimal. Leaf size=96

$$\frac{(A-7B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{48cf(c-c \sin(e+fx))^{7/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{8f(c-c \sin(e+fx))^{9/2}}$$

[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(9/2)+1/48*(A-7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)

Rubi [A] time = 0.28, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2972, 2742}

$$\frac{(A-7B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{48cf(c-c \sin(e+fx))^{7/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{8f(c-c \sin(e+fx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(8*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(48*c*f*(c - c*Sin[e + f*x])^(7/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{5/2}}{8f(c-c \sin(e+fx))^{9/2}} + \frac{(A-7B) \int \frac{(a+a \sin(e+fx))^{5/2}}{(c-c \sin(e+fx))^{9/2}} dx}{8c} \\ &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{5/2}}{8f(c-c \sin(e+fx))^{9/2}} + \frac{(A-7B) \cos(e+fx)(a+a \sin(e+fx))^{5/2}}{48cf(c-c \sin(e+fx))^{7/2}} \end{aligned}$$

Mathematica [A] time = 2.96, size = 145, normalized size = 1.51

$$\frac{a^2 \sqrt{a(\sin(e+fx)+1)} \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right) \left((4A+17B) \sin(e+fx) - 3(A-B) \cos(2(e+fx)) \right) + 12c^4 f (\sin(e+fx)-1)^4 \sqrt{c-c \sin(e+fx)} \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)}{12c^4 f (\sin(e+fx)-1)^4 \sqrt{c-c \sin(e+fx)} \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(5*A - 5*B - 3*(A - B)*Cos[2*(e + f*x)] + (4*A + 17*B)*Sin[e + f*x] - 3*B*Sin[3*(e + f*x)]))/(12*c^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^4*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.45, size = 165, normalized size = 1.72

$$\frac{\left(3(A-B)a^2 \cos^2(fx+e) - 4(A-B)a^2 + 2\left(3Ba^2 \cos(fx+e)^2 - (A+5B)a^2\right) \sin(fx+e)\right) \sqrt{a \sin(fx+e)}}{6\left(c^5 f \cos(fx+e)^5 - 8c^5 f \cos(fx+e)^3 + 8c^5 f \cos(fx+e) + 4\left(c^5 f \cos(fx+e)^3 - 2c^5 f \cos(fx+e)\right) \sin(fx+e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2), x, algorithm="fricas")

[Out] -1/6*(3*(A - B)*a^2*cos(f*x + e)^2 - 4*(A - B)*a^2 + 2*(3*B*a^2*cos(f*x + e)^2 - (A + 5*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^5*f*cos(f*x + e)^5 - 8*c^5*f*cos(f*x + e)^3 + 8*c^5*f*cos(f*x + e) + 4*(c^5*f*cos(f*x + e)^3 - 2*c^5*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.68, size = 309, normalized size = 3.22

$$\frac{\sin(fx+e) \left(a(1+\sin(fx+e))\right)^{\frac{5}{2}} \left(A(\cos^4(fx+e)) - A(\cos^3(fx+e)) \sin(fx+e) - B(\cos^4(fx+e))\right)}{(-c \sin(fx+e) + c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2), x)

[Out] -1/6/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(A*cos(f*x+e)^4-A*cos(f*x+e)^3*sin(f*x+e)-B*cos(f*x+e)^4+B*cos(f*x+e)^3*sin(f*x+e)+4*A*cos(f*x+e)^3+5*A*cos(f*x+e)^2*sin(f*x+e)+2*B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)-9*A*cos(f*x+e)^2+4*A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-10*A*cos(f*x+e)-14*A*sin(f*x+e)-2*B*cos(f*x+e)+2*B*sin(f*x+e)+14*A-2*B)/(-c*(sin(f*x+e)-1))^(9/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(a \sin(fx+e) + a)^{\frac{5}{2}}}{(-c \sin(fx+e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e)
+ c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2}}{(c - c \sin(e + f x))^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)

[Out] Timed out

$$3.158 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$$

Optimal. Leaf size=146

$$\frac{(A-4B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{240c^2 f(c-c \sin(e+fx))^{7/2}} + \frac{(A-4B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{40cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{10f(c-c \sin(e+fx))^{11/2}}$$

[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(11/2)+1/40*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(9/2)+1/240*(A-4*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(7/2)

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(A-4B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{240c^2 f(c-c \sin(e+fx))^{7/2}} + \frac{(A-4B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{40cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{10f(c-c \sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 4*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(240*c^2*f*(c - c*Sin[e + f*x])^(7/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{11/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{(A - 4B) \int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{11/2}} dx}{5c}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{(A - 4B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{40cf(c - c \sin(e + fx))^{11/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{10f(c - c \sin(e + fx))^{11/2}} + \frac{(A - 4B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{40cf(c - c \sin(e + fx))^{11/2}}$$

Mathematica [A] time = 4.08, size = 146, normalized size = 1.00

$$\frac{a^2 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (-5(8A + 13B) \sin(e + fx) + 10(2A + B) \cos(2(e + fx)))}{120c^5 f (\sin(e + fx) - 1)^5 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(-36*A - 6*B + 10*(2*A + B)*Cos[2*(e + f*x)] - 5*(8*A + 13*B)*Sin[e + f*x] + 15*B*Sin[3*(e + f*x)]))/(120*c^5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^5*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.46, size = 182, normalized size = 1.25

$$\frac{\left(5(2A + B)a^2 \cos^2(fx + e) - 2(7A + 2B)a^2 + 5\left(3Ba^2 \cos^2(fx + e) - 2(A + 2B)a^2\right) \sin(fx + e)\right) \sqrt{a \sin(fx + e)}}{30\left(5c^6 f \cos^5(fx + e) - 20c^6 f \cos^3(fx + e) + 16c^6 f \cos(fx + e) - \left(c^6 f \cos^5(fx + e) - 12c^6 f \cos^3(fx + e) + 16c^6 f \cos(fx + e)\right) \sin(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="fricas")

[Out] -1/30*(5*(2*A + B)*a^2*cos(f*x + e)^2 - 2*(7*A + 2*B)*a^2 + 5*(3*B*a^2*cos(f*x + e)^2 - 2*(A + 2*B)*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e)^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.70, size = 368, normalized size = 2.52

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{5/2} \left(4A \left(\cos^5(fx + e) \right) + 4A \left(\cos^4(fx + e) \right) \sin(fx + e) - B \left(\cos^5(fx + e) \right) - \dots \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x)`

[Out] $\frac{1}{30} \frac{f \sin(fx+e) (a(1+\sin(fx+e)))^{5/2} (4A \cos(fx+e)^5 + 4A \cos(fx+e)^4 \sin(fx+e) - B \cos(fx+e)^5 - B \sin(fx+e) \cos(fx+e)^4 - 24A \cos(fx+e)^4 + 20A \cos(fx+e)^3 \sin(fx+e) + 6B \cos(fx+e)^4 - 5B \cos(fx+e)^3 \sin(fx+e) - 48A \cos(fx+e)^3 - 68A \cos(fx+e)^2 \sin(fx+e) - 3B \cos(fx+e)^3 + 2B \cos(fx+e)^2 \sin(fx+e) + 118A \cos(fx+e)^2 - 50A \sin(fx+e) \cos(fx+e) - 22B \cos(fx+e)^2 + 20B \sin(fx+e) \cos(fx+e) + 74A \cos(fx+e) + 124A \sin(fx+e) + 4B \cos(fx+e) - 16B \sin(fx+e) - 124A + 16B)}{(-c(\sin(fx+e)-1))^{11/2} (\cos(fx+e)^3 - \cos(fx+e)^2 \sin(fx+e) - 3\cos(fx+e)^2 - 2\sin(fx+e) \cos(fx+e) - 2\cos(fx+e) + 4\sin(fx+e) + 4)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}}{(-c \sin(fx + e) + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(-c*sin(f*x + e) + c)^(11/2), x)`

mupad [B] time = 20.76, size = 341, normalized size = 2.34

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\frac{16 a^2 e^{e 6i + f x 6i} (A 6i + B 1i) \sqrt{a + a \sin(e + fx)}}{5 c^6 f} - \frac{16 a^2 e^{e 6i + f x 6i} \cos(2e + 2fx) (A 2i + B 1i) \sqrt{a + a \sin(e + fx)}}{3 c^6 f} + \dots \right)}{\cos(e + fx) e^{e 6i + f x 6i} 264i - e^{e 6i + f x 6i} \cos(3e + 3fx) 220i + e^{e 6i + f x 6i} \cos(5e + 5fx) 20i - e^{e 6i + f x 6i} \sin(e + fx) a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(11/2),x)`

[Out] $((c - c \sin(e + f x))^{1/2} ((16 a^2 \exp(e 6i + f x 6i) (A 6i + B 1i) (a + a \sin(e + f x))^{1/2}) / (5 c^6 f) - (16 a^2 \exp(e 6i + f x 6i) \cos(2e + 2 f x) (A 2i + B 1i) (a + a \sin(e + f x))^{1/2}) / (3 c^6 f) + (a^2 \exp(e 6i + f x 6i) \sin(e + f x) (8 A + 13 B) (a + a \sin(e + f x))^{1/2} 8i) / (3 c^6 f) - (B a^2 \exp(e 6i + f x 6i) \sin(3e + 3 f x) (a + a \sin(e + f x))^{1/2} 8i) / (c^6 f)) / (\cos(e + f x) \exp(e 6i + f x 6i) 264i - \exp(e 6i + f x 6i) \cos(3e + 3 f x) 220i + \exp(e 6i + f x 6i) \cos(5e + 5 f x) 20i - \exp(e 6i + f x 6i) \sin(2e + 2 f x) 330i + \exp(e 6i + f x 6i) \sin(4e + 4 f x) 88i - \exp(e 6i + f x 6i) \sin(6e + 6 f x) 2i)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2),x)`

[Out] Timed out

$$3.159 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{13/2}} dx$$

Optimal. Leaf size=196

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{960c^3 f(c-c \sin(e+fx))^{7/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{160c^2 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{40cf(c-c \sin(e+fx))^{11/2}}$$

[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f/(c-c*sin(f*x+e))^(13/2)+1/40*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/160*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)+1/960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c^3/f/(c-c*sin(f*x+e))^(7/2)

Rubi [A] time = 0.48, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{960c^3 f(c-c \sin(e+fx))^{7/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{160c^2 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{5/2}}{40cf(c-c \sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(13/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(40*c*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(160*c^2*f*(c - c*Sin[e + f*x])^(9/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(960*c^3*f*(c - c*Sin[e + f*x])^(7/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{13/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{12f(c - c \sin(e + fx))^{13/2}} + \frac{(A - 3B) \int \frac{(a + a \sin(e + fx))^{5/2}}{(c - c \sin(e + fx))^{13/2}} dx}{40cf(c - c \sin(e + fx))^{13/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{12f(c - c \sin(e + fx))^{13/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{40cf(c - c \sin(e + fx))^{13/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{12f(c - c \sin(e + fx))^{13/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{40cf(c - c \sin(e + fx))^{13/2}} \\
&= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{12f(c - c \sin(e + fx))^{13/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{5/2}}{40cf(c - c \sin(e + fx))^{13/2}}
\end{aligned}$$

Mathematica [A] time = 5.62, size = 144, normalized size = 0.73

$$\frac{a^2 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (6(6A + 7B) \sin(e + fx) - 15(A + B) \cos(2(e + fx)))}{120c^6 f (\sin(e + fx) - 1)^6 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(13/2), x]

[Out] (a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(29*A + 13*B - 15*(A + B)*Cos[2*(e + f*x)] + 6*(6*A + 7*B)*Sin[e + f*x] - 10*B*Sin[3*(e + f*x)]))/(120*c^6*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^6*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.47, size = 196, normalized size = 1.00

$$\frac{\left(15(A + B)a^2 \cos(fx + e)^2 - 2(11A + 7B)a^2 + 2\left(10Ba^2 \cos(fx + e)^2 - (9A + 13B)a^2\right) \sin(fx + e)\right) \sin(fx + e)}{60\left(c^7 f \cos(fx + e)^7 - 18c^7 f \cos(fx + e)^5 + 48c^7 f \cos(fx + e)^3 - 32c^7 f \cos(fx + e) + 2\left(3c^7 f \cos(fx + e)^5 - 16c^7 f \cos(fx + e)^3 + 16c^7 f \cos(fx + e)\right) \sin(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2), x, algorithm="fricas")

[Out] 1/60*(15*(A + B)*a^2*cos(f*x + e)^2 - 2*(11*A + 7*B)*a^2 + 2*(10*B*a^2*cos(f*x + e)^2 - (9*A + 13*B)*a^2)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^7*f*cos(f*x + e)^7 - 18*c^7*f*cos(f*x + e)^5 + 48*c^7*f*cos(f*x + e)^3 - 32*c^7*f*cos(f*x + e) + 2*(3*c^7*f*cos(f*x + e)^5 - 16*c^7*f*cos(f*x + e)^3 + 16*c^7*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.72, size = 423, normalized size = 2.16

$$\frac{\sin(fx + e) \left(a(1 + \sin(fx + e)) \right)^{\frac{5}{2}} \left(444A \sin(fx + e) - 202A \sin(fx + e) \cos(fx + e) + 29B \left(\cos^2(fx + e) \right) \right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x)

[Out] 1/60/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(5/2)*(-17*B*cos(f*x+e)^3*sin(f*x+e)+44*A*sin(f*x+e)-202*A*sin(f*x+e)*cos(f*x+e)+119*A*cos(f*x+e)^3*sin(f*x+e)+49*A*cos(f*x+e)^4*sin(f*x+e)-7*B*sin(f*x+e)*cos(f*x+e)^4-343*A*cos(f*x+e)^2*sin(f*x+e)+29*B*cos(f*x+e)^2*sin(f*x+e)+24*B*cos(f*x+e)^4-444*A+52*B-224*A*cos(f*x+e)^3+46*B*sin(f*x+e)*cos(f*x+e)+42*A*cos(f*x+e)^5-6*B*cos(f*x+e)^5-B*cos(f*x+e)^6+B*cos(f*x+e)^5*sin(f*x+e)-7*A*cos(f*x+e)^5*sin(f*x+e)-75*B*cos(f*x+e)^2-168*A*cos(f*x+e)^4+12*B*cos(f*x+e)^3-52*B*sin(f*x+e)+242*A*cos(f*x+e)-6*B*cos(f*x+e)+7*A*cos(f*x+e)^6+545*A*cos(f*x+e)^2)/(-c*(sin(f*x+e)-1))^(13/2)/(cos(f*x+e)^3-cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^2-2*sin(f*x+e)*cos(f*x+e)-2*cos(f*x+e)+4*sin(f*x+e)+4)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 20.71, size = 357, normalized size = 1.82

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\frac{a^2 e^{7i+fx7i} (A29i+B13i) \sqrt{a+a \sin(e+fx)} 16i}{15c^7 f} - \frac{a^2 e^{7i+fx7i} \cos(2e+2fx) (A1i+B1i) \sqrt{a+a \sin(e+fx)}}{c^7 f} \right)}{-858 \cos(e + fx) e^{7i+fx7i} + 858 e^{7i+fx7i} \cos(3e + 3fx) - 130 e^{7i+fx7i} \cos(5e + 5fx) + 2 e^{7i+fx7i} \cos(7e + 7fx) + 1144 \exp(e*7i + f*x*7i) \sin(2e + 2fx) - 416 \exp(e*7i + f*x*7i) \sin(4e + 4fx) + 24 \exp(e*7i + f*x*7i) \sin(6e + 6fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c - c*sin(e + f*x))^(13/2),x)

[Out] ((c - c*sin(e + f*x))^(1/2)*((a^2*exp(e*7i + f*x*7i)*(A*29i + B*13i)*(a + a*sin(e + f*x))^(1/2)*16i)/(15*c^7*f) - (a^2*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*16i)/(c^7*f) - (32*a^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(6*A + 7*B)*(a + a*sin(e + f*x))^(1/2))/(5*c^7*f) + (32*B*a^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(3*c^7*f)))/(858*exp(e*7i + f*x*7i)*cos(3*e + 3*f*x) - 858*cos(e + f*x)*exp(e*7i + f*x*7i) - 130*exp(e*7i + f*x*7i)*cos(5*e + 5*f*x) + 2*exp(e*7i + f*x*7i)*cos(7*e + 7*f*x) + 1144*exp(e*7i + f*x*7i)*sin(2*e + 2*f*x) - 416*exp(e*7i + f*x*7i)*sin(4*e + 4*f*x) + 24*exp(e*7i + f*x*7i)*sin(6*e + 6*f*x))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2),x)

[Out] Timed out

$$3.160 \quad \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{9/2} dx$$

Optimal. Leaf size=250

$$\frac{a^4(9A - B) \cos(e + fx)(c - c \sin(e + fx))^{9/2}}{315f\sqrt{a \sin(e + fx) + a}} - \frac{a^3(9A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}(c - c \sin(e + fx))^{9/2}}{126f}$$

[Out] $-1/84*a^2*(9*A-B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{3/2}*(c-c*\sin(f*x+e))^{9/2}/f-1/72*a*(9*A-B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{5/2}*(c-c*\sin(f*x+e))^{9/2}/f-1/9*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{7/2}*(c-c*\sin(f*x+e))^{9/2}/f-1/315*a^4*(9*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{9/2}/f/(a+a*\sin(f*x+e))^{1/2}-1/126*a^3*(9*A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{9/2}*(a+a*\sin(f*x+e))^{1/2}/f$

Rubi [A] time = 0.57, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{a^2(9A - B) \cos(e + fx)(a \sin(e + fx) + a)^{3/2}(c - c \sin(e + fx))^{9/2}}{84f} - \frac{a^3(9A - B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{126f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2), x]

[Out] $-(a^4*(9*A - B)*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{9/2})/(315*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (a^3*(9*A - B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{9/2})/(126*f) - (a^2*(9*A - B)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2}*(c - c*\text{Sin}[e + f*x])^{9/2})/(84*f) - (a*(9*A - B)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{5/2}*(c - c*\text{Sin}[e + f*x])^{9/2})/(72*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{7/2}*(c - c*\text{Sin}[e + f*x])^{9/2})/(9*f)$

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2

$\wedge(-1)] \ \&\& \ \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{9/2} dx &= -\frac{B \cos(e + fx) (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{9/2}}{9f} \\ &= -\frac{a(9A - B) \cos(e + fx) (a + a \sin(e + fx))^5}{72f} \\ &= -\frac{a^2(9A - B) \cos(e + fx) (a + a \sin(e + fx))}{84f} \\ &= -\frac{a^3(9A - B) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{126f} \\ &= -\frac{a^4(9A - B) \cos(e + fx) (c - c \sin(e + fx))}{315f \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 6.85, size = 269, normalized size = 1.08

$$\frac{a^3 c^4 (\sin(e + fx) - 1)^4 (\sin(e + fx) + 1)^3 \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (17640(A - B) \cos(2(e + fx)) + 8$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2), x]

[Out] (a^3*c^4*(-1 + Sin[e + f*x])^4*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(17640*(A - B)*Cos[2*(e + f*x)] + 8820*(A - B)*Cos[4*(e + f*x)] + 2520*A*Cos[6*(e + f*x)] - 2520*B*Cos[6*(e + f*x)] + 315*A*Cos[8*(e + f*x)] - 315*B*Cos[8*(e + f*x)] + 176400*A*Sin[e + f*x] - 176400*B*Sin[e + f*x] + 35280*A*Sin[3*(e + f*x)] + 7056*A*Sin[5*(e + f*x)] + 2016*B*Sin[5*(e + f*x)] + 720*A*Sin[7*(e + f*x)] + 900*B*Sin[7*(e + f*x)] + 1400*B*Sin[9*(e + f*x)]))/(322560*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)

fricas [A] time = 0.53, size = 183, normalized size = 0.73

$$\frac{(315(A - B)a^3c^4 \cos(fx + e))^8 - 315(A - B)a^3c^4 + 8(35Ba^3c^4 \cos(fx + e))^8 + 5(9A - B)a^3c^4 \cos(fx + e)^6 + 6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2), x, algorithm="fricas")

[Out] 1/2520*(315*(A - B)*a^3*c^4*cos(f*x + e)^8 - 315*(A - B)*a^3*c^4 + 8*(35*B*a^3*c^4*cos(f*x + e)^8 + 5*(9*A - B)*a^3*c^4*cos(f*x + e)^6 + 6*(9*A - B)*a^3*c^4*cos(f*x + e)^4 + 8*(9*A - B)*a^3*c^4*cos(f*x + e)^2 + 16*(9*A - B)*a^3*c^4)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x,
algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
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>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*sqrt(2*c)
*(-2048*f*(A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))-B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(
f*x+exp(1))-1/4*pi)))*cos(8*f*x+8*exp(1))/(2048*f)^2-512*f*(5*A*a^3*c^4*sig
n(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*
c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*
cos(2*f*x+2*exp(1))/(512*f)^2-1536*f*(5*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))
-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*c^4*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(6*f*x+6*exp(1))/(1
536*f)^2-1024*f*(9*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))-9*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(4*f*x+4*exp(1))/(1024*f)^2+1536*f*(-A*a
^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)+B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/
4*pi))*cos(-6*f*x-6*exp(1))/(-1536*f)^2+3584*f*(-4*A*a^3*c^4*sign(sin(1/2*
(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-5*B*a^3*c^4*sign(s
in(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(7*f*x+
7*exp(1))/(3584*f)^2+1024*f*(-5*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi)
)*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+5*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-
1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(-4*f*x-4*exp(1))/(-1024*f)
^2+640*f*(-7*A*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))-2*B*a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1
/2*(f*x+exp(1))-1/4*pi))*sin(5*f*x+5*exp(1))/(640*f)^2+512*f*(-9*A*a^3*c^4
*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+9*B*
a^3*c^4*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi
))*cos(-2*f*x-2*exp(1))/(-512*f)^2+256*f*(-70*A*a^3*c^4*sign(sin(1/2*(f*x+
exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+7*B*a^3*c^4*sign(sin(1/
2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(f*x+exp(1))
/(256*f)^2-8064*A*a^3*c^4*f*sign(sin(1/2*(f*x+exp(1))-1/4*pi))*sign(cos(1/2
```

$(f*x+\exp(1))-1/4*\pi)) * \sin(3*f*x+3*\exp(1))/(384*f)^2 - 4608*B*a^3*c^4*f*\sin(\sin(1/2*(f*x+\exp(1))-1/4*\pi)) * \sin(\cos(1/2*(f*x+\exp(1))-1/4*\pi)) * \sin(9*f*x+9*\exp(1))/(4608*f)^2$

maple [A] time = 0.83, size = 259, normalized size = 1.04

$$\frac{(-280B(\cos^8(fx+e)) + 315A(\cos^6(fx+e))\sin(fx+e) - 315B(\cos^6(fx+e))\sin(fx+e) - 360A(\cos^6(fx+e))\sin^2(fx+e) - 360B(\cos^6(fx+e))\sin^2(fx+e) + 40B^2(\cos^6(fx+e))\sin^2(fx+e) + 315A^2(\cos^6(fx+e))\sin^2(fx+e) - 315B^2(\cos^6(fx+e))\sin^2(fx+e) - 576A^2(\cos^6(fx+e))\sin^2(fx+e) + 64B^2(\cos^6(fx+e))\sin^2(fx+e) + 315A^2(\cos^6(fx+e))\sin^2(fx+e) - 315B^2(\cos^6(fx+e))\sin^2(fx+e) - 1152A(128B)(-c(\sin(fx+e)-1))^{9/2}\sin(fx+e)(a(1+\sin(fx+e)))^{7/2}/(\sin(fx+e)-1)/\cos(fx+e))^7}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x)

[Out] $\frac{1}{2520}f * (-280*B*\cos(f*x+e)^8 + 315*A*\cos(f*x+e)^6*\sin(f*x+e) - 315*B*\cos(f*x+e)^6*\sin(f*x+e) - 360*A*\cos(f*x+e)^6 + 40*B*\cos(f*x+e)^6 + 315*A*\cos(f*x+e)^4*\sin(f*x+e) - 315*B*\sin(f*x+e)*\cos(f*x+e)^4 - 432*A*\cos(f*x+e)^4 + 48*B*\cos(f*x+e)^4 + 315*A*\cos(f*x+e)^2*\sin(f*x+e) - 315*B*\cos(f*x+e)^2*\sin(f*x+e) - 576*A*\cos(f*x+e)^2 + 64*B*\cos(f*x+e)^2 + 315*A*\sin(f*x+e) - 315*B*\sin(f*x+e) - 1152*A(128*B)*(-c*(\sin(f*x+e)-1))^{9/2}*\sin(f*x+e)*(a*(1+\sin(f*x+e)))^{7/2}/(\sin(f*x+e)-1)/\cos(f*x+e))^7$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}(-c \sin(fx + e) + c)^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(9/2), x)

mupad [B] time = 20.03, size = 482, normalized size = 1.93

$$e^{-9i-fx9i} \sqrt{c - c \sin(e + fx)} \left(-\frac{a^3 c^4 e^{9i+fx9i} \cos(2e+2fx) (A1i-B1i) \sqrt{a+a \sin(e+fx)} 7i}{64f} - \frac{a^3 c^4 e^{9i+fx9i} \cos(4e+4fx) (A1i-B1i) \sqrt{a+a \sin(e+fx)} 7i}{128f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(9/2),x)

[Out] $(\exp(-e*9i - f*x*9i)*(c - c*\sin(e + f*x))^{1/2}*((a^3*c^4*\exp(e*9i + f*x*9i)*\sin(5*e + 5*f*x)*(7*A + 2*B)*(a + a*\sin(e + f*x))^{1/2})/(160*f) - (a^3*c^4*\exp(e*9i + f*x*9i)*\cos(4*e + 4*f*x)*(A*1i - B*1i)*(a + a*\sin(e + f*x))^{1/2})*7i)/(128*f) - (a^3*c^4*\exp(e*9i + f*x*9i)*\cos(6*e + 6*f*x)*(A*1i - B*1i)*(a + a*\sin(e + f*x))^{1/2})*1i)/(64*f) - (a^3*c^4*\exp(e*9i + f*x*9i)*\cos(8*e + 8*f*x)*(A*1i - B*1i)*(a + a*\sin(e + f*x))^{1/2})*1i)/(512*f) - (a^3*c^4*\exp(e*9i + f*x*9i)*\cos(2*e + 2*f*x)*(A*1i - B*1i)*(a + a*\sin(e + f*x))^{1/2})*7i)/(64*f) + (a^3*c^4*\exp(e*9i + f*x*9i)*\sin(7*e + 7*f*x)*(4*A + 5*B)*(a + a*\sin(e + f*x))^{1/2})/(896*f) + (7*A*a^3*c^4*\exp(e*9i + f*x*9i)*\sin(3*e + 3*f*x)*(a + a*\sin(e + f*x))^{1/2})/(32*f) + (7*a^3*c^4*\exp(e*9i + f*x*9i)*\sin(e + f*x)*(10*A - B)*(a + a*\sin(e + f*x))^{1/2})/(64*f) + (B*a^3*c^4*\exp(e*9i + f*x*9i)*\sin(9*e + 9*f*x)*(a + a*\sin(e + f*x))^{1/2})/(1152*f)))/(2*\cos(e + f*x))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2),  
x)
```

```
[Out] Timed out
```

$$3.161 \quad \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2} dx$$

Optimal. Leaf size=226

$$\frac{2a^4 A \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{35f \sqrt{a \sin(e + fx) + a}} - \frac{4a^3 A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{7/2}}{35f} - \frac{a^2 A \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{35f}$$

[Out] $-1/7*a^2*A*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-1/7*a*A*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-1/8*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}*(c-c*\sin(f*x+e))^{(7/2)}/f-2/35*a^4*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}-4/35*a^3*A*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.56, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{2a^4 A \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{35f \sqrt{a \sin(e + fx) + a}} - \frac{4a^3 A \cos(e + fx) \sqrt{a \sin(e + fx) + a} (c - c \sin(e + fx))^{7/2}}{35f} - \frac{a^2 A \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{35f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(7/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(7/2)}, x]$

[Out] $(-2*a^4*A*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(35*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (4*a^3*A*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(35*f) - (a^2*A*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(7*f) - (a*A*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(7*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)}*(c - c*\text{Sin}[e + f*x])^{(7/2)})/(8*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[n, -2^{(-1)}]$

Rule 2740

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m - 1/2, 0] \ \&\& \ !\text{LtQ}[n, -1] \ \&\& \ !(\text{IGtQ}[n - 1/2, 0] \ \&\& \ \text{LtQ}[n, m]) \ \&\& \ !(\text{ILtQ}[m + n, 0] \ \&\& \ \text{GtQ}[2*m + n + 1, 0])$

Rule 2973

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2]$

$\wedge(-1)] \ \&\& \ \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{7/2} dx &= -\frac{B \cos(e + fx) (a + a \sin(e + fx))^{7/2} (c - c \sin(e + fx))^{7/2}}{8f} \\ &= -\frac{aA \cos(e + fx) (a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{7/2}}{7f} \\ &= -\frac{a^2 A \cos(e + fx) (a + a \sin(e + fx))^{3/2} (c - c \sin(e + fx))^{7/2}}{7f} \\ &= -\frac{4a^3 A \cos(e + fx) \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{7/2}}{35f} \\ &= -\frac{2a^4 A \cos(e + fx) (c - c \sin(e + fx))^{7/2}}{35f \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.58, size = 135, normalized size = 0.60

$$\frac{a^3 c^3 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (19600A \sin(e + fx) + 3920A \sin(3(e + fx)) + 784A \sin(5(e + fx)) + 80A \sin(7(e + fx)))}{(35840f)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2),x]

[Out] (a^3*c^3*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-1960*B*Cos[2*(e + f*x)] - 980*B*Cos[4*(e + f*x)] - 280*B*Cos[6*(e + f*x)] - 35*B*Cos[8*(e + f*x)] + 19600*A*Sin[e + f*x] + 3920*A*Sin[3*(e + f*x)] + 784*A*Sin[5*(e + f*x)] + 80*A*Sin[7*(e + f*x)]))/(35840*f)

fricas [A] time = 0.48, size = 134, normalized size = 0.59

$$\frac{(35Ba^3c^3 \cos(fx + e))^8 - 35Ba^3c^3 - 8(5Aa^3c^3 \cos(fx + e))^6 + 6Aa^3c^3 \cos(fx + e)^4 + 8Aa^3c^3 \cos(fx + e)^2 + 16Aa^3c^3 \cos(fx + e)}{280f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="fricas")

[Out] -1/280*(35*B*a^3*c^3*cos(f*x + e)^8 - 35*B*a^3*c^3 - 8*(5*A*a^3*c^3*cos(f*x + e))^6 + 6*A*a^3*c^3*cos(f*x + e)^4 + 8*A*a^3*c^3*cos(f*x + e)^2 + 16*A*a^3*c^3*cos(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4

mupad [B] time = 17.65, size = 384, normalized size = 1.70

$$e^{-e8i-fx8i} \sqrt{c - c \sin(e + fx)} \left(\frac{35 A a^3 c^3 e^{e8i+fx8i} \sin(e+fx) \sqrt{a+a \sin(e+fx)}}{32f} - \frac{7 B a^3 c^3 e^{e8i+fx8i} \cos(2e+2fx) \sqrt{a+a \sin(e+fx)}}{64f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(7/2), x)

[Out] (exp(- e*8i - f*x*8i)*(c - c*sin(e + f*x))^(1/2)*((35*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(e + f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (7*B*a^3*c^3*exp(e*8i + f*x*8i)*cos(2*e + 2*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (7*B*a^3*c^3*exp(e*8i + f*x*8i)*cos(4*e + 4*f*x)*(a + a*sin(e + f*x))^(1/2))/(128*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(64*f) - (B*a^3*c^3*exp(e*8i + f*x*8i)*cos(8*e + 8*f*x)*(a + a*sin(e + f*x))^(1/2))/(512*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(3*e + 3*f*x)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (7*A*a^3*c^3*exp(e*8i + f*x*8i)*sin(5*e + 5*f*x)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (A*a^3*c^3*exp(e*8i + f*x*8i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2), x)

[Out] Timed out

$$3.162 \quad \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=192

$$\frac{c^3(7A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{105f\sqrt{c - c \sin(e + fx)}} + \frac{2c^2(7A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}\sqrt{c - c \sin(e + fx)}}{105f}$$

[Out] 1/42*(7*A+B)*c*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(3/2)/f-1/7*B*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(5/2)/f+1/105*(7*A+B)*c^3*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(1/2)+2/105*(7*A+B)*c^2*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)*(c-c*sin(f*x+e))^(1/2)/f

Rubi [A] time = 0.46, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{2c^2(7A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}\sqrt{c - c \sin(e + fx)}}{105f} + \frac{c^3(7A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{105f\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] ((7*A + B)*c^3*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(105*f*Sqrt[c - c*Sin[e + f*x]]) + (2*(7*A + B)*c^2*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*Sqrt[c - c*Sin[e + f*x]])/(105*f) + ((7*A + B)*c*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(3/2))/(42*f) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2)*(c - c*Sin[e + f*x])^(5/2))/(7*f)

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2} dx = -\frac{B \cos(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{5/2}}{7f}$$

$$= \frac{(7A + B)c \cos(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{5/2}}{42f}$$

$$= \frac{2(7A + B)c^2 \cos(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{5/2}}{105f}$$

$$= \frac{(7A + B)c^3 \cos(e + fx)(a + a \sin(e + fx))^{7/2}(c - c \sin(e + fx))^{5/2}}{105f \sqrt{c - c \sin(e + fx)}}$$

Mathematica [A] time = 2.07, size = 232, normalized size = 1.21

$$\frac{a^3 c^2 (\sin(e + fx) - 1)^2 (\sin(e + fx) + 1)^3 \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (-525(A + B) \cos(2(e + fx)) -$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]

[Out] (a^3*c^2*(-1 + Sin[e + f*x])^2*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]]*(-525*(A + B)*Cos[2*(e + f*x)] - 210*(A + B)*Cos[4*(e + f*x)] - 35*A*Cos[6*(e + f*x)] - 35*B*Cos[6*(e + f*x)] + 4200*A*Sin[e + f*x] + 525*B*Sin[e + f*x] + 700*A*Sin[3*(e + f*x)] - 35*B*Sin[3*(e + f*x)] + 84*A*Sin[5*(e + f*x)] - 63*B*Sin[5*(e + f*x)] - 15*B*Sin[7*(e + f*x)]))/(6720*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7)

fricas [A] time = 0.48, size = 150, normalized size = 0.78

$$\frac{\left(35(A + B)a^3 c^2 \cos(fx + e)^6 - 35(A + B)a^3 c^2 + 2\left(15Ba^3 c^2 \cos(fx + e)^6 - 3(7A + B)a^3 c^2 \cos(fx + e)^4 - 210f \cos(fx + e)\right)\right)}{210f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] -1/210*(35*(A + B)*a^3*c^2*cos(f*x + e)^6 - 35*(A + B)*a^3*c^2 + 2*(15*B*a^3*c^2*cos(f*x + e)^6 - 3*(7*A + B)*a^3*c^2*cos(f*x + e)^4 - 4*(7*A + B)*a^3*c^2*cos(f*x + e)^2 - 8*(7*A + B)*a^3*c^2*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(f*cos(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}(-c \sin(fx + e) + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*(-c*sin(f*x + e) + c)^(5/2), x)

mupad [B] time = 18.43, size = 383, normalized size = 1.99

$$e^{-e7i-fx7i} \sqrt{c - c \sin(e + fx)} \left(\frac{a^3 c^2 e^{e7i+fx7i} \cos(2e+2fx) (A1i+B1i) \sqrt{a+a \sin(e+fx)} 5i}{32f} + \frac{a^3 c^2 e^{e7i+fx7i} \cos(4e+4fx) (A1i+B1i) \sqrt{a+a \sin(e+fx)} 5i}{16f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(5/2), x)

[Out] (exp(- e*7i - f*x*7i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c^2*exp(e*7i + f*x*7i)*cos(2*e + 2*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*5i)/(32*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(4*e + 4*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(16*f) + (a^3*c^2*exp(e*7i + f*x*7i)*cos(6*e + 6*f*x)*(A*1i + B*1i)*(a + a*sin(e + f*x))^(1/2)*1i)/(96*f) + (a^3*c^2*exp(e*7i + f*x*7i)*sin(5*e + 5*f*x)*(4*A - 3*B)*(a + a*sin(e + f*x))^(1/2))/(160*f) + (a^3*c^2*exp(e*7i + f*x*7i)*sin(3*e + 3*f*x)*(20*A - B)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (5*a^3*c^2*exp(e*7i + f*x*7i)*sin(e + f*x)*(8*A + B)*(a + a*sin(e + f*x))^(1/2))/(32*f) - (B*a^3*c^2*exp(e*7i + f*x*7i)*sin(7*e + 7*f*x)*(a + a*sin(e + f*x))^(1/2))/(224*f)))/(2*cos(e + f*x))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2), x)

[Out] Timed out

$$3.163 \quad \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=142

$$\frac{c^2(3A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c \sin(e + fx)}} + \frac{c(3A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}\sqrt{c - c \sin(e + fx)}}{15f} - Bc$$

[Out] $-1/6*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}*(c-c*\sin(f*x+e))^{(3/2)}/f+1/30*(3*A+B)*c^2*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}+1/15*(3*A+B)*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}*(c-c*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.36, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2973, 2740, 2738}

$$\frac{c^2(3A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{30f\sqrt{c - c \sin(e + fx)}} + \frac{c(3A + B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}\sqrt{c - c \sin(e + fx)}}{15f} - Bc$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(7/2)}*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(3/2)}, x]$

[Out] $((3*A + B)*c^2*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)})/(30*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((3*A + B)*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)}*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(15*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)}*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(6*f)$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{(n)})/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[n, -2^{(-1)}]$

Rule 2740

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^{(n)})/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^{(n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m - 1/2, 0] \&\& !\text{LtQ}[n, -1] \&\& !(\text{IGtQ}[n - 1/2, 0] \&\& \text{LtQ}[n, m]) \&\& !(\text{ILtQ}[m + n, 0] \&\& \text{GtQ}[2*m + n + 1, 0])$

Rule 2973

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m)}*(c + d*\text{Sin}[e + f*x])^{(n)})/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m)}*(c + d*\text{Sin}[e + f*x])^{(n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{LtQ}[m, -2^{(-1)}] \&\& \text{NeQ}[m + n + 1, 0]$

Rubi steps

mupad [B] time = 18.17, size = 321, normalized size = 2.26

$$e^{-e6i-fx6i} \sqrt{c - c \sin(e + fx)} \left(\frac{a^3 c e^{e6i+fx6i} \cos(4e+4fx) (2A+B) \sqrt{a+a \sin(e+fx)}}{16f} - \frac{B a^3 c e^{e6i+fx6i} \cos(6e+6fx) \sqrt{a+a \sin(e+fx)}}{96f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(3/2), x)

[Out] -(exp(- e*6i - f*x*6i)*(c - c*sin(e + f*x))^(1/2)*((a^3*c*exp(e*6i + f*x*6i)*cos(4*e + 4*f*x)*(2*A + B)*(a + a*sin(e + f*x))^(1/2))/(16*f) - (B*a^3*c*exp(e*6i + f*x*6i)*cos(6*e + 6*f*x)*(a + a*sin(e + f*x))^(1/2))/(96*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(e + f*x)*(A*7i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(4*f) + (a^3*c*exp(e*6i + f*x*6i)*cos(2*e + 2*f*x)*(16*A + 11*B)*(a + a*sin(e + f*x))^(1/2))/(32*f) + (a^3*c*exp(e*6i + f*x*6i)*sin(3*e + 3*f*x)*(A*3i - B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(24*f) - (a^3*c*exp(e*6i + f*x*6i)*sin(5*e + 5*f*x)*(A*1i + B*2i)*(a + a*sin(e + f*x))^(1/2)*1i)/(40*f)))/(2*cos(e + f*x))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2), x)

[Out] Timed out

3.164 $\int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$

Optimal. Leaf size=96

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{4f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{9/2}}{5af\sqrt{c - c \sin(e + fx)}}$$

[Out] $1/4*(A-B)*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}+1/5*B*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(9/2)}/a/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^{7/2}}{4f\sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx)(a \sin(e + fx) + a)^{9/2}}{5af\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(7/2)}*(A + B*\text{Sin}[e + f*x])*\text{Sqrt}[c - c*\text{Sin}[e + f*x]], x]$

[Out] $((A - B)*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(7/2)})/(4*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + (B*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(9/2)})/(5*a*f*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[n, -2^{(-1)}]$

Rule 2971

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Dist}[B/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx &= \frac{B \int (a + a \sin(e + fx))^{9/2} \sqrt{c - c \sin(e + fx)} dx}{a} \\ &= \frac{(A - B)c \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f\sqrt{c - c \sin(e + fx)}} + \end{aligned}$$

Mathematica [A] time = 0.97, size = 121, normalized size = 1.26

$$\frac{a^3 \sec(e + fx) \sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)} (4(60A + 23B) \sin(e + fx) + \cos(4(e + fx)))(5A + 4B \sin(e + fx))}{160f}$$

Antiderivative was successfully verified.

maple [B] time = 0.82, size = 174, normalized size = 1.81

$$\frac{(-4B(\cos^4(fx + e)) + 5A(\cos^2(fx + e))\sin(fx + e) + 15B(\cos^2(fx + e))\sin(fx + e) + 20A(\cos^2(fx + e)))}{20f((\cos^2(fx + e))\sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)

[Out] 1/20/f*(-4*B*cos(f*x+e)^4+5*A*cos(f*x+e)^2*sin(f*x+e)+15*B*cos(f*x+e)^2*sin(f*x+e)+20*A*cos(f*x+e)^2+28*B*cos(f*x+e)^2-35*A*sin(f*x+e)-25*B*sin(f*x+e)-40*A-24*B)*(-c*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^2*sin(f*x+e)+3*cos(f*x+e)^2-4*sin(f*x+e)-4)/cos(f*x+e)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}} \sqrt{-c \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)*sqrt(-c*sin(f*x + e) + c), x)

mupad [B] time = 16.68, size = 173, normalized size = 1.80

$$\frac{a^3 \sqrt{a(\sin(e + fx) + 1)} \sqrt{-c(\sin(e + fx) - 1)} (140A \cos(e + fx) + 100B \cos(e + fx) + 135A \cos(3e + 3fx) - 5A \cos(5e + 5fx) + 85B \cos(3e + 3fx) - 15B \cos(5e + 5fx) - 240A \sin(2e + 2fx) + 40A \sin(4e + 4fx) - 90B \sin(2e + 2fx) + 48B \sin(4e + 4fx) - 2B \sin(6e + 6fx))}{(160f(\cos(2e + 2fx) + 1))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2)*(c - c*sin(e + f*x))^(1/2),x)

[Out] -(a^3*(a*(sin(e + f*x) + 1))^(1/2)*(-c*(sin(e + f*x) - 1))^(1/2)*(140*A*cos(e + f*x) + 100*B*cos(e + f*x) + 135*A*cos(3*e + 3*f*x) - 5*A*cos(5*e + 5*f*x) + 85*B*cos(3*e + 3*f*x) - 15*B*cos(5*e + 5*f*x) - 240*A*sin(2*e + 2*f*x) + 40*A*sin(4*e + 4*f*x) - 90*B*sin(2*e + 2*f*x) + 48*B*sin(4*e + 4*f*x) - 2*B*sin(6*e + 6*f*x)))/(160*f*(cos(2*e + 2*f*x) + 1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)

[Out] Timed out

$$3.165 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=239

$$\frac{8a^4(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{f\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} - \frac{4a^3(A+B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{a^2(A+B) \cos(e+fx)}{f\sqrt{c-c \sin(e+fx)}}$$

[Out] $-a^2*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-1/3*a*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(5/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-1/4*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(7/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}-8*a^4*(A+B)*\cos(f*x+e)*\ln(1-\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-4*a^3*(A+B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{4a^3(A+B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{f\sqrt{c-c \sin(e+fx)}} - \frac{a^2(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{f\sqrt{c-c \sin(e+fx)}} - \frac{8a^4(A+B) \cos(e+fx)}{f\sqrt{a \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] $(-8*a^4*(A+B)*\text{Cos}[e+f*x]*\text{Log}[1-\text{Sin}[e+f*x]])/(f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (4*a^3*(A+B)*\text{Cos}[e+f*x]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])/(f*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (a^2*(A+B)*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^{(3/2)})/(f*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (a*(A+B)*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^{(5/2)})/(3*f*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (B*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^{(7/2)})/(4*f*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ

[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{7/2}(A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx = -\frac{B \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f\sqrt{c - c \sin(e + fx)}} + (A + B) \int \frac{(a + a \sin(e + fx))^{7/2}}{\sqrt{c - c \sin(e + fx)}} dx$$

$$= -\frac{a(A + B) \cos(e + fx)(a + a \sin(e + fx))^{5/2}}{3f\sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f\sqrt{c - c \sin(e + fx)}}$$

$$= -\frac{a^2(A + B) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{f\sqrt{c - c \sin(e + fx)}} - \frac{a(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{3f\sqrt{c - c \sin(e + fx)}}$$

$$= -\frac{4a^3(A + B) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{f\sqrt{c - c \sin(e + fx)}} - \frac{a^2(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{f\sqrt{c - c \sin(e + fx)}}$$

$$= -\frac{4a^3(A + B) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{f\sqrt{c - c \sin(e + fx)}} - \frac{a^2(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{f\sqrt{c - c \sin(e + fx)}}$$

$$= -\frac{4a^3(A + B) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{f\sqrt{c - c \sin(e + fx)}} - \frac{a^2(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{f\sqrt{c - c \sin(e + fx)}}$$

$$= -\frac{8a^4(A + B) \cos(e + fx) \log(1 - \sin(e + fx))}{f\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} - \frac{4a^3(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{f\sqrt{c - c \sin(e + fx)}}$$

Mathematica [A] time = 2.84, size = 183, normalized size = 0.77

$$\frac{a^3(\sin(e + fx) + 1)^3 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(24(29A + 36B) \sin(e + fx) - 8(A + B) \right)}{96f\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]], x]

[Out] -1/96*(a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(1 + Sin[e + f*x])^3*Sqrt[a*(1 + Sin[e + f*x])]*(-12*(8*A + 15*B)*Cos[2*(e + f*x)] + 3*B*Cos[4*(e + f*x)] + 1536*(A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + 24*(29*A + 36*B)*Sin[e + f*x] - 8*(A + 4*B)*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(Ba^3 \cos(fx + e))^4 - (3A + 5B)a^3 \cos(fx + e)^2 + 4(A + B)a^3 - ((A + 3B)a^3 \cos(fx + e))^2 - 4(A + B)a^3}{c \sin(fx + e) - c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)
)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a
*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c*sin(f*x + e) - c), x
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.69, size = 671, normalized size = 2.81

$$\frac{\left(-64A \sin(fx + e) + 88A \sin(fx + e) \cos(fx + e) - 32B \left(\cos^2(fx + e)\right) \sin(fx + e) + 192A \sin(fx + e)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] -1/12/f*(-16*B*cos(f*x+e)^3*sin(f*x+e)-64*A*sin(f*x+e)+88*A*sin(f*x+e)*cos(
f*x+e)-4*A*cos(f*x+e)^3*sin(f*x+e)+3*B*sin(f*x+e)*cos(f*x+e)^4+192*A*sin(f*
x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*A*cos(f*x+e)^2*sin(f*x+e
)-32*B*cos(f*x+e)^2*sin(f*x+e)+13*B*cos(f*x+e)^4+64*A+67*B-24*A*cos(f*x+e)^
3-96*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+192*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+
sin(f*x+e))/sin(f*x+e))-96*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+112*B*sin(f*x+
e)*cos(f*x+e)+192*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-9
6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+192*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-96*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+3*B*cos(f*x+e)^5-
80*B*cos(f*x+e)^2+4*A*cos(f*x+e)^4-48*B*cos(f*x+e)^3-67*B*sin(f*x+e)+24*A*c
os(f*x+e)+45*B*cos(f*x+e)-68*A*cos(f*x+e)^2-192*A*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+96*A*ln(2/(cos(f*x+e)+1))-192*B*ln(-(-1+cos(f*x+e)+sin(f*
x+e))/sin(f*x+e))+96*B*ln(2/(cos(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(
f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8
*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin
(f*x+e)-1))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/sqrt(-c*sin(f*x +
e) + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{7/2}}{\sqrt{c - c \sin(e + f x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2), x)

[Out] Timed out

$$3.166 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=271

$$\frac{4a^4(3A+5B) \cos(e+fx) \log(1-\sin(e+fx))}{cf\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} + \frac{2a^3(3A+5B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{cf\sqrt{c-c \sin(e+fx)}} + \frac{a^2(3A+5B) \cos(e+fx)}{cf\sqrt{a \sin(e+fx)+a}}$$

[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(3/2)+1/2*a^4*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c/f/(c-c*sin(f*x+e))^(1/2)+1/6*a*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(1/2)+4*a^4*(3*A+5*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*a^3*(3*A+5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.59, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2a^3(3A+5B) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{cf\sqrt{c-c \sin(e+fx)}} + \frac{a^2(3A+5B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{2cf\sqrt{c-c \sin(e+fx)}} + \frac{4a^4(3A+5B) \cos(e+fx)}{cf\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(2*f*(c - c*Sin[e + f*x])^(3/2)) + (4*a^4*(3*A + 5*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*a^3*(3*A + 5*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c*f*Sqrt[c - c*Sin[e + f*x]]) + (a^2*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c*f*Sqrt[c - c*Sin[e + f*x]]) + (a*(3*A + 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(6*c*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])]

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)

```
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} - \frac{(3A + 5B) \int \frac{(a + a \sin(e + fx))^{7/2}}{\sqrt{c - c \sin(e + fx)}} dx}{2c}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a(3A + 5B) \cos(e + fx)}{6cf\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{a^2(3A + 5B) \cos(e + fx)}{2cf\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^3(3A + 5B) \cos(e + fx)}{cf\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^3(3A + 5B) \cos(e + fx)}{cf\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{2a^3(3A + 5B) \cos(e + fx)}{cf\sqrt{c - c \sin(e + fx)}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{2f(c - c \sin(e + fx))^{3/2}} + \frac{4a^4(3A + 5B) \cos(e + fx)}{cf\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 3.50, size = 292, normalized size = 1.08

$$\frac{a^3 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-2(27A + 59B) \cos(2(e + fx)) - 117A \sin(e + fx) - \dots \right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e +
f*x])^(3/2), x]
```

```
[Out] (a^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x]))*(-132
*A - 45*B - 2*(27*A + 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] - 576*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 117*A*Sin[e + f*x] - 279*B*Sin[e + f*x] + 576*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]])/((c - c*Sin[e + f*x])^(3/2))
```

$+ f*x)/2] - \text{Sin}[(e + f*x)/2]]*\text{Sin}[e + f*x] + 960*B*\text{Log}[\text{Cos}[(e + f*x)/2] - \text{Sin}[(e + f*x)/2]]*\text{Sin}[e + f*x] - 3*A*\text{Sin}[3*(e + f*x)] - 13*B*\text{Sin}[3*(e + f*x)]])/(24*c*f*(\text{Cos}[(e + f*x)/2] + \text{Sin}[(e + f*x)/2])*(-1 + \text{Sin}[e + f*x])*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

fricas [F] time = 1.17, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left(B a^3 \cos(fx + e)^4 - (3A + 5B)a^3 \cos(fx + e)^2 + 4(A + B)a^3 - \left((A + 3B)a^3 \cos(fx + e)^2 - 4(A + B)a^3 \right) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.65, size = 927, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2), x)

[Out] $\frac{1}{6}f*(240*B*\cos(f*x+e)^2*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))+13*B*\cos(f*x+e)^3*\sin(f*x+e)-102*A*\sin(f*x+e)+75*A*\sin(f*x+e)*\cos(f*x+e)+3*A*\cos(f*x+e)^3*\sin(f*x+e)-2*B*\sin(f*x+e)*\cos(f*x+e)^4+288*A*\sin(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-120*B*\cos(f*x+e)^2*\ln(2/(\cos(f*x+e)+1))+24*A*\cos(f*x+e)^2*\sin(f*x+e)+48*B*\cos(f*x+e)^2*\sin(f*x+e)-11*B*\cos(f*x+e)^4+102*A+166*B+27*A*\cos(f*x+e)^3-72*A*\cos(f*x+e)^2*\ln(2/(\cos(f*x+e)+1))+144*A*\cos(f*x+e)^2*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-144*A*\sin(f*x+e)*\ln(2/(\cos(f*x+e)+1))+144*A*\cos(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-72*A*\cos(f*x+e)*\ln(2/(\cos(f*x+e)+1))+107*B*\sin(f*x+e)*\cos(f*x+e)+480*B*\sin(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-240*B*\sin(f*x+e)*\ln(2/(\cos(f*x+e)+1))+240*B*\cos(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-120*B*\cos(f*x+e)*\ln(2/(\cos(f*x+e)+1))+72*A*\cos(f*x+e)*\sin(f*x+e)*\ln(2/(\cos(f*x+e)+1))-144*A*\cos(f*x+e)*\sin(f*x+e)*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))-2*B*\cos(f*x+e)^5+120*B*\ln(2/(\cos(f*x+e)+1))*\sin(f*x+e)*\cos(f*x+e)-240*B*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))*\sin(f*x+e)*\cos(f*x+e)-155*B*\cos(f*x+e)^2-3*A*\cos(f*x+e)^4+61*B*\cos(f*x+e)^3-166*B*\sin(f*x+e)-27*A*\cos(f*x+e)-59*B*\cos(f*x+e)-99*A*\cos(f*x+e)^2-288*A*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))+144*A*\ln(2/(\cos(f*x+e)+1))-480*B*\ln(-(-1+\cos(f*x+e)+\sin(f*x+e))/\sin(f*x+e))+240*B*\ln(2/(\cos(f*x+e)+1)))*(a*(1+\sin(f*x+e)))^(7/2)/(\cos(f*x+e)^4+\sin(f*x+e)*\cos(f*x+e)^3+3*\cos(f*x+e)^3-4*\cos(f*x+e)^2*\sin(f*x+e)-8*\cos(f*x+e)^2-4*\sin(f*x+e)*\cos(f*x+e)-4*\cos(f*x+e)+8*\sin(f*x+e)+8)/(-c*(\sin(f*x+e)-1))^(3/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
3/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),
x)

[Out] Timed out

$$3.167 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=263

$$\frac{6a^4(A+3B) \cos(e+fx) \log(1-\sin(e+fx))}{c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{3a^3(A+3B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{3a^2(A+3B) \cos(e+fx)}{4c^2 f \sqrt{a \sin(e+fx)+a}}$$

[Out] $\frac{1}{4}(A+B) \cos(fx+e) (a+a \sin(fx+e))^{7/2} / (c-c \sin(fx+e))^{5/2} - \frac{1}{2} a^4 (A+3B) \cos(fx+e) (a+a \sin(fx+e))^{5/2} / c / f / (c-c \sin(fx+e))^{3/2} - \frac{3}{4} a^2 (A+3B) \cos(fx+e) (a+a \sin(fx+e))^{3/2} / c^2 / f / (c-c \sin(fx+e))^{1/2} - 6 a^4 (A+3B) \cos(fx+e) \ln(1-\sin(fx+e)) / c^2 / f / (a+a \sin(fx+e))^{1/2} / (c-c \sin(fx+e))^{1/2} - 3 a^3 (A+3B) \cos(fx+e) (a+a \sin(fx+e))^{1/2} / c^2 / f / (c-c \sin(fx+e))^{1/2}$

Rubi [A] time = 0.60, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{3a^3(A+3B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{3a^2(A+3B) \cos(e+fx) (a \sin(e+fx)+a)^{3/2}}{4c^2 f \sqrt{c-c \sin(e+fx)}} - \frac{6a^4(A+3B) \cos(e+fx)}{c^2 f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a \sin[e + fx])^{7/2} (A + B \sin[e + fx]) / (c - c \sin[e + fx])^{5/2}, x]$

[Out] $((A + B) \cos[e + fx] (a + a \sin[e + fx])^{7/2}) / (4 f (c - c \sin[e + fx])^{5/2}) - (a (A + 3B) \cos[e + fx] (a + a \sin[e + fx])^{5/2}) / (2 c f (c - c \sin[e + fx])^{3/2}) - (6 a^4 (A + 3B) \cos[e + fx] \log[1 - \sin[e + fx]]) / (c^2 f \sqrt{a + a \sin[e + fx]} \sqrt{c - c \sin[e + fx]}) - (3 a^3 (A + 3B) \cos[e + fx] \sqrt{a + a \sin[e + fx]}) / (c^2 f \sqrt{c - c \sin[e + fx]}) - (3 a^2 (A + 3B) \cos[e + fx] (a + a \sin[e + fx])^{3/2}) / (4 c^2 f \sqrt{c - c \sin[e + fx]})$

Rule 31

$\text{Int}[(a_ + (b_.) (x_))^{-1}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b x, x]] / b, x] / ; \text{FreeQ}\{a, b\}, x]$

Rule 2667

$\text{Int}[\cos[(e_.) + (f_.) (x_)]^{(p_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.) (x_)])^{(m_.)}, x_Symbol] \rightarrow \text{Dist}[1 / (b^p f), \text{Subst}[\text{Int}[(a + x)^{(m + (p - 1) / 2)} (a - x)^{((p - 1) / 2)}, x], x, b \sin[e + fx]], x] / ; \text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{IntegerQ}[(p - 1) / 2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] \parallel \text{!IntegerQ}[m + 1 / 2])]$

Rule 2737

$\text{Int}[\sqrt{(a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]}] / \sqrt{(c_.) + (d_.) \sin[(e_.) + (f_.) (x_)]}, x_Symbol] \rightarrow \text{Dist}[(a c \cos[e + fx]) / (\sqrt{a + b \sin[e + fx]} \sqrt{c + d \sin[e + fx]}), \text{Int}[\cos[e + fx] / (c + d \sin[e + fx]), x], x] / ; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{EqQ}[b c + a d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2739

$\text{Int}[(a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]^{(m_.)} ((c_.) + (d_.) \sin[(e_.) + (f_.) (x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(-2 b \cos[e + fx] (a + b \sin[e + fx])^{m-1} (c + d \sin[e + fx])^n) / (f (2 n + 1)), x] - \text{Dist}[(b (2 m - 1)) / (d$

```
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
+ 1, 0])
```

Rule 2740

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{7/2}(A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx = \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{(A + 3B) \int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{5/2}} dx}{2c}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{4f(c - c \sin(e + fx))^{5/2}} - \frac{a(A + 3B) \cos(e + fx)}{2cf(c - c \sin(e + fx))^{5/2}}$$

Mathematica [A] time = 2.57, size = 251, normalized size = 0.95

$$(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-4(A + 6B) \sin(e + fx) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(16*(A + B) - 16*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + B*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 48*(A + 3*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 4*(A + 6*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(5/2))
```

fricas [F] time = 25.36, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(B a^3 \cos(fx + e)^4 - (3A + 5B) a^3 \cos(fx + e)^2 + 4(A + B) a^3 - \left((A + 3B) a^3 \cos(fx + e)^2 - 4(A + B) a^3 \right) \sin(fx + e) \right) \sqrt{-c \sin(fx + e) + c}}{3 c^3 \cos(fx + e)^2 - 4 c^3 - \left(c^3 \cos(fx + e)^2 - 4 c^3 \right) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, algorithm="fricas")
```

```
[Out] integral(-(B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.65, size = 1191, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x)
```

```
[Out] 1/2/f*(-216*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+10*B*cos(f*x+e)^3*sin(f*x+e)+32*A*sin(f*x+e)-12*A*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)^3*sin(f*x+e)-B*sin(f*x+e)*cos(f*x+e)^4-96*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+108*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-22*A*cos(f*x+e)^2*sin(f*x+e)-63*B*cos(f*x+e)^2*sin(f*x+e)-9*B*cos(f*x+e)^4-32*A-100*B-20*A*cos(f*x+e)^3+72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3-36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+36*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-72*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-46*B*sin(f*x+e)*cos(f*x+e)-288*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+144*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-144*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+72*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+48*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^5+72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
```

```
n(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*si
n(f*x+e)-72*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+144*B*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+109*B*cos(f*x+e)^2+24*
A*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)^3
*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^4-12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(c
os(f*x+e)+1))+24*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/s
in(f*x+e))-53*B*cos(f*x+e)^3+100*B*sin(f*x+e)+20*A*cos(f*x+e)+54*B*cos(f*x+
e)+34*A*cos(f*x+e)^2+96*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*A*ln
(2/(cos(f*x+e)+1))+288*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*
ln(2/(cos(f*x+e)+1))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos
(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x
+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e)
+ c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{7}{2}}}{(c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

$$3.168 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{7/2}} dx$$

Optimal. Leaf size=264

$$\frac{a^4(A+7B) \cos(e+fx) \log(1-\sin(e+fx))}{c^3 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{a^3(A+7B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2c^3 f \sqrt{c-c \sin(e+fx)}} + \frac{a^2(A+7B) \cos(e+fx)}{4c^2 f (c-c \sin(e+fx))^{3/2}}$$

[Out] 1/6*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(7/2)-1/12*a*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(5/2)+1/4*a^2*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(3/2)+a^4*(A+7*B)*cos(f*x+e)*ln(1-sin(f*x+e))/c^3/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+1/2*a^3*(A+7*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.61, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{a^3(A+7B) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{2c^3 f \sqrt{c-c \sin(e+fx)}} + \frac{a^2(A+7B) \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{4c^2 f (c-c \sin(e+fx))^{3/2}} + \frac{a^4(A+7B) \cos(e+fx)}{c^3 f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6*f*(c - c*Sin[e + f*x])^(7/2)) - (a*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(12*c*f*(c - c*Sin[e + f*x])^(5/2)) + (a^2*(A + 7*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(4*c^2*f*(c - c*Sin[e + f*x])^(3/2)) + (a^4*(A + 7*B)*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^3*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (a^3*(A + 7*B)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(2*c^3*f*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])]

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*

```
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
+ 1, 0])
```

Rule 2740

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{7/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{(A + 7B) \int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{7/2}} dx}{6c}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{6f(c - c \sin(e + fx))^{7/2}} - \frac{a(A + 7B) \cos(e + fx)}{12cf(c - c \sin(e + fx))^{7/2}}$$

Mathematica [A] time = 2.90, size = 244, normalized size = 0.92

$$(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(18(A + 3B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)^4 - 60$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(7/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2)*(8*(A + B) - 6*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 18*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 6*(A + 7*B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 + 3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6*Sin[e + f*x]))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(7/2))
```

fricas [F] time = 44.32, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(B a^3 \cos(fx + e)^4 - (3A + 5B) a^3 \cos(fx + e)^2 + 4(A + B) a^3 - \left((A + 3B) a^3 \cos(fx + e)^2 - 4(A + B) a^3 \right) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{c^4 \cos(fx + e)^4 - 8c^4 \cos(fx + e)^2 + 8c^4 + 4 \left(c^4 \cos(fx + e)^2 - 2c^4 \sin(fx + e) \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x, algorithm="fricas")
```

```
[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^4*cos(f*x + e)^4 - 8*c^4*cos(f*x + e)^2 + 8*c^4 + 4*(c^4*cos(f*x + e)^2 - 2*c^4)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 1455, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2), x)
```

```
[Out] 1/3/f*(336*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-41*B*cos(f*x+e)^3*sin(f*x+e)-20*A*sin(f*x+e)+14*A*sin(f*x+e)*cos(f*x+e)-8*A*cos(f*x+e)^3*sin(f*x+e)-3*B*sin(f*x+e)*cos(f*x+e)^4+48*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-168*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+14*A*cos(f*x+e)^2*sin(f*x+e)+98*B*cos(f*x+e)^2*sin(f*x+e)+44*B*cos(f*x+e)^4+20*A+116*B+6*A*cos(f*x+e)^3-126*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+63*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-24*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+48*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*A*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+62*B*sin(f*x+e)*cos(f*x+e)+336*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-168*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+168*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-84*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+12*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^5-168*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))
```

```

)))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+84*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)
^2*sin(f*x+e)+84*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-168*B*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)-6*A*cos(f*x+e)^4*
ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*cos(f*x+e)^4*ln(2/(cos(f*x+e
)+1))+21*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-42*B*cos(f*x+e)^4*ln(-(-1+cos(
f*x+e)+sin(f*x+e))/sin(f*x+e))-160*B*cos(f*x+e)^2-18*A*cos(f*x+e)^3*ln(-(-1
+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+9*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+8
*A*cos(f*x+e)^4+12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(
f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+57*B*cos(f*x
+e)^3-116*B*sin(f*x+e)-3*A*sin(f*x+e)*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+6*A
*sin(f*x+e)*cos(f*x+e)^3*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+42*B*co
s(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-21*B*cos(f
*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-6*A*cos(f*x+e)-54*B*cos(f*x+e)-28*A
*cos(f*x+e)^2-48*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*ln(2/(co
s(f*x+e)+1))-336*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+168*B*ln(2/(c
os(f*x+e)+1)))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)
^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos
(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(7/2)

```

maxima [B] time = 1.19, size = 749, normalized size = 2.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(7/2),x,
algorithm="maxima")

```

```

[Out] -1/3*(B*(42*a^(7/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 21*a
^(7/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/c^(7/2) + 2*(21*a^(7/2)
*sin(f*x + e)/(cos(f*x + e) + 1) - 102*a^(7/2)*sin(f*x + e)^2/(cos(f*x + e)
+ 1)^2 + 227*a^(7/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 228*a^(7/2)*sin
(f*x + e)^4/(cos(f*x + e) + 1)^4 + 227*a^(7/2)*sin(f*x + e)^5/(cos(f*x + e)
+ 1)^5 - 102*a^(7/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6 + 21*a^(7/2)*sin(
f*x + e)^7/(cos(f*x + e) + 1)^7)/(c^(7/2) - 6*c^(7/2)*sin(f*x + e)/(cos(f*x
+ e) + 1) + 16*c^(7/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 26*c^(7/2)*si
n(f*x + e)^3/(cos(f*x + e) + 1)^3 + 30*c^(7/2)*sin(f*x + e)^4/(cos(f*x + e)
+ 1)^4 - 26*c^(7/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 16*c^(7/2)*sin(f
*x + e)^6/(cos(f*x + e) + 1)^6 - 6*c^(7/2)*sin(f*x + e)^7/(cos(f*x + e) + 1
)^7 + c^(7/2)*sin(f*x + e)^8/(cos(f*x + e) + 1)^8)) + A*(6*a^(7/2)*log(sin(
f*x + e)/(cos(f*x + e) + 1) - 1)/c^(7/2) - 3*a^(7/2)*log(sin(f*x + e)^2/(co
s(f*x + e) + 1)^2 + 1)/c^(7/2) + 4*(3*a^(7/2)*sqrt(c)*sin(f*x + e)/(cos(f*x
+ e) + 1) - 6*a^(7/2)*sqrt(c)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 22*a^(
7/2)*sqrt(c)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 - 6*a^(7/2)*sqrt(c)*sin(f*
x + e)^4/(cos(f*x + e) + 1)^4 + 3*a^(7/2)*sqrt(c)*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5)/(c^4 - 6*c^4*sin(f*x + e)/(cos(f*x + e) + 1) + 15*c^4*sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 - 20*c^4*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1
5*c^4*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 6*c^4*sin(f*x + e)^5/(cos(f*x +
e) + 1)^5 + c^4*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
7/2),x)

```



```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(7/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(7/2), x)
```

```
[Out] Timed out
```

$$3.169 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{9/2}} dx$$

Optimal. Leaf size=247

$$-\frac{a^4 B \cos(e+fx) \log(1-\sin(e+fx))}{c^4 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{a^3 B \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^3 f (c-c \sin(e+fx))^{3/2}} + \frac{a^2 B \cos(e+fx)(a \sin(e+fx)+a)}{2c^2 f (c-c \sin(e+fx))^{5/2}}$$

[Out] 1/8*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(9/2)-1/3*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(5/2)/c/f/(c-c*sin(f*x+e))^(7/2)+1/2*a^2*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/c^2/f/(c-c*sin(f*x+e))^(5/2)-a^3*B*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/c^3/f/(c-c*sin(f*x+e))^(3/2)-a^4*B*cos(f*x+e)*ln(1-sin(f*x+e))/c^4/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.60, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2739, 2737, 2667, 31}

$$-\frac{a^3 B \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{c^3 f (c-c \sin(e+fx))^{3/2}} + \frac{a^2 B \cos(e+fx)(a \sin(e+fx)+a)^{3/2}}{2c^2 f (c-c \sin(e+fx))^{5/2}} - \frac{a^4 B \cos(e+fx) \log(1-\sin(e+fx))}{c^4 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8*f*(c - c*Sin[e + f*x])^(9/2)) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*c*f*(c - c*Sin[e + f*x])^(7/2)) + (a^2*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(2*c^2*f*(c - c*Sin[e + f*x])^(5/2)) - (a^3*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(c^3*f*(c - c*Sin[e + f*x])^(3/2)) - (a^4*B*Cos[e + f*x]*Log[1 - Sin[e + f*x]])/(c^4*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{9/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{B \int \frac{(a + a \sin(e + fx))}{(c - c \sin(e + fx))} dx}{c} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{8f(c - c \sin(e + fx))^{9/2}} - \frac{aB \cos(e + fx)}{3cf(c - c \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 2.68, size = 238, normalized size = 0.96

$$\frac{(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-3(A + 7B) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)^6}{(c - c \sin(e + fx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(9/2), x]

[Out] ((6*(A + B) - 4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + 9*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 - 3*(A + 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^6 - 6*B*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^8)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(9/2))

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(B a^3 \cos(fx + e)^4 - (3A + 5B) a^3 \cos(fx + e)^2 + 4(A + B) a^3 - \left((A + 3B) a^3 \cos(fx + e)^2 - 4(A + B) a^3 \right) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{5c^5 \cos(fx + e)^4 - 20c^5 \cos(fx + e)^2 + 16c^5 - \left(c^5 \cos(fx + e)^4 - 12c^5 \cos(fx + e)^2 + 16c^5 \right) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
algorithm="fricas")

[Out] integral((B*a^3*cos(f*x + e)^4 - (3*A + 5*B)*a^3*cos(f*x + e)^2 + 4*(A + B)*a^3 - ((A + 3*B)*a^3*cos(f*x + e)^2 - 4*(A + B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^5*cos(f*x + e)^4 - 20*c^5*cos(f*x + e)^2 + 16*c^5 - (c^5*cos(f*x + e)^4 - 12*c^5*cos(f*x + e)^2 + 16*c^5)*sin(f*x + e)), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
algorithm="giac")

[Out] Timed out

maple [B] time = 0.72, size = 1019, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x)

[Out]
$$\begin{aligned} & -1/3/f*(120*B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-11*B*cos(f*x+e)^3*sin(f*x+e)-3*B*cos(f*x+e)^5*ln(2/(cos(f*x+e)+1))+6*A*sin(f*x+e)-6*A*sin(f*x+e)*cos(f*x+e)+3*A*cos(f*x+e)^3*sin(f*x+e)-8*B*sin(f*x+e)*cos(f*x+e)^4-60*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-3*A*cos(f*x+e)^2*sin(f*x+e)+39*B*cos(f*x+e)^2*sin(f*x+e)+19*B*cos(f*x+e)^4-6*A+34*B-48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^3+24*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+14*B*sin(f*x+e)*cos(f*x+e)+96*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+48*B*cos(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*cos(f*x+e)^5*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*B*cos(f*x+e)^5-72*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)+36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+24*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-48*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)+15*B*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))-30*B*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-53*B*cos(f*x+e)^2-3*A*cos(f*x+e)^4+6*B*sin(f*x+e)*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*sin(f*x+e)*cos(f*x+e)^4*ln(2/(cos(f*x+e)+1))+28*B*cos(f*x+e)^3-34*B*sin(f*x+e)+24*B*cos(f*x+e)^3*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*cos(f*x+e)^3*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-20*B*cos(f*x+e)+9*A*cos(f*x+e)^2-96*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*B*ln(2/(cos(f*x+e)+1))*(a*(1+sin(f*x+e)))^(7/2)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)/(-c*(sin(f*x+e)-1))^(9/2) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}}{(-c \sin(fx + e) + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(9/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(9/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{7}{2}}}{(c - c \sin(e + fx))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(9/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(9/2),
x)

[Out] Timed out

$$3.170 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx$$

Optimal. Leaf size=96

$$\frac{(A-9B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{80cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{10f(c-c \sin(e+fx))^{11/2}}$$

[Out] 1/10*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(11/2)+1/80*(A-9*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(9/2)

Rubi [A] time = 0.27, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2972, 2742}

$$\frac{(A-9B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{80cf(c-c \sin(e+fx))^{9/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{10f(c-c \sin(e+fx))^{11/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(10*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 9*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(80*c*f*(c - c*Sin[e + f*x])^(9/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{11/2}} dx &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{7/2}}{10f(c-c \sin(e+fx))^{11/2}} + \frac{(A-9B) \int \frac{(a+a \sin(e+fx))^{7/2}}{(c-c \sin(e+fx))^{11/2}} dx}{10c} \\ &= \frac{(A+B) \cos(e+fx)(a+a \sin(e+fx))^{7/2}}{10f(c-c \sin(e+fx))^{11/2}} + \frac{(A-9B) \cos(e+fx)(a+a \sin(e+fx))^{7/2}}{80cf(c-c \sin(e+fx))^{9/2}} \end{aligned}$$

Mathematica [B] time = 6.94, size = 434, normalized size = 4.52

$$\frac{(-A-7B)(a(\sin(e+fx)+1))^{7/2} \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right)^7}{2f(c-c \sin(e+fx))^{11/2} \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)^7} + \frac{2(A+3B)(a(\sin(e+fx)+1))^{7/2} \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right)^7}{f(c-c \sin(e+fx))^{11/2} \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(11/2), x]
```

```
[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + ((-3*A - 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (2*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(2*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(11/2))
```

fricas [B] time = 0.48, size = 199, normalized size = 2.07

$$\frac{(10Ba^3 \cos(fx + e))^4 - 5(A + 7B)a^3 \cos(fx + e)^2 + 2(3A + 13B)a^3 - 5((A - B)a^3 \cos(fx + e))^2 - 2(A - B)a^3}{10(5c^6 f \cos(fx + e)^5 - 20c^6 f \cos(fx + e)^3 + 16c^6 f \cos(fx + e) - (c^6 f \cos(fx + e))^5 - 12c^6 f \cos(fx + e)) \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="fricas")
```

```
[Out] 1/10*(10*B*a^3*cos(f*x + e)^4 - 5*(A + 7*B)*a^3*cos(f*x + e)^2 + 2*(3*A + 13*B)*a^3 - 5*((A - B)*a^3*cos(f*x + e)^2 - 2*(A - B)*a^3)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(5*c^6*f*cos(f*x + e)^5 - 20*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e) - (c^6*f*cos(f*x + e))^5 - 12*c^6*f*cos(f*x + e)^3 + 16*c^6*f*cos(f*x + e))*sin(f*x + e)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.70, size = 389, normalized size = 4.05

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{\frac{7}{2}} \left(A \left(\cos^5(fx + e) \right) + A \left(\cos^4(fx + e) \right) \sin(fx + e) + B \left(\cos^5(fx + e) \right) \right) + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x)
```

```
[Out] 1/10/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(A*cos(f*x+e)^5+A*cos(f*x+e)^4*sin(f*x+e)+B*cos(f*x+e)^5+B*sin(f*x+e)*cos(f*x+e)^4-6*A*cos(f*x+e)^4+5*A*cos(f*x+e)^3*sin(f*x+e)+4*B*cos(f*x+e)^4-5*B*cos(f*x+e)^3*sin(f*x+e)-17*A*cos(f*x+e)^3-22*A*cos(f*x+e)^2*sin(f*x+e)-7*B*cos(f*x+e)^3-2*B*cos(f*x+e)^2*sin(f*x+e)+32*A*cos(f*x+e)^2-10*A*sin(f*x+e)*cos(f*x+e)-8*B*cos(f*x+e)^2+10*B*sin(f*x+e)*cos(f*x+e)+26*A*cos(f*x+e)+36*A*sin(f*x+e)+6*B*cos(f*x+e)-4*B*si
```

$n(f*x+e)-36*A+4*B)/(-c*(\sin(f*x+e)-1))^{(11/2)}/(\cos(f*x+e)^4+\sin(f*x+e)*\cos(f*x+e)^3+3*\cos(f*x+e)^3-4*\cos(f*x+e)^2*\sin(f*x+e)-8*\cos(f*x+e)^2-4*\sin(f*x+e)*\cos(f*x+e)-4*\cos(f*x+e)+8*\sin(f*x+e)+8)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{7}{2}}}{(-c \sin(fx + e) + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(11/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(7/2)/(-c*sin(f*x + e) + c)^(11/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{11/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(11/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(11/2), x)

[Out] Timed out

$$3.171 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{13/2}} dx$$

Optimal. Leaf size=146

$$\frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{480c^2 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{60cf(c-c \sin(e+fx))^{11/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{12f(c-c \sin(e+fx))^{13/2}}$$

[Out] 1/12*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(13/2)+1/60*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(11/2)+1/480*(A-5*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(9/2)

Rubi [A] time = 0.38, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{480c^2 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-5B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{60cf(c-c \sin(e+fx))^{11/2}} + \frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{12f(c-c \sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(13/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(12*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(60*c*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 5*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(480*c^2*f*(c - c*Sin[e + f*x])^(9/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.73, size = 393, normalized size = 2.69

$$\frac{\sin(fx + e) (a(1 + \sin(fx + e)))^{\frac{7}{2}} (3A(\cos^5(fx + e)) \sin(fx + e) - 3A(\cos^6(fx + e)) - 21A(\cos^4(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2), x)

[Out]
$$-1/30/f*\sin(f*x+e)*(a*(1+\sin(f*x+e)))^{7/2}*(3*A*\cos(f*x+e)^5*\sin(f*x+e)-3*A*\cos(f*x+e)^6-21*A*\cos(f*x+e)^4*\sin(f*x+e)-18*A*\cos(f*x+e)^5-51*A*\cos(f*x+e)^3*\sin(f*x+e)+72*A*\cos(f*x+e)^4+15*B*\cos(f*x+e)^3*\sin(f*x+e)-15*B*\cos(f*x+e)^4+157*A*\cos(f*x+e)^2*\sin(f*x+e)+106*A*\cos(f*x+e)^3-5*B*\cos(f*x+e)^2*\sin(f*x+e)+10*B*\cos(f*x+e)^3+78*A*\sin(f*x+e)*\cos(f*x+e)-235*A*\cos(f*x+e)^2-30*B*\sin(f*x+e)*\cos(f*x+e)+35*B*\cos(f*x+e)^2-196*A*\sin(f*x+e)-118*A*\cos(f*x+e)+20*B*\sin(f*x+e)-10*B*\cos(f*x+e)+196*A-20*B)/(-c*(\sin(f*x+e)-1))^{13/2}/(\cos(f*x+e)^4+\sin(f*x+e)*\cos(f*x+e)^3+3*\cos(f*x+e)^3-4*\cos(f*x+e)^2*\sin(f*x+e)-8*\cos(f*x+e)^2-4*\sin(f*x+e)*\cos(f*x+e)-4*\cos(f*x+e)+8*\sin(f*x+e)+8)$$

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(13/2), x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 22.85, size = 406, normalized size = 2.78

$$\frac{\sqrt{c - c \sin(e + f x)} \left(\frac{56 a^3 e^{7i + f x 7i} (4 A + 5 B) \sqrt{a + a \sin(e + f x)}}{5 c^7 f} + \frac{a^3 e^{7i + f x 7i} \sin(3 e + 3 f x) (A 1i + B 1i) \sqrt{a + a \sin(e + f x)} 32i}{3 c^7 f} - \frac{32i}{3 c^7 f} \right)}{-858 \cos(e + f x) e^{7i + f x 7i} + 858 e^{7i + f x 7i} \cos(3 e + 3 f x) - 130 e^{7i + f x 7i} \cos(5 e + 5 f x) + 2 e^{7i + f x 7i}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(13/2), x)

[Out]
$$-((c - c*\sin(e + f*x))^{1/2}*((56*a^3*\exp(e*7i + f*x*7i))*(4*A + 5*B)*(a + a*\sin(e + f*x))^{1/2}))/((5*c^7*f) + (a^3*\exp(e*7i + f*x*7i)*\sin(3*e + 3*f*x)*(A*1i + B*1i)*(a + a*\sin(e + f*x))^{1/2})*32i)/(3*c^7*f) - (32*a^3*\exp(e*7i + f*x*7i)*\cos(2*e + 2*f*x)*(A + 2*B)*(a + a*\sin(e + f*x))^{1/2}))/((c^7*f) + (8*B*a^3*\exp(e*7i + f*x*7i)*\cos(4*e + 4*f*x)*(a + a*\sin(e + f*x))^{1/2}))/((c^7*f) - (a^3*\exp(e*7i + f*x*7i)*\sin(e + f*x)*(A*13i + B*5i)*(a + a*\sin(e + f*x))^{1/2})*32i)/(5*c^7*f)))/((858*\exp(e*7i + f*x*7i)*\cos(3*e + 3*f*x) - 858*\cos(e + f*x)*\exp(e*7i + f*x*7i) - 130*\exp(e*7i + f*x*7i)*\cos(5*e + 5*f*x) + 2*\exp(e*7i + f*x*7i)*\cos(7*e + 7*f*x) + 1144*\exp(e*7i + f*x*7i)*\sin(2*e + 2*f*x) - 416*\exp(e*7i + f*x*7i)*\sin(4*e + 4*f*x) + 24*\exp(e*7i + f*x*7i)*\sin(6*e + 6*f*x))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(13/2),x)
```

```
[Out] Timed out
```

$$3.172 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{15/2}} dx$$

Optimal. Leaf size=202

$$\frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{6720c^3 f(c-c \sin(e+fx))^{9/2}} + \frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{840c^2 f(c-c \sin(e+fx))^{11/2}} + \frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{168cf(c-c \sin(e+fx))^{13/2}}$$

[Out] 1/14*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(15/2)+1/168*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(13/2)+1/840*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(11/2)+1/6720*(3*A-11*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(9/2)

Rubi [A] time = 0.49, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{6720c^3 f(c-c \sin(e+fx))^{9/2}} + \frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{840c^2 f(c-c \sin(e+fx))^{11/2}} + \frac{(3A-11B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{168cf(c-c \sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(15/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(14*f*(c - c*Sin[e + f*x])^(15/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(168*c*f*(c - c*Sin[e + f*x])^(13/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(840*c^2*f*(c - c*Sin[e + f*x])^(11/2)) + ((3*A - 11*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(6720*c^3*f*(c - c*Sin[e + f*x])^(9/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{15/2}} dx = \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{14f(c - c \sin(e + fx))^{15/2}} + \frac{(3A - 11B) \int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{15/2}} dx}{14c}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{14f(c - c \sin(e + fx))^{15/2}} + \frac{(3A - 11B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{168cf(c - c \sin(e + fx))^{15/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{14f(c - c \sin(e + fx))^{15/2}} + \frac{(3A - 11B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{168cf(c - c \sin(e + fx))^{15/2}}$$

$$= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{14f(c - c \sin(e + fx))^{15/2}} + \frac{(3A - 11B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{168cf(c - c \sin(e + fx))^{15/2}}$$

Mathematica [B] time = 7.11, size = 442, normalized size = 2.19

$$\frac{(-A - 7B)(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7}{4f(c - c \sin(e + fx))^{15/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^7} + \frac{6(A + 3B)(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7}{5f(c - c \sin(e + fx))^{15/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^7}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(15/2), x]

[Out] (8*(A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) - (2*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (6*(A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(15/2))

fricas [A] time = 0.54, size = 234, normalized size = 1.16

$$\frac{\left(140 B a^3 \cos(fx + e)^4 - 7(27 A + 61 B) a^3 \cos(fx + e)^2 + 4(57 A + 71 B) a^3 - 7(5(3 A + 5 B) a^3 \cos(fx + e) - c^8 f \cos(fx + e)^7 - 56 c^8 f \cos(fx + e)^5 + 112 c^8 f \cos(fx + e)^3 - 64 c^8 f \cos(fx + e) - c^8 f \cos(fx + e)\right)}{420 \left(7 c^8 f \cos(fx + e)^7 - 56 c^8 f \cos(fx + e)^5 + 112 c^8 f \cos(fx + e)^3 - 64 c^8 f \cos(fx + e) - c^8 f \cos(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2), x, algorithm="fricas")

[Out] -1/420*(140*B*a^3*cos(f*x + e)^4 - 7*(27*A + 61*B)*a^3*cos(f*x + e)^2 + 4*(57*A + 71*B)*a^3 - 7*(5*(3*A + 5*B)*a^3*cos(f*x + e)^2 - 4*(9*A + 7*B)*a^3)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(7*c^8*f*cos(f*x + e)^7 - 56*c^8*f*cos(f*x + e)^5 + 112*c^8*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e) - (c^8*f*cos(f*x + e)^7 - 24*c^8*f*cos(f*x + e)^5 + 80*c^8*f*cos(f*x + e)^3 - 64*c^8*f*cos(f*x + e))*sin(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.76, size = 505, normalized size = 2.50

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{\frac{7}{2}} \left(-5016A \sin(fx + e) + 2268A \sin(fx + e) \cos(fx + e) - 352B \left(\cos^2 \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x)
```

```
[Out] -1/420/f*x*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(287*B*cos(f*x+e)^3*sin(f*x+e)
-5016*A*sin(f*x+e)+2268*A*sin(f*x+e)*cos(f*x+e)+39*A*cos(f*x+e)^6*sin(f*x+e)
)-1911*A*cos(f*x+e)^3*sin(f*x+e)-1209*A*cos(f*x+e)^4*sin(f*x+e)+93*B*sin(f*
x+e)*cos(f*x+e)^4+5136*A*cos(f*x+e)^2*sin(f*x+e)-352*B*cos(f*x+e)^2*sin(f*x
+e)-380*B*cos(f*x+e)^4+5016*A-472*B+3225*A*cos(f*x+e)^3-476*B*sin(f*x+e)*co
s(f*x+e)-3*B*cos(f*x+e)^6*sin(f*x+e)-936*A*cos(f*x+e)^5+72*B*cos(f*x+e)^5+2
4*B*cos(f*x+e)^6-21*B*cos(f*x+e)^5*sin(f*x+e)+273*A*cos(f*x+e)^5*sin(f*x+e)
+828*B*cos(f*x+e)^2+3120*A*cos(f*x+e)^4-65*B*cos(f*x+e)^3+472*B*sin(f*x+e)-
2748*A*cos(f*x+e)-4*B*cos(f*x+e)-312*A*cos(f*x+e)^6-7404*A*cos(f*x+e)^2+39*
A*cos(f*x+e)^7-3*B*cos(f*x+e)^7)/(-c*(sin(f*x+e)-1))^(15/2)/(cos(f*x+e)^4+s
in(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)
)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(15/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 25.26, size = 827, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
15/2),x)
```

```
[Out] -((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*
(B*a^3*exp(e*4i + f*x*4i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^8*f) + (B*a^3*exp(e*12i + f*x*12i)*(a + a*
((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*16i)/(3*c^
8*f) - (a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) + (a^3*exp(e*11i +
f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(
1/2)*(A*3i + B*5i)*8i)/(3*c^8*f) - (a^3*exp(e*6i + f*x*6i)*(a + a*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)
/(15*c^8*f) - (a^3*exp(e*10i + f*x*10i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(27*A + 41*B)*16i)/(15*c^8*f) + (a^3*exp
(e*7i + f*x*7i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)
```

```

*1i)/2))^(1/2)*(A*43i + B*29i)*8i)/(5*c^8*f) - (a^3*exp(e*9i + f*x*9i)*(a +
a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*43i
+ B*29i)*8i)/(5*c^8*f) + (a^3*exp(e*8i + f*x*8i)*(a + a*((exp(- e*1i - f*x*
1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(89*A + 82*B)*32i)/(35*c^8*f
))/((exp(e*1i + f*x*1i)*14i - 90*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*350
i + 910*exp(e*4i + f*x*4i) + exp(e*5i + f*x*5i)*1638i - 2002*exp(e*6i + f*x
*6i) - exp(e*7i + f*x*7i)*1430i - exp(e*9i + f*x*9i)*1430i + 2002*exp(e*10i
+ f*x*10i) + exp(e*11i + f*x*11i)*1638i - 910*exp(e*12i + f*x*12i) - exp(e
*13i + f*x*13i)*350i + 90*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*14i -
exp(e*16i + f*x*16i) + 1)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(15/2)
,x)

```

[Out] Timed out

$$3.173 \quad \int \frac{(a+a \sin(e+fx))^{7/2}(A+B \sin(e+fx))}{(c-c \sin(e+fx))^{17/2}} dx$$

Optimal. Leaf size=246

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{8960c^4 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{1120c^3 f(c-c \sin(e+fx))^{11/2}} + \frac{(A-3B) \cos(e+fx)}{224c^2 f(c-c \sin(e+fx))^{13/2}}$$

[Out] 1/16*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/f/(c-c*sin(f*x+e))^(17/2)+1/56*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c/f/(c-c*sin(f*x+e))^(15/2)+1/224*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^2/f/(c-c*sin(f*x+e))^(13/2)+1/1120*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^3/f/(c-c*sin(f*x+e))^(11/2)+1/8960*(A-3*B)*cos(f*x+e)*(a+a*sin(f*x+e))^(7/2)/c^4/f/(c-c*sin(f*x+e))^(9/2)

Rubi [A] time = 0.59, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{8960c^4 f(c-c \sin(e+fx))^{9/2}} + \frac{(A-3B) \cos(e+fx)(a \sin(e+fx)+a)^{7/2}}{1120c^3 f(c-c \sin(e+fx))^{11/2}} + \frac{(A-3B) \cos(e+fx)}{224c^2 f(c-c \sin(e+fx))^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(17/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(16*f*(c - c*Sin[e + f*x])^(17/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(56*c*f*(c - c*Sin[e + f*x])^(15/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(224*c^2*f*(c - c*Sin[e + f*x])^(13/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(1120*c^3*f*(c - c*Sin[e + f*x])^(11/2)) + ((A - 3*B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(8960*c^4*f*(c - c*Sin[e + f*x])^(9/2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^{7/2} (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{17/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{16f(c - c \sin(e + fx))^{17/2}} + \frac{(A - 3B) \int \frac{(a + a \sin(e + fx))^{7/2}}{(c - c \sin(e + fx))^{17/2}} dx}{4c} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{16f(c - c \sin(e + fx))^{17/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{56cf(c - c \sin(e + fx))^{17/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{16f(c - c \sin(e + fx))^{17/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{56cf(c - c \sin(e + fx))^{17/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{16f(c - c \sin(e + fx))^{17/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{56cf(c - c \sin(e + fx))^{17/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{16f(c - c \sin(e + fx))^{17/2}} + \frac{(A - 3B) \cos(e + fx) (a + a \sin(e + fx))^{7/2}}{56cf(c - c \sin(e + fx))^{17/2}} \end{aligned}$$

Mathematica [A] time = 7.14, size = 436, normalized size = 1.77

$$\frac{(-A - 7B)(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7}{5f(c - c \sin(e + fx))^{17/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^7} + \frac{(A + 3B)(a(\sin(e + fx) + 1))^{7/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)^7}{f(c - c \sin(e + fx))^{17/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^7}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^(7/2)*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(17/2), x]

[Out] ((A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) - (4*(3*A + 5*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a*(1 + Sin[e + f*x]))^(7/2))/(7*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((A + 3*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(7/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + ((-A - 7*B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(7/2))/(5*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2)) + (B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(7/2))/(4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^7*(c - c*Sin[e + f*x])^(17/2))

fricas [A] time = 0.54, size = 243, normalized size = 0.99

$$\frac{(35Ba^3 \cos(fx + e))^4 - 56(A + 2B)a^3 \cos(fx + e)^2 + 4(17A + 19B)a^3 - 4(7(A + 2B)a^3)}{140(c^9 f \cos(fx + e))^9 - 32c^9 f \cos(fx + e)^7 + 160c^9 f \cos(fx + e)^5 - 256c^9 f \cos(fx + e)^3 + 128c^9 f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2), x, algorithm="fricas")

[Out] 1/140*(35*B*a^3*cos(f*x + e)^4 - 56*(A + 2*B)*a^3*cos(f*x + e)^2 + 4*(17*A + 19*B)*a^3 - 4*(7*(A + 2*B)*a^3*cos(f*x + e)^2 - 2*(9*A + 8*B)*a^3)*sin(f*x + e)

```
x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(c^9*f*cos(f*x +
e)^9 - 32*c^9*f*cos(f*x + e)^7 + 160*c^9*f*cos(f*x + e)^5 - 256*c^9*f*cos(
f*x + e)^3 + 128*c^9*f*cos(f*x + e) + 8*(c^9*f*cos(f*x + e)^7 - 10*c^9*f*co
s(f*x + e)^5 + 24*c^9*f*cos(f*x + e)^3 - 16*c^9*f*cos(f*x + e))*sin(f*x + e
))
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.81, size = 560, normalized size = 2.28

$$\frac{\sin(fx + e) \left(a \left(1 + \sin(fx + e) \right) \right)^{\frac{7}{2}} \left(-3076A \sin(fx + e) + 1468A \sin(fx + e) \cos(fx + e) - 300B \left(\cos^2(fx + e) \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x)
```

```
[Out] -1/140/f*sin(f*x+e)*(a*(1+sin(f*x+e)))^(7/2)*(164*B*cos(f*x+e)^3*sin(f*x+e)
-3076*A*sin(f*x+e)+1468*A*sin(f*x+e)*cos(f*x+e)+108*A*cos(f*x+e)^6*sin(f*x+
e)-1548*A*cos(f*x+e)^3*sin(f*x+e)-1332*A*cos(f*x+e)^4*sin(f*x+e)+111*B*sin(
f*x+e)*cos(f*x+e)^4+3880*A*cos(f*x+e)^2*sin(f*x+e)-300*B*cos(f*x+e)^2*sin(f
*x+e)-275*B*cos(f*x+e)^4+3076*A-268*B+2332*A*cos(f*x+e)^3-204*B*sin(f*x+e)*
cos(f*x+e)-9*B*cos(f*x+e)^6*sin(f*x+e)-960*A*cos(f*x+e)^5+80*B*cos(f*x+e)^5
+40*B*cos(f*x+e)^6-31*B*cos(f*x+e)^5*sin(f*x+e)+372*A*cos(f*x+e)^5*sin(f*x+
e)-B*cos(f*x+e)^8+504*B*cos(f*x+e)^2+2880*A*cos(f*x+e)^4-136*B*cos(f*x+e)^3
+268*B*sin(f*x+e)-1608*A*cos(f*x+e)-12*A*cos(f*x+e)^7*sin(f*x+e)+B*cos(f*x+
e)^7*sin(f*x+e)+64*B*cos(f*x+e)-480*A*cos(f*x+e)^6-5348*A*cos(f*x+e)^2+96*A
*cos(f*x+e)^7-8*B*cos(f*x+e)^7+12*A*cos(f*x+e)^8)/(-c*(sin(f*x+e)-1))^(17/2
)/(cos(f*x+e)^4+sin(f*x+e)*cos(f*x+e)^3+3*cos(f*x+e)^3-4*cos(f*x+e)^2*sin(f
*x+e)-8*cos(f*x+e)^2-4*sin(f*x+e)*cos(f*x+e)-4*cos(f*x+e)+8*sin(f*x+e)+8)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(17/2),x
, algorithm="maxima")
```

[Out] Timed out

mupad [B] time = 28.34, size = 841, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(7/2))/(c - c*sin(e + f*x))^(
17/2),x)
```

```
[Out] ((c - c*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*((
8*B*a^3*exp(e*5i + f*x*5i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
```

```

+ f*x*1i)*1i)/2))^(1/2))/(c^9*f) + (8*B*a^3*exp(e*13i + f*x*13i)*(a + a*((e
xp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2))/(c^9*f) - (6
4*a^3*exp(e*6i + f*x*6i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i +
f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f) - (32*a^3*exp(e*7i + f*x*7i)
*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(8
*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*12i + f*x*12i)*(a + a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*1i + B*2i))/(5*c^9*f)
- (32*a^3*exp(e*11i + f*x*11i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e
*1i + f*x*1i)*1i)/2))^(1/2)*(8*A + 11*B))/(5*c^9*f) + (64*a^3*exp(e*8i + f
*x*8i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/
2)*(A*13i + B*10i))/(7*c^9*f) - (64*a^3*exp(e*10i + f*x*10i)*(a + a*((exp(-
e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(A*13i + B*10i))/
(7*c^9*f) + (16*a^3*exp(e*9i + f*x*9i)*(a + a*((exp(- e*1i - f*x*1i)*1i)/2
- (exp(e*1i + f*x*1i)*1i)/2))^(1/2)*(64*A + 53*B))/(7*c^9*f)))/(exp(e*1i +
f*x*1i)*16i - 119*exp(e*2i + f*x*2i) - exp(e*3i + f*x*3i)*544i + 1700*exp(e
*4i + f*x*4i) + exp(e*5i + f*x*5i)*3808i - 6188*exp(e*6i + f*x*6i) - exp(e*
7i + f*x*7i)*7072i + 4862*exp(e*8i + f*x*8i) + 4862*exp(e*10i + f*x*10i) +
exp(e*11i + f*x*11i)*7072i - 6188*exp(e*12i + f*x*12i) - exp(e*13i + f*x*13
i)*3808i + 1700*exp(e*14i + f*x*14i) + exp(e*15i + f*x*15i)*544i - 119*exp(
e*16i + f*x*16i) - exp(e*17i + f*x*17i)*16i + exp(e*18i + f*x*18i) + 1)

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+a*sin(f*x+e))**(7/2)*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(17/2)
,x)

```

```

[Out] Timed out

```

$$3.174 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=197

$$\frac{4c^3(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{2c^2(A-B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f\sqrt{a \sin(e+fx)+a}} + \frac{c(A-B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)}}$$

[Out] 1/2*(A-B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/f/(a+a*sin(f*x+e))^(1/2)-1/3*B*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/f/(a+a*sin(f*x+e))^(1/2)+4*(A-B)*c^3*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+2*(A-B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.46, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f\sqrt{a \sin(e+fx)+a}} + \frac{4c^3(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{c(A-B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*x]], x]

[Out] (4*(A - B)*c^3*Cos[e + f*x]*Log[1 + Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (2*(A - B)*c^2*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]]) + ((A - B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(2*f*Sqrt[a + a*Sin[e + f*x]]) - (B*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*f*Sqrt[a + a*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n]/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILT

$Q[m + n, 0] \ \&\& \ GtQ[2*m + n + 1, 0]$

Rule 2973

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x])^{(m_.)}((A_.) + (B_.)\sin[(e_.) + (f_.)x])^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(B\cos[e + fx](a + b\sin[e + fx])^m(c + d\sin[e + fx])^n)/(f(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b\sin[e + fx])^m(c + d\sin[e + fx])^n, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx &= -\frac{B \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3f\sqrt{a + a \sin(e + fx)}} + (A - B) \int \frac{(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx \\ &= \frac{(A - B)c \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}} - \frac{B \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3f\sqrt{a + a \sin(e + fx)}} \\ &= \frac{2(A - B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} + \frac{(A - B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2f\sqrt{a + a \sin(e + fx)}} \\ &= \frac{2(A - B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} + \frac{(A - B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2f\sqrt{a + a \sin(e + fx)}} \\ &= \frac{2(A - B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} + \frac{(A - B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2f\sqrt{a + a \sin(e + fx)}} \\ &= \frac{4(A - B)c^3 \cos(e + fx) \log(1 + \sin(e + fx))}{f\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} + \frac{2(A - B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{f\sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.31, size = 185, normalized size = 0.94

$$\frac{c^2(\sin(e + fx) - 1)^2\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((36A - 51B) \sin(e + fx) + 3(A - 3B) \right)}{12f\sqrt{a}(\sin(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/Sqrt[a + a*Sin[e + f*x]], x]

[Out] -1/12*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])^2*Sqrt[c - c*Sin[e + f*x]]*(3*(A - 3*B)*Cos[2*(e + f*x)] - 96*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 96*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (36*A - 51*B)*Sin[e + f*x] + B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5*Sqrt[a*(1 + Sin[e + f*x])])

fricas [F] time = 4.08, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A - 2B)c^2 \cos^2(fx + e) - 2(A - B)c^2 + (Bc^2 \cos^2(fx + e) + 2(A - B)c^2) \sin(fx + e) \right) \sqrt{-c \sin(fx + e)}}{\sqrt{a \sin(fx + e) + a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="fricas")

[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="giac")

[Out] Timed out

maple [B] time = 0.72, size = 595, normalized size = 3.02

$$\frac{\left(48A \ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 48B \ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 15A \sin(fx+e) + 18A \sin(fx+e) \cos(fx+e)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x)

[Out] -1/6/f*(2*B*cos(f*x+e)^3*sin(f*x+e)+48*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-15*A*sin(f*x+e)+18*A*sin(f*x+e)*cos(f*x+e)-3*A*cos(f*x+e)^2*sin(f*x+e)+7*B*cos(f*x+e)^2*sin(f*x+e)+2*B*cos(f*x+e)^4-15*A+17*B+3*A*cos(f*x+e)^3-24*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-26*B*sin(f*x+e)*cos(f*x+e)+24*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-48*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+48*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-48*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-19*B*cos(f*x+e)^2-9*B*cos(f*x+e)^3+17*B*sin(f*x+e)-3*A*cos(f*x+e)+9*B*cos(f*x+e)+15*A*cos(f*x+e)^2-24*A*ln(2/(cos(f*x+e)+1))+24*B*ln(2/(cos(f*x+e)+1))) * (-c*(sin(f*x+e)-1))^(5/2)/(cos(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2-4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{5}{2}}}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{\frac{5}{2}}}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(1/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2), x)
```

```
[Out] Timed out
```


$$3.175 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=146

$$\frac{2c^2(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{c(A-B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f\sqrt{a \sin(e+fx)+a}} - \frac{B \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{2f\sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/2*B*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}+2*(A-B)*c^{(2)*\cos(f*x+e)*\ln(1+\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)/(c-c*\sin(f*x+e))^{(1/2)}+(A-B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{c(A-B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f\sqrt{a \sin(e+fx)+a}} - \frac{B \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{2f\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*x]], x]

[Out] $(2*(A - B)*c^2*\text{Cos}[e + f*x]*\text{Log}[1 + \text{Sin}[e + f*x]])/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((A - B)*c*\text{Cos}[e + f*x]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (B*\text{Cos}[e + f*x]*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n]/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx = -\frac{B \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}} + (A - B) \int \frac{(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

$$= \frac{(A - B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} - \frac{B \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}}$$

$$= \frac{(A - B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} - \frac{B \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}}$$

$$= \frac{(A - B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} - \frac{B \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f\sqrt{a + a \sin(e + fx)}}$$

$$= \frac{2(A - B)c^2 \cos(e + fx) \log(1 + \sin(e + fx))}{f\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{(A - B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.66, size = 146, normalized size = 1.00

$$\frac{c(\sin(e + fx) - 1)\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(B \cos(2(e + fx)) - 4 \left((A - 2B) \sin(e + fx) \right) \right)}{4f\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/Sqrt[a + a*Sin[e + f*x]], x]
```

```
[Out] -1/4*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-1 + Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)] - 4*(4*(-A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (A - 2*B)*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [F] time = 2.82, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Bc \cos^2(fx + e) - (A - B)c \sin(fx + e) + (A - B)c \right) \sqrt{-c \sin(fx + e) + c}}{\sqrt{a \sin(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="fricas")
```

```
[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2), x,
algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable
to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(A*c*sqrt(a*tan(1/2*exp(
1))^2+a)*(27021597764222976*tan(1/2*exp(1))^5-81064793292668928*tan(1/2*exp(
1))^4-90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan(1/2*exp(1)
)^2+27021597764222976*tan(1/2*exp(1))-9007199254740992)+B*c*sqrt(a*tan(1/2*
exp(1))^2+a)*(-27021597764222976*tan(1/2*exp(1))^5+81064793292668928*tan(1/
2*exp(1))^4+90071992547409920*tan(1/2*exp(1))^3-54043195528445952*tan(1/2*e
xp(1))^2-27021597764222976*tan(1/2*exp(1))+9007199254740992)+A*c*sqrt(a*tan
(1/2*exp(1))^2+a)*(54043195528445952*tan(1/2*exp(1))^6-162129586585337856*t
an(1/2*exp(1))^5-324259173170675712*tan(1/2*exp(1))^4+540431955284459520*t
an(1/2*exp(1))^3+486388759756013568*tan(1/2*exp(1))^2-162129586585337856*tan
(1/2*exp(1))*tan(1/4*exp(1))^5+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(5404319552
8445952*tan(1/2*exp(1))^6-162129586585337856*tan(1/2*exp(1))^5-324259173170
675712*tan(1/2*exp(1))^4+540431955284459520*tan(1/2*exp(1))^3+4863887597560
13568*tan(1/2*exp(1))^2-162129586585337856*tan(1/2*exp(1))*tan(1/4*exp(1))
+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(405323966463344640*tan(1/2*exp(1))^5-1215
971899390033920*tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3+810
647932926689280*tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))-135107
988821114880)*tan(1/4*exp(1))^4+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-270215977
64222976*tan(1/2*exp(1))^5+81064793292668928*tan(1/2*exp(1))^4+900719925474
09920*tan(1/2*exp(1))^3-54043195528445952*tan(1/2*exp(1))^2-270215977642229
76*tan(1/2*exp(1))+9007199254740992)*tan(1/4*exp(1))^6+A*c*sqrt(a*tan(1/2*e
xp(1))^2+a)*(-180143985094819840*tan(1/2*exp(1))^6+540431955284459520*tan(1
/2*exp(1))^5+1080863910568919040*tan(1/2*exp(1))^4-1801439850948198400*tan(
1/2*exp(1))^3-1621295865853378560*tan(1/2*exp(1))^2+540431955284459520*tan(
1/2*exp(1))*tan(1/4*exp(1))^3+A*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-4053239664
63344640*tan(1/2*exp(1))^5+1215971899390033920*tan(1/2*exp(1))^4+1351079888
211148800*tan(1/2*exp(1))^3-810647932926689280*tan(1/2*exp(1))^2-4053239664
63344640*tan(1/2*exp(1))+135107988821114880)*tan(1/4*exp(1))^2+B*c*sqrt(a*t
an(1/2*exp(1))^2+a)*(27021597764222976*tan(1/2*exp(1))^5-81064793292668928*
tan(1/2*exp(1))^4-90071992547409920*tan(1/2*exp(1))^3+54043195528445952*tan
(1/2*exp(1))^2+27021597764222976*tan(1/2*exp(1))-9007199254740992)*tan(1/4*
exp(1))^6+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(180143985094819840*tan(1/2*exp(1)
))^6-540431955284459520*tan(1/2*exp(1))^5-1080863910568919040*tan(1/2*exp(1)
))^4+1801439850948198400*tan(1/2*exp(1))^3+1621295865853378560*tan(1/2*exp(
1))^2-540431955284459520*tan(1/2*exp(1))*tan(1/4*exp(1))^3+B*c*sqrt(a*tan(
1/2*exp(1))^2+a)*(405323966463344640*tan(1/2*exp(1))^5-1215971899390033920*
tan(1/2*exp(1))^4-1351079888211148800*tan(1/2*exp(1))^3+810647932926689280*
tan(1/2*exp(1))^2+405323966463344640*tan(1/2*exp(1))-135107988821114880)*t
an(1/4*exp(1))^2+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-54043195528445952*tan(1/2
*exp(1))^6+162129586585337856*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*
exp(1))^4-540431955284459520*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*e
xp(1))^2+162129586585337856*tan(1/2*exp(1))*tan(1/4*exp(1))^5+B*c*sqrt(a*t
an(1/2*exp(1))^2+a)*(-54043195528445952*tan(1/2*exp(1))^6+16212958658533785
6*tan(1/2*exp(1))^5+324259173170675712*tan(1/2*exp(1))^4-540431955284459520
*tan(1/2*exp(1))^3-486388759756013568*tan(1/2*exp(1))^2+162129586585337856*
tan(1/2*exp(1))*tan(1/4*exp(1))+B*c*sqrt(a*tan(1/2*exp(1))^2+a)*(-40532396
6463344640*tan(1/2*exp(1))^5+1215971899390033920*tan(1/2*exp(1))^4+13510798
88211148800*tan(1/2*exp(1))^3-810647932926689280*tan(1/2*exp(1))^2-40532396
6463344640*tan(1/2*exp(1))+135107988821114880)*tan(1/4*exp(1))^4)*ln(abs(2*
tan(1/2*exp(1))^3+6*tan(1/2*exp(1))^2+(tan(1/2*(1/2*f*x+2*exp(1))))-1/tan(1/
```

```

2*(1/2*f*x+2*exp(1)))*(tan(1/2*exp(1))^3-3*tan(1/2*exp(1))^2-3*tan(1/2*exp(1))
(1))+1)-6*tan(1/2*exp(1))-2)/f/(-9007199254740992*sqrt(2)*a*tan(1/2*exp(1))
)^7-9007199254740992*sqrt(2)*a+(-9007199254740992*sqrt(2)*a*tan(1/2*exp(1))
)^7+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199254740992*sqrt(2)*a
*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^4+4503599627
3704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqrt(2)*a*tan(1/2*exp(
1))^2-9007199254740992*sqrt(2)*a+27021597764222976*sqrt(2)*a*tan(1/2*exp(1)
))*tan(1/4*exp(1))^6+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^7+810647
93292668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*a*tan(1/2
*exp(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+13510798882111488
0*sqrt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^2
-27021597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*exp(1)))*t
an(1/4*exp(1))^2+(-27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^7+8106479329
2668928*sqrt(2)*a*tan(1/2*exp(1))^6+27021597764222976*sqrt(2)*a*tan(1/2*exp
(1))^5+135107988821114880*sqrt(2)*a*tan(1/2*exp(1))^4+135107988821114880*sq
rt(2)*a*tan(1/2*exp(1))^3+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^2-270
21597764222976*sqrt(2)*a+81064793292668928*sqrt(2)*a*tan(1/2*exp(1)))*tan(1
/4*exp(1))^4+27021597764222976*sqrt(2)*a*tan(1/2*exp(1))^6+9007199254740992
*sqrt(2)*a*tan(1/2*exp(1))^5+45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^4+
45035996273704960*sqrt(2)*a*tan(1/2*exp(1))^3+9007199254740992*sqrt(2)*a*ta
n(1/2*exp(1))^2+27021597764222976*sqrt(2)*a*tan(1/2*exp(1)))

```

maple [B] time = 0.68, size = 501, normalized size = 3.43

$$\left(-B(\cos^3(fx + e)) + B(\cos^2(fx + e))\sin(fx + e) + 2A(\cos^2(fx + e)) + 2A\sin(fx + e)\cos(fx + e) + 4A\cos(fx + e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/2/f*(-B*cos(f*x+e)^3+B*cos(f*x+e)^2*sin(f*x+e)+2*A*cos(f*x+e)^2+2*A*sin(f*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-8*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-4*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+8*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*B*cos(f*x+e)^2-4*B*sin(f*x+e)*cos(f*x+e)-4*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+8*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-8*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)-4*A*ln(2/(cos(f*x+e)+1))+8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)+3*B*sin(f*x+e)+4*B*ln(2/(cos(f*x+e)+1))-8*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A+3*B)*(-c*(sin(f*x+e)-1))^(3/2)/(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a*(1+sin(f*x+e)))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{3}{2}}}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e + fx) - 1))^{\frac{3}{2}} (A + B \sin(e + fx))}{\sqrt{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(1/2), x)

[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.176 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=96

$$\frac{c(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{B \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f \sqrt{a \sin(e+fx)+a}}$$

[Out] (A-B)*c*cos(f*x+e)*ln(1+sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)-B*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/f/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.32, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2971, 2738, 2737, 2667, 31}

$$\frac{c(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{B \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]],x]

[Out] ((A - B)*c*cos[e + f*x]*Log[1 + Sin[e + f*x]]/(f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) - (B*cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]])/(f*Sqrt[a + a*Sin[e + f*x]])

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2738

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_) + (B_.)*sin[(e_) + (f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[B/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] - Dist[(B*c - A*d)/d, Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x],

$x]$ /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))\sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx &= \frac{B \int \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)} dx}{a} - (-A + B) \int \frac{\sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx \\ &= -\frac{B \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} - \frac{(a(-A + B)c \cos(e + fx))}{\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} \\ &= -\frac{B \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} - \frac{((-A + B)c \cos(e + fx))}{f\sqrt{a + a \sin(e + fx)}} \\ &= \frac{(A - B)c \cos(e + fx) \log(1 + \sin(e + fx))}{f\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} - \frac{B \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f\sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.13, size = 119, normalized size = 1.24

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(B \sin(e + fx) + (A - B) \left(2 \log\left(e^{i(e+fx)} + i\right) - ifx \right) \right)}{f\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/Sqrt[a + a*Sin[e + f*x]], x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*((-I)*f*x + 2*Log[I + E^(I*(e + f*x))]) + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])

fricas [F] time = 1.36, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A)\sqrt{-c \sin(fx + e) + c}}{\sqrt{a \sin(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (8*pi/x/2)>(-8*pi/x/2)4*sqrt(2*c)*(A*sqrt(a*tan(1/2*exp(1)

$$\begin{aligned}
&)^2+a)*(12582912*\tan(1/2*\exp(1))^5-37748736*\tan(1/2*\exp(1))^4-41943040*\tan(\\
& 1/2*\exp(1))^3+25165824*\tan(1/2*\exp(1))^2+12582912*\tan(1/2*\exp(1))-4194304)+ \\
& B*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(-12582912*\tan(1/2*\exp(1))^5+37748736*\tan(1/2 \\
& *\exp(1))^4+41943040*\tan(1/2*\exp(1))^3-25165824*\tan(1/2*\exp(1))^2-12582912* \\
& \tan(1/2*\exp(1))+4194304)+A*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(25165824*\tan(1/2*\exp \\
& (1))^6-75497472*\tan(1/2*\exp(1))^5-150994944*\tan(1/2*\exp(1))^4+251658240*\tan \\
& (1/2*\exp(1))^3+226492416*\tan(1/2*\exp(1))^2-75497472*\tan(1/2*\exp(1)))*\tan(1/ \\
& 4*\exp(1))^5+A*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(25165824*\tan(1/2*\exp(1))^6-75497 \\
& 472*\tan(1/2*\exp(1))^5-150994944*\tan(1/2*\exp(1))^4+251658240*\tan(1/2*\exp(1)) \\
& ^3+226492416*\tan(1/2*\exp(1))^2-75497472*\tan(1/2*\exp(1)))*\tan(1/4*\exp(1))+A* \\
& \sqrt{a*\tan(1/2*\exp(1))^2+a)*(188743680*\tan(1/2*\exp(1))^5-566231040*\tan(1/2* \\
& \exp(1))^4-629145600*\tan(1/2*\exp(1))^3+377487360*\tan(1/2*\exp(1))^2+188743680 \\
& *\tan(1/2*\exp(1))-62914560)*\tan(1/4*\exp(1))^4+A*\sqrt{a*\tan(1/2*\exp(1))^2+a)* \\
& (-12582912*\tan(1/2*\exp(1))^5+37748736*\tan(1/2*\exp(1))^4+41943040*\tan(1/2*\exp \\
& (1))^3-25165824*\tan(1/2*\exp(1))^2-12582912*\tan(1/2*\exp(1))+4194304)*\tan(1/ \\
& 4*\exp(1))^6+A*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(-83886080*\tan(1/2*\exp(1))^6+2516 \\
& 58240*\tan(1/2*\exp(1))^5+503316480*\tan(1/2*\exp(1))^4-838860800*\tan(1/2*\exp(1) \\
&)^3-754974720*\tan(1/2*\exp(1))^2+251658240*\tan(1/2*\exp(1)))*\tan(1/4*\exp(1)) \\
& ^3+A*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(-188743680*\tan(1/2*\exp(1))^5+566231040*\tan \\
& (1/2*\exp(1))^4+629145600*\tan(1/2*\exp(1))^3-377487360*\tan(1/2*\exp(1))^2-188 \\
& 743680*\tan(1/2*\exp(1))+62914560)*\tan(1/4*\exp(1))^2+B*\sqrt{a*\tan(1/2*\exp(1)) \\
& ^2+a)*(12582912*\tan(1/2*\exp(1))^5-37748736*\tan(1/2*\exp(1))^4-41943040*\tan(1 \\
& /2*\exp(1))^3+25165824*\tan(1/2*\exp(1))^2+12582912*\tan(1/2*\exp(1))-4194304)*\tan \\
& (1/4*\exp(1))^6+B*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(83886080*\tan(1/2*\exp(1))^6- \\
& 251658240*\tan(1/2*\exp(1))^5-503316480*\tan(1/2*\exp(1))^4+838860800*\tan(1/2*\exp \\
& (1))^3+754974720*\tan(1/2*\exp(1))^2-251658240*\tan(1/2*\exp(1)))*\tan(1/4*\exp \\
& (1))^3+B*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(188743680*\tan(1/2*\exp(1))^5-566231040 \\
& *\tan(1/2*\exp(1))^4-629145600*\tan(1/2*\exp(1))^3+377487360*\tan(1/2*\exp(1))^2+ \\
& 188743680*\tan(1/2*\exp(1))-62914560)*\tan(1/4*\exp(1))^2+B*\sqrt{a*\tan(1/2*\exp(\\
& 1))^2+a)*(-25165824*\tan(1/2*\exp(1))^6+75497472*\tan(1/2*\exp(1))^5+150994944* \\
& \tan(1/2*\exp(1))^4-251658240*\tan(1/2*\exp(1))^3-226492416*\tan(1/2*\exp(1))^2+7 \\
& 5497472*\tan(1/2*\exp(1)))*\tan(1/4*\exp(1))^5+B*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(- \\
& 25165824*\tan(1/2*\exp(1))^6+75497472*\tan(1/2*\exp(1))^5+150994944*\tan(1/2*\exp \\
& (1))^4-251658240*\tan(1/2*\exp(1))^3-226492416*\tan(1/2*\exp(1))^2+75497472*\tan \\
& (1/2*\exp(1)))*\tan(1/4*\exp(1))+B*\sqrt{a*\tan(1/2*\exp(1))^2+a)*(-188743680*\tan \\
& (1/2*\exp(1))^5+566231040*\tan(1/2*\exp(1))^4+629145600*\tan(1/2*\exp(1))^3-3774 \\
& 87360*\tan(1/2*\exp(1))^2-188743680*\tan(1/2*\exp(1))+62914560)*\tan(1/4*\exp(1)) \\
& ^4)*\ln(\text{abs}(2*\tan(1/2*\exp(1))^3+6*\tan(1/2*\exp(1))^2+(\tan(1/2*(1/2*f*x+2*\exp(\\
& 1))))-1/\tan(1/2*(1/2*f*x+2*\exp(1))))*(\tan(1/2*\exp(1))^3-3*\tan(1/2*\exp(1))^2- \\
& 3*\tan(1/2*\exp(1))+1)-6*\tan(1/2*\exp(1))-2)/f/(-8388608*\sqrt{2}*a*\tan(1/2*\exp \\
& (1))^7-8388608*\sqrt{2}*a+(-8388608*\sqrt{2}*a*\tan(1/2*\exp(1))^7+25165824*\sqrt{2} \\
& *a*\tan(1/2*\exp(1))^6+8388608*\sqrt{2}*a*\tan(1/2*\exp(1))^5+41943040*\sqrt{2} \\
& *a*\tan(1/2*\exp(1))^4+41943040*\sqrt{2}*a*\tan(1/2*\exp(1))^3+8388608*\sqrt{2} \\
&)*a*\tan(1/2*\exp(1))^2-8388608*\sqrt{2}*a+25165824*\sqrt{2}*a*\tan(1/2*\exp(1))) \\
& *\tan(1/4*\exp(1))^6+(-25165824*\sqrt{2}*a*\tan(1/2*\exp(1))^7+75497472*\sqrt{2}* \\
& a*\tan(1/2*\exp(1))^6+25165824*\sqrt{2}*a*\tan(1/2*\exp(1))^5+125829120*\sqrt{2}* \\
& a*\tan(1/2*\exp(1))^4+125829120*\sqrt{2}*a*\tan(1/2*\exp(1))^3+25165824*\sqrt{2}* \\
& a*\tan(1/2*\exp(1))^2-25165824*\sqrt{2}*a+75497472*\sqrt{2}*a*\tan(1/2*\exp(1)))* \\
& \tan(1/4*\exp(1))^2+(-25165824*\sqrt{2}*a*\tan(1/2*\exp(1))^7+75497472*\sqrt{2}*a \\
& *\tan(1/2*\exp(1))^6+25165824*\sqrt{2}*a*\tan(1/2*\exp(1))^5+125829120*\sqrt{2}*a \\
& *\tan(1/2*\exp(1))^4+125829120*\sqrt{2}*a*\tan(1/2*\exp(1))^3+25165824*\sqrt{2}*a \\
& *\tan(1/2*\exp(1))^2-25165824*\sqrt{2}*a+75497472*\sqrt{2}*a*\tan(1/2*\exp(1)))*\tan \\
& (1/4*\exp(1))^4+25165824*\sqrt{2}*a*\tan(1/2*\exp(1))^6+8388608*\sqrt{2}*a*\tan \\
& (1/2*\exp(1))^5+41943040*\sqrt{2}*a*\tan(1/2*\exp(1))^4+41943040*\sqrt{2}*a*\tan(\\
& 1/2*\exp(1))^3+8388608*\sqrt{2}*a*\tan(1/2*\exp(1))^2+25165824*\sqrt{2}*a*\tan(1/ \\
& 2*\exp(1)))
\end{aligned}$$

maple [B] time = 0.68, size = 399, normalized size = 4.16

$$\left(2A \cos(fx + e) \ln\left(\frac{1 - \cos(fx+e) + \sin(fx+e)}{\sin(fx+e)}\right) - A \cos(fx + e) \ln\left(\frac{2}{\cos(fx+e)+1}\right) - 2A \sin(fx + e) \ln\left(\frac{1 - \cos(fx+e) + \sin(fx+e)}{\sin(fx+e)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)

[Out] -1/f*(2*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+B*cos(f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)-2*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+2*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln(2/(cos(f*x+e)+1))-B*sin(f*x+e)+2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))-B*(-c*(sin(f*x+e)-1))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(1/2)

maxima [A] time = 0.79, size = 176, normalized size = 1.83

$$B \left[\frac{2\sqrt{c} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{\sqrt{a}} - \frac{\sqrt{c} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{a}} - \frac{2\sqrt{a}\sqrt{c} \sin(fx+e)}{\left(a + \frac{a \sin(fx+e)^2}{(\cos(fx+e)+1)^2}\right)(\cos(fx+e)+1)} \right] - A \left[\frac{2\sqrt{c} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{\sqrt{a}} - \frac{\sqrt{c} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{\sqrt{a}} \right] / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] (B*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a) - 2*sqrt(a)*sqrt(c)*sin(f*x + e)/((a + a*sin(f*x + e)^2/(cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1))) - A*(2*sqrt(c)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/sqrt(a) - sqrt(c)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/sqrt(a)))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e + fx) - 1)} (A + B \sin(e + fx))}{\sqrt{a(\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2),  
x)
```

```
[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/sqrt(a*(sin(e + f  
*x) + 1)), x)
```

$$3.177 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} \sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=113

$$\frac{(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{2f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{2f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

[Out] $-1/2*(A+B)*\cos(f*x+e)*\ln(1-\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}+1/2*(A-B)*\cos(f*x+e)*\ln(1+\sin(f*x+e))/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.36, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2969, 2737, 2667, 31}

$$\frac{(A-B) \cos(e+fx) \log(\sin(e+fx)+1)}{2f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(A+B) \cos(e+fx) \log(1-\sin(e+fx))}{2f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] $-((A+B)*\text{Cos}[e+f*x]*\text{Log}[1-\text{Sin}[e+f*x]])/(2*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) + ((A-B)*\text{Cos}[e+f*x]*\text{Log}[1+\text{Sin}[e+f*x]])/(2*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^{(p_)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^{(m + (p - 1)/2)}*(a - x)^{((p - 1)/2)}, x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a² - b², 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])}

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a² - b², 0]

Rule 2969

Int[((A_) + (B_.)*sin[(e_) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]]), x_Symbol] :> Dist[(A*b + a*B)/(2*a*b), Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[(B*c + A*d)/(2*c*d), Int[Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[b*c + a*d, 0] && EqQ[a² - b², 0]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx &= \frac{(A + B) \int \frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c - c \sin(e + fx)}} dx}{2a} + \frac{(A - B) \int \frac{\sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx}{2c} \\
&= \frac{(a(A - B) \cos(e + fx)) \int \frac{\cos(e + fx)}{a + a \sin(e + fx)} dx}{2\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{((A + B)c \cos(e + fx))}{2\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} \\
&= \frac{((A - B) \cos(e + fx)) \text{Subst}\left(\int \frac{1}{a+x} dx, x, a \sin(e + fx)\right)}{2f\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} - \frac{((A + B)c \cos(e + fx))}{2f\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} \\
&= -\frac{(A + B) \cos(e + fx) \log(1 - \sin(e + fx))}{2f\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{(A - B) \cos(e + fx)}{2f\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 97, normalized size = 0.86

$$\frac{\cos(e + fx) \left((A + B) \log \left(\cos \left(\frac{1}{2}(e + fx) \right) - \sin \left(\frac{1}{2}(e + fx) \right) \right) + (B - A) \log \left(\sin \left(\frac{1}{2}(e + fx) \right) + \cos \left(\frac{1}{2}(e + fx) \right) \right) \right)}{f\sqrt{a(\sin(e + fx) + 1)} \sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]), x]

[Out] -((Cos[e + f*x]*((A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + (-A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]))/(f*Sqrt[a*(1 + Sin[e + f*x]]]*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{ac \cos(fx + e)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)

maple [A] time = 0.59, size = 165, normalized size = 1.46

$$\frac{\left(A \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - A \ln \left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) + B \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - B \ln \left(\frac{2}{\cos(fx + e) + 1} \right) \right)}{f\sqrt{a(1 + \sin(fx + e))} \sqrt{-c(\sin(fx + e) - 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x)`

[Out] `-1/f*(A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(1/2)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)),x)`

[Out] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a(\sin(e + fx) + 1)} \sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(1/2), x)`

[Out] `Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*sqrt(-c*(sin(e + f*x) - 1))), x)`

$$3.178 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=103

$$\frac{(A+B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

[Out] 1/2*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/2*(A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.25, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2741, 3770}

$$\frac{(A+B) \cos(e+fx)}{2f\sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{2cf\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)), x]

[Out] ((A + B)*Cos[e + f*x])/(2*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*c*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])

Rule 2741

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx = \frac{(A + B) \cos(e + fx)}{2f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{2f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}}$$

$$= \frac{(A + B) \cos(e + fx)}{2f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} + \frac{((A - B) \cos(e + fx))}{2c \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}}$$

$$= \frac{(A + B) \cos(e + fx)}{2f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} + \frac{(A - B) \tanh^{-1} \left(\frac{\cos(e + fx)}{c} \right)}{2cf \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}}$$

Mathematica [A] time = 0.54, size = 191, normalized size = 1.85

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((B - A) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}{2f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)),x]

[Out] ((A + B + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(2*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(3/2))

fricas [A] time = 0.52, size = 337, normalized size = 3.27

$$\frac{\left((A - B) \cos(fx + e) \sin(fx + e) - (A - B) \cos(fx + e) \right) \sqrt{ac} \log \left(-\frac{ac \cos(fx + e)^3 - 2ac \cos(fx + e) + 2\sqrt{ac} \sqrt{a \sin(fx + e)}}{\cos(fx + e)^3} \right)}{4(ac^2 f \cos(fx + e) \sin(fx + e) - ac^2 f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/4*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e))^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) + 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e)), -1/2*(((A - B)*cos(f*x + e)*sin(f*x + e) - (A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A + B))/(a*c^2*f*cos(f*x + e)*sin(f*x + e) - a*c^2*f*cos(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2)), x)

maple [B] time = 0.60, size = 302, normalized size = 2.93

$$\frac{\left(A \sin(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - A \sin(fx + e) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - B \sin(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) \right)}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x)

[Out] 1/2/f*(A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)
*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+s
in(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)-A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln((1-cos(f*x+e)+sin(f*x+e
))/sin(f*x+e))+A*sin(f*x+e)+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*
ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e))*cos(f*x+e)/(a*(1+sin
(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (-c \sin(fx + e) + c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) +
c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(
3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a (\sin(e + fx) + 1)} (-c (\sin(e + fx) - 1))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2),
x)

[Out] Integral((A + B*sin(e + f*x))/(sqrt(a*(sin(e + f*x) + 1))*(-c*(sin(e + f*x)
- 1))^(3/2)), x)

$$3.179 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=153

$$\frac{(A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{4c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(A-B) \cos(e+fx)}{4cf \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(A+B)}{4f \sqrt{a \sin(e+fx)+a}}$$

[Out] 1/4*(A+B)*cos(f*x+e)/f/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B)*cos(f*x+e)/c/f/(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(1/2)+1/4*(A-B)*arctanh(sin(f*x+e))*cos(f*x+e)/c^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.36, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{4c^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(A-B) \cos(e+fx)}{4cf \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(A+B)}{4f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)), x]

[Out] ((A + B)*Cos[e + f*x])/(4*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)) + ((A - B)*Cos[e + f*x])/(4*c*f*Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(3/2)) + ((A - B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(4*c^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])

Rule 2741

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} dx = \frac{(A + B) \cos(e + fx)}{4f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{4cf \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)}{4f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{4cf \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)}{4f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{4cf \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}}$$

$$= \frac{(A + B) \cos(e + fx)}{4f \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{4cf \sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}}$$

Mathematica [A] time = 0.63, size = 222, normalized size = 1.45

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left((A - B)\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c - c*Sin[e + f*x])^(5/2)),x]
```

```
[Out] ((A + B + (A - B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (-A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + (A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(4*f*Sqrt[a*(1 + Sin[e + f*x])]*(c - c*Sin[e + f*x])^(5/2))
```

fricas [A] time = 0.54, size = 424, normalized size = 2.77

$$\frac{\left((A - B) \cos(fx + e)^3 + 2(A - B) \cos(fx + e) \sin(fx + e) - 2(A - B) \cos(fx + e)\right) \sqrt{ac} \log\left(-\frac{ac \cos(fx + e)^3 - 2ac \cos(fx + e) \sin(fx + e) + ac}{8(ac^3 f \cos(fx + e)^3 + 2ac^3 f \sin(fx + e))}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c^3*f*cos(f*x + e)^3 + 2*a*c^3*f*cos(f*x + e)*sin(f*x + e) - 2*a*c^3*f*cos(f*x + e)), -1/4*(((A - B)*cos(f*x + e)^3 + 2*(A - B)*cos(f*x + e)*sin(f*x + e) - 2*(A - B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - ((A - B)*sin(f*x + e) - 2*A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))]
```

$e) + a) \cdot \sqrt{-c \cdot \sin(f \cdot x + e) + c}) / (a \cdot c^3 \cdot f \cdot \cos(f \cdot x + e)^3 + 2 \cdot a \cdot c^3 \cdot f \cdot \cos(f \cdot x + e) \cdot \sin(f \cdot x + e) - 2 \cdot a \cdot c^3 \cdot f \cdot \cos(f \cdot x + e))]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a(-c \sin(fx + e) + c)}^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)

maple [B] time = 0.62, size = 465, normalized size = 3.04

$$\frac{\left(A \left(\cos^2(fx + e) \right) \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - A \left(\cos^2(fx + e) \right) \ln \left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - B \left(\cos^2(fx + e) \right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2), x)

[Out] 1/4/f*(A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)^2+2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*sin(f*x+e)-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)+2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)/(a*(1+sin(f*x+e)))^(1/2)/(-c*(sin(f*x+e)-1))^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a(-c \sin(fx + e) + c)}^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(1/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(-c*sin(f*x + e) + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(1/2), x)
```

```
[Out] Timed out
```

$$3.180 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=271

$$\frac{4c^4(3A-5B) \cos(e+fx) \log(\sin(e+fx)+1)}{af\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{2c^3(3A-5B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af\sqrt{a \sin(e+fx)+a}} - \frac{c^2(3A-5B) \cos(e+fx)}{af\sqrt{a \sin(e+fx)+a}} - \frac{4c^4(3A-5B) \cos(e+fx)}{af\sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(7/2)}/f/(a+a*\sin(f*x+e))^{(3/2)}-1/2*(3*A-5*B)*c^2*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}-1/6*(3*A-5*B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}-4*(3*A-5*B)*c^4*\cos(f*x+e)*\ln(1+\sin(f*x+e))/a/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-2*(3*A-5*B)*c^3*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.58, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2c^3(3A-5B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af\sqrt{a \sin(e+fx)+a}} - \frac{c^2(3A-5B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{2af\sqrt{a \sin(e+fx)+a}} - \frac{4c^4(3A-5B) \cos(e+fx)}{af\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $(-4*(3*A-5*B)*c^4*\cos[e+f*x]*\log[1+\sin[e+f*x]]/(a*f*\sqrt{a+a*\sin[e+f*x]}*\sqrt{c-c*\sin[e+f*x]}) - (2*(3*A-5*B)*c^3*\cos[e+f*x]*\sqrt{c-c*\sin[e+f*x]})/(a*f*\sqrt{a+a*\sin[e+f*x]}) - ((3*A-5*B)*c^2*\cos[e+f*x]*(c-c*\sin[e+f*x])^{(3/2)})/(2*a*f*\sqrt{a+a*\sin[e+f*x]}) - ((3*A-5*B)*c*\cos[e+f*x]*(c-c*\sin[e+f*x])^{(5/2)})/(6*a*f*\sqrt{a+a*\sin[e+f*x]}) - ((A-B)*\cos[e+f*x]*(c-c*\sin[e+f*x])^{(7/2)})/(2*f*(a+a*\sin[e+f*x])^{(3/2)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])]

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)

, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{3/2}} dx = \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{2f(a + a \sin(e + fx))^{3/2}} - \frac{(3A - 5B) \int \frac{(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx}{2a}$$

$$= -\frac{(3A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{6af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= -\frac{(3A - 5B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2af\sqrt{a + a \sin(e + fx)}} - \frac{(3A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{1/2}}{6af}$$

$$= -\frac{2(3A - 5B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(3A - 5B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{1/2}}{2af}$$

$$= -\frac{2(3A - 5B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(3A - 5B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{1/2}}{2af}$$

$$= -\frac{2(3A - 5B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(3A - 5B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{1/2}}{2af}$$

$$= -\frac{4(3A - 5B)c^4 \cos(e + fx) \log(1 + \sin(e + fx))}{af\sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} - \frac{2(3A - 5B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 3.48, size = 271, normalized size = 1.00

$$c^3 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(2(27A - 59B) \cos(2(e + fx)) - 117A \sin(e + fx) - 3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] -1/24*(c^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]])*(132*A - 45*B + 2*(27*A - 59*B)*Cos[2*(e + f*x)] + B*Cos[4*(e + f*x)] + 576*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 960*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 117*A*Sin[e + f*x] + 279*B*Sin[e + f*x] + 576*A*Log[Cos[

```
(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 960*B*Log[Cos[(e + f*x)/2]
+ Sin[(e + f*x)/2]]*Sin[e + f*x] - 3*A*Sin[3*(e + f*x)] + 13*B*Sin[3*(e + f
*x]])))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2
))
```

fricas [F] time = 14.90, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Bc^3 \cos(fx + e)^4 + (3A - 5B)c^3 \cos(fx + e)^2 - 4(A - B)c^3 - \left((A - 3B)c^3 \cos(fx + e)^2 - 4(A - B)c^3 \right) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)
*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3)*sin(f*x + e))*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.66, size = 939, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] 1/6/f*(13*B*cos(f*x+e)^3*sin(f*x+e)-288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(
f*x+e))+480*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+102*A*sin(f*x+e)-75*
A*sin(f*x+e)*cos(f*x+e)-3*A*cos(f*x+e)^3*sin(f*x+e)-2*B*sin(f*x+e)*cos(f*x+
e)^4+120*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)^2*sin(f*x+e)+4
8*B*cos(f*x+e)^2*sin(f*x+e)+11*B*cos(f*x+e)^4+102*A-166*B+27*A*cos(f*x+e)^3
-72*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+144*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))-72*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+107*B*sin(f*x+e)*cos(f*x+e)-240*B*s
in(f*x+e)*ln(2/(cos(f*x+e)+1))+120*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+144*A*
cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*cos(f*x+e)^2*ln
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-72*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos
(f*x+e)+1))+2*B*cos(f*x+e)^5+120*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+
e)+144*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-240*B*cos(f*x+
e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-288*A*sin(f*x+e)*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))+480*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))+155*B*cos(f*x+e)^2-3*A*cos(f*x+e)^4-61*B*cos(f*x+e)^3-166*B*sin(f
*x+e)+144*A*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-
240*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-27*A*c
os(f*x+e)+59*B*cos(f*x+e)-99*A*cos(f*x+e)^2+144*A*ln(2/(cos(f*x+e)+1))-240*
B*ln(2/(cos(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-
cos(f*x+e)^4-4*cos(f*x+e)^2*sin(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+
e)+8*cos(f*x+e)^2+8*sin(f*x+e)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{7}{2}}}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{\frac{7}{2}}}{(a + a \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(
3/2),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(
3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(3/2),
x)

[Out] Timed out

$$3.181 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=210

$$\frac{4c^3(A-2B) \cos(e+fx) \log(\sin(e+fx)+1)}{af\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{2c^2(A-2B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af\sqrt{a \sin(e+fx)+a}} - \frac{c(A-2B) \cos(e+fx)}{2af\sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(5/2)}/f/(a+a*\sin(f*x+e))^{(3/2)}-1/2*(A-2*B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}-4*(A-2*B)*c^3*\cos(f*x+e)*\ln(1+\sin(f*x+e))/a/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-2*(A-2*B)*c^2*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{2c^2(A-2B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af\sqrt{a \sin(e+fx)+a}} - \frac{4c^3(A-2B) \cos(e+fx) \log(\sin(e+fx)+1)}{af\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{c(A-2B) \cos(e+fx)}{2af\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $(-4*(A-2*B)*c^3*\text{Cos}[e+f*x]*\text{Log}[1+\text{Sin}[e+f*x]])/(a*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (2*(A-2*B)*c^2*\text{Cos}[e+f*x]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])/(a*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]) - ((A-2*B)*c*\text{Cos}[e+f*x]*(c-c*\text{Sin}[e+f*x])^{(3/2)})/(2*a*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]) - ((A-B)*\text{Cos}[e+f*x]*(c-c*\text{Sin}[e+f*x])^{(5/2)})/(2*f*(a+a*\text{Sin}[e+f*x])^{(3/2)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n]/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILT

Q[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{3/2}} dx = \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2f(a + a \sin(e + fx))^{3/2}} - \frac{(A - 2B) \int \frac{(c - c \sin(e + fx))^{5/2}}{\sqrt{a + a \sin(e + fx)}} dx}{a}$$

$$= -\frac{(A - 2B)c \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= -\frac{2(A - 2B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(A - 2B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2af\sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{2(A - 2B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(A - 2B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2af\sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{2(A - 2B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af\sqrt{a + a \sin(e + fx)}} - \frac{(A - 2B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2af\sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{4(A - 2B)c^3 \cos(e + fx) \log(1 + \sin(e + fx))}{af\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} - \frac{2(A - 2B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af}$$

Mathematica [A] time = 1.60, size = 212, normalized size = 1.01

$$c^2 \sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(2(2A - 7B) \cos(2(e + fx)) + \sin(e + fx) \left(64(A - 2B) \cos^2(e + fx) - 16B \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e +
f*x])^(3/2), x]
```

```
[Out] -1/8*(c^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(2
8*A - 16*B + 2*(2*A - 7*B)*Cos[2*(e + f*x)] + 64*A*Log[Cos[(e + f*x)/2] + S
in[(e + f*x)/2]] - 128*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + (-8*A +
31*B + 64*(A - 2*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x]
+ B*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Si
n[e + f*x]))^(3/2))
```

fricas [F] time = 8.94, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A - 2B)c^2 \cos^2(fx + e) - 2(A - B)c^2 + (Bc^2 \cos^2(fx + e) + 2(A - B)c^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{a^2 \cos^2(fx + e) - 2a^2 \sin(fx + e) - 2a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e)
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 853, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f*(B*cos(f*x+e)^3*sin(f*x+e)-32*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+64*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+12*A*sin(f*x+e)-10*A*sin
(f*x+e)*cos(f*x+e)+16*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*
sin(f*x+e)+6*B*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^4+12*A-22*B+2*A*cos(f*x
+e)^3-8*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+16*A*sin(f*x+e)*ln(2/(cos(f*x+e
)+1))-8*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+15*B*sin(f*x+e)*cos(f*x+e)-32*B*s
in(f*x+e)*ln(2/(cos(f*x+e)+1))+16*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+16*A*co
s(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*cos(f*x+e)^2*ln((1
-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x
+e)+1))+16*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+16*A*cos(f*x+e)*ln(
(1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-32*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(
f*x+e))/sin(f*x+e))-32*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+64*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+21*B*cos(f*x+e)^
2-7*B*cos(f*x+e)^3-22*B*sin(f*x+e)+16*A*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x
+e)+sin(f*x+e))/sin(f*x+e))-32*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin
(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)+7*B*cos(f*x+e)-12*A*cos(f*x+e)^2+16*A*ln
(2/(cos(f*x+e)+1))-32*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(5/2)/(c
os(f*x+e)^2*sin(f*x+e)+cos(f*x+e)^3+2*sin(f*x+e)*cos(f*x+e)-3*cos(f*x+e)^2-
4*sin(f*x+e)-2*cos(f*x+e)+4)/(a*(1+sin(f*x+e)))^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{5}{2}}}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)/(a*sin(f*x + e)
+ a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (c - c \sin(e + f x))^{5/2}}{(a + a \sin(e + f x))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2)/(a+a*sin(f*x+e))**(3/2), x)

[Out] Timed out

$$3.182 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=159

$$\frac{c^2(A-3B) \cos(e+fx) \log(\sin(e+fx)+1)}{af\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{c(A-3B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{2af\sqrt{a \sin(e+fx)+a}} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/f/(a+a*\sin(f*x+e))^{(3/2)}-(A-3*B)*c^2*\cos(f*x+e)*\ln(1+\sin(f*x+e))/a/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-1/2*(A-3*B)*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.39, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2740, 2737, 2667, 31}

$$\frac{c^2(A-3B) \cos(e+fx) \log(\sin(e+fx)+1)}{af\sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{c(A-3B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{2af\sqrt{a \sin(e+fx)+a}} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $-(((A-3*B)*c^2*\cos[e+f*x]*\log[1+\sin[e+f*x]])/(a*f*\sqrt{a+a*\sin[e+f*x]}*\sqrt{c-c*\sin[e+f*x]}) - ((A-3*B)*c*\cos[e+f*x]*\sqrt{c-c*\sin[e+f*x]})/(2*a*f*\sqrt{a+a*\sin[e+f*x]}) - ((A-B)*\cos[e+f*x]*(c-c*\sin[e+f*x])^{(3/2)})/(2*f*(a+a*\sin[e+f*x])^{(3/2)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2740

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n]/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2}} dx = -\frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{2f(a + a \sin(e + fx))^{3/2}} - \frac{(A - 3B) \int \frac{(c - c \sin(e + fx))^{3/2}}{\sqrt{a + a \sin(e + fx)}} dx}{2a}$$

$$= -\frac{(A - 3B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))}$$

$$= -\frac{(A - 3B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))}$$

$$= -\frac{(A - 3B)c \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))}$$

$$= -\frac{(A - 3B)c^2 \cos(e + fx) \log(1 + \sin(e + fx))}{af\sqrt{a + a \sin(e + fx)}\sqrt{c - c \sin(e + fx)}} - \frac{(A - 3B)c \cos(e + fx)}{2af\sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.86, size = 190, normalized size = 1.19

$$\frac{c\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(2 \sin(e + fx) \left(2(A - 3B) \log\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \right) \right)}{2f(a(\sin(e + fx) + \cos(e + fx)))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(3/2), x]
```

```
[Out] -1/2*(c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(4*A - 3*B - B*Cos[2*(e + f*x)] + 4*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - 12*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 2*(B + 2*(A - 3*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left(Bc \cos(fx + e)^2 - (A - B)c \sin(fx + e) + (A - B)c \right) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2), x, algorithm="fricas")
```

[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*
sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin
(f*x + e) - 2*a^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2), x,
algorithm="giac")

[Out] Timed out

maple [B] time = 0.64, size = 759, normalized size = 4.77

$$\frac{\left(4A \ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 12B \ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - 2A \sin(fx+e) + 2A \sin(fx+e) \cos(fx+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2), x)

[Out] -1/f*(4*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)+2*A*sin(f*x+e)*cos(f*x+e)-3*B*cos(f*x+
e)^2*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^2*sin(f*x+e)-2*A+4*B+A*cos(f*x+e)^2*
ln(2/(cos(f*x+e)+1))-2*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+A*cos(f*x+e)*ln(2/
(cos(f*x+e)+1))-3*B*sin(f*x+e)*cos(f*x+e)+6*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1
))-3*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))+6*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x
+e))+A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-3*B*ln(2/(cos(f*x+e)+1))*
sin(f*x+e)*cos(f*x+e)-2*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e
))+6*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+4*A*sin(f*x+e)*l
n((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-12*B*sin(f*x+e)*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))-4*B*cos(f*x+e)^2+B*cos(f*x+e)^3+4*B*sin(f*x+e)-2*A*si
n(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+6*B*sin(f*x+e)
*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)+2*A*cos(f
*x+e)^2-2*A*ln(2/(cos(f*x+e)+1))+6*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-
1))^(3/2)/(cos(f*x+e)^2-sin(f*x+e)*cos(f*x+e)+cos(f*x+e)+2*sin(f*x+e)-2)/(a
*(1+sin(f*x+e)))^(3/2)

maxima [B] time = 0.55, size = 367, normalized size = 2.31

$$\frac{B \left(\frac{6c^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{a^{\frac{3}{2}}} - \frac{3c^{\frac{3}{2}} \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}{a^{\frac{3}{2}}} - \frac{2 \left(\frac{3c^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{2c^{\frac{3}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3c^{\frac{3}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right)}{a^{\frac{3}{2}} + \frac{2a^{\frac{3}{2}} \sin(fx+e)}{\cos(fx+e)+1} + \frac{2a^{\frac{3}{2}} \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2a^{\frac{3}{2}} \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a^{\frac{3}{2}} \sin(fx+e)^4}{(\cos(fx+e)+1)^4}} \right) - A \left(\frac{3}{2c^{\frac{3}{2}}} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(3/2), x,
algorithm="maxima")

[Out] -(B*(6*c^(3/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(3/2) - 3*c^(3/2)
*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(3/2) - 2*(3*c^(3/2)*sin(f*
x + e)/(cos(f*x + e) + 1) + 2*c^(3/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 +
3*c^(3/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3)/(a^(3/2) + 2*a^(3/2)*sin(f*

$x + e)/(\cos(f*x + e) + 1) + 2*a^{(3/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 +$
 $2*a^{(3/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + a^{(3/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4) -$
 $A*(2*c^{(3/2)}*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1)/a^{(3/2)} - c^{(3/2)}*\log(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/a^{(3/2)} - 4$
 $*\sqrt{a}*c^{(3/2)}*\sin(f*x + e)/((a^2 + 2*a^2*\sin(f*x + e)/(\cos(f*x + e) + 1) + a^2*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2)*(cos(f*x + e) + 1)))/f$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c(\sin(e + fx) - 1))^{3/2} (A + B \sin(e + fx))}{(a(\sin(e + fx) + 1))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(3/2), x)

[Out] Integral((-c*(sin(e + f*x) - 1))**(3/2)*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)

$$3.183 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=100

$$\frac{Bc \cos(e+fx) \log(\sin(e+fx)+1)}{af \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{c(A-B) \cos(e+fx)}{f(a \sin(e+fx)+a)^{3/2} \sqrt{c-c \sin(e+fx)}}$$

[Out] $-(A-B)*c*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(1/2)}+B*c*\cos(f*x+e)*\ln(1+\sin(f*x+e))/a/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2971, 2737, 2667, 31, 2738}

$$\frac{Bc \cos(e+fx) \log(\sin(e+fx)+1)}{af \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{c(A-B) \cos(e+fx)}{f(a \sin(e+fx)+a)^{3/2} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sin}[e+f*x])*Sqrt[c-c*\text{Sin}[e+f*x]]/(a+a*\text{Sin}[e+f*x])^{(3/2)},x]$

[Out] $-(((A-B)*c*\text{Cos}[e+f*x])/(f*(a+a*\text{Sin}[e+f*x])^{(3/2)}*Sqrt[c-c*\text{Sin}[e+f*x]]))+(B*c*\text{Cos}[e+f*x]*\text{Log}[1+\text{Sin}[e+f*x]])/(a*f*Sqrt[a+a*\text{Sin}[e+f*x]]*Sqrt[c-c*\text{Sin}[e+f*x]])$

Rule 31

$\text{Int}[(a_+ + (b_+)*(x_+))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 2667

$\text{Int}[\cos[(e_+) + (f_+)*(x_+)]^{(p_+)}*((a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}, x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a+x)^{(m+(p-1)/2)}*(a-x)^{((p-1)/2)}, x], x, b*\text{Sin}[e+f*x], x] /; \text{FreeQ}[\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] \mid \mid \text{!IntegerQ}[m + 1/2])]$

Rule 2737

$\text{Int}[Sqrt[(a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)]]/Sqrt[(c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)]], x_Symbol] \rightarrow \text{Dist}[(a*c*\text{Cos}[e+f*x])/(Sqrt[a+b*\text{Sin}[e+f*x]]*Sqrt[c+d*\text{Sin}[e+f*x]]), \text{Int}[\text{Cos}[e+f*x]/(c+d*\text{Sin}[e+f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2738

$\text{Int}[Sqrt[(a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)]]*((c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)])^{(n_+)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^n)/(f*(2*n+1)*Sqrt[a+b*\text{Sin}[e+f*x]]), x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[n, -2^{(-1)}]$

Rule 2971

$\text{Int}[Sqrt[(a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)]]*((A_+) + (B_+)*\sin[(e_+) + (f_+)*(x_+)])^{(n_+)}, x_Symbol] \rightarrow \text{Dist}[B/d, \text{Int}[Sqrt[a+b*\text{Sin}[e+f*x]]*(c+d*\text{Sin}[e+f*x])^{(n+1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[Sqrt[a+b*\text{Sin}[e+f*x]]*(c+d*\text{Sin}[e+f*x])^n, x], x]$

x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{3/2}} dx &= \frac{B \int \frac{\sqrt{c - c \sin(e + fx)}}{\sqrt{a + a \sin(e + fx)}} dx}{a} - (-A + B) \int \frac{\sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{3/2}} dx \\ &= -\frac{(A - B)c \cos(e + fx)}{f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{(Bc \cos(e + fx))}{\sqrt{a + a \sin(e + fx)}} \\ &= -\frac{(A - B)c \cos(e + fx)}{f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{(Bc \cos(e + fx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a + a \sin(x)}} dx\right)}{af \sqrt{a + a \sin(e + fx)}} \\ &= -\frac{(A - B)c \cos(e + fx)}{f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{Bc \cos(e + fx) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a + a \sin(x)}} dx\right)}{af \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [C] time = 1.18, size = 143, normalized size = 1.43

$$\frac{\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(-A + 2B \log\left(e^{i(e + fx)} + i\right) + B \left(2 \log\left(e^{i(e + fx)} + i\right) - ifx \right) \right)}{f(a(\sin(e + fx) + 1))^{3/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a + a*Sin[e + f*x])^(3/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(-A + B - I*B*f*x + 2*B*Log[I + E^(I*(e + f*x))] + B*((-I)*f*x + 2*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(3/2))

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.65, size = 408, normalized size = 4.08

$$\left(2B \left(\cos^2(fx + e)\right) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - B \left(\cos^2(fx + e)\right) \ln\left(\frac{2}{\cos(fx + e) + 1}\right) + 2B \sin(fx + e) \cos(fx + e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2), x)

[Out] -1/f*(2*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+A*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)-B*cos(f*x+e)^2-B*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-4*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-A*sin(f*x+e)+B*sin(f*x+e)-4*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*ln(2/(cos(f*x+e)+1))-A+B)*(-c*(sin(f*x+e)-1))^(1/2)/(-1+cos(f*x+e)+sin(f*x+e))/(a*(1+sin(f*x+e)))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{-c \sin(fx + e) + c}}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(3/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e + fx) - 1)} (A + B \sin(e + fx))}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(3/2), x)

[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)

$$3.184 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2} \sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=103

$$\frac{(A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{2af\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}\sqrt{c-c \sin(e+fx)}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(1/2)}+1/2*(A+B)*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2741, 3770}

$$\frac{(A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{2af\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]), x]`

[Out] `-((A - B)*Cos[e + f*x])/((2*f*(a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]) + ((A + B)*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/((2*a*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]])`

Rule 2741

`Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] := Dist[Cos[e + f*x]/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[1/Cos[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]`

Rule 2972

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]`

Rule 3770

`Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} dx = -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{(A + B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{2a \sqrt{a + a \sin(e + fx)}} \\ = -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{((A + B) \cos(e + fx))}{2a \sqrt{a + a \sin(e + fx)}} \\ = -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{(A + B) \tan\left(\frac{1}{2}(e + fx)\right)}{2af \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 0.56, size = 186, normalized size = 1.81

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(-\left((A + B)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{2f(a + a \sin(e + fx))^{3/2} \sqrt{c - c \sin(e + fx)}} + \frac{(A + B) \tan\left(\frac{1}{2}(e + fx)\right)}{2af \sqrt{a + a \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*Sqrt[c - c*Sin[e + f*x]]),x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-A + B - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(2*f*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.52, size = 329, normalized size = 3.19

$$\frac{\left((A + B) \cos(fx + e) \sin(fx + e) + (A + B) \cos(fx + e)\right) \sqrt{ac} \log\left(-\frac{ac \cos(fx+e)^3 - 2ac \cos(fx+e) - 2\sqrt{ac} \sqrt{a \sin(fx+e)}}{\cos(fx+e)^3}\right)}{4(a^2cf \cos(fx + e) \sin(fx + e) + a^2cf c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/4*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3) - 2*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e)), -1/2*(((A + B)*cos(f*x + e)*sin(f*x + e) + (A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*(A - B))/(a^2*c*f*cos(f*x + e)*sin(f*x + e) + a^2*c*f*cos(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)

maple [B] time = 0.60, size = 303, normalized size = 2.94

$$\frac{\left(A \sin(fx + e) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - A \sin(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) + B \sin(fx + e) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x)

[Out] 1/2/f*(A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)*ln((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)*ln((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*ln((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*sin(f*x+e)+B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*ln((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e))*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(1/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*sqrt(-c*sin(f*x
+ e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{3}{2}} \sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}} \sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(1/2),
x)

[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*sqrt(-c*(sin(e
+ f*x) - 1))), x)

$$3.185 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=150

$$\frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}(c-c \sin(e+fx))^{3/2}} + \frac{A \cos(e+fx)}{2af \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{A \cos(e+fx)}{2acf \sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{3/2}/(c-c*\sin(f*x+e))^{3/2}+1/2*A*\cos(f*x+e)/a/f/(c-c*\sin(f*x+e))^{3/2}/(a+a*\sin(f*x+e))^{1/2}+1/2*A*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a/c/f/(a+a*\sin(f*x+e))^{1/2}/(c-c*\sin(f*x+e))^{1/2}$

Rubi [A] time = 0.37, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}(c-c \sin(e+fx))^{3/2}} + \frac{A \cos(e+fx)}{2af \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{A \cos(e+fx)}{2acf \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A+B*\operatorname{Sin}[e+f*x])/((a+a*\operatorname{Sin}[e+f*x])^{3/2}*(c-c*\operatorname{Sin}[e+f*x])^{3/2}),x]$

[Out] $-((A-B)*\operatorname{Cos}[e+f*x])/(2*f*(a+a*\operatorname{Sin}[e+f*x])^{3/2}*(c-c*\operatorname{Sin}[e+f*x])^{3/2})+(A*\operatorname{Cos}[e+f*x])/(2*a*f*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*(c-c*\operatorname{Sin}[e+f*x])^{3/2})+(A*\operatorname{ArcTanh}[\operatorname{Sin}[e+f*x]]*\operatorname{Cos}[e+f*x])/(2*a*c*f*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c-c*\operatorname{Sin}[e+f*x]])$

Rule 2741

$\operatorname{Int}[1/(\operatorname{Sqrt}[(a_)+(b_)*\sin[(e_)+(f_)*(x_)])* \operatorname{Sqrt}[(c_)+(d_)*\sin[(e_)+(f_)*(x_)]]), x_Symbol] \rightarrow \operatorname{Dist}[\operatorname{Cos}[e+f*x]/(\operatorname{Sqrt}[a+b*\operatorname{Sin}[e+f*x]]*\operatorname{Sqrt}[c+d*\operatorname{Sin}[e+f*x]]), \operatorname{Int}[1/\operatorname{Cos}[e+f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c+a*d, 0] && EqQ[a^2-b^2, 0]

Rule 2743

$\operatorname{Int}(((a_)+(b_)*\sin[(e_)+(f_)*(x_)])^{(m_)}*((c_)+(d_)*\sin[(e_)+(f_)*(x_)])^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(b*\operatorname{Cos}[e+f*x]*(a+b*\operatorname{Sin}[e+f*x])^{m_}*(c+d*\operatorname{Sin}[e+f*x])^{n_})/(a*f*(2*m+1)), x] + \operatorname{Dist}[(m+n+1)/(a*(2*m+1)), \operatorname{Int}[(a+b*\operatorname{Sin}[e+f*x])^{(m+1)}*(c+d*\operatorname{Sin}[e+f*x])^{n_}], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c+a*d, 0] && EqQ[a^2-b^2, 0] && ILtQ[Simplify[m+n+1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 2972

$\operatorname{Int}(((a_)+(b_)*\sin[(e_)+(f_)*(x_)])^{(m_)}*((A_)+(B_)*\sin[(e_)+(f_)*(x_)])^{(n_)}), x_Symbol] \rightarrow \operatorname{Simp}[(A*b-a*B)*\operatorname{Cos}[e+f*x]*(a+b*\operatorname{Sin}[e+f*x])^{m_}*(c+d*\operatorname{Sin}[e+f*x])^{n_})/(a*f*(2*m+1)), x] + \operatorname{Dist}[(a*B*(m-n)+A*b*(m+n+1))/(a*b*(2*m+1)), \operatorname{Int}[(a+b*\operatorname{Sin}[e+f*x])^{(m+1)}*(c+d*\operatorname{Sin}[e+f*x])^{n_}], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c+a*d, 0] && EqQ[a^2-b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m+n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m+1, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{3/2}} dx = -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{3/2}} + \frac{A \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx}{2af\sqrt{a+a \sin(e+fx)}} + \dots$$

Mathematica [A] time = 0.67, size = 178, normalized size = 1.19

$$\frac{\cos(e + fx) \left(2A \sin(e + fx) - A \log \left(\cos \left(\frac{1}{2}(e + fx) \right) - \sin \left(\frac{1}{2}(e + fx) \right) \right) + A \log \left(\sin \left(\frac{1}{2}(e + fx) \right) + \cos \left(\frac{1}{2}(e + fx) \right) \right) \right)}{4cf(\sin(e + fx) - 1)(a(\sin(e + fx) - 1))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(3/2)), x]
```

```
[Out] -1/4*(Cos[e + f*x]*(2*B - A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + A*Cos[2*(e + f*x)]*(-Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] + Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])) + 2*A*Sin[e + f*x])/((c*f*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^(3/2)*Sqrt[c - c*Sin[e + f*x]])
```

fricas [A] time = 0.51, size = 272, normalized size = 1.81

$$\frac{\sqrt{ac} A \cos(fx + e)^3 \log \left(\frac{ac \cos(fx+e)^3 - 2ac \cos(fx+e) - 2\sqrt{ac} \sqrt{a \sin(fx+e)+a} \sqrt{-c \sin(fx+e)+c} \sin(fx+e)}{\cos(fx+e)^3} \right) + 2(A \sin(fx + e) + B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{4a^2c^2f \cos(fx + e)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*c)*A*cos(f*x + e)^3*log(-(a*c*cos(f*x + e))^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3 + 2*(A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3), -1/2*(sqrt(-a*c)*A*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e)))*cos(f*x + e)^3 - (A*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^2*c^2*f*cos(f*x + e)^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}}(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

maple [A] time = 0.56, size = 130, normalized size = 0.87

$$\frac{\left(A \left(\cos^2(fx + e) \right) \ln \left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - A \left(\cos^2(fx + e) \right) \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - B \left(\cos^2(fx + e) \right) \right)}{2f \left(a \left(1 + \sin(fx + e) \right) \right)^{\frac{3}{2}} \left(-c \left(\sin(fx + e) - 1 \right) \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2), x)

[Out] 1/2/f*(A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2+A*sin(f*x+e)+B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(3/2)/(-c*(sin(f*x+e)-1))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\left(a \sin(fx + e) + a \right)^{\frac{3}{2}} \left(-c \sin(fx + e) + c \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{\left(a + a \sin(e + fx) \right)^{\frac{3}{2}} \left(c - c \sin(e + fx) \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)), x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{\left(a \left(\sin(e + fx) + 1 \right) \right)^{\frac{3}{2}} \left(-c \left(\sin(e + fx) - 1 \right) \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(3/2), x)

[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(3/2)*(-c*(sin(e + f*x) - 1))**(3/2)), x)

$$3.186 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=217

$$\frac{(3A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8ac^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(3A-B) \cos(e+fx)}{8acf \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(3A-B)}{8af \sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(5/2)}+1/8*(3*A-B)*\cos(f*x+e)/a/f/(c-c*\sin(f*x+e))^{(5/2)}/(a+a*\sin(f*x+e))^{(1/2)}+1/8*(3*A-B)*\cos(f*x+e)/a/c/f/(c-c*\sin(f*x+e))^{(3/2)}/(a+a*\sin(f*x+e))^{(1/2)}+1/8*(3*A-B)*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a/c^2/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A-B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8ac^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{(3A-B) \cos(e+fx)}{8acf \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(3A-B)}{8af \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)})], x]$

[Out] $-((A - B)*\text{Cos}[e + f*x])/((2*f*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)}) + ((3*A - B)*\text{Cos}[e + f*x])/((8*a*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)}) + ((3*A - B)*\text{Cos}[e + f*x])/((8*a*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(3/2)}) + ((3*A - B)*\text{ArcTanh}[\text{Sin}[e + f*x]]*\text{Cos}[e + f*x])/((8*a*c^2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]))$

Rule 2741

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)])], x_Symbol] :> \text{Dist}[\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[1/\text{Cos}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2743

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m_}*(c + d*\text{Sin}[e + f*x])^{n_})/(a*f*(2*m + 1)), x] + \text{Dist}[(m + n + 1)/(a*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*(c + d*\text{Sin}[e + f*x])^{n_}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}(((A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m_}*(c + d*\text{Sin}[e + f*x])^{n_})/(a*f*(2*m + 1)), x] + \text{Dist}[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m+1}*(c + d*\text{Sin}[e + f*x])^{n_}, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} dx = -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} + \frac{(3A - B)}{8af\sqrt{a + \dots}}$$

$$= -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \cos(e + fx)}{8af\sqrt{a + \dots}}$$

$$= -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \cos(e + fx)}{8af\sqrt{a + \dots}}$$

$$= -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \cos(e + fx)}{8af\sqrt{a + \dots}}$$

$$= -\frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}(c - c \sin(e + fx))^{5/2}} + \frac{(A - B) \cos(e + fx)}{8af\sqrt{a + \dots}}$$

Mathematica [A] time = 0.95, size = 306, normalized size = 1.41

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left((B - A)\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) - \dots\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c - c*Sin[e + f*x])^(5/2)),x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (-3*A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (3*A - B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(8*f*(a*(1 + Sin[e + f*x]))^(3/2)*(c - c*Sin[e + f*x])^(5/2))

fricas [A] time = 0.53, size = 437, normalized size = 2.01

$$\left[\frac{\left((3A - B) \cos(fx + e)^3 \sin(fx + e) - (3A - B) \cos(fx + e)^3\right) \sqrt{ac} \log\left(-\frac{ac \cos(fx+e)^3 - 2ac \cos(fx+e) + 2\sqrt{ac} \sqrt{a \dots}}{\cos(f \dots)}\right)}{16 \left(a^2 c^3 f \cos \dots\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

[Out] [-1/16*(((3*A - B)*cos(f*x + e)^3*sin(f*x + e) - (3*A - B)*cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) + 2*sqrt(a*c)*sqrt(

$a \sin(fx + e) + a) \sqrt{-c \sin(fx + e) + c} \sin(fx + e) / \cos(fx + e)^3$
 $+ 2 * ((3A - B) \cos(fx + e)^2 + (3A - B) \sin(fx + e) - A + 3B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} / (a^2 c^3 f \cos(fx + e)^3 \sin(fx + e) - a^2 c^3 f \cos(fx + e)^3)$, $-1/8 * (((3A - B) \cos(fx + e)^3 \sin(fx + e) - (3A - B) \cos(fx + e)^3) \sqrt{-ac} \arctan(\sqrt{-ac} \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} / (ac \cos(fx + e) \sin(fx + e))) + ((3A - B) \cos(fx + e)^2 + (3A - B) \sin(fx + e) - A + 3B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} / (a^2 c^3 f \cos(fx + e)^3 \sin(fx + e) - a^2 c^3 f \cos(fx + e)^3)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)

maple [B] time = 0.55, size = 429, normalized size = 1.98

$$\left(3A \ln \left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) \sin(fx + e) (\cos^2(fx + e)) - 3A (\cos^2(fx + e)) \sin(fx + e) \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x)

[Out] $-1/8/f * (3A * \ln((1 - \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) * \sin(fx + e) * \cos(fx + e)^2 - 3A * \cos(fx + e)^2 * \sin(fx + e) * \ln(-(-1 + \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) - B * \ln((1 - \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) * \sin(fx + e) * \cos(fx + e)^2 + B * \ln(-(-1 + \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) * \cos(fx + e)^2 * \sin(fx + e) - 3A * \cos(fx + e)^2 * \ln((1 - \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) - 2A * \cos(fx + e)^2 * \sin(fx + e) + 3A * \cos(fx + e)^2 * \ln(-(-1 + \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) + B * \cos(fx + e)^2 * \ln((1 - \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) - 2B * \cos(fx + e)^2 * \sin(fx + e) - B * \cos(fx + e)^2 * \ln(-(-1 + \cos(fx + e) + \sin(fx + e)) / \sin(fx + e)) - A * \cos(fx + e)^2 + 3B * \cos(fx + e)^2 - 3A * \sin(fx + e) + B * \sin(fx + e) + A - 3B) * \cos(fx + e) / (a * (1 + \sin(fx + e)))^(3/2) / (-c * (\sin(fx + e) - 1))^(5/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(-c*sin(f*x + e) + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{3}{2}} (c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c-c*sin(f*x+e))**(5/2), x)
```

```
[Out] Timed out
```

$$3.187 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{9/2}}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=323

$$\frac{8c^5(3A-7B) \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{4c^4(3A-7B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{c^3(3A-7B) \cos(e+fx)}{a^2 f \sqrt{a \sin(e+fx)+a}}$$

[Out] 1/4*(3*A-7*B)*c*cos(f*x+e)*(c-c*sin(f*x+e))^(7/2)/a/f/(a+a*sin(f*x+e))^(3/2)-1/4*(A-B)*cos(f*x+e)*(c-c*sin(f*x+e))^(9/2)/f/(a+a*sin(f*x+e))^(5/2)+(3*A-7*B)*c^3*cos(f*x+e)*(c-c*sin(f*x+e))^(3/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+1/3*(3*A-7*B)*c^2*cos(f*x+e)*(c-c*sin(f*x+e))^(5/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)+8*(3*A-7*B)*c^5*cos(f*x+e)*ln(1+sin(f*x+e))/a^2/f/(a+a*sin(f*x+e))^(1/2)/(c-c*sin(f*x+e))^(1/2)+4*(3*A-7*B)*c^4*cos(f*x+e)*(c-c*sin(f*x+e))^(1/2)/a^2/f/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.71, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{4c^4(3A-7B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{c^3(3A-7B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{c^2(3A-7B) \cos(e+fx)}{3a^2 f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (8*(3*A - 7*B)*c^5*Cos[e + f*x]*Log[1 + Sin[e + f*x]]/(a^2*f*Sqrt[a + a*Sin[e + f*x]]*Sqrt[c - c*Sin[e + f*x]]) + (4*(3*A - 7*B)*c^4*Cos[e + f*x]*Sqrt[c - c*Sin[e + f*x]]/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^3*Cos[e + f*x]*(c - c*Sin[e + f*x])^(3/2))/(a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c^2*Cos[e + f*x]*(c - c*Sin[e + f*x])^(5/2))/(3*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + ((3*A - 7*B)*c*Cos[e + f*x]*(c - c*Sin[e + f*x])^(7/2))/(4*a*f*(a + a*Sin[e + f*x])^(3/2)) - ((A - B)*Cos[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(a + a*Sin[e + f*x])^(5/2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])

$(m-1)(c+d\sin[e+fx])^n/(f(2n+1)), x] - \text{Dist}[(b(2m-1))/(d(2n+1)), \text{Int}[(a+b\sin[e+fx])^{m-1}(c+d\sin[e+fx])^{n+1}, x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2740

$\text{Int}[(a + b\sin[e + fx])^m(c + d\sin[e + fx])^n, x_Symbol] :> -\text{Simp}[(b\cos[e + fx](a + b\sin[e + fx])^{m-1}(c + d\sin[e + fx])^n/(f(m + n)), x] + \text{Dist}[(a(2m - 1))/(m + n), \text{Int}[(a + b\sin[e + fx])^{m-1}(c + d\sin[e + fx])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2972

$\text{Int}[(a + b\sin[e + fx])^m(A + B\sin[e + fx])^n(c + d\sin[e + fx])^n, x_Symbol] :> \text{Simp}[(A*b - a*B)\cos[e + fx](a + b\sin[e + fx])^m(c + d\sin[e + fx])^n/(a*f(2*m + 1)), x] + \text{Dist}[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b\sin[e + fx])^{m+1}(c + d\sin[e + fx])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{9/2}}{(a + a \sin(e + fx))^{5/2}} dx &= -\frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{9/2}}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3A - 7B) \int \frac{c - c \sin(e + fx)}{a + a \sin(e + fx)} dx}{4} \\ &= \frac{(3A - 7B)c \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{9/2}}{4f(a + a \sin(e + fx))^{5/2}} \\ &= \frac{(3A - 7B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)c \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4af(a + a \sin(e + fx))^{3/2}} \\ &= \frac{(3A - 7B)c^3 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{3a^2 f \sqrt{a + a \sin(e + fx)}} \\ &= \frac{4(3A - 7B)c^4 \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)c^3 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{a^2} \\ &= \frac{4(3A - 7B)c^4 \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)c^3 \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{a^2} \\ &= \frac{4(3A - 7B)c^4 \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)c^3 \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{a^2} \\ &= \frac{8(3A - 7B)c^5 \cos(e + fx) \log(1 + \sin(e + fx))}{a^2 f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{4(3A - 7B)c^4 \cos(e + fx) \sqrt{c - c \sin(e + fx)}}{a^2} \end{aligned}$$

Mathematica [A] time = 7.01, size = 573, normalized size = 1.77

$$\frac{(28A - 97B) \sin(e + fx)(c - c \sin(e + fx))^{9/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^5 (A - 7B) \cos(2(e + fx))(c - c \sin(e + fx))^{9/2}}{4f(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^9} \frac{4f(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^9}{4f(a(\sin(e + fx) + 1))^{5/2} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^9}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(9/2))/(a + a*Sin[e + f*x])^(5/2), x]

[Out] (-8*(A - B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (16*(2*A - 3*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - ((A - 7*B)*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) + (16*(3*A - 7*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - ((28*A - 97*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*Sin[e + f*x]*(c - c*Sin[e + f*x])^(9/2))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2)) - (B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*(c - c*Sin[e + f*x])^(9/2)*Sin[3*(e + f*x)]/(12*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^9*(a*(1 + Sin[e + f*x]))^(5/2))

fricas [F] time = 45.50, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left((A - 4B)c^4 \cos(fx + e)^4 - 4(2A - 3B)c^4 \cos(fx + e)^2 + 8(A - B)c^4 + (Bc^4 \cos(fx + e)^4 + 4(A - B)c^4) \right)}{3a^3 \cos(fx + e)^2 - 4a^3 + (a^3 \cos(fx + e)^2 - 4a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] integral(-((A - 4*B)*c^4*cos(f*x + e)^4 - 4*(2*A - 3*B)*c^4*cos(f*x + e)^2 + 8*(A - B)*c^4 + (B*c^4*cos(f*x + e)^4 + 4*(A - 2*B)*c^4*cos(f*x + e)^2 - 8*(A - B)*c^4)*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.68, size = 1287, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2), x)


```
[Out] -1/6/f*(-108*B*cos(f*x+e)^3*sin(f*x+e)-1152*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2688*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+396*A*sin(f*x+e)-174*A*sin(f*x+e)*cos(f*x+e)+36*A*cos(f*x+e)^3*sin(f*x+e)-3*A*cos(f*x+e)^4*sin(f*x+e)+15*B*sin(f*x+e)*cos(f*x+e)^4+1008*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-255*A*cos(f*x+e)^2*sin(f*x+e)+581*B*cos(f*x+e)^2*sin(f*x+e)-93*B*cos(f*x+e)^4+396*A-932*B+219*A*cos(f*x+e)^3-336*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-432*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+576*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-288*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+442*B*sin(f*x+e)*cos(f*x+e)-1344*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+672*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+864*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2016*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-288*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+3*A*cos(f*x+e)^5-17*B*cos(f*x+e)^5+2*B*cos(f*x+e)^6+336*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+672*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+2*B*cos(f*x+e)^5*sin(f*x+e)+576*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1344*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1152*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2688*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+1023*B*cos(f*x+e)^2+144*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+33*A*cos(f*x+e)^4-144*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-288*A*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+672*B*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-473*B*cos(f*x+e)^3-932*B*sin(f*x+e)+576*A*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-1344*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-672*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-222*A*cos(f*x+e)+490*B*cos(f*x+e)-429*A*cos(f*x+e)^2+576*A*ln(2/(cos(f*x+e)+1))-1344*B*ln(2/(cos(f*x+e)+1)))*(-c*(sin(f*x+e)-1))^(9/2)/(sin(f*x+e)*cos(f*x+e)^4+cos(f*x+e)^5+4*sin(f*x+e)*cos(f*x+e)^3-5*cos(f*x+e)^4-12*cos(f*x+e)^2*sin(f*x+e)-8*cos(f*x+e)^3-8*sin(f*x+e)*cos(f*x+e)+20*cos(f*x+e)^2+16*sin(f*x+e)+8*cos(f*x+e)-16)/(a*(1+sin(f*x+e)))^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{9}{2}}}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(9/2)/(a+a*sin(f*x+e))^(5/2),x,algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x+e) + A)*(-c*sin(f*x+e) + c)^(9/2)/(a*sin(f*x+e) + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{\frac{9}{2}}}{(a + a \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(9/2))/(a + a*sin(e + f*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(9/2)/(a+a*sin(f*x+e))**(5/2),  
x)
```

```
[Out] Timed out
```

$$3.188 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{7/2}}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=263

$$\frac{6c^4(A-3B) \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{3c^3(A-3B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{3c^2(A-3B) \cos(e+fx)}{4a^2 f \sqrt{a \sin(e+fx)+a}}$$

[Out] $\frac{1}{2}(A-3B)c \cos(fx+e)(c-c \sin(fx+e))^{5/2}/a/f/(a+a \sin(fx+e))^{3/2} - \frac{1}{4}(A-B) \cos(fx+e)(c-c \sin(fx+e))^{7/2}/f/(a+a \sin(fx+e))^{5/2} + \frac{3}{4}(A-3B)c^2 \cos(fx+e)(c-c \sin(fx+e))^{3/2}/a^2/f/(a+a \sin(fx+e))^{1/2} + 6(A-3B)c^4 \cos(fx+e) \ln(1+\sin(fx+e))/a^2/f/(a+a \sin(fx+e))^{1/2} / (c-c \sin(fx+e))^{1/2} + 3(A-3B)c^3 \cos(fx+e)(c-c \sin(fx+e))^{1/2}/a^2/f/(a+a \sin(fx+e))^{1/2}$

Rubi [A] time = 0.61, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{3c^3(A-3B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{3c^2(A-3B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{4a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{6c^4(A-3B) \cos(e+fx)}{a^2 f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2), x]

[Out] $(6(A-3B)c^4 \cos[e+fx] \log[1+\sin[e+fx]])/(a^2 f \sqrt{a+a \sin[e+fx]}) \sqrt{c-c \sin[e+fx]} + (3(A-3B)c^3 \cos[e+fx] \sqrt{c-c \sin[e+fx]})/(a^2 f \sqrt{a+a \sin[e+fx]}) + (3(A-3B)c^2 \cos[e+fx] (c-c \sin[e+fx])^{3/2})/(4a^2 f \sqrt{a+a \sin[e+fx]}) + ((A-3B)c \cos[e+fx] (c-c \sin[e+fx])^{5/2})/(2a^2 f (a+a \sin[e+fx])^{3/2}) - ((A-B) \cos[e+fx] (c-c \sin[e+fx])^{7/2})/(4f (a+a \sin[e+fx])^{5/2})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_.) + (f_.)*(x_)]], x_Symbol] := Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*

```
(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^
2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n
+ 1, 0])
```

Rule 2740

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(
m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n)
, Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[
{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ
[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILt
Q[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\begin{aligned}
 \int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{5/2}} dx &= -\frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(A - 3B) \int \frac{(c - c \sin(e + fx))^{7/2}}{(a + a \sin(e + fx))^{5/2}} dx}{2a} \\
 &= \frac{(A - 3B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{7/2}}{4f(a + a \sin(e + fx))^{5/2}} \\
 &= \frac{3(A - 3B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(A - 3B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2af(a + a \sin(e + fx))^{3/2}} \\
 &= \frac{3(A - 3B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{3(A - 3B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4a^2 f \sqrt{a + a \sin(e + fx)}} \\
 &= \frac{3(A - 3B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{3(A - 3B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4a^2 f \sqrt{a + a \sin(e + fx)}} \\
 &= \frac{3(A - 3B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{3(A - 3B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4a^2 f \sqrt{a + a \sin(e + fx)}} \\
 &= \frac{6(A - 3B)c^4 \cos(e + fx) \log(1 + \sin(e + fx))}{a^2 f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{3(A - 3B)c^3 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{a^2 f \sqrt{a + a \sin(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 2.54, size = 243, normalized size = 0.92

$$(c - c \sin(e + fx))^{7/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(-4(A - 6B) \sin(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) + \dots \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(7/2))/(a + a*Sin[e + f*x])^(5/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(7/2)*(-16*(A - B) + 16*(3*A - 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + B*Cos[2*(e + f*x)]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + 48*(A - 3*B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 4*(A - 6*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(4*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left(Bc^3 \cos(fx + e)^4 + (3A - 5B)c^3 \cos(fx + e)^2 - 4(A - B)c^3 - \left((A - 3B)c^3 \cos(fx + e)^2 - 4(A - 6B)c^3 \cos(fx + e) \right) \sin(fx + e) \right) \sqrt{a + a \sin(fx + e)}}{3a^3 \cos(fx + e)^2 - 4a^3 + \left(a^3 \cos(fx + e)^2 - 4a^3 \right) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")
```

```
[Out] integral((B*c^3*cos(f*x + e)^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e)^2 - 4*(A - 6*B)*c^3*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.64, size = 1206, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2), x)
```

```
[Out] -1/2/f*(-10*B*cos(f*x+e)^3*sin(f*x+e)-96*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+32*A*sin(f*x+e)-12*A*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)^3*sin(f*x+e)+B*sin(f*x+e)*cos(f*x+e)^4+108*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))-22*A*cos(f*x+e)^2*sin(f*x+e)+63*B*cos(f*x+e)^2*sin(f*x+e)-9*B*cos(f*x+e)^4+32*A-100*B+20*A*cos(f*x+e)^3-36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3-36*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+48*A*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+46*B*sin(f*x+e)*cos(f*x+e)-144*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+72*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+72*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-216*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-24*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-B*cos(f*x+e)^5+36*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+72*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+48*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-96*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+288*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))
```

```

in(f*x+e))+109*B*cos(f*x+e)^2+12*A*cos(f*x+e)^3*ln(2/(cos(f*x+e)+1))+2*A*cos
s(f*x+e)^4-12*A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-24*A*cos(f*x+e
)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+72*B*cos(f*x+e)^3*ln((1-cos(f*
x+e)+sin(f*x+e))/sin(f*x+e))-53*B*cos(f*x+e)^3-100*B*sin(f*x+e)+48*A*sin(f*
x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-144*B*sin(f*x+e)*c
os(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+24*A*ln((1-cos(f*x+e)+si
n(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-72*B*ln((1-cos(f*x+e)+sin(f*x
+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-20*A*cos(f*x+e)+54*B*cos(f*x+e)-34
*A*cos(f*x+e)^2+48*A*ln(2/(cos(f*x+e)+1))-144*B*ln(2/(cos(f*x+e)+1)))*(-c*(
sin(f*x+e)-1))^(7/2)/(sin(f*x+e)*cos(f*x+e)^3-cos(f*x+e)^4-4*cos(f*x+e)^2*s
in(f*x+e)-3*cos(f*x+e)^3-4*sin(f*x+e)*cos(f*x+e)+8*cos(f*x+e)^2+8*sin(f*x+e
)+4*cos(f*x+e)-8)/(a*(1+sin(f*x+e)))^(5/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{7}{2}}}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(7/2)/(a+a*sin(f*x+e))^(5/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(7/2)/(a*sin(f*x + e)
+ a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{\frac{7}{2}}}{(a + a \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(
5/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(7/2))/(a + a*sin(e + f*x))^(
5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(7/2)/(a+a*sin(f*x+e))**(5/2),
x)
```

```
[Out] Timed out
```

$$3.189 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{5/2}}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=211

$$\frac{c^3(A-5B) \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{c^2(A-5B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{2a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{c(A-5B) \cos(e+fx)}{4af(a \sin(e+fx)+a)}$$

[Out] $\frac{1}{4}(A-5B) * c * \cos(f*x+e) * (c-c*\sin(f*x+e))^{(3/2)} / a / f / (a+a*\sin(f*x+e))^{(3/2)} - \frac{1}{4}(A-B) * \cos(f*x+e) * (c-c*\sin(f*x+e))^{(5/2)} / f / (a+a*\sin(f*x+e))^{(5/2)} + (A-5*B) * c^3 * \cos(f*x+e) * \ln(1+\sin(f*x+e)) / a^2 / f / (a+a*\sin(f*x+e))^{(1/2)} / (c-c*\sin(f*x+e))^{(1/2)} + 1/2 * (A-5*B) * c^2 * \cos(f*x+e) * (c-c*\sin(f*x+e))^{(1/2)} / a^2 / f / (a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.49, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2972, 2739, 2740, 2737, 2667, 31}

$$\frac{c^2(A-5B) \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{2a^2 f \sqrt{a \sin(e+fx)+a}} + \frac{c^3(A-5B) \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} + \frac{c(A-5B) \cos(e+fx)}{4af(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(5/2), x]

[Out] $((A - 5*B) * c^3 * \text{Cos}[e + f*x] * \text{Log}[1 + \text{Sin}[e + f*x]]) / (a^2 * f * \text{Sqrt}[a + a * \text{Sin}[e + f*x]] * \text{Sqrt}[c - c * \text{Sin}[e + f*x]]) + ((A - 5*B) * c^2 * \text{Cos}[e + f*x] * \text{Sqrt}[c - c * \text{Sin}[e + f*x]]) / (2 * a^2 * f * \text{Sqrt}[a + a * \text{Sin}[e + f*x]]) + ((A - 5*B) * c * \text{Cos}[e + f*x] * (c - c * \text{Sin}[e + f*x])^{(3/2)}) / (4 * a * f * (a + a * \text{Sin}[e + f*x])^{(3/2)}) - ((A - B) * \text{Cos}[e + f*x] * (c - c * \text{Sin}[e + f*x])^{(5/2)}) / (4 * f * (a + a * \text{Sin}[e + f*x])^{(5/2)})$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2667

Int[cos[(e_) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2737

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/Sqrt[(c_) + (d_.)*sin[(e_) + (f_.)*(x_)]], x_Symbol] :> Dist[(a*c*Cos[e + f*x])/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), Int[Cos[e + f*x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2739

Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(-2*b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)), x] - Dist[(b*(2*m - 1))/(d*(2*n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && LtQ[n, -1] && !(ILtQ[m + n, 0] && GtQ[2*m + n, 0])

+ 1, 0])

Rule 2740

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n)/(f*(m + n)), x] + Dist[(a*(2*m - 1))/(m + n), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])
```

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx = -\frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(A - 5B) \int \frac{(c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx}{4a}$$

$$= \frac{(A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{4af(a + a \sin(e + fx))^{5/2}}$$

$$= \frac{(A - 5B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{4af(a + a \sin(e + fx))^{5/2}}$$

$$= \frac{(A - 5B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{4af(a + a \sin(e + fx))^{5/2}}$$

$$= \frac{(A - 5B)c^2 \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{2a^2 f \sqrt{a + a \sin(e + fx)}} + \frac{(A - 5B)c \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{4af(a + a \sin(e + fx))^{5/2}}$$

$$= \frac{(A - 5B)c^3 \cos(e + fx) \log(1 + \sin(e + fx))}{a^2 f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} + \frac{(A - 5B)c^2 \cos(e + fx)(c - c \sin(e + fx))^{5/2}}{2a^2 f \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 1.15, size = 199, normalized size = 0.94

$$\frac{(c - c \sin(e + fx))^{5/2} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(4(A - 2B) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)^2 + 2(A - 2B) \cos(e + fx) (c - c \sin(e + fx))^{5/2}}{f(a \sin(e + fx) + a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2))/(a + a*Sin[e + f*x])^(5/2), x]
```



```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c - c*Sin[e + f*x])^(5/2)*(-2*A + 2
*B + 4*(A - 2*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 2*(A - 5*B)*Log[
Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^
4 + B*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[e + f*x]))/(f*(Cos[(e + f
*x)/2] - Sin[(e + f*x)/2])^5*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{\left((A - 2B)c^2 \cos(fx + e)^2 - 2(A - B)c^2 + (Bc^2 \cos(fx + e)^2 + 2(A - B)c^2) \sin(fx + e) \right) \sqrt{a \sin(fx + e)}}{3a^3 \cos(fx + e)^2 - 4a^3 + (a^3 \cos(fx + e)^2 - 4a^3) \sin(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
algorithm="fricas")
```

```
[Out] integral(((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e)
)^2 + 2*(A - B)*c^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x
+ e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e)), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x,
algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.63, size = 1106, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] 1/f*(-B*cos(f*x+e)^3*sin(f*x+e)-8*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e)
)+40*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f*x+e)+15*B*cos(f*x
+e)^2*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^2*sin(f*x+e)+9*B*cos(f*x+e)^2*sin
(f*x+e)-B*cos(f*x+e)^4+2*A-14*B+2*A*cos(f*x+e)^3-5*B*ln(2/(cos(f*x+e)+1))*c
os(f*x+e)^3-3*A*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+4*A*sin(f*x+e)*ln(2/(cos(
f*x+e)+1))-2*A*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*B*sin(f*x+e)*cos(f*x+e)-20
*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+10*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))+6*A
*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-30*B*cos(f*x+e)^2*ln
((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)*sin(f*x+e)*ln(2/(cos(
f*x+e)+1))+5*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+10*B*ln(2/(cos(
f*x+e)+1))*sin(f*x+e)*cos(f*x+e)+4*A*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e)
)/sin(f*x+e))-20*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-8*A*
sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+40*B*sin(f*x+e)*ln((1-c
os(f*x+e)+sin(f*x+e))/sin(f*x+e))+15*B*cos(f*x+e)^2+A*cos(f*x+e)^3*ln(2/(co
s(f*x+e)+1))-A*cos(f*x+e)^2*sin(f*x+e)*ln(2/(cos(f*x+e)+1))-2*A*cos(f*x+e)^
3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+10*B*cos(f*x+e)^3*ln((1-cos(f*x+
e)+sin(f*x+e))/sin(f*x+e))-8*B*cos(f*x+e)^3-14*B*sin(f*x+e)+4*A*sin(f*x+e)*
cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-20*B*sin(f*x+e)*cos(f*x
+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e)
))/sin(f*x+e)*sin(f*x+e)*cos(f*x+e)^2-10*B*ln((1-cos(f*x+e)+sin(f*x+e))/si
n(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2*A*cos(f*x+e)+8*B*cos(f*x+e)-2*A*cos(f*x
```

$+e)^2+4*A*\ln(2/(\cos(f*x+e)+1))-20*B*\ln(2/(\cos(f*x+e)+1)))*(-c*(\sin(f*x+e)-1))^2)^{5/2}/(\cos(f*x+e)^2*\sin(f*x+e)+\cos(f*x+e)^3+2*\sin(f*x+e)*\cos(f*x+e)-3*\cos(f*x+e)^2-4*\sin(f*x+e)-2*\cos(f*x+e)+4)/(a*(1+\sin(f*x+e)))^{5/2}$

maxima [B] time = 0.49, size = 504, normalized size = 2.39

$$\left(\frac{8\sqrt{a}c^{\frac{5}{2}}\sin(fx+e)^2}{\left(a^3+\frac{4a^3\sin(fx+e)}{\cos(fx+e)+1}+\frac{6a^3\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+\frac{4a^3\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{a^3\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)(\cos(fx+e)+1)^2} - \frac{2c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}+1\right)}{a^{\frac{5}{2}}} + \frac{c^{\frac{5}{2}}\log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)}{a^{\frac{5}{2}}} \right) A +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] ((8*sqrt(a)*c^(5/2)*sin(f*x + e)^2/((a^3 + 4*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 6*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 4*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)*(cos(f*x + e) + 1)^2) - 2*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) + c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2))*A + B*(10*c^(5/2)*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1)/a^(5/2) - 5*c^(5/2)*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/a^(5/2) - 2*(5*c^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 16*c^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 14*c^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 16*c^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 5*c^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5)/(a^(5/2) + 4*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 7*a^(5/2)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 8*a^(5/2)*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 7*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 4*a^(5/2)*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + a^(5/2)*sin(f*x + e)^6/(cos(f*x + e) + 1)^6))/f

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{5/2}}{(a + a \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2),x)

[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(5/2))/(a + a*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2)/(a+a*sin(f*x+e))^(5/2), x)

[Out] Timed out

$$3.190 \quad \int \frac{(A+B \sin(e+fx))(c-c \sin(e+fx))^{3/2}}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=149

$$\frac{Bc^2 \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{4f(a \sin(e+fx)+a)^{5/2}} - \frac{Bc \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af(a \sin(e+fx)+a)}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(3/2)}/f/(a+a*\sin(f*x+e))^{(5/2)}-B*c^2*\cos(f*x+e)*\ln(1+\sin(f*x+e))/a^2/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}-B*c*\cos(f*x+e)*(c-c*\sin(f*x+e))^{(1/2)}/a/f/(a+a*\sin(f*x+e))^{(3/2)}$

Rubi [A] time = 0.39, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2739, 2737, 2667, 31}

$$\frac{Bc^2 \cos(e+fx) \log(\sin(e+fx)+1)}{a^2 f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)(c-c \sin(e+fx))^{3/2}}{4f(a \sin(e+fx)+a)^{5/2}} - \frac{Bc \cos(e+fx) \sqrt{c-c \sin(e+fx)}}{af(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sin}[e+f*x])*(c-c*\text{Sin}[e+f*x])^{(3/2)})/(a+a*\text{Sin}[e+f*x])^{(5/2)},x]$

[Out] $-((B*c^2*\text{Cos}[e+f*x]*\text{Log}[1+\text{Sin}[e+f*x]])/(a^2*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])) - (B*c*\text{Cos}[e+f*x]*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])/(a*f*(a+a*\text{Sin}[e+f*x])^{(3/2)}) - ((A-B)*\text{Cos}[e+f*x]*(c-c*\text{Sin}[e+f*x])^{(3/2)})/(4*f*(a+a*\text{Sin}[e+f*x])^{(5/2)})$

Rule 31

$\text{Int}[(a_+ + (b_+)*(x_+))^{(-1)}, x_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a_+ + b_+*x_+, x_+]]/b_+, x_+] /; \text{FreeQ}[\{a_+, b_+\}, x_+]$

Rule 2667

$\text{Int}[\cos[(e_+) + (f_+)*(x_+)]^{(p_+)}*((a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}, x_Symbol] \rightarrow \text{Dist}[1/(b_+^{p_+}*f_+), \text{Subst}[\text{Int}[(a_+ + x_+)^{(m_+ + (p_+ - 1)/2)}*(a_+ - x_+)^{((p_+ - 1)/2)}, x_+, x_+, b_+*\sin[e_+ + f_+*x_+], x_+] /; \text{FreeQ}[\{a_+, b_+, e_+, f_+, m_+\}, x_+] \&\& \text{IntegerQ}[(p_+ - 1)/2] \&\& \text{EqQ}[a_+^2 - b_+^2, 0] \&\& (\text{GeQ}[p_+, -1] \mid \mid \text{!IntegerQ}[m_+ + 1/2])]$

Rule 2737

$\text{Int}[\text{Sqrt}[(a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)]]/\text{Sqrt}[(c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)]], x_Symbol] \rightarrow \text{Dist}[(a_+*c_+*\text{Cos}[e_+ + f_+*x_+])]/(\text{Sqrt}[a_+ + b_+*\sin[e_+ + f_+*x_+]]*\text{Sqrt}[c_+ + d_+*\sin[e_+ + f_+*x_+]]), \text{Int}[\text{Cos}[e_+ + f_+*x_+]/(c_+ + d_+*\sin[e_+ + f_+*x_+]), x_+, x_+] /; \text{FreeQ}[\{a_+, b_+, c_+, d_+, e_+, f_+\}, x_+] \&\& \text{EqQ}[b_+*c_+ + a_+*d_+, 0] \&\& \text{EqQ}[a_+^2 - b_+^2, 0]$

Rule 2739

$\text{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}*((c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)])^{(n_+)}, x_Symbol] \rightarrow \text{Simp}[(-2*b_+*\text{Cos}[e_+ + f_+*x_+]*(a_+ + b_+*\sin[e_+ + f_+*x_+])^{(m_+ - 1)}*(c_+ + d_+*\sin[e_+ + f_+*x_+])^{(n_+)})/(f_+*(2*n_+ + 1)), x_+] - \text{Dist}[(b_+*(2*m_+ - 1))/(d_+*(2*n_+ + 1)), \text{Int}[(a_+ + b_+*\sin[e_+ + f_+*x_+])^{(m_+ - 1)}*(c_+ + d_+*\sin[e_+ + f_+*x_+])^{(n_+ + 1)}, x_+, x_+] /; \text{FreeQ}[\{a_+, b_+, c_+, d_+, e_+, f_+\}, x_+] \&\& \text{EqQ}[b_+*c_+ + a_+*d_+, 0] \&\& \text{EqQ}[a_+^2 - b_+^2, 0] \&\& \text{IGtQ}[m_+ - 1/2, 0] \&\& \text{LtQ}[n_+, -1] \&\& \text{!(ILtQ}[m_+ + n_+, 0] \&\& \text{GtQ}[2*m_+ + n_+ + 1, 0])]$

Rule 2972

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{5/2}} dx = -\frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4f(a + a \sin(e + fx))^{5/2}} + \frac{B \int \frac{(c - c \sin(e + fx))^{3/2}}{(a + a \sin(e + fx))^{3/2}}}{a}$$

$$= -\frac{Bc \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{Bc \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{Bc \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)(c - c \sin(e + fx))^{3/2}}{4f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{Bc^2 \cos(e + fx) \log(1 + \sin(e + fx))}{a^2 f \sqrt{a + a \sin(e + fx)} \sqrt{c - c \sin(e + fx)}} - \frac{Bc \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{af(a + a \sin(e + fx))^{3/2}}$$

Mathematica [A] time = 0.98, size = 179, normalized size = 1.20

$$\frac{c\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(\sin(e + fx) \left(A - 4B \log\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \right) \right)}{f(a(\sin(e + fx) + 1))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(((A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2))/(a + a*Sin[e + f*x])^(5/2)), x]
```

```
[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[c - c*Sin[e + f*x]]*(B*Cos[2*(e + f*x)]*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] - B*(2 + 3*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + (A - 3*B - 4*B*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]])*Sin[e + f*x))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [F] time = 2.71, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{\left(Bc \cos(fx + e) \right)^2 - (A - B)c \sin(fx + e) + (A - B)c \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{3a^3 \cos(fx + e)^2 - 4a^3 + \left(a^3 \cos(fx + e) \right)^2 - 4a^3} \sin(fx + e), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")
```

```
[Out] integral(-(B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e))^2 - 4*a^3)*sin(f*x + e), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.64, size = 606, normalized size = 4.07

$$\left(8B \ln\left(\frac{1-\cos(fx+e)+\sin(fx+e)}{\sin(fx+e)}\right) - A \sin(fx+e) + A \sin(fx+e) \cos(fx+e) + 2B (\cos^2(fx+e)) \sin(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2), x)

[Out] 1/f*(8*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-A*sin(f*x+e)+A*sin(f*x+e)*cos(f*x+e)+3*B*cos(f*x+e)^2*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)^2*sin(f*x+e)-A-3*B*B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^3+B*sin(f*x+e)*cos(f*x+e)-4*B*sin(f*x+e)*ln(2/(cos(f*x+e)+1))+2*B*cos(f*x+e)*ln(2/(cos(f*x+e)+1))-6*B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*ln(2/(cos(f*x+e)+1))*cos(f*x+e)^2*sin(f*x+e)+2*B*ln(2/(cos(f*x+e)+1))*sin(f*x+e)*cos(f*x+e)-4*B*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+8*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*B*cos(f*x+e)^2+2*B*cos(f*x+e)^3*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^3-3*B*sin(f*x+e)-4*B*sin(f*x+e)*cos(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+2*B*cos(f*x+e)+A*cos(f*x+e)^2-4*B*ln(2/(cos(f*x+e)+1))*(-c*(sin(f*x+e)-1))^(3/2)/(sin(f*x+e)*cos(f*x+e)-cos(f*x+e)^2-2*sin(f*x+e)-cos(f*x+e)+2)/(a*(1+sin(f*x+e)))^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx+e) + A)(-c \sin(fx+e) + c)^{\frac{3}{2}}}{(a \sin(fx+e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)/(a*sin(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c - c \sin(e + fx))^{\frac{3}{2}}}{(a + a \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2), x)

```
[Out] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(3/2))/(a + a*sin(e + f*x))^(5/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2)/(a+a*sin(f*x+e))**(5/2), x)
```

```
[Out] Timed out
```

$$3.191 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=94

$$\frac{c(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{5/2} \sqrt{c-c \sin(e+fx)}} - \frac{Bc \cos(e+fx)}{af(a \sin(e+fx)+a)^{3/2} \sqrt{c-c \sin(e+fx)}}$$

[Out] $-1/2*(A-B)*c*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(5/2)}/(c-c*\sin(f*x+e))^{(1/2)}-B*c*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.33, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2971, 2738}

$$\frac{c(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{5/2} \sqrt{c-c \sin(e+fx)}} - \frac{Bc \cos(e+fx)}{af(a \sin(e+fx)+a)^{3/2} \sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sin}[e+f*x])*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]/(a+a*\text{Sin}[e+f*x])^{(5/2)},x]$

[Out] $-((A-B)*c*\text{Cos}[e+f*x])/(2*f*(a+a*\text{Sin}[e+f*x])^{(5/2)}*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (B*c*\text{Cos}[e+f*x])/(a*f*(a+a*\text{Sin}[e+f*x])^{(3/2)}*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Simp}[(-2*b*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^n)/(f*(2*n+1)*\text{Sqrt}[a+b*\text{Sin}[e+f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x_Symbol] := \text{Dist}[B/d, \text{Int}[\text{Sqrt}[a+b*\text{Sin}[e+f*x]]*(c+d*\text{Sin}[e+f*x])^{(n+1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[\text{Sqrt}[a+b*\text{Sin}[e+f*x]]*(c+d*\text{Sin}[e+f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A+B \sin(e+fx)) \sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{5/2}} dx &= \frac{B \int \frac{\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{3/2}} dx}{a} - (-A+B) \int \frac{\sqrt{c-c \sin(e+fx)}}{(a+a \sin(e+fx))^{5/2}} dx \\ &= -\frac{(A-B)c \cos(e+fx)}{2f(a+a \sin(e+fx))^{5/2} \sqrt{c-c \sin(e+fx)}} - \frac{B}{af(a+a \sin(e+fx))^{3/2} \sqrt{c-c \sin(e+fx)}} \end{aligned}$$

Mathematica [A] time = 0.51, size = 99, normalized size = 1.05

$$\frac{\sqrt{a(\sin(e+fx)+1)} \sqrt{c-c \sin(e+fx)} (A+2B \sin(e+fx)+B)}{2a^3 f \left(\cos\left(\frac{1}{2}(e+fx)\right) - \sin\left(\frac{1}{2}(e+fx)\right) \right) \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]]/(a + a*Sin[e + f*x]))^(5/2), x]

[Out] -1/2*(Sqrt[a*(1 + Sin[e + f*x])]*(A + B + 2*B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]])/(a^3*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.43, size = 85, normalized size = 0.90

$$\frac{(2B \sin(fx + e) + A + B) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c}}{2(a^3 f \cos(fx + e)^3 - 2a^3 f \cos(fx + e) \sin(fx + e) - 2a^3 f \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] 1/2*(2*B*sin(f*x + e) + A + B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*f*cos(f*x + e)^3 - 2*a^3*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*f*cos(f*x + e))

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.66, size = 135, normalized size = 1.44

$$\frac{\sin(fx + e) \sqrt{-c(\sin(fx + e) - 1)} (A(\cos^2(fx + e)) + A \sin(fx + e) \cos(fx + e) + B(\cos^2(fx + e)) + B \sin(fx + e) \cos(fx + e))}{2f(a(1 + \sin(fx + e)))^{\frac{5}{2}}(-1 + \cos(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2), x)

[Out] -1/2/f*sin(f*x+e)*(-c*(sin(f*x+e)-1))^(1/2)*(A*cos(f*x+e)^2+A*sin(f*x+e)*cos(f*x+e)+B*cos(f*x+e)^2+B*sin(f*x+e)*cos(f*x+e)+2*A*cos(f*x+e)-3*A*sin(f*x+e)-B*sin(f*x+e)-3*A-B)/(a*(1+sin(f*x+e)))^(5/2)/(-1+cos(f*x+e)+sin(f*x+e))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{-c \sin(fx + e) + c}}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2)/(a+a*sin(f*x+e))^(5/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)

mupad [B] time = 14.86, size = 156, normalized size = 1.66

$$\frac{2\sqrt{-c(\sin(e+fx)-1)}\left(A\sin(2e+2fx)+3B\sin(2e+2fx)-2A\left(2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)-3B\left(2\sin\left(\frac{e}{2}+\frac{fx}{2}\right)^2-1\right)\right)}{a^2f\sqrt{a(\sin(e+fx)+1)}\left(-8\sin(e+fx)^2+4\sin(e+fx)+2\sin(2e+2fx)^2+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c - c*sin(e + f*x))^(1/2))/(a + a*sin(e + f*x))^(5/2), x)

[Out] -(2*(-c*(sin(e + f*x) - 1))^(1/2)*(A*sin(2*e + 2*f*x) + 3*B*sin(2*e + 2*f*x) - 2*A*(2*sin(e/2 + (f*x)/2)^2 - 1) - 3*B*(2*sin(e/2 + (f*x)/2)^2 - 1) + B*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1)))/(a^2*f*(a*(sin(e + f*x) + 1))^(1/2)*(4*sin(e + f*x) + 4*sin(3*e + 3*f*x) + 2*sin(2*e + 2*f*x)^2 - 8*sin(e + f*x)^2 + 8))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c(\sin(e+fx)-1)}(A+B\sin(e+fx))}{(a(\sin(e+fx)+1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1/2)/(a+a*sin(f*x+e))**(5/2), x)

[Out] Integral(sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)

$$3.192 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2} \sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=151

$$\frac{(A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{4a^2 f \sqrt{a \sin(e+fx) + a} \sqrt{c-c \sin(e+fx)}} - \frac{(A+B) \cos(e+fx)}{4af(a \sin(e+fx) + a)^{3/2} \sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)}{4f(a \sin(e+fx) + a)^{5/2}}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(5/2)}/(c-c*\sin(f*x+e))^{(1/2)}-1/4*(A+B)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(1/2)}+1/4*(A+B)*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a^2/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{4a^2 f \sqrt{a \sin(e+fx) + a} \sqrt{c-c \sin(e+fx)}} - \frac{(A+B) \cos(e+fx)}{4af(a \sin(e+fx) + a)^{3/2} \sqrt{c-c \sin(e+fx)}} - \frac{(A-B) \cos(e+fx)}{4f(a \sin(e+fx) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^{(5/2)}*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])], x]$

[Out] $-((A - B)*\text{Cos}[e + f*x])/((4*f*(a + a*\text{Sin}[e + f*x])^{(5/2)}*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) - ((A + B)*\text{Cos}[e + f*x])/((4*a*f*(a + a*\text{Sin}[e + f*x])^{(3/2)}*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((A + B)*\text{ArcTanh}[\text{Sin}[e + f*x]]*\text{Cos}[e + f*x])/((4*a^2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 2741

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)])], x_Symbol] :> \text{Dist}[\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[1/\text{Cos}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2743

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}), x_Symbol] :> \text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m_)}*(c + d*\text{Sin}[e + f*x])^{(n_)} / (a*f*(2*m + 1)), x] + \text{Dist}[(m + n + 1) / (a*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n_)}], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}), x_Symbol] :> \text{Simp}(((A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m_)}*(c + d*\text{Sin}[e + f*x])^{(n_)} / (a*f*(2*m + 1)), x] + \text{Dist}[(a*B*(m - n) + A*b*(m + n + 1)) / (a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n_)}], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} + \frac{(A + B) \int \frac{1}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx}{4f(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} - \frac{(A + B) \int \frac{1}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx}{4af(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} - \frac{(A + B) \int \frac{1}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx}{4af(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} - \frac{(A + B) \int \frac{1}{(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} dx}{4af(a + a \sin(e + fx))^{5/2} \sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.67, size = 214, normalized size = 1.42

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(- (A + B)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*Sqrt[c - c*Sin[e + f*x]]),x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-A + B - (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(4*f*(a*(1 + Sin[e + f*x]))^(5/2)*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.53, size = 408, normalized size = 2.70

$$\frac{\left(\left((A + B) \cos(fx + e)^3 - 2(A + B) \cos(fx + e) \sin(fx + e) - 2(A + B) \cos(fx + e)\right) \sqrt{ac} \log\left(-\frac{ac \cos(fx + e)^3}{8(a^3 c f \cos(fx + e)^3 - 2a^3 c)}\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/8*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))*sin(f*x + e))/cos(f*x + e)^3 + 2*((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e)), -1/4*(((A + B)*cos(f*x + e)^3 - 2*(A + B)*cos(f*x + e)*sin(f*x + e) - 2*(A + B)*cos(f*x + e))*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) - ((A + B)*sin(f*x + e) + 2*A)*sqrt(a*sin(f*x + e)

) + a)*sqrt(-c*sin(f*x + e) + c))/(a^3*c*f*cos(f*x + e)^3 - 2*a^3*c*f*cos(f*x + e)*sin(f*x + e) - 2*a^3*c*f*cos(f*x + e))]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)

maple [B] time = 0.63, size = 466, normalized size = 3.09

$$\left(-A \left(\cos^2(fx + e) \right) \ln \left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) + A \left(\cos^2(fx + e) \right) \ln \left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)} \right) - B \left(\cos^2(fx + e) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x)

[Out] 1/4/f*(-A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*A*cos(f*x+e)^2+2*B*sin(f*x+e)*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+3*A*sin(f*x+e)-2*A*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*sin(f*x+e)-2*B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*A*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} \sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*sqrt(-c*sin(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{5}{2}} \sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{5}{2}} \sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(1/2), x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a*(sin(e + f*x) + 1))**(5/2)*sqrt(-c*(sin(e + f*x) - 1))), x)
```

$$3.193 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=208

$$\frac{(3A+B) \cos(e+fx)}{8a^2 f \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(3A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8a^2 c f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(3A+B)}{8af(a \sin(e+fx)+a)}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{5/2}/(c-c*\sin(f*x+e))^{3/2}-1/8*(3*A+B)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{3/2}/(c-c*\sin(f*x+e))^{3/2}+1/8*(3*A+B)*\cos(f*x+e)/a^2/f/(c-c*\sin(f*x+e))^{3/2}/(a+a*\sin(f*x+e))^{1/2}+1/8*(3*A+B)*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a^2/c/f/(a+a*\sin(f*x+e))^{1/2}/(c-c*\sin(f*x+e))^{1/2}$

Rubi [A] time = 0.48, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{(3A+B) \cos(e+fx)}{8a^2 f \sqrt{a \sin(e+fx)+a} (c-c \sin(e+fx))^{3/2}} + \frac{(3A+B) \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8a^2 c f \sqrt{a \sin(e+fx)+a} \sqrt{c-c \sin(e+fx)}} - \frac{(3A+B)}{8af(a \sin(e+fx)+a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^{5/2}*(c - c*\text{Sin}[e + f*x])^{3/2}), x]$

[Out] $-((A - B)*\text{Cos}[e + f*x])/(4*f*(a + a*\text{Sin}[e + f*x])^{5/2}*(c - c*\text{Sin}[e + f*x])^{3/2}) - ((3*A + B)*\text{Cos}[e + f*x])/(8*a*f*(a + a*\text{Sin}[e + f*x])^{3/2}*(c - c*\text{Sin}[e + f*x])^{3/2}) + ((3*A + B)*\text{Cos}[e + f*x])/(8*a^2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])*(c - c*\text{Sin}[e + f*x])^{3/2}) + ((3*A + B)*\text{ArcTanh}[\text{Sin}[e + f*x]])* \text{Cos}[e + f*x])/(8*a^2*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 2741

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)])], x_Symbol] :> \text{Dist}[\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[1/\text{Cos}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 2743

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}), x_Symbol] :> \text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(a*f*(2*m + 1)), x] + \text{Dist}[(m + n + 1)/(a*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || !SumSimplerQ[n, 1])

Rule 2972

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}), x_Symbol] :> \text{Simp}(((A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(a*f*(2*m + 1)), x] + \text{Dist}[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} dx &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} + \frac{(3A + B)}{8af(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} - \frac{(3A + B)}{8af(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} - \frac{(3A + B)}{8af(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} - \frac{(3A + B)}{8af(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} - \frac{(3A + B)}{8af(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.96, size = 305, normalized size = 1.47

$$\frac{\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left((A + B)\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(3/2)), x]

[Out] ((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-2*A*Cos[e + f*x]^2 + (-A + B)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2 + (A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - (3*A + B)*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (3*A + B)*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(8*f*(a*(1 + Sin[e + f*x]))^(5/2)*(c - c*Sin[e + f*x])^(3/2))

fricas [A] time = 0.55, size = 419, normalized size = 2.01

$$\left[\frac{\left((3A + B) \cos(fx + e)^3 \sin(fx + e) + (3A + B) \cos(fx + e)^3\right) \sqrt{ac} \log\left(-\frac{ac \cos(fx + e)^3 - 2ac \cos(fx + e) - 2\sqrt{ac} \sqrt{a \sin(fx + e)}}{\cos(fx + e)}\right)}{16 \left(a^3 c^2 f \cos(fx + e)\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] [1/16*(((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(a*c)*log(-(a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e))/cos(f*x + e)^3)

- 2*((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3), -1/8*((3*A + B)*cos(f*x + e)^3*sin(f*x + e) + (3*A + B)*cos(f*x + e)^3)*sqrt(-a*c)*arctan(sqrt(-a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a*c*cos(f*x + e)*sin(f*x + e))) + ((3*A + B)*cos(f*x + e)^2 - (3*A + B)*sin(f*x + e) - A - 3*B)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)/(a^3*c^2*f*cos(f*x + e)^3*sin(f*x + e) + a^3*c^2*f*cos(f*x + e)^3)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

maple [B] time = 0.57, size = 431, normalized size = 2.07

$$\frac{\left(3A \left(\cos^2(fx + e)\right) \sin(fx + e) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - 3A \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) \sin(fx + e) \left(\cos^2(fx + e)\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x)

[Out] -1/8/f*(3*A*cos(f*x+e)^2*sin(f*x+e)*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2+B*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*cos(f*x+e)^2*sin(f*x+e)-B*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))*sin(f*x+e)*cos(f*x+e)^2-2*A*cos(f*x+e)^2*sin(f*x+e)+3*A*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+2*B*cos(f*x+e)^2*sin(f*x+e)+B*cos(f*x+e)^2*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-B*cos(f*x+e)^2*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))+A*cos(f*x+e)^2+3*B*cos(f*x+e)^2-3*A*sin(f*x+e)-B*sin(f*x+e)-A-3*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{5}{2}} (c - c \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(3/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(3/2), x)
```

```
[Out] Timed out
```

$$3.194 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=245

$$\frac{3A \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} + \frac{3A \cos(e+fx)}{8a^2cf\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{3/2}} + \frac{3A}{8a^2f\sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(5/2)}/(c-c*\sin(f*x+e))^{(5/2)}-1/2*A*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(3/2)}/(c-c*\sin(f*x+e))^{(5/2)}+3/8*A*\cos(f*x+e)/a^2/f/(c-c*\sin(f*x+e))^{(5/2)}/(a+a*\sin(f*x+e))^{(1/2)}+3/8*A*\cos(f*x+e)/a^2/c/f/(c-c*\sin(f*x+e))^{(3/2)}/(a+a*\sin(f*x+e))^{(1/2)}+3/8*A*\operatorname{arctanh}(\sin(f*x+e))*\cos(f*x+e)/a^2/c^2/f/(a+a*\sin(f*x+e))^{(1/2)}/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.57, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2972, 2743, 2741, 3770}

$$\frac{3A \cos(e+fx) \tanh^{-1}(\sin(e+fx))}{8a^2c^2f\sqrt{a \sin(e+fx)+a}\sqrt{c-c \sin(e+fx)}} + \frac{3A \cos(e+fx)}{8a^2cf\sqrt{a \sin(e+fx)+a}(c-c \sin(e+fx))^{3/2}} + \frac{3A}{8a^2f\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)})], x]$

[Out] $-((A - B)*\text{Cos}[e + f*x])/(4*f*(a + a*\text{Sin}[e + f*x])^{(5/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)}) - (A*\text{Cos}[e + f*x])/(2*a*f*(a + a*\text{Sin}[e + f*x])^{(3/2)}*(c - c*\text{Sin}[e + f*x])^{(5/2)}) + (3*A*\text{Cos}[e + f*x])/(8*a^2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*(c - c*\text{Sin}[e + f*x])^{(5/2)}) + (3*A*\text{Cos}[e + f*x])/(8*a^2*c*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])*(c - c*\text{Sin}[e + f*x])^{(3/2)}) + (3*A*\text{ArcTanh}[\text{Sin}[e + f*x]]*\text{Cos}[e + f*x])/(8*a^2*c^2*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 2741

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)])], x_Symbol] :> \text{Dist}[\text{Cos}[e + f*x]/(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]), \text{Int}[1/\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2743

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m_)}*(c + d*\text{Sin}[e + f*x])^{(n_)})/(a*f*(2*m + 1)), x] + \text{Dist}[(m + n + 1)/(a*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n_)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{ILtQ}[\text{Simplify}[m + n + 1], 0] \&\& \text{NeQ}[m, -2^{(-1)}] \&\& (\text{SumSimplerQ}[m, 1] || ! \text{SumSimplerQ}[n, 1])$

Rule 2972

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] :> \text{Simp}[(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m_)}*(c + d*\text{Sin}[e + f*x])^{(n_)})/(a*f*(2*m + 1)), x] + \text{Dist}[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n_)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{LtQ}[m, -2^{(-1)}] || (\text{ILtQ}[m + n, 0] \&\& !\text{SumSimplerQ}[n, 1])) \&\& \text{NeQ}[2*m + 1, 0]$

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} dx &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} + \frac{A \int \frac{1}{(a + a \sin(e + fx))^{5/2}} dx}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} - \frac{2af(a + a \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} - \frac{2af(a + a \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} - \frac{2af(a + a \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} - \frac{2af(a + a \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \\ &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} - \frac{2af(a + a \sin(e + fx))^{5/2}}{4f(a + a \sin(e + fx))^{5/2} (c - c \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.94, size = 246, normalized size = 1.00

$$\sec^3(e + fx) \left(22A \sin(e + fx) + 6A \sin(3(e + fx)) - 9A \log \left(\cos \left(\frac{1}{2}(e + fx) \right) - \sin \left(\frac{1}{2}(e + fx) \right) \right) - 12A \cos(2(e + fx)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c - c*Sin[e + f*x])^(5/2)), x]

[Out] (Sec[e + f*x]^3*(16*B - 9*A*Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 12*A*Cos[2*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) - 3*A*Cos[4*(e + f*x)]*(Log[Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]) + 9*A*Log[Cos[(e + f*x)/2] + Sin[(e + f*x)/2]] + 22*A*Sin[e + f*x] + 6*A*Sin[3*(e + f*x)])/(64*a^2*c^2*f*Sqrt[a*(1 + Sin[e + f*x])]*Sqrt[c - c*Sin[e + f*x]])

fricas [A] time = 0.56, size = 306, normalized size = 1.25

$$\frac{3 \sqrt{ac} A \cos(fx + e)^5 \log \left(-\frac{ac \cos(fx+e)^3 - 2ac \cos(fx+e) - 2\sqrt{ac} \sqrt{a \sin(fx+e)+a} \sqrt{-c \sin(fx+e)+c} \sin(fx+e)}{\cos(fx+e)^3} \right) + 2 \left((3A \cos(fx + e) - 2\sqrt{ac}) \sqrt{a \sin(fx + e) + a} \sqrt{-c \sin(fx + e) + c} \sin(fx + e) \right)}{16 a^3 c^3 f \cos(fx + e)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] [1/16*(3*sqrt(a*c)*A*cos(f*x + e)^5*log(-a*c*cos(f*x + e)^3 - 2*a*c*cos(f*x + e) - 2*sqrt(a*c)*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)) + 2*(3*A*cos(f*x + e) - 2*sqrt(a*c))*sqrt(a*sin(f*x + e) + a)*sqrt(-c*sin(f*x + e) + c)*sin(f*x + e)]/16*a^3*c^3*f*cos(f*x + e)^5

$(f*x + e))/\cos(f*x + e)^3) + 2*((3*A*\cos(f*x + e)^2 + 2*A)*\sin(f*x + e) + 2*B)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{-c*\sin(f*x + e) + c})/(a^3*c^3*f*\cos(f*x + e)^5), -1/8*(3*\sqrt{-a*c}*A*\arctan(\sqrt{-a*c})*\sqrt{a*\sin(f*x + e) + a}*\sqrt{-c*\sin(f*x + e) + c})/(a*c*\cos(f*x + e)*\sin(f*x + e))*\cos(f*x + e)^5 - ((3*A*\cos(f*x + e)^2 + 2*A)*\sin(f*x + e) + 2*B)*\sqrt{a*\sin(f*x + e) + a}*\sqrt{-c*\sin(f*x + e) + c})/(a^3*c^3*f*\cos(f*x + e)^5]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)

maple [A] time = 0.60, size = 151, normalized size = 0.62

$$\frac{\left(3A \left(\cos^4(fx + e)\right) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - 3A \left(\cos^4(fx + e)\right) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - 2B \left(\cos^4(fx + e)\right) \ln\left(\frac{1 - \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right) - 2B \left(\cos^4(fx + e)\right) \ln\left(-\frac{-1 + \cos(fx + e) + \sin(fx + e)}{\sin(fx + e)}\right)\right)}{8f \left(a \left(1 + \sin(fx + e)\right)\right)^{\frac{5}{2}} \left(-c \left(\sin(fx + e) - 1\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x)

[Out] 1/8/f*(3*A*cos(f*x+e)^4*ln((1-cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-3*A*cos(f*x+e)^4*ln(-(-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))-2*B*cos(f*x+e)^4+3*A*cos(f*x+e)^2*sin(f*x+e)+2*A*sin(f*x+e)+2*B)*cos(f*x+e)/(a*(1+sin(f*x+e)))^(5/2)/(-c*(sin(f*x+e)-1))^(5/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{5}{2}} (-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(5/2)*(-c*sin(f*x + e) + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{5}{2}} (c - c \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)),x)

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c - c*sin(e + f*x))^(5/2)), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c-c*sin(f*x+e))**(5/2), x)
```

```
[Out] Timed out
```

$$3.195 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^n dx$$

Optimal. Leaf size=174

$$c2^{n+\frac{1}{2}}(A(m+n+1) + B(m-n)) \cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n} (a \sin(e+fx) + a)^m (c - c \sin(e+fx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}+m; \frac{3}{2}+m; \frac{f(2m+1)(m+n+1)}{f(2m+1)(m+n+1)}\right)$$

[Out] $2^{(1/2+n)} * c * (B * (m-n) + A * (1+m+n)) * \cos(f*x+e) * \text{hypergeom}([1/2-n, 1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e)) * (1-\sin(f*x+e))^{(1/2-n)} * (a+a*\sin(f*x+e))^m * (c-c*\sin(f*x+e))^{(-1+n)} / f / (1+2*m) / (1+m+n) - B * \cos(f*x+e) * (a+a*\sin(f*x+e))^m * (c-c*\sin(f*x+e))^n / f / (1+m+n)$

Rubi [A] time = 0.31, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2973, 2745, 2689, 70, 69}

$$c2^{n+\frac{1}{2}}(A(m+n+1) + B(m-n)) \cos(e+fx)(1-\sin(e+fx))^{\frac{1}{2}-n} (a \sin(e+fx) + a)^m (c - c \sin(e+fx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}+m; \frac{3}{2}+m; \frac{f(2m+1)(m+n+1)}{f(2m+1)(m+n+1)}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m * (A + B*\text{Sin}[e + f*x]) * (c - c*\text{Sin}[e + f*x])^n, x]$

[Out] $(2^{(1/2 + n)} * c * (B * (m - n) + A * (1 + m + n)) * \text{Cos}[e + f*x] * \text{Hypergeometric2F1}[(1 + 2*m)/2, (1 - 2*n)/2, (3 + 2*m)/2, (1 + \text{Sin}[e + f*x])/2] * (1 - \text{Sin}[e + f*x])^{(1/2 - n)} * (a + a*\text{Sin}[e + f*x])^m * (c - c*\text{Sin}[e + f*x])^{(-1 + n)}) / (f * (1 + 2*m) * (1 + m + n)) - (B * \text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m * (c - c*\text{Sin}[e + f*x])^n) / (f * (1 + m + n))$

Rule 69

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * \text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a+b*x))/(b*c-a*d)] / (b*(m+1)*(b/(b*c-a*d))^n), x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x_Symbol] \rightarrow \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * \text{Simp}[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

$\text{Int}[(\cos[e + f*x] + (f*x)*g)^p * (a + b*\sin[e + f*x])^m, x_Symbol] \rightarrow \text{Dist}[(a^2 * (g*\cos[e + f*x])^{p+1}) / (f*g*(a + b*\sin[e + f*x])^{(p+1)/2} * (a - b*\sin[e + f*x])^{(p+1)/2}), \text{Subst}[\text{Int}[(a + b*x)^{m+(p-1)/2} * (a - b*x)^{(p-1)/2}, x], x, \sin[e + f*x]], x] /;$ FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2745

$\text{Int}[(a + b*\sin[e + f*x])^m * (c + d*\sin[e + f*x])^n, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]} * c^{\text{IntPart}[m]} * (a + b*\sin[e + f*x])^{\text{FracPart}[m]} * (c + d*\sin[e + f*x])^{\text{FracPart}[n]}), x]$

```

+ f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] || !FractionQ[n])

```

Rule 2973

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
+ 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^n dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^n}{f(1 + m + n)} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^n}{f(1 + m + n)} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^n}{f(1 + m + n)} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^n}{f(1 + m + n)} \\
&= \frac{2^{\frac{1}{2}+n} c \left(A + \frac{B(m-n)}{1+m+n} \right) \cos(e + fx) {}_2F_1 \left(\frac{1}{2}(1 + m + n), -n; \frac{3}{2} + n; -\tan^2 \left(\frac{-e + \pi/2 - fx}{4} \right) \right)}{f(1 + m + n)}
\end{aligned}$$

Mathematica [C] time = 14.24, size = 2903, normalized size = 16.68

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^
n,x]

```

```

[Out] (4*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)
/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 +
2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^
2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f
*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2])*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*(m
+ n))*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(A*Cos[(-e + Pi/2 - f*x
)/2]^(2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*n) + B*Cos[(-e + Pi/2 - f*x)/2]^(2*m
)*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Sin[e + f*x]*Tan[(-e + Pi/2 - f*x)/4])/(f
*(1 + 2*n)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((-4*m*(8*B*AppellF1[1/2
+ n, -2*m, 2*(1 + m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + P
i/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n,
Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2
+ n, -2*m, 3 + 2*(m + n), 3/2 + n, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e +
Pi/2 - f*x)/4]^2])*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*(Sec[(-e + Pi/2 - f*x)/4]
^2)^(1 + 2*(m + n))*Sin[(-e + Pi/2 - f*x)/2]^(2*n)*Tan[(-e + Pi/2 - f*x)/4]

```

$$\begin{aligned} & ^2*(1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{-1 - 2*m})/(1 + 2*n) - ((8*B*AppellF1[\\ & 1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e \\ & + \pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + \\ & n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2] - 8*B*AppellF1 \\ & [1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(- \\ & e + \pi/2 - f*x)/4]^2])*Cos[(-e + \pi/2 - f*x)/2]^{2*m}*(Sec[(-e + \pi/2 - f*x \\ &)/4]^2)^{(1 + 2*(m + n))*Sin[(-e + \pi/2 - f*x)/2]^{2*n})/((1 + 2*n)*(1 - \tan \\ & [(-e + \pi/2 - f*x)/4]^2)^{2*m}) - (4*n*(8*B*AppellF1[1/2 + n, -2*m, 2*(1 + \\ & m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2] - \\ & (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, \tan[(-e + \pi/2 - f \\ & *x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m, 3 + 2* \\ & (m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]) \\ & *Cos[(-e + \pi/2 - f*x)/2]^{(1 + 2*m)}*(Sec[(-e + \pi/2 - f*x)/4]^2)^{(2*(m + n) \\ &)}*Sin[(-e + \pi/2 - f*x)/2]^{(-1 + 2*n)}*\tan[(-e + \pi/2 - f*x)/4]/((1 + 2*n)* \\ & (1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{2*m}) + (4*m*(8*B*AppellF1[1/2 + n, -2*m, \\ & 2*(1 + m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x) \\ & /4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + n, \tan[(-e + \\ & \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2] - 8*B*AppellF1[1/2 + n, -2*m \\ & , 3 + 2*(m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x \\ &)/4]^2])*Cos[(-e + \pi/2 - f*x)/2]^{(-1 + 2*m)}*(Sec[(-e + \pi/2 - f*x)/4]^2)^{(\\ & 2*(m + n))*Sin[(-e + \pi/2 - f*x)/2]^{(1 + 2*n)}*\tan[(-e + \pi/2 - f*x)/4]/((1 \\ & + 2*n)*(1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{2*m}) - (4*(m + n)*(8*B*AppellF1[\\ & 1/2 + n, -2*m, 2*(1 + m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e \\ & + \pi/2 - f*x)/4]^2] - (A + B)*AppellF1[1/2 + n, -2*m, 1 + 2*(m + n), 3/2 + \\ & n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2] - 8*B*AppellF1 \\ & [1/2 + n, -2*m, 3 + 2*(m + n), 3/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(- \\ & e + \pi/2 - f*x)/4]^2])*Cos[(-e + \pi/2 - f*x)/2]^{2*m}*(Sec[(-e + \pi/2 - f*x \\ &)/4]^2)^{(2*(m + n))*Sin[(-e + \pi/2 - f*x)/2]^{2*n)}*\tan[(-e + \pi/2 - f*x)/4] \\ & ^2)/((1 + 2*n)*(1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{2*m}) - (4*Cos[(-e + \pi/2 \\ & - f*x)/2]^{2*m}*(Sec[(-e + \pi/2 - f*x)/4]^2)^{(2*(m + n))*Sin[(-e + \pi/2 - f \\ & *x)/2]^{2*n)}*\tan[(-e + \pi/2 - f*x)/4]*(-(A + B)*(-(m*(1/2 + n)*AppellF1[3 \\ & /2 + n, 1 - 2*m, 1 + 2*(m + n), 5/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(- \\ & e + \pi/2 - f*x)/4]^2]*Sec[(-e + \pi/2 - f*x)/4]^2*\tan[(-e + \pi/2 - f*x)/4]) \\ & /((3/2 + n)) - ((1/2 + n)*(1 + 2*(m + n))*AppellF1[3/2 + n, -2*m, 2 + 2*(m + \\ & n), 5/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]*Sec[\\ & (-e + \pi/2 - f*x)/4]^2*\tan[(-e + \pi/2 - f*x)/4])/(2*(3/2 + n)))) - 8*B*(-((\\ & m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 3 + 2*(m + n), 5/2 + n, \tan[(-e + \pi \\ & /2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]*Sec[(-e + \pi/2 - f*x)/4]^2*\tan \\ & [(-e + \pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(3 + 2*(m + n))*AppellF1[3/2 \\ & + n, -2*m, 4 + 2*(m + n), 5/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \\ & \pi/2 - f*x)/4]^2]*Sec[(-e + \pi/2 - f*x)/4]^2*\tan[(-e + \pi/2 - f*x)/4])/(2*(\\ & 3/2 + n))) + 8*B*(-((m*(1/2 + n)*AppellF1[3/2 + n, 1 - 2*m, 2*(1 + m + n), \\ & 5/2 + n, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]*Sec[(-e + \\ & \pi/2 - f*x)/4]^2*\tan[(-e + \pi/2 - f*x)/4])/(3/2 + n)) - ((1/2 + n)*(1 + m \\ & + n)*AppellF1[3/2 + n, -2*m, 1 + 2*(1 + m + n), 5/2 + n, \tan[(-e + \pi/2 - f \\ & *x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]*Sec[(-e + \pi/2 - f*x)/4]^2*\tan[(-e + \\ & \pi/2 - f*x)/4])/(3/2 + n))))/(1 + 2*n)*(1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{(\\ & 2*m)))) \end{aligned}$$

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(-c \sin(fx + e) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)

maple [F] time = 5.77, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m (-c(\sin(e + fx) - 1))^n (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**n,x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(-c*(sin(e + f*x) - 1))**n*(A + B*sin(e + f*x)), x)

$$3.196 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^3 dx$$

Optimal. Leaf size=145

$$\frac{a^4 c^3 2^{m+\frac{1}{2}} (B(3-m) - A(m+4)) \cos^7(e+fx) (\sin(e+fx)+1)^{\frac{1}{2}-m} (a \sin(e+fx)+a)^{m-4} {}_2F_1\left(\frac{7}{2}, \frac{1}{2}-m; \frac{9}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{7f(m+4)}$$

[Out] 1/7*2^(1/2+m)*a^4*c^3*(B*(3-m)-A*(4+m))*cos(f*x+e)^7*hypergeom([7/2, 1/2-m], [9/2], 1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-4+m)/f/(4+m)-a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(-3+m)/f/(4+m)

Rubi [A] time = 0.34, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^4 c^3 2^{m+\frac{1}{2}} (B(3-m) - A(m+4)) \cos^7(e+fx) (\sin(e+fx)+1)^{\frac{1}{2}-m} (a \sin(e+fx)+a)^{m-4} {}_2F_1\left(\frac{7}{2}, \frac{1}{2}-m; \frac{9}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{7f(m+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]

[Out] (2^(1/2 + m)*a^4*c^3*(B*(3 - m) - A*(4 + m))*Cos[e + f*x]^7*Hypergeometric2F1[7/2, 1/2 - m, 9/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-4 + m))/(7*f*(4 + m)) - (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(-3 + m))/(f*(4 + m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/((b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*

$\text{Sin}[e + f*x]^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * (A + B*\text{sin}[e + f*x]) * (c + d*\text{sin}[e + f*x])^n, x_Symbol] :> \text{Dist}[a^m * c^m, \text{Int}[\text{Cos}[e + f*x]^{2*m} * (c + d*\text{Sin}[e + f*x])^{n-m} * (A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& !(\text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) (c - c \sin(e + fx))^3 dx &= (a^3 c^3) \int \cos^6(e + fx) (a + a \sin(e + fx))^{-3+m} dx \\ &= -\frac{a^3 B c^3 \cos^7(e + fx) (a + a \sin(e + fx))^{-3+m}}{f(4 + m)} \\ &= -\frac{a^3 B c^3 \cos^7(e + fx) (a + a \sin(e + fx))^{-3+m}}{f(4 + m)} \\ &= -\frac{a^3 B c^3 \cos^7(e + fx) (a + a \sin(e + fx))^{-3+m}}{f(4 + m)} \\ &= -\frac{2^{\frac{1}{2}+m} a^4 c^3 \left(A - \frac{B(3-m)}{4+m} \right) \cos^7(e + fx) {}_2F_1}{f(4 + m)} \end{aligned}$$

Mathematica [F] time = 180.06, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^3,x]

[Out] \$Aborted

fricas [F] time = 0.48, size = 0, normalized size = 0.00

integral(-((B*c^3*cos(f*x + e))^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e))^2 - 4*(A

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] integral(-(B*c^3*cos(f*x + e))^4 + (3*A - 5*B)*c^3*cos(f*x + e)^2 - 4*(A - B)*c^3 - ((A - 3*B)*c^3*cos(f*x + e))^2 - 4*(A - B)*c^3)*sin(f*x + e)*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(B \sin(fx + e) + A)(c \sin(fx + e) - c)^3 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 8.10, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (B \sin(fx + e) + A)(c \sin(fx + e) - c)^3 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)

mapad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^3,x)

[Out] Timed out

$$3.197 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^2 dx$$

Optimal. Leaf size=145

$$\frac{a^3 c^2 2^{m+\frac{1}{2}} (B(2-m) - A(m+3)) \cos^5(e+fx) (\sin(e+fx) + 1)^{\frac{1}{2}-m} (a \sin(e+fx) + a)^{m-3} {}_2F_1\left(\frac{5}{2}, \frac{1}{2} - m; \frac{7}{2}; \frac{1}{2}(1 + \sin(e+fx))\right)}{5f(m+3)}$$

[Out] 1/5*2^(1/2+m)*a^3*c^2*(B*(2-m)-A*(3+m))*cos(f*x+e)^5*hypergeom([5/2, 1/2-m], [7/2], 1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(m-3)/f/(3+m)-a^2*B*c^2*cos(f*x+e)^5*(a+a*sin(f*x+e))^(m-2)/f/(3+m)

Rubi [A] time = 0.33, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^3 c^2 2^{m+\frac{1}{2}} (B(2-m) - A(m+3)) \cos^5(e+fx) (\sin(e+fx) + 1)^{\frac{1}{2}-m} (a \sin(e+fx) + a)^{m-3} {}_2F_1\left(\frac{5}{2}, \frac{1}{2} - m; \frac{7}{2}; \frac{1}{2}(1 + \sin(e+fx))\right)}{5f(m+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]

[Out] (2^(1/2 + m)*a^3*c^2*(B*(2 - m) - A*(3 + m))*Cos[e + f*x]^5*Hypergeometric2F1[5/2, 1/2 - m, 7/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(m - 3))/(5*f*(3 + m)) - (a^2*B*c^2*Cos[e + f*x]^5*(a + a*Sin[e + f*x])^(m - 2))/(f*(3 + m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/((b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*(c + d*x)/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*

$\text{Sin}[e + f*x]^m, x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a + b*\text{sin}[e + f*x])^m * (A + B*\text{sin}[e + f*x]) * (c + d*\text{sin}[e + f*x])^n, x_Symbol] := \text{Dist}[a^m * c^m, \text{Int}[\text{Cos}[e + f*x]^{2*m} * (c + d*\text{Sin}[e + f*x])^{n-m} * (A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& ((\text{LtQ}[m, 0] \& \& \text{GtQ}[n, 0]) || \text{LtQ}[0, n, m] || \text{LtQ}[m, n, 0])$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) (c - c \sin(e + fx))^2 dx &= (a^2 c^2) \int \cos^4(e + fx) (a + a \sin(e + fx))^{-2+m} \\ &= -\frac{a^2 B c^2 \cos^5(e + fx) (a + a \sin(e + fx))^{-2+m}}{f(3 + m)} + \\ &= -\frac{a^2 B c^2 \cos^5(e + fx) (a + a \sin(e + fx))^{-2+m}}{f(3 + m)} + \\ &= -\frac{a^2 B c^2 \cos^5(e + fx) (a + a \sin(e + fx))^{-2+m}}{f(3 + m)} + \\ &= -\frac{2^{\frac{1}{2}+m} a^3 c^2 \left(A - \frac{B(2-m)}{3+m} \right) \cos^5(e + fx) {}_2F_1\left(\frac{5}{2}, \right)}{f(3 + m)} \end{aligned}$$

Mathematica [F] time = 180.08, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^2,x]

[Out] \$Aborted

fricas [F] time = 0.48, size = 0, normalized size = 0.00

integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e)^2 + 2*(A - B)*c^2)*sin(f*x + e))(a*sin(f*x + e) + a)^m, x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-((A - 2*B)*c^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + (B*c^2*cos(f*x + e)^2 + 2*(A - B)*c^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(c \sin(fx + e) - c)^2 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 8.31, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(c \sin(fx + e) - c)^2 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)^2*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int A (a \sin(e + fx) + a)^m dx + \int \left(-2A (a \sin(e + fx) + a)^m \sin(e + fx) \right) dx + \int A (a \sin(e + fx) + a)^m dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**2,x)

[Out] c**2*(Integral(A*(a*sin(e + f*x) + a)**m, x) + Integral(-2*A*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-2*B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**3, x))

$$3.198 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx)) dx$$

Optimal. Leaf size=139

$$\frac{a^2 c 2^{m+\frac{1}{2}} (B(1-m) - A(m+2)) \cos^3(e+fx) (\sin(e+fx) + 1)^{\frac{1}{2}-m} (a \sin(e+fx) + a)^{m-2} {}_2F_1\left(\frac{3}{2}, \frac{1}{2}-m; \frac{5}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{3f(m+2)}$$

[Out] 1/3*2^(1/2+m)*a^2*c*(B*(1-m)-A*(2+m))*cos(f*x+e)^3*hypergeom([3/2, 1/2-m], [5/2], 1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(-2+m)/f/(2+m)-a*B*c*cos(f*x+e)^3*(a+a*sin(f*x+e))^(-1+m)/f/(2+m)

Rubi [A] time = 0.29, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{a^2 c 2^{m+\frac{1}{2}} (B(1-m) - A(m+2)) \cos^3(e+fx) (\sin(e+fx) + 1)^{\frac{1}{2}-m} (a \sin(e+fx) + a)^{m-2} {}_2F_1\left(\frac{3}{2}, \frac{1}{2}-m; \frac{5}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{3f(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]),x]

[Out] (2^(1/2 + m)*a^2*c*(B*(1 - m) - A*(2 + m))*Cos[e + f*x]^3*Hypergeometric2F1[3/2, 1/2 - m, 5/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(-2 + m))/(3*f*(2 + m)) - (a*B*c*Cos[e + f*x]^3*(a + a*Sin[e + f*x])^(-1 + m))/(f*(2 + m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/((b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*

$\text{Sin}[e + f*x]^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[\{(a_) + (b_)*\text{sin}[(e_) + (f_)*(x_)]\}^{(m_)}*\{(A_) + (B_)*\text{sin}[(e_) + (f_)*(x_)]\}*\{(c_) + (d_)*\text{sin}[(e_) + (f_)*(x_)]\}^{(n_)}, x_Symbol] :> \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n - m)}*(A + B*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& (\text{LtQ}[m, 0] \&\& \text{GtQ}[n, 0]) \text{ || } \text{LtQ}[0, n, m] \text{ || } \text{LtQ}[m, n, 0])$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx)) dx &= (ac) \int \cos^2(e + fx)(a + a \sin(e + fx))^{-1+m} \\ &= -\frac{aBc \cos^3(e + fx)(a + a \sin(e + fx))^{-1+m}}{f(2 + m)} + \\ &= -\frac{aBc \cos^3(e + fx)(a + a \sin(e + fx))^{-1+m}}{f(2 + m)} + \\ &= -\frac{aBc \cos^3(e + fx)(a + a \sin(e + fx))^{-1+m}}{f(2 + m)} + \\ &= -\frac{2^{\frac{1}{2}+m} a^2 c \left(A - \frac{B(1-m)}{2+m} \right) \cos^3(e + fx) {}_2F_1\left(\frac{3}{2}, \dots\right)}{f(2 + m)} \end{aligned}$$

Mathematica [C] time = 4.22, size = 462, normalized size = 3.32

$$ic4^{-m-1} e^{ifmx} (1 + ie^{-i(e+fx)})^{-2m} \left((-1)^{3/4} e^{-\frac{1}{2}i(e+fx)} (e^{i(e+fx)} + i) \right)^{2m} (\sin(e + fx) - 1) \sin^{-2m} \left(\frac{1}{4}(2e + 2fx + \pi) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x]), x]

[Out] $(I*4^{(-1 - m)*c}*E^{(I*f*m*x)}*(-(((-1)^{(3/4)}*(I + E^{(I*(e + f*x))}))/E^{((I/2)*(e + f*x))})^{(2*m)}*(((-I)*B*Hypergeometric2F1[-2 - m, -2*m, -1 - m, (-I)/E^{(I*(e + f*x))}]/(E^{(I*(2*e + f*(2 + m)*x)})*(2 + m)) + (2*((-I)*A + B)*Hypergeometric2F1[-1 - m, -2*m, -m, (-I)/E^{(I*(e + f*x))}]/(E^{(I*(e + f*(1 + m)*x)})*(1 + m)) + ((2*I)*A*E^{(I*(e - f*(-1 + m)*x)})*Hypergeometric2F1[1 - m, -2*m, 2 - m, (-I)/E^{(I*(e + f*x))}]/(-1 + m) + (2*B*E^{(I*(e - f*(-1 + m)*x)})*Hypergeometric2F1[1 - m, -2*m, 2 - m, (-I)/E^{(I*(e + f*x))}]/(-1 + m) + (I*B*E^{((2*I)*e - I*f*(-2 + m)*x)}*Hypergeometric2F1[2 - m, -2*m, 3 - m, (-I)/E^{(I*(e + f*x))}]/(-2 + m) + (4*A*Hypergeometric2F1[-2*m, -m, 1 - m, (-I)/E^{(I*(e + f*x))}]/(E^{(I*f*m*x)}*m))*(-1 + Sin[e + f*x])*(a*(1 + Sin[e + f*x]))^m)/((1 + I/E^{(I*(e + f*x))})^{(2*m)}*f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^2*Sin[(2*e + Pi + 2*f*x)/4]^{(2*m)})$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Bc \cos(fx + e)^2 - (A - B)c \sin(fx + e) + (A - B)c\right)(a \sin(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*c*cos(f*x + e)^2 - (A - B)*c*sin(f*x + e) + (A - B)*c)*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(B \sin(fx + e) + A)(c \sin(fx + e) - c)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] integrate(-(B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 4.98, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))(c - c \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (B \sin(fx + e) + A)(c \sin(fx + e) - c)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) - c)*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c \left(\int (-A(a \sin(e + fx) + a)^m) dx + \int A(a \sin(e + fx) + a)^m \sin(e + fx) dx + \int (-B(a \sin(e + fx) + a))^m \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e)),x)

[Out] -c*(Integral(-A*(a*sin(e + f*x) + a)**m, x) + Integral(A*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(-B*(a*sin(e + f*x) + a)**m*sin(e + f*x), x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)**2, x))

3.199 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx$

Optimal. Leaf size=117

$$\frac{2^{m+\frac{1}{2}}(Am + A + Bm) \cos(e + fx)(\sin(e + fx) + 1)^{-m-\frac{1}{2}}(a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(m+1)}$$

[Out] $-B*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(1+m)-2^{(1/2+m)}*(A*m+B*m+A)*\cos(f*x+e)*\text{hypergeom}([1/2, 1/2-m], [3/2], 1/2-1/2*\sin(f*x+e))*(1+\sin(f*x+e))^{(-1/2-m)}*(a+a*\sin(f*x+e))^m/f/(1+m)$

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}}(Am + A + Bm) \cos(e + fx)(\sin(e + fx) + 1)^{-m-\frac{1}{2}}(a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x]), x]$

[Out] $-((B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + m))) - (2^{(1/2 + m)}*(A + A*m + B*m)*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Sin}[e + f*x])/2]*(1 + \text{Sin}[e + f*x])^{(-1/2 - m)}*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + m))$

Rule 2651

$\text{Int}(((a_) + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol) \rightarrow -\text{Simp}[(2^{(n + 1/2)}*a^{(n - 1/2)}*b*\text{Cos}[c + d*x]*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1*(1 - (b*\text{Sin}[c + d*x])/a))/2])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2652

$\text{Int}(((a_) + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol) \rightarrow \text{Dist}[(a^{\text{IntPart}[n]}*(a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]})/(1 + (b*\text{Sin}[c + d*x])/a)^{\text{FracPart}[n]}], \text{Int}[(1 + (b*\text{Sin}[c + d*x])/a)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{IntegerQ}[2*n] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 2751

$\text{Int}(((a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol) \rightarrow -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} + \frac{(A + Am + Bm) \int (a + a \sin(e + fx))^m dx}{1 + m} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} + \frac{((A + Am + Bm)(1 + \sin(e + fx))^{1/2+m} - (A + Am + Bm) \cos(e + fx)(a + a \sin(e + fx))^m)}{2^{1/2+m} f(1 + m)} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} - \frac{2^{1/2+m} (A + Am + Bm) \cos(e + fx)(a + a \sin(e + fx))^m}{2^{1/2+m} f(1 + m)} \end{aligned}$$

Mathematica [C] time = 1.83, size = 275, normalized size = 2.35

$$\sin^{-2m}\left(\frac{1}{4}(2e+2fx+\pi)\right)(a(\sin(e+fx)+1))^m \left(\frac{2\sqrt{2}A\sin\left(\frac{1}{4}(2e+2fx-\pi)\right)\cos^{2m+1}\left(\frac{1}{4}(2e+2fx-\pi)\right) {}_2F_1\left(\frac{1}{2}, m+\frac{1}{2}; m+\frac{3}{2}; \sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)\right)}{(2m+1)\sqrt{1-\sin(e+fx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]), x]

[Out] -(((a*(1 + Sin[e + f*x]))^m*(((-1)^(1/4)*2^(-1 - 2*m)*B*(-(((-1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))*(-1 + m)*Hypergeometric2F1[1, m, -m, (-I)/E^(I*(e + f*x))] - (1 + m)*Hypergeometric2F1[1, 2 + m, 2 - m, (-I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))*(-1 + m^2)) + (2*sqrt[2]*A*cos[(2*e - Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e - Pi + 2*f*x)/4])/((1 + 2*m)*sqrt[1 - Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]^(2*m)))

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B\sin(fx+e)+A\right)\left(a\sin(fx+e)+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)), x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 2.53, size = 0, normalized size = 0.00

$$\int (a + a\sin(fx+e))^m (A + B\sin(fx+e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)), x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e)+A)(a\sin(fx+e)+a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin(e + f x) + 1))^m (A + B \sin(e + f x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)), x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)

$$3.200 \quad \int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{c-c \sin(e+fx)} dx$$

Optimal. Leaf size=123

$$\frac{2^{m+\frac{1}{2}}(Am + Bm + B) \sec(e + fx)(\sin(e + fx) + 1)^{\frac{1}{2}-m}(a \sin(e + fx) + a)^m {}_2F_1\left(-\frac{1}{2}, \frac{1}{2} - m; \frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{cfm}$$

[Out] $2^{(1/2+m)}*(A*m+B*m+B)*\text{hypergeom}([-1/2, 1/2-m], [1/2], 1/2-1/2*\sin(f*x+e))*\sec(f*x+e)*(1+\sin(f*x+e))^{(1/2-m)}*(a+a*\sin(f*x+e))^{m/c/f/m-B*\sec(f*x+e)*(a+a*\sin(f*x+e))^{(1+m)}/a/c/f/m}$

Rubi [A] time = 0.30, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(Am + Bm + B) \sec(e + fx)(\sin(e + fx) + 1)^{\frac{1}{2}-m}(a \sin(e + fx) + a)^m {}_2F_1\left(-\frac{1}{2}, \frac{1}{2} - m; \frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{cfm}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x])]/(c - c*\text{Sin}[e + f*x]), x]$

[Out] $(2^{(1/2 + m)}*(B + A*m + B*m)*\text{Hypergeometric2F1}[-1/2, 1/2 - m, 1/2, (1 - \text{Sin}[e + f*x])/2]*\text{Sec}[e + f*x]*(1 + \text{Sin}[e + f*x])^{(1/2 - m)}*(a + a*\text{Sin}[e + f*x])^m)/(c*f*m) - (B*\text{Sec}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(1 + m)})/(a*c*f*m)$

Rule 69

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1}*\text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x))/(b*c - a*d)]/(b*(m+1)*(b/(b*c - a*d))^n), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{IntegerQ}\{m\} \&\& \text{IntegerQ}\{n\} \&\& \text{GtQ}\{b/(b*c - a*d), 0\} \&\& (\text{RationalQ}\{m\} \parallel \text{IntegerQ}\{n\} \&\& \text{GtQ}\{-d/(b*c - a*d), 0\})$

Rule 70

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \text{Dist}[(c + d*x)^{\text{FracPart}[n]}/(b/(b*c - a*d))^{\text{IntPart}[n]}*(b*(c + d*x)/(b*c - a*d))^{\text{FracPart}[n]}, \text{Int}[(a + b*x)^m*\text{Simp}[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{IntegerQ}\{m\} \&\& \text{IntegerQ}\{n\} \&\& (\text{RationalQ}\{m\} \parallel \text{SimplerQ}\{n + 1, m + 1\})$

Rule 2689

$\text{Int}[(\cos[e + f*x] + (f*x)*g)^p*(a + b*\sin[e + f*x])^m, x_Symbol] \rightarrow \text{Dist}[(a^2*(g*\cos[e + f*x])^{p+1})/(f*g*(a + b*\sin[e + f*x])^{(p+1)/2}*(a - b*\sin[e + f*x])^{(p+1)/2}), \text{Subst}[\text{Int}[(a + b*x)^{m+(p-1)/2}*(a - b*x)^{(p-1)/2}, x], x, \sin[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, g, m, p\}, x \&\& \text{EqQ}\{a^2 - b^2, 0\} \&\& \text{IntegerQ}\{m\}$

Rule 2860

$\text{Int}[(\cos[e + f*x] + (f*x)*g)^p*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x]), x_Symbol] \rightarrow -\text{Simp}[(d*(g*\cos[e + f*x])^{p+1}*(a + b*\sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), \text{Int}[(g*\cos[e + f*x])^p*(a + b*\sin[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x \&\& \text{EqQ}\{a^2 - b^2, 0\} \&\& \text{NeQ}\{m + p + 1, 0\}$

Rule 2967

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[
e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d
, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] &
& GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{c - c \sin(e + fx)} dx &= \frac{\int \sec^2(e + fx) (a + a \sin(e + fx))^{1+m} (A + B \sin(e + fx)) dx}{ac} \\ &= -\frac{B \sec(e + fx) (a + a \sin(e + fx))^{1+m}}{acfm} + \frac{(B + Am + Bm) \int \sec^2(e + fx) (a + a \sin(e + fx))^{1+m} dx}{acfm} \\ &= -\frac{B \sec(e + fx) (a + a \sin(e + fx))^{1+m}}{acfm} + \frac{(a(B + Am + Bm) \int \sec^2(e + fx) (a + a \sin(e + fx))^{1+m} dx)}{acfm} \\ &= -\frac{B \sec(e + fx) (a + a \sin(e + fx))^{1+m}}{acfm} + \frac{\left(2^{-\frac{1}{2}+m} a(B + Am + Bm) \int \sec^2(e + fx) (a + a \sin(e + fx))^{1+m} dx\right)}{acfm} \\ &= \frac{2^{\frac{1}{2}+m} (B + Am + Bm) {}_2F_1\left(-\frac{1}{2}, \frac{1}{2} - m; \frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) \int \sec^2(e + fx) (a + a \sin(e + fx))^{1+m} dx}{cfm} \end{aligned}$$

Mathematica [C] time = 25.57, size = 7409, normalized size = 60.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x]),x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{c \sin(fx + e) - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{c \sin(fx + e) - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="giac")

[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)

maple [F] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{c - c \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{c \sin(fx + e) - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x, algorithm="maxima")

[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{c - c \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A(a \sin(e+fx)+a)^m}{\sin(e+fx)-1} dx + \int \frac{B(a \sin(e+fx)+a)^m \sin(e+fx)}{\sin(e+fx)-1} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e)),x)

[Out] -(Integral(A*(a*sin(e + f*x) + a)^m/(sin(e + f*x) - 1), x) + Integral(B*(a*sin(e + f*x) + a)^m*sin(e + f*x)/(sin(e + f*x) - 1), x))/c

$$3.201 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^2} dx$$

Optimal. Leaf size=148

$$\frac{B \sec^3(e+fx)(a \sin(e+fx)+a)^{m+2}}{a^2 c^2 f(1-m)} + \frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2)) \sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a \sin(e+fx)+a)^{m+2}}{3ac^2 f(1-m)}$$

[Out] 1/3*2^(1/2+m)*(A*(1-m)-B*(2+m))*hypergeom([-3/2, 1/2-m], [-1/2], 1/2-1/2*sin(f*x+e))*sec(f*x+e)^3*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(1+m)/a/c^2/f/(1-m)+B*sec(f*x+e)^3*(a+a*sin(f*x+e))^(2+m)/a^2/c^2/f/(1-m)

Rubi [A] time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{B \sec^3(e+fx)(a \sin(e+fx)+a)^{m+2}}{a^2 c^2 f(1-m)} + \frac{2^{m+\frac{1}{2}}(A(1-m)-B(m+2)) \sec^3(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a \sin(e+fx)+a)^{m+2}}{3ac^2 f(1-m)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] (2^(1/2 + m)*(A*(1 - m) - B*(2 + m))*Hypergeometric2F1[-3/2, 1/2 - m, -1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^3*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(1 + m))/(3*a*c^2*f*(1 - m)) + (B*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(2 + m))/(a^2*c^2*f*(1 - m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2

- b^2, 0] && NeQ[m + p + 1, 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c - c \sin(e + fx))^2} dx &= \frac{\int \sec^4(e + fx) (a + a \sin(e + fx))^{2+m} (A + B \sin(e + fx)) dx}{a^2 c^2} \\ &= \frac{B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{a^2 c^2 f (1 - m)} + \frac{\left(A - \frac{B(2+m)}{1-m}\right) \int \sec^4(e + fx) dx}{a^2 c^2 f (1 - m)} \\ &= \frac{B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{a^2 c^2 f (1 - m)} + \frac{\left(\left(A - \frac{B(2+m)}{1-m}\right) \sec^3(e + fx) + \frac{B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{a^2 c^2 f (1 - m)}\right)}{a^2 c^2 f (1 - m)} \\ &= \frac{B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{a^2 c^2 f (1 - m)} + \frac{\left(2^{-\frac{1}{2}+m} \left(A - \frac{B(2+m)}{1-m}\right) \sec^3(e + fx) + \frac{B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{a^2 c^2 f (1 - m)}\right)}{a^2 c^2 f (1 - m)} \\ &= \frac{2^{\frac{1}{2}+m} \left(A - \frac{B(2+m)}{1-m}\right) {}_2F_1\left(-\frac{3}{2}, \frac{1}{2} - m; -\frac{1}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) \sec^3(e + fx) + B \sec^3(e + fx) (a + a \sin(e + fx))^{2+m}}{3 a^2 c^2 f} \end{aligned}$$

Mathematica [C] time = 6.93, size = 9240, normalized size = 62.43

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^2,x]

[Out] Result too large to show

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(c \sin(fx + e) - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)

maple [F] time = 8.47, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c - c \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(c \sin(fx + e) - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A(a \sin(e+fx)+a)^m}{\sin^2(e+fx)-2 \sin(e+fx)+1} dx + \int \frac{B(a \sin(e+fx)+a)^m \sin(e+fx)}{\sin^2(e+fx)-2 \sin(e+fx)+1} dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**2,x)

[Out] (Integral(A*(a*sin(e + f*x) + a)**m/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1), x) + Integral(B*(a*sin(e + f*x) + a)**m*sin(e + f*x)/(sin(e + f*x)**2 - 2*sin(e + f*x) + 1), x))/c**2

$$3.202 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^3} dx$$

Optimal. Leaf size=148

$$\frac{B \sec^5(e+fx)(a \sin(e+fx)+a)^{m+3}}{a^3 c^3 f(2-m)} + \frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3)) \sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a \sin(e+fx)+a)^{m+2}}{5a^2 c^3 f(2-m)}$$

[Out] 1/5*2^(1/2+m)*(A*(2-m)-B*(3+m))*hypergeom([-5/2, 1/2-m], [-3/2], 1/2-1/2*sin(f*x+e))*sec(f*x+e)^5*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^(2+m)/a^2/c^3/f/(2-m)+B*sec(f*x+e)^5*(a+a*sin(f*x+e))^(3+m)/a^3/c^3/f/(2-m)

Rubi [A] time = 0.33, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2967, 2860, 2689, 70, 69}

$$\frac{2^{m+\frac{1}{2}}(A(2-m)-B(m+3)) \sec^5(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a \sin(e+fx)+a)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{1}{2}-m; -\frac{3}{2}; \frac{1}{2}(1-\sin(e+fx))\right)}{5a^2 c^3 f(2-m)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] (2^(1/2 + m)*(A*(2 - m) - B*(3 + m))*Hypergeometric2F1[-5/2, 1/2 - m, -3/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]^5*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^(2 + m))/(5*a^2*c^3*f*(2 - m)) + (B*Sec[e + f*x]^5*(a + a*Sin[e + f*x])^(3 + m))/(a^3*c^3*f*(2 - m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2860

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dist[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1)), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2

$-b^2, 0] \ \&\& \ \text{NeQ}[m + p + 1, 0]$

Rule 2967

$\text{Int}[(a + b \sin(e + f x))^m (A + B \sin(e + f x)) (c + d \sin(e + f x))^n, x_Symbol] \rightarrow \text{Dist}[a^m c^m, \text{Int}[\text{Cos}[e + f x]^{2m} (c + d \sin[e + f x])^{n-m} (A + B \sin[e + f x]), x], x] /;$ $\text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{EqQ}[b c + a d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]) \ || \ \text{LtQ}[0, n, m] \ || \ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c - c \sin(e + fx))^3} dx &= \frac{\int \sec^6(e + fx) (a + a \sin(e + fx))^{3+m} (A + B \sin(e + fx)) dx}{a^3 c^3} \\ &= \frac{B \sec^5(e + fx) (a + a \sin(e + fx))^{3+m}}{a^3 c^3 f (2 - m)} + \frac{\left(A - \frac{B(3+m)}{2-m}\right) \int \sec^6(e + fx) (a + a \sin(e + fx))^{3+m} dx}{a^3 c^3 f (2 - m)} \\ &= \frac{B \sec^5(e + fx) (a + a \sin(e + fx))^{3+m}}{a^3 c^3 f (2 - m)} + \frac{\left(\left(A - \frac{B(3+m)}{2-m}\right) \sec^5(e + fx) \int \sec^5(e + fx) (a + a \sin(e + fx))^{3+m} dx\right)}{a^3 c^3 f (2 - m)} \\ &= \frac{B \sec^5(e + fx) (a + a \sin(e + fx))^{3+m}}{a^3 c^3 f (2 - m)} + \frac{\left(2^{-\frac{1}{2}+m} \left(A - \frac{B(3+m)}{2-m}\right) \sec^5(e + fx) \int \sec^5(e + fx) (a + a \sin(e + fx))^{3+m} dx\right)}{a^3 c^3 f (2 - m)} \\ &= \frac{2^{\frac{1}{2}+m} \left(A - \frac{B(3+m)}{2-m}\right) {}_2F_1\left(-\frac{5}{2}, \frac{1}{2} - m; -\frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) \sec^5(e + fx) (a + a \sin(e + fx))^{3+m}}{5a^2 c^3 f} \end{aligned}$$

Mathematica [C] time = 7.18, size = 12580, normalized size = 85.00

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^3,x]

[Out] Result too large to show

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{3c^3 \cos(fx + e)^2 - 4c^3 - (c^3 \cos(fx + e)^2 - 4c^3) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(c \sin(fx + e) - c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)

maple [F] time = 9.30, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c - c \sin(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(c \sin(fx + e) - c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^3,x, algorithm="maxima")

[Out] -integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx))(a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**3,x)

[Out] Timed out

$$3.203 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=118

$$\frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c \sin(e+fx)}} - \frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

[Out] $-2*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{(1/2)}+(A+B)*\cos(f*x+e)*\text{hypergeom}([1, 1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e))*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c \sin(e+fx)}} - \frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x])/ \text{Sqrt}[c - c*\text{Sin}[e + f*x]], x]$

[Out] $(-2*B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + 2*m)*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((A + B)*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1, 1/2 + m, 3/2 + m, (1 + \text{Sin}[e + f*x])/2]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + 2*m)*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 68

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*(a + b*x)^{m+1}*\text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x)/(b*c - a*d))]/(b^{n+1}*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 2667

$\text{Int}[\cos[(e + f*x)]^{p+1}, x_Symbol] \rightarrow \text{Dist}[1/(b^{p+1}), \text{Subst}[\text{Int}[(a + x)^{m+(p-1)/2}*(a-x)^{-(p-1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] \mid \mid !\text{IntegerQ}[m + 1/2])$

Rule 2745

$\text{Int}[(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^n, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]}*c^{\text{IntPart}[m]}*(a + b*\sin[e + f*x])^{\text{FracPart}[m]}*(c + d*\sin[e + f*x])^{\text{FracPart}[m]})/\text{Cos}[e + f*x]^{(2*\text{FracPart}[m])}, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\sin[e + f*x])^{n-m}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{FractionQ}[m] \mid \mid !\text{FractionQ}[n])$

Rule 2973

$\text{Int}[(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x])^n, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a +$

$b \sin[e + f x]^m (c + d \sin[e + f x])^n, x, x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x \ \&\& \text{EqQ}[b c + a d, 0] \ \&\& \text{EqQ}[a^2 - b^2, 0] \ \&\& !\text{LtQ}[m, -2^{-1}] \ \&\& \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + f x))^m (A + B \sin(e + f x))}{\sqrt{c - c \sin(e + f x)}} dx &= -\frac{2B \cos(e + f x)(a + a \sin(e + f x))^m}{f(1 + 2m)\sqrt{c - c \sin(e + f x)}} + (A + B) \int \frac{(a + a \sin(e + f x))^m}{\sqrt{c - c \sin(e + f x)}} dx \\ &= -\frac{2B \cos(e + f x)(a + a \sin(e + f x))^m}{f(1 + 2m)\sqrt{c - c \sin(e + f x)}} + \frac{((A + B) \cos(e + f x)) \int \frac{(a + a \sin(e + f x))^m}{\sqrt{a + a \sin(e + f x)}} dx}{\sqrt{a + a \sin(e + f x)}} \\ &= -\frac{2B \cos(e + f x)(a + a \sin(e + f x))^m}{f(1 + 2m)\sqrt{c - c \sin(e + f x)}} + \frac{(a(A + B) \cos(e + f x)) \int \frac{(a + a \sin(e + f x))^m}{\sqrt{a + a \sin(e + f x)}} dx}{f \sqrt{a + a \sin(e + f x)}} \\ &= -\frac{2B \cos(e + f x)(a + a \sin(e + f x))^m}{f(1 + 2m)\sqrt{c - c \sin(e + f x)}} + \frac{(A + B) \cos(e + f x) {}_2F_1\left(1, 2m; 2m + 1; \frac{a + a \sin(e + f x)}{a}\right)}{f} \end{aligned}$$

Mathematica [A] time = 4.87, size = 200, normalized size = 1.69

$$2^{-2m - \frac{3}{2}} \sin\left(\frac{1}{4}(2e + 2fx + \pi)\right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) (a(\sin(e + fx) + 1))^m \left(2^{2m+1}(A + B) {}_2F_1\left(1, 2m; 2m + 1; \frac{a + a \sin(e + fx)}{a}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(B \sin(fx + e) + A)\sqrt{-c \sin(fx + e) + c}(a \sin(fx + e) + a)^m}{c \sin(fx + e) - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{\sqrt{c - c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{\sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*x) - 1)), x)

$$3.204 \quad \int \frac{(A+B \sin(e+fx))(c+c \sin(e+fx))^m}{\sqrt{a-a \sin(e+fx)}} dx$$

Optimal. Leaf size=118

$$\frac{(A+B) \cos(e+fx)(c \sin(e+fx)+c)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a \sin(e+fx)}} - \frac{2B \cos(e+fx)(c \sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a \sin(e+fx)}}$$

[Out] $-2*B*\cos(f*x+e)*(c+c*\sin(f*x+e))^m/f/(1+2*m)/(a-a*\sin(f*x+e))^{1/2}+(A+B)*\cos(f*x+e)*\text{hypergeom}([1, 1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e))*(c+c*\sin(f*x+e))^m/f/(1+2*m)/(a-a*\sin(f*x+e))^{1/2}$

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B) \cos(e+fx)(c \sin(e+fx)+c)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{a-a \sin(e+fx)}} - \frac{2B \cos(e+fx)(c \sin(e+fx)+c)^m}{f(2m+1)\sqrt{a-a \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sin}[e+f*x])*(c+c*\text{Sin}[e+f*x])^m/\text{Sqrt}[a-a*\text{Sin}[e+f*x]], x]$

[Out] $(-2*B*\text{Cos}[e+f*x]*(c+c*\text{Sin}[e+f*x])^m)/(f*(1+2*m)*\text{Sqrt}[a-a*\text{Sin}[e+f*x]]) + ((A+B)*\text{Cos}[e+f*x]*\text{Hypergeometric2F1}[1, 1/2+m, 3/2+m, (1+\text{Sin}[e+f*x])/2]*(c+c*\text{Sin}[e+f*x])^m)/(f*(1+2*m)*\text{Sqrt}[a-a*\text{Sin}[e+f*x]])$

Rule 68

$\text{Int}[(a_+ + (b_+)*(x_+))^{(m_+)}*((c_+ + (d_+)*(x_+))^{(n_+)}, x_Symbol] :> \text{Simp}[(b*c - a*d)^n*(a + b*x)^{(m+1)}*\text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x))/(b*c - a*d)]]/(b^{(n+1)}*(m+1)), x] /; \text{FreeQ}\{a, b, c, d, m\}, x \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& !\text{IntegerQ}\{m\} \&\& \text{IntegerQ}\{n\}$

Rule 2667

$\text{Int}[\cos[(e_+) + (f_+)*(x_+)]^{(p_+)}*((a_+) + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}, x_Symbol] :> \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a+x)^{(m+(p-1)/2)}*(a-x)^{(p-1)/2}, x], x, b*\text{Sin}[e+f*x]], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{GeQ}[p, -1] || !\text{IntegerQ}[m+1/2])]$

Rule 2745

$\text{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}*((c_+) + (d_+)*\sin[(e_+) + (f_+)*(x_+)])^{(n_+)}, x_Symbol] :> \text{Dist}[(a^{\text{IntPart}[m]}*c^{\text{IntPart}[m]}*(a + b*\text{Sin}[e+f*x])^{\text{FracPart}[m]}*(c + d*\text{Sin}[e+f*x])^{\text{FracPart}[m]})/\text{Cos}[e+f*x]^{(2*\text{FracPart}[m])}, \text{Int}[\text{Cos}[e+f*x]^{(2*m)}*(c + d*\text{Sin}[e+f*x])^{(n-m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& (\text{FractionQ}[m] || !\text{FractionQ}[n])]$

Rule 2973

$\text{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)])^{(m_+)}*((A_+) + (B_+)*\sin[(e_+) + (f_+)*(x_+)])^{(n_+)}, x_Symbol] :> -\text{Simp}[(B*\text{Cos}[e+f*x]*(a + b*\text{Sin}[e+f*x])^m*(c + d*\text{Sin}[e+f*x])^n)/(f*(m+n+1)), x] - \text{Dist}[(B*c*(m-n) - A*d*(m+n+1))/(d*(m+n+1)), \text{Int}[(a +$

$b \sin(e + fx)^m (c + d \sin(e + fx))^n, x, x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2] \ \&\& \ \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + c \sin(e + fx))^m}{\sqrt{a - a \sin(e + fx)}} dx &= -\frac{2B \cos(e + fx)(c + c \sin(e + fx))^m}{f(1 + 2m)\sqrt{a - a \sin(e + fx)}} - (-A - B) \int \frac{(c + c \sin(e + fx))^m}{\sqrt{a - a \sin(e + fx)}} dx \\ &= -\frac{2B \cos(e + fx)(c + c \sin(e + fx))^m}{f(1 + 2m)\sqrt{a - a \sin(e + fx)}} - \frac{((-A - B) \cos(e + fx)) \int (c + c \sin(e + fx))^m dx}{\sqrt{a - a \sin(e + fx)}} \\ &= -\frac{2B \cos(e + fx)(c + c \sin(e + fx))^m}{f(1 + 2m)\sqrt{a - a \sin(e + fx)}} - \frac{((-A - B)c \cos(e + fx)) \int (c + c \sin(e + fx))^m dx}{f\sqrt{a - a \sin(e + fx)}} \\ &= -\frac{2B \cos(e + fx)(c + c \sin(e + fx))^m}{f(1 + 2m)\sqrt{a - a \sin(e + fx)}} + \frac{(A + B) \cos(e + fx) \int (c + c \sin(e + fx))^m dx}{f\sqrt{a - a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 2.38, size = 200, normalized size = 1.69

$$2^{-2m-\frac{3}{2}} \sin\left(\frac{1}{4}(2e + 2fx + \pi)\right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) (c(\sin(e + fx) + 1))^m \left(2^{2m+1}(A + B) {}_2F_1\left(1, 1 + 2m, 2(1 + m), \sin\left(\frac{1}{2}(e + fx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + c*Sin[e + f*x])^m)/Sqrt[a - a*Sin[e + f*x]],x]

[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(c*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[a - a*Sin[e + f*x]])

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(B \sin(fx + e) + A)\sqrt{-a \sin(fx + e) + a}(c \sin(fx + e) + c)^m}{a \sin(fx + e) - a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^m/(a*sin(f*x + e) - a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(c \sin(fx + e) + c)^m}{\sqrt{-a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)

maple [F] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(fx + e))(c + c \sin(fx + e))^m}{\sqrt{a - a \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)

[Out] int((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(c \sin(fx + e) + c)^m}{\sqrt{-a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))^m/(a-a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(c*sin(f*x + e) + c)^m/sqrt(-a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx))(c + c \sin(e + fx))^m}{\sqrt{a - a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(c + c*sin(e + f*x))^m)/(a - a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{\sqrt{-a(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+c*sin(f*x+e))**m/(a-a*sin(f*x+e))**(1/2),x)

[Out] Integral((c*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-a*(sin(e + f*x) - 1)), x)

$$3.205 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx$$

Optimal. Leaf size=275

$$\frac{64c^3(B(5-2m) - A(2m+7)) \cos(e+fx)(a \sin(e+fx) + a)^m}{f(2m+5)(2m+7)(4m^2+8m+3)\sqrt{c-c \sin(e+fx)}} - \frac{16c^2(B(5-2m) - A(2m+7)) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{f(2m+7)(4m^2+8m+3)}$$

[Out] $-2*c*(B*(5-2*m)-A*(7+2*m))*\cos(f*x+e)*(a+a*\sin(f*x+e))^m*(c-c*\sin(f*x+e))^{3/2}/f/(4*m^2+24*m+35)-2*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^m*(c-c*\sin(f*x+e))^{5/2}/f/(7+2*m)-64*c^3*(B*(5-2*m)-A*(7+2*m))*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(5+2*m)/(7+2*m)/(4*m^2+8*m+3)/(c-c*\sin(f*x+e))^{1/2}-16*c^2*(B*(5-2*m)-A*(7+2*m))*\cos(f*x+e)*(a+a*\sin(f*x+e))^m*(c-c*\sin(f*x+e))^{1/2}/f/(7+2*m)/(4*m^2+16*m+15)$

Rubi [A] time = 0.50, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2973, 2740, 2738}

$$\frac{16c^2(B(5-2m) - A(2m+7)) \cos(e+fx)\sqrt{c-c \sin(e+fx)}(a \sin(e+fx) + a)^m}{f(2m+7)(4m^2+16m+15)} - \frac{64c^3(B(5-2m) - A(2m+7)) \cos(e+fx)\sqrt{c-c \sin(e+fx)}}{f(2m+5)(2m+7)(4m^2+8m+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2), x]

[Out] $(-64*c^3*(B*(5-2*m)-A*(7+2*m))*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^m)/(f*(5+2*m)*(7+2*m)*(3+8*m+4*m^2)*\text{Sqrt}[c-c*\text{Sin}[e+f*x]]) - (16*c^2*(B*(5-2*m)-A*(7+2*m))*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^m*\text{Sqrt}[c-c*\text{Sin}[e+f*x]])/(f*(7+2*m)*(15+16*m+4*m^2)) - (2*c*(B*(5-2*m)-A*(7+2*m))*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^m*(c-c*\text{Sin}[e+f*x])^{3/2})/(f*(5+2*m)*(7+2*m)) - (2*B*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^m*(c-c*\text{Sin}[e+f*x])^{5/2})/(f*(7+2*m))$

Rule 2738

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^n)/(f*(m+n)), x] + Dist[(a*(2*m-1))/(m+n), Int[(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m-1/2, 0] && !LtQ[n, -1] && !(IGtQ[n-1/2, 0] && LtQ[n, m]) && !(ILtQ[m+n, 0] && GtQ[2*m+n+1, 0])

Rule 2973

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m+n+1)), x] - Dist[(B*c*(m-n) - A*d*(m+n+1))/(d*(m+n+1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,

f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{5/2} dx = -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{5/2}}{f(7 + 2m)}$$

$$= -\frac{2c(B(5 - 2m) - A(7 + 2m)) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{5/2}}{f(5 + 2m)(7 + 2m)}$$

$$= -\frac{16c^2(B(5 - 2m) - A(7 + 2m)) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{5/2}}{f(3 + 2m)(5 + 2m)(7 + 2m)}$$

$$= -\frac{64c^3(B(5 - 2m) - A(7 + 2m)) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{5/2}}{f(1 + 2m)(3 + 2m)(5 + 2m)(7 + 2m)\sqrt{c}}$$

Mathematica [C] time = 6.82, size = 667, normalized size = 2.43

$$(c - c \sin(e + fx))^{5/2} (a(\sin(e + fx) + 1))^m \left(\frac{(24Am^2 + 184Am + 350A - 12Bm^2 - 104Bm - 385B) \left(\left(\frac{1}{8} - \frac{i}{8} \right) \cos\left(\frac{3}{2}(e + fx)\right) - \left(\frac{1}{8} + \frac{i}{8} \right) \sin\left(\frac{3}{2}(e + fx)\right) \right)}{(2m+3)(2m+5)(2m+7)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(5/2),x]

[Out] ((a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^(5/2)*(((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 + I/8)*Cos[(e + f*x)/2] + (1/8 - I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((2100*A - 1575*B + 1272*A*m - 110*B*m + 304*A*m^2 - 68*B*m^2 + 32*A*m^3 - 8*B*m^3)*((1/8 - I/8)*Cos[(e + f*x)/2] + (1/8 + I/8)*Sin[(e + f*x)/2]))/((1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 - I/8)*Cos[(3*(e + f*x))/2] - (1/8 + I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((350*A - 385*B + 184*A*m - 104*B*m + 24*A*m^2 - 12*B*m^2)*((1/8 + I/8)*Cos[(3*(e + f*x))/2] - (1/8 - I/8)*Sin[(3*(e + f*x))/2]))/((3 + 2*m)*(5 + 2*m)*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 + I/8)*Cos[(5*(e + f*x))/2] - (1/8 + I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((14*A - 35*B + 4*A*m - 6*B*m)*((-1/8 - I/8)*Cos[(5*(e + f*x))/2] - (1/8 - I/8)*Sin[(5*(e + f*x))/2]))/((5 + 2*m)*(7 + 2*m)) + ((1/8 - I/8)*B*Cos[(7*(e + f*x))/2] - (1/8 + I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m) + ((1/8 + I/8)*B*Cos[(7*(e + f*x))/2] - (1/8 - I/8)*B*Sin[(7*(e + f*x))/2])/(7 + 2*m)))/(f*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^5)

fricas [B] time = 0.49, size = 562, normalized size = 2.04

$$2 \left((8 Bc^2m^3 + 36 Bc^2m^2 + 46 Bc^2m + 15 Bc^2) \cos(fx + e)^4 + 64 (A + B)c^2m - (8 (A - 2B)c^2m^3 + 4 (11A - 28B)c^2m^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, algorithm="fricas")

```
[Out] 2*((8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^4 + 64
*(A + B)*c^2*m - (8*(A - 2*B)*c^2*m^3 + 4*(11*A - 28*B)*c^2*m^2 + 2*(31*A -
86*B)*c^2*m + 3*(7*A - 20*B)*c^2)*cos(f*x + e)^3 + 32*(7*A - 5*B)*c^2 + (8
*(A - B)*c^2*m^3 + 4*(19*A - 11*B)*c^2*m^2 + 190*(A - B)*c^2*m + (77*A - 85
*B)*c^2)*cos(f*x + e)^2 + 2*(8*(A - B)*c^2*m^3 + 60*(A - B)*c^2*m^2 + 2*(79
*A - 63*B)*c^2*m + (161*A - 145*B)*c^2)*cos(f*x + e) + (64*(A + B)*c^2*m -
(8*B*c^2*m^3 + 36*B*c^2*m^2 + 46*B*c^2*m + 15*B*c^2)*cos(f*x + e)^3 + 32*(7
*A - 5*B)*c^2 - (8*(A - B)*c^2*m^3 + 4*(11*A - 19*B)*c^2*m^2 + 2*(31*A - 63
*B)*c^2*m + 3*(7*A - 15*B)*c^2)*cos(f*x + e)^2 - 2*(8*(A - B)*c^2*m^3 + 60*
(A - B)*c^2*m^2 + 2*(63*A - 79*B)*c^2*m + (49*A - 65*B)*c^2)*cos(f*x + e))*
sin(f*x + e))*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(16*f*m^4 +
128*f*m^3 + 344*f*m^2 + 352*f*m + (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f
*m + 105*f)*cos(f*x + e) - (16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 10
5*f)*sin(f*x + e) + 105*f)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{5}{2}}(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(5/2)*(a*sin(f*x + e)
+ a)^m, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x)
```

maxima [B] time = 0.55, size = 725, normalized size = 2.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] -2*(((4*m^2 + 24*m + 43)*a^m*c^(5/2) - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin
(f*x + e)/(cos(f*x + e) + 1) + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)
)^2/(cos(f*x + e) + 1)^2 + 2*(4*m^2 + 8*m + 35)*a^m*c^(5/2)*sin(f*x + e)^3/
(cos(f*x + e) + 1)^3 - (12*m^2 + 40*m - 15)*a^m*c^(5/2)*sin(f*x + e)^4/(cos
(f*x + e) + 1)^4 + (4*m^2 + 24*m + 43)*a^m*c^(5/2)*sin(f*x + e)^5/(cos(f*x
+ e) + 1)^5)*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(
f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((8*m^3 + 36*m^2 + 46*m + 15)*(sin(f*
x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2)) - 2*((4*m^2 + 40*m + 115)*a^m*c^(
5/2) - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*sin(f*x + e)/(cos(f*x + e) +
1) + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)^3/(co
s(f*x + e) + 1)^3 - (16*m^3 + 76*m^2 + 260*m - 175)*a^m*c^(5/2)*sin(f*x + e)
)^4/(cos(f*x + e) + 1)^4 + 2*(12*m^3 + 76*m^2 + 97*m + 175)*a^m*c^(5/2)*sin
(f*x + e)^5/(cos(f*x + e) + 1)^5 - 2*(4*m^3 + 40*m^2 + 115*m)*a^m*c^(5/2)*s
```

```
in(f*x + e)^6/(cos(f*x + e) + 1)^6 + (4*m^2 + 40*m + 115)*a^m*c^(5/2)*sin(f
*x + e)^7/(cos(f*x + e) + 1)^7)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1
) + 1) - m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((16*m^4 + 128*m^3
+ 344*m^2 + 352*m + (16*m^4 + 128*m^3 + 344*m^2 + 352*m + 105)*sin(f*x + e
)^2/(cos(f*x + e) + 1)^2 + 105)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(
5/2)))/f
```

mupad [B] time = 21.06, size = 749, normalized size = 2.72

$$\frac{\sqrt{c - c \sin(e + f x)} \left(\frac{B c^2 (a + a \sin(e + f x))^m (m^3 8i + m^2 36i + m 46i + 15i)}{4 f (16 m^4 + 128 m^3 + 344 m^2 + 352 m + 105)} - \frac{c^2 e^{e^{3i + f x 3i}} (a + a \sin(e + f x))^m (2100 A - 1575 B + 1272 A m - 110 B m + 304 A m^2 + 32 A m^3 - 68 B m^2 - 8 B m^3)}{4 f (16 m^4 + 128 m^3 + 344 m^2 + 352 m + 105)} \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(5/2),
x)
```

```
[Out] -((c - c*sin(e + f*x))^(1/2)*((B*c^2*(a + a*sin(e + f*x))^m*(m*46i + m^2*36
i + m^3*8i + 15i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*
exp(e*3i + f*x*3i)*(a + a*sin(e + f*x))^m*(2100*A - 1575*B + 1272*A*m - 110
*B*m + 304*A*m^2 + 32*A*m^3 - 68*B*m^2 - 8*B*m^3))/(4*f*(352*m + 344*m^2 +
128*m^3 + 16*m^4 + 105)) + (c^2*exp(e*4i + f*x*4i)*(a + a*sin(e + f*x))^m*(
A*2100i - B*1575i + A*m*1272i - B*m*110i + A*m^2*304i + A*m^3*32i - B*m^2*6
8i - B*m^3*8i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105)) - (c^2*exp
(e*5i + f*x*5i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(350*A - 385*B + 184*A*m -
104*B*m + 24*A*m^2 - 12*B*m^2))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
105)) + (c^2*exp(e*2i + f*x*2i)*(2*m + 1)*(a + a*sin(e + f*x))^m*(A*350i -
B*385i + A*m*184i - B*m*104i + A*m^2*24i - B*m^2*12i))/(4*f*(352*m + 344*m
^2 + 128*m^3 + 16*m^4 + 105)) - (B*c^2*exp(e*7i + f*x*7i)*(a + a*sin(e + f*
x))^m*(46*m + 36*m^2 + 8*m^3 + 15))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^
4 + 105)) + (c^2*exp(e*1i + f*x*1i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3
)*(14*A - 35*B + 4*A*m - 6*B*m))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4 +
105)) - (c^2*exp(e*6i + f*x*6i)*(a + a*sin(e + f*x))^m*(8*m + 4*m^2 + 3)*(
A*14i - B*35i + A*m*4i - B*m*6i))/(4*f*(352*m + 344*m^2 + 128*m^3 + 16*m^4
+ 105))))/(exp(e*4i + f*x*4i) - (exp(e*3i + f*x*3i)*(m*352i + m^2*344i + m^
3*128i + m^4*16i + 105i))/(352*m + 344*m^2 + 128*m^3 + 16*m^4 + 105))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```


3.206 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx$

Optimal. Leaf size=166

$$\frac{2Bc^2 \cos(e + fx)(a \sin(e + fx) + a)^{m+2}}{a^2 f(2m + 5)\sqrt{c - c \sin(e + fx)}} + \frac{4c^2(A - B) \cos(e + fx)(a \sin(e + fx) + a)^m}{f(2m + 1)\sqrt{c - c \sin(e + fx)}} - \frac{2c^2(A - 3B) \cos(e + fx)}{af(2m + 3)\sqrt{c - c \sin(e + fx)}}$$

[Out] $4*(A-B)*c^2*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{(1/2)}-2*(A-3*B)*c^2*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1+m)}/a/f/(3+2*m)/(c-c*\sin(f*x+e))^{(1/2)}-2*B*c^2*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(2+m)}/a^2/f/(5+2*m)/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.35, antiderivative size = 192, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2973, 2740, 2738}

$$\frac{8c^2(B(3 - 2m) - A(2m + 5)) \cos(e + fx)(a \sin(e + fx) + a)^m}{f(2m + 5)(4m^2 + 8m + 3)\sqrt{c - c \sin(e + fx)}} - \frac{2c(B(3 - 2m) - A(2m + 5)) \cos(e + fx)\sqrt{c - c \sin(e + fx)}}{f(2m + 3)(2m + 5)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x])*(c - c*\text{Sin}[e + f*x])^{(3/2)}, x]$

[Out] $(-8*c^2*(B*(3 - 2*m) - A*(5 + 2*m))*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m)/(f*(5 + 2*m)*(3 + 8*m + 4*m^2)*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) - (2*c*(B*(3 - 2*m) - A*(5 + 2*m))*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])/(f*(3 + 2*m)*(5 + 2*m)) - (2*B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m*(c - c*\text{Sin}[e + f*x])^{(3/2)})/(f*(5 + 2*m))$

Rule 2738

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2740

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n)), x] + \text{Dist}[(a*(2*m - 1))/(m + n), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IGtQ[m - 1/2, 0] && !LtQ[n, -1] && !(IGtQ[n - 1/2, 0] && LtQ[n, m]) && !(ILtQ[m + n, 0] && GtQ[2*m + n + 1, 0])

Rule 2973

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)}*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{3/2} dx = -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{3/2}}{f(5 + 2m)}$$

$$= -\frac{2c(B(3 - 2m) - A(5 + 2m)) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{3/2}}{f(3 + 2m)(5 + 2m)}$$

$$= -\frac{8c^2(B(3 - 2m) - A(5 + 2m)) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{3/2}}{f(1 + 2m)(3 + 2m)(5 + 2m)\sqrt{c - c \sin(e + fx)}}$$

Mathematica [A] time = 1.77, size = 174, normalized size = 1.05

$$\frac{c\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (a(\sin(e + fx) + 1))^m \left(-2(2m + 1)(2Am + 5A - 2Bm - 9A) \right)}{f(2m + 1)(2m + 3)(2m + 5) \left(\cos\left(\frac{1}{2}(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3/2),x]

[Out] (c*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c - c*Sin[e + f*x]]*(50*A - 39*B + 40*A*m - 16*B*m + 8*A*m^2 - 4*B*m^2 + B*(3 + 8*m + 4*m^2)*Cos[2*(e + f*x)] - 2*(1 + 2*m)*(5*A - 9*B + 2*A*m - 2*B*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [A] time = 0.50, size = 313, normalized size = 1.89

$$2 \left((4Bcm^2 + 8Bcm + 3Bc) \cos(fx + e)^3 + 8(A + B)cm + (4Acm^2 + 12(A - B)cm + (5A - 6B)c) \cos(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] 2*((4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^3 + 8*(A + B)*c*m + (4*A*c*m^2 + 12*(A - B)*c*m + (5*A - 6*B)*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c + (4*(A - B)*c*m^2 + 4*(5*A - 3*B)*c*m + (25*A - 21*B)*c)*cos(f*x + e) + (8*(A + B)*c*m + (4*B*c*m^2 + 8*B*c*m + 3*B*c)*cos(f*x + e)^2 + 4*(5*A - 3*B)*c - (4*(A - B)*c*m^2 + 4*(3*A - 5*B)*c*m + (5*A - 9*B)*c)*cos(f*x + e))*sin(f*x + e)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(8*f*m^3 + 36*f*m^2 + 46*f*m + (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*cos(f*x + e) - (8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)*sin(f*x + e) + 15*f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(-c \sin(fx + e) + c)^{\frac{3}{2}} (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(-c*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x)

maxima [B] time = 0.53, size = 498, normalized size = 3.00

$$2 \frac{\left(a^m c^{\frac{3}{2}} (2m+5) - \frac{a^m c^{\frac{3}{2}} (2m-3) \sin(fx+e)}{\cos(fx+e)+1} - \frac{a^m c^{\frac{3}{2}} (2m-3) \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^m c^{\frac{3}{2}} (2m+5) \sin(fx+e)^3}{(\cos(fx+e)+1)^3} \right) A e^{2m \log\left(\frac{\sin(fx+e)}{\cos(fx+e)+1} + 1\right) - m \log\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1\right)}}{(4m^2 + 8m + 3) \left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 1 \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] $-2*((a^m*c^{(3/2)}*(2*m + 5) - a^m*c^{(3/2)}*(2*m - 3)*\sin(f*x + e)/(\cos(f*x + e) + 1) - a^m*c^{(3/2)}*(2*m - 3)*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a^m*c^{(3/2)}*(2*m + 5)*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3)*A*e^{(2*m*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1) - m*\log(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1))}/((4*m^2 + 8*m + 3)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(3/2)}) - 2*(a^m*c^{(3/2)}*(2*m + 9) - 2*(2*m^2 + 9*m)*a^m*c^{(3/2)}*\sin(f*x + e)/(\cos(f*x + e) + 1) + (4*m^2 + 15)*a^m*c^{(3/2)}*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + (4*m^2 + 15)*a^m*c^{(3/2)}*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 - 2*(2*m^2 + 9*m)*a^m*c^{(3/2)}*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^m*c^{(3/2)}*(2*m + 9)*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5)*B*e^{(2*m*\log(\sin(f*x + e)/(\cos(f*x + e) + 1) + 1) - m*\log(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1))}/((8*m^3 + 36*m^2 + 46*m + (8*m^3 + 36*m^2 + 46*m + 15)*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15)*(\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)^{(3/2)})/f$

mupad [B] time = 19.20, size = 480, normalized size = 2.89

$$\sqrt{c - c \sin(e + f x)} \left(\frac{c e^{3i + f x 3i} (a + a \sin(e + f x))^m (45 A - 30 B + 28 A m + 4 B m + 4 A m^2)}{f (m^3 8i + m^2 36i + m 46i + 15i)} + \frac{c e^{2i + f x 2i} (a + a \sin(e + f x))^m (A 45i - B 30i + A m^2 28i + B m^4 i + A m^2 4i)}{f (m^3 8i + m^2 36i + m 46i + 15i)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(3/2),x)

[Out] $((c - c \sin(e + f x))^{(1/2)} * ((c \exp(e * 3i + f x * 3i) * (a + a \sin(e + f x)))^m * (45 * A - 30 * B + 28 * A * m + 4 * B * m + 4 * A * m^2)) / (f * (m^4 6i + m^2 * 36i + m^3 * 8i + 15i)) + (c \exp(e * 2i + f x * 2i) * (a + a \sin(e + f x)))^m * (A * 45i - B * 30i + A * m^2 28i + B * m^4 i + A * m^2 4i)) / (f * (m^4 6i + m^2 * 36i + m^3 * 8i + 15i)) + (B * c * (a + a \sin(e + f x))^m * (m^8 i + m^2 * 4i + 3i)) / (2 * f * (m^4 6i + m^2 * 36i + m^3 * 8i + 15i)) + (B * c \exp(e * 5i + f x * 5i) * (a + a \sin(e + f x)))^m * (8 * m + 4 * m^2 + 3)) / (2 * f * (m^4 6i + m^2 * 36i + m^3 * 8i + 15i)) + (c \exp(e * 1i + f x * 1i) * (2 * m + 1) * (a + a \sin(e + f x)))^m * (10 * A - 15 * B + 4 * A * m - 2 * B * m)) / (2 * f * (m^4 6i + m^2 * 36i + m^3 * 8i + 15i)) + (c \exp(e * 4i + f x * 4i) * (2 * m + 1) * (a + a \sin(e + f x)))^m * (A * 10i -$

```
B*15i + A*m*4i - B*m*2i))/(2*f*(m*46i + m^2*36i + m^3*8i + 15i))))/(exp(e*3
i + f*x*3i) + (exp(e*2i + f*x*2i)*(46*m + 36*m^2 + 8*m^3 + 15))/(m*46i + m^
2*36i + m^3*8i + 15i))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.207 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)}$

Optimal. Leaf size=104

$$\frac{2c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^m}{f(2m + 1)\sqrt{c - c \sin(e + fx)}} + \frac{2Bc \cos(e + fx)(a \sin(e + fx) + a)^{m+1}}{af(2m + 3)\sqrt{c - c \sin(e + fx)}}$$

[Out] $2*(A-B)*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{(1/2)+2*B*c*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1+m)}/a/f/(3+2*m)/(c-c*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2971, 2738}

$$\frac{2c(A - B) \cos(e + fx)(a \sin(e + fx) + a)^m}{f(2m + 1)\sqrt{c - c \sin(e + fx)}} + \frac{2Bc \cos(e + fx)(a \sin(e + fx) + a)^{m+1}}{af(2m + 3)\sqrt{c - c \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x])*Sqrt[c - c*\text{Sin}[e + f*x]], x]$

[Out] $(2*(A - B)*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[c - c*\text{Sin}[e + f*x]]) + (2*B*c*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(1 + m)})/(a*f*(3 + 2*m)*Sqrt[c - c*\text{Sin}[e + f*x]])$

Rule 2738

$\text{Int}[Sqrt[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] := \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*\text{Sin}[e + f*x]]), x] /;$ FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[n, -2^(-1)]

Rule 2971

$\text{Int}[Sqrt[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])^{(n_)}, x_Symbol] := \text{Dist}[B/d, \text{Int}[Sqrt[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{(n + 1)}, x], x] - \text{Dist}[(B*c - A*d)/d, \text{Int}[Sqrt[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) \sqrt{c - c \sin(e + fx)} dx &= \frac{B \int (a + a \sin(e + fx))^{1+m} \sqrt{c - c \sin(e + fx)} dx}{a} \\ &= \frac{2(A - B)c \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.44, size = 116, normalized size = 1.12

$$\frac{2\sqrt{c - c \sin(e + fx)} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (a(\sin(e + fx) + 1))^m (A(2m + 3) + B(2m + 1) \sin(e + fx))}{f(2m + 1)(2m + 3) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c - c*Sin[e + f*x]], x]

[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sqrt[c - c*Sin[e + f*x]]*(-2*B + A*(3 + 2*m) + B*(1 + 2*m)*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))

fricas [A] time = 0.47, size = 165, normalized size = 1.59

$$\frac{2 \left((2 B m + B) \cos (f x + e)^2 - 2 (A + B) m - (2 A m + 3 A - 2 B) \cos (f x + e) - (2 (A + B) m + (2 B m + B) \cos (f x + e)) \right)}{4 f m^2 + 8 f m + (4 f m^2 + 8 f m + 3 f) \cos (f x + e) - (4 f m^2 + 8 f m + 3 f) \sin (f x + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2), x, algorithm="fricas")

[Out] -2*((2*B*m + B)*cos(f*x + e)^2 - 2*(A + B)*m - (2*A*m + 3*A - 2*B)*cos(f*x + e) - (2*(A + B)*m + (2*B*m + B)*cos(f*x + e) + 3*A - B)*sin(f*x + e) - 3*(A + B)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(4*f*m^2 + 8*f*m + (4*f*m^2 + 8*f*m + 3*f)*cos(f*x + e) - (4*f*m^2 + 8*f*m + 3*f)*sin(f*x + e) + 3*f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (f x + e) + A) \sqrt{-c \sin (f x + e) + c} (a \sin (f x + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int (a + a \sin (f x + e))^m (A + B \sin (f x + e)) \sqrt{c - c \sin (f x + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2), x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2), x)

maxima [B] time = 0.54, size = 323, normalized size = 3.11

$$\frac{2 \left(\frac{2 a^m \sqrt{c} m \sin (f x + e)}{\cos (f x + e) + 1} + \frac{2 a^m \sqrt{c} m \sin (f x + e)^2}{(\cos (f x + e) + 1)^2} - a^m \sqrt{c} - \frac{a^m \sqrt{c} \sin (f x + e)^3}{(\cos (f x + e) + 1)^3} \right) B e^{\left(2 m \log \left(\frac{\sin (f x + e)}{\cos (f x + e) + 1} + 1 \right) - m \log \left(\frac{\sin (f x + e)^2}{(\cos (f x + e) + 1)^2} + 1 \right) \right)} + \frac{\left(a^m \sqrt{c} + \frac{a^m \sqrt{c} \sin (f x + e)}{\cos (f x + e) + 1} \right)}{\left(4 m^2 + 8 m + \frac{(4 m^2 + 8 m + 3) \sin (f x + e)^2}{(\cos (f x + e) + 1)^2} + 3 \right) \sqrt{\frac{\sin (f x + e)^2}{(\cos (f x + e) + 1)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2), x, algorithm="maxima")

```
[Out] -2*(2*(2*a^m*sqrt(c))*m*sin(f*x + e)/(cos(f*x + e) + 1) + 2*a^m*sqrt(c))*m*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - a^m*sqrt(c) - a^m*sqrt(c)*sin(f*x + e)^
3/(cos(f*x + e) + 1)^3)*B*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) -
m*log(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1))/((4*m^2 + 8*m + (4*m^2 + 8
*m + 3)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 3)*sqrt(sin(f*x + e)^2/(cos(f
*x + e) + 1)^2 + 1)) + (a^m*sqrt(c) + a^m*sqrt(c)*sin(f*x + e)/(cos(f*x + e
) + 1))*A*e^(2*m*log(sin(f*x + e)/(cos(f*x + e) + 1) + 1) - m*log(sin(f*x +
e)^2/(cos(f*x + e) + 1)^2 + 1))/((2*m + 1)*sqrt(sin(f*x + e)^2/(cos(f*x +
e) + 1)^2 + 1)))/f
```

mupad [B] time = 1.44, size = 105, normalized size = 1.01

$$\frac{\left(a \left(\sin(e + fx) + 1\right)\right)^m \sqrt{-c \left(\sin(e + fx) - 1\right)} \left(6A \cos(e + fx) - 4B \cos(e + fx) + B \sin(2e + 2fx)\right)}{f \left(\sin(e + fx) - 1\right) \left(4m^2 + 8m + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1/2),
x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^(1/2)*(6*A*cos(e + f*x)
- 4*B*cos(e + f*x) + B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*
e + 2*f*x)))/(f*(sin(e + f*x) - 1)*(8*m + 4*m^2 + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sin(e + fx) + 1\right)\right)^m \sqrt{-c \left(\sin(e + fx) - 1\right)} \left(A + B \sin(e + fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1/2),x)
```

```
[Out] Integral((a*(sin(e + f*x) + 1))^m*sqrt(-c*(sin(e + f*x) - 1))*(A + B*sin(e
+ f*x)), x)
```

$$3.208 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{\sqrt{c-c \sin(e+fx)}} dx$$

Optimal. Leaf size=118

$$\frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c \sin(e+fx)}} - \frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

[Out] $-2*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{1/2}+(A+B)*\cos(f*x+e)*\text{hypergeom}([1, 1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e))*(a+a*\sin(f*x+e))^m/f/(1+2*m)/(c-c*\sin(f*x+e))^{1/2}$

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2973, 2745, 2667, 68}

$$\frac{(A+B) \cos(e+fx)(a \sin(e+fx)+a)^m {}_2F_1\left(1, m+\frac{1}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{c-c \sin(e+fx)}} - \frac{2B \cos(e+fx)(a \sin(e+fx)+a)^m}{f(2m+1)\sqrt{c-c \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(A + B*\text{Sin}[e + f*x])/ \text{Sqrt}[c - c*\text{Sin}[e + f*x]], x]$

[Out] $(-2*B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + 2*m)*\text{Sqrt}[c - c*\text{Sin}[e + f*x]]) + ((A + B)*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1, 1/2 + m, 3/2 + m, (1 + \text{Sin}[e + f*x])/2]*(a + a*\text{Sin}[e + f*x])^m)/(f*(1 + 2*m)*\text{Sqrt}[c - c*\text{Sin}[e + f*x]])$

Rule 68

$\text{Int}[(a + b*x)^m*((c + d*x)^n), x_Symbol] \rightarrow \text{Simp}[(b*c - a*d)^n*(a + b*x)^{m+1}*\text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x)/(b*c - a*d))]/(b^{n+1}*(m+1)), x] /;$ FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 2667

$\text{Int}[\cos[(e + f*x)]^{p-1}*((a + b*\sin[e + f*x])^m), x_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^{m+(p-1)/2}*(a - x)^{(p-1)/2}, x], x, b*\text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p-1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2745

$\text{Int}[(a + b*\sin[e + f*x])^m*((c + d*\sin[e + f*x])^n), x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]}*c^{\text{IntPart}[m]}*(a + b*\sin[e + f*x])^{\text{FracPart}[m]}*(c + d*\sin[e + f*x])^{\text{FracPart}[m]})/\text{Cos}[e + f*x]^{2*\text{FracPart}[m]}, \text{Int}[\text{Cos}[e + f*x]^{2*m}*(c + d*\sin[e + f*x])^{n-m}, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (FractionQ[m] || !FractionQ[n])

Rule 2973

$\text{Int}[(a + b*\sin[e + f*x])^m*((A + B*\sin[e + f*x])^n), x_Symbol] \rightarrow -\text{Simp}[(B*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^n)/(f*(m + n + 1)), x] - \text{Dist}[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), \text{Int}[(a +$

$b*\sin[e + f*x]^m*(c + d*\sin[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m, n\}, x] \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{-1}] \ \&\& \ \text{NeQ}[m + n + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{\sqrt{c - c \sin(e + fx)}} dx &= -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c \sin(e + fx)}} + (A + B) \int \frac{(a + a \sin(e + fx))^m}{\sqrt{c - c \sin(e + fx)}} dx \\ &= -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c \sin(e + fx)}} + \frac{((A + B) \cos(e + fx))}{\sqrt{a + a \sin(e + fx)}} \\ &= -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c \sin(e + fx)}} + \frac{(a(A + B) \cos(e + fx))}{f\sqrt{a + a \sin(e + fx)}} \\ &= -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + 2m)\sqrt{c - c \sin(e + fx)}} + \frac{(A + B) \cos(e + fx)}{f\sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 2.23, size = 200, normalized size = 1.69

$$2^{-2m-\frac{3}{2}} \sin\left(\frac{1}{4}(2e + 2fx + \pi)\right) \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) (a(\sin(e + fx) + 1))^m \left(2^{2m+1}(A + B) {}_2F_1\left(1, \dots\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c - c*Sin[e + f*x]],x]

[Out] (2^(-3/2 - 2*m)*(-(2^(3 + 2*m)*B) + 2^(1 + 2*m)*(A + B)*Hypergeometric2F1[1, 1 + 2*m, 2*(1 + m), Sin[(2*e + Pi + 2*f*x)/4]] + (A + B)*Hypergeometric2F1[1 + 2*m, 1 + 2*m, 2*(1 + m), (1 - Tan[(2*e - Pi + 2*f*x)/8]^2)/2]*(Sec[(2*e - Pi + 2*f*x)/8]^2)^(1 + 2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^m*Sin[(2*e + Pi + 2*f*x)/4])/((f + 2*f*m)*Sqrt[c - c*Sin[e + f*x]])

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(B \sin(fx + e) + A)\sqrt{-c \sin(fx + e) + c}(a \sin(fx + e) + a)^m}{c \sin(fx + e) - c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c*sin(f*x + e) - c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{\sqrt{c - c \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{-c \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(-c*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{\sqrt{c - c \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{\sqrt{-c(\sin(e + fx) - 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(1/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(-c*(sin(e + f*x) - 1)), x)

$$3.209 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=134

$$\frac{(A(1-2m) - B(2m+3)) \cos(e+fx) (a \sin(e+fx) + a)^m {}_2F_1\left(1, m + \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e+fx) + 1)\right)}{4cf(2m+1)\sqrt{c-c \sin(e+fx)}} + \frac{(A+B)c}{2}$$

[Out] 1/2*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(3/2)+1/4*(A*(1-2*m)-B*(3+2*m))*cos(f*x+e)*hypergeom([1, 1/2+m], [3/2+m], 1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m/c/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.30, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(1-2m) - B(2m+3)) \cos(e+fx) (a \sin(e+fx) + a)^m {}_2F_1\left(1, m + \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e+fx) + 1)\right)}{4cf(2m+1)\sqrt{c-c \sin(e+fx)}} + \frac{(A+B)c}{2}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*f*(c - c*Sin[e + f*x])^(3/2)) + ((A*(1 - 2*m) - B*(3 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[1, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(4*c*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])

Rule 68

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 2667

Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2745

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (FractionQ[m] || !FractionQ[n])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{3/2}} dx = \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{2f(c - c \sin(e + fx))^{3/2}} + \frac{\left(Bc \left(-\frac{3}{2} - m \right) - Ac \right)}{2f(c - c \sin(e + fx))^{3/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{2f(c - c \sin(e + fx))^{3/2}} + \frac{\left(\left(Bc \left(-\frac{3}{2} - m \right) - Ac \right) \right)}{2f(c - c \sin(e + fx))^{3/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{2f(c - c \sin(e + fx))^{3/2}} + \frac{\left(a \left(Bc \left(-\frac{3}{2} - m \right) - Ac \right) \right)}{2f(c - c \sin(e + fx))^{3/2}}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{2f(c - c \sin(e + fx))^{3/2}} + \frac{(A(1 - 2m) - B(3 + 2m))}{2f(c - c \sin(e + fx))^{3/2}}$$

Mathematica [C] time = 6.76, size = 3178, normalized size = 23.72

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(3/2), x]
```

```
[Out] (2^(-3/2 - 2*m)*B*(-(4^m*Hypergeometric2F1[1, 2*m, 1 + 2*m, Cos[(-e + Pi/2 - f*x)/2]]) + Hypergeometric2F1[2*m, 2*m, 1 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a + a*Sin[e + f*x])^m)/(f*m*(c - c*Sin[e + f*x])^(3/2)) - ((A + B)*(Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3*(a + a*Sin[e + f*x])^m*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/(8*sqrt[2]*f*(c - c*Sin[e + f*x])^(3/2)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^3*(-1/8*(m*cos[(-e + Pi/2 - f*x)/4]*(Cos[(-e + Pi/2 - f*x)/4]^2)^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/4]*(AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(2*m)*Tan[(-e + Pi/2 - f*x)/4]^2 - (AppellF1[1, -2*m, 2*m, 2, Cot[(-e + Pi/2 - f*x)/4]^2, -Cot[(-e + Pi/2 - f*x)/4]^2]*Cot[(-e + Pi/2 - f*x)/4]^2*(Csc[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m))/(1 - Cot[(-e + Pi/2 - f*x)/4]^2)^(2*m) + (2^(1 - 2*m)*AppellF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - Tan[(-e + Pi/2 - f*x)/4]^2)/2, 1 - Tan[(-e + Pi/2 - f*x)/4]^2]*(-1 + Tan[(-e + Pi/2 - f*x)/4]^2)*(1 - Tan[(-e + Pi/2 - f*x)/4]^4)^(2*m))/(1 + 2*m)))/sqrt[2] + ((Cos[(-e + Pi/2 - f*x)/4]^2)^(2*m)*((AppellF1[1, -2*m, 2*m, 2, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*(Sec[(-e + Pi/2 - f*x)/4]^2)^(1 + 2*m)*Tan[(-e + Pi/2
```

$$\begin{aligned}
& -f*x)/4])/2 + m*AppellF1[1, -2*m, 2*m, 2, \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Tan} \\
& [(-e + \text{Pi}/2 - f*x)/4]^2*(\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)}*\text{Tan}[(-e + \text{Pi}/2 \\
& - f*x)/4]^3 + (\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)}*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2 \\
& *(-1/2*(m*AppellF1[2, 1 - 2*m, 2*m, 3, \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Tan}[(-e \\
& + \text{Pi}/2 - f*x)/4]^2)*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4]) - \\
& (m*AppellF1[2, -2*m, 1 + 2*m, 3, \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Tan}[(-e + \text{Pi} \\
& /2 - f*x)/4]^2)*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4])/2) + (\\
& m*AppellF1[1, -2*m, 2*m, 2, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot}[(-e + \text{Pi}/2 - f \\
& *x)/4]^2]*\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^3*(\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)}*(1 \\
& - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)})/(1 - \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)} \\
& + m*AppellF1[1, -2*m, 2*m, 2, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot}[(-e + \text{Pi}/2 \\
& - f*x)/4]^2]*\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^3*(1 - \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(- \\
& 1 - 2*m)}*(\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(1 + 2*m)}*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/ \\
& 4]^2)^{(2*m)} + (\text{AppellF1}[1, -2*m, 2*m, 2, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot}[(- \\
& -e + \text{Pi}/2 - f*x)/4]^2]*\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]*(\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^2 \\
&)^{(1 + 2*m)}*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)})/(2*(1 - \text{Cot}[(-e + \text{Pi}/2 \\
& - f*x)/4]^2)^{(2*m)}) - (\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2*(\text{Csc}[(-e + \text{Pi}/2 - f*x)/4] \\
& ^2)^{(2*m)}*(m*AppellF1[2, 1 - 2*m, 2*m, 3, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot} \\
& [(-e + \text{Pi}/2 - f*x)/4]^2]*\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]*\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^ \\
& 2)/2 + (m*AppellF1[2, -2*m, 1 + 2*m, 3, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot}[(- \\
& e + \text{Pi}/2 - f*x)/4]^2]*\text{Cot}[(-e + \text{Pi}/2 - f*x)/4]*\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^2)/ \\
& 2)*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)})/(1 - \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2) \\
& ^{(2*m)} + (m*AppellF1[1, -2*m, 2*m, 2, \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2, -\text{Cot}[(-e \\
& + \text{Pi}/2 - f*x)/4]^2]*\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]*(\text{Csc}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(\\
& 2*m)}*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(-1 + 2*m)})/ \\
& (1 - \text{Cot}[(-e + \text{Pi}/2 - f*x)/4]^2)^{(2*m)} + (\text{AppellF1}[1 + 2*m, 2*m, 1, 2 + 2*m \\
& , (1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)/2, 1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2]*\text{Sec}[(- \\
& -e + \text{Pi}/2 - f*x)/4]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4]*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4 \\
&]^4)^{(2*m)})/(2^{(2*m)}*(1 + 2*m)) + (2^{(1 - 2*m)}*(-1/2*((1 + 2*m)*\text{AppellF1}[2 \\
& + 2*m, 2*m, 2, 3 + 2*m, (1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)/2, 1 - \text{Tan}[(-e + P \\
& i/2 - f*x)/4]^2]*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4])/(2 + \\
& 2*m) - (m*(1 + 2*m)*\text{AppellF1}[2 + 2*m, 1 + 2*m, 1, 3 + 2*m, (1 - \text{Tan}[(-e + P \\
& i/2 - f*x)/4]^2)/2, 1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2]*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4 \\
&]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4])/(2*(2 + 2*m)))*(-1 + \text{Tan}[(-e + \text{Pi}/2 - f*x)/4] \\
& ^2)*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^4)^{(2*m)})/(1 + 2*m) - (2^{(2 - 2*m)}*m*Appel \\
& llF1[1 + 2*m, 2*m, 1, 2 + 2*m, (1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)/2, 1 - \text{Tan}[\\
& (-e + \text{Pi}/2 - f*x)/4]^2]*\text{Sec}[(-e + \text{Pi}/2 - f*x)/4]^2*\text{Tan}[(-e + \text{Pi}/2 - f*x)/4] \\
& ^3*(-1 + \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^2)*(1 - \text{Tan}[(-e + \text{Pi}/2 - f*x)/4]^4)^{(-1 + \\
& 2*m)})/(1 + 2*m)))/(8*\text{Sqrt}[2]))
\end{aligned}$$

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{-c \sin(fx + e) + c} (a \sin(fx + e) + a)^m}{c^2 \cos(fx + e)^2 + 2c^2 \sin(fx + e) - 2c^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(c^2*cos(f*x + e)^2 + 2*c^2*sin(f*x + e) - 2*c^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c - c \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{(-c(\sin(e + fx) - 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))**(3/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(-c*(sin(e + f*x) - 1))**(3/2), x)

$$3.210 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c-c \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=134

$$\frac{(A(3-2m) - B(2m+5)) \cos(e+fx)(a \sin(e+fx) + a)^m {}_2F_1\left(2, m + \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e+fx) + 1)\right)}{16c^2 f(2m+1) \sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)}{4c}$$

[Out] 1/4*(A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(c-c*sin(f*x+e))^(5/2)+1/16*(A*(3-2*m)-B*(5+2*m))*cos(f*x+e)*hypergeom([2, 1/2+m], [3/2+m], 1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^m/c^2/f/(1+2*m)/(c-c*sin(f*x+e))^(1/2)

Rubi [A] time = 0.33, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2972, 2745, 2667, 68}

$$\frac{(A(3-2m) - B(2m+5)) \cos(e+fx)(a \sin(e+fx) + a)^m {}_2F_1\left(2, m + \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e+fx) + 1)\right)}{16c^2 f(2m+1) \sqrt{c-c \sin(e+fx)}} + \frac{(A+B) \cos(e+fx)}{4c}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^(5/2), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(4*f*(c - c*Sin[e + f*x])^(5/2)) + ((A*(3 - 2*m) - B*(5 + 2*m))*Cos[e + f*x]*Hypergeometric2F1[2, 1/2 + m, 3/2 + m, (1 + Sin[e + f*x])/2]*(a + a*Sin[e + f*x])^m)/(16*c^2*f*(1 + 2*m)*Sqrt[c - c*Sin[e + f*x]])

Rule 68

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^(n+1)*(a + b*x)^(m+1)*Hypergeometric2F1[-n, m+1, m+2, -((d*(a + b*x))/(b*c - a*d))])/(b^(n+1)*(m+1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rule 2667

Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

Rule 2745

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (FractionQ[m] || !FractionQ[n])

Rule 2972

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)),

```
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x] /; FreeQ[{
a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m
+ 1, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c - c \sin(e + fx))^{5/2}} dx &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^m}{4f(c - c \sin(e + fx))^{5/2}} + \frac{\left(Bc \left(-\frac{5}{2} - m \right) - Ac \right)}{4f(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^m}{4f(c - c \sin(e + fx))^{5/2}} + \frac{\left(\left(Bc \left(-\frac{5}{2} - m \right) - Ac \right) \right)}{4f(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^m}{4f(c - c \sin(e + fx))^{5/2}} + \frac{\left(a^2 \left(Bc \left(-\frac{5}{2} - m \right) - Ac \right) \right)}{4f(c - c \sin(e + fx))^{5/2}} \\ &= \frac{(A + B) \cos(e + fx) (a + a \sin(e + fx))^m}{4f(c - c \sin(e + fx))^{5/2}} + \frac{(A(3 - 2m) - B(5 + 2m))}{4f(c - c \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [C] time = 6.87, size = 8147, normalized size = 60.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])
^(5/2), x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{-c \sin(fx + e) + c} (a \sin(fx + e) + a)^m}{3c^3 \cos(fx + e)^2 - 4c^3 - (c^3 \cos(fx + e)^2 - 4c^3) \sin(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, alg
orithm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*sqrt(-c*sin(f*x + e) + c)*(a*sin(f*x + e) +
a)^m/(3*c^3*cos(f*x + e)^2 - 4*c^3 - (c^3*cos(f*x + e)^2 - 4*c^3)*sin(f*x +
e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2), x, alg
orithm="giac")
```


[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)

maple [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c - c \sin(fx + e))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c-c*sin(f*x+e))^(5/2),x)

[Out] Timed out

$$3.211 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-4-m} dx$$

Optimal. Leaf size=267

$$\frac{2(3A - 2B(m + 2)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1}}{c^3 f(2m + 5)(2m + 7)(4m^2 + 8m + 3)} + \frac{2(3A - 2B(m + 2)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-2}}{c^2 f(2m + 7)(4m^2 + 16m + 15)}$$

[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(4-m)/f/(7+2*m)+(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3-m)/c/f/(4*m^2+24*m+35)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)/c^2/f/(8*m^3+60*m^2+142*m+105)+2*(3*A-2*B*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/c^3/f/(16*m^4+128*m^3+344*m^2+352*m+105)

Rubi [A] time = 0.43, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{2(3A - 2B(m + 2)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-2}}{c^2 f(2m + 7)(4m^2 + 16m + 15)} + \frac{2(3A - 2B(m + 2)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1}}{c^3 f(2m + 5)(2m + 7)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(4 - m), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(4 - m))/(f*(7 + 2*m)) + ((3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3 - m))/(c*f*(5 + 2*m)*(7 + 2*m)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(c^2*f*(7 + 2*m)*(15 + 16*m + 4*m^2)) + (2*(3*A - 2*B*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(c^3*f*(5 + 2*m)*(7 + 2*m)*(3 + 8*m + 4*m^2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{

a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-4-m} dx &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{f(7 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{f(7 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{f(7 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m}{f(7 + 2m)} \end{aligned}$$

Mathematica [A] time = 14.13, size = 353, normalized size = 1.32

$$\frac{2^{-m-18} \cos\left(\frac{1}{2}(-e - fx + \frac{\pi}{2})\right) \csc^{21}\left(\frac{1}{8}(-e - fx + \frac{\pi}{2})\right) \sec^7\left(\frac{1}{8}(-e - fx + \frac{\pi}{2})\right) \sin^{-2m}\left(\frac{1}{2}(-e - fx + \frac{\pi}{2})\right) (a \sin(e + fx))^m}{1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-4 - m), x]

[Out] -((2^(-18 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^21*Sec[(-e + Pi/2 - f*x)/8]^7*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-4 - m)*(-96*A + 58*B - 176*A*m + 64*B*m - 96*A*m^2 + 16*B*m^2 - 16*A*m^3 + 4*(2 + m)*(-3*A + 2*B*(2 + m))*Cos[2*(-e + Pi/2 - f*x)] + 3*A*Cos[3*(-e + Pi/2 - f*x)] - 4*B*Cos[3*(-e + Pi/2 - f*x)] - 2*B*m*Cos[3*(-e + Pi/2 - f*x)] + (29 + 32*m + 8*m^2)*(3*A - 2*B*(2 + m))*Sin[e + f*x]))/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(7 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^7*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-4 - m))))

fricas [A] time = 0.49, size = 205, normalized size = 0.77

$$\frac{\left(4(2Bm^2 - (3A - 8B)m - 6A + 8B) \cos(fx + e)^3 + (8Am^3 + 12(4A - B)m^2 + 2(47A - 24B)m + 60A - 45B) \cos(fx + e) - (2(2Bm - 3A + 4B) \cos(fx + e)^3 - (8Bm^3 - 12(A - 4B)m^2 - 2(24A - 47B)m - 45A + 60B) \cos(fx + e)) \sin(fx + e)\right) (a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-4}}{(16f^4m^4 + 128f^3m^3 + 344f^2m^2 + 352f^1m + 105f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m), x, algorithm="fricas")

[Out] (4*(2*B*m^2 - (3*A - 8*B)*m - 6*A + 8*B)*cos(f*x + e)^3 + (8*A*m^3 + 12*(4*A - B)*m^2 + 2*(47*A - 24*B)*m + 60*A - 45*B)*cos(f*x + e) - (2*(2*B*m - 3*A + 4*B)*cos(f*x + e)^3 - (8*B*m^3 - 12*(A - 4*B)*m^2 - 2*(24*A - 47*B)*m - 45*A + 60*B)*cos(f*x + e))*sin(f*x + e)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 4)/(16*f*m^4 + 128*f*m^3 + 344*f*m^2 + 352*f*m + 105*f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(4-m), x)

maple [F] time = 5.94, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{-4-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(4-m), x)

mupad [B] time = 22.23, size = 368, normalized size = 1.38

$$\frac{\sin(4e + 4fx) (a + a \sin(e + fx))^m (4B - 3A + 2Bm) \operatorname{li}}{4f(c - c \sin(e + fx))^{m+4} (m^4 16i + m^3 128i + m^2 344i + m 352i + 105i)} + \frac{\cos(e + fx) (a + a \sin(e + fx))^m}{4f(c - c \sin(e + fx))^{m+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 4),x)

[Out] (cos(e + f*x)*(a + a*sin(e + f*x))^m*(A*168i - B*84i + A*m*340i - B*m*96i + A*m^2*192i + A*m^3*32i - B*m^2*24i))/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) - (sin(4*e + 4*f*x)*(a + a*sin(e + f*x))^m*(4*B - 3*A + 2*B*m)*1i)/(4*f*(c - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (sin(2*e + 2*f*x)*(a + a*sin(e + f*x))^m*(8*m + 2*m^2 + 7)*(4*B - 3*A + 2*B*m)*1i)/(f*(c - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i)) + (cos(3*e + 3*f*x)*(m + 2)*(a + a*sin(e + f*x))^m*(B*4i - A*3i + B*m*2i))/(f*(c - c*sin(e + f*x))^(m + 4)*(m*352i + m^2*344i + m^3*128i + m^4*16i + 105i))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(4-m),x)

[Out] Timed out

$$3.212 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-3-m} dx$$

Optimal. Leaf size=191

$$\frac{(2A - B(2m + 3)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1}}{c^2 f(2m + 5)(4m^2 + 8m + 3)} + \frac{(A + B) \cos(e + fx)(a \sin(e + fx) - a)^m (c - c \sin(e + fx))^{-m-1}}{f(2m + 5)(4m^2 + 8m + 3)}$$

[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(3-m)/f/(5+2*m)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)/c/f/(4*m^2+16*m+15)+(2*A-B*(3+2*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/c^2/f/(8*m^3+36*m^2+46*m+15)

Rubi [A] time = 0.31, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2972, 2743, 2742}

$$\frac{(2A - B(2m + 3)) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1}}{c^2 f(2m + 5)(4m^2 + 8m + 3)} + \frac{(A + B) \cos(e + fx)(a \sin(e + fx) - a)^m (c - c \sin(e + fx))^{-m-1}}{f(2m + 5)(4m^2 + 8m + 3)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(3 - m), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(3 - m))/(f*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(c*f*(3 + 2*m)*(5 + 2*m)) + ((2*A - B*(3 + 2*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(1 - m))/(c^2*f*(5 + 2*m)*(3 + 8*m + 4*m^2))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2743

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(m + n + 1)/(a*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + n + 1], 0] && NeQ[m, -2^(-1)] && (SumSimplerQ[m, 1] || ! SumSimplerQ[n, 1])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-3-m} dx &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-3-m}}{f(5 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-3-m}}{f(5 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-3-m}}{f(5 + 2m)} \end{aligned}$$

Mathematica [A] time = 10.73, size = 269, normalized size = 1.41

$$2^{-m-13} \cos\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right) \csc^{15}\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right) \sec^5\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right) \sin^{-2m}\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right) (a \sin(e + fx))^m (c - c \sin(e + fx))^{-3-m}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-3 - m), x]

[Out] (2^(-13 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^15*Sec[(-e + Pi/2 - f*x)/8]^5*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-3 - m)*(16*A - 9*B + 24*A*m - 6*B*m + 8*A*m^2 + (2*A - 3*B - 2*B*m)*Cos[2*(-e + Pi/2 - f*x)] + 2*(3 + 2*m)*(-2*A + B*(3 + 2*m))*Sin[e + f*x])/(f*(1 + 2*m)*(3 + 2*m)*(5 + 2*m)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^5*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-3 - m)))

fricas [A] time = 0.48, size = 137, normalized size = 0.72

$$\frac{\left((2Bm - 2A + 3B) \cos(fx + e)^3 + (4Bm^2 - 4(A - 3B)m - 6A + 9B) \cos(fx + e) \sin(fx + e) + (4Am^2 + 4A - 9B) \sin(fx + e)\right) (a \sin(fx + e) + a)^m (c - c \sin(fx + e))^{-3-m}}{8fm^3 + 36fm^2 + 46fm + 15f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^-3-m), x, algorithm="fricas")

[Out] ((2*B*m - 2*A + 3*B)*cos(f*x + e)^3 + (4*B*m^2 - 4*(A - 3*B)*m - 6*A + 9*B)*cos(f*x + e)*sin(f*x + e) + (4*A*m^2 + 4*(3*A - B)*m + 9*A - 6*B)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(c - c*sin(f*x + e))^-3-m/(8*f*m^3 + 36*f*m^2 + 46*f*m + 15*f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (c - c \sin(fx + e))^{-3-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^-3-m), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(c - c*sin(f*x + e))^-3-m), x)

maple [F] time = 6.18, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))(c - c \sin(fx + e))^{-3-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3-m),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3-m),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3-m),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(3-m - 3), x)
```

mupad [B] time = 15.48, size = 239, normalized size = 1.25

$$\frac{(a(\sin(e + fx) + 1))^m (30A \cos(e + fx) - 15B \cos(e + fx) - 2A \cos(3e + 3fx) + 3B \cos(3e + 3fx))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 3),x)
```

```
[Out] -((a*(sin(e + f*x) + 1))^m*(30*A*cos(e + f*x) - 15*B*cos(e + f*x) - 2*A*cos(3*e + 3*f*x) + 3*B*cos(3*e + 3*f*x) - 12*A*sin(2*e + 2*f*x) + 18*B*sin(2*e + 2*f*x) + 8*B*m^2*sin(2*e + 2*f*x) + 48*A*m*cos(e + f*x) - 10*B*m*cos(e + f*x) + 16*A*m^2*cos(e + f*x) + 2*B*m*cos(3*e + 3*f*x) - 8*A*m*sin(2*e + 2*f*x) + 24*B*m*sin(2*e + 2*f*x)))/(c^3*f*(-c*(sin(e + f*x) - 1))^m*(46*m + 36*m^2 + 8*m^3 + 15)*(15*sin(e + f*x) + 6*cos(2*e + 2*f*x) - sin(3*e + 3*f*x) - 10))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(3-m),x)
```

```
[Out] Timed out
```

$$3.213 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-2-m} dx$$

Optimal. Leaf size=114

$$\frac{(A + B) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-2}}{f(2m + 3)} + \frac{(A - 2B(m + 1)) \cos(e + fx)(a \sin(e + fx) + a)^m}{cf(2m + 1)(2m + 3)}$$

[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)/f/(3+2*m)+(A-2*B*(1+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/c/f/(4*m^2+8*m+3)

Rubi [A] time = 0.22, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2972, 2742}

$$\frac{(A + B) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-2}}{f(2m + 3)} + \frac{(A - 2B(m + 1)) \cos(e + fx)(a \sin(e + fx) + a)^m}{cf(2m + 1)(2m + 3)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 - m), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m))/(f*(3 + 2*m)) + ((A - 2*B*(1 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(c*f*(1 + 2*m)*(3 + 2*m))

Rule 2742

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && NeQ[m, -2^(-1)]

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-2-m} dx &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-2-m}}{f(3 + 2m)} \\ &= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-2-m}}{f(3 + 2m)} \end{aligned}$$

Mathematica [A] time = 8.79, size = 211, normalized size = 1.85

$$\frac{2^{-m-7} \cos\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right) \csc^9\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right) \sec^3\left(\frac{1}{8}\left(-e - fx + \frac{\pi}{2}\right)\right) \sin^{-2m}\left(\frac{1}{2}\left(-e - fx + \frac{\pi}{2}\right)\right) (a \sin(\dots))}{f(4m^2 + 8m - \dots)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-2 - m), x]

[Out] -((2^(-7 - m)*Cos[(-e + Pi/2 - f*x)/2]*Csc[(-e + Pi/2 - f*x)/8]^9*Sec[(-e + Pi/2 - f*x)/8]^3*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-2 - m)*(B - 2*A*(1 + m) + (A - 2*B*(1 + m))*Sin[e + f*x]))/(f*(3 + 8*m + 4*m^2)*(-1 + Cot[(-e + Pi/2 - f*x)/8]^2)^3*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-2 - m))))

fricas [A] time = 0.48, size = 89, normalized size = 0.78

$$\frac{((2Bm - A + 2B) \cos(fx + e) \sin(fx + e) + (2Am + 2A - B) \cos(fx + e))(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-2}}{4fm^2 + 8fm + 3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m), x, algorithm="fricas")

[Out] ((2*B*m - A + 2*B)*cos(f*x + e)*sin(f*x + e) + (2*A*m + 2*A - B)*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2)/(4*f*m^2 + 8*f*m + 3*f)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m - 2), x)

maple [F] time = 5.41, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e))(c - c \sin(fx + e))^{-2-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m), x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(m - 2), x)

mupad [B] time = 14.16, size = 134, normalized size = 1.18

$$\frac{(a(\sin(e+fx)+1))^m(4A\cos(e+fx)-2B\cos(e+fx)-A\sin(2e+2fx)+2B\sin(2e+2fx)+4A^2\sin(e+fx))}{c^2f(-c(\sin(e+fx)-1))^m(4m^2+8m+3)(4\sin(e+fx)+\cos(2e+2fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 2),x)

[Out] -((a*(sin(e + f*x) + 1))^m*(4*A*cos(e + f*x) - 2*B*cos(e + f*x) - A*sin(2*e + 2*f*x) + 2*B*sin(2*e + 2*f*x) + 4*A*m*cos(e + f*x) + 2*B*m*sin(2*e + 2*f*x)))/(c^2*f*(-c*(sin(e + f*x) - 1))^m*(8*m + 4*m^2 + 3)*(4*sin(e + f*x) + cos(2*e + 2*f*x) - 3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)

[Out] Timed out

$$3.214 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-1-m} dx$$

Optimal. Leaf size=163

$$\frac{(A + B) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} B 2^{\frac{1}{2}-m} \cos(e + fx)(1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^{\frac{1}{2}}}{f(2m + 1)}$$

[Out] (A+B)*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/f/(1+2*m)-2^(1/2-1/2-m)*B*cos(f*x+e)*hypergeom([1/2+m, 1/2+m],[3/2+m],1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(1-m)/f/(1+2*m)

Rubi [A] time = 0.31, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2972, 2745, 2689, 70, 69}

$$\frac{(A + B) \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} B 2^{\frac{1}{2}-m} \cos(e + fx)(1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^{\frac{1}{2}}}{f(2m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m), x]

[Out] ((A + B)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/((f*(1 + 2*m)) - (2^(1/2 - m)*B*Cos[e + f*x]*Hypergeometric2F1[(1 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(f*(1 + 2*m))

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/((b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[e_] + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[e_] + (f_.)*(x_))^(m_), x_Symbol] :> Dist[(a^2*(g*cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^(p + 1)/2*(a - b*Sin[e + f*x])^(p + 1)/2), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^(p - 1)/2, x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2745

Int[((a_) + (b_.)*sin[e_] + (f_.)*(x_)]^(m_)*((c_) + (d_.)*sin[e_] + (f_.)*(x_))^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && (IntegerQ[m] || !IntegerQ[n])

Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (FractionQ[m] || !FractionQ[n])

Rule 2972

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] + Dist[(a*B*(m - n) + A*b*(m + n + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (LtQ[m, -2^(-1)] || (ILtQ[m + n, 0] && !SumSimplerQ[n, 1])) && NeQ[2*m + 1, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-1-m} dx = \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1-m}}{f(1 + 2m)}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1-m}}{f(1 + 2m)}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1-m}}{f(1 + 2m)}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1-m}}{f(1 + 2m)}$$

$$= \frac{(A + B) \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-1-m}}{f(1 + 2m)}$$

Mathematica [C] time = 11.62, size = 675, normalized size = 4.14

$$\frac{2^{-m}(2m - 3) \cos^2\left(\frac{1}{4}(-e - fx + \frac{\pi}{2})\right) \cot\left(\frac{1}{4}(-e - fx + \frac{\pi}{2})\right) \sin^{-2m}\left(\frac{1}{2}(-e - fx + \frac{\pi}{2})\right) (a \sin(e + fx) + a)^m (A + B \sin(e + fx))^{m-1}}{f(4m^2 - 1) \left((2m - 3) \left((A + B) \left(\cos\left(\frac{1}{2}(-e - fx + \frac{\pi}{2})\right) + 1 \right) \left(1 - \tan^2\left(\frac{1}{4}(-e - fx + \frac{\pi}{2})\right) \right) \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m), x]

[Out] -((((-3 + 2*m)*Cos[(-e + Pi/2 - f*x)/4]^2*Cot[(-e + Pi/2 - f*x)/4]*(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(-1 - m)*(8*B*(1 + 2*m)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2 - (A + B)*((-1 + 2*m)*Hypergeometric2F1[-1/2 - m, -2*m, 1/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2] + (1 + 2*m)*Hypergeometric2F1[1/2 - m, -2*m, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2]*Tan[(-e + Pi/2 - f*x)/4]^2)))/(2^m*f*(-1 + 4*m^2)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*(-1 - m))*(-64*B*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + Pi/2 - f*x)/4]^4 - 32*B*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sin[(-e + P

$i/2 - f*x)/4]^4 + (-3 + 2*m)*(-4*B*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, \tan[(-e + \pi/2 - f*x)/4]^2, -\tan[(-e + \pi/2 - f*x)/4]^2]*\sin[(-e + \pi/2 - f*x)/2]^2 + (A + B)*(1 + \cos[(-e + \pi/2 - f*x)/2])*(1 - \tan[(-e + \pi/2 - f*x)/4]^2)^{(2*m)}))$

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(-c \sin(fx + e) + c\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^{(-1-m)},x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^{-m - 1}, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^{(-1-m)},x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^{-m - 1}, x)

maple [F] time = 2.87, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{-1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^{(-1-m)},x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^{(-1-m)},x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^{(-1-m)},x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^{-m - 1}, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^{m+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^{(m + 1)},x)

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^(m + 1), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)
```

```
[Out] Timed out
```

$$3.215 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-m} dx$$

Optimal. Leaf size=158

$$\frac{c2^{\frac{1}{2}-m}(A + 2Bm) \cos(e + fx)(1 - \sin(e + fx))^{m+\frac{1}{2}}(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}(2m + 1), \dots\right)}{f(2m + 1)}$$

[Out] $2^{(1/2-m)} * c * (2*B*m+A) * \cos(f*x+e) * \text{hypergeom}([1/2+m, 1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e)) * (1-\sin(f*x+e))^{(1/2+m)} * (a+a*\sin(f*x+e))^m * (c-c*\sin(f*x+e))^{(-1-m)} / f / (1+2*m) - B*\cos(f*x+e) * (a+a*\sin(f*x+e))^m / f / ((c-c*\sin(f*x+e))^m)$

Rubi [A] time = 0.27, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c2^{\frac{1}{2}-m}(A + 2Bm) \cos(e + fx)(1 - \sin(e + fx))^{m+\frac{1}{2}}(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}(2m + 1), \dots\right)}{f(2m + 1)}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]

[Out] $(2^{(1/2 - m)} * c * (A + 2*B*m) * \text{Cos}[e + f*x] * \text{Hypergeometric2F1}[(1 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + \text{Sin}[e + f*x])/2] * (1 - \text{Sin}[e + f*x])^{(1/2 + m)} * (a + a*\text{Sin}[e + f*x])^m * (c - c*\text{Sin}[e + f*x])^{(-1 - m)}) / (f*(1 + 2*m)) - (B*\text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m) / (f*(c - c*\text{Sin}[e + f*x])^m)$

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))]/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] := Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2745

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*FracPart[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr

eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && (FractionQ[m] || !FractionQ[n])

Rule 2973

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 1, 0]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{-m} dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-m}}{f} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-m}}{f} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-m}}{f} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{-m}}{f} \\ &= \frac{2^{\frac{1}{2}-m} c (A + 2Bm) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, 1 + 2m; \frac{3}{2} + 2m; -\tan^2\left(\frac{e + fx}{2}\right)\right)}{f} \end{aligned}$$

Mathematica [C] time = 17.20, size = 2552, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c - c*Sin[e + f*x])^m,x]

[Out] (2^(2 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^(2*m)*(a + a*Sin[e + f*x])^m*((A*Cos[(-e + Pi/2 - f*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x])/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m))*Tan[(-e + Pi/2 - f*x)/4]/(f*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(c - c*Sin[e + f*x])^m*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)*(-(2^(2 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m))) - ((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4]^2*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(-1 - 2*m))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)))


```

- m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sec[(-e + Pi/2 - f*x)/4]^2)/(2^m*(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(-1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) + (2^(2 - m)*m*((A + B)*AppellF1[1/2 - m, -2*m, 1, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(-AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + AppellF1[1/2 - m, -2*m, 3, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(1 - 2*m)*Tan[(-e + Pi/2 - f*x)/4])/((-1 + 2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)) - (2^(2 - m)*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Tan[(-e + Pi/2 - f*x)/4]*((A + B)*(-((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 1, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/((3/2 - m)) - ((1/2 - m)*AppellF1[3/2 - m, -2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m)))) + 8*B*(((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 2, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/((3/2 - m)) - ((1/2 - m)*m*AppellF1[3/2 - m, 1 - 2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/((3/2 - m)) + ((1/2 - m)*AppellF1[3/2 - m, -2*m, 3, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/((3/2 - m)) - (3*(1/2 - m)*AppellF1[3/2 - m, -2*m, 4, 5/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]*Sec[(-e + Pi/2 - f*x)/4]^2*Tan[(-e + Pi/2 - f*x)/4])/(2*(3/2 - m)))))/((-1 + 2*m)*Sin[(-e + Pi/2 - f*x)/2]^(2*m)*(1 - Tan[(-e + Pi/2 - f*x)/4]^2)^(2*m)))

```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^m}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)
```

maple [F] time = 4.37, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(-c \sin(fx + e) + c)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))^m),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(-c*sin(f*x + e) + c)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c - c \sin(e + fx))^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c - c*sin(e + f*x))^m, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/((c-c*sin(f*x+e))**m),x)

[Out] Timed out

$$3.216 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{1-m} dx$$

Optimal. Leaf size=170

$$\frac{c^2 2^{\frac{1}{2}-m} (2A - B(1 - 2m)) \cos(e + fx) (1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}, 2m+1\right)}{f(2m+1)}$$

[Out] $2^{(1/2-m)} * c^2 * (2*A - B*(1-2*m)) * \cos(f*x+e) * \text{hypergeom}([1/2+m, -1/2+m], [3/2+m], 1/2+1/2*\sin(f*x+e)) * (1-\sin(f*x+e))^{(1/2+m)} * (a+a*\sin(f*x+e))^m * (c-c*\sin(f*x+e))^{(-1-m)} / f / (1+2*m) - 1/2*B*\cos(f*x+e) * (a+a*\sin(f*x+e))^m * (c-c*\sin(f*x+e))^{(1-m)} / f$

Rubi [A] time = 0.33, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^2 2^{\frac{1}{2}-m} (2A - B(1 - 2m)) \cos(e + fx) (1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}, 2m+1\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m * (A + B*\text{Sin}[e + f*x]) * (c - c*\text{Sin}[e + f*x])^{(1 - m)}, x]$

[Out] $(2^{(1/2 - m)} * c^2 * (2*A - B*(1 - 2*m)) * \text{Cos}[e + f*x] * \text{Hypergeometric2F1}[(-1 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + \text{Sin}[e + f*x])/2] * (1 - \text{Sin}[e + f*x])^{(1/2 + m)} * (a + a*\text{Sin}[e + f*x])^m * (c - c*\text{Sin}[e + f*x])^{(-1 - m)}) / (f*(1 + 2*m)) - (B*\text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m * (c - c*\text{Sin}[e + f*x])^{(1 - m)}) / (2*f)$

Rule 69

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{m+1} * \text{Hypergeometric2F1}[-n, m+1, m+2, -(d*(a + b*x))/(b*c - a*d)] / (b*(m+1)*(b/(b*c - a*d))^n), x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x_Symbol] \rightarrow \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * \text{Simp}[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

$\text{Int}[(\cos(e + f*x) + (g + h*\sin(e + f*x))^p * (a + b*\sin(e + f*x)))^m, x_Symbol] \rightarrow \text{Dist}[(a^2 * (g*\cos[e + f*x])^{p+1}) / (f*g*(a + b*\sin[e + f*x])^{(p+1)/2} * (a - b*\sin[e + f*x])^{(p+1)/2}), \text{Subst}[\text{Int}[(a + b*x)^{m + (p-1)/2} * (a - b*x)^{(p-1)/2}, x], x, \text{Sin}[e + f*x]], x] /;$ FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2745

$\text{Int}[(a + b*\sin(e + f*x))^m * (c + d*\sin(e + f*x))^n, x_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[m]} * c^{\text{IntPart}[m]} * (a + b*\sin[e + f*x])^m * (c + d*\sin[e + f*x])^n), x]$

```

+ f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] || !FractionQ[n])

```

Rule 2973

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
+ 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{1-m} dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m}}{2f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m}}{2f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m}}{2f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m}}{2f} \\
&= \frac{2^{\frac{1}{2}-m} c^2 (2A - B(1 - 2m)) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, 1 - m; \frac{3}{2} - m; -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m}}{2f}\right)}{2f}
\end{aligned}$$

Mathematica [C] time = 92.47, size = 3601, normalized size = 21.18

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
1 - m),x]

```

```

[Out] (2^(5 - m)*((A + B)*AppellF1[1/2 - m, -2*m, 2, 3/2 - m, Tan[(-e + Pi/2 - f*
x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] - (A + 9*B)*AppellF1[1/2 - m, -2*m, 3
, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2] + 8*B*(
2*AppellF1[1/2 - m, -2*m, 4, 3/2 - m, Tan[(-e + Pi/2 - f*x)/4]^2, -Tan[(-e
+ Pi/2 - f*x)/4]^2] - AppellF1[1/2 - m, -2*m, 5, 3/2 - m, Tan[(-e + Pi/2 -
f*x)/4]^2, -Tan[(-e + Pi/2 - f*x)/4]^2]))*(a + a*Sin[e + f*x])^m*(c - c*Sin
[e + f*x])^(1 - m)*(Cos[Pi/4 + (e - Pi/2 + f*x)/2]^2*((A*Cos[(-e + Pi/2 - f
*x)/2]^(2*m))/(Cos[Pi/4 + (e - Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)
/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[e + f*x])/(Cos[Pi/4 + (e
- Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m)) + (A*Cos[(-e + P
i/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e - Pi/2
+ f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + (B*Cos[(-e + Pi/2 - f*
x)/2]^(2*m)*Sin[e + f*x]*Sin[Pi/4 + (e - Pi/2 + f*x)/2]^2)/(Cos[Pi/4 + (e -
Pi/2 + f*x)/2] - Sin[Pi/4 + (e - Pi/2 + f*x)/2])^(2*m) + Cos[Pi/4 + (e - P
i/2 + f*x)/2]*((-2*A*Cos[(-e + Pi/2 - f*x)/2]^(2*m)*Sin[Pi/4 + (e - Pi/2 +

```


fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(-c \sin(fx + e) + c\right)^{-m+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)

maple [F] time = 3.81, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(1-m),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^{1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(1 - m), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(1-m),x)
```

```
[Out] Timed out
```

$$3.217 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{2-m} dx$$

Optimal. Leaf size=173

$$\frac{c^3 2^{\frac{5}{2}-m} (3A - 2B(1-m)) \cos(e + fx) (1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}(2m - 1), \frac{3}{2} + m, 1/2 + 1/2 \sin(fx + e) * (1 - \sin(fx + e))^{1/2 + m} * (a + a \sin(fx + e))^m * (c - c \sin(fx + e))^{-1 - m} / f / (1 + 2m) - 1/3 * B * \cos(fx + e) * (a + a \sin(fx + e))^m * (c - c \sin(fx + e))^{2 - m} / f\right)}{3f(2m + 1)}$$

[Out] 1/3*2^(5/2-m)*c^3*(3*A-2*B*(1-m))*cos(f*x+e)*hypergeom([1/2+m, -3/2+m], [3/2+m], 1/2+1/2*sin(f*x+e))*(1-sin(f*x+e))^(1/2+m)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^(2-m)/f

Rubi [A] time = 0.34, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2973, 2745, 2689, 70, 69}

$$\frac{c^3 2^{\frac{5}{2}-m} (3A - 2B(1-m)) \cos(e + fx) (1 - \sin(e + fx))^{m+\frac{1}{2}} (a \sin(e + fx) + a)^m (c - c \sin(e + fx))^{-m-1} {}_2F_1\left(\frac{1}{2}(2m - 1), \frac{3}{2} + m, 1/2 + 1/2 \sin(fx + e) * (1 - \sin(fx + e))^{1/2 + m} * (a + a \sin(fx + e))^m * (c - c \sin(fx + e))^{-1 - m} / f / (1 + 2m) - 1/3 * B * \cos(fx + e) * (a + a \sin(fx + e))^m * (c - c \sin(fx + e))^{2 - m} / f\right)}{3f(2m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(2 - m), x]

[Out] (2^(5/2 - m)*c^3*(3*A - 2*B*(1 - m))*Cos[e + f*x]*Hypergeometric2F1[(-3 + 2*m)/2, (1 + 2*m)/2, (3 + 2*m)/2, (1 + Sin[e + f*x])/2]*(1 - Sin[e + f*x])^(1/2 + m)*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(-1 - m))/(3*f*(1 + 2*m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^(2 - m))/(3*f)

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)])/((b*(m + 1)*(b*(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 70

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])

Rule 2689

Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[(a^2*(g*Cos[e + f*x])^(p + 1))/(f*g*(a + b*Sin[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)), Subst[Int[(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]

Rule 2745

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[m]*c^IntPart[m]*(a + b*Sin[e


```

+ f*x])^FracPart[m]*(c + d*Sin[e + f*x])^FracPart[m])/Cos[e + f*x]^(2*Frac
Part[m]), Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; Fr
eeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0]
&& (FractionQ[m] || !FractionQ[n])

```

Rule 2973

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m +
n + 1)), x] - Dist[(B*c*(m - n) - A*d*(m + n + 1))/(d*(m + n + 1)), Int[(a +
b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e,
f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2
^(-1)] && NeQ[m + n + 1, 0]

```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c - c \sin(e + fx))^{2-m} dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{2-m}}{3f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{2-m}}{3f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{2-m}}{3f} \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^{2-m}}{3f} \\
&= \frac{2^{\frac{5}{2}-m} c^3 (3A - 2B(1 - m)) \cos(e + fx) {}_2F_1}{\dots}
\end{aligned}$$

Mathematica [C] time = 49.22, size = 5163, normalized size = 29.84

Result too large to show

Warning: Unable to verify antiderivative.

```

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c - c*Sin[e + f*x])^(
2 - m), x]

```

```

[Out] Result too large to show

```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(-c \sin(fx + e) + c\right)^{-m+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m), x, alg
orithm="fricas")

```

```

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(
-m + 2), x)

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)

maple [F] time = 6.07, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c - c \sin(fx + e))^{2-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^{-m+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))^(2-m),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^(-m + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^{2-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^(2 - m), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c-c*sin(f*x+e))**(2-m),x)

[Out] Timed out

$$3.218 \quad \int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^n (B(3 - n) - B(4 + n) \sin(e + fx)) dx$$

Optimal. Leaf size=34

$$\frac{a^3 B c^3 \cos^7(e + fx) (c - c \sin(e + fx))^{n-3}}{f}$$

[Out] $a^3 B c^3 \cos(f x + e)^7 (c - c \sin(f x + e))^{(-3 + n)} / f$

Rubi [A] time = 0.27, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2967, 2854}

$$\frac{a^3 B c^3 \cos^7(e + fx) (c - c \sin(e + fx))^{n-3}}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Sin[e + f*x]),x]

[Out] (a^3*B*c^3*Cos[e + f*x]^7*(c - c*Sin[e + f*x])^(-3 + n))/f

Rule 2854

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int (a + a \sin(e + fx))^3 (c - c \sin(e + fx))^n (B(3 - n) - B(4 + n) \sin(e + fx)) dx = (a^3 c^3) \int \cos^6(e + fx) (c - c \sin(e + fx))^n dx = \frac{a^3 B c^3 \cos^7(e + fx) (c - c \sin(e + fx))^{n-3}}{f}$$

Mathematica [A] time = 0.54, size = 63, normalized size = 1.85

$$\frac{a^3 B (14 \sin(2(e + fx)) - \sin(4(e + fx)) + 14 \cos(e + fx) - 6 \cos(3(e + fx))) (c - c \sin(e + fx))^n}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^3*(c - c*Sin[e + f*x])^n*(B*(3 - n) - B*(4 + n)*Sin[e + f*x]),x]

[Out] $(a^3 B (c - c \sin[e + f x])^n (14 \cos[e + f x] - 6 \cos[3(e + f x)] + 14 \sin[2(e + f x)] - \sin[4(e + f x)]) / (8 f)$

fricas [B] time = 0.48, size = 78, normalized size = 2.29

$$\frac{\left(3 B a^3 \cos(f x + e)^3 - 4 B a^3 \cos(f x + e) + \left(B a^3 \cos(f x + e)^3 - 4 B a^3 \cos(f x + e)\right) \sin(f x + e)\right) (-c \sin(f x + e))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x, algorithm="fricas")`

[Out] $-(3 B a^3 \cos(f x + e)^3 - 4 B a^3 \cos(f x + e) + (B a^3 \cos(f x + e)^3 - 4 B a^3 \cos(f x + e)) \sin(f x + e)) (-c \sin(f x + e) + c)^n / f$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(B(n+4) \sin(f x + e) + B(n-3)) (a \sin(f x + e) + a)^3 (-c \sin(f x + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x, algorithm="giac")`

[Out] `integrate(-(B*(n+4)*sin(f*x+e)+B*(n-3))*(a*sin(f*x+e)+a)^3*(-c*sin(f*x+e)+c)^n,x)`

maple [F] time = 8.04, size = 0, normalized size = 0.00

$$\int (a + a \sin(f x + e))^3 (c - c \sin(f x + e))^n (B(3 - n) - B(4 + n) \sin(f x + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)`

[Out] `int((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (B(n+4) \sin(f x + e) + B(n-3)) (a \sin(f x + e) + a)^3 (-c \sin(f x + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^3*(c-c*sin(f*x+e))^n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x, algorithm="maxima")`

[Out] `-integrate((B*(n+4)*sin(f*x+e)+B*(n-3))*(a*sin(f*x+e)+a)^3*(-c*sin(f*x+e)+c)^n,x)`

mupad [B] time = 14.45, size = 64, normalized size = 1.88

$$\frac{B a^3 (-c (\sin(e + f x) - 1))^n (14 \cos(e + f x) - 6 \cos(3e + 3 f x) + 14 \sin(2e + 2 f x) - \sin(4e + 4 f x))}{8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(B*(n-3)+B*sin(e+f*x)*(n+4))*(a+a*sin(e+f*x))^3*(c-c*sin(e+f*x))^n,x)`

[Out] $(B*a^3*(-c*(\sin(e + f*x) - 1))^n*(14*\cos(e + f*x) - 6*\cos(3*e + 3*f*x) + 14*\sin(2*e + 2*f*x) - \sin(4*e + 4*f*x)))/(8*f)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**3*(c-c*sin(f*x+e))**n*(B*(3-n)-B*(4+n)*sin(f*x+e)),x)`

[Out] Timed out

$$3.219 \quad \int (a - a \sin(e + fx))^3 (c + c \sin(e + fx))^n (B(3 - n) + B(4 + n) \sin(e + fx)) dx$$

Optimal. Leaf size=34

$$\frac{a^3 B c^3 \cos^7(e + fx) (c \sin(e + fx) + c)^{n-3}}{f}$$

[Out] $-a^3 B c^3 \cos^7(e + fx) (c \sin(e + fx) + c)^{n-3} / f$

Rubi [A] time = 0.24, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2967, 2854}

$$\frac{a^3 B c^3 \cos^7(e + fx) (c \sin(e + fx) + c)^{n-3}}{f}$$

Antiderivative was successfully verified.

[In] `Int[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Sin[e + f*x]), x]`

[Out] $-(a^3 B c^3 \cos^7(e + fx) (c \sin(e + fx) + c)^{n-3}) / f$

Rule 2854

`Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]`

Rule 2967

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))`

Rubi steps

$$\int (a - a \sin(e + fx))^3 (c + c \sin(e + fx))^n (B(3 - n) + B(4 + n) \sin(e + fx)) dx = (a^3 c^3) \int \cos^6(e + fx) (c + c \sin(e + fx))^n dx = \frac{a^3 B c^3 \cos^7(e + fx) (c + c \sin(e + fx))^{n-3}}{f}$$

Mathematica [A] time = 1.14, size = 67, normalized size = 1.97

$$\frac{a^3 B \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) (c(\sin(e + fx) + 1))^n}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(a - a*Sin[e + f*x])^3*(c + c*Sin[e + f*x])^n*(B*(3 - n) + B*(4 + n)*Sin[e + f*x]),x]

[Out] -((a^3*B*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(c*(1 + Sin[e + f*x]))^n)/f)

fricas [B] time = 0.47, size = 77, normalized size = 2.26

$$\frac{\left(3Ba^3 \cos(fx + e)^3 - 4Ba^3 \cos(fx + e) - \left(Ba^3 \cos(fx + e)^3 - 4Ba^3 \cos(fx + e)\right) \sin(fx + e)\right) (c \sin(fx + e) + c)^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x, algorithm="fricas")

[Out] (3*B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e) - (B*a^3*cos(f*x + e)^3 - 4*B*a^3*cos(f*x + e))*sin(f*x + e))*(c*sin(f*x + e) + c)^n/f

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -(B(n + 4) \sin(fx + e) - B(n - 3))(a \sin(fx + e) - a)^3 (c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x, algorithm="giac")

[Out] integrate(-(B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*sin(f*x + e) + c)^n, x)

maple [F] time = 8.06, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))^3 (c + c \sin(fx + e))^n (B(3 - n) + B(4 + n) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)

[Out] int((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int (B(n + 4) \sin(fx + e) - B(n - 3))(a \sin(fx + e) - a)^3 (c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+e)),x, algorithm="maxima")

[Out] -integrate((B*(n + 4)*sin(f*x + e) - B*(n - 3))*(a*sin(f*x + e) - a)^3*(c*sin(f*x + e) + c)^n, x)

mupad [B] time = 14.46, size = 61, normalized size = 1.79

$$\frac{Ba^3 (c (\sin(e + fx) + 1))^n (14 \cos(e + fx) - 6 \cos(3e + 3fx) - 14 \sin(2e + 2fx) + \sin(4e + 4fx))}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(B*(n - 3) - B*sin(e + f*x))*(n + 4))*(a - a*sin(e + f*x))^3*(c + c*sin
(e + f*x))^n,x)
```

```
[Out] -(B*a^3*(c*(sin(e + f*x) + 1))^n*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14
*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^3*(c+c*sin(f*x+e))^n*(B*(3-n)+B*(4+n)*sin(f*x+
e)),x)
```

```
[Out] Timed out
```


$$3.220 \quad \int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^3 (B(-3 + m) - B(4 + m) \sin(e + fx)) dx$$

Optimal. Leaf size=33

$$\frac{a^3 B c^3 \cos^7(e + fx) (a \sin(e + fx) + a)^{m-3}}{f}$$

[Out] a^3*B*c^3*cos(f*x+e)^7*(a+a*sin(f*x+e))^(m-3)/f

Rubi [A] time = 0.24, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2967, 2854}

$$\frac{a^3 B c^3 \cos^7(e + fx) (a \sin(e + fx) + a)^{m-3}}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*Sin[e + f*x]), x]

[Out] (a^3*B*c^3*Cos[e + f*x]^7*(a + a*Sin[e + f*x])^(m-3))/f

Rule 2854

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p_*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]

Rule 2967

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m_*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^n, x_Symbol] := Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^3 (B(-3 + m) - B(4 + m) \sin(e + fx)) dx = (a^3 c^3) \int \cos^6(e + fx) (a + a \sin(e + fx))^m (c - c \sin(e + fx))^3 dx = \frac{a^3 B c^3 \cos^7(e + fx) (a + a \sin(e + fx))^{m-3}}{f}$$

Mathematica [A] time = 1.11, size = 66, normalized size = 2.00

$$\frac{B c^3 \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right)^7 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^7 (a(\sin(e + fx) + 1))^m}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^3*(B*(-3 + m) - B*(4 + m)*Sin[e + f*x]),x]

[Out] (B*c^3*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^7*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))*(a*(1 + Sin[e + f*x]))^m)/f

fricas [B] time = 0.49, size = 78, normalized size = 2.36

$$\frac{\left(3 B c^3 \cos (f x+e)^3-4 B c^3 \cos (f x+e)-\left(B c^3 \cos (f x+e)^3-4 B c^3 \cos (f x+e)\right) \sin (f x+e)\right)\left(a \sin (f x+e)\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x, algorithm="fricas")

[Out] -(3*B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e) - (B*c^3*cos(f*x + e)^3 - 4*B*c^3*cos(f*x + e))*sin(f*x + e))*(a*sin(f*x + e) + a)^m/f

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B(m+4) \sin (f x+e)-B(m-3))(c \sin (f x+e)-c)^3(a \sin (f x+e)+a)^m d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 7.86, size = 0, normalized size = 0.00

$$\int (a+a \sin (f x+e))^m(c-c \sin (f x+e))^3(B(m-3)-B(4+m) \sin (f x+e)) d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(m-3)-B*(4+m)*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B(m+4) \sin (f x+e)-B(m-3))(c \sin (f x+e)-c)^3(a \sin (f x+e)+a)^m d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*(m + 4)*sin(f*x + e) - B*(m - 3))*(c*sin(f*x + e) - c)^3*(a*sin(f*x + e) + a)^m, x)

mupad [B] time = 14.38, size = 61, normalized size = 1.85

$$\frac{B c^3\left(a\left(\sin (e+f x)+1\right)\right)^m\left(14 \cos (e+f x)-6 \cos (3 e+3 f x)-14 \sin (2 e+2 f x)+\sin (4 e+4 f x)\right)}{8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*(m - 3) - B*sin(e + f*x)*(m + 4))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^3,x)
```

```
[Out] (B*c^3*(a*(sin(e + f*x) + 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) - 14*sin(2*e + 2*f*x) + sin(4*e + 4*f*x)))/(8*f)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^3*(B*(-3+m)-B*(4+m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

$$3.221 \quad \int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^3 (B(-3 + m) + B(4 + m) \sin(e + fx)) dx$$

Optimal. Leaf size=35

$$-\frac{a^3 B c^3 \cos^7(e + fx) (a - a \sin(e + fx))^{m-3}}{f}$$

[Out] $-a^3 B c^3 \cos^7(fx + e)^7 (a - a \sin(fx + e))^{(-3 + m)} / f$

Rubi [A] time = 0.24, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2967, 2854}

$$-\frac{a^3 B c^3 \cos^7(e + fx) (a - a \sin(e + fx))^{m-3}}{f}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*Sin[e + f*x]),x]

[Out] $-((a^3 B c^3 \cos^7[e + f*x])^7 (a - a \sin[e + f*x])^{(-3 + m)}) / f$

Rule 2854

Int[(cos[(e_) + (f_)*(x_)]*(g_))^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[a*d*m + b*c*(m + p + 1), 0]

Rule 2967

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m)*(A + B*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] & GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^3 (B(-3 + m) + B(4 + m) \sin(e + fx)) dx = (a^3 c^3) \int \cos^6(e + fx) (a - a \sin(e + fx))^m dx = -\frac{a^3 B c^3 \cos^7(e + fx) (a - a \sin(e + fx))^m}{f}$$

Mathematica [A] time = 0.54, size = 61, normalized size = 1.74

$$\frac{B c^3 (-14 \sin(2(e + fx)) + \sin(4(e + fx)) - 14 \cos(e + fx) + 6 \cos(3(e + fx))) (a - a \sin(e + fx))^m}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^3*(B*(-3 + m) + B*(4 + m)*Sin[e + f*x]),x]

[Out] $(B*c^3*(a - a*\sin[e + f*x])^m*(-14*\cos[e + f*x] + 6*\cos[3*(e + f*x)] - 14*\sin[2*(e + f*x)] + \sin[4*(e + f*x)]))/(8*f)$

fricas [B] time = 0.48, size = 77, normalized size = 2.20

$$\frac{(3Bc^3 \cos(fx + e)^3 - 4Bc^3 \cos(fx + e) + (Bc^3 \cos(fx + e)^3 - 4Bc^3 \cos(fx + e)) \sin(fx + e))(-a \sin(fx + e) + a)^m}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e))),x, algorithm="fricas")

[Out] $(3*B*c^3*\cos(f*x + e)^3 - 4*B*c^3*\cos(f*x + e) + (B*c^3*\cos(f*x + e)^3 - 4*B*c^3*\cos(f*x + e))*\sin(f*x + e))*(-a*\sin(f*x + e) + a)^m/f$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B(m + 4) \sin(fx + e) + B(m - 3))(c \sin(fx + e) + c)^3 (-a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e))),x, algorithm="giac")

[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*sin(f*x + e) + a)^m, x)

maple [F] time = 8.12, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))^m (c + c \sin(fx + e))^3 (B(m - 3) + B(4 + m) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)

[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(m-3)+B*(4+m)*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B(m + 4) \sin(fx + e) + B(m - 3))(c \sin(fx + e) + c)^3 (-a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e))),x, algorithm="maxima")

[Out] integrate((B*(m + 4)*sin(f*x + e) + B*(m - 3))*(c*sin(f*x + e) + c)^3*(-a*sin(f*x + e) + a)^m, x)

mupad [B] time = 14.36, size = 64, normalized size = 1.83

$$\frac{Bc^3(-a(\sin(e + fx) - 1))^m(14\cos(e + fx) - 6\cos(3e + 3fx) + 14\sin(2e + 2fx) - \sin(4e + 4fx))}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*(m - 3) + B*sin(e + f*x)*(m + 4))*(a - a*sin(e + f*x))^m*(c + c*sin(e + f*x))^3,x)

```
[Out] -(B*c^3*(-a*(sin(e + f*x) - 1))^m*(14*cos(e + f*x) - 6*cos(3*e + 3*f*x) + 14*sin(2*e + 2*f*x) - sin(4*e + 4*f*x)))/(8*f)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^3*(B*(-3+m)+B*(4+m)*sin(f*x+e)),x)
```

```
[Out] Timed out
```

$$3.222 \quad \int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n (B(m - n) - B(1 + m + n) \sin(e + fx)) dx$$

Optimal. Leaf size=36

$$\frac{B \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^n}{f}$$

[Out] B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n/f

Rubi [A] time = 0.13, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$, Rules used = {2970}

$$\frac{B \cos(e + fx)(a \sin(e + fx) + a)^m (c - c \sin(e + fx))^n}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n)*Sin[e + f*x]),x]

[Out] (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n)/f

Rule 2970

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2^(-1)]

Rubi steps

$$\int (a + a \sin(e + fx))^m (c - c \sin(e + fx))^n (B(m - n) - B(1 + m + n) \sin(e + fx)) dx = \frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c - c \sin(e + fx))^n}{f}$$

Mathematica [A] time = 0.49, size = 36, normalized size = 1.00

$$\frac{B \cos(e + fx)(a(\sin(e + fx) + 1))^m (c - c \sin(e + fx))^n}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^m*(c - c*Sin[e + f*x])^n*(B*(m - n) - B*(1 + m + n)*Sin[e + f*x]),x]

[Out] (B*Cos[e + f*x]*(a*(1 + Sin[e + f*x]))^m*(c - c*Sin[e + f*x])^n)/f

fricas [A] time = 0.49, size = 36, normalized size = 1.00

$$\frac{(a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^n B \cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x, algorithm="fricas")

[Out] (a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n*B*cos(f*x + e)/f

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [F] time = 7.66, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (c - c \sin(fx + e))^n (B(m - n) - B(n + m + 1) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(n+m+1)*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(n+m+1)*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int (B(m + n + 1) \sin(fx + e) - B(m - n)) (a \sin(fx + e) + a)^m (-c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x, algorithm="maxima")

[Out] -integrate((B*(m + n + 1)*sin(f*x + e) - B*(m - n))*(a*sin(f*x + e) + a)^m*(-c*sin(f*x + e) + c)^n, x)

mupad [B] time = 13.54, size = 36, normalized size = 1.00

$$\frac{B \cos(e + fx) (a (\sin(e + fx) + 1))^m (-c (\sin(e + fx) - 1))^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*(m - n) - B*sin(e + f*x)*(m + n + 1))*(a + a*sin(e + f*x))^m*(c - c*sin(e + f*x))^n,x)

[Out] (B*cos(e + f*x)*(a*(sin(e + f*x) + 1))^m*(-c*(sin(e + f*x) - 1))^n)/f

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c-c*sin(f*x+e))^n*(B*(m-n)-B*(1+m+n)*sin(f*x+e)),x)

[Out] Timed out

$$3.223 \quad \int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^n (B(m - n) + B(1 + m + n) \sin(e + fx)) dx$$

Optimal. Leaf size=37

$$\frac{B \cos(e + fx)(a - a \sin(e + fx))^m (c \sin(e + fx) + c)^n}{f}$$

[Out] $-B \cos(fx + e) (a - a \sin(fx + e))^m (c + c \sin(fx + e))^n / f$

Rubi [A] time = 0.12, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.022$, Rules used = {2970}

$$\frac{B \cos(e + fx)(a - a \sin(e + fx))^m (c \sin(e + fx) + c)^n}{f}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n)*Sin[e + f*x]),x]

[Out] $-((B \cos[e + f*x] (a - a \sin[e + f*x])^m (c + c \sin[e + f*x])^n) / f)$

Rule 2970

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + 1)), x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[A*b*(m + n + 1) + a*B*(m - n), 0] && NeQ[m, -2^(-1)]

Rubi steps

$$\int (a - a \sin(e + fx))^m (c + c \sin(e + fx))^n (B(m - n) + B(1 + m + n) \sin(e + fx)) dx = -\frac{B \cos(e + fx)(a - a \sin(e + fx))^m (c \sin(e + fx) + c)^n}{f}$$

Mathematica [A] time = 0.47, size = 37, normalized size = 1.00

$$\frac{B \cos(e + fx)(a - a \sin(e + fx))^m (c(\sin(e + fx) + 1))^n}{f}$$

Antiderivative was successfully verified.

[In] Integrate[(a - a*Sin[e + f*x])^m*(c + c*Sin[e + f*x])^n*(B*(m - n) + B*(1 + m + n)*Sin[e + f*x]),x]

[Out] $-((B \cos[e + f*x] (c*(1 + \sin[e + f*x]))^n (a - a \sin[e + f*x])^m) / f)$

fricas [A] time = 0.50, size = 37, normalized size = 1.00

$$\frac{(-a \sin(fx + e) + a)^m (c \sin(fx + e) + c)^n B \cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x, algorithm="fricas")

[Out] $-(-a*\sin(f*x + e) + a)^m*(c*\sin(f*x + e) + c)^n*B*\cos(f*x + e)/f$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [F] time = 8.40, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))^m (c + c \sin(fx + e))^n (B(m - n) + B(n + m + 1) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(n+m+1)*sin(f*x+e)),x)

[Out] int((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(n+m+1)*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B(m + n + 1) \sin(fx + e) + B(m - n))(-a \sin(fx + e) + a)^m (c \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*(m + n + 1)*sin(f*x + e) + B*(m - n))*(-a*sin(f*x + e) + a)^m*(c*sin(f*x + e) + c)^n, x)

mupad [B] time = 13.50, size = 37, normalized size = 1.00

$$\frac{B \cos(e + fx) (-a (\sin(e + fx) - 1))^m (c (\sin(e + fx) + 1))^n}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*(m - n) + B*sin(e + f*x)*(m + n + 1))*(a - a*sin(e + f*x))^m*(c + c*sin(e + f*x))^n,x)

[Out] $-(B*\cos(e + f*x)*(-a*(\sin(e + f*x) - 1))^m*(c*(\sin(e + f*x) + 1))^n)/f$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+c*sin(f*x+e))^n*(B*(m-n)+B*(1+m+n)*sin(f*x+e)),x)

[Out] Timed out

3.224 $\int \sin^3(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=140

$$-\frac{a^3 A \cos^7(c+dx)}{7d} + \frac{3a^3 A \cos^5(c+dx)}{5d} - \frac{2a^3 A \cos^3(c+dx)}{3d} + \frac{a^3 A \sin^5(c+dx) \cos(c+dx)}{3d} - \frac{a^3 A \sin^3(c+dx)}{12d}$$

[Out] $1/8*a^3*A*x-2/3*a^3*A*\cos(d*x+c)^3/d+3/5*a^3*A*\cos(d*x+c)^5/d-1/7*a^3*A*\cos(d*x+c)^7/d-1/8*a^3*A*\cos(d*x+c)*\sin(d*x+c)/d-1/12*a^3*A*\cos(d*x+c)*\sin(d*x+c)^3/d+1/3*a^3*A*\cos(d*x+c)*\sin(d*x+c)^5/d$

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 2633, 2635, 8}

$$-\frac{a^3 A \cos^7(c+dx)}{7d} + \frac{3a^3 A \cos^5(c+dx)}{5d} - \frac{2a^3 A \cos^3(c+dx)}{3d} + \frac{a^3 A \sin^5(c+dx) \cos(c+dx)}{3d} - \frac{a^3 A \sin^3(c+dx)}{12d}$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] $(a^3*A*x)/8 - (2*a^3*A*\cos[c + d*x]^3)/(3*d) + (3*a^3*A*\cos[c + d*x]^5)/(5*d) - (a^3*A*\cos[c + d*x]^7)/(7*d) - (a^3*A*\cos[c + d*x]*\sin[c + d*x])/(8*d) - (a^3*A*\cos[c + d*x]*\sin[c + d*x]^3)/(12*d) + (a^3*A*\cos[c + d*x]*\sin[c + d*x]^5)/(3*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \sin^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (a^3A\sin^3(c+dx)+2a^3A\sin^4(c+dx)-2a^3A\sin^5(c+dx) \\
&= (a^3A)\int \sin^3(c+dx)dx - (a^3A)\int \sin^7(c+dx)dx - \\
&= -\frac{a^3A\cos(c+dx)\sin^3(c+dx)}{2d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{3d} \\
&= -\frac{2a^3A\cos^3(c+dx)}{3d} + \frac{3a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos^7(c+dx)}{7d} \\
&= \frac{3}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{3a^3A\cos^5(c+dx)}{5d} - \\
&= \frac{1}{8}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{3a^3A\cos^5(c+dx)}{5d} -
\end{aligned}$$

Mathematica [A] time = 0.14, size = 87, normalized size = 0.62

$$\frac{a^3A(-210\sin(2(c+dx)) - 210\sin(4(c+dx)) + 70\sin(6(c+dx)) - 1365\cos(c+dx) - 175\cos(3(c+dx)) + 147\cos(5(c+dx)) - 15\cos(7(c+dx)) - 210\sin(2(c+dx)) - 210\sin(4(c+dx)) + 70\sin(6(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(840*c + 840*d*x - 1365*Cos[c + d*x] - 175*Cos[3*(c + d*x)] + 147*Cos[5*(c + d*x)] - 15*Cos[7*(c + d*x)] - 210*Sin[2*(c + d*x)] - 210*Sin[4*(c + d*x)] + 70*Sin[6*(c + d*x)]))/(6720*d)

fricas [A] time = 0.46, size = 105, normalized size = 0.75

$$\frac{120Aa^3\cos(dx+c)^7 - 504Aa^3\cos(dx+c)^5 + 560Aa^3\cos(dx+c)^3 - 105Aa^3dx - 35(8Aa^3\cos(dx+c)^5 - 14Aa^3\cos(dx+c)^3 + 3Aa^3\cos(dx+c))\sin(dx+c)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/840*(120*A*a^3*cos(d*x + c)^7 - 504*A*a^3*cos(d*x + c)^5 + 560*A*a^3*cos(d*x + c)^3 - 105*A*a^3*d*x - 35*(8*A*a^3*cos(d*x + c)^5 - 14*A*a^3*cos(d*x + c)^3 + 3*A*a^3*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.17, size = 131, normalized size = 0.94

$$\frac{1}{8}Aa^3x - \frac{Aa^3\cos(7dx+7c)}{448d} + \frac{7Aa^3\cos(5dx+5c)}{320d} - \frac{5Aa^3\cos(3dx+3c)}{192d} - \frac{13Aa^3\cos(dx+c)}{64d} + \frac{Aa^3\sin(6dx+6c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/8*A*a^3*x - 1/448*A*a^3*cos(7*d*x + 7*c)/d + 7/320*A*a^3*cos(5*d*x + 5*c)/d - 5/192*A*a^3*cos(3*d*x + 3*c)/d - 13/64*A*a^3*cos(d*x + c)/d + 1/96*A*a^3*sin(6*d*x + 6*c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d - 1/32*A*a^3*sin(2*d*x + 2*c)/d

maple [A] time = 0.60, size = 158, normalized size = 1.13

$$\frac{a^3A\left(\frac{16}{5}\sin^6(dx+c) + \frac{6\sin^4(dx+c)}{5} + \frac{8\sin^2(dx+c)}{5}\right)\cos(dx+c)}{7} - 2a^3A\left(\frac{\left(\sin^5(dx+c) + \frac{5\sin^3(dx+c)}{4} + \frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16}\right) + 2a^3A\left(\frac{1}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)`

[Out] $\frac{1}{d} \left(\frac{1}{7} a^3 A (16/5 + \sin(dx+c)^6 + 6/5 \sin(dx+c)^4 + 8/5 \sin(dx+c)^2) \cos(dx+c) - 2 a^3 A (-1/6 (\sin(dx+c)^5 + 5/4 \sin(dx+c)^3 + 15/8 \sin(dx+c)) \cos(dx+c) + 5/16 dx + 5/16 c) + 2 a^3 A (-1/4 (\sin(dx+c)^3 + 3/2 \sin(dx+c)) \cos(dx+c) + 3/8 dx + 3/8 c) - 1/3 a^3 A (2 + \sin(dx+c)^2) \cos(dx+c) \right)$

maxima [A] time = 0.43, size = 157, normalized size = 1.12

$$\frac{96 \left(5 \cos(dx+c)^7 - 21 \cos(dx+c)^5 + 35 \cos(dx+c)^3 - 35 \cos(dx+c) \right) A a^3 - 1120 \left(\cos(dx+c)^3 - 3 \cos(dx+c) \right) A a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")`

[Out] $-\frac{1}{3360} (96 (5 \cos(dx+c)^7 - 21 \cos(dx+c)^5 + 35 \cos(dx+c)^3 - 35 \cos(dx+c)) A a^3 - 1120 (\cos(dx+c)^3 - 3 \cos(dx+c)) A a^3 + 35 (4 \sin(2dx+2c)^3 + 60 dx + 60 c + 9 \sin(4dx+4c) - 48 \sin(2dx+2c)) A a^3 - 210 (12 dx + 12 c + \sin(4dx+4c) - 8 \sin(2dx+2c)) A a^3) / d$

mupad [B] time = 15.30, size = 300, normalized size = 2.14

$$A a^3 \left(105 c - 210 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 2464 \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) - 1400 \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right) - 4032 \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 6790 \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right) + 2240 \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) - 14560 \tan^8\left(\frac{c}{2} + \frac{dx}{2}\right) - 6790 \tan^9\left(\frac{c}{2} + \frac{dx}{2}\right) + 210 \tan^{13}\left(\frac{c}{2} + \frac{dx}{2}\right) + 105 dx + 735 \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 2205 \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 3675 \tan^6\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 3675 \tan^8\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 2205 \tan^{10}\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 735 \tan^{12}\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) + 105 \tan^{14}\left(\frac{c}{2} + \frac{dx}{2}\right) (c + dx) - 352 \right) / (840 d (\tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 1)^7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c+d*x)^3*(A-A*sin(c+d*x))*(a+a*sin(c+d*x))^3,x)`

[Out] $(A a^3 (105 c - 210 \tan(c/2 + (dx)/2) - 2464 \tan^2(c/2 + (dx)/2) - 1400 \tan^3(c/2 + (dx)/2) - 4032 \tan^4(c/2 + (dx)/2) + 6790 \tan^5(c/2 + (dx)/2) + 2240 \tan^6(c/2 + (dx)/2) - 14560 \tan^8(c/2 + (dx)/2) - 6790 \tan^9(c/2 + (dx)/2) + 210 \tan^{13}(c/2 + (dx)/2) + 105 dx + 735 \tan^2(c/2 + (dx)/2) (c + dx) + 2205 \tan^4(c/2 + (dx)/2) (c + dx) + 3675 \tan^6(c/2 + (dx)/2) (c + dx) + 3675 \tan^8(c/2 + (dx)/2) (c + dx) + 2205 \tan^{10}(c/2 + (dx)/2) (c + dx) + 735 \tan^{12}(c/2 + (dx)/2) (c + dx) + 105 \tan^{14}(c/2 + (dx)/2) (c + dx) - 352) / (840 d (\tan^2(c/2 + (dx)/2) + 1)^7)$

sympy [A] time = 8.49, size = 440, normalized size = 3.14

$$\left\{ \begin{array}{l} \frac{5 A a^3 x \sin^6(c+dx)}{8} - \frac{15 A a^3 x \sin^4(c+dx) \cos^2(c+dx)}{8} + \frac{3 A a^3 x \sin^4(c+dx)}{4} - \frac{15 A a^3 x \sin^2(c+dx) \cos^4(c+dx)}{8} + \frac{3 A a^3 x \sin^2(c+dx) \cos^2(c+dx)}{2} \\ x (-A \sin(c) + A) (a \sin(c) + a)^3 \sin^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)`

[Out] $\text{Piecewise} \left(\left(-5 A a^3 x \sin(c+dx)^6 / 8 - 15 A a^3 x \sin(c+dx)^4 \cos^2(c+dx) / 8 + 3 A a^3 x \sin(c+dx)^2 \cos^4(c+dx) / 8 + 3 A a^3 x \sin^2(c+dx) \cos^2(c+dx) / 2 - 5 A a^3 x \cos^6(c+dx) / 8 + 3 A a^3 x \cos^4(c+dx) / 4 + A a^3 \sin^2(c+dx) \cos^2(c+dx) / 2 + 11 A a^3 \sin^5(c+dx) \cos(c+dx) / (8 d) + 2 A a^3 \sin^4(c+dx) \cos(c+dx) / (8 d) \right) \right)$

```

sin(c + d*x)**4*cos(c + d*x)**3/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)**
3/(3*d) - 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 8*A*a**3*sin(c + d*
x)**2*cos(c + d*x)**5/(5*d) - A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 5*A*a
**3*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 3*A*a**3*sin(c + d*x)*cos(c + d*x)
**3/(4*d) + 16*A*a**3*cos(c + d*x)**7/(35*d) - 2*A*a**3*cos(c + d*x)**3/(3*
d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**3, True))

```

3.225 $\int \sin^2(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=121

$$\frac{2a^3 A \cos^5(c+dx)}{5d} - \frac{2a^3 A \cos^3(c+dx)}{3d} + \frac{a^3 A \sin^5(c+dx) \cos(c+dx)}{6d} + \frac{5a^3 A \sin^3(c+dx) \cos(c+dx)}{24d} - \frac{3a^3 A \sin(c+dx)}{6d}$$

[Out] $3/16*a^3*A*x-2/3*a^3*A*\cos(d*x+c)^3/d+2/5*a^3*A*\cos(d*x+c)^5/d-3/16*a^3*A*\cos(d*x+c)*\sin(d*x+c)/d+5/24*a^3*A*\cos(d*x+c)*\sin(d*x+c)^3/d+1/6*a^3*A*\cos(d*x+c)*\sin(d*x+c)^5/d$

Rubi [A] time = 0.17, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 2635, 8, 2633}

$$\frac{2a^3 A \cos^5(c+dx)}{5d} - \frac{2a^3 A \cos^3(c+dx)}{3d} + \frac{a^3 A \sin^5(c+dx) \cos(c+dx)}{6d} + \frac{5a^3 A \sin^3(c+dx) \cos(c+dx)}{24d} - \frac{3a^3 A \sin(c+dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[c + d*x]^2*(a + a*\text{Sin}[c + d*x])^3*(A - A*\text{Sin}[c + d*x]), x]$

[Out] $(3*a^3*A*x)/16 - (2*a^3*A*\text{Cos}[c + d*x]^3)/(3*d) + (2*a^3*A*\text{Cos}[c + d*x]^5)/(5*d) - (3*a^3*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(16*d) + (5*a^3*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^3)/(24*d) + (a^3*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x]^5)/(6*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2966

$\text{Int}[\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\text{sin}[e + f*x]^n*(a + b*\text{sin}[e + f*x])^m*(A + B*\text{sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{EqQ}[A*b + a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rubi steps

$$\begin{aligned}
\int \sin^2(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (a^3A\sin^2(c+dx)+2a^3A\sin^3(c+dx)-2a^3A\sin^4(c+dx))dx \\
&= (a^3A)\int \sin^2(c+dx)dx - (a^3A)\int \sin^6(c+dx)dx \\
&= -\frac{a^3A\cos(c+dx)\sin(c+dx)}{2d} + \frac{a^3A\cos(c+dx)\sin^5(c+dx)}{6d} \\
&= \frac{1}{2}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} \\
&= \frac{1}{2}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d} \\
&= \frac{3}{16}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{2a^3A\cos^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 77, normalized size = 0.64

$$\frac{a^3A(-15\sin(2(c+dx)) - 45\sin(4(c+dx)) + 5\sin(6(c+dx)) - 240\cos(c+dx) - 40\cos(3(c+dx)) + 24\cos(5(c+dx)))}{960d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(180*c + 180*d*x - 240*Cos[c + d*x] - 40*Cos[3*(c + d*x)] + 24*Cos[5*(c + d*x)] - 15*Sin[2*(c + d*x)] - 45*Sin[4*(c + d*x)] + 5*Sin[6*(c + d*x)])/(960*d)

fricas [A] time = 0.46, size = 91, normalized size = 0.75

$$\frac{96Aa^3\cos(dx+c)^5 - 160Aa^3\cos(dx+c)^3 + 45Aa^3dx + 5(8Aa^3\cos(dx+c)^5 - 26Aa^3\cos(dx+c)^3 + 9Aa^3\sin(dx+c))}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/240*(96*A*a^3*cos(d*x + c)^5 - 160*A*a^3*cos(d*x + c)^3 + 45*A*a^3*d*x + 5*(8*A*a^3*cos(d*x + c)^5 - 26*A*a^3*cos(d*x + c)^3 + 9*A*a^3*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.17, size = 113, normalized size = 0.93

$$\frac{3}{16}Aa^3x + \frac{Aa^3\cos(5dx+5c)}{40d} - \frac{Aa^3\cos(3dx+3c)}{24d} - \frac{Aa^3\cos(dx+c)}{4d} + \frac{Aa^3\sin(6dx+6c)}{192d} - \frac{3Aa^3\sin(4dx+4c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 3/16*A*a^3*x + 1/40*A*a^3*cos(5*d*x + 5*c)/d - 1/24*A*a^3*cos(3*d*x + 3*c)/d - 1/4*A*a^3*cos(d*x + c)/d + 1/192*A*a^3*sin(6*d*x + 6*c)/d - 3/64*A*a^3*sin(4*d*x + 4*c)/d - 1/64*A*a^3*sin(2*d*x + 2*c)/d

maple [A] time = 0.50, size = 136, normalized size = 1.12

$$-a^3A\left(\frac{\left(\sin^5(dx+c) + \frac{5\sin^3(dx+c)}{4} + \frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6} + \frac{5dx}{16} + \frac{5c}{16}\right) + \frac{2a^3A\left(\frac{8}{3}\sin^4(dx+c) + \frac{4\sin^2(dx+c)}{3}\right)\cos(dx+c)}{5} - \frac{2a^3A(2+\sin^2(dx+c))}{3}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)`

[Out] $\frac{1}{d}(-a^3A(-\frac{1}{6}(\sin(dx+c))^5+\frac{5}{4}\sin(dx+c)^3+\frac{15}{8}\sin(dx+c))\cos(dx+c)+\frac{5}{16}dx+\frac{5}{16}c)+\frac{2}{5}a^3A(\frac{8}{3}+\sin(dx+c)^4+\frac{4}{3}\sin(dx+c)^2)\cos(dx+c)-\frac{2}{3}a^3A(2+\sin(dx+c)^2)\cos(dx+c)+a^3A(-\frac{1}{2}\cos(dx+c)\sin(dx+c)+\frac{1}{2}dx+\frac{1}{2}c)$

maxima [A] time = 0.40, size = 138, normalized size = 1.14

$$\frac{128(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c))Aa^3+640(\cos(dx+c)^3-3\cos(dx+c))Aa^3-5(\dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")`

[Out] $\frac{1}{960}(128(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c))Aa^3+640(\cos(dx+c)^3-3\cos(dx+c))Aa^3-5(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c))Aa^3+240(2dx+2c-\sin(2dx+2c))Aa^3)/d$

mupad [B] time = 15.29, size = 256, normalized size = 2.12

$$Aa^3\left(45c-90\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-768\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+130\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+1500\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-1280\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c+d*x)^2*(A-A*sin(c+d*x))*(a+a*sin(c+d*x))^3,x)`

[Out] $(Aa^3(45c-90\tan(c/2+(dx)/2)-768\tan(c/2+(dx)/2)^2+130\tan(c/2+(dx)/2)^3+1500\tan(c/2+(dx)/2)^5-1280\tan(c/2+(dx)/2)^6-1500\tan(c/2+(dx)/2)^7-1920\tan(c/2+(dx)/2)^8-130\tan(c/2+(dx)/2)^9+90\tan(c/2+(dx)/2)^{11}+45dx+270\tan(c/2+(dx)/2)^2(c+dx)+675\tan(c/2+(dx)/2)^4(c+dx)+900\tan(c/2+(dx)/2)^6(c+dx)+675\tan(c/2+(dx)/2)^8(c+dx)+270\tan(c/2+(dx)/2)^{10}(c+dx)+45\tan(c/2+(dx)/2)^{12}(c+dx)-128)/(240d(\tan(c/2+(dx)/2)^2+1)^6)$

sympy [A] time = 5.69, size = 359, normalized size = 2.97

$$\left\{ \begin{array}{l} -\frac{5Aa^3x\sin^6(c+dx)}{16} - \frac{15Aa^3x\sin^4(c+dx)\cos^2(c+dx)}{16} - \frac{15Aa^3x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{Aa^3x\sin^2(c+dx)}{2} - \frac{5Aa^3x\cos^6(c+dx)}{16} + \frac{Aa^3x\cos^6(c+dx)}{16} \\ x(-A\sin(c)+A)(a\sin(c)+a)^3\sin^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)`

[Out] `Piecewise((-5*A*a**3*x*sin(c+d*x)**6/16-15*A*a**3*x*sin(c+d*x)**4*cos(c+d*x)**2/16-15*A*a**3*x*sin(c+d*x)**2*cos(c+d*x)**4/16+A*a**3*x*sin(c+d*x)**2/2-5*A*a**3*x*cos(c+d*x)**6/16+A*a**3*x*cos(c+d*x)**2/2+11*A*a**3*sin(c+d*x)**5*cos(c+d*x)/(16*d)+2*A*a**3*sin(c+d*x)**4*cos(c+d*x)/d+5*A*a**3*sin(c+d*x)**3*cos(c+d*x)**3/(6*d)+8*A*a**3*sin(c+d*x)**2*cos(c+d*x)**3/(3*d)-2*A*a**3*sin(c+d*x)**2*cos(c+d*x)/d+5*A*a**3*sin(c+d*x)*cos(c+d*x)**5/(16*d)-A*a**3*sin(c+d*x)`

```
*x)*cos(c + d*x)/(2*d) + 16*A*a**3*cos(c + d*x)**5/(15*d) - 4*A*a**3*cos(c  
+ d*x)**3/(3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c)**2,  
True))
```

$$3.226 \quad \int \sin(c + dx)(a + a \sin(c + dx))^3(A - A \sin(c + dx)) dx$$

Optimal. Leaf size=96

$$\frac{a^3 A \cos^5(c + dx)}{5d} - \frac{2a^3 A \cos^3(c + dx)}{3d} + \frac{a^3 A \sin^3(c + dx) \cos(c + dx)}{2d} - \frac{a^3 A \sin(c + dx) \cos(c + dx)}{4d} + \frac{1}{4} a^3 A x$$

[Out] $\frac{1}{4} a^3 A x - \frac{2}{3} a^3 A \cos(d x + c)^3 / d + \frac{1}{5} a^3 A \cos(d x + c)^5 / d - \frac{1}{4} a^3 A \cos(d x + c) \sin(d x + c) / d + \frac{1}{2} a^3 A \cos(d x + c) \sin(d x + c)^3 / d$

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2966, 2638, 2635, 8, 2633}

$$\frac{a^3 A \cos^5(c + dx)}{5d} - \frac{2a^3 A \cos^3(c + dx)}{3d} + \frac{a^3 A \sin^3(c + dx) \cos(c + dx)}{2d} - \frac{a^3 A \sin(c + dx) \cos(c + dx)}{4d} + \frac{1}{4} a^3 A x$$

Antiderivative was successfully verified.

[In] Int[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] $(a^3 A x) / 4 - (2 a^3 A \cos[c + d x]^3) / (3 d) + (a^3 A \cos[c + d x]^5) / (5 d) - (a^3 A \cos[c + d x] \sin[c + d x]) / (4 d) + (a^3 A \cos[c + d x] \sin[c + d x]^3) / (2 d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \sin(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (a^3A\sin(c+dx)+2a^3A\sin^2(c+dx)-2a^3A\sin^4(c+dx))dx \\
&= (a^3A)\int \sin(c+dx)dx - (a^3A)\int \sin^5(c+dx)dx + \dots \\
&= -\frac{a^3A\cos(c+dx)}{d} - \frac{a^3A\cos(c+dx)\sin(c+dx)}{d} + \frac{a^3A\cos^3(c+dx)}{3d} - \frac{a^3A\cos^5(c+dx)}{5d} \\
&= a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos^7(c+dx)}{7d} \\
&= \frac{1}{4}a^3Ax - \frac{2a^3A\cos^3(c+dx)}{3d} + \frac{a^3A\cos^5(c+dx)}{5d} - \frac{a^3A\cos^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.48, size = 55, normalized size = 0.57

$$\frac{a^3A(-90\cos(c+dx) - 25\cos(3(c+dx)) + 3(-5\sin(4(c+dx)) + \cos(5(c+dx)) + 20dx))}{240d}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(-90*Cos[c + d*x] - 25*Cos[3*(c + d*x)] + 3*(20*d*x + Cos[5*(c + d*x)] - 5*Sin[4*(c + d*x)])))/(240*d)

fricas [A] time = 0.44, size = 77, normalized size = 0.80

$$\frac{12Aa^3\cos(dx+c)^5 - 40Aa^3\cos(dx+c)^3 + 15Aa^3dx - 15(2Aa^3\cos(dx+c)^3 - Aa^3\cos(dx+c))\sin(dx+c)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/60*(12*A*a^3*cos(d*x + c)^5 - 40*A*a^3*cos(d*x + c)^3 + 15*A*a^3*d*x - 15*(2*A*a^3*cos(d*x + c)^3 - A*a^3*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.16, size = 77, normalized size = 0.80

$$\frac{1}{4}Aa^3x + \frac{Aa^3\cos(5dx+5c)}{80d} - \frac{5Aa^3\cos(3dx+3c)}{48d} - \frac{3Aa^3\cos(dx+c)}{8d} - \frac{Aa^3\sin(4dx+4c)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/4*A*a^3*x + 1/80*A*a^3*cos(5*d*x + 5*c)/d - 5/48*A*a^3*cos(3*d*x + 3*c)/d - 3/8*A*a^3*cos(d*x + c)/d - 1/16*A*a^3*sin(4*d*x + 4*c)/d

maple [A] time = 0.40, size = 117, normalized size = 1.22

$$\frac{a^3A\left(\frac{8}{3} + \sin^4(dx+c) + \frac{4(\sin^2(dx+c))}{3}\right)\cos(dx+c)}{5} - 2a^3A\left(-\frac{(\sin^3(dx+c) + \frac{3\sin(dx+c)}{2})\cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + 2a^3A\left(-\frac{\cos(dx+c)\sin(dx+c)}{2} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $1/d*(1/5*a^3*A*(8/3+\sin(d*x+c)^4+4/3*\sin(d*x+c)^2)*\cos(d*x+c)-2*a^3*A*(-1/4*(\sin(d*x+c)^3+3/2*\sin(d*x+c))*\cos(d*x+c)+3/8*d*x+3/8*c)+2*a^3*A*(-1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)-a^3*A*\cos(d*x+c))$

maxima [A] time = 0.37, size = 112, normalized size = 1.17

$$\frac{16(3 \cos(dx + c)^5 - 10 \cos(dx + c)^3 + 15 \cos(dx + c))Aa^3 - 15(12dx + 12c + \sin(4dx + 4c) - 8 \sin(2dx + 2c))Aa^3 + 120(2dx + 2c - \sin(2dx + 2c))Aa^3 - 240Aa^3\cos(dx + c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")`

[Out] $1/240*(16*(3*\cos(d*x + c)^5 - 10*\cos(d*x + c)^3 + 15*\cos(d*x + c))*A*a^3 - 15*(12*d*x + 12*c + \sin(4*d*x + 4*c) - 8*\sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c - \sin(2*d*x + 2*c))*A*a^3 - 240*A*a^3*\cos(d*x + c))/d$

mupad [B] time = 15.03, size = 292, normalized size = 3.04

$$\frac{Aa^3x}{4} \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 \left(\frac{Aa^3(15c+15dx)}{12} - \frac{Aa^3(75c+75dx-120)}{60}\right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\frac{Aa^3(15c+15dx)}{12} - \frac{Aa^3(75c+75dx-160)}{60}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(c + d*x)*(A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)`

[Out] $(A*a^3*x)/4 - (\tan(c/2 + (d*x)/2)^8*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(75*c + 75*d*x - 120))/60) + \tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/12 - (A*a^3*(75*c + 75*d*x - 160))/60) + \tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 80))/60) + \tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(150*c + 150*d*x - 480))/60) + (A*a^3*\tan(c/2 + (d*x)/2))/2 - 3*A*a^3*\tan(c/2 + (d*x)/2)^3 + 3*A*a^3*\tan(c/2 + (d*x)/2)^7 - (A*a^3*\tan(c/2 + (d*x)/2)^9)/2 + (A*a^3*(15*c + 15*d*x))/60 - (A*a^3*(15*c + 15*d*x - 56))/60)/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^5)$

sympy [A] time = 3.65, size = 267, normalized size = 2.78

$$\left\{ \begin{array}{l} \frac{3Aa^3x \sin^4(c+dx)}{4} - \frac{3Aa^3x \sin^2(c+dx) \cos^2(c+dx)}{2} + Aa^3x \sin^2(c+dx) - \frac{3Aa^3x \cos^4(c+dx)}{4} + Aa^3x \cos^2(c+dx) + \frac{Aa^3 \sin^5(c)}{5} \\ x(-A \sin(c) + A)(a \sin(c) + a)^3 \sin(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)`

[Out] `Piecewise((-3*A*a**3*x*sin(c + d*x)**4/4 - 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/2 + A*a**3*x*sin(c + d*x)**2 - 3*A*a**3*x*cos(c + d*x)**4/4 + A*a**3*x*cos(c + d*x)**2 + A*a**3*sin(c + d*x)**4*cos(c + d*x)/d + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(4*d) + 4*A*a**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(4*d) - A*a**3*sin(c + d*x)*cos(c + d*x)/d + 8*A*a**3*cos(c + d*x)**5/(15*d) - A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3*sin(c), True))`

3.227 $\int (a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx$

Optimal. Leaf size=82

$$-\frac{5a^3 A \cos^3(c + dx)}{12d} - \frac{A \cos^3(c + dx) (a^3 \sin(c + dx) + a^3)}{4d} + \frac{5a^3 A \sin(c + dx) \cos(c + dx)}{8d} + \frac{5}{8} a^3 Ax$$

[Out] $5/8*a^3*A*x-5/12*a^3*A*\cos(d*x+c)^3/d+5/8*a^3*A*\cos(d*x+c)*\sin(d*x+c)/d-1/4*A*\cos(d*x+c)^3*(a^3+a^3*\sin(d*x+c))/d$

Rubi [A] time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {2736, 2678, 2669, 2635, 8}

$$-\frac{5a^3 A \cos^3(c + dx)}{12d} - \frac{A \cos^3(c + dx) (a^3 \sin(c + dx) + a^3)}{4d} + \frac{5a^3 A \sin(c + dx) \cos(c + dx)}{8d} + \frac{5}{8} a^3 Ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[c + d*x])^3*(A - A*\text{Sin}[c + d*x]), x]$

[Out] $(5*a^3*A*x)/8 - (5*a^3*A*\text{Cos}[c + d*x]^3)/(12*d) + (5*a^3*A*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) - (A*\text{Cos}[c + d*x]^3*(a^3 + a^3*\text{Sin}[c + d*x]))/(4*d)$

Rule 8

$\text{Int}[a_, x_Symbol] := \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2669

$\text{Int}[(\text{cos}[(e_*) + (f_*)*(x_)]*(g_*)^{(p_*)}*((a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)])), x_Symbol] := -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)})/(f*g*(p+1)), x] + \text{Dist}[a, \text{Int}[(g*\text{Cos}[e + f*x])^p, x], x] /; \text{FreeQ}\{a, b, e, f, g, p\}, x \ \&\& \ (\text{IntegerQ}[2*p] \ \|\ \text{NeQ}[a^2 - b^2, 0])$

Rule 2678

$\text{Int}[(\text{cos}[(e_*) + (f_*)*(x_)]*(g_*)^{(p_*)}*((a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]))^{(m_)}, x_Symbol] := -\text{Simp}[(b*(g*\text{Cos}[e + f*x])^{(p+1)}*(a + b*\text{Sin}[e + f*x])^{(m-1)})/(f*g*(m+p)), x] + \text{Dist}[(a*(2*m+p-1))/(m+p), \text{Int}[(g*\text{Cos}[e + f*x])^p*(a + b*\text{Sin}[e + f*x])^{(m-1)}, x], x] /; \text{FreeQ}\{a, b, e, f, g, m, p\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[m, 0] \ \&\& \ \text{NeQ}[m+p, 0] \ \&\& \ \text{IntegersQ}[2*m, 2*p]$

Rule 2736

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_*)}*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)])^{(n_)}, x_Symbol] := \text{Dist}[a^m*c^m, \text{Int}[\text{Cos}[e + f*x]^{(2*m)}*(c + d*\text{Sin}[e + f*x])^{(n-m)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \ \&\& \ \text{EqQ}[b*c + a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ !(\text{IntegerQ}[n] \ \&\& \ ((\text{LtQ}[m, 0] \ \&\& \ \text{GtQ}[n, 0]) \ \|\ \text{LtQ}[0, n, m] \ \|\ \text{LtQ}[m, n, 0]))$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(c + dx))^3 (A - A \sin(c + dx)) dx &= (aA) \int \cos^2(c + dx) (a + a \sin(c + dx))^2 dx \\
&= -\frac{A \cos^3(c + dx) (a^3 + a^3 \sin(c + dx))}{4d} + \frac{1}{4} (5a^2 A) \int \cos^2(c + dx) dx \\
&= -\frac{5a^3 A \cos^3(c + dx)}{12d} - \frac{A \cos^3(c + dx) (a^3 + a^3 \sin(c + dx))}{4d} + \frac{5a^2 A}{4} \int \cos^2(c + dx) dx \\
&= -\frac{5a^3 A \cos^3(c + dx)}{12d} + \frac{5a^3 A \cos(c + dx) \sin(c + dx)}{8d} - \frac{A \cos^3(c + dx)}{4d} \\
&= \frac{5}{8} a^3 A x - \frac{5a^3 A \cos^3(c + dx)}{12d} + \frac{5a^3 A \cos(c + dx) \sin(c + dx)}{8d} - \frac{A \cos^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 54, normalized size = 0.66

$$\frac{a^3 A (24 \sin(2(c + dx)) - 3 \sin(4(c + dx)) - 48 \cos(c + dx) - 16 \cos(3(c + dx)) + 60 dx)}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(60*d*x - 48*Cos[c + d*x] - 16*Cos[3*(c + d*x)] + 24*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)])/(96*d)

fricas [A] time = 0.45, size = 63, normalized size = 0.77

$$\frac{16 A a^3 \cos(dx + c)^3 - 15 A a^3 dx + 3 (2 A a^3 \cos(dx + c)^3 - 5 A a^3 \cos(dx + c)) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/24*(16*A*a^3*cos(d*x + c)^3 - 15*A*a^3*d*x + 3*(2*A*a^3*cos(d*x + c)^3 - 5*A*a^3*cos(d*x + c))*sin(d*x + c))/d

giac [A] time = 0.17, size = 77, normalized size = 0.94

$$\frac{5}{8} A a^3 x - \frac{A a^3 \cos(3 dx + 3 c)}{6 d} - \frac{A a^3 \cos(dx + c)}{2 d} - \frac{A a^3 \sin(4 dx + 4 c)}{32 d} + \frac{A a^3 \sin(2 dx + 2 c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 5/8*A*a^3*x - 1/6*A*a^3*cos(3*d*x + 3*c)/d - 1/2*A*a^3*cos(d*x + c)/d - 1/32*A*a^3*sin(4*d*x + 4*c)/d + 1/4*A*a^3*sin(2*d*x + 2*c)/d

maple [A] time = 0.41, size = 89, normalized size = 1.09

$$\frac{-a^3 A \left(-\frac{(\sin^3(dx+c) + \frac{3 \sin(dx+c)}{2}) \cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8} \right) + \frac{2a^3 A (2 + \sin^2(dx+c)) \cos(dx+c)}{3} - 2a^3 A \cos(dx+c) + a^3 A (dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $1/d*(-a^3*A*(-1/4*(\sin(d*x+c))^3+3/2*\sin(d*x+c))*\cos(d*x+c)+3/8*d*x+3/8*c)+2/3*a^3*A*(2+\sin(d*x+c)^2)*\cos(d*x+c)-2*a^3*A*\cos(d*x+c)+a^3*A*(d*x+c)$

maxima [A] time = 0.39, size = 86, normalized size = 1.05

$$\frac{64(\cos(dx+c)^3 - 3\cos(dx+c))Aa^3 + 3(12dx + 12c + \sin(4dx + 4c) - 8\sin(2dx + 2c))Aa^3 - 96(dx+c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")`

[Out] $-1/96*(64*(\cos(d*x+c)^3 - 3*\cos(d*x+c))*A*a^3 + 3*(12*d*x + 12*c + \sin(4*d*x + 4*c) - 8*\sin(2*d*x + 2*c))*A*a^3 - 96*(d*x + c)*A*a^3 + 192*A*a^3*\cos(d*x+c))/d$

mupad [B] time = 15.19, size = 250, normalized size = 3.05

$$\frac{5Aa^3x}{8} \frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\frac{Aa^3(15c+15dx)}{6} - \frac{Aa^3(60c+60dx-32)}{24}\right) + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \left(\frac{Aa^3(15c+15dx)}{6} - \frac{Aa^3(60c+60dx-96)}{24}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3,x)`

[Out] $(5*A*a^3*x)/8 - (\tan(c/2 + (d*x)/2)^2*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(60*c + 60*d*x - 32))/24) + \tan(c/2 + (d*x)/2)^6*((A*a^3*(15*c + 15*d*x))/6 - (A*a^3*(60*c + 60*d*x - 96))/24) + \tan(c/2 + (d*x)/2)^4*((A*a^3*(15*c + 15*d*x))/4 - (A*a^3*(90*c + 90*d*x - 96))/24) - (3*A*a^3*\tan(c/2 + (d*x)/2))/4 - (11*A*a^3*\tan(c/2 + (d*x)/2)^3)/4 + (11*A*a^3*\tan(c/2 + (d*x)/2)^5)/4 + (3*A*a^3*\tan(c/2 + (d*x)/2)^7)/4 + (A*a^3*(15*c + 15*d*x))/24 - (A*a^3*(15*c + 15*d*x - 32))/24)/(d*(\tan(c/2 + (d*x)/2)^2 + 1)^4)$

sympy [A] time = 1.37, size = 196, normalized size = 2.39

$$\left\{ \begin{array}{l} -\frac{3Aa^3x\sin^4(c+dx)}{8} - \frac{3Aa^3x\sin^2(c+dx)\cos^2(c+dx)}{4} - \frac{3Aa^3x\cos^4(c+dx)}{8} + Aa^3x + \frac{5Aa^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{2Aa^3\sin^2(c+dx)\cos(c+dx)}{d} \\ x(-A\sin(c) + A)(a\sin(c) + a)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)`

[Out] `Piecewise((-3*A*a**3*x*sin(c + d*x)**4/8 - 3*A*a**3*x*sin(c + d*x)**2*cos(c + d*x)**2/4 - 3*A*a**3*x*cos(c + d*x)**4/8 + A*a**3*x + 5*A*a**3*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 2*A*a**3*sin(c + d*x)**2*cos(c + d*x)/d + 3*A*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 4*A*a**3*cos(c + d*x)**3/(3*d) - 2*A*a**3*cos(c + d*x)/d, Ne(d, 0)), (x*(-A*sin(c) + A)*(a*sin(c) + a)**3, True))`

$$3.228 \quad \int \csc(c + dx)(a + a \sin(c + dx))^3(A - A \sin(c + dx)) dx$$

Optimal. Leaf size=76

$$-\frac{a^3 A \cos^3(c + dx)}{3d} + \frac{a^3 A \cos(c + dx)}{d} + \frac{a^3 A \sin(c + dx) \cos(c + dx)}{d} - \frac{a^3 A \tanh^{-1}(\cos(c + dx))}{d} + a^3 Ax$$

[Out] $a^3 A x - a^3 A \operatorname{arctanh}(\cos(dx+c))/d + a^3 A \cos(dx+c)/d - 1/3 a^3 A \cos(dx+c)^3/d + a^3 A \cos(dx+c) \sin(dx+c)/d$

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2966, 3770, 2635, 8, 2633}

$$-\frac{a^3 A \cos^3(c + dx)}{3d} + \frac{a^3 A \cos(c + dx)}{d} + \frac{a^3 A \sin(c + dx) \cos(c + dx)}{d} - \frac{a^3 A \tanh^{-1}(\cos(c + dx))}{d} + a^3 Ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]*(a + a*\text{Sin}[c + d*x])^3*(A - A*\text{Sin}[c + d*x]), x]$

[Out] $a^3 A x - (a^3 A \operatorname{ArcTanh}[\text{Cos}[c + d*x]])/d + (a^3 A \text{Cos}[c + d*x])/d - (a^3 A * \text{Cos}[c + d*x]^3)/(3*d) + (a^3 A \text{Cos}[c + d*x] * \text{Sin}[c + d*x])/d$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2966

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^n*(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{EqQ}[A*b + a*B, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \csc(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (2a^3A+a^3A\csc(c+dx)-2a^3A\sin^2(c+dx)-a^3A\sin^3(c+dx))dx \\
&= 2a^3Ax+(a^3A)\int \csc(c+dx)dx-(a^3A)\int \sin^3(c+dx)dx \\
&= 2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3A\cos(c+dx)}{d} \\
&= a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{d}+\frac{a^3A\cos(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 74, normalized size = 0.97

$$\frac{a^3A\left(9\cos(c+dx)-\cos(3(c+dx))+6\left(\sin(2(c+dx))+2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)-2c}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] (a^3*A*(9*Cos[c + d*x] - Cos[3*(c + d*x)] + 6*(-2*c + 2*d*x - 2*Log[Cos[(c + d*x)/2]] + 2*Log[Sin[(c + d*x)/2]] + Sin[2*(c + d*x)])))/(12*d)

fricas [A] time = 0.48, size = 92, normalized size = 1.21

$$\frac{2Aa^3\cos(dx+c)^3-6Aa^3dx-6Aa^3\cos(dx+c)\sin(dx+c)-6Aa^3\cos(dx+c)+3Aa^3\log\left(\frac{1}{2}\cos(dx+c)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(2*A*a^3*cos(d*x + c)^3 - 6*A*a^3*d*x - 6*A*a^3*cos(d*x + c)*sin(d*x + c) - 6*A*a^3*cos(d*x + c) + 3*A*a^3*log(1/2*cos(d*x + c) + 1/2) - 3*A*a^3*log(-1/2*cos(d*x + c) + 1/2))/d

giac [A] time = 0.16, size = 107, normalized size = 1.41

$$\frac{3(dx+c)Aa^3+3Aa^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)-\frac{2\left(3Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Aa^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*A*a^3 + 3*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))) - 2*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*A*a^3*tan(1/2*d*x + 1/2*c) - 2*A*a^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3/d

maple [A] time = 0.43, size = 99, normalized size = 1.30

$$\frac{A\cos(dx+c)\left(\sin^2(dx+c)\right)a^3}{3d}+\frac{2a^3A\cos(dx+c)}{3d}+\frac{a^3A\cos(dx+c)\sin(dx+c)}{d}+a^3Ax+\frac{a^3Ac}{d}+\frac{a^3A\ln(\csc(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $1/3/d*A*\cos(d*x+c)*\sin(d*x+c)^2*a^3+2/3*a^3*A*\cos(d*x+c)/d+a^3*A*\cos(d*x+c)*\sin(d*x+c)/d+a^3*A*x+1/d*a^3*A*c+1/d*a^3*A*\ln(\csc(d*x+c)-\cot(d*x+c))$

maxima [A] time = 0.39, size = 85, normalized size = 1.12

$$\frac{2(\cos(dx+c)^3 - 3\cos(dx+c))Aa^3 + 3(2dx+2c - \sin(2dx+2c))Aa^3 - 12(dx+c)Aa^3 + 6Aa^3 \log(\csc(dx+c) - \cot(dx+c))}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/6*(2*(\cos(dx+c)^3 - 3*\cos(dx+c))*A*a^3 + 3*(2*d*x + 2*c - \sin(2*d*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 + 6*A*a^3*\log(\cot(dx+c) + \csc(dx+c)))/d$

mupad [B] time = 13.23, size = 212, normalized size = 2.79

$$\frac{-2Aa^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 4Aa^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 2Aa^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \frac{4Aa^3}{3}}{d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1 \right)} + \frac{2Aa^3 \operatorname{atan}\left(\frac{4A^2a^6}{4A^2a^6 - 4A^2a^6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x),x)

[Out] $((4*A*a^3)/3 + 2*A*a^3*\tan(c/2 + (d*x)/2) + 4*A*a^3*\tan(c/2 + (d*x)/2)^2 - 2*A*a^3*\tan(c/2 + (d*x)/2)^5)/(d*(3*\tan(c/2 + (d*x)/2)^2 + 3*\tan(c/2 + (d*x)/2)^4 + \tan(c/2 + (d*x)/2)^6 + 1)) + (2*A*a^3*\operatorname{atan}((4*A^2*a^6)/(4*A^2*a^6 - 4*A^2*a^6*\tan(c/2 + (d*x)/2))) + (4*A^2*a^6*\tan(c/2 + (d*x)/2)))/(4*A^2*a^6 - 4*A^2*a^6*\tan(c/2 + (d*x)/2)))/d + (A*a^3*\log(\tan(c/2 + (d*x)/2)))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3 \left(\int (-2 \sin(c + dx) \csc(c + dx)) dx + \int 2 \sin^3(c + dx) \csc(c + dx) dx + \int \sin^4(c + dx) \csc(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $-A*a^3*(\operatorname{Integral}(-2*\sin(c + d*x)*\csc(c + d*x), x) + \operatorname{Integral}(2*\sin(c + d*x)^3*\csc(c + d*x), x) + \operatorname{Integral}(\sin(c + d*x)^4*\csc(c + d*x), x) + \operatorname{Integral}(-\csc(c + d*x), x))$

3.229 $\int \csc^2(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=79

$$\frac{2a^3 A \cos(c+dx)}{d} - \frac{a^3 A \cot(c+dx)}{d} + \frac{a^3 A \sin(c+dx) \cos(c+dx)}{2d} - \frac{2a^3 A \tanh^{-1}(\cos(c+dx))}{d} - \frac{1}{2} a^3 A x$$

[Out] $-1/2*a^3*A*x-2*a^3*A*\operatorname{arctanh}(\cos(d*x+c))/d+2*a^3*A*\cos(d*x+c)/d-a^3*A*\cot(d*x+c)/d+1/2*a^3*A*\cos(d*x+c)*\sin(d*x+c)/d$

Rubi [A] time = 0.18, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2950, 2709, 3770, 3767, 8, 2638, 2635}

$$\frac{2a^3 A \cos(c+dx)}{d} - \frac{a^3 A \cot(c+dx)}{d} + \frac{a^3 A \sin(c+dx) \cos(c+dx)}{2d} - \frac{2a^3 A \tanh^{-1}(\cos(c+dx))}{d} - \frac{1}{2} a^3 A x$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c+d*x]^2*(a+a*\text{Sin}[c+d*x])^3*(A-A*\text{Sin}[c+d*x]),x]$

[Out] $-(a^3*A*x)/2 - (2*a^3*A*\text{ArcTanh}[\text{Cos}[c+d*x]])/d + (2*a^3*A*\text{Cos}[c+d*x])/d - (a^3*A*\text{Cot}[c+d*x])/d + (a^3*A*\text{Cos}[c+d*x]*\text{Sin}[c+d*x])/(2*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2635

$\text{Int}[(b_.*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Sin}[c+d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c+d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c+d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2709

$\text{Int}[(a_.) + (b_.*\sin[(e_.) + (f_.)*(x_)])^{(m_)*\tan[(e_.) + (f_.)*(x_)]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[a^p, \text{Int}[\text{ExpandIntegrand}[(\text{Sin}[e+f*x])^p*(a+b*\text{Sin}[e+f*x])^{(m-p/2)})/(a-b*\text{Sin}[e+f*x])^{(p/2)}, x], x], x] /; \text{FreeQ}\{a, b, e, f\}, x \} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegersQ}[m, p/2] \&\& (\text{LtQ}[p, 0] \mid \mid \text{GtQ}[m - p/2, 0])$

Rule 2950

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]^{(p_)*((a_.) + (b_.*\sin[(e_.) + (f_.)*(x_)])^{(m_.)*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[a^n*c^n, \text{Int}[\text{Tan}[e+f*x]^p*(a+b*\text{Sin}[e+f*x])^{(m-n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \} \&\& \text{EqQ}[b*c + a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{EqQ}[p + 2*n, 0] \&\& \text{IntegerQ}[n]$

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \csc^2(c + dx)(a + a \sin(c + dx))^3(A - A \sin(c + dx)) dx &= (aA) \int \cot^2(c + dx)(a + a \sin(c + dx))^2 dx \\ &= \frac{A \int (2a^4 \csc(c + dx) + a^4 \csc^2(c + dx) - 2a^4 \sin(c + dx)) dx}{a} \\ &= (a^3 A) \int \csc^2(c + dx) dx - (a^3 A) \int \sin^2(c + dx) dx \\ &= -\frac{2a^3 A \tanh^{-1}(\cos(c + dx))}{d} + \frac{2a^3 A \cos(c + dx)}{d} \\ &= -\frac{1}{2}a^3 Ax - \frac{2a^3 A \tanh^{-1}(\cos(c + dx))}{d} + \frac{2a^3 A \cos(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.97

$$\frac{a^3 A \left(-8 \sin(c) \sin(dx) + \sin(2(c + dx)) + 8 \cos(c) \cos(dx) - 4 \cot(c + dx) + 8 \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 8 \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^2*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]), x]
 [Out] (a^3*A*(-2*c - 2*d*x + 8*Cos[c]*Cos[d*x] - 4*Cot[c + d*x] - 8*Log[Cos[(c + d*x)/2]] + 8*Log[Sin[(c + d*x)/2]] - 8*Sin[c]*Sin[d*x] + Sin[2*(c + d*x)])) / (4*d)

fricas [A] time = 0.49, size = 111, normalized size = 1.41

$$\frac{Aa^3 \cos(dx + c)^3 + 2Aa^3 \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c) - 2Aa^3 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) \sin(dx + c)}{2d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)), x, algorithm="fricas")

[Out] -1/2*(A*a^3*cos(d*x + c)^3 + 2*A*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 2*A*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + A*a^3*cos(d*x + c) + (A*a^3*d*x - 4*A*a^3*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))

giac [B] time = 0.19, size = 153, normalized size = 1.94

$$\frac{(dx + c)Aa^3 - 4Aa^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) - Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + \frac{4Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + Aa^3}{\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)} + \frac{2\left(Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{2d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out]
$$-1/2*((d*x + c)*A*a^3 - 4*A*a^3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c)))) - A*a^3*\tan(1/2*d*x + 1/2*c) + (4*A*a^3*\tan(1/2*d*x + 1/2*c) + A*a^3)/\tan(1/2*d*x + 1/2*c) + 2*(A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 4*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - A*a^3*\tan(1/2*d*x + 1/2*c) - 4*A*a^3)/(\tan(1/2*d*x + 1/2*c)^2 + 1)^2/d$$

maple [A] time = 0.36, size = 95, normalized size = 1.20

$$\frac{a^3 A \cos(dx + c) \sin(dx + c)}{2d} - \frac{a^3 A x}{2} - \frac{a^3 A c}{2d} + \frac{2a^3 A \cos(dx + c)}{d} + \frac{2a^3 A \ln(\csc(dx + c) - \cot(dx + c))}{d} - \frac{a^3 A \cot(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out]
$$1/2*a^3*A*\cos(d*x+c)*\sin(d*x+c)/d - 1/2*a^3*A*x - 1/2/d*a^3*A*c + 2*a^3*A*\cos(d*x+c)/d + 2/d*a^3*A*\ln(\csc(d*x+c) - \cot(d*x+c)) - a^3*A*\cot(d*x+c)/d$$

maxima [A] time = 0.55, size = 83, normalized size = 1.05

$$\frac{(2 dx + 2 c - \sin(2 dx + 2 c)) A a^3 + 4 A a^3 (\log(\cos(dx + c) + 1) - \log(\cos(dx + c) - 1)) - 8 A a^3 \cos(dx + c) + 4 A a^3 \tan(dx + c)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out]
$$-1/4*((2*d*x + 2*c - \sin(2*d*x + 2*c))*A*a^3 + 4*A*a^3*(\log(\cos(d*x + c) + 1) - \log(\cos(d*x + c) - 1)) - 8*A*a^3*\cos(d*x + c) + 4*A*a^3/\tan(d*x + c))/d$$

mupad [B] time = 13.19, size = 226, normalized size = 2.86

$$\frac{A a^3 \operatorname{atan}\left(\frac{A^2 a^6}{4 A^2 a^6 + A^2 a^6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)} - \frac{4 A^2 a^6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{4 A^2 a^6 + A^2 a^6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} - \frac{3 A a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 - 8 A a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 8 A a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{d \left(2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^2,x)

[Out]
$$(A*a^3*\operatorname{atan}\left(\frac{A^2*a^6}{4*A^2*a^6 + A^2*a^6*\tan(c/2 + (d*x)/2)}\right) - (4*A^2*a^6*\tan(c/2 + (d*x)/2))/(4*A^2*a^6 + A^2*a^6*\tan(c/2 + (d*x)/2)))/d - (A*a^3 - 8*A*a^3*\tan(c/2 + (d*x)/2) - 8*A*a^3*\tan(c/2 + (d*x)/2)^3 + 3*A*a^3*\tan(c/2 + (d*x)/2)^4)/(d*(2*\tan(c/2 + (d*x)/2) + 4*\tan(c/2 + (d*x)/2)^3 + 2*\tan(c/2 + (d*x)/2)^5)) + (2*A*a^3*\log(\tan(c/2 + (d*x)/2)))/d + (A*a^3*\tan(c/2 + (d*x)/2))/(2*d)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-Aa^3 \left(\int (-2 \sin(c + dx) \csc^2(c + dx)) dx + \int 2 \sin^3(c + dx) \csc^2(c + dx) dx + \int \sin^4(c + dx) \csc^2(c + dx) dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

```
[Out] -A*a**3*(Integral(-2*sin(c + d*x)*csc(c + d*x)**2, x) + Integral(2*sin(c +  
d*x)**3*csc(c + d*x)**2, x) + Integral(sin(c + d*x)**4*csc(c + d*x)**2, x)  
+ Integral(-csc(c + d*x)**2, x))
```

$$3.230 \quad \int \csc^3(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$$

Optimal. Leaf size=78

$$\frac{a^3 A \cos(c+dx)}{d} - \frac{2a^3 A \cot(c+dx)}{d} - \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3 A \cot(c+dx) \csc(c+dx)}{2d} - 2a^3 Ax$$

[Out] $-2*a^3*A*x-1/2*a^3*A*\operatorname{arctanh}(\cos(d*x+c))/d+a^3*A*\cos(d*x+c)/d-2*a^3*A*\cot(d*x+c)/d-1/2*a^3*A*\cot(d*x+c)*\csc(d*x+c)/d$

Rubi [A] time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2966, 3767, 8, 3768, 3770, 2638}

$$\frac{a^3 A \cos(c+dx)}{d} - \frac{2a^3 A \cot(c+dx)}{d} - \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{2d} - \frac{a^3 A \cot(c+dx) \csc(c+dx)}{2d} - 2a^3 Ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c+d*x]^3*(a+a*\text{Sin}[c+d*x])^3*(A-A*\text{Sin}[c+d*x]),x]$

[Out] $-2*a^3*A*x - (a^3*A*\text{ArcTanh}[\text{Cos}[c+d*x]])/(2*d) + (a^3*A*\text{Cos}[c+d*x])/d - (2*a^3*A*\text{Cot}[c+d*x])/d - (a^3*A*\text{Cot}[c+d*x]*\text{Csc}[c+d*x])/(2*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c+d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2966

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e+f*x]^{n*}(a+b*\sin[e+f*x])^m*(A+B*\sin[e+f*x]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \&\& \text{EqQ}[A*b+a*B, 0] \&\& \text{EqQ}[a^2-b^2, 0] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n]$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1+x^2)^{(n/2-1)}, x], x], x, \text{Cot}[c+d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c+d*x])*(b*\text{Csc}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c+d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c+d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \csc^3(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (-2a^3A+2a^3A\csc^2(c+dx)+a^3A\csc^3(c+dx) \\
&= -2a^3Ax+(a^3A)\int \csc^3(c+dx)dx-(a^3A)\int \sin \\
&= -2a^3Ax+\frac{a^3A\cos(c+dx)}{d}-\frac{a^3A\cot(c+dx)\csc}{2d} \\
&= -2a^3Ax-\frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d}+\frac{a^3A\cos(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 142, normalized size = 1.82

$$-\frac{a^3A\sin(c)\sin(dx)}{d}+\frac{a^3A\cos(c)\cos(dx)}{d}-\frac{2a^3A\cot(c+dx)}{d}-\frac{a^3A\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3A\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]
[Out] -2*a^3*A*x + (a^3*A*Cos[c]*Cos[d*x])/d - (2*a^3*A*Cot[c + d*x])/d - (a^3*A*
Csc[(c + d*x)/2]^2)/(8*d) - (a^3*A*Log[Cos[(c + d*x)/2]])/(2*d) + (a^3*A*Lo
g[Sin[(c + d*x)/2]])/(2*d) + (a^3*A*Sec[(c + d*x)/2]^2)/(8*d) - (a^3*A*Sin[
c]*Sin[d*x])/d
```

fricas [B] time = 0.48, size = 152, normalized size = 1.95

$$\frac{8Aa^3dx\cos(dx+c)^2-4Aa^3\cos(dx+c)^3-8Aa^3dx-8Aa^3\cos(dx+c)\sin(dx+c)+2Aa^3\cos(dx+c)}{4(d\cos(dx+c)+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fr
icas")
[Out] -1/4*(8*A*a^3*d*x*cos(d*x + c)^2 - 4*A*a^3*cos(d*x + c)^3 - 8*A*a^3*d*x - 8
*A*a^3*cos(d*x + c)*sin(d*x + c) + 2*A*a^3*cos(d*x + c) + (A*a^3*cos(d*x +
c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) - (A*a^3*cos(d*x + c)^2 - A*a^3)*
log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [A] time = 0.20, size = 137, normalized size = 1.76

$$\frac{Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-16(dx+c)Aa^3+4Aa^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)+8Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{16Aa^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="gi
ac")
[Out] 1/8*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*(d*x + c)*A*a^3 + 4*A*a^3*log(abs(ta
n(1/2*d*x + 1/2*c))) + 8*A*a^3*tan(1/2*d*x + 1/2*c) + 16*A*a^3/(tan(1/2*d*x
+ 1/2*c)^2 + 1) - (6*A*a^3*tan(1/2*d*x + 1/2*c)^2 + 8*A*a^3*tan(1/2*d*x +
1/2*c) + A*a^3)/tan(1/2*d*x + 1/2*c)^2)/d
```

maple [A] time = 0.56, size = 94, normalized size = 1.21

$$\frac{a^3 A \cos(dx+c)}{d} - 2a^3 Ax - \frac{2a^3 Ac}{d} - \frac{2a^3 A \cot(dx+c)}{d} - \frac{a^3 A \cot(dx+c) \csc(dx+c)}{2d} + \frac{a^3 A \ln(\csc(dx+c) - \cot(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] a^3*A*cos(d*x+c)/d-2*a^3*A*x-2/d*a^3*A*c-2*a^3*A*cot(d*x+c)/d-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))

maxima [A] time = 0.40, size = 90, normalized size = 1.15

$$\frac{8(dx+c)Aa^3 - Aa^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1) \right) - 4Aa^3 \cos(dx+c) + \frac{8Aa^3}{\tan(dx+c)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] -1/4*(8*(d*x+c)*A*a^3 - A*a^3*(2*cos(d*x+c)/(cos(d*x+c)^2-1) - log(cos(d*x+c)+1) + log(cos(d*x+c)-1)) - 4*A*a^3*cos(d*x+c) + 8*A*a^3/tan(d*x+c))/d

mupad [B] time = 13.50, size = 220, normalized size = 2.82

$$Aa^3 \left(\frac{\cos(c+dx)}{2} - 4 \operatorname{atan} \left(\frac{\sqrt{17} \left(4 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) - \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \right)}{17 \cos\left(\frac{c}{2} - \operatorname{atan}(4) + \frac{dx}{2}\right)} \right) - \frac{\ln\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{2} + \cos(2c+2dx) + \frac{\cos(3c+3dx)}{2} + 2 \sin(2c+2dx) \right) / (2d(\cos(c+dx)^2-1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^3,x)

[Out] (A*a^3*(cos(c+d*x)/2 - 4*atan((17^(1/2)*(4*cos(c/2+(d*x)/2) - sin(c/2+(d*x)/2)))/(17*cos(c/2-atan(4)+(d*x)/2))) - log(sin(c/2+(d*x)/2)/cos(c/2+(d*x)/2))/2 + cos(2*c+2*d*x) + cos(3*c+3*d*x)/2 + 2*sin(2*c+2*d*x) + 4*atan((17^(1/2)*(4*cos(c/2+(d*x)/2) - sin(c/2+(d*x)/2)))/(17*cos(c/2-atan(4)+(d*x)/2)))*cos(2*c+2*d*x) + (log(sin(c/2+(d*x)/2)/cos(c/2+(d*x)/2))*cos(2*c+2*d*x))/2 - 1)/(2*d*(cos(c+d*x)^2-1))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**3*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

[Out] Timed out

$$3.231 \quad \int \csc^4(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$$

Optimal. Leaf size=78

$$\frac{a^3 A \cot^3(c+dx)}{3d} - \frac{a^3 A \cot(c+dx)}{d} + \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3 A \cot(c+dx) \csc(c+dx)}{d} - a^3 Ax$$

[Out] $-a^3 A x + a^3 A \operatorname{arctanh}(\cos(d x + c)) / d - a^3 A \cot(d x + c) / d - 1 / 3 a^3 A \cot(d x + c)^3 / d - a^3 A \cot(d x + c) \csc(d x + c) / d$

Rubi [A] time = 0.13, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 3770, 3768, 3767}

$$\frac{a^3 A \cot^3(c+dx)}{3d} - \frac{a^3 A \cot(c+dx)}{d} + \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3 A \cot(c+dx) \csc(c+dx)}{d} - a^3 Ax$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^4*(a + a*\text{Sin}[c + d*x])^3*(A - A*\text{Sin}[c + d*x]), x]$

[Out] $-(a^3 A x) + (a^3 A \operatorname{ArcTanh}[\text{Cos}[c + d*x]]) / d - (a^3 A \cot[c + d*x]) / d - (a^3 A \cot[c + d*x]^3) / (3*d) - (a^3 A \cot[c + d*x] * \text{Csc}[c + d*x]) / d$

Rule 2966

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^n*(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x], x] /;$ FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \cot[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\cos[c + d*x]*(b*\csc[c + d*x])^{(n - 1)}) / (d*(n - 1)), x] + \text{Dist}[(b^2*(n - 2)) / (n - 1), \text{Int}[(b*\csc[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\operatorname{ArcTanh}[\cos[c + d*x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (-a^3A-2a^3A\csc(c+dx)+2a^3A\csc^3(c+dx)+ \\
&= -a^3Ax + (a^3A) \int \csc^4(c+dx)dx - (2a^3A) \int \csc(c \\
&= -a^3Ax + \frac{2a^3A \tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3A \cot(c+dx)}{d} \\
&= -a^3Ax + \frac{a^3A \tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3A \cot(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.49, size = 141, normalized size = 1.81

$$\frac{a^3A \left(-8 \tan\left(\frac{1}{2}(c+dx)\right) + 8 \cot\left(\frac{1}{2}(c+dx)\right) + 6 \csc^2\left(\frac{1}{2}(c+dx)\right) - 6 \sec^2\left(\frac{1}{2}(c+dx)\right) + 24 \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) \right)}{24a}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^4*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] -1/24*(a^3*A*(24*c + 24*d*x + 8*Cot[(c + d*x)/2] + 6*Csc[(c + d*x)/2]^2 - 24*Log[Cos[(c + d*x)/2]] + 24*Log[Sin[(c + d*x)/2]] - 6*Sec[(c + d*x)/2]^2 - 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + (Csc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 8*Tan[(c + d*x)/2])/d

fricas [B] time = 0.46, size = 175, normalized size = 2.24

$$\frac{4Aa^3 \cos(dx+c)^3 - 6Aa^3 \cos(dx+c) - 3(Aa^3 \cos(dx+c)^2 - Aa^3) \log\left(\frac{1}{2} \cos(dx+c) + \frac{1}{2}\right) \sin(dx+c) + 3}{6(d \cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/6*(4*A*a^3*cos(d*x + c)^3 - 6*A*a^3*cos(d*x + c) - 3*(A*a^3*cos(d*x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 6*(A*a^3*d*x*cos(d*x + c)^2 - A*a^3*d*x - A*a^3*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))

giac [A] time = 0.18, size = 150, normalized size = 1.92

$$\frac{Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 6Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 24(dx+c)Aa^3 - 24Aa^3 \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right) + 9Aa^3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/24*(A*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 24*(d*x + c)*A*a^3 - 24*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 9*A*a^3*tan(1/2*d*x + 1/2*c) + (44*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 6*A*a^3*tan(1/2*d*x + 1/2*c) - A*a^3)/tan(1/2*d*x + 1/2*c)^3)/d

maple [A] time = 0.53, size = 103, normalized size = 1.32

$$-a^3 Ax - \frac{a^3 Ac}{d} - \frac{a^3 A \ln(\csc(dx+c) - \cot(dx+c))}{d} - \frac{a^3 A \cot(dx+c) \csc(dx+c)}{d} - \frac{2a^3 A \cot(dx+c)}{3d} - \frac{a^3 A \cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $-a^3 A x - 1/d a^3 A c - 1/d a^3 A \ln(\csc(d*x+c) - \cot(d*x+c)) - a^3 A \cot(d*x+c) * \csc(d*x+c)/d - 2/3 a^3 A \cot(d*x+c)/d - 1/3/d a^3 A \cot(d*x+c) * \csc(d*x+c)^2$

maxima [A] time = 0.45, size = 117, normalized size = 1.50

$$\frac{6(dx+c)Aa^3 - 3Aa^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2 - 1} - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1) \right) - 6Aa^3 \left(\log(\cos(dx+c) + 1) - \log(\cos(dx+c) - 1) \right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^4*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] $-1/6*(6*(d*x+c)*A*a^3 - 3*A*a^3*(2*\cos(d*x+c)/(\cos(d*x+c)^2 - 1) - \log(\cos(d*x+c) + 1) + \log(\cos(d*x+c) - 1)) - 6*A*a^3*(\log(\cos(d*x+c) + 1) - \log(\cos(d*x+c) - 1)) + 2*(3*\tan(d*x+c)^2 + 1)*A*a^3/\tan(d*x+c)^3)/d$

mupad [B] time = 13.32, size = 245, normalized size = 3.14

$$\frac{\frac{Aa^3 \sin(2c+2dx)}{2} - \frac{Aa^3 \cos(3c+3dx)}{6} + \frac{Aa^3 \cos(c+dx)}{2} - \frac{Aa^3 \sin(3c+3dx) \operatorname{atan}\left(\frac{\sqrt{2}\left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{2 \cos\left(\frac{c}{2} + \frac{\pi}{4} + \frac{dx}{2}\right)}\right)}{2} + \frac{3Aa^3 \sin(c+dx)}{4}}{\frac{3d \sin(c+dx)}{4} - \frac{d \sin(3c+3dx)}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^4,x)

[Out] $-((A*a^3*\sin(2*c + 2*d*x))/2 - (A*a^3*\cos(3*c + 3*d*x))/6 + (A*a^3*\cos(c + d*x))/2 - (A*a^3*\sin(3*c + 3*d*x)*\operatorname{atan}((2^{1/2}*(\cos(c/2 + (d*x)/2) + \sin(c/2 + (d*x)/2)))/(2*\cos(c/2 + \pi/4 + (d*x)/2))))/2 + (3*A*a^3*\sin(c + d*x)*\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/4 + (3*A*a^3*\sin(c + d*x)*\operatorname{atan}((2^{1/2}*(\cos(c/2 + (d*x)/2) + \sin(c/2 + (d*x)/2)))/(2*\cos(c/2 + \pi/4 + (d*x)/2))))/2 - (A*a^3*\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2))*\sin(3*c + 3*d*x))/4)/((3*d*\sin(c + d*x))/4 - (d*\sin(3*c + 3*d*x))/4)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**4*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

[Out] Timed out

3.232 $\int \csc^5(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=86

$$-\frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{5a^3 A \tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{4d} - \frac{3a^3 A \cot(c+dx) \csc(c+dx)}{8d}$$

[Out] $5/8*a^3*A*\operatorname{arctanh}(\cos(d*x+c))/d-2/3*a^3*A*\cot(d*x+c)^3/d-3/8*a^3*A*\cot(d*x+c)*\csc(d*x+c)/d-1/4*a^3*A*\cot(d*x+c)*\csc(d*x+c)^3/d$

Rubi [A] time = 0.15, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2966, 3770, 3767, 8, 3768}

$$-\frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{5a^3 A \tanh^{-1}(\cos(c+dx))}{8d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{4d} - \frac{3a^3 A \cot(c+dx) \csc(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^5*(a + a*\text{Sin}[c + d*x])^3*(A - A*\text{Sin}[c + d*x]), x]$

[Out] $(5*a^3*A*\text{ArcTanh}[\text{Cos}[c + d*x]])/(8*d) - (2*a^3*A*\text{Cot}[c + d*x]^3)/(3*d) - (3*a^3*A*\text{Cot}[c + d*x]*\text{Csc}[c + d*x])/(8*d) - (a^3*A*\text{Cot}[c + d*x]*\text{Csc}[c + d*x]^3)/(4*d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2966

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{n*}(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \ \&\& \ \text{EqQ}[A*b + a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \csc^5(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (-a^3A\csc(c+dx)-2a^3A\csc^2(c+dx)+2a^3A\csc^3(c+dx))dx \\
&= -\left((a^3A)\int \csc(c+dx)dx\right) + (a^3A)\int \csc^5(c+dx)dx \\
&= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} - \frac{a^3A\cot(c+dx)\csc^3(c+dx)}{4d} \\
&= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot^5(c+dx)}{5d} \\
&= \frac{5a^3A\tanh^{-1}(\cos(c+dx))}{8d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{3a^3A\cot^5(c+dx)}{5d}
\end{aligned}$$

Mathematica [B] time = 0.07, size = 210, normalized size = 2.44

$$a^3A\left(-\frac{\tan\left(\frac{1}{2}(c+dx)\right)}{3d} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{3d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^5*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]), x]

[Out] a^3*A*(Cot[(c + d*x)/2]/(3*d) - (3*Csc[(c + d*x)/2]^2)/(32*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(12*d) - Csc[(c + d*x)/2]^4/(64*d) + (5*Log[Cos[(c + d*x)/2]])/(8*d) - (5*Log[Sin[(c + d*x)/2]])/(8*d) + (3*Sec[(c + d*x)/2]^2)/(32*d) + Sec[(c + d*x)/2]^4/(64*d) - Tan[(c + d*x)/2]/(3*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(12*d))

fricas [B] time = 0.45, size = 166, normalized size = 1.93

$$\frac{32Aa^3\cos(dx+c)^3\sin(dx+c) - 18Aa^3\cos(dx+c)^3 + 30Aa^3\cos(dx+c) - 15(Aa^3\cos(dx+c)^4 - 2Aa^3\cos(dx+c)^2 + Aa^3)}{48(d\cos(dx+c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)), x, algorithm="fricas")

[Out] -1/48*(32*A*a^3*cos(d*x + c)^3*sin(d*x + c) - 18*A*a^3*cos(d*x + c)^3 + 30*A*a^3*cos(d*x + c) - 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(A*a^3*cos(d*x + c)^4 - 2*A*a^3*cos(d*x + c)^2 + A*a^3)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)

giac [B] time = 0.20, size = 174, normalized size = 2.02

$$\frac{3Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 16Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 24Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 120Aa^3\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{48(d\cos(dx+c) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)), x, algorithm="giac")

[Out] 1/192*(3*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 16*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 24*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c)))) -

$$48Aa^3 \tan(1/2dx + 1/2c) + (250Aa^3 \tan(1/2dx + 1/2c)^4 + 48Aa^3 \tan(1/2dx + 1/2c)^3 - 24Aa^3 \tan(1/2dx + 1/2c)^2 - 16Aa^3 \tan(1/2dx + 1/2c) - 3Aa^3) / \tan(1/2dx + 1/2c)^4 / d$$

maple [A] time = 0.62, size = 109, normalized size = 1.27

$$\frac{5a^3 A \ln(\csc(dx+c) - \cot(dx+c))}{8d} + \frac{2a^3 A \cot(dx+c)}{3d} - \frac{2a^3 A \cot(dx+c) (\csc^2(dx+c))}{3d} - \frac{a^3 A \cot(dx+c) (\csc^2(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $-\frac{5}{8} \frac{a^3 A \ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{2}{3} \frac{a^3 A \cot(dx+c)}{d} - \frac{2}{3} \frac{a^3 A \cot(dx+c) (\csc^2(dx+c))}{d} - \frac{1}{4} \frac{a^3 A \cot(dx+c) \csc(dx+c)^2}{d} - \frac{1}{8} \frac{a^3 A \cot(dx+c) \csc(dx+c)^3}{d} - \frac{3}{8} \frac{a^3 A \cot(dx+c) \csc(dx+c)^4}{d}$

maxima [A] time = 0.43, size = 145, normalized size = 1.69

$$\frac{3 A a^3 \left(\frac{2(3 \cos(dx+c)^3 - 5 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right) + 24 A a^3 (\log(\cos(dx+c) + 1) - \log(\cos(dx+c) - 1))}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^5*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{48} \frac{(3Aa^3(2(3\cos(dx+c)^3 - 5\cos(dx+c)) / (\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1) - 3\log(\cos(dx+c) + 1) + 3\log(\cos(dx+c) - 1)) + 24Aa^3(\log(\cos(dx+c) + 1) - \log(\cos(dx+c) - 1)) + 96Aa^3/\tan(dx+c) - 32(3\tan(dx+c)^2 + 1)Aa^3/\tan(dx+c)^3)/d}$

mupad [B] time = 13.12, size = 244, normalized size = 2.84

$$A a^3 \left(3 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 3 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^8 - 16 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + 16 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^7 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 24 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 24 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^5 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3 - 12 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \right) / (192 d \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^5,x)

[Out] $-\frac{Aa^3(3\cos(c/2 + (dx)/2)^8 - 3\sin(c/2 + (dx)/2)^8 - 16\cos(c/2 + (dx)/2)\sin(c/2 + (dx)/2)^7 + 16\cos(c/2 + (dx)/2)^7\sin(c/2 + (dx)/2) - 24\cos(c/2 + (dx)/2)^6\sin(c/2 + (dx)/2)^2 + 24\cos(c/2 + (dx)/2)^5\sin(c/2 + (dx)/2)^3 - 12\cos(c/2 + (dx)/2)^4\sin(c/2 + (dx)/2)^4)}{(192d\cos(c/2 + (dx)/2)^4\sin(c/2 + (dx)/2)^4)}$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**5*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

[Out] Timed out

3.233 $\int \csc^6(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=105

$$\frac{a^3 A \cot^5(c+dx)}{5d} - \frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{4d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{2d} + \frac{a^3 A \cot(c+dx)}{d}$$

[Out] $1/4*a^3*A*\operatorname{arctanh}(\cos(d*x+c))/d-2/3*a^3*A*\cot(d*x+c)^3/d-1/5*a^3*A*\cot(d*x+c)^5/d+1/4*a^3*A*\cot(d*x+c)*\csc(d*x+c)/d-1/2*a^3*A*\cot(d*x+c)*\csc(d*x+c)^3/d$

Rubi [A] time = 0.23, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2950, 2709, 3767, 8, 3768, 3770}

$$\frac{a^3 A \cot^5(c+dx)}{5d} - \frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{a^3 A \tanh^{-1}(\cos(c+dx))}{4d} - \frac{a^3 A \cot(c+dx) \csc^3(c+dx)}{2d} + \frac{a^3 A \cot(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[c+d*x]^6*(a+a*\operatorname{Sin}[c+d*x])^3*(A-A*\operatorname{Sin}[c+d*x]),x]$

[Out] $(a^3*A*\operatorname{ArcTanh}[\operatorname{Cos}[c+d*x]])/(4*d) - (2*a^3*A*\operatorname{Cot}[c+d*x]^3)/(3*d) - (a^3*A*\operatorname{Cot}[c+d*x]^5)/(5*d) + (a^3*A*\operatorname{Cot}[c+d*x]*\operatorname{Csc}[c+d*x])/(4*d) - (a^3*A*\operatorname{Cot}[c+d*x]*\operatorname{Csc}[c+d*x]^3)/(2*d)$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 2709

$\operatorname{Int}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])^{(m_)}*\tan[(e_.) + (f_)*(x_)]^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[a^p, \operatorname{Int}[\operatorname{ExpandIntegrand}[(\operatorname{Sin}[e + f*x])^p*(a + b*\operatorname{Sin}[e + f*x])^{(m-p/2)}]/(a - b*\operatorname{Sin}[e + f*x])^{(p/2)}, x], x] /; \operatorname{FreeQ}[\{a, b, e, f\}, x] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{IntegersQ}[m, p/2] \ \&\& (\operatorname{LtQ}[p, 0] \ || \ \operatorname{GtQ}[m - p/2, 0])$

Rule 2950

$\operatorname{Int}[\sin[(e_.) + (f_)*(x_)]^{(p_)}*((a_ + (b_)*\sin[(e_.) + (f_)*(x_)])^{(m_)}*((c_.) + (d_)*\sin[(e_.) + (f_)*(x_)]^{(n_)}), x_Symbol] \rightarrow \operatorname{Dist}[a^n*c^n, \operatorname{Int}[\operatorname{Tan}[e + f*x]^p*(a + b*\operatorname{Sin}[e + f*x])^{(m-n)}, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \operatorname{EqQ}[b*c + a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{EqQ}[p + 2*n, 0] \ \&\& \operatorname{IntegerQ}[n]$

Rule 3767

$\operatorname{Int}[\csc[(c_.) + (d_)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], \operatorname{Cot}[c + d*x], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 3768

$\operatorname{Int}[(\csc[(c_.) + (d_)*(x_)]*(b_))^{(n_)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n-2)}, x], x] /; \operatorname{FreeQ}[\{b, c, d\}, x] \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{IntegerQ}[2*n]$

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \csc^6(c + dx)(a + a \sin(c + dx))^3(A - A \sin(c + dx)) dx &= (a^3 A^3) \int \frac{\cot^6(c + dx)}{(A - A \sin(c + dx))^2} dx \\ &= \frac{a^3 \int (-A^4 \csc^2(c + dx) - 2A^4 \csc^3(c + dx) + 2A^4 \csc^4(c + dx)) dx}{A^3} \\ &= -\left((a^3 A) \int \csc^2(c + dx) dx \right) + (a^3 A) \int \csc^6(c + dx) dx \\ &= \frac{a^3 A \cot(c + dx) \csc(c + dx)}{d} - \frac{a^3 A \cot(c + dx) \csc^3(c + dx)}{2d} \\ &= \frac{a^3 A \tanh^{-1}(\cos(c + dx))}{d} - \frac{2a^3 A \cot^3(c + dx)}{3d} - \frac{a^3 A}{3d} \\ &= \frac{a^3 A \tanh^{-1}(\cos(c + dx))}{4d} - \frac{2a^3 A \cot^3(c + dx)}{3d} - \frac{a^3 A}{3d} \end{aligned}$$

Mathematica [B] time = 0.07, size = 268, normalized size = 2.55

$$a^3 A \left(-\frac{7 \tan\left(\frac{1}{2}(c + dx)\right)}{30d} + \frac{7 \cot\left(\frac{1}{2}(c + dx)\right)}{30d} - \frac{\csc^4\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{\csc^2\left(\frac{1}{2}(c + dx)\right)}{16d} + \frac{\sec^4\left(\frac{1}{2}(c + dx)\right)}{32d} - \frac{\sec^2\left(\frac{1}{2}(c + dx)\right)}{16d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^6*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] $a^3 A \left(\frac{7 \cot\left(\frac{c + dx}{2}\right)}{30d} + \frac{\csc^2\left(\frac{c + dx}{2}\right)}{16d} - \frac{19 \cot\left(\frac{c + dx}{2}\right) \csc\left(\frac{c + dx}{2}\right)^2}{480d} - \frac{\csc\left(\frac{c + dx}{2}\right)^4}{32d} - \frac{\cot\left(\frac{c + dx}{2}\right) \csc\left(\frac{c + dx}{2}\right)^4}{160d} + \frac{\log\left[\cos\left(\frac{c + dx}{2}\right)\right]}{4d} - \frac{\log\left[\sin\left(\frac{c + dx}{2}\right)\right]}{4d} - \frac{\sec\left(\frac{c + dx}{2}\right)^2}{16d} + \frac{\sec\left(\frac{c + dx}{2}\right)^4}{32d} - \frac{7 \tan\left(\frac{c + dx}{2}\right)}{30d} + \frac{19 \sec\left(\frac{c + dx}{2}\right)^2 \tan\left(\frac{c + dx}{2}\right)}{480d} + \frac{\sec\left(\frac{c + dx}{2}\right)^4 \tan\left(\frac{c + dx}{2}\right)}{160d} \right)$

fricas [B] time = 0.43, size = 201, normalized size = 1.91

$$\frac{56 A a^3 \cos(dx + c)^5 - 80 A a^3 \cos(dx + c)^3 + 15 (A a^3 \cos(dx + c)^4 - 2 A a^3 \cos(dx + c)^2 + A a^3) \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2} \sin(dx + c)\right)}{d^4 \cos(dx + c)^4 - 2 d^2 \cos(dx + c)^2 + d^2 \sin(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] $\frac{1}{120} (56 A a^3 \cos(dx + c)^5 - 80 A a^3 \cos(dx + c)^3 + 15 (A a^3 \cos(dx + c)^4 - 2 A a^3 \cos(dx + c)^2 + A a^3) \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2} \sin(dx + c)\right) - 15 (A a^3 \cos(dx + c)^4 - 2 A a^3 \cos(dx + c)^2 + A a^3) \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2} \sin(dx + c)\right) - 30 (A a^3 \cos(dx + c)^3 + A a^3 \cos(dx + c)) \sin(dx + c)}{(d \cos(dx + c))^4 - 2 d^2 \cos(dx + c)^2 + d^2 \sin(dx + c)^2}$

giac [A] time = 0.19, size = 174, normalized size = 1.66

$$3 A a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^5 + 15 A a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 25 A a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 120 A a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)$$

48

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] 1/480*(3*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*tan(1/2*d*x + 1/2*c)^4 + 25*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^3*log(abs(tan(1/2*d*x + 1/2*c))) - 90*A*a^3*tan(1/2*d*x + 1/2*c) + (274*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 90*A*a^3*tan(1/2*d*x + 1/2*c)^4 - 25*A*a^3*tan(1/2*d*x + 1/2*c)^2 - 15*A*a^3*tan(1/2*d*x + 1/2*c) - 3*A*a^3)/tan(1/2*d*x + 1/2*c)^5)/d

maple [A] time = 0.72, size = 132, normalized size = 1.26

$$\frac{7a^3 A \cot(dx + c)}{15d} + \frac{a^3 A \cot(dx + c) \csc(dx + c)}{4d} - \frac{a^3 A \ln(\csc(dx + c) - \cot(dx + c))}{4d} - \frac{a^3 A \cot(dx + c) (\csc^3(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] 7/15*a^3*A*cot(d*x+c)/d+1/4*a^3*A*cot(d*x+c)*csc(d*x+c)/d-1/4/d*a^3*A*ln(csc(d*x+c)-cot(d*x+c))-1/2*a^3*A*cot(d*x+c)*csc(d*x+c)^3/d-1/5/d*a^3*A*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*A*cot(d*x+c)*csc(d*x+c)^2

maxima [A] time = 0.44, size = 175, normalized size = 1.67

$$\frac{15 A a^3 \left(\frac{2(3 \cos(dx+c)^3 - 5 \cos(dx+c))}{\cos(dx+c)^4 - 2 \cos(dx+c)^2 + 1} - 3 \log(\cos(dx+c) + 1) + 3 \log(\cos(dx+c) - 1) \right) - 60 A a^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2 - 1} - 1 \right)}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^6*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] 1/120*(15*A*a^3*(2*(3*cos(d*x + c)^3 - 5*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 3*log(cos(d*x + c) + 1) + 3*log(cos(d*x + c) - 1)) - 60*A*a^3*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)) + 120*A*a^3/tan(d*x + c) - 8*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*A*a^3/tan(d*x + c)^5)/d

mupad [B] time = 13.16, size = 244, normalized size = 2.32

$$A a^3 \left(3 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 3 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^{10} - 15 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^9 + 15 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^9 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^6,x)

[Out] -(A*a^3*(3*cos(c/2 + (d*x)/2)^10 - 3*sin(c/2 + (d*x)/2)^10 - 15*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^9 + 15*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2) -

$$25\cos(c/2 + (d*x)/2)^2\sin(c/2 + (d*x)/2)^8 + 90\cos(c/2 + (d*x)/2)^4\sin(c/2 + (d*x)/2)^6 - 90\cos(c/2 + (d*x)/2)^6\sin(c/2 + (d*x)/2)^4 + 25\cos(c/2 + (d*x)/2)^8\sin(c/2 + (d*x)/2)^2 + 120\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2))\cos(c/2 + (d*x)/2)^5\sin(c/2 + (d*x)/2)^5)/(480*d*\cos(c/2 + (d*x)/2)^5*\sin(c/2 + (d*x)/2)^5)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**6*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)

[Out] Timed out

3.234 $\int \csc^7(c+dx)(a+a \sin(c+dx))^3(A-A \sin(c+dx)) dx$

Optimal. Leaf size=130

$$\frac{2a^3 A \cot^5(c+dx)}{5d} - \frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{3a^3 A \tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d} - \frac{5a^3 A \cot(c+dx)}{6d}$$

[Out] $3/16*a^3*A*\operatorname{arctanh}(\cos(d*x+c))/d-2/3*a^3*A*\cot(d*x+c)^3/d-2/5*a^3*A*\cot(d*x+c)^5/d+3/16*a^3*A*\cot(d*x+c)*\csc(d*x+c)/d-5/24*a^3*A*\cot(d*x+c)*\csc(d*x+c)^3/d-1/6*a^3*A*\cot(d*x+c)*\csc(d*x+c)^5/d$

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 3768, 3770, 3767}

$$\frac{2a^3 A \cot^5(c+dx)}{5d} - \frac{2a^3 A \cot^3(c+dx)}{3d} + \frac{3a^3 A \tanh^{-1}(\cos(c+dx))}{16d} - \frac{a^3 A \cot(c+dx) \csc^5(c+dx)}{6d} - \frac{5a^3 A \cot(c+dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csc}[c+d*x]^7*(a+a*\operatorname{Sin}[c+d*x])^3*(A-A*\operatorname{Sin}[c+d*x]),x]$

[Out] $(3*a^3*A*\operatorname{ArcTanh}[\operatorname{Cos}[c+d*x]])/(16*d) - (2*a^3*A*\operatorname{Cot}[c+d*x]^3)/(3*d) - (2*a^3*A*\operatorname{Cot}[c+d*x]^5)/(5*d) + (3*a^3*A*\operatorname{Cot}[c+d*x]*\operatorname{Csc}[c+d*x])/(16*d) - (5*a^3*A*\operatorname{Cot}[c+d*x]*\operatorname{Csc}[c+d*x]^3)/(24*d) - (a^3*A*\operatorname{Cot}[c+d*x]*\operatorname{Csc}[c+d*x]^5)/(6*d)$

Rule 2966

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e+f*x]^n*(a+b*\sin[e+f*x])^m*(A+B*\sin[e+f*x]), x], x] /;$ FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1+x^2)^{(n/2-1)}, x], x], x, \operatorname{Cot}[c+d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c+d*x]*(b*\operatorname{Csc}[c+d*x])^{(n-1)})/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c+d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c+d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \csc^7(c+dx)(a+a\sin(c+dx))^3(A-A\sin(c+dx))dx &= \int (-a^3A\csc^3(c+dx) - 2a^3A\csc^4(c+dx) + 2a^3A\csc^5(c+dx) \\
&= -\left((a^3A)\int \csc^3(c+dx)dx\right) + (a^3A)\int \csc^7(c+dx)dx \\
&= \frac{a^3A\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^3A\cot(c+dx)\csc^5(c+dx)}{6d} \\
&= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a^3A\cot^5(c+dx)}{3d} \\
&= \frac{a^3A\tanh^{-1}(\cos(c+dx))}{2d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a^3A\cot^5(c+dx)}{3d} \\
&= \frac{3a^3A\tanh^{-1}(\cos(c+dx))}{16d} - \frac{2a^3A\cot^3(c+dx)}{3d} - \frac{2a^3A\cot^5(c+dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 306, normalized size = 2.35

$$a^3A\left(-\frac{2\tan\left(\frac{1}{2}(c+dx)\right)}{15d} + \frac{2\cot\left(\frac{1}{2}(c+dx)\right)}{15d} - \frac{\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{\sec^6\left(\frac{1}{2}(c+dx)\right)}{64d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^7*(a + a*Sin[c + d*x])^3*(A - A*Sin[c + d*x]),x]

[Out] a^3*A*((2*Cot[(c + d*x)/2])/(15*d) + (3*Csc[(c + d*x)/2]^2)/(64*d) + (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(240*d) - Csc[(c + d*x)/2]^4/(64*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^4)/(80*d) - Csc[(c + d*x)/2]^6/(384*d) + (3*Log[Cos[(c + d*x)/2]])/(16*d) - (3*Log[Sin[(c + d*x)/2]])/(16*d) - (3*Sec[(c + d*x)/2]^2)/(64*d) + Sec[(c + d*x)/2]^4/(64*d) + Sec[(c + d*x)/2]^6/(384*d) - (2*Tan[(c + d*x)/2])/(15*d) - (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(40*d) + (Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(80*d))

fricas [B] time = 0.47, size = 240, normalized size = 1.85

$$90Aa^3\cos(dx+c)^5 - 80Aa^3\cos(dx+c)^3 - 90Aa^3\cos(dx+c) - 45\left(Aa^3\cos(dx+c)^6 - 3Aa^3\cos(dx+c)^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="fricas")

[Out] -1/480*(90*A*a^3*cos(d*x + c)^5 - 80*A*a^3*cos(d*x + c)^3 - 90*A*a^3*cos(d*x + c) - 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(1/2*cos(d*x + c) + 1/2) + 45*(A*a^3*cos(d*x + c)^6 - 3*A*a^3*cos(d*x + c)^4 + 3*A*a^3*cos(d*x + c)^2 - A*a^3)*log(-1/2*cos(d*x + c) + 1/2) + 64*(2*A*a^3*cos(d*x + c)^5 - 5*A*a^3*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2 - d)

giac [B] time = 0.20, size = 242, normalized size = 1.86

$$5Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^6 + 24Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^5 + 45Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 40Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 15Aa^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="giac")

[Out] $\frac{1}{1920}*(5*A*a^3*\tan(1/2*d*x + 1/2*c)^6 + 24*A*a^3*\tan(1/2*d*x + 1/2*c)^5 + 45*A*a^3*\tan(1/2*d*x + 1/2*c)^4 + 40*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 15*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 360*A*a^3*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - 240*A*a^3*\tan(1/2*d*x + 1/2*c) + (882*A*a^3*\tan(1/2*d*x + 1/2*c)^6 + 240*A*a^3*\tan(1/2*d*x + 1/2*c)^5 + 15*A*a^3*\tan(1/2*d*x + 1/2*c)^4 - 40*A*a^3*\tan(1/2*d*x + 1/2*c)^3 - 45*A*a^3*\tan(1/2*d*x + 1/2*c)^2 - 24*A*a^3*\tan(1/2*d*x + 1/2*c) - 5*A*a^3)/\tan(1/2*d*x + 1/2*c)^6)/d$

maple [A] time = 0.73, size = 155, normalized size = 1.19

$$\frac{3a^3 A \cot(dx + c) \csc(dx + c)}{16d} - \frac{3a^3 A \ln(\csc(dx + c) - \cot(dx + c))}{16d} + \frac{4a^3 A \cot(dx + c)}{15d} + \frac{2a^3 A \cot(dx + c) (\csc(dx + c) - \cot(dx + c))}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x)

[Out] $\frac{3}{16}*a^3*A*\cot(d*x+c)*\csc(d*x+c)/d - \frac{3}{16}/d*a^3*A*\ln(\csc(d*x+c) - \cot(d*x+c)) + \frac{4}{15}*a^3*A*\cot(d*x+c)/d + \frac{2}{15}/d*a^3*A*\cot(d*x+c)*\csc(d*x+c)^2 - \frac{2}{5}/d*a^3*A*\cot(d*x+c)*\csc(d*x+c)^4 - \frac{1}{6}*a^3*A*\cot(d*x+c)*\csc(d*x+c)^5/d - \frac{5}{24}*a^3*A*\cot(d*x+c)*\csc(d*x+c)^3/d$

maxima [A] time = 0.60, size = 207, normalized size = 1.59

$$\frac{5 A a^3 \left(\frac{2(15 \cos(dx+c)^5 - 40 \cos(dx+c)^3 + 33 \cos(dx+c))}{\cos(dx+c)^6 - 3 \cos(dx+c)^4 + 3 \cos(dx+c)^2 - 1} - 15 \log(\cos(dx+c) + 1) + 15 \log(\cos(dx+c) - 1) \right) - 120 A a^3}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^7*(a+a*sin(d*x+c))^3*(A-A*sin(d*x+c)),x, algorithm="maxima")

[Out] $\frac{1}{480}*(5*A*a^3*(2*(15*\cos(d*x + c)^5 - 40*\cos(d*x + c)^3 + 33*\cos(d*x + c)) / (\cos(d*x + c)^6 - 3*\cos(d*x + c)^4 + 3*\cos(d*x + c)^2 - 1) - 15*\log(\cos(d*x + c) + 1) + 15*\log(\cos(d*x + c) - 1)) - 120*A*a^3*(2*\cos(d*x + c) / (\cos(d*x + c)^2 - 1) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1)) + 320*(3*\tan(d*x + c)^2 + 1)*A*a^3/\tan(d*x + c)^3 - 64*(15*\tan(d*x + c)^4 + 10*\tan(d*x + c)^2 + 3)*A*a^3/\tan(d*x + c)^5)/d$

mupad [B] time = 13.37, size = 340, normalized size = 2.62

$$A a^3 \left(5 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 5 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^{12} - 24 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} + 24 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^{11} \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A - A*sin(c + d*x))*(a + a*sin(c + d*x))^3)/sin(c + d*x)^7,x)

[Out] $-(A*a^3*(5*\cos(c/2 + (d*x)/2)^{12} - 5*\sin(c/2 + (d*x)/2)^{12} - 24*\cos(c/2 + (d*x)/2)*\sin(c/2 + (d*x)/2)^{11} + 24*\cos(c/2 + (d*x)/2)^{11}*\sin(c/2 + (d*x)/2) - 45*\cos(c/2 + (d*x)/2)^2*\sin(c/2 + (d*x)/2)^{10} - 40*\cos(c/2 + (d*x)/2)^3*\sin(c/2 + (d*x)/2)^9 + 15*\cos(c/2 + (d*x)/2)^4*\sin(c/2 + (d*x)/2)^8 + 240*\cos(c/2 + (d*x)/2)^5*\sin(c/2 + (d*x)/2)^7 - 240*\cos(c/2 + (d*x)/2)^7*\sin(c/2 + (d*x)/2)^5 - 15*\cos(c/2 + (d*x)/2)^8*\sin(c/2 + (d*x)/2)^4 + 40*\cos(c/2 + (d*x)/2)^9*\sin(c/2 + (d*x)/2)^3 + 45*\cos(c/2 + (d*x)/2)^{10}*\sin(c/2 + (d*x)/2)^2 - 45*\cos(c/2 + (d*x)/2)^{11}*\sin(c/2 + (d*x)/2) + 5*\sin(c/2 + (d*x)/2)^{12} - 24*\cos(c/2 + (d*x)/2)*\sin(c/2 + (d*x)/2)^{11} + 24*\cos(c/2 + (d*x)/2)^{11}*\sin(c/2 + (d*x)/2) - 45*\cos(c/2 + (d*x)/2)^2*\sin(c/2 + (d*x)/2)^{10} - 40*\cos(c/2 + (d*x)/2)^3*\sin(c/2 + (d*x)/2)^9 + 15*\cos(c/2 + (d*x)/2)^4*\sin(c/2 + (d*x)/2)^8 + 240*\cos(c/2 + (d*x)/2)^5*\sin(c/2 + (d*x)/2)^7 - 240*\cos(c/2 + (d*x)/2)^7*\sin(c/2 + (d*x)/2)^5 - 15*\cos(c/2 + (d*x)/2)^8*\sin(c/2 + (d*x)/2)^4 + 40*\cos(c/2 + (d*x)/2)^9*\sin(c/2 + (d*x)/2)^3 + 45*\cos(c/2 + (d*x)/2)^{10}*\sin(c/2 + (d*x)/2)^2 - 45*\cos(c/2 + (d*x)/2)^{11}*\sin(c/2 + (d*x)/2) + 5*\sin(c/2 + (d*x)/2)^{12})/d$

```
/2)^2 + 360*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(c/2 + (d*x)/2)^6  
*sin(c/2 + (d*x)/2)^6)/(1920*d*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sin(d*x+c))**3*(A-A*sin(d*x+c)),x)
```

```
[Out] Timed out
```


$$3.235 \quad \int \frac{\sin^4(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=129

$$-\frac{4A \cos(c+dx)}{a^3 d} + \frac{A \sin(c+dx) \cos(c+dx)}{2a^3 d} - \frac{199A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} + \frac{41A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3}$$

[Out] $-19/2*A*x/a^3-4*A*\cos(d*x+c)/a^3/d+1/2*A*\cos(d*x+c)*\sin(d*x+c)/a^3/d-2/5*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^3+41/15*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^2-199/15*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))$

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2966, 2638, 2635, 8, 2650, 2648}

$$-\frac{4A \cos(c+dx)}{a^3 d} + \frac{A \sin(c+dx) \cos(c+dx)}{2a^3 d} - \frac{199A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} + \frac{41A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[c + d*x]^4*(A - A*SIN[c + d*x]))/(a + a*SIN[c + d*x])^3,x]

[Out] $(-19*A*x)/(2*a^3) - (4*A*\cos[c + d*x])/(a^3*d) + (A*\cos[c + d*x]*\sin[c + d*x])/(2*a^3*d) - (2*A*\cos[c + d*x])/(5*a^3*d*(1 + \sin[c + d*x])^3) + (41*A*\cos[c + d*x])/(15*a^3*d*(1 + \sin[c + d*x])^2) - (199*A*\cos[c + d*x])/(15*a^3*d*(1 + \sin[c + d*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]*(b*sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2648

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*cos[c + d*x]*(a + b*sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[si

$n[e + f*x]^n*(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x, x$ /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(c + dx)(A - A \sin(c + dx))}{(a + a \sin(c + dx))^3} dx &= \int \left(-\frac{9A}{a^3} + \frac{4A \sin(c + dx)}{a^3} - \frac{A \sin^2(c + dx)}{a^3} + \frac{2A}{a^3(1 + \sin(c + dx))^3} \right) dx \\ &= -\frac{9Ax}{a^3} - \frac{A \int \sin^2(c + dx) dx}{a^3} + \frac{(2A) \int \frac{1}{(1 + \sin(c + dx))^3} dx}{a^3} + \frac{(4A) \int \sin(c + dx) dx}{a^3} \\ &= -\frac{9Ax}{a^3} - \frac{4A \cos(c + dx)}{a^3 d} + \frac{A \cos(c + dx) \sin(c + dx)}{2a^3 d} - \frac{2A \cos(c + dx)}{5a^3 d(1 + \sin(c + dx))} \\ &= -\frac{19Ax}{2a^3} - \frac{4A \cos(c + dx)}{a^3 d} + \frac{A \cos(c + dx) \sin(c + dx)}{2a^3 d} - \frac{2A \cos(c + dx)}{5a^3 d(1 + \sin(c + dx))} \\ &= -\frac{19Ax}{2a^3} - \frac{4A \cos(c + dx)}{a^3 d} + \frac{A \cos(c + dx) \sin(c + dx)}{2a^3 d} - \frac{2A \cos(c + dx)}{5a^3 d(1 + \sin(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.94, size = 254, normalized size = 1.97

$$A \left(-11400dx \sin\left(c + \frac{dx}{2}\right) - 5700dx \sin\left(c + \frac{3dx}{2}\right) + 1830 \sin\left(2c + \frac{3dx}{2}\right) - 4234 \sin\left(2c + \frac{5dx}{2}\right) + 1140dx \sin\left(3c + \frac{5dx}{2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sin[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (A*(-11400*d*x*Cos[(d*x)/2] + 12060*Cos[c + (d*x)/2] - 14090*Cos[c + (3*d*x)/2] + 5700*d*x*Cos[2*c + (3*d*x)/2] + 1140*d*x*Cos[2*c + (5*d*x)/2] + 1050*Cos[3*c + (5*d*x)/2] + 165*Cos[3*c + (7*d*x)/2] + 15*Cos[5*c + (9*d*x)/2] + 19780*Sin[(d*x)/2] - 11400*d*x*Sin[c + (d*x)/2] - 5700*d*x*Sin[c + (3*d*x)/2] + 1830*Sin[2*c + (3*d*x)/2] - 4234*Sin[2*c + (5*d*x)/2] + 1140*d*x*Sin[3*c + (5*d*x)/2] + 165*Sin[4*c + (7*d*x)/2] - 15*Sin[4*c + (9*d*x)/2]))/(80*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)

fricas [B] time = 0.46, size = 248, normalized size = 1.92

$$\frac{15 A \cos(dx + c)^5 + 90 A \cos(dx + c)^4 + (285 A dx + 683 A) \cos(dx + c)^3 - 1140 A dx + (855 A dx - 526 A) \cos(dx + c)^2 - 6*(95 A dx + 191 A) \cos(dx + c) - (15 A \cos(dx + c)^4 - 75 A \cos(dx + c)^3 + 1140 A dx - 19*(15 A dx - 32 A) \cos(dx + c)^2 + 6*(95 A dx + 189 A) \cos(dx + c) - 12 A) \sin(dx + c) - 12 A}{30 (a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d + (a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/30*(15*A*cos(d*x + c)^5 + 90*A*cos(d*x + c)^4 + (285*A*d*x + 683*A)*cos(d*x + c)^3 - 1140*A*d*x + (855*A*d*x - 526*A)*cos(d*x + c)^2 - 6*(95*A*d*x + 191*A)*cos(d*x + c) - (15*A*cos(d*x + c)^4 - 75*A*cos(d*x + c)^3 + 1140*A*d*x - 19*(15*A*d*x - 32*A)*cos(d*x + c)^2 + 6*(95*A*d*x + 189*A)*cos(d*x + c) - 12*A)*sin(d*x + c) - 12*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))

giac [A] time = 0.21, size = 156, normalized size = 1.21

$$\frac{\frac{285(dx+c)A}{a^3} + \frac{30\left(A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 8A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 8A\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^2 a^3} + \frac{4\left(135A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 615A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 1025A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 685A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 164A\right)}{a^3 \left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 1\right)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] $-\frac{1}{30} \cdot \frac{285 \cdot (d \cdot x + c) \cdot A / a^3 + 30 \cdot (A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 8 \cdot A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 - A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 8 \cdot A)}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 1)^2 \cdot a^3} + \frac{4 \cdot (135 \cdot A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 + 615 \cdot A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 + 1025 \cdot A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 685 \cdot A \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 164 \cdot A)}{a^3 \cdot (\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + 1)^5} / d$

maple [B] time = 0.45, size = 257, normalized size = 1.99

$$\frac{A \left(\tan^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d a^3 \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} - \frac{8A \left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d a^3 \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} + \frac{A \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d a^3 \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} - \frac{8A}{d a^3 \left(1 + \tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)

[Out] $-\frac{1}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(1 + \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^2} \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^3 - \frac{8}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(1 + \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^2} \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 + \frac{1}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(1 + \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^2} \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) - \frac{8}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(1 + \tan(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^2} - \frac{19}{d} \cdot \frac{A}{a^3} \cdot \arctan(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c)) - \frac{16}{5} \cdot \frac{A}{a^3} \cdot \frac{1}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^5} + \frac{8}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^4} + \frac{4}{3} \cdot \frac{A}{a^3} \cdot \frac{1}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^3} - \frac{10}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)^2} - \frac{18}{d} \cdot \frac{A}{a^3} \cdot \frac{1}{(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 1)}$

maxima [B] time = 0.59, size = 715, normalized size = 5.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-\frac{1}{15} \cdot \frac{A \cdot ((1325 \cdot \sin(d \cdot x + c) / (\cos(d \cdot x + c) + 1) + 2673 \cdot \sin(d \cdot x + c)^2 / (\cos(d \cdot x + c) + 1)^2 + 3805 \cdot \sin(d \cdot x + c)^3 / (\cos(d \cdot x + c) + 1)^3 + 4329 \cdot \sin(d \cdot x + c)^4 / (\cos(d \cdot x + c) + 1)^4 + 3575 \cdot \sin(d \cdot x + c)^5 / (\cos(d \cdot x + c) + 1)^5 + 2275 \cdot \sin(d \cdot x + c)^6 / (\cos(d \cdot x + c) + 1)^6 + 975 \cdot \sin(d \cdot x + c)^7 / (\cos(d \cdot x + c) + 1)^7 + 195 \cdot \sin(d \cdot x + c)^8 / (\cos(d \cdot x + c) + 1)^8 + 304) / (a^3 + 5 \cdot a^3 \cdot \sin(d \cdot x + c) / (\cos(d \cdot x + c) + 1) + 12 \cdot a^3 \cdot \sin(d \cdot x + c)^2 / (\cos(d \cdot x + c) + 1)^2 + 20 \cdot a^3 \cdot \sin(d \cdot x + c)^3 / (\cos(d \cdot x + c) + 1)^3 + 26 \cdot a^3 \cdot \sin(d \cdot x + c)^4 / (\cos(d \cdot x + c) + 1)^4 + 26 \cdot a^3 \cdot \sin(d \cdot x + c)^5 / (\cos(d \cdot x + c) + 1)^5 + 20 \cdot a^3 \cdot \sin(d \cdot x + c)^6 / (\cos(d \cdot x + c) + 1)^6 + 12 \cdot a^3 \cdot \sin(d \cdot x + c)^7 / (\cos(d \cdot x + c) + 1)^7 + 5 \cdot a^3 \cdot \sin(d \cdot x + c)^8 / (\cos(d \cdot x + c) + 1)^8 + a^3 \cdot \sin(d \cdot x + c)^9 / (\cos(d \cdot x + c) + 1)^9) + 195 \cdot \arctan(\sin(d \cdot x + c) / (\cos(d \cdot x + c) + 1)) / a^3 + 6 \cdot A \cdot ((105 \cdot \sin(d \cdot x + c) / (\cos(d \cdot x + c) + 1) + 189 \cdot \sin(d \cdot x + c)^2 / (\cos(d \cdot x + c) + 1)^2 + 200 \cdot \sin(d \cdot x + c)^3 / (\cos(d \cdot x + c) + 1)^3 + 160 \cdot \sin(d \cdot x + c)^4 / (\cos(d \cdot x + c) + 1)^4 + 75 \cdot \sin(d \cdot x + c)^5 / (\cos(d \cdot x + c) + 1)^5 + 15 \cdot \sin(d \cdot x + c)^6 / (\cos(d \cdot x + c) + 1)^6 + 24) / (a^3 + 5 \cdot a^3 \cdot \sin(d \cdot x + c) / (\cos(d \cdot x + c) + 1) + 11 \cdot a^3 \cdot \sin(d \cdot x + c)^2 / (\cos(d \cdot x + c) + 1)^2 + 15 \cdot a^3 \cdot \sin(d \cdot x + c)^3 / (\cos(d \cdot x + c) + 1)^3$

+ 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d

mupad [B] time = 17.08, size = 326, normalized size = 2.53

$$\left(\frac{95 A(c+dx)}{2} - \frac{A(1425c+1425dx+570)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^8 + \left(114 A(c+dx) - \frac{A(3420c+3420dx+2850)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^7 + \left(190 A(c+dx) - \frac{A(5700c+5700dx+6650)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \left(190 A(c+dx) - \frac{A(5700c+5700dx+11270)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \left(247 A(c+dx) - \frac{A(7410c+7410dx+10450)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \left(247 A(c+dx) - \frac{A(7410c+7410dx+12846)}{30}\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \left(19 A(c+dx)\right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - \frac{A(285c+285dx+896)}{30} \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - \frac{19 A(c+dx)}{2} \ln\left|\frac{\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1}{\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 1}\right| - \frac{19 A(c+dx)}{2} \ln|a^3|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(c + d*x)^4*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 3910))/30) + tan(c/2 + (d*x)/2)^8*((95*A*(c + d*x))/2 - (A*(1425*c + 1425*d*x + 570))/30) + tan(c/2 + (d*x)/2)^7*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 2850))/30) + tan(c/2 + (d*x)/2)^6*(114*A*(c + d*x) - (A*(3420*c + 3420*d*x + 7902))/30) + tan(c/2 + (d*x)/2)^5*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x + 6650))/30) + tan(c/2 + (d*x)/2)^4*(247*A*(c + d*x) - (A*(7410*c + 7410*d*x + 10450))/30) + tan(c/2 + (d*x)/2)^3*(190*A*(c + d*x) - (A*(5700*c + 5700*d*x + 11270))/30) + tan(c/2 + (d*x)/2)^2*(247*A*(c + d*x) - (A*(7410*c + 7410*d*x + 12846))/30) + (19*A*(c + d*x))/2 - (A*(285*c + 285*d*x + 896))/30)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (19*A*x)/(2*a^3)

sympy [A] time = 78.44, size = 3614, normalized size = 28.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((-285*A*d*x*tan(c/2 + d*x/2)**9/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 1425*A*d*x*tan(c/2 + d*x/2)**8/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 3420*A*d*x*tan(c/2 + d*x/2)**7/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 5700*A*d*x*tan(c/2 + d*x/2)**6/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**5/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d*tan(c/2 + d*x/2)**5 + 780*a**3*d*tan(c/2 + d*x/2)**4 + 600*a**3*d*tan(c/2 + d*x/2)**3 + 360*a**3*d*tan(c/2 + d*x/2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d) - 7410*A*d*x*tan(c/2 + d*x/2)**4/(30*a**3*d*tan(c/2 + d*x/2)**9 + 150*a**3*d*tan(c/2 + d*x/2)**8 + 360*a**3*d*tan(c/2 + d*x/2)**7 + 600*a**3*d*tan(c/2 + d*x/2)**6 + 780*a**3*d

$$\begin{aligned}
& \tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + \\
& d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + \\
& 30*a**3*d) - 5700*A*d*x*\tan(c/2 + d*x/2)**3/(30*a**3*d*\tan(c/2 + d*x/2)**9 \\
& + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a** \\
& 3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(\\
& c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x \\
& /2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 3420*A*d*x*\tan(c/2 + d* \\
& x/2)**2/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 3 \\
& 60*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d \\
& *\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 \\
& + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) \\
& + 30*a**3*d) - 1425*A*d*x*\tan(c/2 + d*x/2)/(30*a**3*d*\tan(c/2 + d*x/2)**9 + \\
& 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3 \\
& *d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/ \\
& 2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2 \\
&)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 285*A*d*x/(30*a**3*d*\tan(\\
& c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x \\
& /2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + \\
& 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3* \\
& d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 570*A*\tan \\
& n(c/2 + d*x/2)**8/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x \\
& /2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + \\
& 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3* \\
& d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 \\
& + d*x/2) + 30*a**3*d) - 2850*A*\tan(c/2 + d*x/2)**7/(30*a**3*d*\tan(c/2 + d* \\
& x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + \\
& 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3 \\
& *d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/ \\
& 2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 6650*A*\tan(c/2 + \\
& d*x/2)**6/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 \\
& + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a** \\
& 3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c \\
& /2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/ \\
& 2) + 30*a**3*d) - 10450*A*\tan(c/2 + d*x/2)**5/(30*a**3*d*\tan(c/2 + d*x/2)** \\
& 9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a \\
& **3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan \\
& (c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d* \\
& x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 12846*A*\tan(c/2 + d*x/ \\
& 2)**4/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360 \\
& *a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan \\
& an(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + \\
& d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + \\
& 30*a**3*d) - 11270*A*\tan(c/2 + d*x/2)**3/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 1 \\
& 50*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d \\
& *\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 \\
& + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)* \\
& *2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 7902*A*\tan(c/2 + d*x/2)**2/ \\
& (30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3* \\
& d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 \\
& + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2) \\
& **3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a**3*d*\tan(c/2 + d*x/2) + 30*a** \\
& 3*d) - 3910*A*\tan(c/2 + d*x/2)/(30*a**3*d*\tan(c/2 + d*x/2)**9 + 150*a**3*d* \\
& \tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a**3*d*\tan(c/2 + \\
& d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c/2 + d*x/2)** \\
& 4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/2)**2 + 150*a \\
& **3*d*\tan(c/2 + d*x/2) + 30*a**3*d) - 896*A/(30*a**3*d*\tan(c/2 + d*x/2)**9 \\
& + 150*a**3*d*\tan(c/2 + d*x/2)**8 + 360*a**3*d*\tan(c/2 + d*x/2)**7 + 600*a** \\
& 3*d*\tan(c/2 + d*x/2)**6 + 780*a**3*d*\tan(c/2 + d*x/2)**5 + 780*a**3*d*\tan(c \\
& /2 + d*x/2)**4 + 600*a**3*d*\tan(c/2 + d*x/2)**3 + 360*a**3*d*\tan(c/2 + d*x/
\end{aligned}$$

```
2)**2 + 150*a**3*d*tan(c/2 + d*x/2) + 30*a**3*d), Ne(d, 0)), (x*(-A*sin(c)
+ A)*sin(c)**4/(a*sin(c) + a)**3, True))
```

$$3.236 \quad \int \frac{\sin^3(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=103

$$\frac{A \cos(c+dx)}{a^3 d} + \frac{104A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} - \frac{31A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} + \frac{4Ax}{a^3}$$

[Out] 4*A*x/a^3+A*cos(d*x+c)/a^3/d+2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-31/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+104/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))

Rubi [A] time = 0.19, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2966, 2638, 2650, 2648}

$$\frac{A \cos(c+dx)}{a^3 d} + \frac{104A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} - \frac{31A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} + \frac{4Ax}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (4*A*x)/a^3 + (A*Cos[c + d*x])/(a^3*d) + (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (31*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (104*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2648

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)^(n_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(c+dx)(A - A \sin(c+dx))}{(a + a \sin(c+dx))^3} dx &= \int \left(\frac{4A}{a^3} - \frac{A \sin(c+dx)}{a^3} - \frac{2A}{a^3(1 + \sin(c+dx))^3} + \frac{7A}{a^3(1 + \sin(c+dx))^2} \right) dx \\
&= \frac{4Ax}{a^3} - \frac{A \int \sin(c+dx) dx}{a^3} - \frac{(2A) \int \frac{1}{(1+\sin(c+dx))^3} dx}{a^3} + \frac{(7A) \int \frac{1}{(1+\sin(c+dx))^2} dx}{a^3} \\
&= \frac{4Ax}{a^3} + \frac{A \cos(c+dx)}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{7A \cos(c+dx)}{3a^3 d(1 + \sin(c+dx))} \\
&= \frac{4Ax}{a^3} + \frac{A \cos(c+dx)}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{31A \cos(c+dx)}{15a^3 d(1 + \sin(c+dx))} \\
&= \frac{4Ax}{a^3} + \frac{A \cos(c+dx)}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{31A \cos(c+dx)}{15a^3 d(1 + \sin(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.80, size = 228, normalized size = 2.21

$$A \left(-1200dx \sin \left(c + \frac{dx}{2} \right) - 600dx \sin \left(c + \frac{3dx}{2} \right) + 405 \sin \left(2c + \frac{3dx}{2} \right) - 491 \sin \left(2c + \frac{5dx}{2} \right) + 120dx \sin \left(3c + \frac{5dx}{2} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] -1/120*(A*(-1200*d*x*Cos[(d*x)/2] + 1665*Cos[c + (d*x)/2] - 1675*Cos[c + (3*d*x)/2] + 600*d*x*Cos[2*c + (3*d*x)/2] + 120*d*x*Cos[2*c + (5*d*x)/2] + 75*Cos[3*c + (5*d*x)/2] + 15*Cos[3*c + (7*d*x)/2] + 2495*Sin[(d*x)/2] - 1200*d*x*Sin[c + (d*x)/2] - 600*d*x*Sin[c + (3*d*x)/2] + 405*Sin[2*c + (3*d*x)/2] - 491*Sin[2*c + (5*d*x)/2] + 120*d*x*Sin[3*c + (5*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2]))*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.45, size = 225, normalized size = 2.18

$$\frac{15 A \cos(dx + c)^4 + (60 A dx + 149 A) \cos(dx + c)^3 - 240 A dx + (180 A dx - 103 A) \cos(dx + c)^2 - 3(40 A dx + 81 A) \cos(dx + c) + (15 A \cos(dx + c)^3 - 240 A dx + 2(30 A dx - 67 A) \cos(dx + c)^2 - 3(40 A dx + 79 A) \cos(dx + c) + 6 A) \sin(dx + c) - 6 A}{15(a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d + (a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(15*A*cos(d*x + c)^4 + (60*A*d*x + 149*A)*cos(d*x + c)^3 - 240*A*d*x + (180*A*d*x - 103*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 81*A)*cos(d*x + c) + (15*A*cos(d*x + c)^3 - 240*A*d*x + 2*(30*A*d*x - 67*A)*cos(d*x + c)^2 - 3*(40*A*d*x + 79*A)*cos(d*x + c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.18, size = 113, normalized size = 1.10

$$2 \left(\frac{30(dx+c)A}{a^3} + \frac{15A}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)a^3} + \frac{60A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 285A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 505A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 335A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 79A}{a^3 \left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^5} \right)$$

15d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 2/15*(30*(d*x + c)*A/a^3 + 15*A/((tan(1/2*d*x + 1/2*c)^2 + 1)*a^3) + (60*A*tan(1/2*d*x + 1/2*c)^4 + 285*A*tan(1/2*d*x + 1/2*c)^3 + 505*A*tan(1/2*d*x + 1/2*c)^2 + 335*A*tan(1/2*d*x + 1/2*c) + 79*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

maple [A] time = 0.43, size = 155, normalized size = 1.50

$$\frac{2A}{d a^3 \left(1 + \tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)} + \frac{8A \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^3} + \frac{16A}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5} - \frac{8A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)

[Out] 2/d*A/a^3/(1+tan(1/2*d*x+1/2*c)^2)+8/d*A/a^3*arctan(tan(1/2*d*x+1/2*c))+16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+4/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+6/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.56, size = 543, normalized size = 5.27

$$2 \left(3A \left(\frac{\frac{105 \sin(dx+c)}{\cos(dx+c)+1} + \frac{189 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{200 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{160 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{75 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{15 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + 24}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{11a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{15a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{11a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a^3 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}} + \frac{15 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right) \right) / 15d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] 2/15*(3*A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 189*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 200*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 160*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 75*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 24)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 11*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + A*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d

mupad [B] time = 16.93, size = 261, normalized size = 2.53

$$\frac{4Ax}{a^3} \left(20A(c+dx) - \frac{4A(75c+75dx+30)}{15} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + \left(44A(c+dx) - \frac{4A(165c+165dx+150)}{15} \right) \tan\left(\frac{c}{2} + \frac{dx}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((sin(c + d*x)^3*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)

```
[Out] (4*A*x)/a^3 - (tan(c/2 + (d*x)/2)*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 205))/15) + tan(c/2 + (d*x)/2)^6*(20*A*(c + d*x) - (4*A*(75*c + 75*d*x + 30))/15) + tan(c/2 + (d*x)/2)^5*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 150))/15) + tan(c/2 + (d*x)/2)^2*(44*A*(c + d*x) - (4*A*(165*c + 165*d*x + 367))/15) + tan(c/2 + (d*x)/2)^4*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 320))/15) + tan(c/2 + (d*x)/2)^3*(60*A*(c + d*x) - (4*A*(225*c + 225*d*x + 385))/15) + 4*A*(c + d*x) - (4*A*(15*c + 15*d*x + 47))/15)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5*(tan(c/2 + (d*x)/2)^2 + 1))
```

sympy [A] time = 45.41, size = 2290, normalized size = 22.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((60*A*d*x*tan(c/2 + d*x/2)**7/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 900*A*d*x*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 660*A*d*x*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 300*A*d*x*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 60*A*d*x/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 120*A*tan(c/2 + d*x/2)**6/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 600*A*tan(c/2 + d*x/2)**5/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1280*A*tan(c/2 + d*x/2)**4/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1540*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 1468*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3
```

```

*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/
2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2)
+ 15*a**3*d) + 820*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**7 + 75*
a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2)**5 + 225*a**3*d*ta
n(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 165*a**3*d*tan(c/2 + d
*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 188*A/(15*a**3*d*tan(c
/2 + d*x/2)**7 + 75*a**3*d*tan(c/2 + d*x/2)**6 + 165*a**3*d*tan(c/2 + d*x/2
)**5 + 225*a**3*d*tan(c/2 + d*x/2)**4 + 225*a**3*d*tan(c/2 + d*x/2)**3 + 16
5*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(
d, 0)), (x*(-A*sin(c) + A)*sin(c)**3/(a*sin(c) + a)**3, True))

```

$$3.237 \quad \int \frac{\sin^2(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=89

$$-\frac{13A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} + \frac{7A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} - \frac{Ax}{a^3}$$

[Out] $-A*x/a^3-2/5*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^3+7/5*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^2-13/5*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))$

Rubi [A] time = 0.17, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2966, 2650, 2648}

$$-\frac{13A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} + \frac{7A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} - \frac{Ax}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] $-((A*x)/a^3) - (2*A*\text{Cos}[c + d*x])/(5*a^3*d*(1 + \text{Sin}[c + d*x])^3) + (7*A*\text{Cos}[c + d*x])/(5*a^3*d*(1 + \text{Sin}[c + d*x])^2) - (13*A*\text{Cos}[c + d*x])/(5*a^3*d*(1 + \text{Sin}[c + d*x]))$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_) + (f_)*(x_)]^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)(A - A \sin(c+dx))}{(a + a \sin(c+dx))^3} dx &= \int \left(-\frac{A}{a^3} + \frac{2A}{a^3(1 + \sin(c+dx))^3} - \frac{5A}{a^3(1 + \sin(c+dx))^2} + \frac{4A}{a^3(1 + \sin(c+dx))} \right) dx \\
&= -\frac{Ax}{a^3} + \frac{(2A) \int \frac{1}{(1+\sin(c+dx))^3} dx}{a^3} + \frac{(4A) \int \frac{1}{1+\sin(c+dx)} dx}{a^3} - \frac{(5A) \int \frac{1}{(1+\sin(c+dx))^2} dx}{a^3} \\
&= -\frac{Ax}{a^3} - \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} + \frac{5A \cos(c+dx)}{3a^3 d(1 + \sin(c+dx))^2} - \frac{4A \cos(c+dx)}{a^3 d(1 + \sin(c+dx))} \\
&= -\frac{Ax}{a^3} - \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} + \frac{7A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^2} - \frac{7A \cos(c+dx)}{3a^3 d(1 + \sin(c+dx))} \\
&= -\frac{Ax}{a^3} - \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} + \frac{7A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^2} - \frac{13A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.76, size = 189, normalized size = 2.12

$$\frac{A \left(-50dx \sin \left(c + \frac{dx}{2} \right) - 25dx \sin \left(c + \frac{3dx}{2} \right) + 40 \sin \left(2c + \frac{3dx}{2} \right) - 26 \sin \left(2c + \frac{5dx}{2} \right) + 5dx \sin \left(3c + \frac{5dx}{2} \right) + 110 \right)}{20a^3 d \left(\sin \left(\frac{c}{2} \right) + \cos \left(\frac{c}{2} \right) \right) \left(\sin \left(\frac{dx}{2} \right) + \cos \left(\frac{dx}{2} \right) \right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] (A*(-50*d*x*Cos[(d*x)/2] + 110*Cos[c + (d*x)/2] - 90*Cos[c + (3*d*x)/2] + 2
5*d*x*Cos[2*c + (3*d*x)/2] + 5*d*x*Cos[2*c + (5*d*x)/2] + 150*Sin[(d*x)/2]
- 50*d*x*Sin[c + (d*x)/2] - 25*d*x*Sin[c + (3*d*x)/2] + 40*Sin[2*c + (3*d*x
)/2] - 26*Sin[2*c + (5*d*x)/2] + 5*d*x*Sin[3*c + (5*d*x)/2]))/(20*a^3*d*(Co
s[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.45, size = 204, normalized size = 2.29

$$\frac{(5 A dx + 13 A) \cos(dx + c)^3 - 20 A dx + 3(5 A dx - 2 A) \cos(dx + c)^2 - (10 A dx + 21 A) \cos(dx + c) - (20 A dx - 5 A) \sin(dx + c)}{5(a^3 d \cos(dx + c)^3 + 3 a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d + (a^3 d \cos(dx + c)^2 - 2 a^3 d \cos(dx + c) - 4 a^3 d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
[Out] -1/5*((5*A*d*x + 13*A)*cos(d*x + c)^3 - 20*A*d*x + 3*(5*A*d*x - 2*A)*cos(d*
x + c)^2 - (10*A*d*x + 21*A)*cos(d*x + c) - (20*A*d*x - (5*A*d*x - 13*A)*co
s(d*x + c)^2 + (10*A*d*x + 19*A)*cos(d*x + c) - 2*A)*sin(d*x + c) - 2*A)/(a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3
*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 93, normalized size = 1.04

$$\frac{\frac{5(dx+c)A}{a^3} + \frac{2 \left(5A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 25A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 55A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 35A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 8A \right)}{a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1 \right)^5}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="gi
ac")
```

[Out] $-1/5*(5*(d*x + c)*A/a^3 + 2*(5*A*\tan(1/2*d*x + 1/2*c)^4 + 25*A*\tan(1/2*d*x + 1/2*c)^3 + 55*A*\tan(1/2*d*x + 1/2*c)^2 + 35*A*\tan(1/2*d*x + 1/2*c) + 8*A)/(a^3*(\tan(1/2*d*x + 1/2*c) + 1)^5)/d$

maple [A] time = 0.43, size = 131, normalized size = 1.47

$$\frac{2A \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^3} - \frac{16A}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5} + \frac{8A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^4} - \frac{4A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} - \frac{2A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{8A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{8A}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)`

[Out] $-2/d*A/a^3*\arctan(\tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^4-4/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^3-2/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^2-2/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)$

maxima [B] time = 0.64, size = 392, normalized size = 4.40

$$\frac{2 \left(A \left(\frac{\frac{95 \sin(dx+c)}{\cos(dx+c)+1} + \frac{145 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{75 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 22}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} + \frac{15 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right) + \frac{2A \left(\frac{c}{2} + \frac{dx}{2}\right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")`

[Out] $-2/15*(A*((95*\sin(d*x + c))/(\cos(d*x + c) + 1) + 145*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 75*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 15*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5) + 15*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^3) + 2*A*(5*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 1)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5))/d$

mupad [B] time = 14.92, size = 178, normalized size = 2.00

$$\frac{\left(5 A (c + d x) - \frac{A (25 c + 25 d x + 10)}{5}\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^4 + \left(10 A (c + d x) - \frac{A (50 c + 50 d x + 50)}{5}\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^3 + \left(10 A (c + d x) - \frac{A (25 c + 25 d x + 10)}{5}\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right)^2 + \left(10 A (c + d x) - \frac{A (50 c + 50 d x + 50)}{5}\right) \tan\left(\frac{c}{2} + \frac{d x}{2}\right) + \left(10 A (c + d x) - \frac{A (25 c + 25 d x + 10)}{5}\right)}{a^3 d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(c + d*x)^2*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)`

[Out] $(\tan(c/2 + (d*x)/2)*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 70))/5) + \tan(c/2 + (d*x)/2)^4*(5*A*(c + d*x) - (A*(25*c + 25*d*x + 10))/5) + \tan(c/2 + (d*x)/2)^3*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 50))/5) + \tan(c/2 + (d*x)/2)^2*(10*A*(c + d*x) - (A*(50*c + 50*d*x + 110))/5) + A*(c + d*x) - (A*(5*c + 5*d*x + 16))/5)/(a^3*d*(\tan(c/2 + (d*x)/2) + 1)^5) - (A*x)/a^3$

sympy [A] time = 25.57, size = 1268, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((-5*A*d*x*tan(c/2 + d*x/2)**5/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*d*x*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 25*A*d*x*tan(c/2 + d*x/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 5*A*d*x/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 10*A*tan(c/2 + d*x/2)**4/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 50*A*tan(c/2 + d*x/2)**3/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 110*A*tan(c/2 + d*x/2)**2/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 70*A*tan(c/2 + d*x/2)/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d) - 16*A/(5*a**3*d*tan(c/2 + d*x/2)**5 + 25*a**3*d*tan(c/2 + d*x/2)**4 + 50*a**3*d*tan(c/2 + d*x/2)**3 + 50*a**3*d*tan(c/2 + d*x/2)**2 + 25*a**3*d*tan(c/2 + d*x/2) + 5*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)**2/(a*sin(c) + a)**3, True))

$$3.238 \quad \int \frac{\sin(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=82

$$\frac{4A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} - \frac{11A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3}$$

[Out] 2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-11/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4/15*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))

Rubi [A] time = 0.14, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2966, 2650, 2648}

$$\frac{4A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)} - \frac{11A \cos(c+dx)}{15a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (11*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x])^2) + (4*A*Cos[c + d*x])/(15*a^3*d*(1 + Sin[c + d*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_) + (f_)*(x_)]^(n_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)(A - A \sin(c+dx))}{(a + a \sin(c+dx))^3} dx &= \int \left(-\frac{2A}{a^3(1 + \sin(c+dx))^3} + \frac{3A}{a^3(1 + \sin(c+dx))^2} - \frac{A}{a^3(1 + \sin(c+dx))} \right) dx \\
&= -\frac{A \int \frac{1}{1 + \sin(c+dx)} dx}{a^3} - \frac{(2A) \int \frac{1}{(1 + \sin(c+dx))^3} dx}{a^3} + \frac{(3A) \int \frac{1}{(1 + \sin(c+dx))^2} dx}{a^3} \\
&= \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{A \cos(c+dx)}{a^3 d(1 + \sin(c+dx))^2} + \frac{A \cos(c+dx)}{a^3 d(1 + \sin(c+dx))} \\
&= \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{11A \cos(c+dx)}{15a^3 d(1 + \sin(c+dx))^2} - \frac{(4A) \int \frac{1}{1 + \sin(c+dx)} dx}{15a^3} \\
&= \frac{2A \cos(c+dx)}{5a^3 d(1 + \sin(c+dx))^3} - \frac{11A \cos(c+dx)}{15a^3 d(1 + \sin(c+dx))^2} + \frac{4A \cos(c+dx)}{15a^3 d(1 + \sin(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 107, normalized size = 1.30

$$\frac{A \left(15 \sin \left(2c + \frac{3dx}{2} \right) - 4 \sin \left(2c + \frac{5dx}{2} \right) + 15 \cos \left(c + \frac{dx}{2} \right) - 5 \cos \left(c + \frac{3dx}{2} \right) + 25 \sin \left(\frac{dx}{2} \right) \right)}{30a^3 d \left(\sin \left(\frac{c}{2} \right) + \cos \left(\frac{c}{2} \right) \right) \left(\sin \left(\frac{1}{2}(c+dx) \right) + \cos \left(\frac{1}{2}(c+dx) \right) \right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] -1/30*(A*(15*Cos[c + (d*x)/2] - 5*Cos[c + (3*d*x)/2] + 25*Sin[(d*x)/2] + 15*Sin[2*c + (3*d*x)/2] - 4*Sin[2*c + (5*d*x)/2]))/(a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)
```

fricas [B] time = 0.43, size = 156, normalized size = 1.90

$$\frac{4A \cos(dx+c)^3 + 7A \cos(dx+c)^2 - 3A \cos(dx+c) - (4A \cos(dx+c)^2 - 3A \cos(dx+c) - 6A) \sin(dx+c)}{15(a^3 d \cos(dx+c)^3 + 3a^3 d \cos(dx+c)^2 - 2a^3 d \cos(dx+c) - 4a^3 d + (a^3 d \cos(dx+c)^2 - 2a^3 d \cos(dx+c) - 6a^3 d) \sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/15*(4*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - (4*A*cos(d*x + c)^2 - 3*A*cos(d*x + c) - 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 6*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.19, size = 63, normalized size = 0.77

$$\frac{2 \left(15 A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - 5 A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 5 A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + A \right)}{15 a^3 d \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^3 - 5*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)
```

maple [A] time = 0.42, size = 71, normalized size = 0.87

$$\frac{4A \left(-\frac{1}{2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^2} - \frac{2}{\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^4} + \frac{5}{3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^3} + \frac{4}{5 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^5} \right)}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)`

[Out] $4/d*A/a^3*(-1/2/(\tan(1/2*d*x+1/2*c)+1)^2-2/(\tan(1/2*d*x+1/2*c)+1)^4+5/3/(\tan(1/2*d*x+1/2*c)+1)^3+4/5/(\tan(1/2*d*x+1/2*c)+1)^5)$

maxima [B] time = 0.62, size = 348, normalized size = 4.24

$$\frac{2 \left(\frac{2A \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 1 \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{5 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} \right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")`

[Out] $\frac{2/15*(2*A*(5*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 1)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5) - 3*A*(5*\sin(d*x + c)/(\cos(d*x + c) + 1) + 5*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 5*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5))/d}$

mupad [B] time = 13.45, size = 110, normalized size = 1.34

$$\frac{2A \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 5 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 5 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 15 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \right)}{15a^3 d \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right) \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((sin(c + d*x)*(A - A*sin(c + d*x)))/(a + a*sin(c + d*x))^3,x)`

[Out] $-(2*A*\cos(c/2 + (d*x)/2)^2*(\cos(c/2 + (d*x)/2)^3 + 15*\sin(c/2 + (d*x)/2)^3 - 5*\cos(c/2 + (d*x)/2)*\sin(c/2 + (d*x)/2)^2 + 5*\cos(c/2 + (d*x)/2)^2*\sin(c/2 + (d*x)/2))/((15*a^3*d*(\cos(c/2 + (d*x)/2) + \sin(c/2 + (d*x)/2))^5)$

sympy [A] time = 14.11, size = 461, normalized size = 5.62

$$\left\{ \begin{array}{l} \frac{30A \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right)}{15a^3 d \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3 d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3 d \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3 d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3 d \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 15a^3 d} + \frac{x(-A \sin(c) + A) \sin(c)}{(a \sin(c) + a)^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)

[Out] Piecewise((-30*A*tan(c/2 + d*x/2)**3/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) + 10*A*tan(c/2 + d*x/2)**2/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*tan(c/2 + d*x/2)/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d) - 2*A/(15*a**3*d*tan(c/2 + d*x/2)**5 + 75*a**3*d*tan(c/2 + d*x/2)**4 + 150*a**3*d*tan(c/2 + d*x/2)**3 + 150*a**3*d*tan(c/2 + d*x/2)**2 + 75*a**3*d*tan(c/2 + d*x/2) + 15*a**3*d), Ne(d, 0)), (x*(-A*sin(c) + A)*sin(c)/(a*sin(c) + a)**3, True))

$$3.239 \quad \int \frac{A - A \sin(c + dx)}{(a + a \sin(c + dx))^3} dx$$

Optimal. Leaf size=58

$$-\frac{A \cos^3(c + dx)}{15d(a \sin(c + dx) + a)^3} - \frac{aA \cos^3(c + dx)}{5d(a \sin(c + dx) + a)^4}$$

[Out] $-1/5*a*A*\cos(d*x+c)^3/d/(a+a*\sin(d*x+c))^4-1/15*A*\cos(d*x+c)^3/d/(a+a*\sin(d*x+c))^3$

Rubi [A] time = 0.11, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2736, 2672, 2671}

$$-\frac{A \cos^3(c + dx)}{15d(a \sin(c + dx) + a)^3} - \frac{aA \cos^3(c + dx)}{5d(a \sin(c + dx) + a)^4}$$

Antiderivative was successfully verified.

[In] Int[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]

[Out] $-(a*A*\cos[c + d*x]^3)/(5*d*(a + a*\sin[c + d*x])^4) - (A*\cos[c + d*x]^3)/(15*d*(a + a*\sin[c + d*x])^3)$

Rule 2671

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*m), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]

Rule 2672

Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^p*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m, x_Symbol] :> Simp[(b*(g*cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(a*f*g*Simplify[2*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a*Simplify[2*m + p + 1]), Int[(g*cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2*m + p + 1, 0] && !IGtQ[m, 0]

Rule 2736

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^m*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Dist[a^m*c^m, Int[Cos[e + f*x]^(2*m)*(c + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && !(IntegerQ[n] && ((LtQ[m, 0] && GtQ[n, 0]) || LtQ[0, n, m] || LtQ[m, n, 0]))

Rubi steps

$$\begin{aligned} \int \frac{A - A \sin(c + dx)}{(a + a \sin(c + dx))^3} dx &= (aA) \int \frac{\cos^2(c + dx)}{(a + a \sin(c + dx))^4} dx \\ &= -\frac{aA \cos^3(c + dx)}{5d(a + a \sin(c + dx))^4} + \frac{1}{5}A \int \frac{\cos^2(c + dx)}{(a + a \sin(c + dx))^3} dx \\ &= -\frac{aA \cos^3(c + dx)}{5d(a + a \sin(c + dx))^4} - \frac{A \cos^3(c + dx)}{15d(a + a \sin(c + dx))^3} \end{aligned}$$

Mathematica [A] time = 0.24, size = 92, normalized size = 1.59

$$\frac{A \left(\sin \left(2c + \frac{5dx}{2} \right) - 15 \cos \left(c + \frac{dx}{2} \right) + 5 \cos \left(c + \frac{3dx}{2} \right) + 5 \sin \left(\frac{dx}{2} \right) \right)}{30a^3d \left(\sin \left(\frac{c}{2} \right) + \cos \left(\frac{c}{2} \right) \right) \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A - A*Sin[c + d*x])/(a + a*Sin[c + d*x])^3,x]

[Out] (A*(-15*Cos[c + (d*x)/2] + 5*Cos[c + (3*d*x)/2] + 5*Sin[(d*x)/2] + Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)

fricas [B] time = 0.41, size = 154, normalized size = 2.66

$$\frac{A \cos(dx + c)^3 - 2A \cos(dx + c)^2 + 3A \cos(dx + c) - (A \cos(dx + c)^2 + 3A \cos(dx + c) + 6A) \sin(dx + c)}{15(a^3d \cos(dx + c)^3 + 3a^3d \cos(dx + c)^2 - 2a^3d \cos(dx + c) - 4a^3d + (a^3d \cos(dx + c)^2 - 2a^3d \cos(dx + c) + 6a^3d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/15*(A*cos(d*x + c)^3 - 2*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) - (A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + 6*a^3*d)*sin(d*x + c))

giac [A] time = 0.17, size = 79, normalized size = 1.36

$$\frac{2 \left(15A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^4 + 15A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 25A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 5A \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 4A \right)}{15a^3d \left(\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] -2/15*(15*A*tan(1/2*d*x + 1/2*c)^4 + 15*A*tan(1/2*d*x + 1/2*c)^3 + 25*A*tan(1/2*d*x + 1/2*c)^2 + 5*A*tan(1/2*d*x + 1/2*c) + 4*A)/(a^3*d*(tan(1/2*d*x + 1/2*c) + 1)^5)

maple [A] time = 0.41, size = 86, normalized size = 1.48

$$\frac{2A \left(-\frac{8}{5 \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)^5} + \frac{3}{\left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)^2} - \frac{1}{\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1} + \frac{4}{\left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)^4} - \frac{14}{3 \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)^3} \right)}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)

[Out] 2/d*A/a^3*(-8/5/(tan(1/2*d*x+1/2*c)+1)^5+3/(tan(1/2*d*x+1/2*c)+1)^2-1/(tan(1/2*d*x+1/2*c)+1)+4/(tan(1/2*d*x+1/2*c)+1)^4-14/3/(tan(1/2*d*x+1/2*c)+1)^3)

maxima [B] time = 0.53, size = 387, normalized size = 6.67

$$\frac{2 \left(\frac{A \left(\frac{20 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{30 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 7 \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} - \frac{3A \left(\frac{5 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{5 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} \right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out]
$$-2/15*(A*(20*\sin(d*x + c)/(\cos(d*x + c) + 1) + 40*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 30*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 15*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5) - 3*A*(5*\sin(d*x + c)/(\cos(d*x + c) + 1) + 5*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 5*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 1)/(a^3 + 5*a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5))/d$$

mupad [B] time = 13.31, size = 134, normalized size = 2.31

$$\frac{2A \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \left(4 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 5 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^3 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + 25 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 15 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 15 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 15 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) \sin\left(\frac{c}{2} + \frac{dx}{2}\right)^5\right)}{15 a^3 d \left(\cos\left(\frac{c}{2} + \frac{dx}{2}\right) + \sin\left(\frac{c}{2} + \frac{dx}{2}\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A - A*sin(c + d*x))/(a + a*sin(c + d*x))^3,x)

[Out]
$$-(2*A*\cos(c/2 + (d*x)/2)*(4*\cos(c/2 + (d*x)/2)^4 + 15*\sin(c/2 + (d*x)/2)^4 + 15*\cos(c/2 + (d*x)/2)*\sin(c/2 + (d*x)/2)^3 + 5*\cos(c/2 + (d*x)/2)^3*\sin(c/2 + (d*x)/2) + 25*\cos(c/2 + (d*x)/2)^2*\sin(c/2 + (d*x)/2)^2))/(15*a^3*d*(\cos(c/2 + (d*x)/2) + \sin(c/2 + (d*x)/2))^5)$$

sympy [A] time = 9.11, size = 573, normalized size = 9.88

$$\left\{ \begin{array}{l} \frac{30A \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right)}{15a^3d \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 15a^3d} - \frac{15a^3d \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 15a^3d}{15a^3d \tan^5\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan^4\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^3\left(\frac{c}{2} + \frac{dx}{2}\right) + 150a^3d \tan^2\left(\frac{c}{2} + \frac{dx}{2}\right) + 75a^3d \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 15a^3d} \\ \frac{x(-A \sin(c) + A)}{(a \sin(c) + a)^3} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)

[Out]
$$\text{Piecewise}\left(\left(-30*A*\tan(c/2 + d*x/2)**4/(15*a**3*d*\tan(c/2 + d*x/2)**5 + 75*a**3*d*\tan(c/2 + d*x/2)**4 + 150*a**3*d*\tan(c/2 + d*x/2)**3 + 150*a**3*d*\tan(c/2 + d*x/2)**2 + 75*a**3*d*\tan(c/2 + d*x/2) + 15*a**3*d) - 30*A*\tan(c/2 + d*x/2)**3/(15*a**3*d*\tan(c/2 + d*x/2)**5 + 75*a**3*d*\tan(c/2 + d*x/2)**4 + 150*a**3*d*\tan(c/2 + d*x/2)**3 + 150*a**3*d*\tan(c/2 + d*x/2)**2 + 75*a**3*d*\tan(c/2 + d*x/2) + 15*a**3*d) - 50*A*\tan(c/2 + d*x/2)**2/(15*a**3*d*\tan(c/2 + d*x/2)**5 + 75*a**3*d*\tan(c/2 + d*x/2)**4 + 150*a**3*d*\tan(c/2 + d*x/2)**3 + 150*a**3*d*\tan(c/2 + d*x/2)**2 + 75*a**3*d*\tan(c/2 + d*x/2) + 15*a**3*d) - 10*A*\tan(c/2 + d*x/2)/(15*a**3*d*\tan(c/2 + d*x/2)**5 + 75*a**3*d*\tan(c/2 + d*x/2)**4 + 150*a**3*d*\tan(c/2 + d*x/2)**3 + 150*a**3*d*\tan(c/2 + d*x/2)**2 + 75*a**3*d*\tan(c/2 + d*x/2) + 15*a**3*d) - 8*A/(15*a**3*d*\tan(c/2 + d*x/2)**5 + 75*a**3*d*\tan(c/2 + d*x/2)**4 + 150*a**3*d*\tan(c/2 + d*x/2)**3 + 150*a**3*d*\tan(c/2 + d*x/2)**2 + 75*a**3*d*\tan(c/2 + d*x/2) + 15*a**3*d), \text{Ne}(d, 0)), (x*(-A*\sin(c) + A)/(a*\sin(c) + a)**3, \text{True}))$$

$$3.240 \quad \int \frac{\csc(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=98

$$\frac{8A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} + \frac{3A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} - \frac{A \tanh^{-1}(\cos(c+dx))}{a^3 d}$$

[Out] $-A \operatorname{arctanh}(\cos(dx+c))/a^3/d + 2/5 A \cos(dx+c)/a^3/d/(1+\sin(dx+c))^{3+3/5} + A \cos(dx+c)/a^3/d/(1+\sin(dx+c))^{2+8/5} + A \cos(dx+c)/a^3/d/(1+\sin(dx+c))$

Rubi [A] time = 0.16, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2966, 3770, 2650, 2648}

$$\frac{8A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)} + \frac{3A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3 d(\sin(c+dx)+1)^3} - \frac{A \tanh^{-1}(\cos(c+dx))}{a^3 d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Csc}[c+dx]*(A - A*\text{Sin}[c+dx]))/(a + a*\text{Sin}[c+dx])^3, x]$

[Out] $-(A*\text{ArcTanh}[\text{Cos}[c+dx]])/(a^3*d) + (2*A*\text{Cos}[c+dx])/(5*a^3*d*(1 + \text{Sin}[c+dx])^3) + (3*A*\text{Cos}[c+dx])/(5*a^3*d*(1 + \text{Sin}[c+dx])^2) + (8*A*\text{Cos}[c+dx])/(5*a^3*d*(1 + \text{Sin}[c+dx]))$

Rule 2648

$\text{Int}[(a + (b_*)\sin[(c_*) + (d_*)(x_*)])^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + dx]/(d*(b + a*\text{Sin}[c + dx])), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2650

$\text{Int}[(a + (b_*)\sin[(c_*) + (d_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(b*\text{Cos}[c + dx]*(a + b*\text{Sin}[c + dx])^n)/(a*d*(2*n + 1)), x] + \text{Dist}[(n + 1)/(a*(2*n + 1)), \text{Int}[(a + b*\text{Sin}[c + dx])^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2966

$\text{Int}[\sin[(e_*) + (f_*)(x_*)]^{(n_*)}*((a_*) + (b_*)\sin[(e_*) + (f_*)(x_*)])^{(m_*)}*((A_*) + (B_*)\sin[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^n*(a + b*\sin[e + f*x])^m*(A + B*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, e, f, A, B\}, x] \ \&\& \ \text{EqQ}[A*b + a*B, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n]$

Rule 3770

$\text{Int}[\text{csc}[(c_*) + (d_*)(x_*)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + dx]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)(A-A\sin(c+dx))}{(a+a\sin(c+dx))^3} dx &= \int \left(\frac{A\csc(c+dx)}{a^3} - \frac{2A}{a^3(1+\sin(c+dx))^3} - \frac{A}{a^3(1+\sin(c+dx))^2} - \frac{A}{a^3(1+\sin(c+dx))} \right) dx \\
&= \frac{A \int \csc(c+dx) dx}{a^3} - \frac{A \int \frac{1}{(1+\sin(c+dx))^2} dx}{a^3} - \frac{A \int \frac{1}{1+\sin(c+dx)} dx}{a^3} - \frac{(2A) \int \frac{1}{1+\sin(c+dx)} dx}{a^3} \\
&= -\frac{A \tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1+\sin(c+dx))^3} + \frac{A \cos(c+dx)}{3a^3 d(1+\sin(c+dx))} \\
&= -\frac{A \tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1+\sin(c+dx))^3} + \frac{3A \cos(c+dx)}{5a^3 d(1+\sin(c+dx))} \\
&= -\frac{A \tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{2A \cos(c+dx)}{5a^3 d(1+\sin(c+dx))^3} + \frac{3A \cos(c+dx)}{5a^3 d(1+\sin(c+dx))}
\end{aligned}$$

Mathematica [B] time = 1.01, size = 313, normalized size = 3.19

$$(A - A \sin(c + dx)) \left(2 \sin\left(\frac{dx}{2}\right) (-19 \sin(c + dx) + 4 \cos(2(c + dx)) - 17) + \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Csc[c + d*x]*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(2*Cos[c/2] - 2*Sin[c/2] + 3*Cos[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 3*Sin[c/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 5*Log[Cos[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 5*Log[Sin[(c + d*x)/2]]*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 2*Sin[(d*x)/2]*(-17 + 4*Cos[2*(c + d*x)] - 19*Sin[c + d*x]))*(A - A*Sin[c + d*x]))/(5*a^3*d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)

fricas [B] time = 0.44, size = 310, normalized size = 3.16

$$16 A \cos(dx + c)^3 - 22 A \cos(dx + c)^2 - 42 A \cos(dx + c) - 5 \left(A \cos(dx + c)^3 + 3 A \cos(dx + c)^2 - 2 A \cos(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")

[Out] 1/10*(16*A*cos(d*x + c)^3 - 22*A*cos(d*x + c)^2 - 42*A*cos(d*x + c) - 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) + 5*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + (A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) - 4*A)*log(-1/2*cos(d*x + c) + 1/2) - 2*(8*A*cos(d*x + c)^2 + 19*A*cos(d*x + c) - 2*A)*sin(d*x + c) - 4*A)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))

giac [A] time = 0.19, size = 99, normalized size = 1.01

$$\frac{5A \log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right|\right)}{a^3} + \frac{2\left(20A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^4 + 55A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 75A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 45A \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 13A\right)}{a^3\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5*(5*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 + 2*(20*A*tan(1/2*d*x + 1/2*c)^4 + 55*A*tan(1/2*d*x + 1/2*c)^3 + 75*A*tan(1/2*d*x + 1/2*c)^2 + 45*A*tan(1/2*d*x + 1/2*c) + 13*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.73, size = 130, normalized size = 1.33

$$\frac{A \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^3} + \frac{16A}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5} - \frac{8A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^4} + \frac{12A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} - \frac{12A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{12A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/d*A/a^3*ln(tan(1/2*d*x+1/2*c))+16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+12/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-10/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.59, size = 433, normalized size = 4.42

$$A \left(\frac{2 \left(\frac{115 \sin(dx+c)}{\cos(dx+c)+1} + \frac{185 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{135 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{45 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 32 \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} + \frac{15 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right) + \frac{2A \left(\frac{20 \sin(dx+c)}{\cos(dx+c)+1} + \frac{40 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} \right)}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}}$$

15 d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/15*(A*(2*(115*sin(d*x + c)/(cos(d*x + c) + 1) + 185*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 135*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 45*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 32)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) + 2*A*(20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 30*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 7)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5))/d
```

mupad [B] time = 14.77, size = 199, normalized size = 2.03

$$A \left(5 \ln\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) + 90 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 150 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 110 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 40 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 25 \log\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)*(a + a*sin(c + d*x))^3),x)
```

```
[Out] (A*(5*log(tan(c/2 + (d*x)/2)) + 90*tan(c/2 + (d*x)/2) + 150*tan(c/2 + (d*x)/2)^2 + 110*tan(c/2 + (d*x)/2)^3 + 40*tan(c/2 + (d*x)/2)^4 + 25*log(tan(c/2 + (d*x)/2)))/d
```

+ (d*x)/2))*tan(c/2 + (d*x)/2) + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^2 + 50*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 25*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 5*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^5 + 26))/(5*a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{A \left(\int \left(-\frac{\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} \right) dx + \int \frac{\sin(c+dx)\csc(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)

[Out] -A*(Integral(-csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3

$$3.241 \quad \int \frac{\csc^2(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=113

$$-\frac{A \cot(c+dx)}{a^3 d} + \frac{4A \tanh^{-1}(\cos(c+dx))}{a^3 d} - \frac{104A \cot(c+dx)}{15a^3 d(\csc(c+dx)+1)} + \frac{31A \cot(c+dx)}{15a^3 d(\csc(c+dx)+1)^2} - \frac{2A \cot(c+dx)}{5a^3 d(\csc(c+dx)+1)^3}$$

[Out] 4*A*arctanh(cos(d*x+c))/a^3/d-94/15*A*cot(d*x+c)/a^3/d+2/5*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^3+13/15*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))^2+4*A*cot(d*x+c)/a^3/d/(1+sin(d*x+c))

Rubi [A] time = 0.40, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2950, 2709, 3770, 3767, 8, 3777, 3922, 3919, 3794}

$$-\frac{A \cot(c+dx)}{a^3 d} + \frac{4A \tanh^{-1}(\cos(c+dx))}{a^3 d} - \frac{104A \cot(c+dx)}{15a^3 d(\csc(c+dx)+1)} + \frac{31A \cot(c+dx)}{15a^3 d(\csc(c+dx)+1)^2} - \frac{2A \cot(c+dx)}{5a^3 d(\csc(c+dx)+1)^3}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (4*A*ArcTanh[Cos[c + d*x]])/(a^3*d) - (A*Cot[c + d*x])/(a^3*d) - (2*A*Cot[c + d*x])/(5*a^3*d*(1 + Csc[c + d*x])^3) + (31*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x])^2) - (104*A*Cot[c + d*x])/(15*a^3*d*(1 + Csc[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2709

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(p_), x_Symbol] := Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m - p/2, 0])

Rule 2950

Int[sin[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[a^n*c^n, Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n, 0] && IntegerQ[n]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3777

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(Cot[c
+ d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)),
Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rule 3794

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] := -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc^2(c + dx)(A - A \sin(c + dx))}{(a + a \sin(c + dx))^3} dx &= (aA) \int \frac{\cot^2(c + dx)}{(a + a \sin(c + dx))^4} dx \\ &= \frac{A \int \left(\frac{9}{a^2} - \frac{4 \csc(c + dx)}{a^2} + \frac{\csc^2(c + dx)}{a^2} - \frac{2}{a^2(1 + \csc(c + dx))^3} + \frac{9}{a^2(1 + \csc(c + dx))^2} - \frac{1}{a^2(1 + \csc(c + dx))} \right) dx}{a} \\ &= \frac{9Ax}{a^3} + \frac{A \int \csc^2(c + dx) dx}{a^3} - \frac{(2A) \int \frac{1}{(1 + \csc(c + dx))^3} dx}{a^3} - \frac{(4A) \int \csc(c + dx) dx}{a^3} \\ &= \frac{9Ax}{a^3} + \frac{4A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{2A \cot(c + dx)}{5a^3 d(1 + \csc(c + dx))^3} + \frac{3A \cot(c + dx)}{a^3 d(1 + \csc(c + dx))} \\ &= \frac{2Ax}{a^3} + \frac{4A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{A \cot(c + dx)}{a^3 d} - \frac{2A \cot(c + dx)}{5a^3 d(1 + \csc(c + dx))} \\ &= \frac{4A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{A \cot(c + dx)}{a^3 d} - \frac{2A \cot(c + dx)}{5a^3 d(1 + \csc(c + dx))^3} + \frac{3A \cot(c + dx)}{a^3 d} \\ &= \frac{4A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{A \cot(c + dx)}{a^3 d} - \frac{2A \cot(c + dx)}{5a^3 d(1 + \csc(c + dx))^3} + \frac{3A \cot(c + dx)}{a^3 d} \end{aligned}$$

Mathematica [A] time = 2.97, size = 167, normalized size = 1.48

$$A \left(-15 \tan\left(\frac{1}{2}(c + dx)\right) + 15 \cot\left(\frac{1}{2}(c + dx)\right) + 120 \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 120 \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) + \frac{2 \sin\left(\frac{1}{2}(c + dx)\right)}{30a^3 d} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^2*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] -1/30*(A*(15*Cot[(c + d*x)/2] - 120*Log[Cos[(c + d*x)/2]] + 120*Log[Sin[(c + d*x)/2]] + 12/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 + 38/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-287 + 79*Cos[2*(c + d*x)] - 354*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 15*Tan[(c + d*x)/2]))/(a^3*d)
```

fricas [B] time = 0.45, size = 406, normalized size = 3.59

$$94 A \cos(dx + c)^4 + 222 A \cos(dx + c)^3 - 115 A \cos(dx + c)^2 - 237 A \cos(dx + c) + 30 \left(A \cos(dx + c)^4 - 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15*(94*A*cos(d*x + c)^4 + 222*A*cos(d*x + c)^3 - 115*A*cos(d*x + c)^2 - 237*A*cos(d*x + c) + 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(1/2*cos(d*x + c) + 1/2) - 30*(A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) - (A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - 4*A)*sin(d*x + c) + 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (94*A*cos(d*x + c)^3 - 128*A*cos(d*x + c)^2 - 243*A*cos(d*x + c) - 6*A)*sin(d*x + c) + 6*A)/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d - (a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.22, size = 146, normalized size = 1.29

$$\frac{120 A \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} - \frac{15 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^3} - \frac{15 \left(8 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - A\right)}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)} + \frac{4 \left(135 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 435 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 605 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 385 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 104 A\right)}{a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5} + \frac{16 A}{5 d a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5} + \frac{8 A}{d a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/30*(120*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*A*tan(1/2*d*x + 1/2*c)/a^3 - 15*(8*A*tan(1/2*d*x + 1/2*c) - A)/(a^3*tan(1/2*d*x + 1/2*c)) + 4*(135*A*tan(1/2*d*x + 1/2*c)^4 + 435*A*tan(1/2*d*x + 1/2*c)^3 + 605*A*tan(1/2*d*x + 1/2*c)^2 + 385*A*tan(1/2*d*x + 1/2*c) + 104*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

maple [A] time = 0.73, size = 169, normalized size = 1.50

$$\frac{A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{2a^3 d} - \frac{A}{2a^3 d \tan\left(\frac{dx}{2} + \frac{c}{2}\right)} - \frac{4A \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^3} - \frac{16A}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5} + \frac{8A}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

[Out] $1/2*A/a^3/d*\tan(1/2*d*x+1/2*c)-1/2*A/a^3/d/\tan(1/2*d*x+1/2*c)-4/d*A/a^3*\ln(\tan(1/2*d*x+1/2*c))-16/5/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^4-44/3/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^3+14/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)^2-18/d*A/a^3/(\tan(1/2*d*x+1/2*c)+1)$

maxima [B] time = 0.69, size = 519, normalized size = 4.59

$$3A \left(\frac{\frac{121 \sin(dx+c)}{\cos(dx+c)+1} + \frac{410 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{610 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{425 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{125 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 5}{\frac{a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{10a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{5a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{5 \sin(dx+c)}{a^3(\cos(dx+c)+1)} \right) + 2$$

30 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

[Out] $-1/30*(3*A*((121*\sin(d*x + c))/(\cos(d*x + c) + 1) + 410*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 610*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 425*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 125*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + 5)/(a^3*\sin(d*x + c)/(\cos(d*x + c) + 1) + 5*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 10*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 5*a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5 + a^3*\sin(d*x + c)^6/(\cos(d*x + c) + 1)^6) + 30*\log(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^3 - 5*\sin(d*x + c)/(a^3*(\cos(d*x + c) + 1))) + 2*A*(2*(115*\sin(d*x + c)/(\cos(d*x + c) + 1) + 185*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 135*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 45*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + 32)/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^2/(\cos(d*x + c) + 1)^2 + 10*a^3*\sin(d*x + c)^3/(\cos(d*x + c) + 1)^3 + 5*a^3*\sin(d*x + c)^4/(\cos(d*x + c) + 1)^4 + a^3*\sin(d*x + c)^5/(\cos(d*x + c) + 1)^5) + 15*\log(\sin(d*x + c)/(\cos(d*x + c) + 1))/a^3)/d$

mupad [B] time = 15.77, size = 210, normalized size = 1.86

$$\frac{A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{2a^3 d} - \frac{4A \ln\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{a^3 d} - \frac{37A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 121A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + \frac{514A \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{3}}{d \left(2a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6 + 10a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + 20a^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 20 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^2*(a + a*sin(c + d*x))^3),x)

[Out] $(A*\tan(c/2 + (d*x)/2))/(2*a^3*d) - (4*A*\log(\tan(c/2 + (d*x)/2)))/(a^3*d) - (A + (491*A*\tan(c/2 + (d*x)/2))/15 + (338*A*\tan(c/2 + (d*x)/2)^2)/3 + (514*A*\tan(c/2 + (d*x)/2)^3)/3 + 121*A*\tan(c/2 + (d*x)/2)^4 + 37*A*\tan(c/2 + (d*x)/2)^5)/(d*(10*a^3*\tan(c/2 + (d*x)/2)^2 + 20*a^3*\tan(c/2 + (d*x)/2)^3 + 20*a^3*\tan(c/2 + (d*x)/2)^4 + 10*a^3*\tan(c/2 + (d*x)/2)^5 + 2*a^3*\tan(c/2 + (d*x)/2)^6 + 2*a^3*\tan(c/2 + (d*x)/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{A \left(\int \left(-\frac{\csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} \right) dx + \int \frac{\sin(c+dx) \csc^2(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**2*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)

[Out] $-A*(Integral(-\csc(c + d*x)**2/(\sin(c + d*x)**3 + 3*\sin(c + d*x)**2 + 3*\sin(c + d*x) + 1), x) + Integral(\sin(c + d*x)*\csc(c + d*x)**2/(\sin(c + d*x)**3 + 3*\sin(c + d*x)**2 + 3*\sin(c + d*x) + 1), x))/a**3$

$$3.242 \quad \int \frac{\csc^3(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=138

$$\frac{4A \cot(c+dx)}{a^3d} + \frac{164A \cos(c+dx)}{15a^3d(\sin(c+dx)+1)} + \frac{29A \cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{19A \tanh^{-1}(\cos(c+dx))}{2a^3d}$$

[Out] $-19/2*A*\operatorname{arctanh}(\cos(d*x+c))/a^3/d+4*A*\cot(d*x+c)/a^3/d-1/2*A*\cot(d*x+c)*\csc(d*x+c)/a^3/d+2/5*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^3+29/15*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))^2+164/15*A*\cos(d*x+c)/a^3/d/(1+\sin(d*x+c))$

Rubi [A] time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{4A \cot(c+dx)}{a^3d} + \frac{164A \cos(c+dx)}{15a^3d(\sin(c+dx)+1)} + \frac{29A \cos(c+dx)}{15a^3d(\sin(c+dx)+1)^2} + \frac{2A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} - \frac{19A \tanh^{-1}(\cos(c+dx))}{2a^3d}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] $(-19*A*\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]])/(2*a^3*d) + (4*A*\operatorname{Cot}[c + d*x])/(a^3*d) - (A*\operatorname{Cot}[c + d*x]*\operatorname{Csc}[c + d*x])/(2*a^3*d) + (2*A*\operatorname{Cos}[c + d*x])/(5*a^3*d*(1 + \operatorname{Sin}[c + d*x])^3) + (29*A*\operatorname{Cos}[c + d*x])/(15*a^3*d*(1 + \operatorname{Sin}[c + d*x])^2) + (164*A*\operatorname{Cos}[c + d*x])/(15*a^3*d*(1 + \operatorname{Sin}[c + d*x]))$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc^3(c + dx)(A - A \sin(c + dx))}{(a + a \sin(c + dx))^3} dx &= \int \left(\frac{9A \csc(c + dx)}{a^3} - \frac{4A \csc^2(c + dx)}{a^3} + \frac{A \csc^3(c + dx)}{a^3} - \frac{2A}{a^3(1 + \sin(c + dx))} \right) dx \\ &= \frac{A \int \csc^3(c + dx) dx}{a^3} - \frac{(2A) \int \frac{1}{(1 + \sin(c + dx))^3} dx}{a^3} - \frac{(4A) \int \csc^2(c + dx) dx}{a^3} \\ &= -\frac{9A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{A \cot(c + dx) \csc(c + dx)}{2a^3 d} + \frac{2A \cos(c + dx)}{5a^3 d(1 + \sin(c + dx))} \\ &= -\frac{19A \tanh^{-1}(\cos(c + dx))}{2a^3 d} + \frac{4A \cot(c + dx)}{a^3 d} - \frac{A \cot(c + dx) \csc(c + dx)}{2a^3 d} \\ &= -\frac{19A \tanh^{-1}(\cos(c + dx))}{2a^3 d} + \frac{4A \cot(c + dx)}{a^3 d} - \frac{A \cot(c + dx) \csc(c + dx)}{2a^3 d} \end{aligned}$$

Mathematica [A] time = 3.89, size = 245, normalized size = 1.78

$$A \left(-240 \tan\left(\frac{1}{2}(c + dx)\right) + 240 \cot\left(\frac{1}{2}(c + dx)\right) - 15 \csc^2\left(\frac{1}{2}(c + dx)\right) + 15 \sec^2\left(\frac{1}{2}(c + dx)\right) + 1140 \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x]^3*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
[Out] (A*(240*Cot[(c + d*x)/2] - 15*Csc[(c + d*x)/2]^2 - 1140*Log[Cos[(c + d*x)/2]]
) + 1140*Log[Sin[(c + d*x)/2]] + 15*Sec[(c + d*x)/2]^2 - (96*Sin[(c + d*x)
/2]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 48/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^4 - (464*Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^3 + 232/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - (2624*Sin[(c + d*x)/2]
)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) - 240*Tan[(c + d*x)/2]))/(120*a^3*d
)
```

fricas [B] time = 0.45, size = 498, normalized size = 3.61

$$896 A \cos(dx + c)^5 - 1222 A \cos(dx + c)^4 - 3218 A \cos(dx + c)^3 + 1168 A \cos(dx + c)^2 + 2292 A \cos(dx + c) - 285 A^2 \cos(dx + c)^5 + 3 A^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/60*(896*A*cos(d*x + c)^5 - 1222*A*cos(d*x + c)^4 - 3218*A*cos(d*x + c)^3
+ 1168*A*cos(d*x + c)^2 + 2292*A*cos(d*x + c) - 285*(A*cos(d*x + c))^5 + 3*A
```


*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A*log(1/2*cos(d*x + c) + 1/2) + 285*(A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + (A*cos(d*x + c)^4 - 2*A*cos(d*x + c)^3 - 5*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c) + 4*A*log(-1/2*cos(d*x + c) + 1/2) - 2*(448*A*cos(d*x + c)^4 + 1059*A*cos(d*x + c)^3 - 550*A*cos(d*x + c)^2 - 1134*A*cos(d*x + c) + 12*A)*sin(d*x + c) + 24*A)/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d + (a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^3 - 5*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*x + c) + 4*a^3*d)*sin(d*x + c))

giac [A] time = 0.30, size = 180, normalized size = 1.30

$$\frac{1140 A \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} - \frac{15 \left(114 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 16 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + A\right)}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2} + \frac{15 \left(A a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 - 16 A a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a^6} + \frac{16 \left(240 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 825 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 1165 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 755 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 199 A\right)}{a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/120*(1140*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 15*(114*A*tan(1/2*d*x + 1/2*c)^2 - 16*A*tan(1/2*d*x + 1/2*c) + A)/(a^3*tan(1/2*d*x + 1/2*c)^2) + 15*(A*a^3*tan(1/2*d*x + 1/2*c)^2 - 16*A*a^3*tan(1/2*d*x + 1/2*c))/a^6 + 16*(240*A*tan(1/2*d*x + 1/2*c)^4 + 825*A*tan(1/2*d*x + 1/2*c)^3 + 1165*A*tan(1/2*d*x + 1/2*c)^2 + 755*A*tan(1/2*d*x + 1/2*c) + 199*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5))/d

maple [A] time = 0.87, size = 209, normalized size = 1.51

$$\frac{A \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{8d a^3} - \frac{2A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{a^3 d} - \frac{A}{8d a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)^2} + \frac{2A}{a^3 d \tan\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{19A \ln\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d a^3} + \frac{16A \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)

[Out] 1/8/d*A/a^3*tan(1/2*d*x+1/2*c)^2-2*A/a^3/d*tan(1/2*d*x+1/2*c)-1/8/d*A/a^3/tan(1/2*d*x+1/2*c)^2+2*A/a^3/d/tan(1/2*d*x+1/2*c)+19/2/d*A/a^3*ln(tan(1/2*d*x+1/2*c))+16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5-8/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^4+52/3/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3-18/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^2+32/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)

maxima [B] time = 0.41, size = 622, normalized size = 4.51

$$12 A \left(\frac{\frac{121 \sin(dx+c)}{\cos(dx+c)+1} + \frac{410 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{610 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{425 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{125 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + 5}{\frac{a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{5 a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10 a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{10 a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{5 a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{30 \log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} - \frac{5 \sin(dx+c)}{a^3 (\cos(dx+c)+1)} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")

```
[Out] 1/120*(12*A*((121*sin(d*x + c)/(cos(d*x + c) + 1) + 410*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 610*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 425*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 125*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5)/(a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 5*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 30*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3 - 5*sin(d*x + c)/(a^3*(cos(d*x + c) + 1))) + A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 15.74, size = 288, normalized size = 2.09

$$A \left(165 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 4234 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 14090 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 19780 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 12060 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^3*(a + a*sin(c + d*x))^3), x)
```

```
[Out] (A*(165*tan(c/2 + (d*x)/2) + 4234*tan(c/2 + (d*x)/2)^2 + 14090*tan(c/2 + (d*x)/2)^3 + 19780*tan(c/2 + (d*x)/2)^4 + 12060*tan(c/2 + (d*x)/2)^5 + 1830*tan(c/2 + (d*x)/2)^6 - 1050*tan(c/2 + (d*x)/2)^7 - 165*tan(c/2 + (d*x)/2)^8 + 15*tan(c/2 + (d*x)/2)^9 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^2 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 11400*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 11400*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^5 + 5700*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 1140*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 - 15))/(120*a^3*d*tan(c/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{A \left(\int \left(-\frac{\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} \right) dx + \int \frac{\sin(c+dx)\csc^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3,x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

$$3.243 \quad \int \frac{\csc^4(c+dx)(A-A \sin(c+dx))}{(a+a \sin(c+dx))^3} dx$$

Optimal. Leaf size=153

$$\frac{A \cot^3(c+dx)}{3a^3d} - \frac{10A \cot(c+dx)}{a^3d} - \frac{93A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)} - \frac{13A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} + 1$$

[Out] 18*A*arctanh(cos(d*x+c))/a^3/d-10*A*cot(d*x+c)/a^3/d-1/3*A*cot(d*x+c)^3/a^3/d+2*A*cot(d*x+c)*csc(d*x+c)/a^3/d-2/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^3-13/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))^2-93/5*A*cos(d*x+c)/a^3/d/(1+sin(d*x+c))

Rubi [A] time = 0.25, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2966, 3770, 3767, 8, 3768, 2650, 2648}

$$\frac{A \cot^3(c+dx)}{3a^3d} - \frac{10A \cot(c+dx)}{a^3d} - \frac{93A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)} - \frac{13A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^2} - \frac{2A \cos(c+dx)}{5a^3d(\sin(c+dx)+1)^3} + 1$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]

[Out] (18*A*ArcTanh[Cos[c + d*x]]/(a^3*d) - (10*A*Cot[c + d*x])/(a^3*d) - (A*Cot[c + d*x]^3)/(3*a^3*d) + (2*A*Cot[c + d*x]*Csc[c + d*x])/(a^3*d) - (2*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^3) - (13*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])^2) - (93*A*Cos[c + d*x])/(5*a^3*d*(1 + Sin[c + d*x])))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2648

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2966

Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && IntegerQ[n]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(c + dx)(A - A \sin(c + dx))}{(a + a \sin(c + dx))^3} dx &= \int \left(-\frac{16A \csc(c + dx)}{a^3} + \frac{9A \csc^2(c + dx)}{a^3} - \frac{4A \csc^3(c + dx)}{a^3} + \frac{A \csc^4(c + dx)}{a^3} \right) dx \\ &= \frac{A \int \csc^4(c + dx) dx}{a^3} + \frac{(2A) \int \frac{1}{(1 + \sin(c + dx))^3} dx}{a^3} - \frac{(4A) \int \csc^3(c + dx) dx}{a^3} \\ &= \frac{16A \tanh^{-1}(\cos(c + dx))}{a^3 d} + \frac{2A \cot(c + dx) \csc(c + dx)}{a^3 d} - \frac{2A \cos(c + dx)}{5a^3 d (1 + \sin(c + dx))} \\ &= \frac{18A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{10A \cot(c + dx)}{a^3 d} - \frac{A \cot^3(c + dx)}{3a^3 d} + \frac{2A \cot(c + dx)}{5a^3 d} \\ &= \frac{18A \tanh^{-1}(\cos(c + dx))}{a^3 d} - \frac{10A \cot(c + dx)}{a^3 d} - \frac{A \cot^3(c + dx)}{3a^3 d} + \frac{2A \cot(c + dx)}{5a^3 d} \end{aligned}$$

Mathematica [B] time = 6.23, size = 348, normalized size = 2.27

$$A \left(\frac{29 \tan\left(\frac{1}{2}(c + dx)\right)}{6d} - \frac{29 \cot\left(\frac{1}{2}(c + dx)\right)}{6d} + \frac{\csc^2\left(\frac{1}{2}(c + dx)\right)}{2d} - \frac{\sec^2\left(\frac{1}{2}(c + dx)\right)}{2d} - \frac{18 \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{18 \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right)}{d} + \frac{18 \sin\left(\frac{1}{2}(c + dx)\right)}{5d \left(\sin\left(\frac{1}{2}(c + dx)\right)\right)^2} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Csc[c + d*x]^4*(A - A*Sin[c + d*x]))/(a + a*Sin[c + d*x])^3,x]
```

```
[Out] (A*((-29*Cot[(c + d*x)/2])/(6*d) + Csc[(c + d*x)/2]^2/(2*d) - (Cot[(c + d*x)
)/2]*Csc[(c + d*x)/2]^2)/(24*d) + (18*Log[Cos[(c + d*x)/2]])/d - (18*Log[Si
n[(c + d*x)/2]])/d - Sec[(c + d*x)/2]^2/(2*d) + (4*Sin[(c + d*x)/2])/(5*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5) - 2/(5*d*(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^4) + (26*Sin[(c + d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 13/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (186*Sin[(c
+ d*x)/2])/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (29*Tan[(c + d*x)/
2])/(6*d) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d))/a^3
```

fricas [B] time = 0.46, size = 594, normalized size = 3.88

$$424 A \cos(dx + c)^6 + 1002 A \cos(dx + c)^5 - 944 A \cos(dx + c)^4 - 2074 A \cos(dx + c)^3 + 531 A \cos(dx + c)^2 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*(424*A*cos(d*x + c)^6 + 1002*A*cos(d*x + c)^5 - 944*A*cos(d*x + c)^4 -
2074*A*cos(d*x + c)^3 + 531*A*cos(d*x + c)^2 + 1077*A*cos(d*x + c) + 135*(
A*cos(d*x + c)^6 - 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x +
c)^3 + 9*A*cos(d*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(
d*x + c)^4 - 3*A*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4
*A)*sin(d*x + c) - 4*A)*log(1/2*cos(d*x + c) + 1/2) - 135*(A*cos(d*x + c)^6
- 2*A*cos(d*x + c)^5 - 6*A*cos(d*x + c)^4 + 4*A*cos(d*x + c)^3 + 9*A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) - (A*cos(d*x + c)^5 + 3*A*cos(d*x + c)^4 - 3*A
*cos(d*x + c)^3 - 7*A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + 4*A)*sin(d*x + c)
- 4*A)*log(-1/2*cos(d*x + c) + 1/2) + (424*A*cos(d*x + c)^5 - 578*A*cos(d*
x + c)^4 - 1522*A*cos(d*x + c)^3 + 552*A*cos(d*x + c)^2 + 1083*A*cos(d*x +
c) + 6*A)*sin(d*x + c) - 6*A)/(a^3*d*cos(d*x + c)^6 - 2*a^3*d*cos(d*x + c)^
5 - 6*a^3*d*cos(d*x + c)^4 + 4*a^3*d*cos(d*x + c)^3 + 9*a^3*d*cos(d*x + c)^
2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d - (a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*
x + c)^4 - 3*a^3*d*cos(d*x + c)^3 - 7*a^3*d*cos(d*x + c)^2 + 2*a^3*d*cos(d*
x + c) + 4*a^3*d)*sin(d*x + c))
```

giac [A] time = 0.30, size = 213, normalized size = 1.39

$$\frac{2160 A \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right)}{a^3} - \frac{5\left(792 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 117 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 12 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - A\right)}{a^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3} + \frac{48\left(125 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 445 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 635 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 415 A \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 108 A\right)}{a^3 \left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5} - \frac{5\left(A a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 - 12 A a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 117 A a^6 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{a^9 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/120*(2160*A*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 5*(792*A*tan(1/2*d*x +
1/2*c)^3 - 117*A*tan(1/2*d*x + 1/2*c)^2 + 12*A*tan(1/2*d*x + 1/2*c) - A)/(a
^3*tan(1/2*d*x + 1/2*c)^3) + 48*(125*A*tan(1/2*d*x + 1/2*c)^4 + 445*A*tan(1
/2*d*x + 1/2*c)^3 + 635*A*tan(1/2*d*x + 1/2*c)^2 + 415*A*tan(1/2*d*x + 1/2*
c) + 108*A)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5) - 5*(A*a^6*tan(1/2*d*x + 1/2
*c)^3 - 12*A*a^6*tan(1/2*d*x + 1/2*c)^2 + 117*A*a^6*tan(1/2*d*x + 1/2*c))/a
^9)/d
```

maple [A] time = 0.83, size = 249, normalized size = 1.63

$$\frac{A \left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{24d a^3} - \frac{A \left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{2d a^3} + \frac{39A \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{8a^3 d} - \frac{A}{24d a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)^3} + \frac{A}{2d a^3 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)^2} - \frac{A}{8a^3 d \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x)
```

```
[Out] 1/24/d*A/a^3*tan(1/2*d*x+1/2*c)^3-1/2/d*A/a^3*tan(1/2*d*x+1/2*c)^2+39/8*A/a
^3/d*tan(1/2*d*x+1/2*c)-1/24/d*A/a^3/tan(1/2*d*x+1/2*c)^3+1/2/d*A/a^3/tan(1
/2*d*x+1/2*c)^2-39/8*A/a^3/d/tan(1/2*d*x+1/2*c)-18/d*A/a^3*ln(tan(1/2*d*x+1
/2*c))-16/5/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^5+8/d*A/a^3/(tan(1/2*d*x+1/2*c)
+1)^4-20/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)^3+22/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
^2-50/d*A/a^3/(tan(1/2*d*x+1/2*c)+1)
```

maxima [B] time = 0.46, size = 706, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/120*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 2782*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 9410*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 13645*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9285*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2580*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 15)/(a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7) - 15*(12*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/a^3 + 780*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - A*((20*sin(d*x + c)/(cos(d*x + c) + 1) - 230*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4777*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 15785*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 22390*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 14940*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 4005*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 5)/(a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 10*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5*a^3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 5*(81*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^3 - 1380*log(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

mupad [B] time = 15.28, size = 314, normalized size = 2.05

$$A \left(335 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 35 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 7559 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + 24610 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 33170 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^5 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A*sin(c + d*x))/(sin(c + d*x)^4*(a + a*sin(c + d*x))^3), x)
```

```
[Out] -(A*(335*tan(c/2 + (d*x)/2)^2 - 35*tan(c/2 + (d*x)/2) + 7559*tan(c/2 + (d*x)/2)^3 + 24610*tan(c/2 + (d*x)/2)^4 + 33170*tan(c/2 + (d*x)/2)^5 + 18670*tan(c/2 + (d*x)/2)^6 + 1310*tan(c/2 + (d*x)/2)^7 - 2375*tan(c/2 + (d*x)/2)^8 - 335*tan(c/2 + (d*x)/2)^9 + 35*tan(c/2 + (d*x)/2)^10 - 5*tan(c/2 + (d*x)/2)^11 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^3 + 10800*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^4 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^5 + 21600*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^6 + 10800*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^7 + 2160*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^8 + 5))/(120*a^3*d*tan(c/2 + (d*x)/2)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{A \left(\int \left(-\frac{\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} \right) dx + \int \frac{\sin(c+dx)\csc^4(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1} dx \right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(A-A*sin(d*x+c))/(a+a*sin(d*x+c))**3, x)
```

```
[Out] -A*(Integral(-csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x) + Integral(sin(c + d*x)*csc(c + d*x)**4/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x) + 1), x))/a**3
```

$$3.244 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$$

Optimal. Leaf size=327

$$\frac{a(5Ad(6c^2 + 20cd + 9d^2) - B(6c^3 - 30c^2d - 71cd^2 - 45d^3)) \sin(e + fx) \cos(e + fx)}{120f} + \frac{1}{8}ax(A(8c^3 + 12c^2d$$

[Out] 1/8*a*(B*(4*c^3+12*c^2*d+9*c*d^2+3*d^3)+A*(8*c^3+12*c^2*d+12*c*d^2+3*d^3))*x-1/30*a*(5*A*d*(3*c^3+16*c^2*d+12*c*d^2+4*d^3)-B*(3*c^4-15*c^3*d-52*c^2*d^2-60*c*d^3-16*d^4))*cos(f*x+e)/d/f-1/120*a*(5*A*d*(6*c^2+20*c*d+9*d^2)-B*(6*c^3-30*c^2*d-71*c*d^2-45*d^3))*cos(f*x+e)*sin(f*x+e)/f-1/60*a*(4*(5*A+4*B)*d^2-3*c*(B*c-5*(A+B)*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d/f+1/20*a*(B*c-5*(A+B)*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d/f-1/5*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4/d/f

Rubi [A] time = 0.58, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3023, 2753, 2734}

$$\frac{a(5Ad(16c^2d + 3c^3 + 12cd^2 + 4d^3) - B(-52c^2d^2 - 15c^3d + 3c^4 - 60cd^3 - 16d^4)) \cos(e + fx)}{30df} - \frac{a(5Ad(6c^2$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]

[Out] (a*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c*d^2 + 3*d^3))*x)/8 - (a*(5*A*d*(3*c^3 + 16*c^2*d + 12*c*d^2 + 4*d^3) - B*(3*c^4 - 15*c^3*d - 52*c^2*d^2 - 60*c*d^3 - 16*d^4))*Cos[e + f*x])/(30*d*f) - (a*(5*A*d*(6*c^2 + 20*c*d + 9*d^2) - B*(6*c^3 - 30*c^2*d - 71*c*d^2 - 45*d^3))*Cos[e + f*x]*Sin[e + f*x])/(120*f) - (a*(4*(5*A + 4*B)*d^2 - 3*c*(B*c - 5*(A + B)*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d*f) + (a*(B*c - 5*(A + B)*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d*f) - (a*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(5*d*f)

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx &= \int (c + d \sin(e + fx))^3 (aA + (aA + aB) \sin(e + fx) \\
&= -\frac{aB \cos(e + fx)(c + d \sin(e + fx))^4}{5df} + \frac{\int (c + d \sin(e + fx))^3 (aA + (aA + aB) \sin(e + fx)) dx}{5df} \\
&= \frac{a(Bc - 5(A + B)d) \cos(e + fx)(c + d \sin(e + fx))^4}{20df} \\
&= -\frac{a(4(5A + 4B)d^2 - 3c(Bc - 5(A + B)d)) \cos(e + fx)(c + d \sin(e + fx))^3}{60df} \\
&= \frac{1}{8}a(B(4c^3 + 12c^2d + 9cd^2 + 3d^3) + A(8c^3 + 12c^2d + 9cd^2 + 3d^3)) \cos(3(e + fx)) + 15(-8(Ad(3c^2 + 3cd + d^2) + B(12c^2 + 12cd + 5d^2))) \cos(3(e + fx)) + 15(-8(Ad(3c^2 + 3cd + d^2) + B(12c^2 + 12cd + 5d^2))) \cos(3(e + fx))
\end{aligned}$$

Mathematica [A] time = 2.00, size = 267, normalized size = 0.82

$$\frac{a(\sin(e + fx) + 1)(10d(4Ad(3c + d) + B(12c^2 + 12cd + 5d^2)) \cos(3(e + fx)) + 15(-8(Ad(3c^2 + 3cd + d^2) + B(12c^2 + 12cd + 5d^2))) \cos(3(e + fx)) + 15(-8(Ad(3c^2 + 3cd + d^2) + B(12c^2 + 12cd + 5d^2))) \cos(3(e + fx))}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,
x]
```

```
[Out] (a*(1 + Sin[e + f*x])*(-60*(2*A*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + B*(8
*c^3 + 18*c^2*d + 18*c*d^2 + 5*d^3))*Cos[e + f*x] + 10*d*(4*A*d*(3*c + d) +
B*(12*c^2 + 12*c*d + 5*d^2))*Cos[3*(e + f*x)] - 6*B*d^3*Cos[5*(e + f*x)] +
15*(4*(B*(4*c^3 + 12*c^2*d + 9*c*d^2 + 3*d^3) + A*(8*c^3 + 12*c^2*d + 12*c
*d^2 + 3*d^3))*f*x - 8*(B*(c + d)^3 + A*d*(3*c^2 + 3*c*d + d^2))*Sin[2*(e +
f*x)] + d^2*(A*d + B*(3*c + d))*Sin[4*(e + f*x)]))/(480*f*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.47, size = 239, normalized size = 0.73

$$\frac{24Bad^3 \cos(fx + e)^5 - 40(3Bac^2d + 3(A + B)acd^2 + (A + 2B)ad^3) \cos(fx + e)^3 - 15(4(2A + B)ac^3 + 12(A + B)acd^2 + 3Bac^2d) \cos(fx + e) - 15(2(3Bac^2d + 3(A + B)acd^2 + (A + 2B)ad^3)) \cos(fx + e)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] -1/120*(24*B*a*d^3*cos(f*x + e)^5 - 40*(3*B*a*c^2*d + 3*(A + B)*a*c*d^2 + (
A + 2*B)*a*d^3)*cos(f*x + e)^3 - 15*(4*(2*A + B)*a*c^3 + 12*(A + B)*a*c^2*d
+ 3*(4*A + 3*B)*a*c*d^2 + 3*(A + B)*a*d^3)*f*x + 120*((A + B)*a*c^3 + 3*(A
+ B)*a*c^2*d + 3*(A + B)*a*c*d^2 + (A + B)*a*d^3)*cos(f*x + e) - 15*(2*(3*
```


$$B*a*c*d^2 + (A + B)*a*d^3)*\cos(f*x + e)^3 - (4*B*a*c^3 + 12*(A + B)*a*c^2*d + 3*(4*A + 5*B)*a*c*d^2 + 5*(A + B)*a*d^3)*\cos(f*x + e))*\sin(f*x + e))/f$$

giac [A] time = 0.18, size = 314, normalized size = 0.96

$$-\frac{Bad^3 \cos(5fx + 5e)}{80f} + \frac{1}{8} (8Aac^3 + 4Bac^3 + 12Aac^2d + 12Bac^2d + 12Aacd^2 + 9Bacd^2 + 3Aad^3 + 3Bad^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] -1/80*B*a*d^3*cos(5*f*x + 5*e)/f + 1/8*(8*A*a*c^3 + 4*B*a*c^3 + 12*A*a*c^2*d + 12*B*a*c^2*d + 12*A*a*c*d^2 + 9*B*a*c*d^2 + 3*A*a*d^3 + 3*B*a*d^3)*x + 1/48*(12*B*a*c^2*d + 12*A*a*c*d^2 + 12*B*a*c*d^2 + 4*A*a*d^3 + 5*B*a*d^3)*cos(3*f*x + 3*e)/f - 1/8*(8*A*a*c^3 + 8*B*a*c^3 + 24*A*a*c^2*d + 18*B*a*c^2*d + 18*A*a*c*d^2 + 18*B*a*c*d^2 + 6*A*a*d^3 + 5*B*a*d^3)*cos(f*x + e)/f + 1/32*(3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(4*f*x + 4*e)/f - 1/4*(B*a*c^3 + 3*A*a*c^2*d + 3*B*a*c^2*d + 3*A*a*c*d^2 + 3*B*a*c*d^2 + A*a*d^3 + B*a*d^3)*sin(2*f*x + 2*e)/f

maple [A] time = 0.52, size = 422, normalized size = 1.29

$$-Ac^3a \cos(fx + e) + 3Ac^2da \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - Acd^2a (2 + \sin^2(fx + e)) \cos(fx + e) + Ad^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)

[Out] 1/f*(-A*c^3*a*cos(f*x+e)+3*A*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+A*d^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+B*c^3*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*c^2*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*c*d^2*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*d^3*a*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+A*c^3*a*(f*x+e)-3*A*c^2*d*a*cos(f*x+e)+3*A*c*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*A*d^3*a*(2+sin(f*x+e)^2)*cos(f*x+e)-B*c^3*a*cos(f*x+e)+3*B*c^2*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*c*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^3*a*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e))

maxima [A] time = 0.36, size = 406, normalized size = 1.24

$$480(fx + e)Aac^3 + 120(2fx + 2e - \sin(2fx + 2e))Bac^3 + 360(2fx + 2e - \sin(2fx + 2e))Aac^2d + 480$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] 1/480*(480*(f*x + e)*A*a*c^3 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^3 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c^2*d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c^2*d + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2*d + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a*c*d^2 + 360*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d^2 + 480*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*c*d^2 + 45*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*c*d^2 + 160*

$$(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a*d^3 + 15*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a*d^3 - 32*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a*d^3 + 15*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a*d^3 - 480*A*a*c^3*\cos(f*x + e) - 480*B*a*c^3*\cos(f*x + e) - 1440*A*a*c^2*d*\cos(f*x + e))/f$$

mupad [B] time = 15.58, size = 830, normalized size = 2.54

$$a \operatorname{atan} \left(\frac{a \tan\left(\frac{e}{2} + \frac{fx}{2}\right) (8Ac^3 + 3Ad^3 + 4Bc^3 + 3Bd^3 + 12Acd^2 + 12Ac^2d + 9Bcd^2 + 12Bc^2d)}{4 \left(2Aac^3 + \frac{3Aad^3}{4} + Bac^3 + \frac{3Bad^3}{4} + 3Aacd^2 + 3Aac^2d + \frac{9Bacd^2}{4} + 3Bac^2d \right)} \right) (8Ac^3 + 3Ad^3 + 4Bc^3 + 3Bd^3 + 12Acd^2 + 12Ac^2d + 9Bcd^2 + 12Bc^2d)$$

$$4f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3,x)

[Out] (a*atan((a*tan(e/2 + (f*x)/2)*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*(2*A*a*c^3 + (3*A*a*d^3)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d)))*(8*A*c^3 + 3*A*d^3 + 4*B*c^3 + 3*B*d^3 + 12*A*c*d^2 + 12*A*c^2*d + 9*B*c*d^2 + 12*B*c^2*d))/(4*f) - (tan(e/2 + (f*x)/2)*((3*A*a*d^3)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^8*(2*A*a*c^3 + 2*B*a*c^3 + 6*A*a*c^2*d) + tan(e/2 + (f*x)/2)^2*(8*A*a*c^3 + (20*A*a*d^3)/3 + 8*B*a*c^3 + (16*B*a*d^3)/3 + 20*A*a*c*d^2 + 24*A*a*c^2*d + 20*B*a*c*d^2 + 20*B*a*c^2*d) + tan(e/2 + (f*x)/2)^4*(12*A*a*c^3 + (28*A*a*d^3)/3 + 12*B*a*c^3 + (32*B*a*d^3)/3 + 2*8*A*a*c*d^2 + 36*A*a*c^2*d + 28*B*a*c*d^2 + 28*B*a*c^2*d) + tan(e/2 + (f*x)/2)^6*(8*A*a*c^3 + 4*A*a*d^3 + 8*B*a*c^3 + 12*A*a*c*d^2 + 24*A*a*c^2*d + 12*B*a*c*d^2 + 12*B*a*c^2*d) - tan(e/2 + (f*x)/2)^9*((3*A*a*d^3)/4 + B*a*c^3 + (3*B*a*d^3)/4 + 3*A*a*c*d^2 + 3*A*a*c^2*d + (9*B*a*c*d^2)/4 + 3*B*a*c^2*d) + tan(e/2 + (f*x)/2)^3*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) - tan(e/2 + (f*x)/2)^7*((7*A*a*d^3)/2 + 2*B*a*c^3 + (7*B*a*d^3)/2 + 6*A*a*c*d^2 + 6*A*a*c^2*d + (21*B*a*c*d^2)/2 + 6*B*a*c^2*d) + 2*A*a*c^3 + (4*A*a*d^3)/3 + 2*B*a*c^3 + (16*B*a*d^3)/15 + 4*A*a*c*d^2 + 6*A*a*c^2*d + 4*B*a*c*d^2 + 4*B*a*c^2*d)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))

sympy [A] time = 5.37, size = 996, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)

[Out] Piecewise((A*a*c**3*x - A*a*c**3*cos(e + f*x)/f + 3*A*a*c**2*d*x*sin(e + f*x)**2/2 + 3*A*a*c**2*d*x*cos(e + f*x)**2/2 - 3*A*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 3*A*a*c**2*d*cos(e + f*x)/f + 3*A*a*c*d**2*x*sin(e + f*x)**2/2 + 3*A*a*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*c*d**2*cos(e + f*x)**3/f + 3*A*a*d**3*x*sin(e + f*x)**4/8 + 3*A*a*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a*d**3*x*cos(e + f*x)**4/8 - 5*A*a*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - A*a*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*A*a*d**3*cos(e + f*x)**3/(3*f) + B*a*c**3*x*sin(e + f*x)**2/2 + B*a*c**3*x*cos(e + f*x)**2/2 - B*a*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**3*cos(e + f*x)/f + 3*B*a*c**2*d*x*sin(e + f*x)**2/2 + 3*B*a*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) -

```

2*B*a*c**2*d*cos(e + f*x)**3/f + 9*B*a*c*d**2*x*sin(e + f*x)**4/8 + 9*B*a*c
*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*B*a*c*d**2*x*cos(e + f*x)**4/
8 - 15*B*a*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a*c*d**2*sin(e +
f*x)**2*cos(e + f*x)/f - 9*B*a*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) -
2*B*a*c*d**2*cos(e + f*x)**3/f + 3*B*a*d**3*x*sin(e + f*x)**4/8 + 3*B*a*d*
*3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**3*x*cos(e + f*x)**4/8 - B
*a*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a*d**3*sin(e + f*x)**3*cos(e +
f*x)/(8*f) - 4*B*a*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a*d**3
*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a*d**3*cos(e + f*x)**5/(15*f), Ne
(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a), True))

```

3.245 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$

Optimal. Leaf size=213

$$\frac{1}{8}ax(4A(2c^2 + 2cd + d^2) + B(4c^2 + 8cd + 3d^2)) - \frac{a(4Ad(c^2 + 3cd + d^2) - B(c^3 - 4c^2d - 8cd^2 - 4d^3)) \cos(e + fx)}{6df}$$

[Out] $\frac{1}{8}ax(4A(2c^2 + 2cd + d^2) + B(4c^2 + 8cd + 3d^2)) - \frac{a(4Ad(c^2 + 3cd + d^2) - B(c^3 - 4c^2d - 8cd^2 - 4d^3)) \cos(e + fx)}{6df}$

Rubi [A] time = 0.36, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3023, 2753, 2734}

$$\frac{a(4Ad(c^2 + 3cd + d^2) - B(-4c^2d + c^3 - 8cd^2 - 4d^3)) \cos(e + fx)}{6df} + \frac{a(-8cd(A + B) - 3d^2(4A + 3B) + 2Bc^2) \sin(e + fx)}{24f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]

[Out] $\frac{a(4A(2c^2 + 2cd + d^2) + B(4c^2 + 8cd + 3d^2))x}{8} - \frac{a(4Ad(c^2 + 3cd + d^2) - B(c^3 - 4c^2d - 8cd^2 - 4d^3)) \cos(e + fx)}{6df} + \frac{a(2Bc^2 - 8(A + B)cd - 3(4A + 3B)d^2) \cos(e + fx) \sin(e + fx)}{(24f)} + \frac{a(Bc - 4(A + B)d) \cos(e + fx) (c + d \sin(e + fx))^2}{(12df)} - \frac{aB \cos(e + fx) (c + d \sin(e + fx))^3}{(4df)}$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +

2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx &= \int (c + d \sin(e + fx))^2 (aA + (aA + aB) \sin(e + fx) \\ &= -\frac{aB \cos(e + fx)(c + d \sin(e + fx))^3}{4df} + \frac{\int (c + d \sin(e + fx))^2 dx}{4df} \\ &= \frac{a(Bc - 4(A + B)d) \cos(e + fx)(c + d \sin(e + fx))^3}{12df} \\ &= \frac{1}{8}a \left(4A(2c^2 + 2cd + d^2) + B(4c^2 + 8cd + 3d^2)\right) \cos(e + fx) \end{aligned}$$

Mathematica [A] time = 1.12, size = 185, normalized size = 0.87

$$\frac{a(\sin(e + fx) + 1) \left(3 \left(4fx \left(4A(2c^2 + 2cd + d^2) + B(4c^2 + 8cd + 3d^2)\right) - 8 \left(Ad(2c + d) + B(c + d)^2\right) \sin(2(e + fx))\right) + 96f \left(\sin\left(\frac{1}{2}(e + fx)\right)\right)^2}{96f \left(\sin\left(\frac{1}{2}(e + fx)\right)\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2, x]

[Out] (a*(1 + Sin[e + f*x])*(-24*(B*(4*c^2 + 6*c*d + 3*d^2) + A*(4*c^2 + 8*c*d + 3*d^2))*Cos[e + f*x] + 8*d*(A*d + B*(2*c + d))*Cos[3*(e + f*x)] + 3*(4*(4*A*(2*c^2 + 2*c*d + d^2) + B*(4*c^2 + 8*c*d + 3*d^2))*f*x - 8*(B*(c + d)^2 + A*d*(2*c + d))*Sin[2*(e + f*x)] + B*d^2*Sin[4*(e + f*x)]))/(96*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)

fricas [A] time = 0.47, size = 160, normalized size = 0.75

$$\frac{8(2Bacd + (A + B)ad^2) \cos(fx + e)^3 + 3(4(2A + B)ac^2 + 8(A + B)acd + (4A + 3B)ad^2)fx - 24((A + B)ad^2) \sin(fx + e)}{96f \left(\sin\left(\frac{1}{2}(e + fx)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/24*(8*(2*B*a*c*d + (A + B)*a*d^2)*cos(f*x + e)^3 + 3*(4*(2*A + B)*a*c^2 + 8*(A + B)*a*c*d + (4*A + 3*B)*a*d^2)*f*x - 24*((A + B)*a*c^2 + 2*(A + B)*a*c*d + (A + B)*a*d^2)*cos(f*x + e) + 3*(2*B*a*d^2*cos(f*x + e)^3 - (4*B*a*c^2 + 8*(A + B)*a*c*d + (4*A + 5*B)*a*d^2)*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.15, size = 198, normalized size = 0.93

$$\frac{Bad^2 \sin(4fx + 4e)}{32f} + \frac{1}{8} \left(8Aac^2 + 4Bac^2 + 8Aacd + 8Bacd + 4Aad^2 + 3Bad^2\right)x + \frac{(2Bacd + Aad^2 + Bad^2) \cos(fx + e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")

```
[Out] 1/32*B*a*d^2*sin(4*f*x + 4*e)/f + 1/8*(8*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d +
8*B*a*c*d + 4*A*a*d^2 + 3*B*a*d^2)*x + 1/12*(2*B*a*c*d + A*a*d^2 + B*a*d^2)
*cos(3*f*x + 3*e)/f - 1/4*(4*A*a*c^2 + 4*B*a*c^2 + 8*A*a*c*d + 6*B*a*c*d +
3*A*a*d^2 + 3*B*a*d^2)*cos(f*x + e)/f - 1/4*(B*a*c^2 + 2*A*a*c*d + 2*B*a*c*
d + A*a*d^2 + B*a*d^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 274, normalized size = 1.29

$$-A c^2 a \cos (f x+e)+2 A c d a\left(-\frac{\sin (f x+e) \cos (f x+e)}{2}+\frac{f x}{2}+\frac{e}{2}\right)-\frac{A d^2 a\left(2+\sin ^2(f x+e)\right) \cos (f x+e)}{3}+B c^2 a\left(-\frac{\sin (f x+e) \cos (f x+e)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 1/f*(-A*c^2*a*cos(f*x+e)+2*A*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*
e)-1/3*A*d^2*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*c^2*a*(-1/2*sin(f*x+e)*cos(f*x
+e)+1/2*f*x+1/2*e)-2/3*B*c*d*a*(2+sin(f*x+e)^2)*cos(f*x+e)+B*d^2*a*(-1/4*(s
in(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+A*c^2*a*(f*x+e)-2*A*c
*d*a*cos(f*x+e)+A*d^2*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*c^2*a*
cos(f*x+e)+2*B*c*d*a*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*d^2*a
*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.37, size = 264, normalized size = 1.24

$$96(f x+e) A a c^2+24(2 f x+2 e-\sin (2 f x+2 e)) B a c^2+48(2 f x+2 e-\sin (2 f x+2 e)) A a c d+64(\cos (f x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm
="maxima")
```

```
[Out] 1/96*(96*(f*x + e)*A*a*c^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c^2 +
48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a*c*d + 64*(cos(f*x + e)^3 - 3*cos(f*
x + e))*B*a*c*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a*c*d + 32*(cos(f*x
+ e)^3 - 3*cos(f*x + e))*A*a*d^2 + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a
*d^2 + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a*d^2 + 3*(12*f*x + 12*e + si
n(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a*d^2 - 96*A*a*c^2*cos(f*x + e) - 96
*B*a*c^2*cos(f*x + e) - 192*A*a*c*d*cos(f*x + e))/f
```

mupad [B] time = 15.41, size = 547, normalized size = 2.57

$$a \operatorname{atan}\left(\frac{a \tan \left(\frac{e}{2}+\frac{f x}{2}\right)\left(8 A c^2+4 A d^2+4 B c^2+3 B d^2+8 A c d+8 B c d\right)}{4\left(2 A a c^2+A a d^2+B a c^2+\frac{3 B a d^2}{4}+2 A a c d+2 B a c d\right)}\right)\left(8 A c^2+4 A d^2+4 B c^2+3 B d^2+8 A c d+8 B c d\right) \operatorname{tan}$$

$$4 f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a*atan((a*tan(e/2 + (f*x)/2))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*
c*d + 8*B*c*d))/(4*(2*A*a*c^2 + A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c
*d + 2*B*a*c*d)))*(8*A*c^2 + 4*A*d^2 + 4*B*c^2 + 3*B*d^2 + 8*A*c*d + 8*B*c*
d))/(4*f) - (tan(e/2 + (f*x)/2)^6*(2*A*a*c^2 + 2*B*a*c^2 + 4*A*a*c*d) + tan
(e/2 + (f*x)/2)*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d)
```

+ tan(e/2 + (f*x)/2)^4*(6*A*a*c^2 + 4*A*a*d^2 + 6*B*a*c^2 + 4*B*a*d^2 + 12*A*a*c*d + 8*B*a*c*d) + tan(e/2 + (f*x)/2)^2*(6*A*a*c^2 + (16*A*a*d^2)/3 + 6*B*a*c^2 + (16*B*a*d^2)/3 + 12*A*a*c*d + (32*B*a*c*d)/3) - tan(e/2 + (f*x)/2)^7*(A*a*d^2 + B*a*c^2 + (3*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + tan(e/2 + (f*x)/2)^3*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) - tan(e/2 + (f*x)/2)^5*(A*a*d^2 + B*a*c^2 + (11*B*a*d^2)/4 + 2*A*a*c*d + 2*B*a*c*d) + 2*A*a*c^2 + (4*A*a*d^2)/3 + 2*B*a*c^2 + (4*B*a*d^2)/3 + 4*A*a*c*d + (8*B*a*c*d)/3)/(f*(4*tan(e/2 + (f*x)/2)^2 + 6*tan(e/2 + (f*x)/2)^4 + 4*tan(e/2 + (f*x)/2)^6 + tan(e/2 + (f*x)/2)^8 + 1))

sympy [A] time = 2.31, size = 571, normalized size = 2.68

$$\left\{ \begin{array}{l} Aac^2x - \frac{Aac^2 \cos(e+fx)}{f} + Aacd x \sin^2(e+fx) + Aacd x \cos^2(e+fx) - \frac{Aacd \sin(e+fx) \cos(e+fx)}{f} - \frac{2Aacd \cos(e+fx)}{f} \\ x(A+B \sin(e))(c+d \sin(e))^2(a \sin(e)+a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)

[Out] Piecewise((A*a*c**2*x - A*a*c**2*cos(e + f*x)/f + A*a*c*d*x*sin(e + f*x)**2 + A*a*c*d*x*cos(e + f*x)**2 - A*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 2*A*a*c*d*cos(e + f*x)/f + A*a*d**2*x*sin(e + f*x)**2/2 + A*a*d**2*x*cos(e + f*x)**2/2 - A*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - A*a*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a*d**2*cos(e + f*x)**3/(3*f) + B*a*c**2*x*sin(e + f*x)**2/2 + B*a*c**2*x*cos(e + f*x)**2/2 - B*a*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c**2*cos(e + f*x)/f + B*a*c*d*x*sin(e + f*x)**2 + B*a*c*d*x*cos(e + f*x)**2 - 2*B*a*c*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*B*a*c*d*cos(e + f*x)**3/(3*f) + 3*B*a*d**2*x*sin(e + f*x)**4/8 + 3*B*a*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a*d**2*x*cos(e + f*x)**4/8 - 5*B*a*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - B*a*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 2*B*a*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a), True))

3.246 $\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx)) dx$

Optimal. Leaf size=111

$$\frac{a(3A(c+d) + B(3c+d)) \cos(e+fx)}{3f} - \frac{a(3Ad + 3Bc - Bd) \sin(e+fx) \cos(e+fx)}{6f} + \frac{1}{2} ax(A(2c+d) + B(c+d)) - \frac{Bc}{2}$$

[Out] $\frac{1}{2} a (B(c+d) + A(2c+d)) x - \frac{1}{3} a (3A(c+d) + B(3c+d)) \cos(fx+e) / f - \frac{1}{6} a (3Ad + 3Bc - Bd) \cos(fx+e) \sin(fx+e) / f - \frac{1}{3} B d \cos(fx+e) (a + a \sin(fx+e))^2 / a / f$

Rubi [A] time = 0.16, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {2968, 3023, 2734}

$$\frac{a(3A(c+d) + B(3c+d)) \cos(e+fx)}{3f} - \frac{a(3Ad + 3Bc - Bd) \sin(e+fx) \cos(e+fx)}{6f} + \frac{1}{2} ax(A(2c+d) + B(c+d)) - \frac{Bc}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] $\frac{a(B(c+d) + A(2c+d))x}{2} - \frac{a(3A(c+d) + B(3c+d)) \cos[e + fx]}{(3f)} - \frac{a(3Bc + 3Ad - Bd) \cos[e + fx] \sin[e + fx]}{(6f)} - \frac{Bd \cos[e + fx] (a + a \sin[e + f*x])^2}{(3af)}$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[(b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx)) (Ac + (Bc + Ad) \sin(e + fx) \\ &+ Bd \cos(e + fx)(a + a \sin(e + fx))^2 + \frac{1}{2} a (B(c+d) + A(2c+d)) x - \frac{a(3A(c+d) + B(3c+d)) \cos(e + fx)}{3f} \end{aligned}$$

Mathematica [A] time = 0.44, size = 104, normalized size = 0.94

$$\frac{a(-3(4A(c+d) + B(4c+3d))\cos(e+fx) + 12Acfx - 3Ad\sin(2(e+fx)) + 6Adfx - 3Bc\sin(2(e+fx)) + 6Bcfx) + 6Bcfx}{12f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] (a*(12*A*c*f*x + 6*B*c*f*x + 6*A*d*f*x + 6*B*d*f*x - 3*(4*A*(c + d) + B*(4*c + 3*d))*Cos[e + f*x] + B*d*Cos[3*(e + f*x)] - 3*B*c*Sin[2*(e + f*x)] - 3*A*d*Sin[2*(e + f*x)] - 3*B*d*Sin[2*(e + f*x)])/(12*f)

fricas [A] time = 0.47, size = 84, normalized size = 0.76

$$\frac{2Bad\cos(fx+e)^3 + 3((2A+B)ac + (A+B)ad)fx - 3(Bac + (A+B)ad)\cos(fx+e)\sin(fx+e) - 6((A+B)ac + (A+B)ad)\cos(fx+e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/6*(2*B*a*d*cos(f*x + e)^3 + 3*((2*A + B)*a*c + (A + B)*a*d)*f*x - 3*(B*a*c + (A + B)*a*d)*cos(f*x + e)*sin(f*x + e) - 6*((A + B)*a*c + (A + B)*a*d)*cos(f*x + e))/f

giac [A] time = 0.15, size = 101, normalized size = 0.91

$$\frac{Bad\cos(3fx+3e)}{12f} + \frac{1}{2}(2Aac + Bac + Aad + Bad)x - \frac{(4Aac + 4Bac + 4Aad + 3Bad)\cos(fx+e)}{4f} - \frac{(Bac + Aad)\sin(fx+e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 1/12*B*a*d*cos(3*f*x + 3*e)/f + 1/2*(2*A*a*c + B*a*c + A*a*d + B*a*d)*x - 1/4*(4*A*a*c + 4*B*a*c + 4*A*a*d + 3*B*a*d)*cos(f*x + e)/f - 1/4*(B*a*c + A*a*d + B*a*d)*sin(2*f*x + 2*e)/f

maple [A] time = 0.30, size = 147, normalized size = 1.32

$$\frac{-\frac{Bad(2+\sin^2(fx+e))\cos(fx+e)}{3} + Aad\left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + Bac\left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + Bad\left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] 1/f*(-1/3*B*a*d*(2+sin(f*x+e)^2)*cos(f*x+e)+A*a*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+B*a*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-A*a*c*cos(f*x+e)-A*a*d*cos(f*x+e)-B*a*c*cos(f*x+e)+A*a*c*(f*x+e))

maxima [A] time = 0.37, size = 143, normalized size = 1.29

$$\frac{12(fx+e)Aac + 3(2fx+2e-\sin(2fx+2e))Bac + 3(2fx+2e-\sin(2fx+2e))Aad + 4(\cos(fx+e) - \sin(fx+e))}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] $\frac{1}{12} \cdot (12 \cdot (f \cdot x + e) \cdot A \cdot a \cdot c + 3 \cdot (2 \cdot f \cdot x + 2 \cdot e - \sin(2 \cdot f \cdot x + 2 \cdot e)) \cdot B \cdot a \cdot c + 3 \cdot (2 \cdot f \cdot x + 2 \cdot e - \sin(2 \cdot f \cdot x + 2 \cdot e)) \cdot A \cdot a \cdot d + 4 \cdot (\cos(f \cdot x + e)^3 - 3 \cdot \cos(f \cdot x + e)) \cdot B \cdot a \cdot d + 3 \cdot (2 \cdot f \cdot x + 2 \cdot e - \sin(2 \cdot f \cdot x + 2 \cdot e)) \cdot B \cdot a \cdot d - 12 \cdot A \cdot a \cdot c \cdot \cos(f \cdot x + e) - 12 \cdot B \cdot a \cdot c \cdot \cos(f \cdot x + e) - 12 \cdot A \cdot a \cdot d \cdot \cos(f \cdot x + e)) / f$

mupad [B] time = 13.31, size = 134, normalized size = 1.21

$$\frac{\frac{3 A a d \sin(2 e+2 f x)}{2} - \frac{B a d \cos(3 e+3 f x)}{2} + \frac{3 B a c \sin(2 e+2 f x)}{2} + \frac{3 B a d \sin(2 e+2 f x)}{2} + 6 A a c \cos(e+f x) + 6 A a d \cos(e+f x)}{6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x)),x)

[Out] $-\left(\frac{3 A a d \sin(2 e+2 f x)}{2} - \frac{B a d \cos(3 e+3 f x)}{2} + \frac{3 B a c \sin(2 e+2 f x)}{2} + \frac{3 B a d \sin(2 e+2 f x)}{2} + 6 A a c \cos(e+f x) + 6 A a d \cos(e+f x) + 6 B a c \cos(e+f x) + \frac{9 B a d \cos(e+f x)}{2} - 6 A a c f x - 3 A a d f x - 3 B a c f x - 3 B a d f x\right) / (6 f)$

sympy [A] time = 1.00, size = 277, normalized size = 2.50

$$\left\{ \begin{array}{l} A a c x - \frac{A a c \cos(e+f x)}{f} + \frac{A a d x \sin^2(e+f x)}{2} + \frac{A a d x \cos^2(e+f x)}{2} - \frac{A a d \sin(e+f x) \cos(e+f x)}{2 f} - \frac{A a d \cos(e+f x)}{f} + \frac{B a c x \sin^2(e+f x)}{2} + \frac{B a c x \cos^2(e+f x)}{2} \\ x (A + B \sin(e)) (c + d \sin(e)) (a \sin(e) + a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] Piecewise((A*a*c*x - A*a*c*cos(e + f*x)/f + A*a*d*x*sin(e + f*x)**2/2 + A*a*d*x*cos(e + f*x)**2/2 - A*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - A*a*d*cos(e + f*x)/f + B*a*c*x*sin(e + f*x)**2/2 + B*a*c*x*cos(e + f*x)**2/2 - B*a*c*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*c*cos(e + f*x)/f + B*a*d*x*sin(e + f*x)**2/2 + B*a*d*x*cos(e + f*x)**2/2 - B*a*d*sin(e + f*x)**2*cos(e + f*x)/f - B*a*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a), True))

3.247 $\int (a + a \sin(e + fx))(A + B \sin(e + fx)) dx$

Optimal. Leaf size=48

$$-\frac{a(A+B)\cos(e+fx)}{f} + \frac{1}{2}ax(2A+B) - \frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

[Out] $1/2*a*(2*A+B)*x - a*(A+B)*\cos(f*x+e)/f - 1/2*a*B*\cos(f*x+e)*\sin(f*x+e)/f$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2734}

$$-\frac{a(A+B)\cos(e+fx)}{f} + \frac{1}{2}ax(2A+B) - \frac{aB\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]

[Out] $(a*(2*A + B)*x)/2 - (a*(A + B)*\text{Cos}[e + f*x])/f - (a*B*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(2*f)$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx)) dx = \frac{1}{2}a(2A + B)x - \frac{a(A + B)\cos(e + fx)}{f} - \frac{aB\cos(e + fx)\sin(e + fx)}{2f}$$

Mathematica [A] time = 0.10, size = 45, normalized size = 0.94

$$\frac{a(-4(A+B)\cos(e+fx) + 4Afx - B\sin(2(e+fx)) + 2Be + 2Bfx)}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]),x]

[Out] $(a*(2*B*e + 4*A*f*x + 2*B*f*x - 4*(A + B)*\text{Cos}[e + f*x] - B*\text{Sin}[2*(e + f*x)])/(4*f)$

fricas [A] time = 0.43, size = 43, normalized size = 0.90

$$\frac{(2A + B)afx - Ba \cos(fx + e) \sin(fx + e) - 2(A + B)a \cos(fx + e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] $1/2*((2*A + B)*a*f*x - B*a*\cos(f*x + e)*\sin(f*x + e) - 2*(A + B)*a*\cos(f*x + e))/f$

giac [A] time = 0.13, size = 48, normalized size = 1.00

$$\frac{1}{2}(2Aa + Ba)x - \frac{Ba \sin(2fx + 2e)}{4f} - \frac{(Aa + Ba) \cos(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] 1/2*(2*A*a + B*a)*x - 1/4*B*a*sin(2*f*x + 2*e)/f - (A*a + B*a)*cos(f*x + e)/f

maple [A] time = 0.16, size = 59, normalized size = 1.23

$$\frac{aB \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - aA \cos(fx + e) - aB \cos(fx + e) + aA(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

[Out] 1/f*(a*B*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a*A*cos(f*x+e)-a*B*cos(f*x+e)+a*A*(f*x+e))

maxima [A] time = 0.45, size = 57, normalized size = 1.19

$$\frac{4(fx + e)Aa + (2fx + 2e - \sin(2fx + 2e))Ba - 4Aa \cos(fx + e) - 4Ba \cos(fx + e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] 1/4*(4*(f*x + e)*A*a + (2*f*x + 2*e - sin(2*f*x + 2*e))*B*a - 4*A*a*cos(f*x + e) - 4*B*a*cos(f*x + e))/f

mupad [B] time = 13.26, size = 100, normalized size = 2.08

$$Aax - \frac{-Ba \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + (2Aa + 2Ba) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + Ba \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 2Aa + 2Ba}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + 2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 1 \right)} + \frac{Bax}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x)),x)

[Out] A*a*x - (2*A*a + 2*B*a + tan(e/2 + (f*x)/2)^2*(2*A*a + 2*B*a) - B*a*tan(e/2 + (f*x)/2)^3 + B*a*tan(e/2 + (f*x)/2))/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 + 1)) + (B*a*x)/2

sympy [A] time = 0.35, size = 94, normalized size = 1.96

$$\begin{cases} Aax - \frac{Aa \cos(e+fx)}{f} + \frac{Bax \sin^2(e+fx)}{2} + \frac{Bax \cos^2(e+fx)}{2} - \frac{Ba \sin(e+fx) \cos(e+fx)}{2f} - \frac{Ba \cos(e+fx)}{f} & \text{for } f \neq 0 \\ x(A + B \sin(e))(a \sin(e) + a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e)),x)

[Out] Piecewise((A*a*x - A*a*cos(e + f*x)/f + B*a*x*sin(e + f*x)**2/2 + B*a*x*cos(e + f*x)**2/2 - B*a*sin(e + f*x)*cos(e + f*x)/(2*f) - B*a*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a), True))

$$3.248 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=98

$$\frac{2a(c-d)(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2 f \sqrt{c^2-d^2}} - \frac{ax(Bc-d(A+B))}{d^2} - \frac{aB \cos(e+fx)}{df}$$

[Out] $-a*(B*c-(A+B)*d)*x/d^2-a*B*\cos(f*x+e)/d/f+2*a*(c-d)*(-A*d+B*c)*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/d^2/f/(c^2-d^2)^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a(c-d)(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2 f \sqrt{c^2-d^2}} - \frac{ax(Bc-d(A+B))}{d^2} - \frac{aB \cos(e+fx)}{df}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]

[Out] $-((a*(B*c - (A + B)*d)*x)/d^2) + (2*a*(c - d)*(B*c - A*d)*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(d^2*\text{Sqrt}[c^2 - d^2]*f) - (a*B*\text{Cos}[e + f*x])/d^2$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{c + d \sin(e + fx)} dx = \int \frac{aA + (aA + aB) \sin(e + fx) + aB \sin^2(e + fx)}{c + d \sin(e + fx)} dx$$

$$= -\frac{aB \cos(e + fx)}{df} + \frac{\int \frac{aAd - a(Bc - (A+B)d) \sin(e+fx)}{c + d \sin(e+fx)} dx}{d}$$

$$= -\frac{a(Bc - (A + B)d)x}{d^2} - \frac{aB \cos(e + fx)}{df} + \frac{(a(c - d)(Bc - Ad)) \int \frac{1}{c + d \sin(e + fx)} dx}{d^2}$$

$$= -\frac{a(Bc - (A + B)d)x}{d^2} - \frac{aB \cos(e + fx)}{df} + \frac{(2a(c - d)(Bc - Ad)) \operatorname{Subst}\left(\int \frac{1}{c + d \sin(u)} du, e + fx, x\right)}{d^2}$$

$$= -\frac{a(Bc - (A + B)d)x}{d^2} - \frac{aB \cos(e + fx)}{df} - \frac{(4a(c - d)(Bc - Ad)) \operatorname{Subst}\left(\int \frac{1}{c + d \sin(u)} du, e + fx, x\right)}{d^2}$$

$$= -\frac{a(Bc - (A + B)d)x}{d^2} + \frac{2a(c - d)(Bc - Ad) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{d^2 \sqrt{c^2 - d^2}}$$

Mathematica [C] time = 0.65, size = 196, normalized size = 2.00

$$a(\sin(e + fx) + 1) \left(\frac{2(c-d)(\cos(e) - i \sin(e))(Bc - Ad) \tan^{-1}\left(\frac{(\cos(e) - i \sin(e)) \sec\left(\frac{fx}{2}\right) \left(c \sin\left(\frac{fx}{2}\right) + d \cos\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2 - d^2} \sqrt{(\cos(e) - i \sin(e))^2}}\right)}{f \sqrt{c^2 - d^2} \sqrt{(\cos(e) - i \sin(e))^2}} + Adx + Bx(d - c) + \frac{Bd \sin(e) \sin(fx)}{f} \right)$$

$$d^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),
x]
```

```
[Out] (a*(A*d*x + B*(-c + d)*x - (B*d*Cos[e]*Cos[f*x]))/f + (2*(c - d)*(B*c - A*d)
*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)
/2])]/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2])*(Cos[e] - I*Sin[e]))/(
Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + (B*d*Sin[e]*Sin[f*x])/f)*(
1 + Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)
```

fricas [A] time = 0.49, size = 292, normalized size = 2.98

$$\left[\frac{2Bad \cos(fx + e) + 2(Bac - (A + B)ad)fx - (Bac - Aad) \sqrt{\frac{c-d}{c+d}} \log\left(-\frac{(2c^2 - d^2) \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2 - 2(c-d) \sin(fx + e)}{d^2 \cos(fx + e)^2}\right)}{2d^2 f} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*(2*B*a*d*cos(f*x + e) + 2*(B*a*c - (A + B)*a*d)*f*x - (B*a*c - A*a*d)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2))/(d^2*f), -(B*a*d*cos(f*x + e) + (B*a*c - (A + B)*a*d)*f*x + (B*a*c - A*a*d)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e)))/(d^2*f)]

giac [A] time = 0.15, size = 141, normalized size = 1.44

$$\frac{\frac{(Bac - Aad - Bad)(fx + e)}{d^2} + \frac{2Ba}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 1\right)d} - \frac{2(Bac^2 - Aacd - Bacd + Aad^2)\left(\pi\left[\frac{fx+e}{2\pi} + \frac{1}{2}\right]\text{sgn}(c) + \arctan\left(\frac{c\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + d}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2}d^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] -((B*a*c - A*a*d - B*a*d)*(f*x + e)/d^2 + 2*B*a/((tan(1/2*f*x + 1/2*e)^2 + 1)*d) - 2*(B*a*c^2 - A*a*c*d - B*a*c*d + A*a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2))/f

maple [B] time = 0.40, size = 294, normalized size = 3.00

$$\frac{2a \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)Ac}{fd\sqrt{c^2 - d^2}} + \frac{2a \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)A}{f\sqrt{c^2 - d^2}} + \frac{2a \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)Bc^2}{fd^2\sqrt{c^2 - d^2}} - \frac{2a \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right)Bd}{fd\sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] -2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+2/f*a/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/f*a/d*B/(1+tan(1/2*f*x+1/2*e)^2)+2/f*a/d*A*arctan(tan(1/2*f*x+1/2*e))-2/f*a/d^2*B*arctan(tan(1/2*f*x+1/2*e))*c+2/f*a/d*B*arctan(tan(1/2*f*x+1/2*e))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 16.58, size = 3074, normalized size = 31.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x)),x)

[Out] (2*A*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) + (2*B*a*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(f*(c + d)) - (B*a*cos(e + f*x))/(f*(c + d)) + (2*A*a*c*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d*f*(c + d)) - (A*a*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*3i - A^2*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i - B^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + B^2*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*6i + A*B*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i + A*B*c^3*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*4i + A*B*c^5*d*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i - A*B*c^2*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A*B*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^4*d^2*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A*B*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*6i - A*B*c^3*d^3*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*10i + A*B*c^4*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i)/(4*A^2*d^7*sin(e/2 + (f*x)/2) + 2*B^2*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c^2*d^5*cos(e/2 + (f*x)/2) - 2*A^2*c^3*d^4*cos(e/2 + (f*x)/2) - 2*A^2*c^4*d^3*cos(e/2 + (f*x)/2) - 2*B^2*c^3*d^4*cos(e/2 + (f*x)/2) + B^2*c^5*d^2*cos(e/2 + (f*x)/2) - 4*A^2*c^2*d^5*sin(e/2 + (f*x)/2) - 4*A^2*c^3*d^4*sin(e/2 + (f*x)/2) - 4*B^2*c^2*d^5*sin(e/2 + (f*x)/2) + 2*B^2*c^4*d^3*sin(e/2 + (f*x)/2) + 4*A*B*d^7*sin(e/2 + (f*x)/2) + 2*A^2*c*d^6*cos(e/2 + (f*x)/2) + B^2*c*d^6*cos(e/2 + (f*x)/2) + 4*A^2*c*d^6*sin(e/2 + (f*x)/2) - 4*A*B*c^3*d^4*cos(e/2 + (f*x)/2) + 2*A*B*c^5*d^2*cos(e/2 + (f*x)/2) - 8*A*B*c^2*d^5*sin(e/2 + (f*x)/2) + 4*A*B*c^4*d^3*sin(e/2 + (f*x)/2) + 2*A*B*c*d^6*cos(e/2 + (f*x)/2)))*(d^2 - c^2)^(1/2)*2i)/(d*f*(c + d)) - (2*B*a*c^2*atan(sin(e/2 + (f*x)/2)/cos(e/2 + (f*x)/2)))/(d^2*f*(c + d)) - (B*a*c*cos(e + f*x))/(d*f*(c + d)) + (B*a*c*atan((A^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*3i + A^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i - B^2*c^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + B^2*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A*B*d^6*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + A^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c*d^5*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + B^2*c^3*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*1i + B^2*c^5*d*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i + A^2*c*d^5*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*4i + A^2*c^2*d^4*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i + A^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*1i - B^2*c^3*d^3*cos(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)*2i - A^2*c^2*d^2*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(3/2)*2i + A^2*c^2*d^4*sin(e/2 + (f*x)/2)*(d^2 - c^2)^(1/2)

$$\begin{aligned}
&) * 3i - A^2 * c^3 * d^3 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i - A^2 * c^4 * d^2 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i + B^2 * c^2 * d^2 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(3/2)} * 3i - B^2 * c^2 * d^4 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 6i + B^2 * c^4 * d^2 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 6i + A * B * c * d^5 * \cos(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i - A * B * c * d^3 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(3/2)} * 6i + A * B * c * d^5 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 6i + A * B * c^3 * d * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(3/2)} * 4i + A * B * c^5 * d * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 4i - A * B * c^2 * d^2 * \cos(e/2 + (f*x)/2) * (d^2 - c^2)^{(3/2)} * 2i + A * B * c^2 * d^4 * \cos(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i - A * B * c^3 * d^3 * \cos(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i - A * B * c^4 * d^2 * \cos(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i - A * B * c^2 * d^4 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 6i - A * B * c^3 * d^3 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 10i + A * B * c^4 * d^2 * \sin(e/2 + (f*x)/2) * (d^2 - c^2)^{(1/2)} * 2i / (4 * A^2 * d^7 * \sin(e/2 + (f*x)/2) + 2 * B^2 * d^7 * \sin(e/2 + (f*x)/2) + 2 * A^2 * c^2 * d^5 * \cos(e/2 + (f*x)/2) - 2 * A^2 * c^3 * d^4 * \cos(e/2 + (f*x)/2) - 2 * A^2 * c^4 * d^3 * \cos(e/2 + (f*x)/2) - 2 * B^2 * c^3 * d^4 * \cos(e/2 + (f*x)/2) + B^2 * c^5 * d^2 * \cos(e/2 + (f*x)/2) - 4 * A^2 * c^2 * d^5 * \sin(e/2 + (f*x)/2) - 4 * A^2 * c^3 * d^4 * \sin(e/2 + (f*x)/2) - 4 * B^2 * c^2 * d^5 * \sin(e/2 + (f*x)/2) + 2 * B^2 * c^4 * d^3 * \sin(e/2 + (f*x)/2) + 4 * A * B * d^7 * \sin(e/2 + (f*x)/2) + 2 * A^2 * c * d^6 * \cos(e/2 + (f*x)/2) + B^2 * c * d^6 * \cos(e/2 + (f*x)/2) + 4 * A^2 * c * d^6 * \sin(e/2 + (f*x)/2) - 4 * A * B * c^3 * d^4 * \cos(e/2 + (f*x)/2) + 2 * A * B * c^5 * d^2 * \cos(e/2 + (f*x)/2) - 8 * A * B * c^2 * d^5 * \sin(e/2 + (f*x)/2) + 4 * A * B * c^4 * d^3 * \sin(e/2 + (f*x)/2) + 2 * A * B * c * d^6 * \cos(e/2 + (f*x)/2)) * (d^2 - c^2)^{(1/2)} * 2i / (d^2 * f * (c + d))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.249 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=124

$$\frac{2a(d^2(A+B)(c-d) - Bc(c^2 - d^2)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{d^2 f (c^2 - d^2)^{3/2}} + \frac{a(Bc - Ad) \cos(e+fx)}{df(c+d)(c+d \sin(e+fx))} + \frac{aBx}{d^2}$$

[Out] a*B*x/d^2+2*a*((A+B)*(c-d)*d^2-B*c*(c^2-d^2))*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^2/(c^2-d^2)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))

Rubi [A] time = 0.33, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2968, 3021, 2735, 2660, 618, 204}

$$\frac{2a(d^2(A+B)(c-d) - Bc(c^2 - d^2)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{d^2 f (c^2 - d^2)^{3/2}} + \frac{a(Bc - Ad) \cos(e+fx)}{df(c+d)(c+d \sin(e+fx))} + \frac{aBx}{d^2}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] (a*B*x)/d^2 + (2*a*((A + B)*(c - d)*d^2 - B*c*(c^2 - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(d^2*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(d*(c + d)*f*(c + d*Sin[e + f*x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),

x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx = \int \frac{aA + (aA + aB) \sin(e + fx) + aB \sin^2(e + fx)}{(c + d \sin(e + fx))^2} dx$$

$$= \frac{a(Bc - Ad) \cos(e + fx)}{d(c + d)f(c + d \sin(e + fx))} - \frac{\int \frac{-a(A+B)(c-d)d - aB(c^2 - d^2) \sin(e + fx)}{c + d \sin(e + fx)} dx}{d(c^2 - d^2)}$$

$$= \frac{aBx}{d^2} + \frac{a(Bc - Ad) \cos(e + fx)}{d(c + d)f(c + d \sin(e + fx))} + \frac{a(Ad^2 - B(c^2 + cd - d^2)) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{d^2(c + d)\sqrt{c^2 - d^2}}$$

Mathematica [C] time = 1.27, size = 217, normalized size = 1.75

$$a(\sin(e + fx) + 1) \left(\frac{2(\cos(e) - i \sin(e))(Ad^2 - B(c^2 + cd - d^2)) \tan^{-1}\left(\frac{(\cos(e) - i \sin(e)) \sec\left(\frac{fx}{2}\right) \left(c \sin\left(\frac{fx}{2}\right) + d \cos\left(\frac{fx}{2}\right)\right)}{\sqrt{c^2 - d^2} \sqrt{(\cos(e) - i \sin(e))^2}}\right)}{f(c + d)\sqrt{c^2 - d^2} \sqrt{(\cos(e) - i \sin(e))^2}} + \frac{\csc(e)(Ad - B)(c \cos(e) + d \sin(e))}{f(c + d)(c + d \sin(e + fx))} \right) \frac{1}{d^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2, x]

[Out] (a*(1 + Sin[e + f*x])*(B*x + (2*(A*d^2 - B*(c^2 + c*d - d^2))*ArcTan[(Sec[(f*x)/2]*(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2])])/(Sqrt[c^2 - d^2]*Sqrt[(Cos[e] - I*Sin[e])^2]))*(Cos[e] - I*Sin[e]))/((c + d)*Sqrt[c^2 - d^2]*f*Sqrt[(Cos[e] - I*Sin[e])^2]) + ((-B*c) + A*d)*Csc[e]*(c*Cos[e] + d*Sin[f*x])/((c + d)*f*(c + d*Sin[e + f*x]))/(d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)

fricas [B] time = 0.49, size = 655, normalized size = 5.28

$$\frac{2(Bac^3d + Bac^2d^2 - Bacd^3 - Bad^4)fx \sin(fx + e) + 2(Bac^4 + Bac^3d - Bac^2d^2 - Bacd^3)fx + (Bac^3 + Bac^2d -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*(2*(B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + 2*(B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(B*a*c^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f), ((B*a*c^3*d + B*a*c^2*d^2 - B*a*c*d^3 - B*a*d^4)*f*x*sin(f*x + e) + (B*a*c^4 + B*a*c^3*d - B*a*c^2*d^2 - B*a*c*d^3)*f*x + (B*a*c^3 + B*a*c^2*d - (A + B)*a*c*d^2 + (B*a*c^2*d + B*a*c*d^2 - (A + B)*a*d^3)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (B*a*c^3*d - A*a*c^2*d^2 - B*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^3*d^3 + c^2*d^4 - c*d^5 - d^6)*f*sin(f*x + e) + (c^4*d^2 + c^3*d^3 - c^2*d^4 - c*d^5)*f)]

giac [A] time = 0.20, size = 204, normalized size = 1.65

$$\frac{(fx+e)Ba}{d^2} - \frac{2(Bac^2+Bacd-Aad^2-Bad^2)\left(\pi\left[\frac{fx+e}{2\pi}+\frac{1}{2}\right]\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(cd^2+d^3)\sqrt{c^2-d^2}} + \frac{2\left(Bacd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)-Aad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+Bac^2-Aad^2\right)}{(c^2d+cd^2)\left(c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)^2+2d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+c}$$

f

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] ((f*x + e)*B*a/d^2 - 2*(B*a*c^2 + B*a*c*d - A*a*d^2 - B*a*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^2 + d^3)*sqrt(c^2 - d^2)) + 2*(B*a*c*d*tan(1/2*f*x + 1/2*e) - A*a*d^2*tan(1/2*f*x + 1/2*e) + B*a*c^2 - A*a*c*d)/((c^2*d + c*d^2)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c))/f

maple [B] time = 0.49, size = 434, normalized size = 3.50

$$\frac{2ad \tan\left(\frac{fx}{2} + \frac{e}{2}\right) A}{f \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) d + c \right) (c + d) c} + \frac{2a \tan\left(\frac{fx}{2} + \frac{e}{2}\right) B}{f \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) d + c \right) (c + d) f \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right) \right) c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) d + c \right) (c + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

[Out] -2*a/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)*A+2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B-2*a/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)

$$\frac{1}{(c+d)A+2a/f/d/(\tan(1/2fx+1/2e))^2+c+2\tan(1/2fx+1/2e)d+c}/(c+d)B$$

$$\frac{c+2a/f/(c+d)/(c^2-d^2)^{1/2}\arctan(1/2(2c\tan(1/2fx+1/2e)+2d)/(c^2-d^2)^{1/2})}{A-2a/f/d^2/(c+d)/(c^2-d^2)^{1/2}\arctan(1/2(2c\tan(1/2fx+1/2e)+2d)/(c^2-d^2)^{1/2})} \frac{Bc^2-2a/f/d/(c+d)/(c^2-d^2)^{1/2}\arctan(1/2(2c\tan(1/2fx+1/2e)+2d)/(c^2-d^2)^{1/2})}{Bc+2a/f/(c+d)/(c^2-d^2)^{1/2}\arctan(1/2(2c\tan(1/2fx+1/2e)+2d)/(c^2-d^2)^{1/2})} \frac{B+2a/fB/d^2\arctan(\tan(1/2fx+1/2e))}{B+2a/fB/d^2\arctan(\tan(1/2fx+1/2e))}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 20.32, size = 5102, normalized size = 41.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^2,x)

[Out]
$$\frac{(2B^2a^2\operatorname{atan}\left(\frac{B^2a^2(32(B^2a^2c^2d^3 + 2B^2a^2c^3d^2 + B^2a^2c^4d) + (2cd^3 + d^4 + c^2d^2) + (32\tan(e/2 + (fx)/2)*(6B^2a^2c^2d^4 + 2B^2a^2c^3d^3 - 4B^2a^2c^4d^2 - A^2a^2cd^5 + B^2a^2cd^5 - 2B^2a^2c^5d + 2AB^2a^2c^2d^4 + 2AB^2a^2c^3d^3 - 2AB^2a^2cd^5))}{2cd^4 + d^5 + c^2d^3} + (B^2a^2(32\tan(e/2 + (fx)/2)*(2A^2cd^7 + 2B^2a^2cd^7 + 2A^2c^2d^6 - 4B^2a^2c^3d^5 - 2B^2a^2c^4d^4))}{(2cd^4 + d^5 + c^2d^3)} - (32(B^2a^2cd^6 - A^2cd^5 - A^2c^3d^4 + B^2a^2cd^5))}{(2cd^3 + d^4 + c^2d^2)} + (B^2a^2(32(c^2d^7 + 2c^3d^6 + c^4d^5))}{(2cd^3 + d^4 + c^2d^2)} + (32\tan(e/2 + (fx)/2)*(3cd^9 + 6c^2d^8 + c^3d^7 - 4c^4d^6 - 2c^5d^5))}{(2cd^4 + d^5 + c^2d^3)}*1i)/d^2)*1i)/d^2))/d^2 + (B^2a^2(32(B^2a^2c^2d^3 + 2B^2a^2c^3d^2 + B^2a^2c^4d))}{(2cd^3 + d^4 + c^2d^2)} + (32\tan(e/2 + (fx)/2)*(6B^2a^2c^2d^4 + 2B^2a^2c^3d^3 - 4B^2a^2c^4d^2 - A^2a^2cd^5 + B^2a^2cd^5 - 2B^2a^2c^5d + 2AB^2a^2c^2d^4 + 2AB^2a^2c^3d^3 - 2AB^2a^2cd^5))}{(2cd^4 + d^5 + c^2d^3)} + (B^2a^2(32(B^2a^2cd^6 - A^2cd^5 - A^2c^3d^4 + B^2a^2cd^5))}{(2cd^3 + d^4 + c^2d^2)} - (32\tan(e/2 + (fx)/2)*(2A^2cd^7 + 2B^2a^2cd^7 + 2A^2c^2d^6 - 4B^2a^2c^3d^5 - 2B^2a^2c^4d^4))}{(2cd^4 + d^5 + c^2d^3)} + (B^2a^2(32(c^2d^7 + 2c^3d^6 + c^4d^5))}{(2cd^3 + d^4 + c^2d^2)} + (32\tan(e/2 + (fx)/2)*(3cd^9 + 6c^2d^8 + c^3d^7 - 4c^4d^6 - 2c^5d^5))}{(2cd^4 + d^5 + c^2d^3)}*1i)/d^2)*1i)/d^2))/d^2)/((64(B^3a^3c^3 + AB^2a^3c^3 - B^3a^3cd^2 + B^3a^3c^2d - 2AB^2a^3cd^2 + AB^2a^3c^2d - A^2B^2a^3cd^2))}{(2cd^3 + d^4 + c^2d^2)} - (64\tan(e/2 + (fx)/2)*(2B^3a^3cd^3 - 2B^3a^3c^4 - 4B^3a^3c^3d + 2AB^2a^3cd^3 + 2AB^2a^3c^2d^2))}{(2cd^4 + d^5 + c^2d^3)} - (B^2a^2(32(B^2a^2c^2d^3 + 2B^2a^2c^3d^2 + B^2a^2c^4d))}{(2cd^3 + d^4 + c^2d^2)} + (32\tan(e/2 + (fx)/2)*(6B^2a^2c^2d^4 + 2B^2a^2c^3d^3 - 4B^2a^2c^4d^2 - A^2a^2cd^5 + B^2a^2cd^5 - 2B^2a^2c^5d + 2AB^2a^2c^2d^4 + 2AB^2a^2c^3d^3 - 2AB^2a^2cd^5))}{(2cd^4 + d^5 + c^2d^3)} + (B^2a^2(32\tan(e/2 + (fx)/2)*(2A^2cd^7 + 2B^2a^2cd^7 + 2A^2c^2d^6 - 4B^2a^2c^3d^5 - 2B^2a^2c^4d^4))}{(2cd^4 + d^5 + c^2d^3)} - (32(B^2a^2cd^6 - A^2cd^5 - A^2c^3d^4 + B^2a^2cd^5))}{(2cd^3 + d^4 + c^2d^2)} + (B^2a^2(32(c^2d^7 + 2c^3d^6 + c^4d^5))}{(2cd^3 + d^4 + c^2d^2)} + (32\tan(e/2 + (fx)/2)*(3cd^9 + 6c^2d^8 + c^3d^7 - 4c^4d^6 - 2c^5d^5))}{(2cd^4 + d^5 + c^2d^3)}*1i)/d^2)*1i)/d^2))/d^2)$$

$$\begin{aligned} & *c^4*d))/(2*c*d^3 + d^4 + c^2*d^2) + (32*\tan(e/2 + (f*x)/2)*(6*B^2*a^2*c^2*d^4 + 2*B^2*a^2*c^3*d^3 - 4*B^2*a^2*c^4*d^2 - A^2*a^2*c*d^5 + B^2*a^2*c*d^5 \\ & - 2*B^2*a^2*c^5*d + 2*A*B*a^2*c^2*d^4 + 2*A*B*a^2*c^3*d^3 - 2*A*B*a^2*c*d^5))/(2*c*d^4 + d^5 + c^2*d^3) + (a*(-(c + d)^3*(c - d))^{(1/2)}*((32*(B*a*c*d^6 - A*a*c^2*d^5 - A*a*c^3*d^4 + B*a*c^2*d^5))/(2*c*d^3 + d^4 + c^2*d^2) - \\ & (32*\tan(e/2 + (f*x)/2)*(2*A*a*c*d^7 + 2*B*a*c*d^7 + 2*A*a*c^2*d^6 - 4*B*a*c^3*d^5 - 2*B*a*c^4*d^4))/(2*c*d^4 + d^5 + c^2*d^3) + (a*((32*(c^2*d^7 + 2*c^3*d^6 + c^4*d^5))/(2*c*d^3 + d^4 + c^2*d^2) + (32*\tan(e/2 + (f*x)/2)*(3*c*d^9 + 6*c^2*d^8 + c^3*d^7 - 4*c^4*d^6 - 2*c^5*d^5))/(2*c*d^4 + d^5 + c^2*d^3) \\ & 3))*(-(c + d)^3*(c - d))^{(1/2)}*(A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*((A*d^2 - B*c^2 + B*d^2 - B*c*d))/(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2))*(-(c + d)^3*(c - d))^{(1/2)}*(A*d^2 - B*c^2 + B*d^2 - B*c*d)*2i)/(f*(2*c*d^5 + d^6 - 2*c^3*d^3 - c^4*d^2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out

$$3.250 \quad \int \frac{(a+a \sin(e+fx))(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=176

$$\frac{a(2Ac - Ad + Bc - 2Bd) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{f(c+d)(c^2 - d^2)^{3/2}} - \frac{a(Ad(c-2d) + B(c^2 + 2cd - 2d^2)) \cos(e+fx)}{2df(c-d)(c+d)^2(c+d \sin(e+fx))} + \frac{a(Bc - Ad)}{2df(c+d)(c+d \sin(e+fx))}$$

[Out] a*(2*A*c-A*d+B*c-2*B*d)*arctan((d+c*tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/(c+d)/(c^2-d^2)^(3/2)/f+1/2*a*(-A*d+B*c)*cos(f*x+e)/d/(c+d)/f/(c+d*sin(f*x+e))^2-1/2*a*(A*(c-2*d)*d+B*(c^2+2*c*d-2*d^2))*cos(f*x+e)/(c-d)/d/(c+d)^2/f/(c+d*sin(f*x+e))

Rubi [A] time = 0.42, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {2968, 3021, 2754, 12, 2660, 618, 204}

$$\frac{a(2Ac - Ad + Bc - 2Bd) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{f(c+d)(c^2 - d^2)^{3/2}} - \frac{a(Ad(c-2d) + B(c^2 + 2cd - 2d^2)) \cos(e+fx)}{2df(c-d)(c+d)^2(c+d \sin(e+fx))} + \frac{a(Bc - Ad)}{2df(c+d)(c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] (a*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(d + c*Tan[(e + f*x)/2]]/Sqrt[c^2 - d^2])/((c + d)*(c^2 - d^2)^(3/2)*f) + (a*(B*c - A*d)*Cos[e + f*x])/(2*d*(c + d)*f*(c + d*Sin[e + f*x])^2) - (a*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos[e + f*x])/(2*(c - d)*d*(c + d)^2*f*(c + d*Sin[e + f*x]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2754

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m, x]

$*x])^{(m+1)}/(f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*\text{Simp}[(a*c - b*d)*(m+1) - (b*c - a*d)*(m+2)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m]$

Rule 2968

$\text{Int}[(a_.* + (b_.*\text{sin}[(e_.* + (f_.*(x_.*))])^{(m_.*)}*((A_.* + (B_.*\text{sin}[(e_.* + (f_.*(x_.*))])*(c_.* + (d_.*\text{sin}[(e_.* + (f_.*(x_.*))])], x_Symbol] := \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 3021

$\text{Int}[(a_.* + (b_.*\text{sin}[(e_.* + (f_.*(x_.*))])^{(m_.*)}*((A_.* + (B_.*\text{sin}[(e_.* + (f_.*(x_.*))]) + (C_.*\text{sin}[(e_.* + (f_.*(x_.*))])^2), x_Symbol] := -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)}/(b*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))(A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx &= \int \frac{aA + (aA + aB) \sin(e + fx) + aB \sin^2(e + fx)}{(c + d \sin(e + fx))^3} dx \\ &= \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-2a(A+B)(c-d)d - a(c-d)(Ad+B(c+d \sin(e+fx)))}{(c+d \sin(e+fx))^2} dx}{2d(c^2 - d^2)} \\ &= \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a(A(c - 2d)d + B(c^2 + 2cd - d^2))}{2(c - d)d(c + d)^2 f(c + d \sin(e + fx))} \\ &= \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a(A(c - 2d)d + B(c^2 + 2cd - d^2))}{2(c - d)d(c + d)^2 f(c + d \sin(e + fx))} \\ &= \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a(A(c - 2d)d + B(c^2 + 2cd - d^2))}{2(c - d)d(c + d)^2 f(c + d \sin(e + fx))} \\ &= \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a(A(c - 2d)d + B(c^2 + 2cd - d^2))}{2(c - d)d(c + d)^2 f(c + d \sin(e + fx))} \\ &= \frac{a(2Ac + Bc - Ad - 2Bd) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{(c - d)(c + d)^2 \sqrt{c^2 - d^2} f} + \frac{a(Bc - Ad)}{2d(c + d)f(c + d \sin(e + fx))} \end{aligned}$$

Mathematica [C] time = 2.71, size = 345, normalized size = 1.96

$$a(\sin(e + fx) + 1) \left(\frac{d \csc(e) \left((Ad^2(d-2c) + Bc(2c^2 + 2cd - 3d^2)) \sin(2e + fx) - d(Ad(c-2d) + B(c^2 + 2cd - 2d^2)) \cos(e + 2fx) + \sin(fx)(Bc(2c^2 + 6cd - 5d^2) + Ad^2(d-2c) + Bc(2c^2 + 2cd - 3d^2)) \right)}{d^2(c + d \sin(e + fx))^2} \right)$$

$$4f(c - d)(c + d)^2 \left(\sin(e + fx) + 1 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
```

```
[Out] (a*(1 + Sin[e + f*x])*((4*(2*A*c + B*c - A*d - 2*B*d)*ArcTan[(Sec[(f*x)/2]*
(Cos[e] - I*Sin[e])*(d*Cos[e + (f*x)/2] + c*Sin[(f*x)/2])]/(Sqrt[c^2 - d^2]
*Sqrt[(Cos[e] - I*Sin[e])^2]))*(Cos[e] - I*Sin[e]))/(Sqrt[c^2 - d^2]*Sqrt[(
Cos[e] - I*Sin[e])^2]) + ((2*c^2 + d^2)*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2
*d^2))*Cot[e] + d*Csc[e]*(-(d*(A*(c - 2*d)*d + B*(c^2 + 2*c*d - 2*d^2))*Cos
[e + 2*f*x]) + (B*c*(2*c^2 + 6*c*d - 5*d^2) - A*d*(-4*c^2 + 6*c*d + d^2))*S
in[f*x] + (A*d^2*(-2*c + d) + B*c*(2*c^2 + 2*c*d - 3*d^2))*Sin[2*e + f*x]))
/(d^2*(c + d*Sin[e + f*x]^2)))/(4*(c - d)*(c + d)^2*f*(Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])^2)
```

fricas [B] time = 0.53, size = 967, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(2*(B*a*c^4 + (A + 2*B)*a*c^3*d - (2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*
c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*sin(f*x + e) + ((2*A + B)*a*c^3 - (A
+ 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 -
(A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2
)*sin(f*x + e)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*
sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))
*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) +
2*(2*(A + B)*a*c^4 - (2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)
*a*c*d^3 + A*a*d^4)*cos(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d
^5 + c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3
*d^4 + c^2*d^5 + c*d^6)*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 -
c^3*d^4 - c^2*d^5 + c*d^6 + d^7)*f), 1/2*((B*a*c^4 + (A + 2*B)*a*c^3*d - (
2*A + 3*B)*a*c^2*d^2 - (A + 2*B)*a*c*d^3 + 2*(A + B)*a*d^4)*cos(f*x + e)*si
n(f*x + e) + ((2*A + B)*a*c^3 - (A + 2*B)*a*c^2*d + (2*A + B)*a*c*d^2 - (A
+ 2*B)*a*d^3 - ((2*A + B)*a*c*d^2 - (A + 2*B)*a*d^3)*cos(f*x + e)^2 + 2*((2
*A + B)*a*c^2*d - (A + 2*B)*a*c*d^2)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-
(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*(A + B)*a*c^4 - (
2*A + B)*a*c^3*d - (3*A + 2*B)*a*c^2*d^2 + (2*A + B)*a*c*d^3 + A*a*d^4)*cos
(f*x + e))/((c^5*d^2 + c^4*d^3 - 2*c^3*d^4 - 2*c^2*d^5 + c*d^6 + d^7)*f*cos
(f*x + e)^2 - 2*(c^6*d + c^5*d^2 - 2*c^4*d^3 - 2*c^3*d^4 + c^2*d^5 + c*d^6)
*f*sin(f*x + e) - (c^7 + c^6*d - c^5*d^2 - c^4*d^3 - c^3*d^4 - c^2*d^5 + c*
d^6 + d^7)*f)]
```

giac [B] time = 0.23, size = 594, normalized size = 3.38

$$\frac{(2 Aac+Bac-Aad-2 Bad)\left(\pi\left[\frac{fx+e}{2\pi}+\frac{1}{2}\right]\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^3+c^2d-cd^2-d^3)\sqrt{c^2-d^2}} + \frac{Bac^4 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 3 Aac^3 d \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 2 Bac^3 d \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3}{(c^3+c^2d-cd^2-d^3)\sqrt{c^2-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="giac")
```

```
[Out] ((2*A*a*c + B*a*c - A*a*d - 2*B*a*d)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(
c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^3 + c^2*d -
```

$$\begin{aligned} & c*d^2 - d^3)*\text{sqrt}(c^2 - d^2)) + (B*a*c^4*\tan(1/2*f*x + 1/2*e)^3 - 3*A*a*c^3 \\ & *d*\tan(1/2*f*x + 1/2*e)^3 - 2*B*a*c^3*d*\tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c^2* \\ & d^2*\tan(1/2*f*x + 1/2*e)^3 + 2*A*a*c*d^3*\tan(1/2*f*x + 1/2*e)^3 - 2*A*a*c^4 \\ & *\tan(1/2*f*x + 1/2*e)^2 - 2*B*a*c^4*\tan(1/2*f*x + 1/2*e)^2 + 2*A*a*c^3*d*\tan \\ & n(1/2*f*x + 1/2*e)^2 + B*a*c^3*d*\tan(1/2*f*x + 1/2*e)^2 - 3*A*a*c^2*d^2*\tan \\ & (1/2*f*x + 1/2*e)^2 - 4*B*a*c^2*d^2*\tan(1/2*f*x + 1/2*e)^2 + 4*A*a*c*d^3*\tan \\ & n(1/2*f*x + 1/2*e)^2 + 2*B*a*c*d^3*\tan(1/2*f*x + 1/2*e)^2 + 2*A*a*d^4*\tan(1 \\ & /2*f*x + 1/2*e)^2 - B*a*c^4*\tan(1/2*f*x + 1/2*e) - 5*A*a*c^3*d*\tan(1/2*f*x \\ & + 1/2*e) - 6*B*a*c^3*d*\tan(1/2*f*x + 1/2*e) + 6*A*a*c^2*d^2*\tan(1/2*f*x + 1 \\ & /2*e) + 4*B*a*c^2*d^2*\tan(1/2*f*x + 1/2*e) + 2*A*a*c*d^3*\tan(1/2*f*x + 1/2* \\ & e) - 2*A*a*c^4 - 2*B*a*c^4 + 2*A*a*c^3*d + B*a*c^3*d + A*a*c^2*d^2)/((c^5 + \\ & c^4*d - c^3*d^2 - c^2*d^3)*(c*\tan(1/2*f*x + 1/2*e)^2 + 2*d*\tan(1/2*f*x + 1 \\ & /2*e) + c)^2))/f \end{aligned}$$

maple [B] time = 0.52, size = 2021, normalized size = 11.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)`

[Out]
$$\begin{aligned} & -4*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2 \\ & -d^3)*\tan(1/2*f*x+1/2*e)^2*B*d^2+6*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f* \\ & x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)*A*d^2+4*a/f/(\tan(1 \\ & /2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2 \\ & *f*x+1/2*e)*B*d^2-a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c \\ & ^2/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)*B+2*a/f/(\tan(1/2*f*x+1/2*e)^2*c \\ & +2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*c*d+a/f/(\tan(1/2*f*x+1 \\ & /2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*B*c*d+a/f/(\tan(\\ & 1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^3+c^2*d-c*d^2-d^3)*\tan \\ & n(1/2*f*x+1/2*e)^3*B+2*a/f/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^{(1/2)}*\arctan(1/2 \\ & *(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c-a/f/(c^3+c^2*d-c*d^2-d^3 \\ &)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})* \\ & A*d-6*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d- \\ & c*d^2-d^3)*\tan(1/2*f*x+1/2*e)*B*d+2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f \\ & *x+1/2*e)*d+c)^2/c/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)*A*d^3-3*a/f/(\tan \\ & (1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*\tan \\ & n(1/2*f*x+1/2*e)^3*A*d+2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d \\ & +c)^2/c/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)^3*A*d^3-5*a/f/(\tan(1/2*f*x \\ & +1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x \\ & +1/2*e)*A*d-2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^ \\ & 3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)^3*B*d+2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2 \\ & *\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*\tan(1/2*f*x+1/2*e)^2*A*d \\ & +2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2 \\ & -d^3)/c*\tan(1/2*f*x+1/2*e)^2*B*d^3+4*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2* \\ & f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c*\tan(1/2*f*x+1/2*e)^2*A*d^3+2*a/f/ \\ & (\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)/c \\ & ^2*\tan(1/2*f*x+1/2*e)^2*A*d^4+a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2 \\ & *e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c*\tan(1/2*f*x+1/2*e)^2*B*d-2*a/f/(\tan(1/2* \\ & f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A*c^2+a/f/ \\ & (\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*A \\ & *d^2-2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c \\ & *d^2-d^3)*B*c^2+a/f/(c^3+c^2*d-c*d^2-d^3)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*t \\ & an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c-2*a/f/(c^3+c^2*d-c*d^2-d^3)/(c^ \\ & 2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*d-2 \\ & *a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d \\ & ^3)*c^2*\tan(1/2*f*x+1/2*e)^2*A-2*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+ \\ & 1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*c^2*\tan(1/2*f*x+1/2*e)^2*B+2*a/f/(\tan(1 \\ & /2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2 \end{aligned}$$

$*f*x+1/2*e)^3*A*d^2-3*a/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^3+c^2*d-c*d^2-d^3)*\tan(1/2*f*x+1/2*e)^2*A*d^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 15.59, size = 554, normalized size = 3.15

$$\frac{\frac{Aad^2-2Aac^2-2Bac^2+2Aacd+Bacd}{-c^3-c^2d+cd^2+d^3} + \frac{a \tan\left(\frac{e}{2} + \frac{fx}{2}\right) (2Ad^3 - Bc^3 + 6Ac^2d - 5Ac^2d + 4Bcd^2 - 6Bc^2d)}{c(-c^3 - c^2d + cd^2 + d^3)} + \frac{a \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 (2Ad^3 + Bc^3 + 2Acd^2 - 2Bc^3)}{c(-c^3 - c^2d + cd^2 + d^3)}}{f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 (2c^2 + 4d^2) + c^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^4 + c^2 + 4cd \tan\left(\frac{e}{2} + \frac{fx}{2}\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x)))/(c + d*sin(e + f*x))^3,x)

[Out] - ((A*a*d^2 - 2*A*a*c^2 - 2*B*a*c^2 + 2*A*a*c*d + B*a*c*d)/(c*d^2 - c^2*d - c^3 + d^3) + (a*tan(e/2 + (f*x)/2)*(2*A*d^3 - B*c^3 + 6*A*c*d^2 - 5*A*c^2*d + 4*B*c*d^2 - 6*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^3*(2*A*d^3 + B*c^3 + 2*A*c*d^2 - 3*A*c^2*d - 2*B*c^2*d))/(c*(c*d^2 - c^2*d - c^3 + d^3)) + (a*tan(e/2 + (f*x)/2)^2*(c^2 + 2*d^2)*(A*d^2 - 2*A*c^2 - 2*B*c^2 + 2*A*c*d + B*c*d))/(c^2*(c*d^2 - c^2*d - c^3 + d^3)))/(f*(tan(e/2 + (f*x)/2)^2*(2*c^2 + 4*d^2) + c^2*tan(e/2 + (f*x)/2)^4 + c^2 + 4*c*d*tan(e/2 + (f*x)/2)^3 + 4*c*d*tan(e/2 + (f*x)/2))) - (a*atan((((a*(2*A*c - A*d + B*c - 2*B*d)*(2*c*d^3 - 2*c^3*d + 2*d^4 - 2*c^2*d^2))/(2*(c + d)^(5/2)*(c - d)^(3/2)*(c*d^2 - c^2*d - c^3 + d^3)) + (a*c*tan(e/2 + (f*x)/2)*(2*A*c - A*d + B*c - 2*B*d))/((c + d)^(5/2)*(c - d)^(3/2)))*(c*d^2 - c^2*d - c^3 + d^3))/(2*A*a*c - A*a*d + B*a*c - 2*B*a*d))*(2*A*c - A*d + B*c - 2*B*d))/(f*(c + d)^(5/2)*(c - d)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)

[Out] Timed out

3.251 $\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$

Optimal. Leaf size=464

$$\frac{a^2 (6Ad(c - 10d) - B(2c^2 - 12cd + 55d^2)) \cos(e + fx)(c + d \sin(e + fx))^3}{120d^2 f} + \frac{a^2 (6Ad(c^2 - 10cd - 12d^2) - B(2c^5 - 12c^4d + 47c^3d^2 + 208c^2d^3 + 216cd^4 + 64d^5)) \cos(e + fx)(c + d \sin(e + fx))^2}{60d^2 f}$$

```
[Out] 1/16*a^2*(6*A*(4*c^3+8*c^2*d+7*c*d^2+2*d^3)+B*(16*c^3+42*c^2*d+36*c*d^2+11*d^3))*x+1/60*a^2*(6*A*d*(c^4-10*c^3*d-44*c^2*d^2-40*c*d^3-12*d^4)-B*(2*c^5-12*c^4*d+47*c^3*d^2+208*c^2*d^3+216*c*d^4+64*d^5))*cos(f*x+e)/d^2/f+1/240*a^2*(6*A*d*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)-B*(4*c^4-24*c^3*d+96*c^2*d^2+284*c*d^3+165*d^4))*cos(f*x+e)*sin(f*x+e)/d/f+1/120*a^2*(6*A*d*(c^2-10*c*d-12*d^2)-B*(2*c^3-12*c^2*d+51*c*d^2+64*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/120*a^2*(6*A*(c-10*d)*d-B*(2*c^2-12*c*d+55*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f+1/30*a^2*(-6*A*d+2*B*c-7*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^2/f-1/6*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^4/d/f
```

Rubi [A] time = 0.95, antiderivative size = 464, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2 (6Ad(-44c^2d^2 - 10c^3d + c^4 - 40cd^3 - 12d^4) - B(47c^3d^2 + 208c^2d^3 - 12c^4d + 2c^5 + 216cd^4 + 64d^5)) \cos(e + fx)(c + d \sin(e + fx))^3}{60d^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
[Out] (a^2*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*x)/16 + (a^2*(6*A*d*(c^4 - 10*c^3*d - 44*c^2*d^2 - 40*c*d^3 - 12*d^4) - B*(2*c^5 - 12*c^4*d + 47*c^3*d^2 + 208*c^2*d^3 + 216*c*d^4 + 64*d^5))*Cos[e + f*x])/(60*d^2*f) + (a^2*(6*A*d*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3) - B*(4*c^4 - 24*c^3*d + 96*c^2*d^2 + 284*c*d^3 + 165*d^4))*Cos[e + f*x]*Sin[e + f*x])/(240*d*f) + (a^2*(6*A*d*(c^2 - 10*c*d - 12*d^2) - B*(2*c^3 - 12*c^2*d + 51*c*d^2 + 64*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^2*f) + (a^2*(6*A*(c - 10*d)*d - B*(2*c^2 - 12*c*d + 55*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(120*d^2*f) + (a^2*(2*B*c - 6*A*d - 7*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(30*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2*sin[e + f*x])*(c + d*Sin[e + f*x])^4)/(6*d*f)
```

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*SIN[e + f*x])^m*(A*c + (B*c + A*d)*SIN[e + f*x] + B*d*SIN[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 1)*(c + d*SIN[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*SIN[e + f*x])^(m - 1)*(c + d*SIN[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*SIN[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*SIN[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*SIN[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx &= -\frac{B \cos(e + fx) (a^2 + a^2 \sin(e + fx)) (c + d \sin(e + fx))}{6df} \\ &= -\frac{B \cos(e + fx) (a^2 + a^2 \sin(e + fx)) (c + d \sin(e + fx))}{6df} \\ &= \frac{a^2 (2Bc - 6Ad - 7Bd) \cos(e + fx) (c + d \sin(e + fx))}{30d^2 f} \\ &= \frac{a^2 (6A(c - 10d)d - B(2c^2 - 12cd + 55d^2)) \cos(e + fx)}{120d^2 f} \\ &= \frac{a^2 (6Ad(c^2 - 10cd - 12d^2) - B(2c^3 - 12c^2d + 12cd^2 - 11d^3)) \cos(e + fx)}{120d^2 f} \\ &= \frac{1}{16} a^2 (6A(4c^3 + 8c^2d + 7cd^2 + 2d^3) + B(16c^3 + 42c^2d + 36cd^2 + 11d^3)) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{c^2 + d^2} \end{aligned}$$

Mathematica [A] time = 3.13, size = 437, normalized size = 0.94

$$\frac{a^2 \cos(e + fx) \left(60 (6A (4c^3 + 8c^2d + 7cd^2 + 2d^3) + B (16c^3 + 42c^2d + 36cd^2 + 11d^3)) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{c^2 + d^2} \right)}{120d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*SIN[e + f*x])^2*(A + B*SIN[e + f*x])*(c + d*SIN[e + f*x])^3,x]
```

```
[Out] -1/480*(a^2*Cos[e + f*x]*(60*(6*A*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3) + B*(16*c^3 + 42*c^2*d + 36*c*d^2 + 11*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[
```

2]] + Sqrt[Cos[e + f*x]^2]*(960*A*c^3 + 880*B*c^3 + 2640*A*c^2*d + 2400*B*c^2*d + 2400*A*c*d^2 + 2268*B*c*d^2 + 756*A*d^3 + 712*B*d^3 - 16*(3*A*d*(5*c^2 + 10*c*d + 4*d^2) + B*(5*c^3 + 30*c^2*d + 36*c*d^2 + 14*d^3))*Cos[2*(e + f*x)] + 12*d^2*(3*B*c + A*d + 2*B*d)*Cos[4*(e + f*x)] + 240*A*c^3*Sin[e + f*x] + 480*B*c^3*Sin[e + f*x] + 1440*A*c^2*d*Sin[e + f*x] + 1530*B*c^2*d*Sin[e + f*x] + 1530*A*c*d^2*Sin[e + f*x] + 1620*B*c*d^2*Sin[e + f*x] + 540*A*d^3*Sin[e + f*x] + 545*B*d^3*Sin[e + f*x] - 90*B*c^2*d*Sin[3*(e + f*x)] - 90*A*c*d^2*Sin[3*(e + f*x)] - 180*B*c*d^2*Sin[3*(e + f*x)] - 60*A*d^3*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)] + 5*B*d^3*Sin[5*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])

fricas [A] time = 0.50, size = 364, normalized size = 0.78

$$48 \left(3Ba^2cd^2 + (A + 2B)a^2d^3 \right) \cos(fx + e)^5 - 80 \left(Ba^2c^3 + 3(A + 2B)a^2c^2d + 3(2A + 3B)a^2cd^2 + (3A + 4B)a^2d^3 \right) \cos(fx + e)^3 - 15 \left(8(3A + 2B)a^2c^3 + 6(8A + 7B)a^2c^2d + 6(7A + 6B)a^2cd^2 + (12A + 11B)a^2d^3 \right) f \cos(fx + e) + 480 \left((A + B)a^2c^3 + 3(A + B)a^2c^2d + 3(A + B)a^2cd^2 + (A + B)a^2d^3 \right) \cos(fx + e) + 5 \left(8Ba^2d^3 \cos(fx + e)^5 - 2(18Ba^2c^2d + 18(A + 2B)a^2cd^2 + (12A + 19B)a^2d^3) \cos(fx + e)^3 + 3(8(A + 2B)a^2c^3 + 6(8A + 9B)a^2c^2d + 6(9A + 10B)a^2cd^2 + (20A + 21B)a^2d^3) \cos(fx + e) \right) \sin(fx + e) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/240*(48*(3*B*a^2*c*d^2 + (A + 2*B)*a^2*d^3)*cos(f*x + e)^5 - 80*(B*a^2*c^3 + 3*(A + 2*B)*a^2*c^2*d + 3*(2*A + 3*B)*a^2*c*d^2 + (3*A + 4*B)*a^2*d^3)*cos(f*x + e)^3 - 15*(8*(3*A + 2*B)*a^2*c^3 + 6*(8*A + 7*B)*a^2*c^2*d + 6*(7*A + 6*B)*a^2*c*d^2 + (12*A + 11*B)*a^2*d^3)*f*x + 480*((A + B)*a^2*c^3 + 3*(A + B)*a^2*c^2*d + 3*(A + B)*a^2*c*d^2 + (A + B)*a^2*d^3)*cos(f*x + e) + 5*(8*B*a^2*d^3*cos(f*x + e)^5 - 2*(18*B*a^2*c^2*d + 18*(A + 2*B)*a^2*c*d^2 + (12*A + 19*B)*a^2*d^3)*cos(f*x + e)^3 + 3*(8*(A + 2*B)*a^2*c^3 + 6*(8*A + 9*B)*a^2*c^2*d + 6*(9*A + 10*B)*a^2*c*d^2 + (20*A + 21*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.22, size = 474, normalized size = 1.02

$$-\frac{Ba^2d^3 \sin(6fx + 6e)}{192f} + \frac{1}{16} \left(24Aa^2c^3 + 16Ba^2c^3 + 48Aa^2c^2d + 42Ba^2c^2d + 42Aa^2cd^2 + 36Ba^2cd^2 + 12Aa^2d^3 \right) \cos(fx + e)^5 - \frac{1}{80} \left(3Ba^2c^3 + Aa^2d^3 + 2Ba^2d^3 \right) \cos(fx + e)^3 - \frac{1}{48} \left(4Ba^2c^3 + 12Aa^2c^2d + 24Ba^2c^2d + 24Aa^2c^2d + 27Ba^2c^2d + 9Aa^2d^3 + 10Ba^2d^3 \right) \cos(3fx + 3e) / f - \frac{1}{8} \left(16Aa^2c^3 + 14Ba^2c^3 + 42Aa^2c^2d + 36Ba^2c^2d + 36Aa^2c^2d + 33Ba^2c^2d + 11Aa^2d^3 + 10Ba^2d^3 \right) \cos(fx + e) / f + \frac{1}{64} \left(6Ba^2c^2d + 6Aa^2c^2d + 12Ba^2c^2d + 4Aa^2d^3 + 5Ba^2d^3 \right) \sin(4fx + 4e) / f - \frac{1}{64} \left(16Aa^2c^3 + 32Ba^2c^3 + 96Aa^2c^2d + 96Ba^2c^2d + 96Aa^2c^2d + 96Ba^2c^2d + 32Aa^2d^3 + 31Ba^2d^3 \right) \sin(2fx + 2e) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] -1/192*B*a^2*d^3*sin(6*f*x + 6*e)/f + 1/16*(24*A*a^2*c^3 + 16*B*a^2*c^3 + 48*A*a^2*c^2*d + 42*B*a^2*c^2*d + 42*A*a^2*c*d^2 + 36*B*a^2*c*d^2 + 12*A*a^2*d^3 + 11*B*a^2*d^3)*x - 1/80*(3*B*a^2*c*d^2 + A*a^2*d^3 + 2*B*a^2*d^3)*cos(5*f*x + 5*e)/f + 1/48*(4*B*a^2*c^3 + 12*A*a^2*c^2*d + 24*B*a^2*c^2*d + 24*A*a^2*c^2*d + 27*B*a^2*c^2*d + 9*A*a^2*d^3 + 10*B*a^2*d^3)*cos(3*f*x + 3*e)/f - 1/8*(16*A*a^2*c^3 + 14*B*a^2*c^3 + 42*A*a^2*c^2*d + 36*B*a^2*c^2*d + 36*A*a^2*c^2*d + 33*B*a^2*c^2*d + 11*A*a^2*d^3 + 10*B*a^2*d^3)*cos(f*x + e)/f + 1/64*(6*B*a^2*c^2*d + 6*A*a^2*c^2*d + 12*B*a^2*c^2*d + 4*A*a^2*d^3 + 5*B*a^2*d^3)*sin(4*f*x + 4*e)/f - 1/64*(16*A*a^2*c^3 + 32*B*a^2*c^3 + 96*A*a^2*c^2*d + 96*B*a^2*c^2*d + 96*A*a^2*c^2*d + 96*B*a^2*c^2*d + 32*A*a^2*d^3 + 31*B*a^2*d^3)*sin(2*f*x + 2*e)/f

maple [A] time = 0.64, size = 745, normalized size = 1.61

$$a^2Ac^3 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - a^2Ac^2d \left(2 + \sin^2(fx + e) \right) \cos(fx + e) + 3a^2Ac^2d^2 \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2} \right)}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sin(f*x+e))^2*(A+B*\sin(f*x+e))*(c+d*\sin(f*x+e))^3,x)$

[Out] $\frac{1}{f}*(a^2*A*c^3*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-a^2*A*c^2*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)+3*a^2*A*c*d^2*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^2*A*d^3*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)-1/3*B*a^2*c^3*(2+\sin(f*x+e)^2)*\cos(f*x+e)+3*B*a^2*c^2*d*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^2*c*d^2*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)+B*a^2*d^3*(-1/6*(\sin(f*x+e)^5+5/4*\sin(f*x+e)^3+15/8*\sin(f*x+e))*\cos(f*x+e)+5/16*f*x+5/16*e)-2*a^2*A*c^3*\cos(f*x+e)+6*a^2*A*c^2*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-2*a^2*A*c*d^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)+2*a^2*A*d^3*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)+2*B*a^2*c^3*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-2*B*a^2*c^2*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)+6*B*a^2*c*d^2*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^2*d^3*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)+a^2*A*c^3*(f*x+e)-3*a^2*A*c^2*d*\cos(f*x+e)+3*a^2*A*c*d^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d^3*(2+\sin(f*x+e)^2)*\cos(f*x+e)-B*a^2*c^3*\cos(f*x+e)+3*B*a^2*c^2*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c*d^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)+B*a^2*d^3*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e))$

maxima [A] time = 0.46, size = 724, normalized size = 1.56

$240(2fx + 2e - \sin(2fx + 2e))Aa^2c^3 + 960(fx + e)Aa^2c^3 + 320(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2c^3 + 480$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sin(f*x+e))^2*(A+B*\sin(f*x+e))*(c+d*\sin(f*x+e))^3,x, \text{algorithm}="maxima")$

[Out] $\frac{1}{960}*(240*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2*c^3 + 960*(f*x + e)*A*a^2*c^3 + 320*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2*c^3 + 480*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^2*c^3 + 960*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^2*c^2*d + 1440*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2*c^2*d + 1920*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2*c^2*d + 90*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^2*c^2*d + 720*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^2*c^2*d + 1920*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^2*c*d^2 + 90*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^2*c*d^2 + 720*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2*c*d^2 - 192*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a^2*c*d^2 + 960*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2*c*d^2 + 180*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^2*c*d^2 - 64*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a^2*d^3 + 320*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^2*d^3 + 60*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^2*d^3 - 128*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*B*a^2*d^3 + 5*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B*a^2*d^3 + 30*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^2*d^3 - 1920*A*a^2*c^3*\cos(f*x + e) - 960*B*a^2*c^3*\cos(f*x + e) - 2880*A*a^2*c^2*d*\cos(f*x + e))/f$

mupad [B] time = 15.99, size = 1291, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*\sin(e + f*x))*(a + a*\sin(e + f*x))^2*(c + d*\sin(e + f*x))^3,x)$

[Out] $(a^2*\text{atan}((a^2*\tan(e/2 + (f*x)/2)*(24*A*c^3 + 12*A*d^3 + 16*B*c^3 + 11*B*d^3 + 42*A*c*d^2 + 48*A*c^2*d + 36*B*c*d^2 + 42*B*c^2*d)))/(8*(3*A*a^2*c^3 + ($

$$\begin{aligned}
& 3Aa^2d^3)/2 + 2Ba^2c^3 + (11Ba^2d^3)/8 + (21Aa^2cd^2)/4 + 6Aa^2c^2d + (9Ba^2cd^2)/2 + (21Ba^2c^2d)/4)) \cdot (24Ac^3 + 12Ad^3 \\
& + 16Bc^3 + 11Bd^3 + 42Acd^2 + 48Ac^2d + 36Bcd^2 + 42Bc^2d) \\
& / (8f) - (\tan(e/2 + (fx)/2) \cdot (Aa^2c^3 + (3Aa^2d^3)/2 + 2Ba^2c^3 + (11Ba^2d^3)/8 \\
& + (21Aa^2cd^2)/4 + 6Aa^2c^2d + (9Ba^2cd^2)/2 + (21Ba^2c^2d)/4) + \tan(e/2 + (fx)/2)^8 \cdot (20Aa^2c^3 + 4Aa^2d^3 + 14 \\
& Ba^2c^3 + 24Aa^2cd^2 + 42Aa^2c^2d + 12Ba^2cd^2 + 24Ba^2c^2d) - \tan(e/2 + (fx)/2)^{11} \cdot (Aa^2c^3 + (3Aa^2d^3)/2 + 2Ba^2c^3 + (11Ba^2d^3)/8 \\
& + (21Aa^2cd^2)/4 + 6Aa^2c^2d + (9Ba^2cd^2)/2 + (21Ba^2c^2d)/4) + \tan(e/2 + (fx)/2)^5 \cdot (2Aa^2c^3 + 7Aa^2d^3 + 4Ba^2c^3 \\
& + (47Ba^2d^3)/4 + (33Aa^2cd^2)/2 + 12Aa^2c^2d + 21Ba^2cd^2 + (33Ba^2c^2d)/2) - \tan(e/2 + (fx)/2)^7 \cdot (2Aa^2c^3 + 7Aa^2d^3 + 4Ba^2c^3 \\
& + (47Ba^2d^3)/4 + (33Aa^2cd^2)/2 + 12Aa^2c^2d + 21Ba^2cd^2 + (33Ba^2c^2d)/2) + \tan(e/2 + (fx)/2)^3 \cdot (3Aa^2c^3 \\
& + (17Aa^2d^3)/2 + 6Ba^2c^3 + (187Ba^2d^3)/24 + (87Aa^2cd^2)/4 + 18Aa^2c^2d + (51Ba^2cd^2)/2 + (87Ba^2c^2d)/4) - \tan(e/2 + (fx)/2)^9 \cdot (3Aa^2c^3 \\
& + (17Aa^2d^3)/2 + 6Ba^2c^3 + (187Ba^2d^3)/24 + (87Aa^2cd^2)/4 + 18Aa^2c^2d + (51Ba^2cd^2)/2 + (87Ba^2c^2d)/4) + \tan(e/2 + (fx)/2)^4 \cdot (40Aa^2c^3 + 32Aa^2d^3 + 36Ba^2c^3 \\
& + 32Ba^2d^3 + 96Aa^2cd^2 + 108Aa^2c^2d + 96Ba^2cd^2 + 96Ba^2c^2d) + \tan(e/2 + (fx)/2)^2 \cdot (20Aa^2c^3 + (72Aa^2d^3)/5 + 18Ba^2c^3 \\
& + (64Ba^2d^3)/5 + 48Aa^2cd^2 + 54Aa^2c^2d + (216Ba^2cd^2)/5 + 48Ba^2c^2d) + \tan(e/2 + (fx)/2)^6 \cdot (40Aa^2c^3 + 24Aa^2d^3 \\
& + (100Ba^2c^3)/3 + (64Ba^2d^3)/3 + 80Aa^2cd^2 + 100Aa^2c^2d + 72Ba^2cd^2 + 80Ba^2c^2d) + \tan(e/2 + (fx)/2)^{10} \cdot (4Aa^2c^3 + 2Ba^2c^3 \\
& + 6Aa^2c^2d) + 4Aa^2c^3 + (12Aa^2d^3)/5 + (10Ba^2c^3)/3 + (32Ba^2d^3)/15 + 8Aa^2cd^2 + 10Aa^2c^2d + (36Ba^2cd^2)/5 + 8Ba^2c^2d) / (f \cdot (6 \tan(e/2 + (fx)/2)^2 + 15 \tan(e/2 + (fx)/2)^4 + 20 \tan(e/2 + (fx)/2)^6 + 15 \tan(e/2 + (fx)/2)^8 + 6 \tan(e/2 + (fx)/2)^{10} + \tan(e/2 + (fx)/2)^{12} + 1))
\end{aligned}$$

sympy [A] time = 11.95, size = 1865, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)

[Out] Piecewise((Aa**2c**3*x*sin(e + f*x)**2/2 + Aa**2c**3*x*cos(e + f*x)**2/2 + Aa**2c**3*x - Aa**2c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2Aa**2c**3*cos(e + f*x)/f + 3Aa**2c**2*d*x*sin(e + f*x)**2 + 3Aa**2c**2*d*x*cos(e + f*x)**2 - 3Aa**2c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3Aa**2c**2*d*sin(e + f*x)*cos(e + f*x)/f - 2Aa**2c**2*d*cos(e + f*x)**3/f - 3Aa**2c**2*d*cos(e + f*x)/f + 9Aa**2c*d**2*x*sin(e + f*x)**4/8 + 9Aa**2c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3Aa**2c*d**2*x*sin(e + f*x)**2/2 + 9Aa**2c*d**2*x*cos(e + f*x)**4/8 + 3Aa**2c*d**2*x*cos(e + f*x)**2/2 - 15Aa**2c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6Aa**2c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9Aa**2c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3Aa**2c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4Aa**2c*d**2*cos(e + f*x)**3/f + 3Aa**2d**3*x*sin(e + f*x)**4/4 + 3Aa**2d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3Aa**2d**3*x*cos(e + f*x)**4/4 - Aa**2d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5Aa**2d**3*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4Aa**2d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - Aa**2d**3*sin(e + f*x)**2*cos(e + f*x)/f - 3Aa**2d**3*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8Aa**2d**3*cos(e + f*x)**5/(15*f) - 2Aa**2d**3*cos(e + f*x)**3/(3*f) + Ba**2c**3*x*sin(e + f*x)**2 + Ba**2c**3*x*cos(e + f*x)**2 - Ba**2c**3*sin(e + f*x)**2*cos(e + f*x)/f - Ba**2c**3*sin(e + f*x)*cos(e + f*x)/f - 2Ba**2c**3*cos(e + f*x)**3/(3*f) - Ba**2c**3*cos(e + f*x)/f + 9Ba**2c**2*d*x*sin(e + f*x)**4/8 + 9Ba**2c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3Ba**2c**2*d*x*sin(e + f*x)**2/2 +

```

9*B**2*c**2*d*x*cos(e + f*x)**4/8 + 3*B**2*c**2*d*x*cos(e + f*x)**2/2
- 15*B**2*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 6*B**2*c**2*d*sin
(e + f*x)**2*cos(e + f*x)/f - 9*B**2*c**2*d*sin(e + f*x)*cos(e + f*x)**3/
(8*f) - 3*B**2*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B**2*c**2*d*c
os(e + f*x)**3/f + 9*B**2*c*d**2*x*sin(e + f*x)**4/4 + 9*B**2*c*d**2*x*
sin(e + f*x)**2*cos(e + f*x)**2/2 + 9*B**2*c*d**2*x*cos(e + f*x)**4/4 - 3
*B**2*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*B**2*c*d**2*sin(e + f*
x)**3*cos(e + f*x)/(4*f) - 4*B**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/
f - 3*B**2*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*B**2*c*d**2*sin(e
+ f*x)*cos(e + f*x)**3/(4*f) - 8*B**2*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*
a**2*c*d**2*cos(e + f*x)**3/f + 5*B**2*d**3*x*sin(e + f*x)**6/16 + 15*B*a
**2*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B**2*d**3*x*sin(e + f*x
)**4/8 + 15*B**2*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B**2*d**
3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*B**2*d**3*x*cos(e + f*x)**6/16
+ 3*B**2*d**3*x*cos(e + f*x)**4/8 - 11*B**2*d**3*sin(e + f*x)**5*cos(e
+ f*x)/(16*f) - 2*B**2*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*B**2*d**
3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 5*B**2*d**3*sin(e + f*x)**3*cos
(e + f*x)/(8*f) - 8*B**2*d**3*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 5*B
**2*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B**2*d**3*sin(e + f*x)
*cos(e + f*x)**3/(8*f) - 16*B**2*d**3*cos(e + f*x)**5/(15*f), Ne(f, 0)),
(x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a)**2, True))

```

$$3.252 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal. Leaf size=336

$$\frac{a^2 (5Ad(c - 8d) - 2B(c^2 - 5cd + 18d^2)) \cos(e + fx)(c + d \sin(e + fx))^2}{60d^2 f} + \frac{1}{8} a^2 x (12Ac^2 + 16Acd + 7Ad^2 + 8B$$

```
[Out] 1/8*a^2*(12*A*c^2+16*A*c*d+7*A*d^2+8*B*c^2+14*B*c*d+6*B*d^2)*x+1/30*a^2*(5*A*d*(c^3-8*c^2*d-20*c*d^2-8*d^3)-2*B*(c^4-5*c^3*d+16*c^2*d^2+40*c*d^3+18*d^4))*cos(f*x+e)/d^2/f+1/120*a^2*(5*A*d*(2*c^2-16*c*d-21*d^2)-B*(4*c^3-20*c^2*d+66*c*d^2+90*d^3))*cos(f*x+e)*sin(f*x+e)/d/f+1/60*a^2*(5*A*(c-8*d)*d-2*B*(c^2-5*c*d+18*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^2/f+1/20*a^2*(2*B*(c-3*d)-5*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f-1/5*B*cos(f*x+e)*(a^2+a^2*sin(f*x+e))*(c+d*sin(f*x+e))^3/d/f
```

Rubi [A] time = 0.70, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^2 (5Ad(-8c^2d + c^3 - 20cd^2 - 8d^3) - 2B(16c^2d^2 - 5c^3d + c^4 + 40cd^3 + 18d^4)) \cos(e + fx)}{30d^2 f} + \frac{a^2 (5Ad(c - 8d))}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] (a^2*(12*A*c^2 + 8*B*c^2 + 16*A*c*d + 14*B*c*d + 7*A*d^2 + 6*B*d^2)*x)/8 + (a^2*(5*A*d*(c^3 - 8*c^2*d - 20*c*d^2 - 8*d^3) - 2*B*(c^4 - 5*c^3*d + 16*c^2*d^2 + 40*c*d^3 + 18*d^4))*Cos[e + f*x])/(30*d^2*f) + (a^2*(5*A*d*(2*c^2 - 16*c*d - 21*d^2) - B*(4*c^3 - 20*c^2*d + 66*c*d^2 + 90*d^3))*Cos[e + f*x]*Sin[e + f*x])/(120*d*f) + (a^2*(5*A*(c - 8*d)*d - 2*B*(c^2 - 5*c*d + 18*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(60*d^2*f) + (a^2*(2*B*(c - 3*d) - 5*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(20*d^2*f) - (B*Cos[e + f*x]*(a^2 + a^2*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(5*d*f)
```

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2753

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]
```

Rule 2968

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^m*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]) , x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx = -\frac{B \cos(e + fx) (a^2 + a^2 \sin(e + fx)) (c + d \sin(e + fx))}{5df}$$

$$= -\frac{B \cos(e + fx) (a^2 + a^2 \sin(e + fx)) (c + d \sin(e + fx))}{5df}$$

$$= \frac{a^2 (2B(c - 3d) - 5Ad) \cos(e + fx) (c + d \sin(e + fx))}{20d^2 f}$$

$$= \frac{a^2 (5A(c - 8d)d - 2B(c^2 - 5cd + 18d^2)) \cos(e + fx)}{60d^2 f}$$

$$= \frac{1}{8} a^2 (12Ac^2 + 8Bc^2 + 16Acd + 14Bcd + 7Ad^2)$$

Mathematica [A] time = 1.57, size = 296, normalized size = 0.88

$$a^2 \cos(e + fx) \left(60 (A (12c^2 + 16cd + 7d^2) + 2B (4c^2 + 7cd + 3d^2)) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{\cos^2(e + fx)} (-8 (12Ac^2 + 8Bc^2 + 16Acd + 14Bcd + 7Ad^2)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
[Out] -1/240*(a^2*Cos[e + f*x]*(60*(2*B*(4*c^2 + 7*c*d + 3*d^2) + A*(12*c^2 + 16*c*d + 7*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(480*A*c^2 + 440*B*c^2 + 880*A*c*d + 800*B*c*d + 400*A*d^2 + 378*B*d^2 - 8*(10*A*d*(c + d) + B*(5*c^2 + 20*c*d + 12*d^2))*Cos[2*(e + f*x)] + 6*B*d^2*Cos[4*(e + f*x)] + 120*A*c^2*Sin[e + f*x] + 240*B*c^2*Sin[e + f*x] + 480*A*c*d*Sin[e + f*x] + 510*B*c*d*Sin[e + f*x] + 255*A*d^2*Sin[e + f*x] + 270*B*d^2*Sin[e + f*x] - 30*B*c*d*Sin[3*(e + f*x)] - 15*A*d^2*Sin[3*(e + f*x)] - 30*B*d^2*Sin[3*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 245, normalized size = 0.73

$$24Ba^2d^2 \cos(fx + e)^5 - 40(Ba^2c^2 + 2(A + 2B)a^2cd + (2A + 3B)a^2d^2) \cos(fx + e)^3 - 15(4(3A + 2B)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out]
$$-1/120*(24*B*a^2*d^2*\cos(f*x + e)^5 - 40*(B*a^2*c^2 + 2*(A + 2*B)*a^2*c*d + (2*A + 3*B)*a^2*d^2)*\cos(f*x + e)^3 - 15*(4*(3*A + 2*B)*a^2*c^2 + 2*(8*A + 7*B)*a^2*c*d + (7*A + 6*B)*a^2*d^2)*f*x + 240*((A + B)*a^2*c^2 + 2*(A + B)*a^2*c*d + (A + B)*a^2*d^2)*\cos(f*x + e) - 15*(2*(2*B*a^2*c*d + (A + 2*B)*a^2*d^2)*\cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c^2 + 2*(8*A + 9*B)*a^2*c*d + (9*A + 10*B)*a^2*d^2)*\cos(f*x + e))*\sin(f*x + e))/f$$

giac [A] time = 0.18, size = 311, normalized size = 0.93

$$-\frac{Ba^2d^2 \cos(5fx + 5e)}{80f} + \frac{1}{8} (12Aa^2c^2 + 8Ba^2c^2 + 16Aa^2cd + 14Ba^2cd + 7Aa^2d^2 + 6Ba^2d^2)x + \frac{(4Ba^2c^2 + 8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out]
$$-1/80*B*a^2*d^2*\cos(5*f*x + 5*e)/f + 1/8*(12*A*a^2*c^2 + 8*B*a^2*c^2 + 16*A*a^2*c*d + 14*B*a^2*c*d + 7*A*a^2*d^2 + 6*B*a^2*d^2)*x + 1/48*(4*B*a^2*c^2 + 8*A*a^2*c*d + 16*B*a^2*c*d + 8*A*a^2*d^2 + 9*B*a^2*d^2)*\cos(3*f*x + 3*e)/f - 1/8*(16*A*a^2*c^2 + 14*B*a^2*c^2 + 28*A*a^2*c*d + 24*B*a^2*c*d + 12*A*a^2*d^2 + 11*B*a^2*d^2)*\cos(f*x + e)/f + 1/32*(2*B*a^2*c*d + A*a^2*d^2 + 2*B*a^2*d^2)*\sin(4*f*x + 4*e)/f - 1/4*(A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d + 4*B*a^2*c*d + 2*A*a^2*d^2 + 2*B*a^2*d^2)*\sin(2*f*x + 2*e)/f$$

maple [A] time = 0.52, size = 496, normalized size = 1.48

$$a^2Ac^2 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - \frac{2a^2Acd(2+\sin^2(fx+e))\cos(fx+e)}{3} + a^2Ad^2 \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)

[Out]
$$1/f*(a^2*A*c^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*A*c*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)+a^2*A*d^2*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-1/3*B*a^2*c^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)+2*B*a^2*c*d*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^2*d^2*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)-2*a^2*A*c^2*\cos(f*x+e)+4*a^2*A*c*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-2/3*a^2*A*d^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)+2*B*a^2*c^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-4/3*B*a^2*c*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)+2*B*a^2*d^2*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)+a^2*A*c^2*(f*x+e)-2*a^2*A*c*d*\cos(f*x+e)+a^2*A*d^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-B*a^2*c^2*\cos(f*x+e)+2*B*a^2*c*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*d^2*(2+\sin(f*x+e)^2)*\cos(f*x+e))$$

maxima [A] time = 0.44, size = 478, normalized size = 1.42

$$120(2fx + 2e - \sin(2fx + 2e))Aa^2c^2 + 480(fx + e)Aa^2c^2 + 160(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2c^2 + 240$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] 1/480*(120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c^2 + 480*(f*x + e)*A*a^2*c^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c*d + 640*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c*d + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*c*d + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c*d + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d^2 + 15*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^2*d^2 + 120*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*d^2 - 32*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^2*d^2 + 160*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d^2 - 960*A*a^2*c^2*cos(f*x + e) - 480*B*a^2*c^2*cos(f*x + e) - 960*A*a^2*c*d*cos(f*x + e))/f

mupad [B] time = 15.71, size = 765, normalized size = 2.28

$$a^2 \operatorname{atan} \left(\frac{a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) (12Ac^2 + 7Ad^2 + 8Bc^2 + 6Bd^2 + 16Acd + 14Bcd)}{4 \left(3Aa^2c^2 + \frac{7Aa^2d^2}{4} + 2Ba^2c^2 + \frac{3Ba^2d^2}{2} + 4Aa^2cd + \frac{7Ba^2cd}{2} \right)} \right) \frac{(12Ac^2 + 7Ad^2 + 8Bc^2 + 6Bd^2 + 16Acd + 14Bcd)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2,x)

[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*(3*A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2)))*(12*A*c^2 + 7*A*d^2 + 8*B*c^2 + 6*B*d^2 + 16*A*c*d + 14*B*c*d))/(4*f) - (tan(e/2 + (f*x)/2)^8*(4*A*a^2*c^2 + 2*B*a^2*c^2 + 4*A*a^2*c*d) + tan(e/2 + (f*x)/2)*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) - tan(e/2 + (f*x)/2)^9*(A*a^2*c^2 + (7*A*a^2*d^2)/4 + 2*B*a^2*c^2 + (3*B*a^2*d^2)/2 + 4*A*a^2*c*d + (7*B*a^2*c*d)/2) + tan(e/2 + (f*x)/2)^3*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) - tan(e/2 + (f*x)/2)^7*(2*A*a^2*c^2 + (11*A*a^2*d^2)/2 + 4*B*a^2*c^2 + 7*B*a^2*d^2 + 8*A*a^2*c*d + 11*B*a^2*c*d) + tan(e/2 + (f*x)/2)^6*(16*A*a^2*c^2 + 8*A*a^2*d^2 + 12*B*a^2*c^2 + 4*B*a^2*d^2 + 24*A*a^2*c*d + 16*B*a^2*c*d) + tan(e/2 + (f*x)/2)^2*(16*A*a^2*c^2 + (40*A*a^2*d^2)/3 + (44*B*a^2*c^2)/3 + 12*B*a^2*d^2 + (88*A*a^2*c*d)/3 + (80*B*a^2*c*d)/3) + tan(e/2 + (f*x)/2)^4*(24*A*a^2*c^2 + (56*A*a^2*d^2)/3 + (64*B*a^2*c^2)/3 + 20*B*a^2*d^2 + (128*A*a^2*c*d)/3 + (112*B*a^2*c*d)/3) + 4*A*a^2*c^2 + (8*A*a^2*d^2)/3 + (10*B*a^2*c^2)/3 + (12*B*a^2*d^2)/5 + (20*A*a^2*c*d)/3 + (16*B*a^2*c*d)/3)/(f*(5*tan(e/2 + (f*x)/2)^2 + 10*tan(e/2 + (f*x)/2)^4 + 10*tan(e/2 + (f*x)/2)^6 + 5*tan(e/2 + (f*x)/2)^8 + tan(e/2 + (f*x)/2)^10 + 1))

sympy [A] time = 6.00, size = 1129, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)

[Out] Piecewise((A**2*c**2*x*sin(e + f*x)**2/2 + A**2*c**2*x*cos(e + f*x)**2/2 + A**2*c**2*x - A**2*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A**2*c**2*cos(e + f*x)/f + 2*A**2*c*d*x*sin(e + f*x)**2 + 2*A**2*c*d*x*cos(e + f*x)**2 - 2*A**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 2*A**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A**2*c*d*cos(e + f*x)**3/(3*f) - 2*A**2*c*d*cos(e + f*x)/f + 3*A**2*d**2*x*sin(e + f*x)**4/8 + 3*A**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A**2*d**2*x*cos(e + f*x)**2/2 + 3*A**2*d**2*x*cos(e + f*x)**4/8 + A**2*d**2*x*cos(e + f*x)**2/2 - 5*A**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*A**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A**2*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*A**2*d**2*cos(e + f*x)**3/(3*f) + B**2*c**2*x*sin(e + f*x)**2 + B**2*c**2*x*cos(e + f*x)**2 - B**2*c**2*sin(e + f*x)**2*cos(e + f*x)/f - B**2*c**2*sin(e + f*x)*cos(e + f*x)/f - 2*B**2*c**2*cos(e + f*x)**3/(3*f) - B**2*c**2*cos(e + f*x)/f + 3*B**2*c*d*x*sin(e + f*x)**4/4 + 3*B**2*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B**2*c*d*x*sin(e + f*x)**2 + 3*B**2*c*d*x*cos(e + f*x)**4/4 + B**2*c*d*x*cos(e + f*x)**2 - 5*B**2*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B**2*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B**2*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B**2*c*d*sin(e + f*x)*cos(e + f*x)/f - 8*B**2*c*d*cos(e + f*x)**3/(3*f) + 3*B**2*d**2*x*sin(e + f*x)**4/4 + 3*B**2*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*B**2*d**2*x*cos(e + f*x)**4/4 - B**2*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B**2*d**2*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 4*B**2*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - B**2*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B**2*d**2*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 8*B**2*d**2*cos(e + f*x)**5/(15*f) - 2*B**2*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2*(a*sin(e) + a)**2, True))

3.253 $\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$

Optimal. Leaf size=166

$$\frac{a^2(12Ac + 8Ad + 8Bc + 7Bd) \cos(e + fx)}{6f} - \frac{a^2(12Ac + 8Ad + 8Bc + 7Bd) \sin(e + fx) \cos(e + fx)}{24f} + \frac{1}{8} a^2 x (12Ac + 8Ad + 8Bc + 7Bd)$$

[Out] $\frac{1}{8} a^2 x (12 A c + 8 A d + 8 B c + 7 B d) - \frac{1}{6} a^2 (12 A c + 8 A d + 8 B c + 7 B d) \cos(f x + e) / f - \frac{1}{24} a^2 (12 A c + 8 A d + 8 B c + 7 B d) \cos(f x + e) \sin(f x + e) / f - \frac{1}{12} (4 A d + 4 B c - B d) \cos(f x + e) (a + a \sin(f x + e))^2 / f - \frac{1}{4} B d \cos(f x + e) (a + a \sin(f x + e))^3 / a / f$

Rubi [A] time = 0.27, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3023, 2751, 2644}

$$\frac{a^2(12Ac + 8Ad + 8Bc + 7Bd) \cos(e + fx)}{6f} - \frac{a^2(12Ac + 8Ad + 8Bc + 7Bd) \sin(e + fx) \cos(e + fx)}{24f} + \frac{1}{8} a^2 x (12Ac + 8Ad + 8Bc + 7Bd)$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] $(a^2(12A^2c + 8B^2c + 8A^2d + 7B^2d)x)/8 - (a^2(12A^2c + 8B^2c + 8A^2d + 7B^2d) \cos[e + f*x]) / (6f) - (a^2(12A^2c + 8B^2c + 8A^2d + 7B^2d) \cos[e + f*x] \sin[e + f*x]) / (24f) - ((4B^2c + 4A^2d - B^2d) \cos[e + f*x] (a + a \sin[e + f*x])^2) / (12f) - (B^2d \cos[e + f*x] (a + a \sin[e + f*x])^3) / (4af)$

Rule 2644

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b^2)*x)/2, x] + (-Simp[(2*a*b*cos[c + d*x])/d, x] - Simp[(b^2*cos[c + d*x]*Sin[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx))^2 (Ac + (Bc + Ad) \sin(e + fx) \\ &+ Bd \cos(e + fx)(a + a \sin(e + fx))^3 + \int (a + a \sin(e + fx))^2 (4Bc + 4Ad - Bd) \cos(e + fx)(a + a \sin(e + fx)) dx \\ &= \frac{1}{8} a^2 (12Ac + 8Bc + 8Ad + 7Bd)x - \frac{a^2 (12Ac + 8Bc + 8Ad + 7Bd) \cos(e + fx)}{12f} \end{aligned}$$

Mathematica [A] time = 0.76, size = 160, normalized size = 0.96

$$\frac{a^2 \cos(e + fx) \left(6(12Ac + 8Ad + 8Bc + 7Bd) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{\cos^2(e + fx)} (8(Ad + B(c + 2d)) \sin^2(e + fx) + 6Bd \sin(e + fx)^3) \right)}{24f \sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]), x]

[Out] -1/24*(a^2*Cos[e + f*x]*(6*(12*A*c + 8*B*c + 8*A*d + 7*B*d)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(8*(6*A*c + 5*B*c + 5*A*d + 4*B*d) + 3*(4*A*c + 8*B*c + 8*A*d + 7*B*d)*Sin[e + f*x] + 8*(A*d + B*(c + 2*d))*Sin[e + f*x]^2 + 6*B*d*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])

fricas [A] time = 0.45, size = 144, normalized size = 0.87

$$\frac{8(Ba^2c + (A + 2B)a^2d) \cos(fx + e)^3 + 3(4(3A + 2B)a^2c + (8A + 7B)a^2d)fx - 48((A + B)a^2c + (A + B)a^2d) \cos(fx + e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/24*(8*(B*a^2*c + (A + 2*B)*a^2*d)*cos(f*x + e)^3 + 3*(4*(3*A + 2*B)*a^2*c + (8*A + 7*B)*a^2*d)*f*x - 48*((A + B)*a^2*c + (A + B)*a^2*d)*cos(f*x + e) + 3*(2*B*a^2*d*cos(f*x + e)^3 - (4*(A + 2*B)*a^2*c + (8*A + 9*B)*a^2*d)*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.17, size = 172, normalized size = 1.04

$$\frac{Ba^2d \sin(4fx + 4e)}{32f} + \frac{1}{8} (12Aa^2c + 8Ba^2c + 8Aa^2d + 7Ba^2d)x + \frac{(Ba^2c + Aa^2d + 2Ba^2d) \cos(3fx + 3e)}{12f} - \frac{1}{12f} (12Aa^2c + 8Ba^2c + 8Aa^2d + 7Ba^2d) \cos(3fx + 3e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 1/32*B*a^2*d*sin(4*f*x + 4*e)/f + 1/8*(12*A*a^2*c + 8*B*a^2*c + 8*A*a^2*d + 7*B*a^2*d)*x + 1/12*(B*a^2*c + A*a^2*d + 2*B*a^2*d)*cos(3*f*x + 3*e)/f - 1/4*(8*A*a^2*c + 7*B*a^2*c + 7*A*a^2*d + 6*B*a^2*d)*cos(f*x + e)/f - 1/4*(A*a^2*c + 2*B*a^2*c + 2*A*a^2*d + 2*B*a^2*d)*sin(2*f*x + 2*e)/f

maple [A] time = 0.41, size = 278, normalized size = 1.67

$$a^2 A c \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - \frac{a^2 A d (2 + \sin^2(fx+e)) \cos(fx+e)}{3} - \frac{B a^2 c (2 + \sin^2(fx+e)) \cos(fx+e)}{3} + B a^2 d \left(-\frac{(\sin^3(fx+e) + \dots)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] 1/f*(a^2*A*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*a^2*A*d*(2+sin(f*x+e)^2)*cos(f*x+e)-1/3*B*a^2*c*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^2*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2*a^2*A*c*cos(f*x+e)+2*a^2*A*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+2*B*a^2*c*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2/3*B*a^2*d*(2+sin(f*x+e)^2)*cos(f*x+e)+a^2*A*c*(f*x+e)-a^2*A*d*cos(f*x+e)-B*a^2*c*cos(f*x+e)+B*a^2*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))

maxima [A] time = 0.42, size = 268, normalized size = 1.61

$$24(2fx + 2e - \sin(2fx + 2e))Aa^2c + 96(fx + e)Aa^2c + 32(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2c + 48(2fx + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] 1/96*(24*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*c + 96*(f*x + e)*A*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*c + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*c + 32*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^2*d + 48*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^2*d + 64*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^2*d + 3*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^2*d + 24*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^2*d - 192*A*a^2*c*cos(f*x + e) - 96*B*a^2*c*cos(f*x + e) - 96*A*a^2*d*cos(f*x + e))/f

mupad [B] time = 14.60, size = 492, normalized size = 2.96

$$a^2 \operatorname{atan} \left(\frac{a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) (12Ac + 8Ad + 8Bc + 7Bd)}{4 \left(3Aa^2c + 2Aa^2d + 2Ba^2c + \frac{7Ba^2d}{4} \right)} \right) \frac{a^2 \left(\operatorname{atan} \left(\tan \left(\frac{e}{2} + \frac{fx}{2} \right) \right) - \frac{fx}{2} \right) (12A \dots)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x)),x)

[Out] (a^2*atan((a^2*tan(e/2 + (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*(3*A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4)))*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*f) - (a^2*(atan(tan(e/2 + (f*x)/2)) - (f*x)/2)*(12*A*c + 8*A*d + 8*B*c + 7*B*d))/(4*f) - (tan(e/2 + (f*x)/2)^3*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (15*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^7*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4) - tan(e/2 + (f*x)/2)^5*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (15*B*a^2*d)/4) + tan(e/2 + (f*x)/2)^4*(12*A*a^2*c + 10*A*a^2*d + 10*B*a^2*c + 8*B*a^2*d) + tan(e/2 + (f*x)/2)^2*(12*A*a^2*c + (34*A*a^2*d)/3 + (34*B*a^2*c)/3 + (32*B*a^2*d)/3) + tan(e/2 + (f*x)/2)^6*(4*A*a^2*c + 2*A*a^2*d + 2*B*a^2*c) + tan(e/2 + (f*x)/2)*(A*a^2*c + 2*A*a^2*d + 2*B*a^2*c + (7*B*a^2*d)/4) + 4*A*a^2*c + (10*A*a^2*d)/3 + (10*B*a^2*c)/3 + (8*B*a^2*d)/3)/

$(f*(4*\tan(e/2 + (f*x)/2)^2 + 6*\tan(e/2 + (f*x)/2)^4 + 4*\tan(e/2 + (f*x)/2)^6 + \tan(e/2 + (f*x)/2)^8 + 1))$

sympy [A] time = 2.76, size = 571, normalized size = 3.44

$$\left\{ \begin{array}{l} \frac{Aa^2cx \sin^2(e+fx)}{2} + \frac{Aa^2cx \cos^2(e+fx)}{2} + Aa^2cx - \frac{Aa^2c \sin(e+fx) \cos(e+fx)}{2f} - \frac{2Aa^2c \cos(e+fx)}{f} + Aa^2dx \sin^2(e+fx) + A \\ x(A + B \sin(e))(c + d \sin(e))(a \sin(e) + a)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] Piecewise((A*a**2*c*x*sin(e + f*x)**2/2 + A*a**2*c*x*cos(e + f*x)**2/2 + A*a**2*c*x - A*a**2*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*c*cos(e + f*x)/f + A*a**2*d*x*sin(e + f*x)**2 + A*a**2*d*x*cos(e + f*x)**2 - A*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f - A*a**2*d*cos(e + f*x)*cos(e + f*x)/f - 2*A*a**2*d*cos(e + f*x)**3/(3*f) - A*a**2*d*cos(e + f*x)/f + B*a**2*c*x*sin(e + f*x)**2 + B*a**2*c*x*cos(e + f*x)**2 - B*a**2*c*sin(e + f*x)**2*cos(e + f*x)/f - B*a**2*c*cos(e + f*x)*cos(e + f*x)/f - 2*B*a**2*c*cos(e + f*x)**3/(3*f) - B*a**2*c*cos(e + f*x)/f + 3*B*a**2*d*x*sin(e + f*x)**4/8 + 3*B*a**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**2*d*x*sin(e + f*x)**2/2 + 3*B*a**2*d*x*cos(e + f*x)**4/8 + B*a**2*d*x*cos(e + f*x)**2/2 - 5*B*a**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 2*B*a**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 4*B*a**2*d*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*(a*sin(e) + a)**2, True))

3.254 $\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) dx$

Optimal. Leaf size=94

$$\frac{2a^2(3A + 2B) \cos(e + fx)}{3f} - \frac{a^2(3A + 2B) \sin(e + fx) \cos(e + fx)}{6f} + \frac{1}{2}a^2x(3A + 2B) - \frac{B \cos(e + fx)(a \sin(e + fx) + a)}{3f}$$

[Out] $\frac{1}{2}a^2(3A + 2B)x - \frac{2}{3}a^2(3A + 2B)\cos(fx + e)/f - \frac{1}{6}a^2(3A + 2B)\cos(fx + e)\sin(fx + e)/f - \frac{1}{3}B\cos(fx + e)(a + a\sin(fx + e))^2/f$

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2751, 2644}

$$\frac{2a^2(3A + 2B) \cos(e + fx)}{3f} - \frac{a^2(3A + 2B) \sin(e + fx) \cos(e + fx)}{6f} + \frac{1}{2}a^2x(3A + 2B) - \frac{B \cos(e + fx)(a \sin(e + fx) + a)}{3f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]

[Out] $\frac{a^2(3A + 2B)x}{2} - \frac{(2a^2(3A + 2B)\cos[e + f*x])}{(3f)} - \frac{(a^2(3A + 2B)\cos[e + f*x]\sin[e + f*x])}{(6f)} - \frac{(B\cos[e + f*x](a + a\sin[e + f*x])^2)}{(3f)}$

Rule 2644

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx)) dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^2}{3f} + \frac{1}{3}(3A + 2B) \int (a + a \sin(e + fx)) dx \\ &= \frac{1}{2}a^2(3A + 2B)x - \frac{2a^2(3A + 2B) \cos(e + fx)}{3f} - \frac{a^2(3A + 2B) \cos(e + fx) \sin(e + fx)}{6f} \end{aligned}$$

Mathematica [A] time = 0.32, size = 106, normalized size = 1.13

$$\frac{a^2 \cos(e + fx) \left(6(3A + 2B) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{\cos^2(e + fx)} (3(A + 2B) \sin(e + fx) + 2(6A + 5B) + 2B \sin(e + fx)) \right)}{6f \sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]),x]

[Out] $-1/6*(a^2*\cos[e + f*x]*(6*(3*A + 2*B)*\text{ArcSin}[\text{Sqrt}[1 - \sin[e + f*x]]/\text{Sqrt}[2] + \text{Sqrt}[\cos[e + f*x]^2]*(2*(6*A + 5*B) + 3*(A + 2*B)*\sin[e + f*x] + 2*B*\sin[e + f*x]^2)))/(f*\text{Sqrt}[\cos[e + f*x]^2])$

fricas [A] time = 0.46, size = 70, normalized size = 0.74

$$\frac{2Ba^2 \cos(fx + e)^3 + 3(3A + 2B)a^2 fx - 3(A + 2B)a^2 \cos(fx + e) \sin(fx + e) - 12(A + B)a^2 \cos(fx + e)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="fricas")`

[Out] $1/6*(2*B*a^2*\cos(f*x + e)^3 + 3*(3*A + 2*B)*a^2*f*x - 3*(A + 2*B)*a^2*\cos(f*x + e)*\sin(f*x + e) - 12*(A + B)*a^2*\cos(f*x + e))/f$

giac [A] time = 0.16, size = 88, normalized size = 0.94

$$\frac{Ba^2 \cos(3fx + 3e)}{12f} + \frac{1}{2}(3Aa^2 + 2Ba^2)x - \frac{(8Aa^2 + 7Ba^2) \cos(fx + e)}{4f} - \frac{(Aa^2 + 2Ba^2) \sin(2fx + 2e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="giac")`

[Out] $1/12*B*a^2*\cos(3*f*x + 3*e)/f + 1/2*(3*A*a^2 + 2*B*a^2)*x - 1/4*(8*A*a^2 + 7*B*a^2)*\cos(f*x + e)/f - 1/4*(A*a^2 + 2*B*a^2)*\sin(2*f*x + 2*e)/f$

maple [A] time = 0.30, size = 117, normalized size = 1.24

$$\frac{a^2 A \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right) - \frac{Ba^2(2+\sin^2(fx+e))\cos(fx+e)}{3} - 2a^2 A \cos(fx + e) + 2B a^2 \left(-\frac{\sin(fx+e)\cos(fx+e)}{2} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x)`

[Out] $1/f*(a^2*A*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^2*(2+\sin(f*x+e)^2)*\cos(f*x+e)-2*a^2*A*\cos(f*x+e)+2*B*a^2*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)+a^2*A*(f*x+e)-B*a^2*\cos(f*x+e))$

maxima [A] time = 0.44, size = 114, normalized size = 1.21

$$\frac{3(2fx + 2e - \sin(2fx + 2e))Aa^2 + 12(fx + e)Aa^2 + 4(\cos(fx + e)^3 - 3\cos(fx + e))Ba^2 + 6(2fx + 2e)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e)),x, algorithm="maxima")`

[Out] $1/12*(3*(2*f*x + 2*e - \sin(2*f*x + 2*e))*A*a^2 + 12*(f*x + e)*A*a^2 + 4*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^2 + 6*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^2 - 24*A*a^2*\cos(f*x + e) - 12*B*a^2*\cos(f*x + e))/f$

mupad [B] time = 13.16, size = 91, normalized size = 0.97

$$\frac{\frac{3Aa^2 \sin(2e+2fx)}{2} - \frac{Ba^2 \cos(3e+3fx)}{2} + 3Ba^2 \sin(2e + 2fx) + 12Aa^2 \cos(e + fx) + \frac{21Ba^2 \cos(e+fx)}{2} - 9Aa^2}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2,x)
```

```
[Out] -((3*A*a^2*sin(2*e + 2*f*x))/2 - (B*a^2*cos(3*e + 3*f*x))/2 + 3*B*a^2*sin(2
*e + 2*f*x) + 12*A*a^2*cos(e + f*x) + (21*B*a^2*cos(e + f*x))/2 - 9*A*a^2*f
*x - 6*B*a^2*f*x)/(6*f)
```

sympy [A] time = 0.95, size = 199, normalized size = 2.12

$$\left\{ \begin{array}{l} \frac{Aa^2x \sin^2(e+fx)}{2} + \frac{Aa^2x \cos^2(e+fx)}{2} + Aa^2x - \frac{Aa^2 \sin(e+fx) \cos(e+fx)}{2f} - \frac{2Aa^2 \cos(e+fx)}{f} + Ba^2x \sin^2(e + fx) + Ba^2x \cos^2 \\ x(A + B \sin(e))(a \sin(e) + a)^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((A*a**2*x*sin(e + f*x)**2/2 + A*a**2*x*cos(e + f*x)**2/2 + A*a**2
*x - A*a**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**2*cos(e + f*x)/f + B*a
**2*x*sin(e + f*x)**2 + B*a**2*x*cos(e + f*x)**2 - B*a**2*sin(e + f*x)**2*c
os(e + f*x)/f - B*a**2*sin(e + f*x)*cos(e + f*x)/f - 2*B*a**2*cos(e + f*x)*
*3/(3*f) - B*a**2*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) +
a)**2, True))
```

$$3.255 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=171

$$\frac{2a^2(c-d)^2(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f \sqrt{c^2-d^2}} - \frac{a^2 x (2Ad(c-2d) - B(2c^2 - 4cd + 3d^2))}{2d^3} + \frac{a^2(-2Ad + 2Bc - 3B^2)}{2d^2 f}$$

[Out] $-1/2*a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x/d^3+1/2*a^2*(-2*A*d+2*B*c-3*B*d)*\cos(f*x+e)/d^2/f-1/2*B*\cos(f*x+e)*(a^2+a^2*\sin(f*x+e))/d/f-2*a^2*(c-d)^2*(-A*d+B*c)*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/d^3/f/(c^2-d^2)^{(1/2)}$

Rubi [A] time = 0.52, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a^2(c-d)^2(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f \sqrt{c^2-d^2}} - \frac{a^2 x (2Ad(c-2d) - B(2c^2 - 4cd + 3d^2))}{2d^3} + \frac{a^2(-2Ad + 2Bc - 3B^2)}{2d^2 f}$$

Antiderivative was successfully verified.

[In] Int[((a + aSin[e + f*x])^2*(A + BSin[e + f*x]))/(c + dSin[e + f*x]),x]

[Out] $-(a^2*(2*A*(c-2*d)*d-B*(2*c^2-4*c*d+3*d^2))*x)/(2*d^3)-(2*a^2*(c-d)^2*(B*c-A*d)*\text{ArcTan}[(d+c*\text{Tan}[(e+f*x)/2])/ \text{Sqrt}[c^2-d^2]])/(d^3*\text{Sqrt}[c^2-d^2]*f)+(a^2*(2*B*c-2*A*d-3*B*d)*\text{Cos}[e+f*x])/(2*d^2*f)-(B*\text{Cos}[e+f*x]*(a^2+a^2*\sin[e+f*x]))/(2*d*f)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + dSin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a

+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2976

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] & IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2(A + B \sin(e + fx))}{c + d \sin(e + fx)} dx &= -\frac{B \cos(e + fx)(a^2 + a^2 \sin(e + fx))}{2df} + \frac{\int \frac{(a+a \sin(e+fx))(a(Bc+2Ad)-c+d \sin(e+fx))}{c+d \sin(e+fx)} dx}{2} \\ &= -\frac{B \cos(e + fx)(a^2 + a^2 \sin(e + fx))}{2df} + \frac{\int \frac{a^2(Bc+2Ad)+(a^2(Bc+2Ad)-a^2 \sin^2(e+fx))}{c+d \sin(e+fx)} dx}{2} \\ &= \frac{a^2(2Bc - 2Ad - 3Bd) \cos(e + fx)}{2d^2 f} - \frac{B \cos(e + fx)(a^2 + a^2 \sin(e + fx))}{2df} \\ &= -\frac{a^2(2A(c - 2d)d - B(2c^2 - 4cd + 3d^2))x}{2d^3} + \frac{a^2(2Bc - 2Ad - 3Bd)}{2d^2 f} \\ &= -\frac{a^2(2A(c - 2d)d - B(2c^2 - 4cd + 3d^2))x}{2d^3} + \frac{a^2(2Bc - 2Ad - 3Bd)}{2d^2 f} \\ &= -\frac{a^2(2A(c - 2d)d - B(2c^2 - 4cd + 3d^2))x}{2d^3} + \frac{a^2(2Bc - 2Ad - 3Bd)}{2d^2 f} \\ &= -\frac{a^2(2A(c - 2d)d - B(2c^2 - 4cd + 3d^2))x}{2d^3} - \frac{2a^2(c - d)^2(Bc - Ad)}{d^3 \sqrt{c^2 - d^2}} \end{aligned}$$

Mathematica [A] time = 0.62, size = 177, normalized size = 1.04

$$a^2(\sin(e + fx) + 1)^2 \left(2(e + fx)(2Ad(2d - c) + B(2c^2 - 4cd + 3d^2)) - \frac{8(c-d)^2(Bc - Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{\sqrt{c^2 - d^2}} - 4d(Ac + Bc) \right) \\ \frac{4d^3 f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^4}{}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]), x]

[Out] (a^2*(1 + Sin[e + f*x])^2*(2*(2*A*d*(-c + 2*d) + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x) - (8*(c - d)^2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 4*d*(-(B*c) + A*d + 2*B*d)*Cos[e + f*x] - B*d^2*Sin[2*(e + f*x)])/(4*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)

fricas [A] time = 0.51, size = 452, normalized size = 2.64

$$\frac{Ba^2d^2 \cos(fx + e) \sin(fx + e) - (2Ba^2c^2 - 2(A + 2B)a^2cd + (4A + 3B)a^2d^2)fx + (Ba^2c^2 - (A + B)a^2cd)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2*c*d + (4*A + 3*B)*a^2*d^2)*f*x + (B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f), -1/2*(B*a^2*d^2*cos(f*x + e)*sin(f*x + e) - (2*B*a^2*c^2 - 2*(A + 2*B)*a^2*c*d + (4*A + 3*B)*a^2*d^2)*f*x - 2*(B*a^2*c^2 - (A + B)*a^2*c*d + A*a^2*d^2)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d)))/((c - d)*cos(f*x + e)) - 2*(B*a^2*c*d - (A + 2*B)*a^2*d^2)*cos(f*x + e))/(d^3*f)]

giac [A] time = 0.19, size = 314, normalized size = 1.84

$$\frac{(2Ba^2c^2 - 2Aa^2cd - 4Ba^2cd + 4Aa^2d^2 + 3Ba^2d^2)(fx + e)}{d^3} - \frac{4(Ba^2c^3 - Aa^2c^2d - 2Ba^2c^2d + 2Aa^2cd^2 + Ba^2cd^2 - Aa^2d^3)}{\sqrt{c^2 - d^2}d^3} \left(\pi \left[\frac{fx + e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 1/2*((2*B*a^2*c^2 - 2*A*a^2*c*d - 4*B*a^2*c*d + 4*A*a^2*d^2 + 3*B*a^2*d^2)*(f*x + e)/d^3 - 4*(B*a^2*c^3 - A*a^2*c^2*d - 2*B*a^2*c^2*d + 2*A*a^2*c*d^2 + B*a^2*c*d^2 - A*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^3) + 2*

$$(B*a^2*d*\tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c*\tan(1/2*f*x + 1/2*e)^2 - 2*A*a^2*d*\tan(1/2*f*x + 1/2*e)^2 - 4*B*a^2*d*\tan(1/2*f*x + 1/2*e)^2 - B*a^2*d*\tan(1/2*f*x + 1/2*e) + 2*B*a^2*c - 2*A*a^2*d - 4*B*a^2*d)/((\tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^2))/f$$

maple [B] time = 0.43, size = 713, normalized size = 4.17

$$\frac{2a^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) A c^2}{f d^2 \sqrt{c^2 - d^2}} - \frac{4a^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) A c}{f d \sqrt{c^2 - d^2}} + \frac{2a^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) A}{f \sqrt{c^2 - d^2}} - \frac{2a^2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) A}{f d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-4/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-2/f*a^2/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+4/f*a^2/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2/f*a^2/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2+2/f*a^2/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^2-1/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)-2/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*A+2/f*a^2/d^2/(1+tan(1/2*f*x+1/2*e)^2)^2*B*c-4/f*a^2/d/(1+tan(1/2*f*x+1/2*e)^2)^2*B-2/f*a^2/d^2*arctan(tan(1/2*f*x+1/2*e))*A*c+4/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*A+2/f*a^2/d^3*arctan(tan(1/2*f*x+1/2*e))*B*c^2-4/f*a^2/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c+3/f*a^2/d*arctan(tan(1/2*f*x+1/2*e))*B
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 20.07, size = 7371, normalized size = 43.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x)),x)
```

```
[Out] (atan((((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (((32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2))/d^3 - (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^
```


$$\begin{aligned}
& 5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8) \\
&)/d^6)*(B*a^2*c^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)* \\
& 1i)/2))/d^3 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c^3*d^5 - 96*A^3*a^6*c^2* \\
& d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^6*c^5*d^3 - 18*B^3*a^6*c^ \\
& ^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 + 192*B^3*a^6*c^5*d^3 - 1 \\
& 28*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6*c^7*d + 18*A*B^2*a^6*c*d \\
& ^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132*A*B^2*a^6*c^2*d^6 + 354* \\
& A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B^2*a^6*c^5*d^3 - 144*A*B \\
& ^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B*a^6*c^3*d^5 - 336*A^2*B* \\
& a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*c^6*d^2))/d^6))*(B*a^2*c \\
& ^2*1i + (a^2*d^2*(4*A + 3*B)*1i)/2 - (a^2*d*(2*A*c + 4*B*c)*1i)/2)*2i)/(d^3 \\
& *f) - ((2*(A*a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 + (2*tan(e/2 + (f*x)/2)^2*(A \\
& *a^2*d - B*a^2*c + 2*B*a^2*d))/d^2 - (B*a^2*tan(e/2 + (f*x)/2)^3)/d + (B*a^ \\
& 2*tan(e/2 + (f*x)/2))/d)/(f*(2*tan(e/2 + (f*x)/2)^2 + tan(e/2 + (f*x)/2)^4 \\
& + 1)) + (a^2*atan(((a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))*((8*(16*A^2* \\
& a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2*a^4*c^2*d^6 - \\
& 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^ \\
& 6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4*c^4*d^4 - 8*A* \\
& B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4*d^5 - 32*A^2*a^ \\
& 4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2*a^4*c^2*d^7 + 4 \\
& 3*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + 32*B^2*a^4*c^6 \\
& *d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d^8 - 80*A*B*a^4 \\
& *c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a^4*c^5*d^4 + 16 \\
& *A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c)*(-(c + d)*(c - \\
& d)^3)^(1/2))*((8*tan(e/2 + (f*x)/2)*(8*A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A \\
& *a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 - \\
& (8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^2*d^7 - 8*B*a^2*c^2*d^7 + 2* \\
& B*a^2*c^3*d^6))/d^5 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/2)*(12*c*d^10 - \\
& 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c*d^3 + d^4))/(\\
& c*d^3 + d^4)*1i)/(c*d^3 + d^4) + (a^2*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/ \\
& 2))*((8*(16*A^2*a^4*c^2*d^6 - 16*A^2*a^4*c^3*d^5 + 4*A^2*a^4*c^4*d^4 + 9*B^2 \\
& *a^4*c^2*d^6 - 24*B^2*a^4*c^3*d^5 + 28*B^2*a^4*c^4*d^4 - 16*B^2*a^4*c^5*d^3 \\
& + 4*B^2*a^4*c^6*d^2 + 24*A*B*a^4*c^2*d^6 - 44*A*B*a^4*c^3*d^5 + 32*A*B*a^4 \\
& *c^4*d^4 - 8*A*B*a^4*c^5*d^3))/d^5 + (8*tan(e/2 + (f*x)/2)*(32*A^2*a^4*c^4* \\
& d^5 - 32*A^2*a^4*c^3*d^6 - 16*A^2*a^4*c^2*d^7 - 8*A^2*a^4*c^5*d^4 - 48*B^2* \\
& a^4*c^2*d^7 + 43*B^2*a^4*c^3*d^6 + 8*B^2*a^4*c^4*d^5 - 44*B^2*a^4*c^5*d^4 + \\
& 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 + 28*A^2*a^4*c*d^8 + 18*B^2*a^4*c*d \\
& ^8 - 80*A*B*a^4*c^2*d^7 + 8*A*B*a^4*c^3*d^6 + 76*A*B*a^4*c^4*d^5 - 64*A*B*a \\
& ^4*c^5*d^4 + 16*A*B*a^4*c^6*d^3 + 48*A*B*a^4*c*d^8))/d^6 + (a^2*(A*d - B*c) \\
& *(-(c + d)*(c - d)^3)^(1/2))*((8*(8*A*a^2*c*d^8 + 6*B*a^2*c*d^8 - 8*A*a^2*c^ \\
& 2*d^7 - 8*B*a^2*c^2*d^7 + 2*B*a^2*c^3*d^6))/d^5 - (8*tan(e/2 + (f*x)/2)*(8* \\
& A*a^2*c*d^9 - 16*A*a^2*c^2*d^8 + 8*A*a^2*c^3*d^7 - 8*B*a^2*c^2*d^8 + 16*B*a \\
& ^2*c^3*d^7 - 8*B*a^2*c^4*d^6))/d^6 + (a^2*(32*c^2*d^3 + (8*tan(e/2 + (f*x)/ \\
& 2)*(12*c*d^10 - 8*c^3*d^8))/d^6)*(A*d - B*c)*(-(c + d)*(c - d)^3)^(1/2))/(c \\
& *d^3 + d^4))/(c*d^3 + d^4)*1i)/(c*d^3 + d^4))/((16*(2*B^3*a^6*c^7 + 20*A^ \\
& 3*a^6*c^2*d^5 - 16*A^3*a^6*c^3*d^4 + 4*A^3*a^6*c^4*d^3 + 3*B^3*a^6*c^3*d^4 \\
& - 10*B^3*a^6*c^4*d^3 + 13*B^3*a^6*c^5*d^2 - 8*A^3*a^6*c*d^6 - 8*B^3*a^6*c^6 \\
& *d - 6*A^2*B*a^6*c*d^6 + 3*A*B^2*a^6*c^2*d^5 - 6*A*B^2*a^6*c^3*d^4 + 3*A*B^ \\
& 2*a^6*c^4*d^3 + 24*A^2*B*a^6*c^2*d^5 - 36*A^2*B*a^6*c^3*d^4 + 24*A^2*B*a^6* \\
& c^4*d^3 - 6*A^2*B*a^6*c^5*d^2))/d^5 - (16*tan(e/2 + (f*x)/2)*(104*A^3*a^6*c^ \\
& ^3*d^5 - 96*A^3*a^6*c^2*d^6 - 8*B^3*a^6*c^8 - 48*A^3*a^6*c^4*d^4 + 8*A^3*a^ \\
& 6*c^5*d^3 - 18*B^3*a^6*c^2*d^6 + 84*B^3*a^6*c^3*d^5 - 170*B^3*a^6*c^4*d^4 + \\
& 192*B^3*a^6*c^5*d^3 - 128*B^3*a^6*c^6*d^2 + 32*A^3*a^6*c*d^7 + 48*B^3*a^6* \\
& c^7*d + 18*A*B^2*a^6*c*d^7 + 24*A*B^2*a^6*c^7*d + 48*A^2*B*a^6*c*d^7 - 132* \\
& A*B^2*a^6*c^2*d^6 + 354*A*B^2*a^6*c^3*d^5 - 480*A*B^2*a^6*c^4*d^4 + 360*A*B \\
& ^2*a^6*c^5*d^3 - 144*A*B^2*a^6*c^6*d^2 - 216*A^2*B*a^6*c^2*d^6 + 384*A^2*B* \\
& a^6*c^3*d^5 - 336*A^2*B*a^6*c^4*d^4 + 144*A^2*B*a^6*c^5*d^3 - 24*A^2*B*a^6*
\end{aligned}$$

$$\begin{aligned}
& c^6 d^2) / d^6 - (a^2 (A d - B c) * (-(c + d) * (c - d)^3)^{1/2} * ((8 * (16 * A^2 * a^4 * c^2 * d^6 - 16 * A^2 * a^4 * c^3 * d^5 + 4 * A^2 * a^4 * c^4 * d^4 + 9 * B^2 * a^4 * c^2 * d^6 - 24 * B^2 * a^4 * c^3 * d^5 + 28 * B^2 * a^4 * c^4 * d^4 - 16 * B^2 * a^4 * c^5 * d^3 + 4 * B^2 * a^4 * c^6 * d^2 + 24 * A * B * a^4 * c^2 * d^6 - 44 * A * B * a^4 * c^3 * d^5 + 32 * A * B * a^4 * c^4 * d^4 - 8 * A * B * a^4 * c^5 * d^3)) / d^5 + (8 * \tan(e/2 + (f * x) / 2) * (32 * A^2 * a^4 * c^4 * d^5 - 32 * A^2 * a^4 * c^3 * d^6 - 16 * A^2 * a^4 * c^2 * d^7 - 8 * A^2 * a^4 * c^5 * d^4 - 48 * B^2 * a^4 * c^2 * d^7 + 43 * B^2 * a^4 * c^3 * d^6 + 8 * B^2 * a^4 * c^4 * d^5 - 44 * B^2 * a^4 * c^5 * d^4 + 32 * B^2 * a^4 * c^6 * d^3 - 8 * B^2 * a^4 * c^7 * d^2 + 28 * A^2 * a^4 * c * d^8 + 18 * B^2 * a^4 * c * d^8 - 80 * A * B * a^4 * c^2 * d^7 + 8 * A * B * a^4 * c^3 * d^6 + 76 * A * B * a^4 * c^4 * d^5 - 64 * A * B * a^4 * c^5 * d^4 + 16 * A * B * a^4 * c^6 * d^3 + 48 * A * B * a^4 * c * d^8)) / d^6 + (a^2 (A d - B c) * (-(c + d) * (c - d)^3)^{1/2} * ((8 * \tan(e/2 + (f * x) / 2) * (8 * A * a^2 * c * d^9 - 16 * A * a^2 * c^2 * d^8 + 8 * A * a^2 * c^3 * d^7 - 8 * B * a^2 * c^2 * d^8 + 16 * B * a^2 * c^3 * d^7 - 8 * B * a^2 * c^4 * d^6)) / d^6 - (8 * (8 * A * a^2 * c * d^8 + 6 * B * a^2 * c * d^8 - 8 * A * a^2 * c^2 * d^7 - 8 * B * a^2 * c^2 * d^7 + 2 * B * a^2 * c^3 * d^6)) / d^5 + (a^2 (32 * c^2 * d^3 + (8 * \tan(e/2 + (f * x) / 2) * (12 * c * d^10 - 8 * c^3 * d^8)) / d^6) * (A d - B c) * (-(c + d) * (c - d)^3)^{1/2}) / (c * d^3 + d^4)) / (c * d^3 + d^4)) / (c * d^3 + d^4) + (a^2 (A d - B c) * (-(c + d) * (c - d)^3)^{1/2} * ((8 * (16 * A^2 * a^4 * c^2 * d^6 - 16 * A^2 * a^4 * c^3 * d^5 + 4 * A^2 * a^4 * c^4 * d^4 + 9 * B^2 * a^4 * c^2 * d^6 - 24 * B^2 * a^4 * c^3 * d^5 + 28 * B^2 * a^4 * c^4 * d^4 - 16 * B^2 * a^4 * c^5 * d^3 + 4 * B^2 * a^4 * c^6 * d^2 + 24 * A * B * a^4 * c^2 * d^6 - 44 * A * B * a^4 * c^3 * d^5 + 32 * A * B * a^4 * c^4 * d^4 - 8 * A * B * a^4 * c^5 * d^3)) / d^5 + (8 * \tan(e/2 + (f * x) / 2) * (32 * A^2 * a^4 * c^4 * d^5 - 32 * A^2 * a^4 * c^3 * d^6 - 16 * A^2 * a^4 * c^2 * d^7 - 8 * A^2 * a^4 * c^5 * d^4 - 48 * B^2 * a^4 * c^2 * d^7 + 43 * B^2 * a^4 * c^3 * d^6 + 8 * B^2 * a^4 * c^4 * d^5 - 44 * B^2 * a^4 * c^5 * d^4 + 32 * B^2 * a^4 * c^6 * d^3 - 8 * B^2 * a^4 * c^7 * d^2 + 28 * A^2 * a^4 * c * d^8 + 18 * B^2 * a^4 * c * d^8 - 80 * A * B * a^4 * c^2 * d^7 + 8 * A * B * a^4 * c^3 * d^6 + 76 * A * B * a^4 * c^4 * d^5 - 64 * A * B * a^4 * c^5 * d^4 + 16 * A * B * a^4 * c^6 * d^3 + 48 * A * B * a^4 * c * d^8)) / d^6 + (a^2 (A d - B c) * (-(c + d) * (c - d)^3)^{1/2} * ((8 * (8 * A * a^2 * c * d^8 + 6 * B * a^2 * c * d^8 - 8 * A * a^2 * c^2 * d^7 - 8 * B * a^2 * c^2 * d^7 + 2 * B * a^2 * c^3 * d^6)) / d^5 - (8 * \tan(e/2 + (f * x) / 2) * (8 * A * a^2 * c * d^9 - 16 * A * a^2 * c^2 * d^8 + 8 * A * a^2 * c^3 * d^7 - 8 * B * a^2 * c^2 * d^8 + 16 * B * a^2 * c^3 * d^7 - 8 * B * a^2 * c^4 * d^6)) / d^6 + (a^2 (32 * c^2 * d^3 + (8 * \tan(e/2 + (f * x) / 2) * (12 * c * d^10 - 8 * c^3 * d^8)) / d^6) * (A d - B c) * (-(c + d) * (c - d)^3)^{1/2}) / (c * d^3 + d^4)) / (c * d^3 + d^4)) * (A d - B c) * (-(c + d) * (c - d)^3)^{1/2} * 2i) / (f * (c * d^3 + d^4))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.256 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=198

$$\frac{2a^2(c-d)(Ad(c+2d)-B(2c^2+2cd-d^2)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f(c+d)\sqrt{c^2-d^2}} - \frac{a^2 x(-Ad+2Bc-2Bd)}{d^3} + \frac{a^2(Ad-B(2c+d)) \cos(e+fx)}{d^2 f(c+d)}$$

[Out] $-a^2(-A*d+2*B*c-2*B*d)*x/d^3+a^2*(A*d-B*(2*c+d))*\cos(f*x+e)/d^2/(c+d)/f+(-A*d+B*c)*\cos(f*x+e)*(a^2+a^2*\sin(f*x+e))/d/(c+d)/f/(c+d*\sin(f*x+e))-2*a^2*(c-d)*(A*d*(c+2*d)-B*(2*c^2+2*c*d-d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/d^3/(c+d)/f/(c^2-d^2)^{(1/2)}$

Rubi [A] time = 0.58, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{2a^2(c-d)(Ad(c+2d)-B(2c^2+2cd-d^2)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f(c+d)\sqrt{c^2-d^2}} + \frac{a^2(Ad-B(2c+d)) \cos(e+fx)}{d^2 f(c+d)} - \frac{a^2 x(-Ad+2Bc-2Bd)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] $-((a^2*(2*B*c - A*d - 2*B*d)*x)/d^3) - (2*a^2*(c - d)*(A*d*(c + 2*d) - B*(2*c^2 + 2*c*d - d^2))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(d^3*(c + d)*\text{Sqrt}[c^2 - d^2]*f) + (a^2*(A*d - B*(2*c + d))*\text{Cos}[e + f*x])/(d^2*(c + d)*f) + ((B*c - A*d)*\text{Cos}[e + f*x]*(a^2 + a^2*\text{Sin}[e + f*x]))/(d*(c + d)*f*(c + d*\text{Sin}[e + f*x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2975

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.)), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx &= \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} + \int \frac{(a + a \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} \\
 &= \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} + \int \frac{-a^2(B(c-d)-2A)}{d(c + d)f(c + d \sin(e + fx))} \\
 &= \frac{a^2(Ad - B(2c + d)) \cos(e + fx)}{d^2(c + d)f} + \frac{(Bc - Ad) \cos(e + fx) (a^2 - a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} \\
 &= -\frac{a^2(2Bc - Ad - 2Bd)x}{d^3} + \frac{a^2(Ad - B(2c + d)) \cos(e + fx)}{d^2(c + d)f} + \frac{(Bc - Ad) \cos(e + fx) (a^2 - a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} \\
 &= -\frac{a^2(2Bc - Ad - 2Bd)x}{d^3} + \frac{a^2(Ad - B(2c + d)) \cos(e + fx)}{d^2(c + d)f} + \frac{(Bc - Ad) \cos(e + fx) (a^2 - a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} \\
 &= -\frac{a^2(2Bc - Ad - 2Bd)x}{d^3} + \frac{a^2(Ad - B(2c + d)) \cos(e + fx)}{d^2(c + d)f} + \frac{(Bc - Ad) \cos(e + fx) (a^2 - a^2 \sin(e + fx))}{d(c + d)f(c + d \sin(e + fx))} \\
 &= -\frac{a^2(2Bc - Ad - 2Bd)x}{d^3} - \frac{2a^2(c - d) (Ad(c + 2d) - B(2c^2 + 2cd + d^2))}{d^3(c + d)\sqrt{c^2 - d^2}}
 \end{aligned}$$

Mathematica [A] time = 1.00, size = 192, normalized size = 0.97

$$a^2(\sin(e + fx) + 1)^2 \left(\frac{2(c-d)(B(2c^2+2cd-d^2)-Ad(c+2d)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} + (e + fx)(Ad - 2Bc + 2Bd) - \frac{d(d-c)(Ad-Bc) \cos\left(\frac{1}{2}(e+fx)\right)}{(c+d)(c+d \sin(e+fx))} \right) \frac{d^3 f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^4}{d^3 f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^4}$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] (a^2*(1 + Sin[e + f*x])^2*((-2*B*c + A*d + 2*B*d)*(e + f*x) + (2*(c - d)*(-A*d*(c + 2*d)) + B*(2*c^2 + 2*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - B*d*Cos[e + f*x] - (d*(-c + d)*(-B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x]))/(d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)

fricas [A] time = 0.57, size = 731, normalized size = 3.69

$$\frac{2(2Ba^2c^3 - Aa^2c^2d - (A + 2B)a^2cd^2)fx + (2Ba^2c^3 - (A - 2B)a^2c^2d - (2A + B)a^2cd^2 + (2Ba^2c^2d - (A - 2B)a^2cd^2))}{(cd^3 + d^4)\sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [-1/2*(2*(2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*B*a^2*c^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + 2*((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f), -((2*B*a^2*c^3 - A*a^2*c^2*d - (A + 2*B)*a^2*c*d^2)*f*x + (2*B*a^2*c^3 - (A - 2*B)*a^2*c^2*d - (2*A + B)*a^2*c*d^2 + (2*B*a^2*c^2*d - (A - 2*B)*a^2*c*d^2 - (2*A + B)*a^2*d^3)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d))/((c - d)*cos(f*x + e))) + (2*B*a^2*c^2*d - A*a^2*c*d^2 + A*a^2*d^3)*cos(f*x + e) + ((2*B*a^2*c^2*d - A*a^2*c*d^2 - (A + 2*B)*a^2*d^3)*f*x + (B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e))*sin(f*x + e))/((c*d^4 + d^5)*f*sin(f*x + e) + (c^2*d^3 + c*d^4)*f)]

giac [B] time = 0.20, size = 498, normalized size = 2.52

$$\frac{2(2Ba^2c^3 - Aa^2c^2d - Aa^2cd^2 - 3Ba^2cd^2 + 2Aa^2d^3 + Ba^2d^3) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + d}{\sqrt{c^2 - d^2}}\right) \right)}{(cd^3 + d^4)\sqrt{c^2 - d^2}} - \frac{2(Ba^2c^2d \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - Aa^2cd^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right))}{(cd^3 + d^4)\sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")


```
[Out] (2*(2*B*a^2*c^3 - A*a^2*c^2*d - A*a^2*c*d^2 - 3*B*a^2*c*d^2 + 2*A*a^2*d^3 +
B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*
x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^3 + d^4)*sqrt(c^2 - d^2)) - 2*(B*a^
2*c^2*d*tan(1/2*f*x + 1/2*e)^3 - A*a^2*c*d^2*tan(1/2*f*x + 1/2*e)^3 - B*a^2
*c*d^2*tan(1/2*f*x + 1/2*e)^3 + A*a^2*d^3*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*
c^3*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d*tan(1/2*f*x + 1/2*e)^2 + A*a^2*c*d
^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^2*d*tan(1/2*f*x + 1/2*e) - A*a^2*c*d^
2*tan(1/2*f*x + 1/2*e) + B*a^2*c*d^2*tan(1/2*f*x + 1/2*e) + A*a^2*d^3*tan(1
/2*f*x + 1/2*e) + 2*B*a^2*c^3 - A*a^2*c^2*d + A*a^2*c*d^2)/((c*tan(1/2*f*x
+ 1/2*e)^4 + 2*d*tan(1/2*f*x + 1/2*e)^3 + 2*c*tan(1/2*f*x + 1/2*e)^2 + 2*d*
tan(1/2*f*x + 1/2*e) + c)*(c^2*d^2 + c*d^3)) - (2*B*a^2*c - A*a^2*d - 2*B*a
^2*d)*(f*x + e)/d^3)/f
```

maple [B] time = 0.51, size = 848, normalized size = 4.28

$$\frac{2a^2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) A}{f\left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right)d + c\right)(c+d)} - \frac{2a^2 d \tan\left(\frac{fx}{2} + \frac{e}{2}\right) A}{f\left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right)d + c\right)(c+d)c} - f a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x
+1/2*e)*A-2*a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2
*e)*d+c)/(c+d)*c*tan(1/2*f*x+1/2*e)*B+2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2*a^2/f/d/(tan(1/2*f*x+1/2*
e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c-2*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+
2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2*a^2/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan
(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/
2*f*x+1/2*e)*d+c)/(c+d)*B*c-2*a^2/f/d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2*a^2/f/d/(c+d)/(c^2-d^2)
^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+4*a^2/f
/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1
/2))*A+4*a^2/f/d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)
+2*d)/(c^2-d^2)^(1/2))*B*c^3-6*a^2/f/d/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*
c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2*a^2/f/(c+d)/(c^2-d^2)^(1/2)
)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-2*a^2/f/d^2*B/
(1+tan(1/2*f*x+1/2*e)^2)+2*a^2/f/d^2*A*arctan(tan(1/2*f*x+1/2*e))-4*a^2/f/d
^3*B*arctan(tan(1/2*f*x+1/2*e))*c+4*a^2/f/d^2*B*arctan(tan(1/2*f*x+1/2*e))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 21.58, size = 8706, normalized size = 43.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)
[Out] - ((2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2)^2*(A*a^2*d^2 + 2*B*a^2*c^2 - A*a^2*c*d))/(d^2*(c + d)) + (2*tan(e/2 + (f*x)/2)*(A*a^2*d^2 + 3*B*a^2*c^2 - A*a^2*c*d + B*a^2*c*d))/(c*d*(c + d)) + (2*tan(e/2 + (f*x)/2)^3*(A*a^2*d^2 + B*a^2*c^2 - A*a^2*c*d - B*a^2*c*d))/(c*d*(c + d)))/(f*(c + 2*d*tan(e/2 + (f*x)/2) + 2*c*tan(e/2 + (f*x)/2)^2 + c*tan(e/2 + (f*x)/2)^4 + 2*d*tan(e/2 + (f*x)/2)^3)) - (atan((((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*(((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)))/(2*c*d^7 + d^8 + c^2*d^6)))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3 + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)))/(2*c*d^7 + d^8 + c^2*d^6)))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8))/(2*c*d^7 + d^8 + c^2*d^6))*1i)/d^3)/((64*(4*B^3*a^6*c^6 - 2*A^3*a^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 - 2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5*d + 9*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2*a^6*c^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)))/(2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i)*(((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)))/(2*c*d^7 + d^8 + c^2*d^6)))*(B*a^2*c*2i - a^2*d*(A + 2*B)*1i))/d^3 - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6))/d^3 + (32*tan(e/2 + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2
```

$$\begin{aligned}
& *a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - \\
& 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - \\
& 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8*d^8)) / (2*c*d^7 + d^8 + c^2*d^6)) / d^3 - ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i) \\
&) * ((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B* \\
& a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)) / (2*c*d^6 + d^7 + c^2*d^5) + ((B*a^2*c*2i - a^2*d*(A + 2*B)*1i) * ((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 \\
& - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6)) / (2*c*d^6 + d^7 + c^2*d^5) + ((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8)) / (2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2) * (3*c*d^12 + 6*c^2*d^11 + c^3*d^10 - 4 \\
& *c^4*d^9 - 2*c^5*d^8)) / (2*c*d^7 + d^8 + c^2*d^6)) * (B*a^2*c*2i - a^2*d*(A + 2*B)*1i)) / d^3 - (32*tan(e/2 + (f*x)/2) * (4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2 \\
& *A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B* \\
& a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6)) / (2*c*d^7 + d^8 + c^2*d^6) \\
&)) / d^3 + (32*tan(e/2 + (f*x)/2) * (8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4* \\
& A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c^8*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B* \\
& a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8*d^8)) / (2* \\
& c*d^7 + d^8 + c^2*d^6)) / d^3 + (64*tan(e/2 + (f*x)/2) * (16*B^3*a^6*c^7 + 2*A \\
& ^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*c^4*d^3 - 32*B^3*a^6*c^2*d^5 \\
& + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40*B^3*a^6*c^5*d^2 + 4*A^3*a^6 \\
& *c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 24*A*B^2*a^6*c*d^6 - 24*A*B^2 \\
& *a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*c^2*d^5 - 24*A*B^2*a^6*c^3*d^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^5 - 30*A^2*B*a^6*c^3*d^4 + 1 \\
& 2*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2)) / (2*c*d^7 + d^8 + c^2*d^6)) * (B \\
& *a^2*c*2i - a^2*d*(A + 2*B)*1i) * 2i) / (d^3*f) - (a^2*atan(((a^2*(-(c + d))^3*(\\
& c - d))^(1/2) * ((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + \\
& 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2* \\
& d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)) / (2*c*d^6 \\
& + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2) * (8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3 \\
& d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2* \\
& a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - \\
& 2*A^2*a^4*c^8*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - \\
& 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8 \\
& *d^8)) / (2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d))^3*(c - d))^(1/2) * ((32*tan \\
& (e/2 + (f*x)/2) * (4*A*a^2*c*d^10 + 2*B*a^2*c*d^10 + 2*A*a^2*c^2*d^9 - 4*A*a^2 \\
& *c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4 \\
& *d^7 + 4*B*a^2*c^5*d^6)) / (2*c*d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2 \\
& *B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4* \\
& d^6)) / (2*c*d^6 + d^7 + c^2*d^5) + (a^2*((32*(c^2*d^10 + 2*c^3*d^9 + c^4*d^8) \\
&)) / (2*c*d^6 + d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2) * (3*c*d^12 + 6*c^2*d^11 \\
& + c^3*d^10 - 4*c^4*d^9 - 2*c^5*d^8)) / (2*c*d^7 + d^8 + c^2*d^6)) * (-(c + d) \\
& ^3*(c - d))^(1/2) * (2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)) / (3*c*d^5 + \\
& d^6 + 3*c^2*d^4 + c^3*d^3)) * (2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d) \\
&) / (3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)) * (2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d \\
& - 2*B*c*d) * 1i) / (3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3) + (a^2*(-(c + d))^3*(c \\
& - d))^(1/2) * ((32*(A^2*a^4*c^2*d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4 \\
& *B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 \\
& + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3)) / (2*c*d^6 + \\
& d^7 + c^2*d^5) + (32*tan(e/2 + (f*x)/2) * (8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3* \\
& d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4 \\
& *c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2* \\
& A^2*a^4*c^8*d^8 + 7*B^2*a^4*c*d^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - \\
& 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c^8 \\
& *d^8)) / (2*c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d))^3*(c - d))^(1/2) * ((32*(A*a^2 \\
& *c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - \\
& B*a^2*c^4*d^6)) / (2*c*d^6 + d^7 + c^2*d^5) - (32*tan(e/2 + (f*x)/2) * (4*A*a^
\end{aligned}$$

$$\begin{aligned}
& 2*c*d^{10} + 2*B*a^2*c*d^{10} + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6 \\
&)/(2*c*d^7 + d^8 + c^2*d^6) + (a^2*((32*(c^2*d^{10} + 2*c^3*d^9 + c^4*d^8)) \\
&)/(2*c*d^6 + d^7 + c^2*d^5) + (32*\tan(e/2 + (f*x)/2)*(3*c*d^{12} + 6*c^2*d^{11} \\
& + c^3*d^{10} - 4*c^4*d^9 - 2*c^5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3 \\
& *(c - d))^{(1/2)}*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(\\
& 3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)*1i)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))/((64*(4*B^3*a^6*c^6 - 2 \\
& *A^3*a^6*c^2*d^4 - 2*A^3*a^6*c^3*d^3 - 10*B^3*a^6*c^2*d^4 + 14*B^3*a^6*c^3*d^3 - 2*B^3*a^6*c^4*d^2 + 4*A^3*a^6*c*d^5 + 2*B^3*a^6*c*d^5 - 8*B^3*a^6*c^5 \\
& *d + 9*A*B^2*a^6*c*d^5 - 12*A*B^2*a^6*c^5*d + 12*A^2*B*a^6*c*d^5 - 30*A*B^2 \\
& *a^6*c^2*d^4 + 21*A*B^2*a^6*c^3*d^3 + 12*A*B^2*a^6*c^4*d^2 - 21*A^2*B*a^6*c^2*d^4 + 9*A^2*B*a^6*c^4*d^2))/(2*c*d^6 + d^7 + c^2*d^5) + (64*\tan(e/2 + (f \\
& *x)/2)*(16*B^3*a^6*c^7 + 2*A^3*a^6*c^2*d^5 - 4*A^3*a^6*c^3*d^4 - 2*A^3*a^6*c^4*d^3 - 32*B^3*a^6*c^2*d^5 + 16*B^3*a^6*c^3*d^4 + 48*B^3*a^6*c^4*d^3 - 40 \\
& *B^3*a^6*c^5*d^2 + 4*A^3*a^6*c*d^6 + 8*B^3*a^6*c*d^6 - 16*B^3*a^6*c^6*d + 2 \\
& 4*A*B^2*a^6*c*d^6 - 24*A*B^2*a^6*c^6*d + 18*A^2*B*a^6*c*d^6 - 48*A*B^2*a^6*c^2*d^5 - 24*A*B^2*a^6*c^3*d^4 + 72*A*B^2*a^6*c^4*d^3 - 12*A^2*B*a^6*c^2*d^ \\
& 5 - 30*A^2*B*a^6*c^3*d^4 + 12*A^2*B*a^6*c^4*d^3 + 12*A^2*B*a^6*c^5*d^2))/(2 \\
& *c*d^7 + d^8 + c^2*d^6) + (a^2*(-(c + d)^3*(c - d))^{(1/2)}*((32*(A^2*a^4*c^2 \\
& *d^6 + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4 \\
& *d^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B \\
& *a^4*c^4*d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*\tan(e/2 \\
& + (f*x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A \\
& ^2*a^4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 \\
& + 28*B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d \\
& ^8 + 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a \\
& ^4*c^5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6 \\
&) + (a^2*(-(c + d)^3*(c - d))^{(1/2)}*((32*\tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^{10} \\
& + 2*B*a^2*c*d^{10} + 2*A*a^2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4 \\
& *B*a^2*c^2*d^9 - 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c \\
& *d^7 + d^8 + c^2*d^6) - (32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^3*d^7 + \\
& B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + c^2*d^5) \\
& + (a^2*((32*(c^2*d^{10} + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + \\
& (32*\tan(e/2 + (f*x)/2)*(3*c*d^{12} + 6*c^2*d^{11} + c^3*d^{10} - 4*c^4*d^9 - 2*c^ \\
& 5*d^8))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^{(1/2)}*(2*A*d^2 - 2* \\
& B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2 \\
& *A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c \\
& ^3*d^3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3* \\
& c^2*d^4 + c^3*d^3) - (a^2*(-(c + d)^3*(c - d))^{(1/2)}*((32*(A^2*a^4*c^2*d^6 \\
& + 2*A^2*a^4*c^3*d^5 + A^2*a^4*c^4*d^4 + 4*B^2*a^4*c^2*d^6 - 8*B^2*a^4*c^4*d^ \\
& ^4 + 4*B^2*a^4*c^6*d^2 + 4*A*B*a^4*c^2*d^6 + 4*A*B*a^4*c^3*d^5 - 4*A*B*a^4*c^4 \\
& *d^4 - 4*A*B*a^4*c^5*d^3))/(2*c*d^6 + d^7 + c^2*d^5) + (32*\tan(e/2 + (f* \\
& x)/2)*(8*A^2*a^4*c^2*d^7 + 4*A^2*a^4*c^3*d^6 - 4*A^2*a^4*c^4*d^5 - 2*A^2*a^ \\
& 4*c^5*d^4 + 6*B^2*a^4*c^2*d^7 - 29*B^2*a^4*c^3*d^6 - 4*B^2*a^4*c^4*d^5 + 28 \\
& *B^2*a^4*c^5*d^4 - 8*B^2*a^4*c^7*d^2 - 2*A^2*a^4*c*d^8 + 7*B^2*a^4*c*d^8 + \\
& 22*A*B*a^4*c^2*d^7 - 16*A*B*a^4*c^3*d^6 - 26*A*B*a^4*c^4*d^5 + 8*A*B*a^4*c^ \\
& 5*d^4 + 8*A*B*a^4*c^6*d^3 + 4*A*B*a^4*c*d^8))/(2*c*d^7 + d^8 + c^2*d^6) + (\\
& a^2*(-(c + d)^3*(c - d))^{(1/2)}*((32*(A*a^2*c*d^9 + 2*B*a^2*c*d^9 - A*a^2*c^ \\
& 3*d^7 + B*a^2*c^2*d^8 - 2*B*a^2*c^3*d^7 - B*a^2*c^4*d^6))/(2*c*d^6 + d^7 + \\
& c^2*d^5) - (32*\tan(e/2 + (f*x)/2)*(4*A*a^2*c*d^{10} + 2*B*a^2*c*d^{10} + 2*A*a^ \\
& 2*c^2*d^9 - 4*A*a^2*c^3*d^8 - 2*A*a^2*c^4*d^7 - 4*B*a^2*c^2*d^9 - 6*B*a^2*c \\
& ^3*d^8 + 4*B*a^2*c^4*d^7 + 4*B*a^2*c^5*d^6))/(2*c*d^7 + d^8 + c^2*d^6) + (a \\
& ^2*((32*(c^2*d^{10} + 2*c^3*d^9 + c^4*d^8))/(2*c*d^6 + d^7 + c^2*d^5) + (32*t \\
& an(e/2 + (f*x)/2)*(3*c*d^{12} + 6*c^2*d^{11} + c^3*d^{10} - 4*c^4*d^9 - 2*c^5*d^8 \\
&))/(2*c*d^7 + d^8 + c^2*d^6))*(-(c + d)^3*(c - d))^{(1/2)}*(2*A*d^2 - 2*B*c^2 \\
& + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(2*A*d^ \\
& 2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d))/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^
\end{aligned}$$

3))*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)/(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))*(-(c + d)^3*(c - d))^(1/2)*(2*A*d^2 - 2*B*c^2 + B*d^2 + A*c*d - 2*B*c*d)*2i)/(f*(3*c*d^5 + d^6 + 3*c^2*d^4 + c^3*d^3))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out

$$3.257 \quad \int \frac{(a+a \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=215

$$\frac{a^2(3Ad^2 - B(2c^2 + 3cd - 2d^2)) \cos(e+fx)}{2d^2 f(c+d)^2(c+d \sin(e+fx))} + \frac{a^2(3Ad^3 - B(2c^3 + 4c^2d + cd^2 - 4d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f(c+d)^2 \sqrt{c^2-d^2}} + \dots$$

[Out] $a^2 B x / d^3 + 1/2 (-A d + B c) \cos(f x + e) (a^2 + a^2 \sin(f x + e)) / d / (c + d) / f / (c + d \sin(f x + e))^2 - 1/2 a^2 (3 A d^3 - B (2 c^3 + 4 c^2 d - 2 d^2)) \cos(f x + e) / d^2 / (c + d)^2 / f / (c + d \sin(f x + e)) + a^2 (3 A d^3 - B (2 c^3 + 4 c^2 d + c d^2 - 4 d^3)) \arctan((d + c \tan(1/2 f x + 1/2 e)) / (c^2 - d^2)^{1/2}) / d^3 / (c + d)^2 / f / (c^2 - d^2)^{1/2}$

Rubi [A] time = 0.62, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2975, 2968, 3021, 2735, 2660, 618, 204}

$$\frac{a^2(3Ad^3 - B(4c^2d + 2c^3 + cd^2 - 4d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^3 f(c+d)^2 \sqrt{c^2-d^2}} - \frac{a^2(3Ad^2 - B(2c^2 + 3cd - 2d^2)) \cos(e+fx)}{2d^2 f(c+d)^2(c+d \sin(e+fx))} + \dots$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] $(a^2 B x) / d^3 + (a^2 (3 A d^3 - B (2 c^3 + 4 c^2 d + c d^2 - 4 d^3)) \text{ArcTan}[(d + c \text{Tan}[(e + f x) / 2]) / \text{Sqrt}[c^2 - d^2]]) / (d^3 (c + d)^2 \text{Sqrt}[c^2 - d^2] * f) + ((B c - A d) \text{Cos}[e + f x] * (a^2 + a^2 \text{Sin}[e + f x])) / (2 d * (c + d) * f * (c + d \text{Sin}[e + f x])^2) - (a^2 (3 A d^3 - B (2 c^3 + 4 c^2 d + c d^2 - 4 d^3)) \text{Cos}[e + f x]) / (2 d^2 * (c + d)^2 * f * (c + d \text{Sin}[e + f x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2975

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)), x] - Dist[b/(d*(n + 1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^2 (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx &= \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{\int \frac{(a + a \sin(e + fx))}{(c + d \sin(e + fx))^3} dx}{2d(c + d)f(c + d \sin(e + fx))^2} \\ &= \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{\int \frac{-a^2(Bc - 3Ad - 2Bc \sin(e + fx))}{(c + d \sin(e + fx))^3} dx}{2d(c + d)f(c + d \sin(e + fx))^2} \\ &= \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a^2 (3Ad^2 - B(c + d)^2)}{2d^2(c + d)^2} \\ &= \frac{a^2 Bx}{d^3} + \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a^2 (3Ad^2 - B(c + d)^2)}{2d^2(c + d)^2} \\ &= \frac{a^2 Bx}{d^3} + \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a^2 (3Ad^2 - B(c + d)^2)}{2d^2(c + d)^2} \\ &= \frac{a^2 Bx}{d^3} + \frac{(Bc - Ad) \cos(e + fx) (a^2 + a^2 \sin(e + fx))}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a^2 (3Ad^2 - B(c + d)^2)}{2d^2(c + d)^2} \\ &= \frac{a^2 Bx}{d^3} - \frac{a^2 (2Bc(c + d)^2 - d^2(3Ad + B(c + 4d))) \tan^{-1} \left(\frac{d + c \tan(e + fx)}{\sqrt{c^2 - d^2}} \right)}{d^3(c + d)^2 \sqrt{c^2 - d^2} f} \end{aligned}$$

Mathematica [A] time = 1.37, size = 226, normalized size = 1.05

$$a^2(\sin(e + fx) + 1)^2 \left(-\frac{d(Ad(c+4d)+B(-3c^2-4cd+2d^2))\cos(e+fx)}{(c+d)^2(c+d\sin(e+fx))} - \frac{2(B(2c^3+4c^2d+cd^2-4d^3)-3Ad^3)\tan^{-1}\left(\frac{c\tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c+d)^2\sqrt{c^2-d^2}} - \frac{d(d-c)(Ad)}{(c+d)(c+d)} \right)$$

$$2d^3 f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)^4$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] (a^2*(1 + Sin[e + f*x])^2*(2*B*(e + f*x) - (2*(-3*A*d^3 + B*(2*c^3 + 4*c^2*d + c*d^2 - 4*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(c + d)^2*Sqrt[c^2 - d^2]) - (d*(-c + d)*(-(B*c) + A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])^2) - (d*(A*d*(c + 4*d) + B*(-3*c^2 - 4*c*d + 2*d^2))*Cos[e + f*x])/((c + d)^2*(c + d*Sin[e + f*x])))/(2*d^3*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)

fricas [B] time = 0.55, size = 1483, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [1/4*(4*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*cos(f*x + e)^2 - 4*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*d^4)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - 2*(4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 - d^9)*f), 1/2*(2*(B*a^2*c^4*d^2 + 2*B*a^2*c^3*d^3 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x*cos(f*x + e)^2 - 2*(B*a^2*c^6 + 2*B*a^2*c^5*d + B*a^2*c^4*d^2 - B*a^2*c^2*d^4 - 2*B*a^2*c*d^5 - B*a^2*d^6)*f*x - (2*B*a^2*c^5 + 4*B*a^2*c^4*d + 3*B*a^2*c^3*d^2 - 3*A*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5 - (2*B*a^2*c^3*d^2 + 4*B*a^2*c^2*d^3 + B*a^2*c*d^4 - (3*A + 4*B)*a^2*d^5)*cos(f*x + e)^2 + 2*(2*B*a^2*c^4*d + 4*B*a^2*c^3*d^2 + B*a^2*c^2*d^3 - (3*A + 4*B)*a^2*c*d^4)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) - (2*B*a^2*c^5*d + 4*B*a^2*c^4*d^2 - (4*A + 3*B)*a^2*c^3*d^3 - (A + 4*B)*a^2*c^2*d^4 + (4*A + B)*a^2*c*d^5 + A*a^2*d^6)*cos(f*x + e) - (4*(B*a^2*c^5*d + 2*B*a^2*c^4*d^2 - 2*B*a^2*c^2*d^4 - B*a^2*c*d^5)*f*x + (3*B*a^2*c^4*d^2 - (A - 4*B)*a^2*c^3*d^3 - (4*A + 5*B)*a^2*c^2*d^4 + (A - 4*B)*a^2*c*d^5 + 2*(2*A + B)*a^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^5 + 2*c^3*d^6 - 2*c*d^8 - d^9)*f*cos(f*x + e)^2 - 2*(c^5*d^4 + 2*c^4*d^5 - 2*c^2*d^7 - c*d^8)*f*sin(f*x + e) - (c^6*d^3 + 2*c^5*d^4 + c^4*d^5 - c^2*d^7 - 2*c*d^8 - d^9)*f)]

giac [B] time = 0.24, size = 703, normalized size = 3.27

$$\frac{(fx+e)Ba^2}{d^3} - \frac{(2Ba^2c^3+4Ba^2c^2d+Ba^2cd^2-3Aa^2d^3-4Ba^2d^3)\left(\pi\left[\frac{fx+e}{2\pi}+\frac{1}{2}\right]\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{(c^2d^3+2cd^4+d^5)\sqrt{c^2-d^2}} + \frac{Ba^2c^4d\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3+Aa^2c^3d}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] ((f*x + e)*B*a^2/d^3 - (2*B*a^2*c^3 + 4*B*a^2*c^2*d + B*a^2*c*d^2 - 3*A*a^2*d^3 - 4*B*a^2*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^3 + 2*c*d^4 + d^5)*sqrt(c^2 - d^2)) + (B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^3 + A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^3 - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*B*a^2*c^5*tan(1/2*f*x + 1/2*e)^2 + 4*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 + 3*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e)^2 - A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 + 8*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e)^2 - 8*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*B*a^2*c*d^4*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^2*d^5*tan(1/2*f*x + 1/2*e)^2 + 7*B*a^2*c^4*d*tan(1/2*f*x + 1/2*e) - A*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) + 12*B*a^2*c^3*d^2*tan(1/2*f*x + 1/2*e) - 12*A*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 4*B*a^2*c^2*d^3*tan(1/2*f*x + 1/2*e) - 2*A*a^2*c*d^4*tan(1/2*f*x + 1/2*e) + 2*B*a^2*c^5 + 4*B*a^2*c^4*d - 4*A*a^2*c^3*d^2 - B*a^2*c^3*d^2 - A*a^2*c^2*d^3)/((c^4*d^2 + 2*c^3*d^3 + c^2*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2))/f

maple [B] time = 0.54, size = 1916, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)

[Out] -a^2/f/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2*a^2/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3-2*a^2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*A-4*a^2/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*B-8*a^2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-2*a^2/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A+2*a^2/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B+4*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-2*a^2/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*A+7*a^2/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c-4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c+3*a^2/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-a^2/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A+a^2/f/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*tan(1/2*f*x+1/2*e)^3*A+4*a^2/f/(tan(1/2*f*x+1/2*e)^2*c+

$$2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^3*B-4*a^2/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*A+3*a^2/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*B-a^2/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A+12*a^2/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B-12*a^2/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-4*a^2/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B+4*a^2/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2+2*a^2/f/d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3-4*a^2/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*A-a^2/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^2*A+8*a^2/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^2*B+4*a^2/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2*a^2/f*B/d^3*\arctan(\tan(1/2*f*x+1/2*e))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 22.50, size = 8632, normalized size = 40.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2)/(c + d*sin(e + f*x))^3,x)

[Out]
$$(2*B*a^2*\operatorname{atan}(((B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8)))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*\tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6)))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8)))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3) + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B^2$$

$$\begin{aligned}
& 2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24* \\
& A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8)/(4*c*d^9 + d^10 + \\
& 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c^ \\
& 2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3* \\
& d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ \\
& 3*d^6 + c^4*d^5) - (8*\tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 \\
& + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d \\
& ^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^1 \\
& 0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^11 \\
& + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ \\
& 3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3* \\
& d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^10 \\
& + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3))/d^3)/((16*(2*B^3*a^ \\
& 6*c^5 + 17*B^3*a^6*c^3*d^2 - 16*B^3*a^6*c*d^4 + 12*B^3*a^6*c^4*d - 24*A*B^2 \\
& *a^6*c*d^4 + 6*A*B^2*a^6*c^4*d - 9*A^2*B*a^6*c*d^4 + 12*A*B^2*a^6*c^3*d^2)) \\
& / (4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) - (16*\tan(e/2 + (f*x)/2) \\
& *(28*B^3*a^6*c^2*d^4 - 8*B^3*a^6*c^6 - 8*B^3*a^6*c^3*d^3 - 44*B^3*a^6*c^4*d \\
& ^2 + 16*B^3*a^6*c*d^5 - 32*B^3*a^6*c^5*d + 12*A*B^2*a^6*c*d^5 + 24*A*B^2*a^ \\
& 6*c^2*d^4 + 12*A*B^2*a^6*c^3*d^3))/ (4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 \\
& + c^4*d^6) - (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^ \\
& 4*c^4*d^4 + 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2)))/(4*c*d^8 + d^9 + 6*c^2 \\
& *d^7 + 4*c^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 7 \\
& 5*B^2*a^4*c^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^ \\
& 6*d^3 - 8*B^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c \\
& ^2*d^7 + 24*A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c* \\
& d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*\tan(e/2 + (f*x)/ \\
& 2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^11 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^ \\
& 9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3*d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d \\
& ^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) - \\
& (8*(4*B*a^2*c*d^10 - 6*A*a^2*c^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 \\
& + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3*d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6)))/(\\
& 4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^3*d^6 + c^4*d^5) + (B*a^2*((8*(4*c^2*d^12 + \\
& 16*c^3*d^11 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8)))/(4*c*d^8 + d^9 + 6*c^ \\
& 2*d^7 + 4*c^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^ \\
& 13 + 64*c^3*d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4* \\
& c*d^9 + d^10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3 \\
& + (B*a^2*((8*(4*B^2*a^4*c^2*d^6 + 16*B^2*a^4*c^3*d^5 + 24*B^2*a^4*c^4*d^4 + \\
& 16*B^2*a^4*c^5*d^3 + 4*B^2*a^4*c^6*d^2)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c^ \\
& 3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(40*B^2*a^4*c^2*d^7 + 75*B^2*a^4*c \\
& ^3*d^6 + 24*B^2*a^4*c^4*d^5 - 36*B^2*a^4*c^5*d^4 - 32*B^2*a^4*c^6*d^3 - 8*B \\
& ^2*a^4*c^7*d^2 - 9*A^2*a^4*c*d^8 - 8*B^2*a^4*c*d^8 + 6*A*B*a^4*c^2*d^7 + 24 \\
& *A*B*a^4*c^3*d^6 + 12*A*B*a^4*c^4*d^5 - 24*A*B*a^4*c*d^8))/(4*c*d^9 + d^10 \\
& + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*B*a^2*c*d^10 - 6*A*a^2*c \\
& ^2*d^9 - 12*A*a^2*c^3*d^8 - 6*A*a^2*c^4*d^7 + 8*B*a^2*c^2*d^9 + 6*B*a^2*c^3 \\
& *d^8 + 4*B*a^2*c^4*d^7 + 2*B*a^2*c^5*d^6)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c \\
& ^3*d^6 + c^4*d^5) - (8*\tan(e/2 + (f*x)/2)*(12*A*a^2*c*d^11 + 16*B*a^2*c*d^1 \\
& 1 + 24*A*a^2*c^2*d^10 + 12*A*a^2*c^3*d^9 + 28*B*a^2*c^2*d^10 - 8*B*a^2*c^3* \\
& d^9 - 44*B*a^2*c^4*d^8 - 32*B*a^2*c^5*d^7 - 8*B*a^2*c^6*d^6))/(4*c*d^9 + d^ \\
& 10 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6) + (B*a^2*((8*(4*c^2*d^12 + 16*c^3*d^1 \\
& 1 + 24*c^4*d^10 + 16*c^5*d^9 + 4*c^6*d^8)))/(4*c*d^8 + d^9 + 6*c^2*d^7 + 4*c \\
& ^3*d^6 + c^4*d^5) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^14 + 48*c^2*d^13 + 64*c^3 \\
& *d^12 + 16*c^4*d^11 - 36*c^5*d^10 - 32*c^6*d^9 - 8*c^7*d^8))/(4*c*d^9 + d^1 \\
& 0 + 6*c^2*d^8 + 4*c^3*d^7 + c^4*d^6))*1i)/d^3)*1i)/d^3)*1i)/d^3))/d^3*f) \\
& - ((A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^2*c*d^2 - 4*B*a^2*c^2*d)/ \\
& (d^2*(2*c*d + c^2 + d^2)) - (\tan(e/2 + (f*x)/2)^3*(B*a^2*c^3 - 2*A*a^2*d^3 \\
& - 4*A*a^2*c*d^2 + A*a^2*c^2*d + 4*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + \\
& (\tan(e/2 + (f*x)/2)*(2*A*a^2*d^3 - 7*B*a^2*c^3 + 12*A*a^2*c*d^2 + A*a^2*c^ \\
& 2*d + 4*B*a^2*c*d^2 - 12*B*a^2*c^2*d))/(c*d*(2*c*d + c^2 + d^2)) + (\tan(e/2 \\
& + (f*x)/2)^2*(c^2 + 2*d^2)*(A*a^2*d^3 - 2*B*a^2*c^3 + 4*A*a^2*c*d^2 + B*a^
\end{aligned}$$

$$\begin{aligned}
& (4ABa^4c^3d^6 + 12A^2B^2a^4c^4d^5 - 24A^2B^2a^4c^4d^8) / (4c^2d^9 + d^{10} \\
& + 6c^2d^8 + 4c^3d^7 + c^4d^6) + (a^2(-c+d)^5(c-d)^{1/2}) \cdot ((8 \tan(e/2 + (f*x)/2) \cdot (12A^2a^2c^2d^{11} + 16B^2a^2c^2d^{11} + 24A^2a^2c^2d^{10} + \\
& 12A^2a^2c^3d^9 + 28B^2a^2c^2d^{10} - 8B^2a^2c^3d^9 - 44B^2a^2c^4d^8 - 32B^2a^2c^5d^7 - 8B^2a^2c^6d^6)) / (4c^2d^9 + d^{10} + 6c^2d^8 + 4c^3d^7 \\
& + c^4d^6) - (8(4B^2a^2c^2d^{10} - 6A^2a^2c^2d^9 - 12A^2a^2c^3d^8 - 6A^2a^2c^4d^7 + 8B^2a^2c^2d^9 + 6B^2a^2c^3d^8 + 4B^2a^2c^4d^7 + 2B^2a^2c^5d^6)) / (4c^2d^8 + d^9 + 6c^2d^7 + 4c^3d^6 + c^4d^5) + (a^2(-c+d)^5(c-d)^{1/2}) \cdot ((8(4c^2d^{12} + 16c^3d^{11} + 24c^4d^{10} + 16c^5d^9 + 4c^6d^8)) / (4c^2d^8 + d^9 + 6c^2d^7 + 4c^3d^6 + c^4d^5) + (8 \tan(e/2 + (f*x)/2) \cdot (12c^2d^{14} + 48c^2d^{13} + 64c^3d^{12} + 16c^4d^{11} - 36c^5d^{10} - 32c^6d^9 - 8c^7d^8)) / (4c^2d^9 + d^{10} + 6c^2d^8 + 4c^3d^7 + c^4d^6)) \cdot (2B^2c^3 - 3A^2d^3 - 4B^2d^3 + B^2c^2d^2 + 4B^2c^2d) / (2(4c^2d^8 + d^9 + 5c^2d^7 - 5c^4d^5 - 4c^5d^4 - c^6d^3)) \cdot (2B^2c^3 - 3A^2d^3 - 4B^2d^3 + B^2c^2d) / (2(4c^2d^8 + d^9 + 5c^2d^7 - 5c^4d^5 - 4c^5d^4 - c^6d^3)) + (a^2(-c+d)^5(c-d)^{1/2}) \cdot ((8(4B^2a^4c^2d^6 + 16B^2a^4c^3d^5 + 24B^2a^4c^4d^4 + 16B^2a^4c^5d^3 + 4B^2a^4c^6d^2)) / (4c^2d^8 + d^9 + 6c^2d^7 + 4c^3d^6 + c^4d^5) + (8 \tan(e/2 + (f*x)/2) \cdot (40B^2a^4c^2d^7 + 75B^2a^4c^3d^6 + 24B^2a^4c^4d^5 - 36B^2a^4c^5d^4 - 32B^2a^4c^6d^3 - 8B^2a^4c^7d^2 - 9A^2a^4c^2d^8 - 8B^2a^4c^2d^8 + 6A^2B^2a^4c^2d^7 + 24A^2B^2a^4c^3d^6 + 12A^2B^2a^4c^4d^5 - 24A^2B^2a^4c^4d^8)) / (4c^2d^9 + d^{10} + 6c^2d^8 + 4c^3d^7 + c^4d^6) + (a^2(-c+d)^5(c-d)^{1/2}) \cdot ((8(4B^2a^2c^2d^{10} - 6A^2a^2c^2d^9 - 12A^2a^2c^3d^8 - 6A^2a^2c^4d^7 + 8B^2a^2c^2d^9 + 6B^2a^2c^3d^8 + 4B^2a^2c^4d^7 + 2B^2a^2c^5d^6)) / (4c^2d^8 + d^9 + 6c^2d^7 + 4c^3d^6 + c^4d^5) - (8 \tan(e/2 + (f*x)/2) \cdot (12A^2a^2c^2d^{11} + 16B^2a^2c^2d^{11} + 24A^2a^2c^2d^{10} + 12A^2a^2c^3d^9 + 28B^2a^2c^2d^{10} - 8B^2a^2c^3d^9 - 44B^2a^2c^4d^8 - 32B^2a^2c^5d^7 - 8B^2a^2c^6d^6)) / (4c^2d^9 + d^{10} + 6c^2d^8 + 4c^3d^7 + c^4d^6) + (a^2(-c+d)^5(c-d)^{1/2}) \cdot ((8(4c^2d^{12} + 16c^3d^{11} + 24c^4d^{10} + 16c^5d^9 + 4c^6d^8)) / (4c^2d^8 + d^9 + 6c^2d^7 + 4c^3d^6 + c^4d^5) + (8 \tan(e/2 + (f*x)/2) \cdot (12c^2d^{14} + 48c^2d^{13} + 64c^3d^{12} + 16c^4d^{11} - 36c^5d^{10} - 32c^6d^9 - 8c^7d^8)) / (4c^2d^9 + d^{10} + 6c^2d^8 + 4c^3d^7 + c^4d^6)) \cdot (2B^2c^3 - 3A^2d^3 - 4B^2d^3 + B^2c^2d^2 + 4B^2c^2d) / (2(4c^2d^8 + d^9 + 5c^2d^7 - 5c^4d^5 - 4c^5d^4 - c^6d^3)) \cdot (2B^2c^3 - 3A^2d^3 - 4B^2d^3 + B^2c^2d) / (2(4c^2d^8 + d^9 + 5c^2d^7 - 5c^4d^5 - 4c^5d^4 - c^6d^3)) \cdot (-c+d)^5(c-d)^{1/2} \cdot (2B^2c^3 - 3A^2d^3 - 4B^2d^3 + B^2c^2d + 4B^2c^2d) \cdot 1i) / (f(4c^2d^8 + d^9 + 5c^2d^7 - 5c^4d^5 - 4c^5d^4 - c^6d^3))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)

[Out] Timed out

$$3.258 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$$

Optimal. Leaf size=604

$$\frac{a^3 (-14Acd + 91Ad^2 + 6Bc^2 - 27Bcd + 87Bd^2) \cos(e + fx)(c + d \sin(e + fx))^4 - a^3 (7Ad(2c^2 - 18cd + 115d^2))}{210d^3 f}$$

[Out] $\frac{1}{16}a^3(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))*x - \frac{1}{420}a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-920c^2d^4-952cd^5-288d^6))*\cos(fx+e)/d^3/f - \frac{1}{1680}a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4-735d^5))*\cos(fx+e)*\sin(fx+e)/d^2/f - \frac{1}{840}a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))*\cos(fx+e)*(c+d*\sin(fx+e))^2/d^3/f - \frac{1}{840}a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))*\cos(fx+e)*(c+d*\sin(fx+e))^3/d^3/f - \frac{1}{210}a^3(-14Acd+91Ad^2+6Bc^2-27Bcd+87Bd^2)*\cos(fx+e)*(c+d*\sin(fx+e))^4/d^3/f - \frac{1}{7}a*B*\cos(fx+e)*(a+a*\sin(fx+e))^2*(c+d*\sin(fx+e))^4/d/f + \frac{1}{42}(3B(c-3d)-7Ad)*\cos(fx+e)*(a^3+a^3*\sin(fx+e))*(c+d*\sin(fx+e))^4/d^2/f$

Rubi [A] time = 1.49, antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^3 (7Ad(107c^3d^2 + 472c^2d^3 - 18c^4d + 2c^5 + 456cd^4 + 136d^5) - 3B(51c^4d^2 - 189c^3d^3 - 920c^2d^4 - 14c^5d + 2c^6))}{420d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]

[Out] $\frac{a^3(3B(10c^3+26c^2d+23cd^2+7d^3)+A(40c^3+90c^2d+78cd^2+23d^3))*x}{16} - \frac{a^3(7Ad(2c^5-18c^4d+107c^3d^2+472c^2d^3+456cd^4+136d^5)-3B(2c^6-14c^5d+51c^4d^2-189c^3d^3-920c^2d^4-952cd^5-288d^6))*\cos[e+fx]}{(420d^3f)} - \frac{a^3(7Ad(4c^4-36c^3d+216c^2d^2+626cd^3+345d^4)-3B(4c^5-28c^4d+104c^3d^2-392c^2d^3-1263cd^4-735d^5))*\cos[e+fx]*\sin[e+fx]}{(1680d^2f)} - \frac{a^3(7Ad(2c^3-18c^2d+111cd^2+136d^3)-B(6c^4-42c^3d+165c^2d^2-651cd^3-864d^4))*\cos[e+fx]*(c+d*\sin[e+fx])^2}{(840d^3f)} - \frac{a^3(7Ad(2c^2-18cd+115d^2)-B(6c^3-42c^2d+177cd^2-735d^3))*\cos[e+fx]*(c+d*\sin[e+fx])^3}{(840d^3f)} - \frac{a^3(6Bc^2-14Ad-27Bcd+91Ad^2+87Bd^2)*\cos[e+fx]*(c+d*\sin[e+fx])^4}{(210d^3f)} - \frac{a*B*\cos[e+fx]*(a+a*\sin[e+fx])^2*(c+d*\sin[e+fx])^4}{(7d*f)} + \frac{(3B(c-3d)-7Ad)*\cos[e+fx]*(a^3+a^3*\sin[e+fx])*(c+d*\sin[e+fx])^4}{(42d^2f)}$

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m]/(f

```
*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m
+ a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&& IntegerQ[2*m]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
 \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^3}{7df} \\
 &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^3}{7df} \\
 &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^3}{7df} \\
 &= -\frac{a^3 (6Bc^2 - 14Acd - 27Bcd + 91Ad^2 + 87Bd^2)}{210d^3} \\
 &= -\frac{a^3 (7Ad (2c^2 - 18cd + 115d^2) - B (6c^3 - 18cd^2 + 115d^3))}{210d^3} \\
 &= -\frac{a^3 (7Ad (2c^3 - 18c^2d + 111cd^2 + 136d^3) - B (6c^3 - 18cd^2 + 115d^3))}{210d^3} \\
 &= \frac{1}{16} a^3 (3B (10c^3 + 26c^2d + 23cd^2 + 7d^3) + \dots)
 \end{aligned}$$

Mathematica [A] time = 4.76, size = 528, normalized size = 0.87

$$a^3 \cos(e + fx) \left(420 \left(A \left(40c^3 + 90c^2d + 78cd^2 + 23d^3 \right) + 3B \left(10c^3 + 26c^2d + 23cd^2 + 7d^3 \right) \right) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]

[Out] -1/3360*(a^3*Cos[e + f*x]*(420*(3*B*(10*c^3 + 26*c^2*d + 23*c*d^2 + 7*d^3) + A*(40*c^3 + 90*c^2*d + 78*c*d^2 + 23*d^3))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(12880*A*c^3 + 11760*B*c^3 + 35280*A*c^2*d + 32676*B*c^2*d + 32676*A*c*d^2 + 30828*B*c*d^2 + 10276*A*d^3 + 9762*B*d^3 - (112*A*(5*c^3 + 45*c^2*d + 66*c*d^2 + 26*d^3) + 3*B*(560*c^3 + 2464*c^2*d + 2912*c*d^2 + 1083*d^3))*Cos[2*(e + f*x)] + 18*d*(14*A*d*(c + d) + B*(14*c^2 + 42*c*d + 23*d^2))*Cos[4*(e + f*x)] - 15*B*d^3*Cos[6*(e + f*x)] + 5040*A*c^3*Sin[e + f*x] + 6930*B*c^3*Sin[e + f*x] + 20790*A*c^2*d*Sin[e + f*x] + 22050*B*c^2*d*Sin[e + f*x] + 22050*A*c*d^2*Sin[e + f*x] + 22785*B*c*d^2*Sin[e + f*x] + 7595*A*d^3*Sin[e + f*x] + 7665*B*d^3*Sin[e + f*x] - 210*B*c^3*Sin[3*(e + f*x)] - 630*A*c^2*d*Sin[3*(e + f*x)] - 1890*B*c^2*d*Sin[3*(e + f*x)] - 1890*A*c*d^2*Sin[3*(e + f*x)] - 2940*B*c*d^2*Sin[3*(e + f*x)] - 980*A*d^3*Sin[3*(e + f*x)] - 1260*B*d^3*Sin[3*(e + f*x)] + 105*B*c*d^2*Sin[5*(e + f*x)] + 35*A*d^3*Sin[5*(e + f*x)] + 105*B*d^3*Sin[5*(e + f*x)])))/(f*Sqrt[Cos[e + f*x]^2])

fricas [A] time = 0.51, size = 432, normalized size = 0.72

$$240 Ba^3 d^3 \cos(fx + e)^7 - 1008 (Ba^3 c^2 d + (A + 3B)a^3 cd^2 + (A + 2B)a^3 d^3) \cos(fx + e)^5 + 560 ((A + 3B)a^3 c^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] 1/1680*(240*B*a^3*d^3*cos(f*x + e)^7 - 1008*(B*a^3*c^2*d + (A + 3*B)*a^3*c*d^2 + (A + 2*B)*a^3*d^3)*cos(f*x + e)^5 + 560*((A + 3*B)*a^3*c^3 + 3*(3*A + 5*B)*a^3*c^2*d + 3*(5*A + 7*B)*a^3*c*d^2 + (7*A + 9*B)*a^3*d^3)*cos(f*x + e)^3 + 105*(10*(4*A + 3*B)*a^3*c^3 + 6*(15*A + 13*B)*a^3*c^2*d + 3*(26*A + 23*B)*a^3*c*d^2 + (23*A + 21*B)*a^3*d^3)*f*x - 6720*((A + B)*a^3*c^3 + 3*(A + B)*a^3*c^2*d + 3*(A + B)*a^3*c*d^2 + (A + B)*a^3*d^3)*cos(f*x + e) - 35*(8*(3*B*a^3*c*d^2 + (A + 3*B)*a^3*d^3)*cos(f*x + e)^5 - 2*(6*B*a^3*c^3 + 18*(A + 3*B)*a^3*c^2*d + 3*(18*A + 31*B)*a^3*c*d^2 + (31*A + 45*B)*a^3*d^3)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^3 + 6*(17*A + 19*B)*a^3*c^2*d + 3*(38*A + 41*B)*a^3*c*d^2 + (41*A + 43*B)*a^3*d^3)*cos(f*x + e))*sin(f*x + e)/f

giac [A] time = 0.25, size = 566, normalized size = 0.94

$$\frac{Ba^3 d^3 \cos(7fx + 7e)}{448f} + \frac{1}{16} (40Aa^3 c^3 + 30Ba^3 c^3 + 90Aa^3 c^2 d + 78Ba^3 c^2 d + 78Aa^3 cd^2 + 69Ba^3 cd^2 + 23Aa^3 d^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] 1/448*B*a^3*d^3*cos(7*f*x + 7*e)/f + 1/16*(40*A*a^3*c^3 + 30*B*a^3*c^3 + 90*A*a^3*c^2*d + 78*B*a^3*c^2*d + 78*A*a^3*c*d^2 + 69*B*a^3*c*d^2 + 23*A*a^3*d^3)

$$d^3 + 21Ba^3d^3) * x - 1/320 * (12Ba^3c^2d + 12Aa^3cd^2 + 36Ba^3c^2d^2 + 12Aa^3d^3 + 19Ba^3d^3) * \cos(5fx + 5e) / f + 1/192 * (16Aa^3c^3 + 48Ba^3c^3 + 144Aa^3c^2d + 204Ba^3c^2d + 204Aa^3cd^2 + 228Ba^3cd^2 + 76Aa^3d^3 + 81Ba^3d^3) * \cos(3fx + 3e) / f - 1/64 * (240Aa^3c^3 + 208Ba^3c^3 + 624Aa^3c^2d + 552Ba^3c^2d + 552Aa^3cd^2 + 504Ba^3cd^2 + 168Aa^3d^3 + 155Ba^3d^3) * \cos(fx + e) / f - 1/192 * (3Ba^3cd^2 + Aa^3d^3 + 3Ba^3d^3) * \sin(6fx + 6e) / f + 1/64 * (2Ba^3c^3 + 6Aa^3c^2d + 18Ba^3c^2d + 18Aa^3cd^2 + 27Ba^3cd^2 + 9Aa^3d^3 + 11Ba^3d^3) * \sin(4fx + 4e) / f - 1/64 * (48Aa^3c^3 + 64Ba^3c^3 + 192Aa^3c^2d + 192Ba^3c^2d + 192Aa^3cd^2 + 189Ba^3cd^2 + 63Aa^3d^3 + 61Ba^3d^3) * \sin(2fx + 2e) / f$$

maple [A] time = 0.65, size = 1077, normalized size = 1.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)

[Out] $1/f * (-Ba^3cd^2 * (2 + \sin(fx+e))^2 * \cos(fx+e) + 3Ba^3c^3 * (-1/2 * \sin(fx+e)) * \cos(fx+e) + 1/2 * fx + 1/2 * e) + a^3 * A * c^3 * (fx+e) + 3Ba^3d^3 * (-1/6 * (\sin(fx+e))^5 + 5/4 * \sin(fx+e)^3 + 15/8 * \sin(fx+e)) * \cos(fx+e) + 5/16 * fx + 5/16 * e) - 3a^3 * A * c^3 * \cos(fx+e) + 3a^3 * A * d^3 * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) + Ba^3c^3 * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) + 3a^3 * A * c^3 * (-1/2 * \sin(fx+e) * \cos(fx+e) + 1/2 * fx + 1/2 * e) + a^3 * A * d^3 * (-1/6 * (\sin(fx+e))^5 + 5/4 * \sin(fx+e)^3 + 15/8 * \sin(fx+e)) * \cos(fx+e) + 5/16 * fx + 5/16 * e) + Ba^3d^3 * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) - Ba^3c^3 * \cos(fx+e) - 3/5 * a^3 * A * d^3 * (8/3 + \sin(fx+e))^4 + 4/3 * \sin(fx+e)^2 * \cos(fx+e) - Ba^3c^3 * (2 + \sin(fx+e))^2 * \cos(fx+e) - 1/3 * a^3 * A * c^3 * (2 + \sin(fx+e))^2 * \cos(fx+e) - 1/3 * a^3 * A * d^3 * (2 + \sin(fx+e))^2 * \cos(fx+e) - 1/7 * Ba^3d^3 * (16/5 + \sin(fx+e))^6 + 6/5 * \sin(fx+e)^4 + 8/5 * \sin(fx+e)^2 * \cos(fx+e) + 9 * Ba^3cd^2 * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) - 3/5 * Ba^3d^3 * (8/3 + \sin(fx+e))^4 + 4/3 * \sin(fx+e)^2 * \cos(fx+e) + 9 * a^3 * A * c^2 * d * (-1/2 * \sin(fx+e) * \cos(fx+e) + 1/2 * fx + 1/2 * e) + 3 * Ba^3c^2 * d * (-1/2 * \sin(fx+e) * \cos(fx+e) + 1/2 * fx + 1/2 * e) + 9 * a^3 * A * c * d^2 * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) + 3 * a^3 * A * c^2 * d * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) + 9 * Ba^3c^2 * d * (-1/4 * (\sin(fx+e))^3 + 3/2 * \sin(fx+e)) * \cos(fx+e) + 3/8 * fx + 3/8 * e) + 3 * Ba^3cd^2 * (-1/6 * (\sin(fx+e))^5 + 5/4 * \sin(fx+e)^3 + 15/8 * \sin(fx+e)) * \cos(fx+e) + 5/16 * fx + 5/16 * e) + 3 * a^3 * A * c * d^2 * (-1/2 * \sin(fx+e) * \cos(fx+e) + 1/2 * fx + 1/2 * e) - 3 * a^3 * A * c^2 * d * \cos(fx+e) - 3 * a^3 * A * c^2 * d * (2 + \sin(fx+e))^2 * \cos(fx+e) - 9/5 * Ba^3cd^2 * (8/3 + \sin(fx+e))^4 + 4/3 * \sin(fx+e)^2 * \cos(fx+e) - 3 * a^3 * A * c * d^2 * (2 + \sin(fx+e))^2 * \cos(fx+e) - 3 * Ba^3c^2 * d * (2 + \sin(fx+e))^2 * \cos(fx+e) - 3/5 * a^3 * A * c * d^2 * (8/3 + \sin(fx+e))^4 + 4/3 * \sin(fx+e)^2 * \cos(fx+e) - 3/5 * Ba^3c^2 * d * (8/3 + \sin(fx+e))^4 + 4/3 * \sin(fx+e)^2 * \cos(fx+e)$

maxima [A] time = 0.63, size = 1056, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $1/6720 * (2240 * (\cos(fx + e))^3 - 3 * \cos(fx + e)) * Aa^3c^3 + 5040 * (2 * fx + 2 * e - \sin(2 * fx + 2 * e)) * Aa^3c^3 + 6720 * (fx + e) * Aa^3c^3 + 6720 * (\cos(fx + e))^3 - 3 * \cos(fx + e)) * Ba^3c^3 + 210 * (12 * fx + 12 * e + \sin(4 * fx + 4 * e) - 8 * \sin(2 * fx + 2 * e)) * Ba^3c^3 + 5040 * (2 * fx + 2 * e - \sin(2 * fx + 2 * e)) * Ba^3c^3 + 20160 * (\cos(fx + e))^3 - 3 * \cos(fx + e)) * Aa^3c^2 * d + 630 * (12 * fx + 12 * e + \sin(4 * fx + 4 * e) - 8 * \sin(2 * fx + 2 * e)) * Aa^3c^2 * d + 15120 * (2 * fx + 2 * e - \sin(2 * fx + 2 * e)) * Aa^3c^2 * d - 1344 * (3 * \cos(fx + e))^5 - 10 * \cos(fx$

$$\begin{aligned}
& + e)^3 + 15\cos(f*x + e))*B*a^3*c^2*d + 20160*(\cos(f*x + e)^3 - 3*\cos(f*x \\
& + e))*B*a^3*c^2*d + 1890*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + \\
& 2*e))*B*a^3*c^2*d + 5040*(2*f*x + 2*e - \sin(2*f*x + 2*e))*B*a^3*c^2*d - 134 \\
& 4*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 15*\cos(f*x + e))*A*a^3*c*d^2 + 20 \\
& 160*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^3*c*d^2 + 1890*(12*f*x + 12*e + s \\
& \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*A*a^3*c*d^2 + 5040*(2*f*x + 2*e - \sin \\
& (2*f*x + 2*e))*A*a^3*c*d^2 - 4032*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 + 1 \\
& 5*\cos(f*x + e))*B*a^3*c*d^2 + 6720*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*B*a^3* \\
& c*d^2 + 105*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48 \\
& *\sin(2*f*x + 2*e))*B*a^3*c*d^2 + 1890*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8 \\
& *\sin(2*f*x + 2*e))*B*a^3*c*d^2 - 1344*(3*\cos(f*x + e)^5 - 10*\cos(f*x + e)^3 \\
& + 15*\cos(f*x + e))*A*a^3*d^3 + 2240*(\cos(f*x + e)^3 - 3*\cos(f*x + e))*A*a^ \\
& 3*d^3 + 35*(4*\sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48* \\
& \sin(2*f*x + 2*e))*A*a^3*d^3 + 630*(12*f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin \\
& (2*f*x + 2*e))*A*a^3*d^3 + 192*(5*\cos(f*x + e)^7 - 21*\cos(f*x + e)^5 + 35*c \\
& \cos(f*x + e)^3 - 35*\cos(f*x + e))*B*a^3*d^3 - 1344*(3*\cos(f*x + e)^5 - 10*co \\
& s(f*x + e)^3 + 15*\cos(f*x + e))*B*a^3*d^3 + 105*(4*\sin(2*f*x + 2*e)^3 + 60* \\
& f*x + 60*e + 9*\sin(4*f*x + 4*e) - 48*\sin(2*f*x + 2*e))*B*a^3*d^3 + 210*(12* \\
& f*x + 12*e + \sin(4*f*x + 4*e) - 8*\sin(2*f*x + 2*e))*B*a^3*d^3 - 20160*A*a^3 \\
& *c^3*\cos(f*x + e) - 6720*B*a^3*c^3*\cos(f*x + e) - 20160*A*a^3*c^2*d*\cos(f*x \\
& + e))/f
\end{aligned}$$

mupad [B] time = 16.18, size = 1395, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^3,x)
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^3 + 23*A*d^3 + 30*B*c^3 + 21*B*d^
3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 + 78*B*c^2*d))/(8*(5*A*a^3*c^3 + (
23*A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4
+ (45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4)))*(40*A*c^3
+ 23*A*d^3 + 30*B*c^3 + 21*B*d^3 + 78*A*c*d^2 + 90*A*c^2*d + 69*B*c*d^2 +
78*B*c^2*d))/(8*f) - (tan(e/2 + (f*x)/2)*(3*A*a^3*c^3 + (23*A*a^3*d^3)/8 +
(15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (45*A*a^3*c^2*d)
/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f*x)/2)^10*(40*A
a^3*c^3 + 4*A*a^3*d^3 + 24*B*a^3*c^3 + 36*A*a^3*c*d^2 + 72*A*a^3*c^2*d + 1
2*B*a^3*c*d^2 + 36*B*a^3*c^2*d) - tan(e/2 + (f*x)/2)^13*(3*A*a^3*c^3 + (23*
A*a^3*d^3)/8 + (15*B*a^3*c^3)/4 + (21*B*a^3*d^3)/8 + (39*A*a^3*c*d^2)/4 + (
45*A*a^3*c^2*d)/4 + (69*B*a^3*c*d^2)/8 + (39*B*a^3*c^2*d)/4) + tan(e/2 + (f
*x)/2)^3*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 + (35*B*a^3*d^3)/
2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 + 57*B*a^3*c^2*d)
- tan(e/2 + (f*x)/2)^11*(12*A*a^3*c^3 + (115*A*a^3*d^3)/6 + 17*B*a^3*c^3 +
(35*B*a^3*d^3)/2 + 57*A*a^3*c*d^2 + 51*A*a^3*c^2*d + (115*B*a^3*c*d^2)/2 +
57*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^8*((322*A*a^3*c^3)/3 + (148*A*a^3*d^3
)/3 + 82*B*a^3*c^3 + 32*B*a^3*d^3 + 188*A*a^3*c*d^2 + 246*A*a^3*c^2*d + 148
*B*a^3*c*d^2 + 188*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^6*((448*A*a^3*c^3)/3 +
(328*A*a^3*d^3)/3 + 128*B*a^3*c^3 + 112*B*a^3*d^3 + 344*A*a^3*c*d^2 + 384*
A*a^3*c^2*d + 328*B*a^3*c*d^2 + 344*B*a^3*c^2*d) + tan(e/2 + (f*x)/2)^2*((1
36*A*a^3*c^3)/3 + (476*A*a^3*d^3)/15 + 40*B*a^3*c^3 + (144*B*a^3*d^3)/5 + (
532*A*a^3*c*d^2)/5 + 120*A*a^3*c^2*d + (476*B*a^3*c*d^2)/5 + (532*B*a^3*c^2
*d)/5) + tan(e/2 + (f*x)/2)^5*(15*A*a^3*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^
3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 +
(841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)/4) - tan(e/2 + (f*x)/2)^9*(15*A*a^3
*c^3 + (841*A*a^3*d^3)/24 + (91*B*a^3*c^3)/4 + (345*B*a^3*d^3)/8 + (339*A*a
^3*c*d^2)/4 + (273*A*a^3*c^2*d)/4 + (841*B*a^3*c*d^2)/8 + (339*B*a^3*c^2*d)
/4) + tan(e/2 + (f*x)/2)^4*(114*A*a^3*c^3 + (456*A*a^3*d^3)/5 + 102*B*a^3*c
^3 + (432*B*a^3*d^3)/5 + (1416*A*a^3*c*d^2)/5 + 306*A*a^3*c^2*d + (1368*B*a
^3*c*d^2)/5 + (1416*B*a^3*c^2*d)/5) + tan(e/2 + (f*x)/2)^12*(6*A*a^3*c^3 +

```

$$2Ba^3c^3 + 6Aa^3c^2d) + (22Aa^3c^3)/3 + (68Aa^3d^3)/15 + 6Ba^3c^3 + (144Ba^3d^3)/35 + (76Aa^3c^2d)/5 + 18Aa^3c^2d + (68Ba^3c^2d)/5 + (76Ba^3c^2d)/5)/(f(7\tan(e/2 + (f*x)/2)^2 + 21\tan(e/2 + (f*x)/2)^4 + 35\tan(e/2 + (f*x)/2)^6 + 35\tan(e/2 + (f*x)/2)^8 + 21\tan(e/2 + (f*x)/2)^{10} + 7\tan(e/2 + (f*x)/2)^{12} + \tan(e/2 + (f*x)/2)^{14} + 1))$$

sympy [A] time = 22.09, size = 2878, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)

[Out] Piecewise((3*A*a**3*c**3*x*sin(e + f*x)**2/2 + 3*A*a**3*c**3*x*cos(e + f*x)**2/2 + A*a**3*c**3*x - A*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*c**3*cos(e + f*x)/f + 9*A*a**3*c**2*d*x*sin(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 9*A*a**3*c**2*d*x*sin(e + f*x)**2/2 + 9*A*a**3*c**2*d*x*cos(e + f*x)**4/8 + 9*A*a**3*c**2*d*x*cos(e + f*x)**2/2 - 15*A*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 9*A*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 6*A*a**3*c**2*d*cos(e + f*x)**3/f - 3*A*a**3*c**2*d*cos(e + f*x)/f + 27*A*a**3*c*d**2*x*sin(e + f*x)**4/8 + 27*A*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*A*a**3*c*d**2*x*sin(e + f*x)**2/2 + 27*A*a**3*c*d**2*x*cos(e + f*x)**4/8 + 3*A*a**3*c*d**2*x*cos(e + f*x)**2/2 - 3*A*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 45*A*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*A*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 27*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*A*a**3*c*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 6*A*a**3*c*d**2*cos(e + f*x)**3/f + 5*A*a**3*d**3*x*sin(e + f*x)**6/16 + 15*A*a**3*d**3*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*A*a**3*d**3*x*sin(e + f*x)**4/8 + 15*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*A*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 5*A*a**3*d**3*x*cos(e + f*x)**6/16 + 9*A*a**3*d**3*x*cos(e + f*x)**4/8 - 11*A*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*A*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - 15*A*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - A*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)/f - 5*A*a**3*d**3*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*A*a**3*d**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*A*a**3*d**3*cos(e + f*x)**5/(5*f) - 2*A*a**3*d**3*cos(e + f*x)**3/(3*f) + 3*B*a**3*c**3*x*sin(e + f*x)**4/8 + 3*B*a**3*c**3*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**3*x*sin(e + f*x)**2/2 + 3*B*a**3*c**3*x*cos(e + f*x)**4/8 + 3*B*a**3*c**3*x*cos(e + f*x)**2/2 - 5*B*a**3*c**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**3*cos(e + f*x)**3/f - B*a**3*c**3*cos(e + f*x)/f + 27*B*a**3*c**2*d*x*sin(e + f*x)**4/8 + 27*B*a**3*c**2*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*d*x*sin(e + f*x)**2/2 + 27*B*a**3*c**2*d*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*d*x*cos(e + f*x)**2/2 - 3*B*a**3*c**2*d*sin(e + f*x)**4*cos(e + f*x)/f - 45*B*a**3*c**2*d*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)**3/f - 9*B*a**3*c**2*d*sin(e + f*x)**2*cos(e + f*x)/f - 27*B*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*c**2*d*cos(e + f*x)**5/(5*f) - 6*B*a**3*c**2*d*cos(e + f*x)**3/f + 15*B*a**3*c*d**2*x*sin(e + f*x)**6/16 + 45*B*a**3*c*d**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**4/8 + 45*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 27*B*a**3*c*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*c*d**2*x*cos(e + f*x)**6/16 + 27*B*a**3*c*d**2*x*cos(e + f*x)**4/8 - 33*B*a**3*c*d**2*sin(e + f*x)*

```

*5*cos(e + f*x)/(16*f) - 9*B*a**3*c*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5
*B*a**3*c*d**2*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 45*B*a**3*c*d**2*sin
(e + f*x)**3*cos(e + f*x)/(8*f) - 12*B*a**3*c*d**2*sin(e + f*x)**2*cos(e +
f*x)**3/f - 3*B*a**3*c*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 15*B*a**3*c*d*
*2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 27*B*a**3*c*d**2*sin(e + f*x)*cos(
e + f*x)**3/(8*f) - 24*B*a**3*c*d**2*cos(e + f*x)**5/(5*f) - 2*B*a**3*c*d**
2*cos(e + f*x)**3/f + 15*B*a**3*d**3*x*sin(e + f*x)**6/16 + 45*B*a**3*d**3*
x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*B*a**3*d**3*x*sin(e + f*x)**4/8 +
45*B*a**3*d**3*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 3*B*a**3*d**3*x*sin(e
+ f*x)**2*cos(e + f*x)**2/4 + 15*B*a**3*d**3*x*cos(e + f*x)**6/16 + 3*B*a*
*3*d**3*x*cos(e + f*x)**4/8 - B*a**3*d**3*sin(e + f*x)**6*cos(e + f*x)/f -
33*B*a**3*d**3*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 2*B*a**3*d**3*sin(e +
f*x)**4*cos(e + f*x)**3/f - 3*B*a**3*d**3*sin(e + f*x)**4*cos(e + f*x)/f -
5*B*a**3*d**3*sin(e + f*x)**3*cos(e + f*x)**3/(2*f) - 5*B*a**3*d**3*sin(e +
f*x)**3*cos(e + f*x)/(8*f) - 8*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**5
/(5*f) - 4*B*a**3*d**3*sin(e + f*x)**2*cos(e + f*x)**3/f - 15*B*a**3*d**3*s
in(e + f*x)*cos(e + f*x)**5/(16*f) - 3*B*a**3*d**3*sin(e + f*x)*cos(e + f*x
)**3/(8*f) - 16*B*a**3*d**3*cos(e + f*x)**7/(35*f) - 8*B*a**3*d**3*cos(e +
f*x)**5/(5*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**3*(a*sin(e) + a
)**3, True))

```

$$3.259 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal. Leaf size=463

$$\frac{1}{16} a^3 x \left(A (40c^2 + 60cd + 26d^2) + B (30c^2 + 52cd + 23d^2) \right) + \frac{a^3 (2Ad(2c - 11d) - B(2c^2 - 8cd + 21d^2)) \cos(e + fx)}{40d^3 f}$$

[Out] 1/16*a^3*(B*(30*c^2+52*c*d+23*d^2)+A*(40*c^2+60*c*d+26*d^2))*x-1/60*a^3*(2*A*d*(2*c^4-15*c^3*d+72*c^2*d^2+180*c*d^3+76*d^4)-B*(2*c^5-12*c^4*d+37*c^3*d^2-112*c^2*d^3-304*c*d^4-136*d^5))*cos(f*x+e)/d^3/f-1/240*a^3*(2*A*d*(4*c^3-30*c^2*d+146*c*d^2+195*d^3)-B*(4*c^4-24*c^3*d+76*c^2*d^2-236*c*d^3-345*d^4))*cos(f*x+e)*sin(f*x+e)/d^2/f-1/120*a^3*(2*A*d*(2*c^2-15*c*d+76*d^2)-B*(2*c^3-12*c^2*d+41*c*d^2-136*d^3))*cos(f*x+e)*(c+d*sin(f*x+e))^2/d^3/f+1/40*a^3*(2*A*(2*c-11*d)*d-B*(2*c^2-8*c*d+21*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f-1/6*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^2*(c+d*sin(f*x+e))^3/d/f+1/30*(-6*A*d+3*B*c-8*B*d)*cos(f*x+e)*(a^3+a^3*sin(f*x+e))*(c+d*sin(f*x+e))^3/d^2/f

Rubi [A] time = 1.13, antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2976, 2968, 3023, 2753, 2734}

$$\frac{a^3 (2Ad(72c^2d^2 - 15c^3d + 2c^4 + 180cd^3 + 76d^4) - B(37c^3d^2 - 112c^2d^3 - 12c^4d + 2c^5 - 304cd^4 - 136d^5)) \cos(e + fx)}{60d^3 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]

[Out] (a^3*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*x)/16 - (a^3*(2*A*d*(2*c^4 - 15*c^3*d + 72*c^2*d^2 + 180*c*d^3 + 76*d^4) - B*(2*c^5 - 12*c^4*d + 37*c^3*d^2 - 112*c^2*d^3 - 304*c*d^4 - 136*d^5))*Cos[e + f*x])/((60*d^3*f) - (a^3*(2*A*d*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3) - B*(4*c^4 - 24*c^3*d + 76*c^2*d^2 - 236*c*d^3 - 345*d^4))*Cos[e + f*x]*Sin[e + f*x])/((240*d^2*f) - (a^3*(2*A*d*(2*c^2 - 15*c*d + 76*d^2) - B*(2*c^3 - 12*c^2*d + 41*c*d^2 - 136*d^3))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(120*d^3*f) + (a^3*(2*A*(2*c - 11*d)*d - B*(2*c^2 - 8*c*d + 21*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(40*d^3*f) - (a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^3)/(6*d*f) + ((3*B*c - 6*A*d - 8*B*d)*Cos[e + f*x]*(a^3 + a^3*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(30*d^2*f)

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx = -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2(c + d \sin(e + fx))}{6df}$$

$$= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2(c + d \sin(e + fx))}{6df}$$

$$= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2(c + d \sin(e + fx))}{6df}$$

$$= \frac{a^3 (2A(2c - 11d)d - B(2c^2 - 8cd + 21d^2)) \cos(e + fx)}{40d^3 f}$$

$$= -\frac{a^3 (2Ad(2c^2 - 15cd + 76d^2) - B(2c^3 - 12c^2d + 12cd^2 - 4d^3)) \cos(e + fx)}{40d^3 f}$$

$$= \frac{1}{16} a^3 (B(30c^2 + 52cd + 23d^2) + A(40c^2 + 60cd + 26d^2)) \sin^{-1}\left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}}\right) + \sqrt{\cos^2(e + fx)}$$

Mathematica [A] time = 2.42, size = 355, normalized size = 0.77

$$a^3 \cos(e + fx) \left(60 (A (40c^2 + 60cd + 26d^2) + B (30c^2 + 52cd + 23d^2)) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{\cos^2(e + fx)} \right) -$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
[Out] -1/480*(a^3*Cos[e + f*x]*(60*(B*(30*c^2 + 52*c*d + 23*d^2) + A*(40*c^2 + 60*c*d + 26*d^2))*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2])
```

$$2] * (1840 * A * c^2 + 1680 * B * c^2 + 3360 * A * c * d + 3112 * B * c * d + 1556 * A * d^2 + 1468 * B * d^2 - 16 * (A * (5 * c^2 + 30 * c * d + 22 * d^2) + B * (15 * c^2 + 44 * c * d + 26 * d^2)) * \text{Cos}[2 * (e + f * x)] + 12 * d * (2 * B * c + A * d + 3 * B * d) * \text{Cos}[4 * (e + f * x)] + 720 * A * c^2 * \text{Sin}[e + f * x] + 990 * B * c^2 * \text{Sin}[e + f * x] + 1980 * A * c * d * \text{Sin}[e + f * x] + 2100 * B * c * d * \text{Sin}[e + f * x] + 1050 * A * d^2 * \text{Sin}[e + f * x] + 1085 * B * d^2 * \text{Sin}[e + f * x] - 30 * B * c^2 * \text{Sin}[3 * (e + f * x)] - 60 * A * c * d * \text{Sin}[3 * (e + f * x)] - 180 * B * c * d * \text{Sin}[3 * (e + f * x)] - 90 * A * d^2 * \text{Sin}[3 * (e + f * x)] - 140 * B * d^2 * \text{Sin}[3 * (e + f * x)] + 5 * B * d^2 * \text{Sin}[5 * (e + f * x)])) / (f * \text{Sqrt}[\text{Cos}[e + f * x]^2])$$

fricas [A] time = 0.49, size = 299, normalized size = 0.65

$$48 (2 B a^3 c d + (A + 3 B) a^3 d^2) \cos(fx + e)^5 - 80 ((A + 3 B) a^3 c^2 + 2 (3 A + 5 B) a^3 c d + (5 A + 7 B) a^3 d^2) \cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/240*(48*(2*B*a^3*c*d + (A + 3*B)*a^3*d^2)*cos(f*x + e)^5 - 80*((A + 3*B)*a^3*c^2 + 2*(3*A + 5*B)*a^3*c*d + (5*A + 7*B)*a^3*d^2)*cos(f*x + e)^3 - 15*(10*(4*A + 3*B)*a^3*c^2 + 4*(15*A + 13*B)*a^3*c*d + (26*A + 23*B)*a^3*d^2)*f*x + 960*((A + B)*a^3*c^2 + 2*(A + B)*a^3*c*d + (A + B)*a^3*d^2)*cos(f*x + e) + 5*(8*B*a^3*d^2*cos(f*x + e)^5 - 2*(6*B*a^3*c^2 + 12*(A + 3*B)*a^3*c*d + (18*A + 31*B)*a^3*d^2)*cos(f*x + e)^3 + 3*(2*(12*A + 17*B)*a^3*c^2 + 4*(17*A + 19*B)*a^3*c*d + (38*A + 41*B)*a^3*d^2)*cos(f*x + e))*sin(f*x + e))/f

giac [A] time = 0.21, size = 380, normalized size = 0.82

$$-\frac{B a^3 d^2 \sin(6 f x + 6 e)}{192 f} + \frac{1}{16} (40 A a^3 c^2 + 30 B a^3 c^2 + 60 A a^3 c d + 52 B a^3 c d + 26 A a^3 d^2 + 23 B a^3 d^2) x - \frac{(2 B a^3 c d + (A + 3 B) a^3 d^2) \cos(fx + e)^5 - 80 ((A + 3 B) a^3 c^2 + 2 (3 A + 5 B) a^3 c d + (5 A + 7 B) a^3 d^2) \cos(fx + e)^3 - 15 (10 (4 A + 3 B) a^3 c^2 + 4 (15 A + 13 B) a^3 c d + (26 A + 23 B) a^3 d^2) f x + 960 ((A + B) a^3 c^2 + 2 (A + B) a^3 c d + (A + B) a^3 d^2) \cos(fx + e) + 5 (8 B a^3 d^2 \cos(fx + e)^5 - 2 (6 B a^3 c^2 + 12 (A + 3 B) a^3 c d + (18 A + 31 B) a^3 d^2) \cos(fx + e)^3 + 3 (2 (12 A + 17 B) a^3 c^2 + 4 (17 A + 19 B) a^3 c d + (38 A + 41 B) a^3 d^2) \cos(fx + e)) \sin(fx + e)}{240 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] -1/192*B*a^3*d^2*sin(6*f*x + 6*e)/f + 1/16*(40*A*a^3*c^2 + 30*B*a^3*c^2 + 60*A*a^3*c*d + 52*B*a^3*c*d + 26*A*a^3*d^2 + 23*B*a^3*d^2)*x - 1/80*(2*B*a^3*c*d + A*a^3*d^2 + 3*B*a^3*d^2)*cos(5*f*x + 5*e)/f + 1/48*(4*A*a^3*c^2 + 12*B*a^3*c^2 + 24*A*a^3*c*d + 34*B*a^3*c*d + 17*A*a^3*d^2 + 19*B*a^3*d^2)*cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c^2 + 26*B*a^3*c^2 + 52*A*a^3*c*d + 46*B*a^3*c*d + 23*A*a^3*d^2 + 21*B*a^3*d^2)*cos(f*x + e)/f + 1/64*(2*B*a^3*c^2 + 4*A*a^3*c*d + 12*B*a^3*c*d + 6*A*a^3*d^2 + 9*B*a^3*d^2)*sin(4*f*x + 4*e)/f - 1/64*(48*A*a^3*c^2 + 64*B*a^3*c^2 + 128*A*a^3*c*d + 128*B*a^3*c*d + 64*A*a^3*d^2 + 63*B*a^3*d^2)*sin(2*f*x + 2*e)/f

maple [A] time = 0.62, size = 725, normalized size = 1.57

$$-\frac{a^3 A c^2 (2 + \sin^2(fx + e)) \cos(fx + e)}{3} + 2 a^3 A c d \left(-\frac{\left(\sin^3(fx + e) + \frac{3 \sin(fx + e)}{2} \right) \cos(fx + e)}{4} + \frac{3 f x}{8} + \frac{3 e}{8} \right) - \frac{a^3 A d^2 \left(\frac{8}{3} + \sin^4(fx + e) + \frac{4 \sin^2(fx + e)}{3} \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)

```
[Out] 1/f*(-1/3*a^3*A*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+2*a^3*A*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-1/5*a^3*A*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*c^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-2/5*B*a^3*c*d*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)+B*a^3*d^2*(-1/6*(sin(f*x+e)^5+5/4*sin(f*x+e)^3+15/8*sin(f*x+e))*cos(f*x+e)+5/16*f*x+5/16*e)+3*a^3*A*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*a^3*A*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*a^3*A*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-B*a^3*c^2*(2+sin(f*x+e)^2)*cos(f*x+e)+6*B*a^3*c*d*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)-3/5*B*a^3*d^2*(8/3+sin(f*x+e)^4+4/3*sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*c^2*cos(f*x+e)+6*a^3*A*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-a^3*A*d^2*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*c^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-2*B*a^3*c*d*(2+sin(f*x+e)^2)*cos(f*x+e)+3*B*a^3*d^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+a^3*A*c^2*(f*x+e)-2*a^3*A*c*d*cos(f*x+e)+a^3*A*d^2*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*c^2*cos(f*x+e)+2*B*a^3*c*d*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-1/3*B*a^3*d^2*(2+sin(f*x+e)^2)*cos(f*x+e))
```

maxima [A] time = 0.46, size = 704, normalized size = 1.52

$$320 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) Aa^3c^2 + 720 (2fx + 2e - \sin(2fx + 2e)) Aa^3c^2 + 960 (fx + e) Aa^3c^2 + 960$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/960*(320*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c^2 + 960*(f*x + e)*A*a^3*c^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c^2 + 30*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c^2 + 720*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c^2 + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*c*d + 60*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*c*d + 1440*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*c*d - 128*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*c*d + 1920*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*c*d + 180*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*c*d + 480*(2*f*x + 2*e - sin(2*f*x + 2*e))*B*a^3*c*d - 64*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*A*a^3*d^2 + 960*(cos(f*x + e)^3 - 3*cos(f*x + e))*A*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*A*a^3*d^2 + 240*(2*f*x + 2*e - sin(2*f*x + 2*e))*A*a^3*d^2 - 192*(3*cos(f*x + e)^5 - 10*cos(f*x + e)^3 + 15*cos(f*x + e))*B*a^3*d^2 + 320*(cos(f*x + e)^3 - 3*cos(f*x + e))*B*a^3*d^2 + 5*(4*sin(2*f*x + 2*e)^3 + 60*f*x + 60*e + 9*sin(4*f*x + 4*e) - 48*sin(2*f*x + 2*e))*B*a^3*d^2 + 90*(12*f*x + 12*e + sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*B*a^3*d^2 - 2880*A*a^3*c^2*cos(f*x + e) - 960*B*a^3*c^2*cos(f*x + e) - 1920*A*a^3*c*d*cos(f*x + e))/f
```

mupad [B] time = 15.63, size = 976, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))^2,x)
```

```
[Out] (a^3*atan((a^3*tan(e/2 + (f*x)/2)*(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*(5*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (15*A*a^3*c*d)/2 + (13*B*a^3*c*d)/2)))/(40*A*c^2 + 26*A*d^2 + 30*B*c^2 + 23*B*d^2 + 60*A*c*d + 52*B*c*d))/(8*f) - (tan(e/2 + (f*x)/2)^10*(6*A*a^3*c^2 + 2*B*a^3*c^2 + 4*A*a^3*c*d) + tan(e/2 + (f*x)/2)*(3*A*a^3*c^2 + (13*A*a^3*d^2)/4 + (15*B*a^3*c^2)/4 + (23*B*a^3*d^2)/8 + (1
```


$$\begin{aligned}
& 5Aa^3cd)/2 + (13Ba^3cd)/2) - \tan(e/2 + (fx)/2)^{11} \cdot (3Aa^3c^2 + (13Aa^3d^2)/4 + (15Ba^3c^2)/4 + (23Ba^3d^2)/8 + (15Aa^3cd)/2 + (13Ba^3cd)/2) \\
& + \tan(e/2 + (fx)/2)^8 \cdot (34Aa^3c^2 + 12Aa^3d^2 + 22Ba^3c^2 + 4Ba^3d^2 + 44Aa^3cd + 24Ba^3cd) + \tan(e/2 + (fx)/2)^5 \cdot (6Aa^3c^2 + (25Aa^3d^2)/2 + (19Ba^3c^2)/2 + (75Ba^3d^2)/4 + 19Aa^3cd + 25Ba^3cd) \\
& - \tan(e/2 + (fx)/2)^7 \cdot (6Aa^3c^2 + (25Aa^3d^2)/2 + (19Ba^3c^2)/2 + (75Ba^3d^2)/4 + 19Aa^3cd + 25Ba^3cd) + \tan(e/2 + (fx)/2)^4 \cdot (76Aa^3c^2 + 64Aa^3d^2 + 68Ba^3c^2 + 64Ba^3d^2 + 136Aa^3cd + 128Ba^3cd) \\
& + \tan(e/2 + (fx)/2)^3 \cdot (9Aa^3c^2 + (63Aa^3d^2)/4 + (53Ba^3c^2)/4 + (391Ba^3d^2)/24 + (53Aa^3cd)/2 + (63Ba^3cd)/2) - \tan(e/2 + (fx)/2)^9 \cdot (9Aa^3c^2 + (63Aa^3d^2)/4 + (53Ba^3c^2)/4 + (391Ba^3d^2)/24 + (53Aa^3cd)/2 + (63Ba^3cd)/2) \\
& + \tan(e/2 + (fx)/2)^2 \cdot (38Aa^3c^2 + (152Aa^3d^2)/5 + 34Ba^3c^2 + (136Ba^3d^2)/5 + 68Aa^3cd + (304Ba^3cd)/5) + \tan(e/2 + (fx)/2)^6 \cdot ((220Aa^3c^2)/3 + (152Aa^3d^2)/3 + 60Ba^3c^2 + (136Ba^3d^2)/3 + 120Aa^3cd + (304Ba^3cd)/3) \\
& + (22Aa^3c^2)/3 + (76Aa^3d^2)/15 + 6Ba^3c^2 + (68Ba^3d^2)/15 + 12Aa^3cd + (152Ba^3cd)/15) / (f \cdot (6 \tan(e/2 + (fx)/2)^2 + 15 \tan(e/2 + (fx)/2)^4 + 20 \tan(e/2 + (fx)/2)^6 + 15 \tan(e/2 + (fx)/2)^8 + 6 \tan(e/2 + (fx)/2)^{10} + \tan(e/2 + (fx)/2)^{12} + 1)
\end{aligned}$$

`sympy [A]` time = 10.85, size = 1804, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)`

[Out] `Piecewise((3*A*a**3*c**2*x*sin(e + f*x)**2/2 + 3*A*a**3*c**2*x*cos(e + f*x)**2/2 + A*a**3*c**2*x - A*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*A*a**3*c**2*cos(e + f*x)**3/(3*f) - 3*A*a**3*c**2*cos(e + f*x)/f + 3*A*a**3*c*d*x*sin(e + f*x)**4/4 + 3*A*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + 3*A*a**3*c*d*x*cos(e + f*x)**2 - 5*A*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 6*A*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - 3*A*a**3*c*d*sin(e + f*x)*cos(e + f*x)/f - 4*A*a**3*c*d*cos(e + f*x)**3/f - 2*A*a**3*c*d*cos(e + f*x)/f + 9*A*a**3*d**2*x*sin(e + f*x)**4/8 + 9*A*a**3*d**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + A*a**3*d**2*x*cos(e + f*x)**2/2 + 9*A*a**3*d**2*x*cos(e + f*x)**4/8 + A*a**3*d**2*x*cos(e + f*x)**2/2 - A*a**3*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 15*A*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*A*a**3*d**2*sin(e + f*x)**2*cos(e + f*x)/f - 9*A*a**3*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - A*a**3*d**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*A*a**3*d**2*cos(e + f*x)**5/(15*f) - 2*A*a**3*d**2*cos(e + f*x)**3/f + 3*B*a**3*c**2*x*sin(e + f*x)**4/8 + 3*B*a**3*c**2*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a**3*c**2*x*cos(e + f*x)**2/2 + 3*B*a**3*c**2*x*cos(e + f*x)**4/8 + 3*B*a**3*c**2*x*cos(e + f*x)**2/2 - 5*B*a**3*c**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3*c**2*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 3*B*a**3*c**2*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c**2*cos(e + f*x)**3/f - B*a**3*c**2*cos(e + f*x)/f + 9*B*a**3*c*d*x*sin(e + f*x)**4/4 + 9*B*a**3*c*d*x*sin(e + f*x)**2*cos(e + f*x)**2/2 + B*a**3*c*d*x*cos(e + f*x)**2 + 9*B*a**3*c*d*x*cos(e + f*x)**4/4 + B*a**3*c*d*x*cos(e + f*x)**2 - 2*B*a**3*c*d*sin(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*c*d*sin(e + f*x)**3*cos(e + f*x)/(4*f) - 8*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 6*B*a**3*c*d*sin(e + f*x)**2*cos(e + f*x)/f - 9*B*a**3*c*d*sin(e + f*x)*cos(e + f*x)**3/(4*f) - B*a**3*c*d*sin(e + f*x)*cos(e + f*x)/f - 16*B*a**3*c*d*cos(e + f*x)**5/(15*f) - 4*B*a**3*c*d*cos(e + f*x)**3/f + 5*B*a**3*d**2*x*sin(e + f*x)**6/16 + 15*B*a**3*d**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 9*B*a**3*d**2*x*sin(e + f*x)**4/8 + 15*B*a**3*d**2`

```

x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 9*B*a**3*d**2*x*sin(e + f*x)**2*cos(
e + f*x)**2/4 + 5*B*a**3*d**2*x*cos(e + f*x)**6/16 + 9*B*a**3*d**2*x*cos(e
+ f*x)**4/8 - 11*B*a**3*d**2*sin(e + f*x)**5*cos(e + f*x)/(16*f) - 3*B*a**3
*d**2*sin(e + f*x)**4*cos(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)**3*cos(e
+ f*x)**3/(6*f) - 15*B*a**3*d**2*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 4*B*a
**3*d**2*sin(e + f*x)**2*cos(e + f*x)**3/f - B*a**3*d**2*sin(e + f*x)**2*co
s(e + f*x)/f - 5*B*a**3*d**2*sin(e + f*x)*cos(e + f*x)**5/(16*f) - 9*B*a**3
*d**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) - 8*B*a**3*d**2*cos(e + f*x)**5/(5
*f) - 2*B*a**3*d**2*cos(e + f*x)**3/(3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c
+ d*sin(e))**2*(a*sin(e) + a)**3, True))

```

$$3.260 \quad \int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$$

Optimal. Leaf size=201

$$\frac{a^3(20Ac + 15Ad + 15Bc + 13Bd) \cos^3(e + fx)}{60f} - \frac{a^3(20Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)}{5f} - \frac{3a^3(20Ac + 15Ad + 15Bc + 13Bd)}{5f}$$

[Out] $1/8*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*x-1/5*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)/f+1/60*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)^3/f-3/40*a^3*(20*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)*\sin(f*x+e)/f-1/20*(5*A*d+5*B*c-B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^3/f-1/5*B*d*\cos(f*x+e)*(a+a*\sin(f*x+e))^4/a/f$

Rubi [A] time = 0.33, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {2968, 3023, 2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(20Ac + 15Ad + 15Bc + 13Bd) \cos^3(e + fx)}{60f} - \frac{a^3(20Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)}{5f} - \frac{3a^3(20Ac + 15Ad + 15Bc + 13Bd)}{5f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] $(a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*x)/8 - (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*\cos[e + f*x])/(5*f) + (a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*\cos[e + f*x]^3)/(60*f) - (3*a^3*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*\cos[e + f*x]*\sin[e + f*x])/(40*f) - ((5*B*c + 5*A*d - B*d)*\cos[e + f*x]*(a + a*\sin[e + f*x])^3)/(20*f) - (B*d*\cos[e + f*x]*(a + a*\sin[e + f*x])^4)/(5*a*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2645

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]

Rule 2751

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f
*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +
f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&
EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a
+ b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3023

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx))^3 (Ac + (Bc + Ad) \sin(e + fx) + Bd \cos(e + fx)) dx \\
&= -\frac{Bd \cos(e + fx)(a + a \sin(e + fx))^4}{5af} + \frac{\int (a + a \sin(e + fx))^3 (Ac + (Bc + Ad) \sin(e + fx)) dx}{5af} \\
&= -\frac{(5Bc + 5Ad - Bd) \cos(e + fx)(a + a \sin(e + fx))^4}{20f} + \frac{\int (a + a \sin(e + fx))^3 (Ac + (Bc + Ad) \sin(e + fx)) dx}{20f} \\
&= -\frac{(5Bc + 5Ad - Bd) \cos(e + fx)(a + a \sin(e + fx))^4}{20f} + \frac{1}{20} a^3 (20Ac + 15Bc + 15Ad + 13Bd)x - \frac{(5Bc + 5Ad - Bd) \cos(e + fx)(a + a \sin(e + fx))^4}{20f} \\
&= \frac{1}{20} a^3 (20Ac + 15Bc + 15Ad + 13Bd)x - \frac{3a^3 (20Ac + 15Bc + 15Ad + 13Bd) \cos(e + fx)(a + a \sin(e + fx))^4}{20f} \\
&= \frac{1}{8} a^3 (20Ac + 15Bc + 15Ad + 13Bd)x - \frac{a^3 (20Ac + 15Bc + 15Ad + 13Bd) \cos(e + fx)(a + a \sin(e + fx))^4}{8f}
\end{aligned}$$

Mathematica [A] time = 0.91, size = 156, normalized size = 0.78

$$\cos(e + fx) \left(-\frac{1}{4} a^4 (5Ad + 5Bc - Bd) (\sin(e + fx) + 1)^3 - \frac{a^4 (20Ac + 15Ad + 15Bc + 13Bd) \left(30 \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + (2 \sin^2(e + fx) + 9) \right)}{24 \sqrt{\cos^2(e + fx)}} \right)$$

$$5af$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] (Cos[e + f*x]*(-1/4*(a^4*(5*B*c + 5*A*d - B*d)*(1 + Sin[e + f*x])^3) - B*d*
(a + a*Sin[e + f*x])^4 - (a^4*(20*A*c + 15*B*c + 15*A*d + 13*B*d)*(30*ArcSi
```

$n[\text{Sqrt}[1 - \text{Sin}[e + f*x]]/\text{Sqrt}[2]] + \text{Sqrt}[\text{Cos}[e + f*x]^2]*(22 + 9*\text{Sin}[e + f*x] + 2*\text{Sin}[e + f*x]^2))/(24*\text{Sqrt}[\text{Cos}[e + f*x]^2]))/(5*a*f)$

fricas [A] time = 0.47, size = 178, normalized size = 0.89

$$24Ba^3d \cos(fx + e)^5 - 40((A + 3B)a^3c + (3A + 5B)a^3d) \cos(fx + e)^3 - 15(5(4A + 3B)a^3c + (15A + 13B)a^3d) \cos(fx + e) + 480((A + B)a^3c + (A + B)a^3d) \cos(fx + e) - 15(2*(B*a^3c + (A + 3B)*a^3d) \cos(fx + e)^3 - ((12*A + 17*B)*a^3c + (17*A + 19*B)*a^3d) \cos(fx + e)) * \sin(fx + e) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] $-1/120*(24*B*a^3*d*\cos(f*x + e)^5 - 40*((A + 3*B)*a^3*c + (3*A + 5*B)*a^3*d) * \cos(f*x + e)^3 - 15*(5*(4*A + 3*B)*a^3*c + (15*A + 13*B)*a^3*d)*f*x + 480 * ((A + B)*a^3*c + (A + B)*a^3*d) * \cos(f*x + e) - 15*(2*(B*a^3*c + (A + 3*B)*a^3*d) * \cos(f*x + e)^3 - ((12*A + 17*B)*a^3*c + (17*A + 19*B)*a^3*d) * \cos(f*x + e)) * \sin(f*x + e) / f$

giac [A] time = 0.18, size = 217, normalized size = 1.08

$$-\frac{Ba^3d \cos(5fx + 5e)}{80f} + \frac{1}{8} (20Aa^3c + 15Ba^3c + 15Aa^3d + 13Ba^3d)x + \frac{(4Aa^3c + 12Ba^3c + 12Aa^3d + 17Ba^3d)}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] $-1/80*B*a^3*d*\cos(5*f*x + 5*e)/f + 1/8*(20*A*a^3*c + 15*B*a^3*c + 15*A*a^3*d + 13*B*a^3*d)*x + 1/48*(4*A*a^3*c + 12*B*a^3*c + 12*A*a^3*d + 17*B*a^3*d) * \cos(3*f*x + 3*e)/f - 1/8*(30*A*a^3*c + 26*B*a^3*c + 26*A*a^3*d + 23*B*a^3*d) * \cos(f*x + e)/f + 1/32*(B*a^3*c + A*a^3*d + 3*B*a^3*d) * \sin(4*f*x + 4*e)/f - 1/4*(3*A*a^3*c + 4*B*a^3*c + 4*A*a^3*d + 4*B*a^3*d) * \sin(2*f*x + 2*e)/f$

maple [B] time = 0.51, size = 414, normalized size = 2.06

$$-\frac{a^3Ac(2+\sin^2(fx+e))\cos(fx+e)}{3} + a^3Ad \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + Ba^3c \left(-\frac{\left(\sin^3(fx+e) + \frac{3\sin(fx+e)}{2}\right)\cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] $1/f*(-1/3*a^3*A*c*(2+\sin(f*x+e)^2)*\cos(f*x+e)+a^3*A*d*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)+B*a^3*c*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-1/5*B*a^3*d*(8/3+\sin(f*x+e)^4+4/3*\sin(f*x+e)^2)*\cos(f*x+e)+3*a^3*A*c*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-a^3*A*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)-B*a^3*c*(2+\sin(f*x+e)^2)*\cos(f*x+e)+3*B*a^3*d*(-1/4*(\sin(f*x+e)^3+3/2*\sin(f*x+e))*\cos(f*x+e)+3/8*f*x+3/8*e)-3*a^3*A*c*\cos(f*x+e)+3*a^3*A*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)+3*B*a^3*c*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*d*(2+\sin(f*x+e)^2)*\cos(f*x+e)+a^3*A*c*(f*x+e)-a^3*A*d*\cos(f*x+e)-B*a^3*c*\cos(f*x+e)+B*a^3*d*(-1/2*\sin(f*x+e)*\cos(f*x+e)+1/2*f*x+1/2*e))$

maxima [B] time = 0.43, size = 398, normalized size = 1.98

$$160 \left(\cos(fx + e)^3 - 3 \cos(fx + e) \right) Aa^3c + 360 (2fx + 2e - \sin(2fx + 2e)) Aa^3c + 480 (fx + e) Aa^3c + 480$$


```

*f) - 2*A*a**3*d*cos(e + f*x)**3/f - A*a**3*d*cos(e + f*x)/f + 3*B*a**3*c*x
*sin(e + f*x)**4/8 + 3*B*a**3*c*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + 3*B*a
**3*c*x*sin(e + f*x)**2/2 + 3*B*a**3*c*x*cos(e + f*x)**4/8 + 3*B*a**3*c*x*c
os(e + f*x)**2/2 - 5*B*a**3*c*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**3
*c*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)**3
/(8*f) - 3*B*a**3*c*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*c*cos(e + f*
x)**3/f - B*a**3*c*cos(e + f*x)/f + 9*B*a**3*d*x*sin(e + f*x)**4/8 + 9*B*a*
**3*d*x*sin(e + f*x)**2*cos(e + f*x)**2/4 + B*a**3*d*x*sin(e + f*x)**2/2 + 9
*B*a**3*d*x*cos(e + f*x)**4/8 + B*a**3*d*x*cos(e + f*x)**2/2 - B*a**3*d*sin
(e + f*x)**4*cos(e + f*x)/f - 15*B*a**3*d*sin(e + f*x)**3*cos(e + f*x)/(8*f
) - 4*B*a**3*d*sin(e + f*x)**2*cos(e + f*x)**3/(3*f) - 3*B*a**3*d*sin(e + f
*x)**2*cos(e + f*x)/f - 9*B*a**3*d*sin(e + f*x)*cos(e + f*x)**3/(8*f) - B*a
**3*d*sin(e + f*x)*cos(e + f*x)/(2*f) - 8*B*a**3*d*cos(e + f*x)**5/(15*f) -
2*B*a**3*d*cos(e + f*x)**3/f, Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))*
(a*sin(e) + a)**3, True))

```

3.261 $\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) dx$

Optimal. Leaf size=127

$$\frac{5a^3(4A+3B)\cos(e+fx)}{6f} - \frac{5a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{24f} + \frac{5}{8}a^3x(4A+3B) - \frac{a(4A+3B)\cos(e+fx)(a\sin(e+fx)+a)}{12f}$$

[Out] $5/8*a^3*(4*A+3*B)*x - 5/6*a^3*(4*A+3*B)*\cos(f*x+e)/f - 5/24*a^3*(4*A+3*B)*\cos(f*x+e)*\sin(f*x+e)/f - 1/12*a*(4*A+3*B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^2/f - 1/4*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^3/f$

Rubi [A] time = 0.10, antiderivative size = 117, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {2751, 2645, 2638, 2635, 8, 2633}

$$\frac{a^3(4A+3B)\cos^3(e+fx)}{12f} - \frac{a^3(4A+3B)\cos(e+fx)}{f} - \frac{3a^3(4A+3B)\sin(e+fx)\cos(e+fx)}{8f} + \frac{5}{8}a^3x(4A+3B) - \frac{B\cos(e+fx)(a+a\sin(e+fx))^3}{12f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]

[Out] $(5*a^3*(4*A + 3*B)*x)/8 - (a^3*(4*A + 3*B)*\text{Cos}[e + f*x])/f + (a^3*(4*A + 3*B)*\text{Cos}[e + f*x]^3)/(12*f) - (3*a^3*(4*A + 3*B)*\text{Cos}[e + f*x]*\text{Sin}[e + f*x])/(8*f) - (B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^3)/(4*f)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2645

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrig[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && IGtQ[n, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(d*cos[e + f*x]*(a + b*sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] &&

EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^3 (A + B \sin(e + fx)) dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^3}{4f} + \frac{1}{4}(4A + 3B) \int (a + a \sin(e + fx))^3 dx \\
&= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^3}{4f} + \frac{1}{4}(4A + 3B) \int (a^3 + 3a^2 \sin(e + fx) + 3a \sin^2(e + fx) + \sin^3(e + fx)) dx \\
&= \frac{1}{4}a^3(4A + 3B)x - \frac{B \cos(e + fx)(a + a \sin(e + fx))^3}{4f} + \frac{1}{4}(a^3(4A + 3B)x + 3a^2 \int \sin(e + fx) dx + 3a \int \sin^2(e + fx) dx + \int \sin^3(e + fx) dx) \\
&= \frac{1}{4}a^3(4A + 3B)x - \frac{3a^3(4A + 3B) \cos(e + fx)}{4f} - \frac{3a^3(4A + 3B) \sin(e + fx)}{4f} + \frac{3a^3(4A + 3B) \sin^2(e + fx)}{4f} + \frac{3a^3(4A + 3B) \sin^3(e + fx)}{4f} \\
&= \frac{5}{8}a^3(4A + 3B)x - \frac{a^3(4A + 3B) \cos(e + fx)}{f} + \frac{a^3(4A + 3B) \cos^2(e + fx)}{12f}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 120, normalized size = 0.94

$$\frac{a^3 \cos(e + fx) \left(30(4A + 3B) \sin^{-1} \left(\frac{\sqrt{1 - \sin(e + fx)}}{\sqrt{2}} \right) + \sqrt{\cos^2(e + fx)} (8(A + 3B) \sin^2(e + fx) + 9(4A + 5B) \sin(e + fx)) \right)}{24f \sqrt{\cos^2(e + fx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]),x]`

```
[Out] -1/24*(a^3*Cos[e + f*x]*(30*(4*A + 3*B)*ArcSin[Sqrt[1 - Sin[e + f*x]]/Sqrt[2]] + Sqrt[Cos[e + f*x]^2]*(88*A + 72*B + 9*(4*A + 5*B)*Sin[e + f*x] + 8*(A + 3*B)*Sin[e + f*x]^2 + 6*B*Sin[e + f*x]^3)))/(f*Sqrt[Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 93, normalized size = 0.73

$$\frac{8(A + 3B)a^3 \cos^3(fx + e) + 15(4A + 3B)a^3 fx - 96(A + B)a^3 \cos(fx + e) + 3(2Ba^3 \cos(fx + e))^3 - (12A + 17B)a^3 \cos(fx + e) \sin(fx + e)}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="fricas")`

```
[Out] 1/24*(8*(A + 3*B)*a^3*cos(f*x + e)^3 + 15*(4*A + 3*B)*a^3*f*x - 96*(A + B)*a^3*cos(f*x + e) + 3*(2*B*a^3*cos(f*x + e))^3 - (12*A + 17*B)*a^3*cos(f*x + e))*sin(f*x + e)/f
```

giac [A] time = 0.17, size = 116, normalized size = 0.91

$$\frac{Ba^3 \sin(4fx + 4e)}{32f} + \frac{5}{8}(4Aa^3 + 3Ba^3)x + \frac{(Aa^3 + 3Ba^3) \cos(3fx + 3e)}{12f} - \frac{(15Aa^3 + 13Ba^3) \cos(fx + e)}{4f} - \frac{(3Aa^3 + 3Ba^3) \sin(fx + e)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="giac")`

```
[Out] 1/32*B*a^3*sin(4*f*x + 4*e)/f + 5/8*(4*A*a^3 + 3*B*a^3)*x + 1/12*(A*a^3 + 3*B*a^3)*cos(3*f*x + 3*e)/f - 1/4*(15*A*a^3 + 13*B*a^3)*cos(f*x + e)/f - 1/4*(3*A*a^3 + 4*B*a^3)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.41, size = 178, normalized size = 1.40

$$-\frac{a^3 A (2 + \sin^2(fx+e)) \cos(fx+e)}{3} + B a^3 \left(-\frac{\left(\sin^3(fx+e) + \frac{3 \sin(fx+e)}{2} \right) \cos(fx+e)}{4} + \frac{3fx}{8} + \frac{3e}{8} \right) + 3a^3 A \left(-\frac{\sin(fx+e) \cos(fx+e)}{2} + \frac{fx}{2} + \frac{e}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x)

[Out] 1/f*(-1/3*a^3*A*(2+sin(f*x+e)^2)*cos(f*x+e)+B*a^3*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+3*a^3*A*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)-B*a^3*(2+sin(f*x+e)^2)*cos(f*x+e)-3*a^3*A*cos(f*x+e)+3*B*a^3*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^3*A*(f*x+e)-B*a^3*cos(f*x+e))

maxima [A] time = 0.38, size = 171, normalized size = 1.35

$$32 \left(\cos(fx+e)^3 - 3 \cos(fx+e) \right) A a^3 + 72 (2fx+2e - \sin(2fx+2e)) A a^3 + 96 (fx+e) A a^3 + 96 \left(\cos(fx+e)^3 - 3 \cos(fx+e) \right) B a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] 1/96*(32*(cos(f*x+e)^3-3*cos(f*x+e))*A*a^3+72*(2*f*x+2*e-sin(2*f*x+2*e))*A*a^3+96*(f*x+e)*A*a^3+96*(cos(f*x+e)^3-3*cos(f*x+e))*B*a^3+3*(12*f*x+12*e+sin(4*f*x+4*e)-8*sin(2*f*x+2*e))*B*a^3+72*(2*f*x+2*e-sin(2*f*x+2*e))*B*a^3-288*A*a^3*cos(f*x+e)-96*B*a^3*cos(f*x+e))/f

mupad [B] time = 14.26, size = 330, normalized size = 2.60

$$\frac{5a^3 \operatorname{atan} \left(\frac{5a^3 \tan \left(\frac{e}{2} + \frac{fx}{2} \right) (4A+3B)}{4 \left(5Aa^3 + \frac{15Ba^3}{4} \right)} \right) (4A+3B)}{4f} \tan \left(\frac{e}{2} + \frac{fx}{2} \right) \left(3Aa^3 + \frac{15Ba^3}{4} \right) + \frac{22Aa^3}{3} + 6Ba^3 + \tan \left(\frac{e}{2} + \frac{fx}{2} \right)^6 \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(e+f*x))*(a+a*sin(e+f*x))^3,x)

[Out] (5*a^3*atan((5*a^3*tan(e/2+(f*x)/2)*(4*A+3*B))/(4*(5*A*a^3+(15*B*a^3)/4)))*(4*A+3*B))/(4*f)-(tan(e/2+(f*x)/2)*(3*A*a^3+(15*B*a^3)/4)+(22*A*a^3)/3+6*B*a^3+tan(e/2+(f*x)/2)^6*(6*A*a^3+2*B*a^3)-tan(e/2+(f*x)/2)^7*(3*A*a^3+(15*B*a^3)/4)+tan(e/2+(f*x)/2)^3*(3*A*a^3+(23*B*a^3)/4)-tan(e/2+(f*x)/2)^5*(3*A*a^3+(23*B*a^3)/4)+tan(e/2+(f*x)/2)^4*(22*A*a^3+18*B*a^3)+tan(e/2+(f*x)/2)^2*((70*A*a^3)/3+22*B*a^3))/(f*(4*tan(e/2+(f*x)/2)^2+6*tan(e/2+(f*x)/2)^4+4*tan(e/2+(f*x)/2)^6+tan(e/2+(f*x)/2)^8+1)-(5*a^3*(4*A+3*B)*(atan(tan(e/2+(f*x)/2))-(f*x)/2))/(4*f)

sympy [A] time = 2.08, size = 371, normalized size = 2.92

$$\left\{ \begin{array}{l} \frac{3Aa^3x \sin^2(e+fx)}{2} + \frac{3Aa^3x \cos^2(e+fx)}{2} + Aa^3x - \frac{Aa^3 \sin^2(e+fx) \cos(e+fx)}{f} - \frac{3Aa^3 \sin(e+fx) \cos(e+fx)}{2f} - \frac{2Aa^3 \cos^3(e+fx)}{3f} - \frac{3Aa^3}{3f} \\ x(A+B \sin(e))(a \sin(e)+a)^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e)),x)
```

```
[Out] Piecewise((3*A*a**3*x*sin(e + f*x)**2/2 + 3*A*a**3*x*cos(e + f*x)**2/2 + A*
a**3*x - A*a**3*sin(e + f*x)**2*cos(e + f*x)/f - 3*A*a**3*sin(e + f*x)*cos(
e + f*x)/(2*f) - 2*A*a**3*cos(e + f*x)**3/(3*f) - 3*A*a**3*cos(e + f*x)/f +
3*B*a**3*x*sin(e + f*x)**4/8 + 3*B*a**3*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*B*a**3*x*sin(e + f*x)**2/2 + 3*B*a**3*x*cos(e + f*x)**4/8 + 3*B*a**3*
x*cos(e + f*x)**2/2 - 5*B*a**3*sin(e + f*x)**3*cos(e + f*x)/(8*f) - 3*B*a**
3*sin(e + f*x)**2*cos(e + f*x)/f - 3*B*a**3*sin(e + f*x)*cos(e + f*x)**3/(8
*f) - 3*B*a**3*sin(e + f*x)*cos(e + f*x)/(2*f) - 2*B*a**3*cos(e + f*x)**3/f
- B*a**3*cos(e + f*x)/f, Ne(f, 0)), (x*(A + B*sin(e))*(a*sin(e) + a)**3, T
rue))
```

$$3.262 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=246

$$\frac{2a^3(c-d)^3(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4 f \sqrt{c^2-d^2}} + \frac{a^3(Ad(2c-5d)-B(2c^2-5cd+5d^2)) \cos(e+fx)}{2d^3 f} + \frac{a^3 x (Ad(2c^2-5cd+5d^2)-B(2c^3-6c^2d+7cd^2-5d^3))}{d^4 f \sqrt{c^2-d^2}}$$

[Out] $\frac{1}{2} a^3 (A d (2 c^2 - 6 c d + 7 d^2) - B (2 c^3 - 6 c^2 d + 7 c d^2 - 5 d^3)) x / d^4 + \frac{1}{2} a^3 (A (2 c - 5 d) d - B (2 c^2 - 5 c d + 5 d^2)) \cos(f x + e) / d^3 / f - \frac{1}{3} a B \cos(f x + e) (a + a \sin(f x + e))^2 / d / f + \frac{1}{6} (-3 A d + 3 B c - 5 B d) \cos(f x + e) (a^3 + a^3 \sin(f x + e)) / d^2 / f + 2 a^3 (c - d)^3 (-A d + B c) \arctan((d + c \tan(1/2 f x + 1/2 e)) / (c^2 - d^2)^{1/2}) / d^4 / f / (c^2 - d^2)^{1/2}$

Rubi [A] time = 0.90, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2976, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{a^3(Ad(2c-5d)-B(2c^2-5cd+5d^2)) \cos(e+fx)}{2d^3 f} + \frac{2a^3(c-d)^3(Bc-Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{d^4 f \sqrt{c^2-d^2}} + \frac{a^3 x (Ad(2c^2-5cd+5d^2)-B(2c^3-6c^2d+7cd^2-5d^3))}{d^4 f \sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]

[Out] $(a^3(A d (2 c^2 - 6 c d + 7 d^2) - B (2 c^3 - 6 c^2 d + 7 c d^2 - 5 d^3)) x) / (2 d^4) + (2 a^3 (c - d)^3 (B c - A d) \text{ArcTan}[(d + c \text{Tan}[(e + f x) / 2]) / \text{Sqrt}[c^2 - d^2]]) / (d^4 \text{Sqrt}[c^2 - d^2] f) + (a^3 (A (2 c - 5 d) d - B (2 c^2 - 5 c d + 5 d^2)) \text{Cos}[e + f x]) / (2 d^3 f) - (a B \text{Cos}[e + f x] (a + a \text{Sin}[e + f x])^2) / (3 d f) + ((3 B c - 3 A d - 5 B d) \text{Cos}[e + f x] (a^3 + a^3 \text{Sin}[e + f x])) / (6 d^2 f)$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a
+ b*SIN[e + f*x])^m*(A*c + (B*c + A*d)*SIN[e + f*x] + B*d*SIN[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -Simp
mp[(b*B*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m - 1)*(c + d*SIN[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*SIN[e + f*x
])^(m - 1)*(c + d*SIN[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*SIN[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*SIN[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*SIN[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*SIN[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2}{3df} + \frac{\int \frac{(a + a \sin(e + fx))^2 (a(2Bc + 3c + d \sin(e + fx)))}{c + d \sin(e + fx)} dx}{c + d \sin(e + fx)} \\ &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2}{3df} + \frac{(3Bc - 3Ad - 5Bd) \cos(e + fx)}{3df} \\ &= -\frac{aB \cos(e + fx)(a + a \sin(e + fx))^2}{3df} + \frac{(3Bc - 3Ad - 5Bd) \cos(e + fx)}{3df} \\ &= \frac{a^3 (A(2c - 5d)d - B(2c^2 - 5cd + 5d^2)) \cos(e + fx)}{2d^3 f} - \frac{aB \cos(e + fx)}{3df} \\ &= \frac{a^3 (Ad(2c^2 - 6cd + 7d^2) - B(2c^3 - 6c^2d + 7cd^2 - 5d^3)) x}{2d^4} + \frac{a^3 (A(2c - 5d)d - B(2c^2 - 5cd + 5d^2)) \cos(e + fx)}{2d^3 f} \\ &= \frac{a^3 (Ad(2c^2 - 6cd + 7d^2) - B(2c^3 - 6c^2d + 7cd^2 - 5d^3)) x}{2d^4} + \frac{a^3 (A(2c - 5d)d - B(2c^2 - 5cd + 5d^2)) \cos(e + fx)}{2d^3 f} \\ &= \frac{a^3 (Ad(2c^2 - 6cd + 7d^2) - B(2c^3 - 6c^2d + 7cd^2 - 5d^3)) x}{2d^4} + \frac{a^3 (A(2c - 5d)d - B(2c^2 - 5cd + 5d^2)) \cos(e + fx)}{2d^3 f} \end{aligned}$$

Mathematica [A] time = 0.98, size = 233, normalized size = 0.95

$$a^3(\sin(e + fx) + 1)^3 \left(-3d \left(4Ad(3d - c) + B(4c^2 - 12cd + 15d^2) \right) \cos(e + fx) + \frac{24(c-d)^3(Bc - Ad) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e + fx) \right) + d}{\sqrt{c^2 - d^2}} \right)}{\sqrt{c^2 - d^2}} \right) + 12d^4 f \left(\sin \left(\frac{1}{2}(e + fx) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]
```

```
[Out] (a^3*(1 + Sin[e + f*x])^3*(6*(A*d*(2*c^2 - 6*c*d + 7*d^2) + B*(-2*c^3 + 6*c^2*d - 7*c*d^2 + 5*d^3))*(e + f*x) + (24*(c - d)^3*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] - 3*d*(4*A*d*(-c + 3*d) + B*(4*c^2 - 12*c*d + 15*d^2))*Cos[e + f*x] + B*d^3*Cos[3*(e + f*x)] - 3*d^2*(-(B*c) + A*d + 3*B*d)*Sin[2*(e + f*x)])/(12*d^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
```

fricas [A] time = 0.55, size = 627, normalized size = 2.55

$$\frac{2Ba^3d^3 \cos^3(fx + e) - 3(2Ba^3c^3 - 2(A + 3B)a^3c^2d + (6A + 7B)a^3cd^2 - (7A + 5B)a^3d^3)fx + 3(Ba^3cd^2 - (A + 3B)a^3c^2d + (6A + 7B)a^3cd^2 - (7A + 5B)a^3d^3) \sin(fx + e)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] [1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) + 3*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt(-(c - d)/(c + d))*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 - 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 6*(B*a^3*c^2*d - (A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e)]/(d^4*f), 1/6*(2*B*a^3*d^3*cos(f*x + e)^3 - 3*(2*B*a^3*c^3 - 2*(A + 3*B)*a^3*c^2*d + (6*A + 7*B)*a^3*c*d^2 - (7*A + 5*B)*a^3*d^3)*f*x + 3*(B*a^3*c*d^2 - (A + 3*B)*a^3*d^3)*cos(f*x + e)*sin(f*x + e) - 6*(B*a^3*c^3 - (A + 2*B)*a^3*c^2*d + (2*A + B)*a^3*c*d^2 - A*a^3*d^3)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d)))/((c - d)*cos(f*x + e)) - 6*(B*a^3*c^2*d - (A + 3*B)*a^3*c*d^2 + (3*A + 4*B)*a^3*d^3)*cos(f*x + e)]/(d^4*f)]
```

giac [B] time = 0.21, size = 617, normalized size = 2.51

$$\frac{3(2Ba^3c^3 - 2Aa^3c^2d - 6Ba^3c^2d + 6Aa^3cd^2 + 7Ba^3cd^2 - 7Aa^3d^3 - 5Ba^3d^3)(fx + e)}{d^4} - \frac{12(Ba^3c^4 - Aa^3c^3d - 3Ba^3c^3d + 3Aa^3c^2d^2 + 3Ba^3c^2d^2 - 3Aa^3cd^3 - Ba^3d^4)}{\sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*B*a^3*c^3 - 2*A*a^3*c^2*d - 6*B*a^3*c^2*d + 6*A*a^3*c*d^2 + 7*B*
a^3*c*d^2 - 7*A*a^3*d^3 - 5*B*a^3*d^3)*(f*x + e)/d^4 - 12*(B*a^3*c^4 - A*a^
3*c^3*d - 3*B*a^3*c^3*d + 3*A*a^3*c^2*d^2 + 3*B*a^3*c^2*d^2 - 3*A*a^3*c*d^3
- B*a^3*c*d^3 + A*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arct
an((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^4) + 2
*(3*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^5 - 3*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 -
9*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^5 + 6*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^4 -
6*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 - 18*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^4 +
18*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^4 +
12*B*a^3*c^2*tan(1/2*f*x + 1/2*e)^2 - 12*A*a^3*c*d*tan(1/2*f*x + 1/2*e)^2
- 36*B*a^3*c*d*tan(1/2*f*x + 1/2*e)^2 + 36*A*a^3*d^2*tan(1/2*f*x + 1/2*e)^2
+ 48*B*a^3*d^2*tan(1/2*f*x + 1/2*e)^2 - 3*B*a^3*c*d*tan(1/2*f*x + 1/2*e) +
3*A*a^3*d^2*tan(1/2*f*x + 1/2*e) + 9*B*a^3*d^2*tan(1/2*f*x + 1/2*e) + 6*B*
a^3*c^2 - 6*A*a^3*c*d - 18*B*a^3*c*d + 18*A*a^3*d^2 + 22*B*a^3*d^2)/((tan(1
/2*f*x + 1/2*e)^2 + 1)^3*d^3))/f
```

maple [B] time = 0.48, size = 1357, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4*c-2/f*a^3/d^3
/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c^2+6/f*a^3/d^2/(1+tan(1
/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^4*c+4/f*a^3/d^2/(1+tan(1/2*f*x+1/2*
e)^2)^3*A*tan(1/2*f*x+1/2*e)^2*c-4/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*t
an(1/2*f*x+1/2*e)^2*c^2+12/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f
*x+1/2*e)^2*c+1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B*c
-16/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B*tan(1/2*f*x+1/2*e)^2-1/f*a^3/d/(1+
tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*A+6/f*a^3/d^3*arctan(tan(1/2*f*x
+1/2*e))*B*c^2-7/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*B*c-3/f*a^3/d/(1+tan(
1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)*B+2/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^
2)^3*A*c-2/f*a^3/d^3/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c^2+2/f*a^3/d^3*arctan(ta
n(1/2*f*x+1/2*e))*A*c^2-6/f*a^3/d^2*arctan(tan(1/2*f*x+1/2*e))*A*c+6/f*a^3/
d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*B*c-2/f*a^3/d^4*arctan(tan(1/2*f*x+1/2*e))*B
*c^3+1/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*A+3/f*a^3/d/
(1+tan(1/2*f*x+1/2*e)^2)^3*tan(1/2*f*x+1/2*e)^5*B-6/f*a^3/d/(1+tan(1/2*f*x+
1/2*e)^2)^3*A*tan(1/2*f*x+1/2*e)^4-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B*t
an(1/2*f*x+1/2*e)^4-12/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A*tan(1/2*f*x+1/2
*e)^2-2/f*a^3/d^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(
c^2-d^2)^(1/2))*A*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f
*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-6/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*
(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+2/f*a^3/d^4/(c^2-d^2)^(1/
2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^4-6/f*a^3/d
^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))
*B*c^3+6/f*a^3/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/
(c^2-d^2)^(1/2))*B*c^2-2/f*a^3/d/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-1/f*a^3/d^2/(1+tan(1/2*f*x+1/2*e)^2)^3*t
an(1/2*f*x+1/2*e)^5*B*c+7/f*a^3/d*arctan(tan(1/2*f*x+1/2*e))*A+5/f*a^3/d*ar
ctan(tan(1/2*f*x+1/2*e))*B-6/f*a^3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*A-22/3/f*a^
3/d/(1+tan(1/2*f*x+1/2*e)^2)^3*B+2/f*a^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*ta
n(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& d^3(7A + 5B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)/d^4 - (8*\tan(e/2 \\
& + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a \\
& ^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B* \\
& a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^ \\
& 3*(7A + 5B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2)/d^4)*(B*a^3*c^3*1i \\
& + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7A + 5B)*1i)/2 - (a^3*d*(2* \\
& A*c^2 + 6*B*c^2)*1i)/2)*1i)/d^4)/((16*(2*B^3*a^9*c^10 - 47*A^3*a^9*c^2*d^8 \\
& + 55*A^3*a^9*c^3*d^7 - 21*A^3*a^9*c^4*d^6 - 7*A^3*a^9*c^5*d^5 + 8*A^3*a^9*c \\
& ^6*d^4 - 2*A^3*a^9*c^7*d^3 - 15*B^3*a^9*c^3*d^7 + 71*B^3*a^9*c^4*d^6 - 148* \\
& B^3*a^9*c^5*d^5 + 180*B^3*a^9*c^6*d^4 - 139*B^3*a^9*c^7*d^3 + 67*B^3*a^9*c^ \\
& 8*d^2 + 14*A^3*a^9*c*d^9 - 18*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B* \\
& a^9*c*d^9 + 5*A*B^2*a^9*c^2*d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4* \\
& d^6 - 280*A*B^2*a^9*c^5*d^5 + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3 \\
& + 44*A*B^2*a^9*c^8*d^2 - 32*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45* \\
& A^2*B*a^9*c^4*d^6 - 97*A^2*B*a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B* \\
& a^9*c^7*d^3 + 6*A^2*B*a^9*c^8*d^2))/d^8 + (((8*(49*A^2*a^6*c^2*d^9 - 84*A^2 \\
& *a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 \\
& + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a \\
& ^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + \\
& 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^ \\
& 6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*\tan(e/2 + (f* \\
& x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 11 \\
& 6*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^ \\
& 2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 1 \\
& 36*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c \\
& ^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258 \\
& *A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c \\
& ^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 \\
& + (((32*c^2*d^3 + (8*\tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(B \\
& *a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7A + 5B)*1i)/2 - \\
& (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c* \\
& d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^ \\
& 3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (8*\tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24* \\
& A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 2 \\
& 4*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9)*(B*a^3*c^3*1i \\
& + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3*(7A + 5B)*1i)/2 - (a^3*d*(2*A \\
& *c^2 + 6*B*c^2)*1i)/2)/d^4)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 \\
& - (a^3*d^3*(7A + 5B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - ((\\
& (8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a \\
& ^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 + \\
& 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6 \\
& *c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 1 \\
& 88*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c \\
& ^7*d^4))/d^8 + (8*\tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2* \\
& d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A \\
& ^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c \\
& ^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 + 4 \\
& 8*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^ \\
& 11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252* \\
& A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8 \\
& *d^4 + 140*A*B*a^6*c*d^11))/d^9 + (((8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - \\
& 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 \\
& - 2*B*a^3*c^4*d^8))/d^8 + ((32*c^2*d^3 + (8*\tan(e/2 + (f*x)/2)*(12*c*d^13 - \\
& 8*c^3*d^11))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^ \\
& 3*(7A + 5B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4 - (8*\tan(e/2 + \\
& (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3 \\
& *c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^ \\
& 3*c^5*d^8))/d^9)*(B*a^3*c^3*1i + (a^3*d^2*(6*A*c + 7*B*c)*1i)/2 - (a^3*d^3* \\
& (7A + 5B)*1i)/2 - (a^3*d*(2*A*c^2 + 6*B*c^2)*1i)/2))/d^4)*(B*a^3*c^3*1i +
\end{aligned}$$

$$\begin{aligned}
& (a^3d^2(6Ac + 7Bc)1i)/2 - (a^3d^3(7A + 5B)1i)/2 - (a^3d(2A^2c^2 + 6B^2c^2)1i)/2)/d^4 + (16\tan(e/2 + (fx)/2)(8B^3a^9c^{11} - 462A^3a^9c^2d^9 + 926A^3a^9c^3d^8 - 1034A^3a^9c^4d^7 + 704A^3a^9c^5d^6 - 296A^3a^9c^6d^5 + 72A^3a^9c^7d^4 - 8A^3a^9c^8d^3 - 50B^3a^9c^2d^9 + 290B^3a^9c^3d^8 - 788B^3a^9c^4d^7 + 1332B^3a^9c^5d^6 - 1546B^3a^9c^6d^5 + 1274B^3a^9c^7d^4 - 744B^3a^9c^8d^3 + 296B^3a^9c^9d^2 + 98A^3a^9c^d^{10} - 72B^3a^9c^{10}d + 50A^2B^2a^9c^d^{10} - 24A^2B^2a^9c^{10}d + 140A^2B^2a^9c^d^{10} - 430A^2B^2a^9c^2d^9 + 1524A^2B^2a^9c^3d^8 - 3076A^2B^2a^9c^4d^7 + 4018A^2B^2a^9c^5d^6 - 3582A^2B^2a^9c^6d^5 + 2192A^2B^2a^9c^7d^4 - 888A^2B^2a^9c^8d^3 + 216A^2B^2a^9c^9d^2 - 834A^2B^2a^9c^2d^9 + 2206A^2B^2a^9c^3d^8 - 3398A^2B^2a^9c^4d^7 + 3342A^2B^2a^9c^5d^6 - 2152A^2B^2a^9c^6d^5 + 888A^2B^2a^9c^7d^4 - 216A^2B^2a^9c^8d^3 + 24A^2B^2a^9c^9d^2))/d^9)) \cdot (B^2a^3c^31i + (a^3d^2(6Ac + 7Bc)1i)/2 - (a^3d^3(7A + 5B)1i)/2 - (a^3d(2A^2c^2 + 6B^2c^2)1i)/2) \cdot 2i)/(d^4f) - (a^3 \cdot \operatorname{atan}(((a^3(A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2}) \cdot ((8(49A^2a^6c^2d^9 - 84A^2a^6c^3d^8 + 64A^2a^6c^4d^7 - 24A^2a^6c^5d^6 + 4A^2a^6c^6d^5 + 25B^2a^6c^2d^9 - 70B^2a^6c^3d^8 + 109B^2a^6c^4d^7 - 104B^2a^6c^5d^6 + 64B^2a^6c^6d^5 - 24B^2a^6c^7d^4 + 4B^2a^6c^8d^3 + 70A^2B^2a^6c^2d^9 - 158A^2B^2a^6c^3d^8 + 188A^2B^2a^6c^4d^7 - 128A^2B^2a^6c^5d^6 + 48A^2B^2a^6c^6d^5 - 8A^2B^2a^6c^7d^4))/d^8 + (8 \cdot \tan(e/2 + (fx)/2) \cdot (19A^2a^6c^3d^9 - 144A^2a^6c^2d^{10} + 116A^2a^6c^4d^8 - 116A^2a^6c^5d^7 + 48A^2a^6c^6d^6 - 8A^2a^6c^7d^5 - 140B^2a^6c^2d^{10} + 189B^2a^6c^3d^9 - 114B^2a^6c^4d^8 - 41B^2a^6c^5d^7 + 136B^2a^6c^6d^6 - 116B^2a^6c^7d^5 + 48B^2a^6c^8d^4 - 8B^2a^6c^9d^3 + 94A^2a^6c^d^{11} + 50B^2a^6c^d^{11} - 308A^2B^2a^6c^2d^{10} + 258A^2B^2a^6c^3d^9 + 22A^2B^2a^6c^4d^8 - 252A^2B^2a^6c^5d^7 + 232A^2B^2a^6c^6d^6 - 96A^2B^2a^6c^7d^5 + 16A^2B^2a^6c^8d^4 + 140A^2B^2a^6c^d^{11}))/d^9 + (a^3 \cdot (A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2} \cdot ((8 \cdot \tan(e/2 + (fx)/2) \cdot (8A^2a^3c^d^{12} - 24A^2a^3c^2d^{11} + 24A^2a^3c^3d^{10} - 8A^2a^3c^4d^9 - 8B^2a^3c^2d^{11} + 24B^2a^3c^3d^{10} - 24B^2a^3c^4d^9 + 8B^2a^3c^5d^8))/d^9 - (8 \cdot (14A^2a^3c^d^{11} + 10B^2a^3c^d^{11} - 16A^2a^3c^2d^{10} + 2A^2a^3c^3d^9 - 14B^2a^3c^2d^{10} + 6B^2a^3c^3d^9 - 2B^2a^3c^4d^8))/d^8 + (a^3 \cdot (32c^2d^3 + (8 \cdot \tan(e/2 + (fx)/2) \cdot (12c^d^{13} - 8c^3d^{11}))/d^9) \cdot (A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2} / (c^d^4 + d^5)) / (c^d^4 + d^5) \cdot 1i) / (c^d^4 + d^5) + (a^3 \cdot (A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2} \cdot ((8 \cdot (49A^2a^6c^2d^9 - 84A^2a^6c^3d^8 + 64A^2a^6c^4d^7 - 24A^2a^6c^5d^6 + 4A^2a^6c^6d^5 + 25B^2a^6c^2d^9 - 70B^2a^6c^3d^8 + 109B^2a^6c^4d^7 - 104B^2a^6c^5d^6 + 64B^2a^6c^6d^5 - 24B^2a^6c^7d^4 + 4B^2a^6c^8d^3 + 70A^2B^2a^6c^2d^9 - 158A^2B^2a^6c^3d^8 + 188A^2B^2a^6c^4d^7 - 128A^2B^2a^6c^5d^6 + 48A^2B^2a^6c^6d^5 - 8A^2B^2a^6c^7d^4))/d^8 + (8 \cdot \tan(e/2 + (fx)/2) \cdot (19A^2a^6c^3d^9 - 144A^2a^6c^2d^{10} + 116A^2a^6c^4d^8 - 116A^2a^6c^5d^7 + 48A^2a^6c^6d^6 - 8A^2a^6c^7d^5 - 140B^2a^6c^2d^{10} + 189B^2a^6c^3d^9 - 114B^2a^6c^4d^8 - 41B^2a^6c^5d^7 + 136B^2a^6c^6d^6 - 116B^2a^6c^7d^5 + 48B^2a^6c^8d^4 - 8B^2a^6c^9d^3 + 94A^2a^6c^d^{11} + 50B^2a^6c^d^{11} - 308A^2B^2a^6c^2d^{10} + 258A^2B^2a^6c^3d^9 + 22A^2B^2a^6c^4d^8 - 252A^2B^2a^6c^5d^7 + 232A^2B^2a^6c^6d^6 - 96A^2B^2a^6c^7d^5 + 16A^2B^2a^6c^8d^4 + 140A^2B^2a^6c^d^{11}))/d^9 + (a^3 \cdot (A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2} \cdot ((8 \cdot (14A^2a^3c^d^{11} + 10B^2a^3c^d^{11} - 16A^2a^3c^2d^{10} + 2A^2a^3c^3d^9 - 14B^2a^3c^2d^{10} + 6B^2a^3c^3d^9 - 2B^2a^3c^4d^8))/d^8 - (8 \cdot \tan(e/2 + (fx)/2) \cdot (8A^2a^3c^d^{12} - 24A^2a^3c^2d^{11} + 24A^2a^3c^3d^{10} - 8A^2a^3c^4d^9 - 8B^2a^3c^2d^{11} + 24B^2a^3c^3d^{10} - 24B^2a^3c^4d^9 + 8B^2a^3c^5d^8))/d^9 + (a^3 \cdot (32c^2d^3 + (8 \cdot \tan(e/2 + (fx)/2) \cdot (12c^d^{13} - 8c^3d^{11}))/d^9) \cdot (A^2d - B^2c) \cdot (-c + d) \cdot (c - d)^5)^{1/2} / (c^d^4 + d^5)) / (c^d^4 + d^5) \cdot 1i) / (c^d^4 + d^5) / ((16(2B^3a^9c^{10} - 47A^3a^9c^2d^8 + 55A^3a^9c^3d^7 - 21A^3a^9c^4d^6 - 7A^3a^9c^5d^5 + 8A^3a^9c^6d^4 - 2A^3a^9c^7d^3 - 15B^3a^9c^3d^7 + 71B^3a^9c^4d^6 - 148B^3a^9c^5d^5 + 180B^3a^9c^6d^4 - 139B^3a^9c^7d^3 + 67B^3a^9c^8d^2 + 14A^3a^9c^d^9 - 1
\end{aligned}$$

$$\begin{aligned}
& 8*B^3*a^9*c^9*d - 6*A*B^2*a^9*c^9*d + 10*A^2*B*a^9*c*d^9 + 5*A*B^2*a^9*c^2*d^8 - 53*A*B^2*a^9*c^3*d^7 + 174*A*B^2*a^9*c^4*d^6 - 280*A*B^2*a^9*c^5*d^5 \\
& + 257*A*B^2*a^9*c^6*d^4 - 141*A*B^2*a^9*c^7*d^3 + 44*A*B^2*a^9*c^8*d^2 - 32*A^2*B*a^9*c^2*d^8 + 21*A^2*B*a^9*c^3*d^7 + 45*A^2*B*a^9*c^4*d^6 - 97*A^2*B \\
& *a^9*c^5*d^5 + 81*A^2*B*a^9*c^6*d^4 - 34*A^2*B*a^9*c^7*d^3 + 6*A^2*B*a^9*c^8*d^2)/d^8 + (16*\tan(e/2 + (f*x)/2)*(8*B^3*a^9*c^11 - 462*A^3*a^9*c^2*d^9 \\
& + 926*A^3*a^9*c^3*d^8 - 1034*A^3*a^9*c^4*d^7 + 704*A^3*a^9*c^5*d^6 - 296*A^3*a^9*c^6*d^5 + 72*A^3*a^9*c^7*d^4 - 8*A^3*a^9*c^8*d^3 - 50*B^3*a^9*c^2*d^9 \\
& + 290*B^3*a^9*c^3*d^8 - 788*B^3*a^9*c^4*d^7 + 1332*B^3*a^9*c^5*d^6 - 1546*B^3*a^9*c^6*d^5 + 1274*B^3*a^9*c^7*d^4 - 744*B^3*a^9*c^8*d^3 + 296*B^3*a^9*c^9*d^2 \\
& + 98*A^3*a^9*c*d^10 - 72*B^3*a^9*c^10*d + 50*A*B^2*a^9*c*d^10 - 24*A*B^2*a^9*c^10*d + 140*A^2*B*a^9*c*d^10 - 430*A*B^2*a^9*c^2*d^9 + 1524*A*B^2 \\
& *a^9*c^3*d^8 - 3076*A*B^2*a^9*c^4*d^7 + 4018*A*B^2*a^9*c^5*d^6 - 3582*A*B^2*a^9*c^6*d^5 + 2192*A*B^2*a^9*c^7*d^4 - 888*A*B^2*a^9*c^8*d^3 + 216*A*B^2*a^9*c^9*d^2 - 834*A^2*B*a^9*c^2*d^9 \\
& + 2206*A^2*B*a^9*c^3*d^8 - 3398*A^2*B*a^9*c^4*d^7 + 3342*A^2*B*a^9*c^5*d^6 - 2152*A^2*B*a^9*c^6*d^5 + 888*A^2*B*a^9*c^7*d^4 - 216*A^2*B*a^9*c^8*d^3 + 24*A^2*B*a^9*c^9*d^2))/d^9 + (a^3*(A*d \\
& - B*c)*(-(c + d)*(c - d)^5)^(1/2))*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 \\
& - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 \\
& + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*\tan(e/2 + (f*x)/2)*(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6 \\
& *c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 \\
& + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 \\
& + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))*((8*\tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 \\
& + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 + 8*B*a^3*c^5*d^8))/d^9 - (8*(14*A*a^3*c*d^11 + 10*B*a^3*c*d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 + (a^3*(32*c^2*d^3 \\
& + (8*\tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5))/(c*d^4 + d^5))/(c*d^4 + d^5) - (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))*((8*(49*A^2*a^6*c^2*d^9 - 84*A^2*a^6*c^3*d^8 + 64*A^2*a^6*c^4*d^7 - 24*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 25*B^2*a^6*c^2*d^9 \\
& - 70*B^2*a^6*c^3*d^8 + 109*B^2*a^6*c^4*d^7 - 104*B^2*a^6*c^5*d^6 + 64*B^2*a^6*c^6*d^5 - 24*B^2*a^6*c^7*d^4 + 4*B^2*a^6*c^8*d^3 + 70*A*B*a^6*c^2*d^9 - 158*A*B*a^6*c^3*d^8 + 188*A*B*a^6*c^4*d^7 - 128*A*B*a^6*c^5*d^6 + 48*A*B*a^6*c^6*d^5 - 8*A*B*a^6*c^7*d^4))/d^8 + (8*\tan(e/2 + (f*x)/2) \\
& *(19*A^2*a^6*c^3*d^9 - 144*A^2*a^6*c^2*d^10 + 116*A^2*a^6*c^4*d^8 - 116*A^2*a^6*c^5*d^7 + 48*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 - 140*B^2*a^6*c^2*d^10 + 189*B^2*a^6*c^3*d^9 - 114*B^2*a^6*c^4*d^8 - 41*B^2*a^6*c^5*d^7 + 136*B^2*a^6*c^6*d^6 - 116*B^2*a^6*c^7*d^5 \\
& + 48*B^2*a^6*c^8*d^4 - 8*B^2*a^6*c^9*d^3 + 94*A^2*a^6*c*d^11 + 50*B^2*a^6*c*d^11 - 308*A*B*a^6*c^2*d^10 + 258*A*B*a^6*c^3*d^9 + 22*A*B*a^6*c^4*d^8 - 252*A*B*a^6*c^5*d^7 + 232*A*B*a^6*c^6*d^6 - 96*A*B*a^6*c^7*d^5 + 16*A*B*a^6*c^8*d^4 + 140*A*B*a^6*c*d^11))/d^9 + (a^3*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))*((8*(14*A*a^3*c*d^11 + 10*B*a^3*c \\
& *d^11 - 16*A*a^3*c^2*d^10 + 2*A*a^3*c^3*d^9 - 14*B*a^3*c^2*d^10 + 6*B*a^3*c^3*d^9 - 2*B*a^3*c^4*d^8))/d^8 - (8*\tan(e/2 + (f*x)/2)*(8*A*a^3*c*d^12 - 24*A*a^3*c^2*d^11 + 24*A*a^3*c^3*d^10 - 8*A*a^3*c^4*d^9 - 8*B*a^3*c^2*d^11 + 24*B*a^3*c^3*d^10 - 24*B*a^3*c^4*d^9 \\
& + 8*B*a^3*c^5*d^8))/d^9 + (a^3*(32*c^2*d^3 + (8*\tan(e/2 + (f*x)/2)*(12*c*d^13 - 8*c^3*d^11))/d^9)*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2))/(c*d^4 + d^5))/(c*d^4 + d^5))*(A*d - B*c)*(-(c + d)*(c - d)^5)^(1/2)*2i)/(f*(c*d^4 + d^5))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.263 \quad \int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=283

$$\frac{2a^3(c-d)^2 \left(Ad(2c+3d) - B(3c^2+3cd-d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}} \right)}{d^4 f(c+d) \sqrt{c^2-d^2}} - \frac{a^3 x \left(2Ad(2c-3d) - B(6c^2-12cd+7d^2) \right)}{2d^4}$$

[Out] $-1/2*a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/d^4-1/2*a^3*(4*A*c*d-B*(6*c^2-3*c*d-5*d^2))*\cos(f*x+e)/d^3/(c+d)/f+1/2*(2*A*d-B*(3*c+d))*\cos(f*x+e)*(a^3+a^3*\sin(f*x+e))/d^2/(c+d)/f+a*(-A*d+B*c)*\cos(f*x+e)*(a+a*\sin(f*x+e))^2/d/(c+d)/f/(c+d*\sin(f*x+e))+2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c*d-d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 0.94, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {2975, 2976, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{a^3 \left(4Acd - B(6c^2 - 3cd - 5d^2) \right) \cos(e+fx)}{2d^3 f(c+d)} + \frac{2a^3(c-d)^2 \left(Ad(2c+3d) - B(3c^2+3cd-d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}} \right)}{d^4 f(c+d) \sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] $-(a^3*(2*A*(2*c-3*d)*d-B*(6*c^2-12*c*d+7*d^2))*x/(2*d^4)+(2*a^3*(c-d)^2*(A*d*(2*c+3*d)-B*(3*c^2+3*c*d-d^2))*\text{ArcTan}[(d+c*\text{Tan}[(e+f*x)/2])/(\text{Sqrt}[c^2-d^2])]/(d^4*(c+d)*\text{Sqrt}[c^2-d^2]*f)-(a^3*(4*A*c*d-B*(6*c^2-3*c*d-5*d^2))*\text{Cos}[e+f*x]/(2*d^3*(c+d)*f)+((2*A*d-B*(3*c+d))*\text{Cos}[e+f*x]*(a^3+a^3*\text{Sin}[e+f*x]))/(2*d^2*(c+d)*f)+(a*(B*c-A*d)*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^2)/(d*(c+d)*f*(c+d*\text{Sin}[e+f*x]))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*

$\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2968

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2975

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}), x_Symbol] \rightarrow -\text{Simp}[(b^2*(B*c - A*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^{(n+1)})/(d*f*(n+1)*(b*c + a*d)), x] - \text{Dist}[b/(d*(n+1)*(b*c + a*d)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^{(n+1)}*\text{Simp}[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1)))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rule 2976

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}), x_Symbol] \rightarrow -\text{Simp}[(b*B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^{(n+1)})/(d*f*(m+n+1)), x] + \text{Dist}[1/(d*(m+n+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-1)}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rule 3023

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \ \&\& \ \text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx &= \frac{a(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^2}{d(c + d) f (c + d \sin(e + fx))} + \frac{\int \frac{(a + a \sin(e + fx))^2}{(c + d \sin(e + fx))^2} dx}{d(c + d) f} \\
&= \frac{(2Ad - B(3c + d)) \cos(e + fx) (a^3 + a^3 \sin(e + fx))}{2d^2(c + d) f} + \frac{a(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^2}{d(c + d) f (c + d \sin(e + fx))} \\
&= \frac{(2Ad - B(3c + d)) \cos(e + fx) (a^3 + a^3 \sin(e + fx))}{2d^2(c + d) f} + \frac{a(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^2}{d(c + d) f (c + d \sin(e + fx))} \\
&= -\frac{a^3 (4Acd - B(6c^2 - 3cd - 5d^2)) \cos(e + fx)}{2d^3(c + d) f} + \frac{(2Ad - B(3c + d)) \cos(e + fx) (a + a \sin(e + fx))^2}{d(c + d) f (c + d \sin(e + fx))} \\
&= -\frac{a^3 (2A(2c - 3d)d - B(6c^2 - 12cd + 7d^2)) x}{2d^4} - \frac{a^3 (4Acd - B(6c^2 - 3cd - 5d^2)) \cos(e + fx)}{2d^3(c + d) f} \\
&= -\frac{a^3 (2A(2c - 3d)d - B(6c^2 - 12cd + 7d^2)) x}{2d^4} - \frac{a^3 (4Acd - B(6c^2 - 3cd - 5d^2)) \cos(e + fx)}{2d^3(c + d) f} \\
&= -\frac{a^3 (2A(2c - 3d)d - B(6c^2 - 12cd + 7d^2)) x}{2d^4} - \frac{a^3 (4Acd - B(6c^2 - 3cd - 5d^2)) \cos(e + fx)}{2d^3(c + d) f} \\
&= -\frac{a^3 (2A(2c - 3d)d - B(6c^2 - 12cd + 7d^2)) x}{2d^4} - \frac{a^3 (4Acd - B(6c^2 - 3cd - 5d^2)) \cos(e + fx)}{2d^3(c + d) f} + \frac{2a^3(c - d)^2}{2d^4}
\end{aligned}$$

Mathematica [A] time = 1.50, size = 244, normalized size = 0.86

$$a^3(\sin(e + fx) + 1)^3 \left(2(e + fx) (2Ad(3d - 2c) + B(6c^2 - 12cd + 7d^2)) - \frac{8(c-d)^2(B(3c^2+3cd-d^2)-Ad(2c+3d)) \tan^{-1}\left(\frac{c \tan(e+fx)}{(c+d)\sqrt{c^2-d^2}}\right)}{(c+d)\sqrt{c^2-d^2}} \right)$$

$$4d^4 f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] (a^3*(1 + Sin[e + f*x])^3*(2*(2*A*d*(-2*c + 3*d) + B*(6*c^2 - 12*c*d + 7*d^2))*(e + f*x) - (8*(c - d)^2*(-(A*d*(2*c + 3*d)) + B*(3*c^2 + 3*c*d - d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/((c + d)*Sqrt[c^2 - d^2]) - 4*d*(-2*B*c + A*d + 3*B*d)*Cos[e + f*x] + (4*(c - d)^2*d*(B*c - A*d)*Cos[e + f*x])/((c + d)*(c + d*Sin[e + f*x])) - B*d^2*Sin[2*(e + f*x)])/(4*d^4*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)

fricas [A] time = 0.59, size = 1027, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

```
[Out] [1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + (3*B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*sin(f*x + e))*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f), 1/2*((B*a^3*c*d^3 + B*a^3*d^4)*cos(f*x + e)^3 + (6*B*a^3*c^4 - 2*(2*A + 3*B)*a^3*c^3*d + (2*A - 5*B)*a^3*c^2*d^2 + (6*A + 7*B)*a^3*c*d^3)*f*x + 2*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - (A + 4*B)*a^3*c^2*d^2 + (3*A + B)*a^3*c*d^3 + (3*B*a^3*c^3*d - 2*A*a^3*c^2*d^2 - (A + 4*B)*a^3*c*d^3 + (3*A + B)*a^3*d^4)*sin(f*x + e))*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d)))/((c - d)*cos(f*x + e))) + (6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 - (2*A + B)*a^3*d^4)*cos(f*x + e) + ((6*B*a^3*c^3*d - 2*(2*A + 3*B)*a^3*c^2*d^2 + (2*A - 5*B)*a^3*c*d^3 + (6*A + 7*B)*a^3*d^4)*f*x + (3*B*a^3*c^2*d^2 - (2*A + 3*B)*a^3*c*d^3 - 2*(A + 3*B)*a^3*d^4)*cos(f*x + e))*sin(f*x + e))/((c*d^5 + d^6)*f*sin(f*x + e) + (c^2*d^4 + c*d^5)*f)]
```

giac [B] time = 0.21, size = 588, normalized size = 2.08

$$\frac{4(3Ba^3c^4 - 2Aa^3c^3d - 3Ba^3c^3d + Aa^3c^2d^2 - 4Ba^3c^2d^2 + 4Aa^3cd^3 + 5Ba^3cd^3 - 3Aa^3d^4 - Ba^3d^4) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} \frac{fx+e}{2} \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(cd^4 + d^5) \sqrt{c^2 - d^2}} - \frac{4(Ba^3c^4 - 2Aa^3c^3d - 3Ba^3c^3d + Aa^3c^2d^2 - 4Ba^3c^2d^2 + 4Aa^3cd^3 + 5Ba^3cd^3 - 3Aa^3d^4 - Ba^3d^4) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} \frac{fx+e}{2} \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(cd^4 + d^5) \sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(3*B*a^3*c^4 - 2*A*a^3*c^3*d - 3*B*a^3*c^3*d + A*a^3*c^2*d^2 - 4*B*a^3*c^2*d^2 + 4*A*a^3*c*d^3 + 5*B*a^3*c*d^3 - 3*A*a^3*d^4 - B*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c*d^4 + d^5)*sqrt(c^2 - d^2)) - 4*(B*a^3*c^3*d*tan(1/2*f*x + 1/2*e) - A*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) - 2*B*a^3*c^2*d^2*tan(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^3*tan(1/2*f*x + 1/2*e) + B*a^3*c*d^3*tan(1/2*f*x + 1/2*e) - A*a^3*d^4*tan(1/2*f*x + 1/2*e) + B*a^3*c^4 - A*a^3*c^3*d - 2*B*a^3*c^3*d + 2*A*a^3*c^2*d^2 + B*a^3*c^2*d^2 - A*a^3*c*d^3)/((c^2*d^3 + c*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (6*B*a^3*c^2 - 4*A*a^3*c*d - 12*B*a^3*c*d + 6*A*a^3*d^2 + 7*B*a^3*d^2)*(f*x + e)/d^4 - 2*(B*a^3*d*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*d*tan(1/2*f*x + 1/2*e)^2 - 6*B*a^3*d*tan(1/2*f*x + 1/2*e)^2 - B*a^3*d*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c - 2*A*a^3*d - 6*B*a^3*d)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*d^3))/f
```

maple [B] time = 0.56, size = 1534, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)
```

```
[Out] -2*a^3/f*d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*tan(1/2*f*x+1/2*e)*A+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c
```


$$\begin{aligned} &)/(c+d)*c^2*\tan(1/2*f*x+1/2*e)*B-4*a^3/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/ \\ &2*f*x+1/2*e)*d+c)/(c+d)*c*\tan(1/2*f*x+1/2*e)*B+4*a^3/f/d^3/(c+d)/(c^2-d^2)^ \\ &(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c^3-2*a^3/ \\ &f/d^2/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^ \\ &2)^{(1/2)})*A*c^2-8*a^3/f/d/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x \\ &+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c-6*a^3/f/d^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(\\ &1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*c^4*B+6*a^3/f/d^3/(c+d)/(\\ &c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c \\ &^3+8*a^3/f/d^2/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d \\ &)/(c^2-d^2)^{(1/2)})*B*c^2-10*a^3/f/d/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*t \\ &an(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c-2*a^3/f/d/(\tan(1/2*f*x+1/2*e)^2 \\ &*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*c*\tan(1/2*f*x+1/2*e)*A+4*a^3/f/d/(\tan(1/ \\ &2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c+2*a^3/f/d^3/(\tan(1/2*f \\ &*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^3-4*a^3/f/d^2/(\tan(1/2*f* \\ &x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c^2+2*a^3/f/d/(\tan(1/2*f*x+1 \\ &/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+4*a^3/f/d^3/(1+\tan(1/2*f*x+1/ \\ &2*e)^2)^2*B*\tan(1/2*f*x+1/2*e)^2*c-2*a^3/f/d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*ta \\ &n(1/2*f*x+1/2*e)*d+c)/(c+d)*A*c^2-a^3/f/d^2/(1+\tan(1/2*f*x+1/2*e)^2)^2*B*ta \\ &n(1/2*f*x+1/2*e)+4*a^3/f/d^3/(1+\tan(1/2*f*x+1/2*e)^2)^2*B*c-4*a^3/f/d^3*arc \\ &\tan(\tan(1/2*f*x+1/2*e))*A*c+6*a^3/f/d^4*\arctan(\tan(1/2*f*x+1/2*e))*B*c^2-12 \\ &*a^3/f/d^3*\arctan(\tan(1/2*f*x+1/2*e))*B*c+6*a^3/f/(c+d)/(c^2-d^2)^{(1/2)}*arc \\ &\tan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A+2*a^3/f/(c+d)/(c^2- \\ &d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B+4*a^3 \\ &/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*\tan(1/2*f*x+1/2* \\ &e)*A+2*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*\tan(1/ \\ &2*f*x+1/2*e)*B+a^3/f/d^2/(1+\tan(1/2*f*x+1/2*e)^2)^2*B*\tan(1/2*f*x+1/2*e)^3- \\ &2*a^3/f/d^2/(1+\tan(1/2*f*x+1/2*e)^2)^2*A*\tan(1/2*f*x+1/2*e)^2-6*a^3/f/d^2/(\\ &1+\tan(1/2*f*x+1/2*e)^2)^2*B*\tan(1/2*f*x+1/2*e)^2-2*a^3/f/(\tan(1/2*f*x+1/2*e \\ &)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2*a^3/f/d^2/(1+\tan(1/2*f*x+1/2*e)^2 \\ &)^2*A-6*a^3/f/d^2/(1+\tan(1/2*f*x+1/2*e)^2)^2*B+6*a^3/f/d^2*\arctan(\tan(1/2*f \\ &*x+1/2*e))*A+7*a^3/f/d^2*\arctan(\tan(1/2*f*x+1/2*e))*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 23.88, size = 11993, normalized size = 42.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^2,x)

[Out]
$$\begin{aligned} &- ((2*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^ \\ &2 + 3*B*a^3*c^2*d))/(d^3*(c + d)) + (2*\tan(e/2 + (f*x)/2)^4*(A*a^3*d^3 - 3* \\ &B*a^3*c^3 - B*a^3*d^3 - A*a^3*c*d^2 + 2*A*a^3*c^2*d + B*a^3*c*d^2 + 3*B*a^3 \\ &*c^2*d))/(d^3*(c + d)) + (2*\tan(e/2 + (f*x)/2)^2*(2*A*a^3*d^3 - 6*B*a^3*c^3 \\ &+ B*a^3*d^3 - 2*A*a^3*c*d^2 + 4*A*a^3*c^2*d + 5*B*a^3*c*d^2 + 6*B*a^3*c^2* \\ &d))/(d^3*(c + d)) + (4*\tan(e/2 + (f*x)/2)^3*(A*a^3*d^3 - 3*B*a^3*c^3 - A*a^ \\ &3*c*d^2 + 2*A*a^3*c^2*d + 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d))/(c*d^2*(c + d)) + \\ &(\tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^3 - 3*B*a^3*c^3 - 4*A*a^3*c*d^2 + 2*A*a^3 \end{aligned}$$

$$\begin{aligned}
& *c^2*d - 2*B*a^3*c*d^2 + 3*B*a^3*c^2*d)/(c*d^2*(c + d)) + (\tan(e/2 + (f*x) \\
& /2)*(2*A*a^3*d^3 - 9*B*a^3*c^3 + 6*A*a^3*c^2*d + 10*B*a^3*c*d^2 + 9*B*a^3*c \\
& ^2*d))/(c*d^2*(c + d))/(f*(c + 2*d*\tan(e/2 + (f*x)/2) + 3*c*\tan(e/2 + (f*x) \\
&)/2)^2 + 3*c*\tan(e/2 + (f*x)/2)^4 + c*\tan(e/2 + (f*x)/2)^6 + 4*d*\tan(e/2 + \\
& (f*x)/2)^3 + 2*d*\tan(e/2 + (f*x)/2)^5)) - (\operatorname{atan}((((8*(36*A^2*a^6*c^2*d^9 + \\
& 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c \\
& ^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 14 \\
& 4*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^ \\
& 8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88* \\
& A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4)))/(2*c*d^9 + d^10 \\
& + c^2*d^8) + (8*\tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3 \\
& *d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32* \\
& A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c \\
& ^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + \\
& 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c \\
& *d^11 + 88*A*B*a^6*c^2*d^10 - 628*A*B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 5 \\
& 72*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6 \\
& *c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c*d^10 + d^11 + c^2*d^9) + (((((8*(4*c^2 \\
& *d^13 + 8*c^3*d^12 + 4*c^4*d^11)))/(2*c*d^9 + d^10 + c^2*d^8) + (8*\tan(e/2 + \\
& (f*x)/2)*(12*c*d^15 + 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11) \\
&))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - \\
& (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 - (8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^1 \\
& 2 + 4*A*a^3*c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d \\
& ^10 + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (8*\tan(e/2 + (f*x)/2)* \\
& (24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + \\
& 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 \\
& + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^ \\
& 3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4 \\
&)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/ \\
& 2)*1i)/d^4 + (((8*(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4 \\
& *d^7 - 16*A^2*a^6*c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^ \\
& 2*a^6*c^3*d^8 - 59*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d \\
& ^5 - 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B* \\
& a^6*c^3*d^8 - 148*A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 \\
& - 48*A*B*a^6*c^7*d^4))/(2*c*d^9 + d^10 + c^2*d^8) + (8*\tan(e/2 + (f*x)/2)* \\
& (144*A^2*a^6*c^2*d^10 - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2 \\
& *a^6*c^5*d^7 + 32*A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^ \\
& 10 - 299*B^2*a^6*c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B \\
& ^2*a^6*c^6*d^6 + 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9 \\
& *d^3 + 36*A^2*a^6*c*d^11 + 94*B^2*a^6*c*d^11 + 88*A*B*a^6*c^2*d^10 - 628*A* \\
& B*a^6*c^3*d^9 + 208*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6 \\
& *d^6 - 144*A*B*a^6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^11))/(2*c \\
& *d^10 + d^11 + c^2*d^9) + (((8*(12*A*a^3*c*d^12 + 14*B*a^3*c*d^12 + 4*A*a^3 \\
& *c^2*d^11 - 12*A*a^3*c^3*d^10 - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^10 + 6*B*a \\
& ^3*c^5*d^8))/(2*c*d^9 + d^10 + c^2*d^8) + (((8*(4*c^2*d^13 + 8*c^3*d^12 + 4 \\
& *c^4*d^11)))/(2*c*d^9 + d^10 + c^2*d^8) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^15 + \\
& 24*c^2*d^14 + 4*c^3*d^13 - 16*c^4*d^12 - 8*c^5*d^11))/(2*c*d^10 + d^11 + c \\
& ^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1i - (a^3*d*(4*A*c + 12*B* \\
& c)*1i)/2))/d^4 - (8*\tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^13 + 8*B*a^3*c*d^13 - \\
& 8*A*a^3*c^2*d^12 - 40*A*a^3*c^3*d^11 + 8*A*a^3*c^4*d^10 + 16*A*a^3*c^5*d^9 \\
& - 32*B*a^3*c^2*d^12 - 8*B*a^3*c^3*d^11 + 56*B*a^3*c^4*d^10 - 24*B*a^3*c^6*d \\
& ^8))/(2*c*d^10 + d^11 + c^2*d^9))*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7*B)/2)*1 \\
& i - (a^3*d*(4*A*c + 12*B*c)*1i)/2))/d^4)*(B*a^3*c^2*3i + a^3*d^2*(3*A + (7* \\
& B)/2)*1i - (a^3*d*(4*A*c + 12*B*c)*1i)/2)*1i)/d^4)/((16*(132*A^3*a^9*c^3*d^ \\
& 6 - 252*A^3*a^9*c^2*d^7 - 54*B^3*a^9*c^9 + 76*A^3*a^9*c^4*d^5 - 80*A^3*a^9*c \\
& ^5*d^4 + 16*A^3*a^9*c^6*d^3 - 115*B^3*a^9*c^2*d^7 + 350*B^3*a^9*c^3*d^6 - \\
& 537*B^3*a^9*c^4*d^5 + 387*B^3*a^9*c^5*d^4 + 36*B^3*a^9*c^6*d^3 - 297*B^3*a^ \\
& 9*c^7*d^2 + 108*A^3*a^9*c*d^8 + 14*B^3*a^9*c*d^8 + 216*B^3*a^9*c^8*d + 96*A \\
& *B^2*a^9*c*d^8 + 108*A*B^2*a^9*c^8*d + 198*A^2*B*a^9*c*d^8 - 573*A*B^2*a^9*c
\end{aligned}$$

$$\begin{aligned}
& A^3c^3d^{13} + 8B^3c^3d^{13} - 8A^3c^2d^{12} - 40A^3c^3d^{11} + 8A^3c^4d^{10} + 16A^3c^5d^9 - 32B^3c^2d^{12} - 8B^3c^3d^{11} + 56 \\
& *B^3c^4d^{10} - 24B^3c^6d^8)/(2c^3d^{10} + d^{11} + c^2d^9)*(B^3c^2 \\
& *3i + a^3d^2(3A + (7*B)/2)*1i - (a^3d(4A*c + 12*B*c)*1i)/2)/d^4*(B \\
& *a^3c^2*3i + a^3d^2(3A + (7*B)/2)*1i - (a^3d(4A*c + 12*B*c)*1i)/2)/ \\
& /d^4)*(B^3c^2*3i + a^3d^2(3A + (7*B)/2)*1i - (a^3d(4A*c + 12*B*c)* \\
& 1i)/2)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d)^3*(c - d)^3)^(1/2))*((8*(36*A \\
& ^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6*c^5*d \\
& ^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 59*B^2* \\
& a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c^7*d^4 \\
& + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148*A*B*a \\
& ^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7*d^4)) \\
& /((2c^3d^9 + d^{10} + c^2d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2*d^{10} - \\
& 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32*A^2*a \\
& ^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^{10} - 299*B^2*a^6*c^3*d^ \\
& 9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + 156*B^ \\
& 2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6*c*d^{1 \\
& 1} + 94*B^2*a^6*c*d^{11} + 88*A*B*a^6*c^2*d^{10} - 628*A*B*a^6*c^3*d^9 + 208*A*B \\
& *a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^6*c^7* \\
& d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^{11}))/((2c^3d^{10} + d^{11} + c^2d^9) \\
& + (a^3*(-(c + d)^3*(c - d)^3)^(1/2))*((8*tan(e/2 + (f*x)/2)*(24*A^3c^3d^{1 \\
& 3} + 8B^3c^3d^{13} - 8A^3c^2d^{12} - 40A^3c^3d^{11} + 8A^3c^4d^{10} + 16 \\
& *A^3c^5d^9 - 32B^3c^2d^{12} - 8B^3c^3d^{11} + 56B^3c^4d^{10} - 24B^3c^6d^8)))/(2c^3d^{10} + d^{11} + c^2d^9) - (8*(12A^3c^3d^{12} \\
& + 14B^3c^3d^{12} + 4A^3c^2d^{11} - 12A^3c^3d^{10} - 4A^3c^4d^9 \\
& - 20B^3c^3d^{10} + 6B^3c^5d^8))/(2c^3d^9 + d^{10} + c^2d^8) + (a^3* \\
& ((8*(4c^2d^{13} + 8c^3d^{12} + 4c^4d^{11}))/((2c^3d^9 + d^{10} + c^2d^8) + (8 \\
& *tan(e/2 + (f*x)/2)*(12c^3d^{15} + 24c^2d^{14} + 4c^3d^{13} - 16c^4d^{12} - 8 \\
& *c^5d^{11}))/((2c^3d^{10} + d^{11} + c^2d^9))*(-(c + d)^3*(c - d)^3)^(1/2)*(3A* \\
& d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d))/((3c^3d^6 + d^7 + 3c^2d^5 + c^ \\
& 3d^4))*(3A*d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d))/((3c^3d^6 + d^7 + 3 \\
& *c^2d^5 + c^3d^4))*(3A*d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d)*1i)/((3 \\
& *c^3d^6 + d^7 + 3c^2d^5 + c^3d^4) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2))*((8 \\
& *(36*A^2*a^6*c^2*d^9 + 24*A^2*a^6*c^3*d^8 - 44*A^2*a^6*c^4*d^7 - 16*A^2*a^6 \\
& *c^5*d^6 + 16*A^2*a^6*c^6*d^5 + 49*B^2*a^6*c^2*d^9 - 70*B^2*a^6*c^3*d^8 - 5 \\
& 9*B^2*a^6*c^4*d^7 + 144*B^2*a^6*c^5*d^6 - 24*B^2*a^6*c^6*d^5 - 72*B^2*a^6*c \\
& ^7*d^4 + 36*B^2*a^6*c^8*d^3 + 84*A*B*a^6*c^2*d^9 - 32*A*B*a^6*c^3*d^8 - 148 \\
& *A*B*a^6*c^4*d^7 + 88*A*B*a^6*c^5*d^6 + 72*A*B*a^6*c^6*d^5 - 48*A*B*a^6*c^7 \\
& *d^4))/((2c^3d^9 + d^{10} + c^2d^8) + (8*tan(e/2 + (f*x)/2)*(144*A^2*a^6*c^2* \\
& d^{10} - 164*A^2*a^6*c^3*d^9 - 136*A^2*a^6*c^4*d^8 + 136*A^2*a^6*c^5*d^7 + 32 \\
& *A^2*a^6*c^6*d^6 - 32*A^2*a^6*c^7*d^5 - 100*B^2*a^6*c^2*d^{10} - 299*B^2*a^6* \\
& c^3*d^9 + 494*B^2*a^6*c^4*d^8 + 91*B^2*a^6*c^5*d^7 - 504*B^2*a^6*c^6*d^6 + \\
& 156*B^2*a^6*c^7*d^5 + 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 + 36*A^2*a^6 \\
& *c*d^{11} + 94*B^2*a^6*c*d^{11} + 88*A*B*a^6*c^2*d^{10} - 628*A*B*a^6*c^3*d^9 + 2 \\
& 08*A*B*a^6*c^4*d^8 + 572*A*B*a^6*c^5*d^7 - 320*A*B*a^6*c^6*d^6 - 144*A*B*a^ \\
& 6*c^7*d^5 + 96*A*B*a^6*c^8*d^4 + 144*A*B*a^6*c*d^{11}))/((2c^3d^{10} + d^{11} + c^ \\
& 2d^9) + (a^3*(-(c + d)^3*(c - d)^3)^(1/2))*((8*(12A^3c^3d^{12} + 14B^3c^3 \\
& *c^3d^{12} + 4A^3c^2d^{11} - 12A^3c^3d^{10} - 4A^3c^4d^9 - 20B^3c^3 \\
& *c^3d^{10} + 6B^3c^5d^8))/((2c^3d^9 + d^{10} + c^2d^8) - (8*tan(e/2 + (f*x) \\
&)/2)*(24A^3c^3d^{13} + 8B^3c^3d^{13} - 8A^3c^2d^{12} - 40A^3c^3d^{11} + 8A^3c^4d^{10} + 16 \\
& *A^3c^5d^9 - 32B^3c^2d^{12} - 8B^3c^3d^{11} + 56B^3c^4d^{10} - 24B^3c^6d^8)))/(2c^3d^{10} + d^{11} + c^2d^9) + \\
& (a^3*((8*(4c^2d^{13} + 8c^3d^{12} + 4c^4d^{11}))/((2c^3d^9 + d^{10} + c^2d^8) \\
&) + (8*tan(e/2 + (f*x)/2)*(12c^3d^{15} + 24c^2d^{14} + 4c^3d^{13} - 16c^4d^{12} - 8 \\
& *c^5d^{11}))/((2c^3d^{10} + d^{11} + c^2d^9))*(-(c + d)^3*(c - d)^3)^(1/2) \\
& *(3A*d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d))/((3c^3d^6 + d^7 + 3c^2d^5 \\
& + c^3d^4))*(3A*d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d))/((3c^3d^6 + d^ \\
& 7 + 3c^2d^5 + c^3d^4))*(3A*d^2 - 3B*c^2 + B*d^2 + 2A*c*d - 3B*c*d)* \\
& 1i)/((3c^3d^6 + d^7 + 3c^2d^5 + c^3d^4))/((16*(132A^3a^9c^3d^6 - 252*
\end{aligned}$$

$$\begin{aligned}
& 44*A*B*a^6*c*d^{11})/(2*c*d^{10} + d^{11} + c^2*d^9) + (a^3*(-(c + d)^3*(c - d)^3)^{(1/2)}*((8*(12*A*a^3*c*d^{12} + 14*B*a^3*c*d^{12} + 4*A*a^3*c^2*d^{11} - 12*A*a^3*c^3*d^{10} - 4*A*a^3*c^4*d^9 - 20*B*a^3*c^3*d^{10} + 6*B*a^3*c^5*d^8))/(2*c*d^9 + d^{10} + c^2*d^8) - (8*\tan(e/2 + (f*x)/2)*(24*A*a^3*c*d^{13} + 8*B*a^3*c*d^{13} - 8*A*a^3*c^2*d^{12} - 40*A*a^3*c^3*d^{11} + 8*A*a^3*c^4*d^{10} + 16*A*a^3*c^5*d^9 - 32*B*a^3*c^2*d^{12} - 8*B*a^3*c^3*d^{11} + 56*B*a^3*c^4*d^{10} - 24*B*a^3*c^6*d^8))/(2*c*d^{10} + d^{11} + c^2*d^9) + (a^3*((8*(4*c^2*d^{13} + 8*c^3*d^{12} + 4*c^4*d^{11}))/((2*c*d^9 + d^{10} + c^2*d^8) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^{15} + 24*c^2*d^{14} + 4*c^3*d^{13} - 16*c^4*d^{12} - 8*c^5*d^{11}))/((2*c*d^{10} + d^{11} + c^2*d^9)))*(-(c + d)^3*(c - d)^3)^{(1/2)}*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d))/(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))*(3*A*d^2 - 3*B*c^2 + B*d^2 + 2*A*c*d - 3*B*c*d)*2i)/(f*(3*c*d^6 + d^7 + 3*c^2*d^5 + c^3*d^4))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))*2,x)

[Out] Timed out

3.264 $\int \frac{(a+a \sin(e+fx))^3(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$

Optimal. Leaf size=305

$$\frac{(Ad(c+4d) - B(3c^2 + 4cd - 2d^2)) \cos(e+fx) (a^3 \sin(e+fx) + a^3)}{2d^2 f(c+d)^2(c+d \sin(e+fx))} \frac{a^3(c-d) (Ad(2c^2 + 6cd + 7d^2) - 3B)}{d^4 f(c+d)^2 \sqrt{c^2 - d^2}}$$

[Out] $-a^3*(-A*d+3*B*c-3*B*d)*x/d^4-1/2*a^3*(3*B*c*(2*c+3*d)-A*d*(2*c+5*d))*\cos(f*x+e)/d^3/(c+d)^2/f+1/2*a*(-A*d+B*c)*\cos(f*x+e)*(a+a*\sin(f*x+e))^2/d/(c+d)/f/(c+d*\sin(f*x+e))^2-1/2*(A*d*(c+4*d)-B*(3*c^2+4*c*d-2*d^2))*\cos(f*x+e)*(a^3+a^3*\sin(f*x+e))/d^2/(c+d)^2/f/(c+d*\sin(f*x+e))-a^3*(c-d)*(A*d*(2*c^2+6*c*d+7*d^2)-3*B*(2*c^3+4*c^2*d+c*d^2-2*d^3))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/d^4/(c+d)^2/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 0.93, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2975, 2968, 3023, 2735, 2660, 618, 204}

$$\frac{a^3(c-d) (Ad(2c^2 + 6cd + 7d^2) - 3B(4c^2d + 2c^3 + cd^2 - 2d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right) (Ad(c+4d) - B(3c^2 + 4cd - 2d^2))}{d^4 f(c+d)^2 \sqrt{c^2 - d^2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] $-((a^3*(3*B*c - A*d - 3*B*d)*x)/d^4) - (a^3*(c - d)*(A*d*(2*c^2 + 6*c*d + 7*d^2) - 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(d^4*(c + d)^2*\text{Sqrt}[c^2 - d^2]*f) - (a^3*(3*B*c*(2*c + 3*d) - A*d*(2*c + 5*d))*\text{Cos}[e + f*x])/(2*d^3*(c + d)^2*f) + (a*(B*c - A*d)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^2)/(2*d*(c + d)*f*(c + d*\text{Sin}[e + f*x])^2) - ((A*d*(c + 4*d) - B*(3*c^2 + 4*c*d - 2*d^2))*\text{Cos}[e + f*x]*(a^3 + a^3*\text{Sin}[e + f*x]))/(2*d^2*(c + d)^2*f*(c + d*\text{Sin}[e + f*x]))$

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*

$\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2968

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \text{:>} \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2975

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}), x_Symbol] \text{:>} -\text{Simp}[(b^2*(B*c - A*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)})/(d*f*(n + 1)*(b*c + a*d)), x] - \text{Dist}[b/(d*(n + 1)*(b*c + a*d)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m - 1)}*(c + d*\text{Sin}[e + f*x])^{(n + 1)}*\text{Simp}[A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(n + 1)))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rule 3023

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \text{:>} -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 2)), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \ \&\& \ !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^3 (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx &= \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^2}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{\int \frac{(a + a \sin(e + fx))^2}{(c + d \sin(e + fx))^3} dx}{2d(c + d)f(c + d \sin(e + fx))^2} \\
&= \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^2}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{(Ad(c + 4d) - 1)}{2d(c + d)f(c + d \sin(e + fx))^2} \\
&= \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^2}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{(Ad(c + 4d) - 1)}{2d(c + d)f(c + d \sin(e + fx))^2} \\
&= -\frac{a^3(3Bc(2c + 3d) - Ad(2c + 5d)) \cos(e + fx)}{2d^3(c + d)^2 f} + \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)} \\
&= -\frac{a^3(3Bc - Ad - 3Bd)x}{d^4} - \frac{a^3(3Bc(2c + 3d) - Ad(2c + 5d)) \cos(e + fx)}{2d^3(c + d)^2 f} \\
&= -\frac{a^3(3Bc - Ad - 3Bd)x}{d^4} - \frac{a^3(3Bc(2c + 3d) - Ad(2c + 5d)) \cos(e + fx)}{2d^3(c + d)^2 f} \\
&= -\frac{a^3(3Bc - Ad - 3Bd)x}{d^4} - \frac{a^3(3Bc(2c + 3d) - Ad(2c + 5d)) \cos(e + fx)}{2d^3(c + d)^2 f} \\
&= -\frac{a^3(3Bc - Ad - 3Bd)x}{d^4} - \frac{a^3(c - d) (Ad(2c^2 + 6cd + 7d^2) - 3)}{d^4}
\end{aligned}$$

Mathematica [B] time = 3.14, size = 830, normalized size = 2.72

$$a^3(\sin(e + fx) + 1)^3 \left(\frac{4(c-d)(3B(2c^3+4dc^2+d^2c-2d^3)-Ad(2c^2+6dc+7d^2)) \tan^{-1}\left(\frac{d+c \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}} + \frac{-12Bec^5-12Bfxc^5+4Adec^4-12Bdec^4}{d^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + a*Sin[e + f*x])^3*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] (a^3*(1 + Sin[e + f*x])^3*((4*(c - d)*(-(A*d*(2*c^2 + 6*c*d + 7*d^2)) + 3*B*(2*c^3 + 4*c^2*d + c*d^2 - 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^2] + (-12*B*c^5*e + 4*A*c^4*d*e - 12*B*c^4*d*e + 8*A*c^3*d^2*e + 6*B*c^3*d^2*e + 6*A*c^2*d^3*e + 6*B*c^2*d^3*e + 4*A*c*d^4*e + 6*B*c*d^4*e + 2*A*d^5*e + 6*B*d^5*e - 12*B*c^5*f*x + 4*A*c^4*d*f*x - 12*B*c^4*d*f*x + 8*A*c^3*d^2*f*x + 6*B*c^3*d^2*f*x + 6*A*c^2*d^3*f*x + 6*B*c^2*d^3*f*x + 4*A*c*d^4*f*x + 6*B*c*d^4*f*x + 2*A*d^5*f*x + 6*B*d^5*f*x - d*(2*A*d*(-2*c^3 - 4*c^2*d + 5*c*d^2 + d^3) + B*(12*c^4 + 12*c^3*d - 9*c^2*d^2 + 4*c*d^3 + d^4))*Cos[e + f*x] - 2*d^2*(c + d)^2*(-3*B*c + A*d + 3*B*d)*(e + f*x)*Cos[2*(e + f*x)] + B*c^2*d^3*Cos[3*(e + f*x)] + 2*B*c*d^4*Cos[3*(e + f*x)] + B*d^5*Cos[3*(e + f*x)] - 24*B*c^4*d*e*Sin[e + f*x] + 8*A*c^3*d^2*e*Sin[e + f*x] - 24*B*c^3*d^2*e*Sin[e + f*x] + 16*A*c^2*d^3*e*Sin[e + f*x] + 24*B*c^2*d^3*e*Sin[e + f*x] + 8*A*c*d^4*e*Sin[e + f*x] + 24*B*c*d^4*e*Sin[e + f*x] - 24*B*c^4*d*f*x*Sin[e + f*x] + 8*A*c^3*d^2*f*x*Sin[e + f*x] - 24*B*c^3*d^2*f*x*Sin[e + f*x] + 16*A*c^2*d^3*f*x*Sin[e + f*x] + 24*B*c^2*d^3*f*x

$$\frac{\begin{aligned} & * \sin[e + f*x] + 8*A*c*d^4*f*x*\sin[e + f*x] + 24*B*c*d^4*f*x*\sin[e + f*x] - \\ & 9*B*c^3*d^2*\sin[2*(e + f*x)] + 3*A*c^2*d^3*\sin[2*(e + f*x)] - 9*B*c^2*d^3*\sin[2*(e + f*x)] \\ & + 3*A*c*d^4*\sin[2*(e + f*x)] + 4*B*c*d^4*\sin[2*(e + f*x)] - \\ & 6*A*d^5*\sin[2*(e + f*x)] - 2*B*d^5*\sin[2*(e + f*x)] \end{aligned}}{(c + d*\sin[e + f*x])^2} \\ \frac{2)}{(4*d^4*(c + d)^2*f*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^6}$$

fricas [B] time = 0.61, size = 1670, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] [-1/4*(4*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 4*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 4*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e)*sqrt(-(c - d)/(c + d))*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*((c^2 + c*d)*cos(f*x + e)*sin(f*x + e) + (c*d + d^2)*cos(f*x + e))*sqrt(-(c - d)/(c + d)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) - 2*(6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - 2*(4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f), -1/2*(2*(3*B*a^3*c^3*d^2 - (A - 3*B)*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x*cos(f*x + e)^2 + 2*(B*a^3*c^2*d^3 + 2*B*a^3*c*d^4 + B*a^3*d^5)*cos(f*x + e)^3 - 2*(3*B*a^3*c^5 - (A - 3*B)*a^3*c^4*d - 2*A*a^3*c^3*d^2 - 2*A*a^3*c^2*d^3 - (2*A + 3*B)*a^3*c*d^4 - (A + 3*B)*a^3*d^5)*f*x - (6*B*a^3*c^5 - 2*(A - 6*B)*a^3*c^4*d - 3*(2*A - 3*B)*a^3*c^3*d^2 - 3*(3*A - 2*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5 - (6*B*a^3*c^3*d^2 - 2*(A - 6*B)*a^3*c^2*d^3 - 3*(2*A - B)*a^3*c*d^4 - (7*A + 6*B)*a^3*d^5)*cos(f*x + e)^2 + 2*(6*B*a^3*c^4*d - 2*(A - 6*B)*a^3*c^3*d^2 - 3*(2*A - B)*a^3*c^2*d^3 - (7*A + 6*B)*a^3*c*d^4)*sin(f*x + e)*sqrt((c - d)/(c + d))*arctan(-(c*sin(f*x + e) + d)*sqrt((c - d)/(c + d)))/((c - d)*cos(f*x + e))) - (6*B*a^3*c^4*d - 2*(A - 3*B)*a^3*c^3*d^2 - (4*A + 3*B)*a^3*c^2*d^3 + 5*(A + B)*a^3*c*d^4 + (A + 2*B)*a^3*d^5)*cos(f*x + e) - (4*(3*B*a^3*c^4*d - (A - 3*B)*a^3*c^3*d^2 - (2*A + 3*B)*a^3*c^2*d^3 - (A + 3*B)*a^3*c*d^4)*f*x + (9*B*a^3*c^3*d^2 - 3*(A - 3*B)*a^3*c^2*d^3 - (3*A + 4*B)*a^3*c*d^4 + 2*(3*A + B)*a^3*d^5)*cos(f*x + e)*sin(f*x + e))/((c^2*d^6 + 2*c*d^7 + d^8)*f*cos(f*x + e)^2 - 2*(c^3*d^5 + 2*c^2*d^6 + c*d^7)*f*sin(f*x + e) - (c^4*d^4 + 2*c^3*d^5 + 2*c^2*d^6 + 2*c*d^7 + d^8)*f)]

giac [B] time = 0.30, size = 986, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")

```
[Out] ((6*B*a^3*c^4 - 2*A*a^3*c^3*d + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 9*B*a^3*c^2*d^2 - A*a^3*c*d^3 - 9*B*a^3*c*d^3 + 7*A*a^3*d^4 + 6*B*a^3*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((c^2*d^4 + 2*c*d^5 + d^6)*sqrt(c^2 - d^2)) - 2*B*a^3/((tan(1/2*f*x + 1/2*e)^2 + 1)*d^3) - (3*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^3 - A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 + 3*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^3 - 5*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 - 6*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^3 + 4*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^3 + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^3 + 4*B*a^3*c^6*tan(1/2*f*x + 1/2*e)^2 - 2*A*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e)^2 - 4*A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e)^2 + A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 + 5*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e)^2 - 7*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 - 14*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e)^2 + 10*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*B*a^3*c*d^5*tan(1/2*f*x + 1/2*e)^2 + 2*A*a^3*d^6*tan(1/2*f*x + 1/2*e)^2 + 13*B*a^3*c^5*d*tan(1/2*f*x + 1/2*e) - 7*A*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) + 5*B*a^3*c^4*d^2*tan(1/2*f*x + 1/2*e) - 11*A*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) - 22*B*a^3*c^3*d^3*tan(1/2*f*x + 1/2*e) + 16*A*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^2*d^4*tan(1/2*f*x + 1/2*e) + 2*A*a^3*c*d^5*tan(1/2*f*x + 1/2*e) + 4*B*a^3*c^6 - 2*A*a^3*c^5*d + 2*B*a^3*c^5*d - 4*A*a^3*c^4*d^2 - 7*B*a^3*c^4*d^2 + 5*A*a^3*c^3*d^3 + B*a^3*c^3*d^3 + A*a^3*c^2*d^4)/((c^4*d^3 + 2*c^3*d^4 + c^2*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)^2) - (3*B*a^3*c - A*a^3*d - 3*B*a^3*d)*(f*x + e)/d^4)/f
```

maple [B] time = 0.56, size = 2906, normalized size = 9.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -3*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*B+2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*A+4*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*A-10*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^2*A-9*a^3/f/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-4*a^3/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^2-2*a^3/f*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c^2*tan(1/2*f*x+1/2*e)^2*A-4*a^3/f/d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^4*tan(1/2*f*x+1/2*e)^2*B-13*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^3/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B+6*a^3/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^3+a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^3*A-2*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*tan(1/2*f*x+1/2*e)^3*A-9*a^3/f/d^2/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-2*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^2*B-3*a^3/f/d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*tan(1/2*f*x+1/2*e)^3*B-2*a^3/f/d^3/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c^3-5*a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*tan(1/2*f*x+1/2*e)*B-a^3/f/d/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c+6*a^3/f/d^4/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*c^4*B-a^3/f/d/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*tan(1/2*f*x+1/2*e)^2*B-2*a^3/f*d^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)^2/
```

$$\frac{(c^2+2*c*d+d^2)/c*\tan(1/2*f*x+1/2*e)^2*B+7*a^3/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-2*a^3/f*d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A-6*a^3/f/d^4*B*\arctan(\tan(1/2*f*x+1/2*e))*c-a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*A+22*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B-5*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*B+11*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A+5*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^3*A+6*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^3*B-2*a^3/f/d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+7*a^3/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2+2*a^3/f/d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^3-4*a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*A+7*a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^2*A+14*a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^2*B-16*a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-4*a^3/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B+4*a^3/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c^2-4*a^3/f/d^3/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^4*B-a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c-5*a^3/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c+7*a^3/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+6*a^3/f/(c^2+2*c*d+d^2)/(c^2-d^2)^(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B-2*a^3/f/d^3*B/(1+\tan(1/2*f*x+1/2*e)^2)+2*a^3/f/d^3*A*\arctan(\tan(1/2*f*x+1/2*e))+6*a^3/f/d^3*B*\arctan(\tan(1/2*f*x+1/2*e))$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 25.41, size = 13891, normalized size = 45.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^3)/(c + d*sin(e + f*x))^3,x)

[Out] - ((A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2)/(d^3*(c + d)^2) + (4*tan(e/2 + (f*x)/2)^3*(A*a^3*d^4 + 6*B*a^3*c^4 + 5*A*a^3*c*d^3 - 2*A*a^3*c^3*d + B*a^3*c*d^3 + 6*B*a^3*c^3*d - 4*A*a^3*c^2*d^2 - 5*B*a^3*c^2*d^2))/(c*d^2*(c + d)^2) + (tan(e/2 + (f*x)/2)^5*(2*A*a^3*d^4 + 3*B*a^3*c^4 + 4*A*a^3*c*d^3 - A*a^3*c^3*d + 3*B*a^3*c^3*d - 5*A*a^3*c^2*d^2 - 6*B*a^3*c^2*d^2))/(c*d^2*(c + d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*a^3*d^6 + 6*B*a^3*c^6 + 5*A*a^3

$$\begin{aligned}
& *c*d^5 - 2*A*a^3*c^5*d + B*a^3*c*d^5 + 6*B*a^3*c^5*d - 3*A*a^3*c^2*d^4 + 3* \\
& A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 3*B*a^3*c^2*d^4 + 11*B*a^3*c^3*d^3 + 3*B* \\
& a^3*c^4*d^2)/(c^2*d^3*(c + d)^2) + (\tan(e/2 + (f*x)/2)^4*(2*A*a^3*d^6 + 6* \\
& B*a^3*c^6 + 10*A*a^3*c*d^5 - 2*A*a^3*c^5*d + 2*B*a^3*c*d^5 + 6*B*a^3*c^5*d \\
& - 7*A*a^3*c^2*d^4 + A*a^3*c^3*d^3 - 4*A*a^3*c^4*d^2 - 14*B*a^3*c^2*d^4 + 5* \\
& B*a^3*c^3*d^3 + 3*B*a^3*c^4*d^2))/(c^2*d^3*(c + d)^2) + (\tan(e/2 + (f*x)/2) \\
& *(2*A*a^3*d^4 + 21*B*a^3*c^4 + 16*A*a^3*c*d^3 - 7*A*a^3*c^3*d + 4*B*a^3*c*d \\
& ^3 + 21*B*a^3*c^3*d - 11*A*a^3*c^2*d^2 - 14*B*a^3*c^2*d^2))/(c*d^2*(c + d)^ \\
& 2))/(f*(\tan(e/2 + (f*x)/2)^2*(3*c^2 + 4*d^2) + \tan(e/2 + (f*x)/2)^4*(3*c^2 \\
& + 4*d^2) + c^2*\tan(e/2 + (f*x)/2)^6 + c^2 + 8*c*d*\tan(e/2 + (f*x)/2)^3 + 4* \\
& c*d*\tan(e/2 + (f*x)/2)^5 + 4*c*d*\tan(e/2 + (f*x)/2))) - (\operatorname{atan}(((B*a^3*c*3i \\
& - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2 \\
& *a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 \\
& + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^ \\
& 6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + \\
& 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c \\
& ^6*d^5 - 24*A*B*a^6*c^7*d^4)))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c \\
& ^4*d^8) + (8*\tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + \\
& 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c \\
& ^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - \\
& 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^ \\
& 6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 28 \\
& 2*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^ \\
& 6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - \\
& 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) \\
& + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*\tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^1 \\
& 4 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d \\
& ^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^ \\
& 3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^ \\
& 3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A \\
& *a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a \\
& ^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B \\
& *a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c \\
& ^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^ \\
& 13 + 16*c^5*d^12 + 4*c^6*d^11)))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + \\
& c^4*d^8) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + \\
& 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11)))/(4*c*d^12 + d^13 + 6 \\
& *c^2*d^11 + 4*c^3*d^10 + c^4*d^9))* (B*a^3*c*3i - a^3*d*(A + 3*B)*1i))/d^4) \\
& /d^4)*1i)/d^4 + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A^2*a^6*c^2*d^9 + \\
& 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c \\
& ^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144 \\
& *B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8 \\
& *d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A* \\
& B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4)))/(4*c*d^11 + d^12 \\
& + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*\tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2 \\
& *d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A \\
& ^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3* \\
& d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180* \\
& B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d \\
& ^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318* \\
& A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^ \\
& 7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d \\
& ^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c*3i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a \\
& ^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3 \\
& *c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a \\
& ^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2 \\
& *d^10 + 4*c^3*d^9 + c^4*d^8) - (8*\tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24* \\
& B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 3 \\
& 2*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12
\end{aligned}$$

$$\begin{aligned}
& *c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 25 \\
& 2*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - \\
& 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + \\
& 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11)/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + ((B*a^3*c^3*i - a^3*d*(A + 3*B)*1i)*((8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8)))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) - (8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11)))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(B*a^3*c^3*i - a^3*d*(A + 3*B)*1i))/d^4)/d^4)/d^4) * (B*a^3*c^3*i - a^3*d*(A + 3*B)*1i)*2i)/(d^4*f) - (a^3*atan(((a^3*(-(c + d))^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(46*A^2*a^6*c^2*d^10 + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^10 - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^11 + 36*B^2*a^6*c*d^11 + 282*A*B*a^6*c^2*d^10 + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^11))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) + (a^3*(-(c + d))^5*(c - d))^(1/2)*((8*tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^14 + 24*B*a^3*c*d^14 + 52*A*a^3*c^2*d^13 + 4*A*a^3*c^3*d^12 - 44*A*a^3*c^4*d^11 - 32*A*a^3*c^5*d^10 - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^13 - 84*B*a^3*c^3*d^12 - 84*B*a^3*c^4*d^11 + 36*B*a^3*c^5*d^10 + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9) - (8*(4*A*a^3*c*d^13 + 12*B*a^3*c*d^13 + 2*A*a^3*c^2*d^12 - 6*A*a^3*c^3*d^11 - 2*A*a^3*c^4*d^10 + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^12 + 6*B*a^3*c^3*d^11 - 18*B*a^3*c^4*d^10 - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (a^3*(-(c + d))^5*(c - d))^(1/2)*((8*(4*c^2*d^15 + 16*c^3*d^14 + 24*c^4*d^13 + 16*c^5*d^12 + 4*c^6*d^11)))/(4*c*d^11 + d^12 + 6*c^2*d^10 + 4*c^3*d^9 + c^4*d^8) + (8*tan(e/2 + (f*x)/2)*(12*c*d^17 + 48*c^2*d^16 + 64*c^3*d^15 + 16*c^4*d^14 - 36*c^5*d^13 - 32*c^6*d^12 - 8*c^7*d^11)))/(4*c*d^12 + d^13 + 6*c^2*d^11 + 4*c^3*d^10 + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/(2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) + (a^3*(-(c + d))^5*(c - d))^(1/2)*((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5
\end{aligned}$$

$$\begin{aligned}
& - 24* A * B * a^6 * c^7 * d^4) / (4 * c * d^{11} + d^{12} + 6 * c^2 * d^{10} + 4 * c^3 * d^9 + c^4 * d^8) \\
& + (8 * \tan(e/2 + (f * x)/2) * (46 * A^2 * a^6 * c^2 * d^{10} + 99 * A^2 * a^6 * c^3 * d^9 + 36 * A^2 \\
& * a^6 * c^4 * d^8 - 36 * A^2 * a^6 * c^5 * d^7 - 32 * A^2 * a^6 * c^6 * d^6 - 8 * A^2 * a^6 * c^7 * d^5 \\
& + 252 * B^2 * a^6 * c^2 * d^{10} - 81 * B^2 * a^6 * c^3 * d^9 - 594 * B^2 * a^6 * c^4 * d^8 - 81 * B^2 * \\
& a^6 * c^5 * d^7 + 504 * B^2 * a^6 * c^6 * d^6 + 180 * B^2 * a^6 * c^7 * d^5 - 144 * B^2 * a^6 * c^8 * d^4 \\
& - 72 * B^2 * a^6 * c^9 * d^3 - 41 * A^2 * a^6 * c * d^{11} + 36 * B^2 * a^6 * c * d^{11} + 282 * A * B * a \\
& ^6 * c^2 * d^{10} + 228 * A * B * a^6 * c^3 * d^9 - 318 * A * B * a^6 * c^4 * d^8 - 372 * A * B * a^6 * c^5 * d \\
& ^7 + 24 * A * B * a^6 * c^6 * d^6 + 144 * A * B * a^6 * c^7 * d^5 + 48 * A * B * a^6 * c^8 * d^4 - 36 * A * B \\
& * a^6 * c * d^{11})) / (4 * c * d^{12} + d^{13} + 6 * c^2 * d^{11} + 4 * c^3 * d^{10} + c^4 * d^9) + (a^3 * \\
& (- (c + d)^5 * (c - d))^{(1/2)} * ((8 * (4 * A * a^3 * c * d^{13} + 12 * B * a^3 * c * d^{13} + 2 * A * a^3 * \\
& c^2 * d^{12} - 6 * A * a^3 * c^3 * d^{11} - 2 * A * a^3 * c^4 * d^{10} + 2 * A * a^3 * c^5 * d^9 + 24 * B * a^3 \\
& * c^2 * d^{12} + 6 * B * a^3 * c^3 * d^{11} - 18 * B * a^3 * c^4 * d^{10} - 18 * B * a^3 * c^5 * d^9 - 6 * B * a \\
& ^3 * c^6 * d^8))) / (4 * c * d^{11} + d^{12} + 6 * c^2 * d^{10} + 4 * c^3 * d^9 + c^4 * d^8) - (8 * \tan(\\
& e/2 + (f * x)/2) * (28 * A * a^3 * c * d^{14} + 24 * B * a^3 * c * d^{14} + 52 * A * a^3 * c^2 * d^{13} + 4 * A \\
& * a^3 * c^3 * d^{12} - 44 * A * a^3 * c^4 * d^{11} - 32 * A * a^3 * c^5 * d^{10} - 8 * A * a^3 * c^6 * d^9 + 1 \\
& 2 * B * a^3 * c^2 * d^{13} - 84 * B * a^3 * c^3 * d^{12} - 84 * B * a^3 * c^4 * d^{11} + 36 * B * a^3 * c^5 * d^{1 \\
& 0} + 72 * B * a^3 * c^6 * d^9 + 24 * B * a^3 * c^7 * d^8))) / (4 * c * d^{12} + d^{13} + 6 * c^2 * d^{11} + 4 \\
& * c^3 * d^{10} + c^4 * d^9) + (a^3 * (- (c + d)^5 * (c - d))^{(1/2)} * ((8 * (4 * c^2 * d^{15} + 16 \\
& * c^3 * d^{14} + 24 * c^4 * d^{13} + 16 * c^5 * d^{12} + 4 * c^6 * d^{11}))) / (4 * c * d^{11} + d^{12} + 6 * c \\
& ^2 * d^{10} + 4 * c^3 * d^9 + c^4 * d^8) + (8 * \tan(e/2 + (f * x)/2) * (12 * c * d^{17} + 48 * c^2 * \\
& d^{16} + 64 * c^3 * d^{15} + 16 * c^4 * d^{14} - 36 * c^5 * d^{13} - 32 * c^6 * d^{12} - 8 * c^7 * d^{11})) \\
& / (4 * c * d^{12} + d^{13} + 6 * c^2 * d^{11} + 4 * c^3 * d^{10} + c^4 * d^9)) * (7 * A * d^3 - 6 * B * c^3 \\
& + 6 * B * d^3 + 6 * A * c * d^2 + 2 * A * c^2 * d - 3 * B * c * d^2 - 12 * B * c^2 * d)) / (2 * (5 * c * d^8 + \\
& d^9 + 10 * c^2 * d^7 + 10 * c^3 * d^6 + 5 * c^4 * d^5 + c^5 * d^4)) * (7 * A * d^3 - 6 * B * c^3 + \\
& 6 * B * d^3 + 6 * A * c * d^2 + 2 * A * c^2 * d - 3 * B * c * d^2 - 12 * B * c^2 * d)) / (2 * (5 * c * d^8 + \\
& d^9 + 10 * c^2 * d^7 + 10 * c^3 * d^6 + 5 * c^4 * d^5 + c^5 * d^4)) * (7 * A * d^3 - 6 * B * c^3 + \\
& 6 * B * d^3 + 6 * A * c * d^2 + 2 * A * c^2 * d - 3 * B * c * d^2 - 12 * B * c^2 * d) * i) / (2 * (5 * c * d^8 + \\
& d^9 + 10 * c^2 * d^7 + 10 * c^3 * d^6 + 5 * c^4 * d^5 + c^5 * d^4)) / ((16 * (54 * B^3 * a^9 * c^ \\
& 8 - 29 * A^3 * a^9 * c^3 * d^5 - 18 * A^3 * a^9 * c^4 * d^4 - 2 * A^3 * a^9 * c^5 * d^3 - 324 * B^3 * a \\
& ^9 * c^2 * d^6 + 81 * B^3 * a^9 * c^3 * d^5 + 405 * B^3 * a^9 * c^4 * d^4 - 135 * B^3 * a^9 * c^5 * d^3 \\
& - 243 * B^3 * a^9 * c^6 * d^2 + 49 * A^3 * a^9 * c * d^7 + 108 * B^3 * a^9 * c * d^7 + 54 * B^3 * a^9 * \\
& c^7 * d + 288 * A * B^2 * a^9 * c * d^7 - 54 * A * B^2 * a^9 * c^7 * d + 231 * A^2 * B * a^9 * c * d^7 - 57 \\
& 6 * A * B^2 * a^9 * c^2 * d^6 - 135 * A * B^2 * a^9 * c^3 * d^5 + 540 * A * B^2 * a^9 * c^4 * d^4 + 135 * A \\
& * B^2 * a^9 * c^5 * d^3 - 198 * A * B^2 * a^9 * c^6 * d^2 - 231 * A^2 * B * a^9 * c^2 * d^6 - 201 * A^2 * \\
& B * a^9 * c^3 * d^5 + 69 * A^2 * B * a^9 * c^4 * d^4 + 114 * A^2 * B * a^9 * c^5 * d^3 + 18 * A^2 * B * a^9 \\
& * c^6 * d^2)) / (4 * c * d^{11} + d^{12} + 6 * c^2 * d^{10} + 4 * c^3 * d^9 + c^4 * d^8) + (16 * \tan(e \\
& /2 + (f * x)/2) * (216 * B^3 * a^9 * c^9 + 52 * A^3 * a^9 * c^2 * d^7 + 4 * A^3 * a^9 * c^3 * d^6 - 4 \\
& 4 * A^3 * a^9 * c^4 * d^5 - 32 * A^3 * a^9 * c^5 * d^4 - 8 * A^3 * a^9 * c^6 * d^3 - 324 * B^3 * a^9 * c^ \\
& 2 * d^7 - 756 * B^3 * a^9 * c^3 * d^6 + 864 * B^3 * a^9 * c^4 * d^5 + 1080 * B^3 * a^9 * c^5 * d^4 - \\
& 756 * B^3 * a^9 * c^6 * d^3 - 756 * B^3 * a^9 * c^7 * d^2 + 28 * A^3 * a^9 * c * d^8 + 216 * B^3 * a^9 * \\
& c * d^8 + 216 * B^3 * a^9 * c^8 * d + 396 * A * B^2 * a^9 * c * d^8 - 216 * A * B^2 * a^9 * c^8 * d + 192 \\
& * A^2 * B * a^9 * c * d^8 - 108 * A * B^2 * a^9 * c^2 * d^7 - 1224 * A * B^2 * a^9 * c^3 * d^6 + 1260 * A * \\
& B^2 * a^9 * c^5 * d^4 + 324 * A * B^2 * a^9 * c^6 * d^3 - 432 * A * B^2 * a^9 * c^7 * d^2 + 156 * A^2 * B \\
& * a^9 * c^2 * d^7 - 372 * A^2 * B * a^9 * c^3 * d^6 - 372 * A^2 * B * a^9 * c^4 * d^5 + 108 * A^2 * B * a^ \\
& 9 * c^5 * d^4 + 216 * A^2 * B * a^9 * c^6 * d^3 + 72 * A^2 * B * a^9 * c^7 * d^2)) / (4 * c * d^{12} + d^{13} \\
& + 6 * c^2 * d^{11} + 4 * c^3 * d^{10} + c^4 * d^9) + (a^3 * (- (c + d)^5 * (c - d))^{(1/2)} * ((8 \\
& * (4 * A^2 * a^6 * c^2 * d^9 + 16 * A^2 * a^6 * c^3 * d^8 + 24 * A^2 * a^6 * c^4 * d^7 + 16 * A^2 * a^6 * \\
& c^5 * d^6 + 4 * A^2 * a^6 * c^6 * d^5 + 36 * B^2 * a^6 * c^2 * d^9 + 72 * B^2 * a^6 * c^3 * d^8 - 36 * \\
& B^2 * a^6 * c^4 * d^7 - 144 * B^2 * a^6 * c^5 * d^6 - 36 * B^2 * a^6 * c^6 * d^5 + 72 * B^2 * a^6 * c^7 \\
& * d^4 + 36 * B^2 * a^6 * c^8 * d^3 + 24 * A * B * a^6 * c^2 * d^9 + 72 * A * B * a^6 * c^3 * d^8 + 48 * A * \\
& B * a^6 * c^4 * d^7 - 48 * A * B * a^6 * c^5 * d^6 - 72 * A * B * a^6 * c^6 * d^5 - 24 * A * B * a^6 * c^7 * d^ \\
& 4)) / (4 * c * d^{11} + d^{12} + 6 * c^2 * d^{10} + 4 * c^3 * d^9 + c^4 * d^8) + (8 * \tan(e/2 + (f * \\
& x)/2) * (46 * A^2 * a^6 * c^2 * d^{10} + 99 * A^2 * a^6 * c^3 * d^9 + 36 * A^2 * a^6 * c^4 * d^8 - 36 * A \\
& ^2 * a^6 * c^5 * d^7 - 32 * A^2 * a^6 * c^6 * d^6 - 8 * A^2 * a^6 * c^7 * d^5 + 252 * B^2 * a^6 * c^2 * d \\
& ^{10} - 81 * B^2 * a^6 * c^3 * d^9 - 594 * B^2 * a^6 * c^4 * d^8 - 81 * B^2 * a^6 * c^5 * d^7 + 504 * B \\
& ^2 * a^6 * c^6 * d^6 + 180 * B^2 * a^6 * c^7 * d^5 - 144 * B^2 * a^6 * c^8 * d^4 - 72 * B^2 * a^6 * c^9 \\
& * d^3 - 41 * A^2 * a^6 * c * d^{11} + 36 * B^2 * a^6 * c * d^{11} + 282 * A * B * a^6 * c^2 * d^{10} + 228 * A \\
& * B * a^6 * c^3 * d^9 - 318 * A * B * a^6 * c^4 * d^8 - 372 * A * B * a^6 * c^5 * d^7 + 24 * A * B * a^6 * c^6 \\
& * d^6 + 144 * A * B * a^6 * c^7 * d^5 + 48 * A * B * a^6 * c^8 * d^4 - 36 * A * B * a^6 * c * d^{11})) / (4 * c *
\end{aligned}$$

$$\begin{aligned}
& d^{12} + d^{13} + 6c^2d^{11} + 4c^3d^{10} + c^4d^9) + (a^3(-(c+d)^5(c-d))^{1/2} * ((8*\tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^{14} + 24*B*a^3*c*d^{14} + 52*A*a^3*c^2*d^{13} + 4*A*a^3*c^3*d^{12} - 44*A*a^3*c^4*d^{11} - 32*A*a^3*c^5*d^{10} - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^{13} - 84*B*a^3*c^3*d^{12} - 84*B*a^3*c^4*d^{11} + 36*B*a^3*c^5*d^{10} + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^{12} + d^{13} + 6*c^2*d^{11} + 4*c^3*d^{10} + c^4*d^9) - (8*(4*A*a^3*c*d^{13} + 12*B*a^3*c*d^{13} + 2*A*a^3*c^2*d^{12} - 6*A*a^3*c^3*d^{11} - 2*A*a^3*c^4*d^{10} + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^{12} + 6*B*a^3*c^3*d^{11} - 18*B*a^3*c^4*d^{10} - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^{11} + d^{12} + 6*c^2*d^{10} + 4*c^3*d^9 + c^4*d^8) + (a^3(-(c+d)^5(c-d))^{1/2} * ((8*(4*c^2*d^{15} + 16*c^3*d^{14} + 24*c^4*d^{13} + 16*c^5*d^{12} + 4*c^6*d^{11}))/((4*c*d^{11} + d^{12} + 6*c^2*d^{10} + 4*c^3*d^9 + c^4*d^8) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^{17} + 48*c^2*d^{16} + 64*c^3*d^{15} + 16*c^4*d^{14} - 36*c^5*d^{13} - 32*c^6*d^{12} - 8*c^7*d^{11}))/((4*c*d^{12} + d^{13} + 6*c^2*d^{11} + 4*c^3*d^{10} + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/((2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/((2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)) - (a^3(-(c+d)^5(c-d))^{1/2} * ((8*(4*A^2*a^6*c^2*d^9 + 16*A^2*a^6*c^3*d^8 + 24*A^2*a^6*c^4*d^7 + 16*A^2*a^6*c^5*d^6 + 4*A^2*a^6*c^6*d^5 + 36*B^2*a^6*c^2*d^9 + 72*B^2*a^6*c^3*d^8 - 36*B^2*a^6*c^4*d^7 - 144*B^2*a^6*c^5*d^6 - 36*B^2*a^6*c^6*d^5 + 72*B^2*a^6*c^7*d^4 + 36*B^2*a^6*c^8*d^3 + 24*A*B*a^6*c^2*d^9 + 72*A*B*a^6*c^3*d^8 + 48*A*B*a^6*c^4*d^7 - 48*A*B*a^6*c^5*d^6 - 72*A*B*a^6*c^6*d^5 - 24*A*B*a^6*c^7*d^4)))/(4*c*d^{11} + d^{12} + 6*c^2*d^{10} + 4*c^3*d^9 + c^4*d^8) + (8*\tan(e/2 + (f*x)/2) * (46*A^2*a^6*c^2*d^{10} + 99*A^2*a^6*c^3*d^9 + 36*A^2*a^6*c^4*d^8 - 36*A^2*a^6*c^5*d^7 - 32*A^2*a^6*c^6*d^6 - 8*A^2*a^6*c^7*d^5 + 252*B^2*a^6*c^2*d^{10} - 81*B^2*a^6*c^3*d^9 - 594*B^2*a^6*c^4*d^8 - 81*B^2*a^6*c^5*d^7 + 504*B^2*a^6*c^6*d^6 + 180*B^2*a^6*c^7*d^5 - 144*B^2*a^6*c^8*d^4 - 72*B^2*a^6*c^9*d^3 - 41*A^2*a^6*c*d^{11} + 36*B^2*a^6*c*d^{11} + 282*A*B*a^6*c^2*d^{10} + 228*A*B*a^6*c^3*d^9 - 318*A*B*a^6*c^4*d^8 - 372*A*B*a^6*c^5*d^7 + 24*A*B*a^6*c^6*d^6 + 144*A*B*a^6*c^7*d^5 + 48*A*B*a^6*c^8*d^4 - 36*A*B*a^6*c*d^{11}))/((4*c*d^{12} + d^{13} + 6*c^2*d^{11} + 4*c^3*d^{10} + c^4*d^9) + (a^3(-(c+d)^5(c-d))^{1/2} * ((8*(4*A*a^3*c*d^{13} + 12*B*a^3*c*d^{13} + 2*A*a^3*c^2*d^{12} - 6*A*a^3*c^3*d^{11} - 2*A*a^3*c^4*d^{10} + 2*A*a^3*c^5*d^9 + 24*B*a^3*c^2*d^{12} + 6*B*a^3*c^3*d^{11} - 18*B*a^3*c^4*d^{10} - 18*B*a^3*c^5*d^9 - 6*B*a^3*c^6*d^8))/(4*c*d^{11} + d^{12} + 6*c^2*d^{10} + 4*c^3*d^9 + c^4*d^8) - (8*\tan(e/2 + (f*x)/2)*(28*A*a^3*c*d^{14} + 24*B*a^3*c*d^{14} + 52*A*a^3*c^2*d^{13} + 4*A*a^3*c^3*d^{12} - 44*A*a^3*c^4*d^{11} - 32*A*a^3*c^5*d^{10} - 8*A*a^3*c^6*d^9 + 12*B*a^3*c^2*d^{13} - 84*B*a^3*c^3*d^{12} - 84*B*a^3*c^4*d^{11} + 36*B*a^3*c^5*d^{10} + 72*B*a^3*c^6*d^9 + 24*B*a^3*c^7*d^8))/(4*c*d^{12} + d^{13} + 6*c^2*d^{11} + 4*c^3*d^{10} + c^4*d^9) + (a^3(-(c+d)^5(c-d))^{1/2} * ((8*(4*c^2*d^{15} + 16*c^3*d^{14} + 24*c^4*d^{13} + 16*c^5*d^{12} + 4*c^6*d^{11}))/((4*c*d^{11} + d^{12} + 6*c^2*d^{10} + 4*c^3*d^9 + c^4*d^8) + (8*\tan(e/2 + (f*x)/2)*(12*c*d^{17} + 48*c^2*d^{16} + 64*c^3*d^{15} + 16*c^4*d^{14} - 36*c^5*d^{13} - 32*c^6*d^{12} - 8*c^7*d^{11}))/((4*c*d^{12} + d^{13} + 6*c^2*d^{11} + 4*c^3*d^{10} + c^4*d^9))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/((2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d))/((2*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4)))*(-(c+d)^5(c-d))^{1/2} * (7*A*d^3 - 6*B*c^3 + 6*B*d^3 + 6*A*c*d^2 + 2*A*c^2*d - 3*B*c*d^2 - 12*B*c^2*d)*i)/(f*(5*c*d^8 + d^9 + 10*c^2*d^7 + 10*c^3*d^6 + 5*c^4*d^5 + c^5*d^4))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**3*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.265 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=220

$$\frac{2d(3A(c^2 - 3cd + d^2) - B(7c^2 - 9cd + 4d^2)) \cos(e + fx)}{3af} + \frac{x(3Ad(2c^2 - 2cd + d^2) + B(2c^3 - 6c^2d + 9cd^2 - 3d^3)) \sin(e + fx)}{2a}$$

[Out] 1/2*(3*A*d*(2*c^2-2*c*d+d^2)+B*(2*c^3-6*c^2*d+9*c*d^2-3*d^3))*x/a+2/3*d*(3*A*(c^2-3*c*d+d^2)-B*(7*c^2-9*c*d+4*d^2))*cos(f*x+e)/a/f+1/6*d^2*(6*A*c-9*A*d-11*B*c+9*B*d)*cos(f*x+e)*sin(f*x+e)/a/f+1/3*(3*A-4*B)*d*cos(f*x+e)*(c+d*sin(f*x+e))^2/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))

Rubi [A] time = 0.36, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2977, 2753, 2734}

$$\frac{2d(3A(c^2 - 3cd + d^2) - B(7c^2 - 9cd + 4d^2)) \cos(e + fx)}{3af} + \frac{x(3Ad(2c^2 - 2cd + d^2) + B(-6c^2d + 2c^3 + 9cd^2 - 3d^3)) \sin(e + fx)}{2a}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]

[Out] ((3*A*d*(2*c^2 - 2*c*d + d^2) + B*(2*c^3 - 6*c^2*d + 9*c*d^2 - 3*d^3))*x)/(2*a) + (2*d*(3*A*(c^2 - 3*c*d + d^2) - B*(7*c^2 - 9*c*d + 4*d^2))*Cos[e + f*x])/(3*a*f) + (d^2*(6*A*c - 11*B*c - 9*A*d + 9*B*d)*Cos[e + f*x]*Sin[e + f*x])/(6*a*f) + ((3*A - 4*B)*d*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(f*(a + a*Sin[e + f*x]))

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2753

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[1/(m + 1), Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && IntegerQ[2*m]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_)], x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{a + a \sin(e + fx)} dx = -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{f(a + a \sin(e + fx))} + \frac{\int (c + d \sin(e + fx))}{f(a + a \sin(e + fx))} dx$$

$$= \frac{(3A - 4B)d \cos(e + fx)(c + d \sin(e + fx))^2}{3af} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{f(a + a \sin(e + fx))}$$

$$= \frac{(3Ad(2c^2 - 2cd + d^2) + B(2c^3 - 6c^2d + 9cd^2 - 3d^3))x}{2a} + \frac{2d(3c^2 - 2cd + d^2)}{2a}$$

Mathematica [B] time = 1.27, size = 788, normalized size = 3.58

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(9d(Ad(d - 4c) + B(-4c^2 + 3cd - 2d^2)) \cos\left(\frac{3}{2}(e + fx)\right) + 3 \cos\left(\frac{1}{2}(e + fx)\right)\right)}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x]),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(3*(4*A*d*(6*c^2*(e + f*x) - 3*c*d*(1 + 2*e + 2*f*x) + d^2*(1 + 3*e + 3*f*x)) + B*(8*c^3*(e + f*x) - 12*c^2*d*(1 + 2*e + 2*f*x) + 12*c*d^2*(1 + 3*e + 3*f*x) - d^3*(7 + 12*e + 12*f*x)))*Cos[(e + f*x)/2] + 9*d*(A*d*(-4*c + d) + B*(-4*c^2 + 3*c*d - 2*d^2))*Cos[(3*(e + f*x))/2] + 9*B*c*d^2*Cos[(5*(e + f*x))/2] + 3*A*d^3*Cos[(5*(e + f*x))/2] - 2*B*d^3*Cos[(5*(e + f*x))/2] + B*d^3*Cos[(7*(e + f*x))/2] + 48*A*c^3*Sin[(e + f*x)/2] - 48*B*c^3*Sin[(e + f*x)/2] - 144*A*c^2*d*Sin[(e + f*x)/2] + 180*B*c^2*d*Sin[(e + f*x)/2] + 180*A*c*d^2*Sin[(e + f*x)/2] - 180*B*c*d^2*Sin[(e + f*x)/2] - 60*A*d^3*Sin[(e + f*x)/2] + 69*B*d^3*Sin[(e + f*x)/2] + 24*B*c^3*e*Sin[(e + f*x)/2] + 72*A*c^2*d*e*Sin[(e + f*x)/2] - 72*B*c^2*d*e*Sin[(e + f*x)/2] - 72*A*c*d^2*e*Sin[(e + f*x)/2] + 108*B*c*d^2*e*Sin[(e + f*x)/2] + 36*A*d^3*e*Sin[(e + f*x)/2] - 36*B*d^3*e*Sin[(e + f*x)/2] + 24*B*c^3*f*x*Sin[(e + f*x)/2] + 72*A*c^2*d*f*x*Sin[(e + f*x)/2] - 72*B*c^2*d*f*x*Sin[(e + f*x)/2] - 72*A*c*d^2*f*x*Sin[(e + f*x)/2] + 108*B*c*d^2*f*x*Sin[(e + f*x)/2] + 36*A*d^3*f*x*Sin[(e + f*x)/2] - 36*B*d^3*f*x*Sin[(e + f*x)/2] - 36*B*c^2*d*Sin[(3*(e + f*x))/2] - 36*A*c*d^2*Sin[(3*(e + f*x))/2] + 27*B*c*d^2*Sin[(3*(e + f*x))/2] + 9*A*d^3*Sin[(3*(e + f*x))/2] - 18*B*d^3*Sin[(3*(e + f*x))/2] - 9*B*c*d^2*Sin[(5*(e + f*x))/2] - 3*A*d^3*Sin[(5*(e + f*x))/2] + 2*B*d^3*Sin[(5*(e + f*x))/2] + B*d^3*Sin[(7*(e + f*x))/2]))/(24*a*f*(1 + Sin[e + f*x]))

fricas [B] time = 0.48, size = 470, normalized size = 2.14

$$2Bd^3 \cos(fx + e)^4 - 6(A - B)c^3 + 18(A - B)c^2d - 18(A - B)cd^2 + 6(A - B)d^3 + (9Bcd^2 + (3A - B)d^3) \cos(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/6*(2*B*d^3*cos(f*x + e)^4 - 6*(A - B)*c^3 + 18*(A - B)*c^2*d - 18*(A - B)*c*d^2 + 6*(A - B)*d^3 + (9*B*c*d^2 + (3*A - B)*d^3)*cos(f*x + e)^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 6*(3*B*c^2*d + 3*(A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*(2*(A - B)*c^3 - 6*(A - 2*B)*c^2*d + 3*(4*A - 3*B)*c*d^2 - (3*A - 5*B)*d^3 - (2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x)*cos(f*x + e) + (

$$2*B*d^3*\cos(f*x + e)^3 + 6*(A - B)*c^3 - 18*(A - B)*c^2*d + 18*(A - B)*c*d^2 - 6*(A - B)*d^3 + 3*(2*B*c^3 + 6*(A - B)*c^2*d - 3*(2*A - 3*B)*c*d^2 + 3*(A - B)*d^3)*f*x - 3*(3*B*c*d^2 + (A - B)*d^3)*\cos(f*x + e)^2 - 3*(6*B*c^2*d + 3*(2*A - B)*c*d^2 - (A - 3*B)*d^3)*\cos(f*x + e))*\sin(f*x + e))/(a*f*\cos(f*x + e) + a*f*\sin(f*x + e) + a*f)$$

giac [B] time = 0.20, size = 479, normalized size = 2.18

$$\frac{3(2Bc^3+6Ac^2d-6Bc^2d-6Acd^2+9Bcd^2+3Ad^3-3Bd^3)(fx+e)}{a} - \frac{12(Ac^3-Bc^3-3Ac^2d+3Bc^2d+3Acd^2-3Bcd^2-Ad^3+Bd^3)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} + \frac{2\left(9Bcd^2\tan\left(\frac{1}{2}f\right)\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] 1/6*(3*(2*B*c^3 + 6*A*c^2*d - 6*B*c^2*d - 6*A*c*d^2 + 9*B*c*d^2 + 3*A*d^3 - 3*B*d^3)*(f*x + e)/a - 12*(A*c^3 - B*c^3 - 3*A*c^2*d + 3*B*c^2*d + 3*A*c*d^2 - 3*B*c*d^2 - A*d^3 + B*d^3)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(9*B*c*d^2*tan(1/2*f*x + 1/2*e)^5 + 3*A*d^3*tan(1/2*f*x + 1/2*e)^5 - 3*B*d^3*tan(1/2*f*x + 1/2*e)^5 - 18*B*c^2*d*tan(1/2*f*x + 1/2*e)^4 - 18*A*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 18*B*c*d^2*tan(1/2*f*x + 1/2*e)^4 + 6*A*d^3*tan(1/2*f*x + 1/2*e)^4 - 6*B*d^3*tan(1/2*f*x + 1/2*e)^4 - 36*B*c^2*d*tan(1/2*f*x + 1/2*e)^2 - 36*A*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 36*B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + 12*A*d^3*tan(1/2*f*x + 1/2*e)^2 - 24*B*d^3*tan(1/2*f*x + 1/2*e)^2 - 9*B*c*d^2*tan(1/2*f*x + 1/2*e) - 3*A*d^3*tan(1/2*f*x + 1/2*e) + 3*B*d^3*tan(1/2*f*x + 1/2*e) - 18*B*c^2*d - 18*A*c*d^2 + 18*B*c*d^2 + 6*A*d^3 - 10*B*d^3)/((tan(1/2*f*x + 1/2*e)^2 + 1)^3*a))/f

maple [B] time = 0.44, size = 1110, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x)

[Out] -12/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^2*c^2*d+12/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^2*c*d^2-3/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)*B*c*d^2+3/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)^5*B*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*A*tan(1/2*f*x+1/2*e)^4*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^4*c^2*d+6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^4*c*d^2-12/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*A*tan(1/2*f*x+1/2*e)^2*c*d^2+1/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)*B*d^3-6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*A*c*d^2-6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*c^2*d+6/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*c*d^2+6/a/f*arctan(tan(1/2*f*x+1/2*e))*A*c^2*d-6/a/f*arctan(tan(1/2*f*x+1/2*e))*A*c*d^2-6/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c^2*d+9/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c*d^2+6/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2*d-6/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c*d^2-6/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c^2*d+6/a/f/(tan(1/2*f*x+1/2*e)+1)*B*c*d^2+1/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)^5*A*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)^5*B*d^3+2/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*A*tan(1/2*f*x+1/2*e)^4*d^3-2/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^4*d^3+4/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*A*tan(1/2*f*x+1/2*e)^2*d^3-8/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*tan(1/2*f*x+1/2*e)^2*d^3-1/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*tan(1/2*f*x+1/2*e)*A*d^3-10/3/a/f/(1+tan(1/2*f*x+1/2*e))^2)^3*B*d^3+3/a/f*arctan(tan(1/2*f*x+1/2*e))*A*d^3+2/a/f*arctan(tan(1/2*f*x+1/2*e))*B*c^3-3/a/f*arctan(tan(1/2*f*x+1/2*e))*B*d^3-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*c^3+2/a/f/(tan(1/2*f*x+1/2*e)+1)*A*d^3+2/a/f/(1+tan

$(1/2*f*x+1/2*e)^2)^3*A*d^3+2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*c^3-2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*d^3$

maxima [B] time = 0.73, size = 1124, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out]
$$-1/3*(B*d^3*((7*\sin(f*x + e)/(\cos(f*x + e) + 1) + 39*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 24*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 24*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 9*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 9*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 16)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + 3*a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3*a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*a*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 3*a*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + a*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 9*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - 9*B*c*d^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + 2*a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2*a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + a*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - 3*A*d^3*((\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + 2*a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2*a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + a*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) + 18*B*c^2*d*((\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) + 18*A*c*d^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - 6*B*c^3*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) - 18*A*c^2*d*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) + 6*A*c^3/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1)))/f$$

mupad [B] time = 14.05, size = 839, normalized size = 3.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3/(a + a*sin(e + f*x)),x)

[Out]
$$-(12*A*c^3*\cos(e/2 + (f*x)/2) - 18*A*d^3*\cos(e/2 + (f*x)/2) - 12*B*c^3*\cos(e/2 + (f*x)/2) + 18*B*d^3*\cos(e/2 + (f*x)/2) + 6*A*d^3*\cos(e/2 + (f*x)/2)^3 - 12*A*d^3*\cos(e/2 + (f*x)/2)^5 - 6*B*d^3*\cos(e/2 + (f*x)/2)^3 + 36*B*d^3*\cos(e/2 + (f*x)/2)^5 - 16*B*d^3*\cos(e/2 + (f*x)/2)^7 - 9*A*d^3*\cos(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*\cos(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*\cos(e/2 + (f*x)/2)*(e + f*x) - 9*A*d^3*\sin(e/2 + (f*x)/2)*(e + f*x) - 6*B*c^3*\sin(e/2 + (f*x)/2)*(e + f*x) + 9*B*d^3*\sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*d^3*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2) + 12*A*d^3*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x)/2) + 18*B*d^3*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2) + 12*B*d^3*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x)/2) - 16*B*d^3*\cos(e/2 + (f*x)/2)^6*\sin(e/2 + (f*x)/2) + 36*A*c*d^2*\cos(e/2 + (f*x)/2) - 36*A*c^2*d*\cos(e/2 + (f$$

$$\begin{aligned} & x)/2) - 54*B*c*d^2*\cos(e/2 + (f*x)/2) + 36*B*c^2*d*\cos(e/2 + (f*x)/2) + 36* \\ & A*c*d^2*\cos(e/2 + (f*x)/2)^3 + 18*B*c*d^2*\cos(e/2 + (f*x)/2)^3 + 36*B*c^2*d \\ & *\cos(e/2 + (f*x)/2)^3 - 36*B*c*d^2*\cos(e/2 + (f*x)/2)^5 + 18*A*c*d^2*\cos(e/ \\ & 2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*\cos(e/2 + (f*x)/2)*(e + f*x) - 27*B*c*d \\ & ^2*\cos(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*\cos(e/2 + (f*x)/2)*(e + f*x) + \\ & 18*A*c*d^2*\sin(e/2 + (f*x)/2)*(e + f*x) - 18*A*c^2*d*\sin(e/2 + (f*x)/2)*(e \\ & + f*x) - 27*B*c*d^2*\sin(e/2 + (f*x)/2)*(e + f*x) + 18*B*c^2*d*\sin(e/2 + (f \\ & *x)/2)*(e + f*x) + 36*A*c*d^2*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2) - 54* \\ & B*c*d^2*\cos(e/2 + (f*x)/2)^2*\sin(e/2 + (f*x)/2) + 36*B*c^2*d*\cos(e/2 + (f*x \\ &)/2)^2*\sin(e/2 + (f*x)/2) + 36*B*c*d^2*\cos(e/2 + (f*x)/2)^4*\sin(e/2 + (f*x) \\ & /2))/(6*a*f*\cos(e/2 + (f*x)/2) + 6*a*f*\sin(e/2 + (f*x)/2)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e)),x)

[Out] Timed out

$$3.266 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=143

$$\frac{x(2Ad(2c-d) + B(2c^2 - 4cd + 3d^2))}{2a} + \frac{2d(A(c-d) - B(2c-d)) \cos(e+fx)}{af} - \frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))}{f(a \sin(e+fx) + a)}$$

[Out] 1/2*(2*A*(2*c-d)*d+B*(2*c^2-4*c*d+3*d^2))*x/a+2*(A*(c-d)-B*(2*c-d))*d*cos(f*x+e)/a/f+1/2*(2*A-3*B)*d^2*cos(f*x+e)*sin(f*x+e)/a/f-(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^2/f/(a+a*sin(f*x+e))

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2977, 2734}

$$\frac{x(d^2(-2A-3B) + 4Acd + 2Bc(c-2d))}{2a} + \frac{2d(A(c-d) - B(2c-d)) \cos(e+fx)}{af} - \frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))}{f(a \sin(e+fx) + a)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]

[Out] ((2*B*c*(c - 2*d) + 4*A*c*d - (2*A - 3*B)*d^2)*x)/(2*a) + (2*(A*(c - d) - B*(2*c - d))*d*Cos[e + f*x])/(a*f) + ((2*A - 3*B)*d^2*Cos[e + f*x]*Sin[e + f*x])/(2*a*f) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(f*(a + a*Sin[e + f*x]))

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{a+a \sin(e+fx)} dx &= -\frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))^2}{f(a+a \sin(e+fx))} + \frac{\int (c+d \sin(e+fx))}{f(a+a \sin(e+fx))} \\ &= \frac{(2Bc(c-2d) + 4Acd - (2A-3B)d^2)x}{2a} + \frac{2(A(c-d) - B(2c-d)) \cos(e+fx)}{af} \end{aligned}$$

Mathematica [A] time = 0.46, size = 200, normalized size = 1.40

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right) \left(2(e+fx) \left(2Ad(2c-d) + B(2c^2 - 4cd + 3d^2)\right) \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right) + \int (c+d \sin(e+fx))}{f(a+a \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2] + 2*(2*A*(2*c - d)*d + B*(2*c^2 - 4*c*d + 3*d^2))*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*d*(-(A*d) + B*(-2*c + d))*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - B*d^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sin[2*(e + f*x)])/(4*a*f*(1 + Sin[e + f*x]))

fricas [B] time = 0.47, size = 303, normalized size = 2.12

$$\frac{Bd^2 \cos\left(fx + e\right)^3 - 2(A - B)c^2 + 4(A - B)cd - 2(A - B)d^2 + \left(2Bc^2 + 4(A - B)cd - (2A - 3B)d^2\right)fx - 2\left(2Bc^2 + 4(A - B)cd - (2A - 3B)d^2\right)}{4af(1 + \sin[e + fx])}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] 1/2*(B*d^2*cos(f*x + e)^3 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 + (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x - 2*(2*B*c*d + (A - B)*d^2)*cos(f*x + e)^2 - (2*(A - B)*c^2 - 4*(A - 2*B)*c*d + (4*A - 3*B)*d^2 - (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x)*cos(f*x + e) - (B*d^2*cos(f*x + e)^2 - 2*(A - B)*c^2 + 4*(A - B)*c*d - 2*(A - B)*d^2 - (2*B*c^2 + 4*(A - B)*c*d - (2*A - 3*B)*d^2)*f*x + (4*B*c*d + (2*A - B)*d^2)*cos(f*x + e))*sin(f*x + e)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

giac [A] time = 0.16, size = 222, normalized size = 1.55

$$\frac{\left(2Bc^2+4Acd-4Bcd-2Ad^2+3Bd^2\right)\left(fx+e\right) - \frac{4\left(Ac^2-Bc^2-2Acd+2Bcd+Ad^2-Bd^2\right)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)} + \frac{2\left(Bd^2 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 4Bcd \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 2Ad^2 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{2f}}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] 1/2*((2*B*c^2 + 4*A*c*d - 4*B*c*d - 2*A*d^2 + 3*B*d^2)*(f*x + e)/a - 4*(A*c^2 - B*c^2 - 2*A*c*d + 2*B*c*d + A*d^2 - B*d^2)/(a*(tan(1/2*f*x + 1/2*e) + 1)) + 2*(B*d^2*tan(1/2*f*x + 1/2*e)^3 - 4*B*c*d*tan(1/2*f*x + 1/2*e)^2 - 2*A*d^2*tan(1/2*f*x + 1/2*e)^2 + 2*B*d^2*tan(1/2*f*x + 1/2*e)^2 - B*d^2*tan(1/2*f*x + 1/2*e) - 4*B*c*d - 2*A*d^2 + 2*B*d^2)/((tan(1/2*f*x + 1/2*e)^2 + 1)^2*a))/f

maple [B] time = 0.42, size = 524, normalized size = 3.66

$$\frac{B\left(\tan^3\left(\frac{fx}{2} + \frac{e}{2}\right)\right)d^2}{af\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} - \frac{2A\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)d^2}{af\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} - \frac{4B\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)cd}{af\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} + \frac{2B\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)d^2}{af\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2} - \frac{Bd^2}{af\left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x)

[Out] 1/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*B*tan(1/2*f*x+1/2*e)^3*d^2-2/a/f/(1+tan(1/2*f*x+1/2*e)^2)^2*A*tan(1/2*f*x+1/2*e)^2*d^2-4/a/f/(1+tan(1/2*f*x+1/2*e)^2)

$\begin{aligned} &^2*B*\tan(1/2*f*x+1/2*e)^2*c*d+2/a/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)^2*d^2-1/a/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)*d^2-2/a/f/(1+\tan(1/2*f*x+1/2*e))^2*A*d^2-4/a/f/(1+\tan(1/2*f*x+1/2*e))^2*B*c*d+2/a/f/(1+\tan(1/2*f*x+1/2*e))^2*B*d^2+4/a/f*\arctan(\tan(1/2*f*x+1/2*e))*A*c*d-2/a/f*\arctan(\tan(1/2*f*x+1/2*e))*A*d^2+2/a/f*\arctan(\tan(1/2*f*x+1/2*e))*B*c^2-4/a/f*\arctan(\tan(1/2*f*x+1/2*e))*B*c*d+3/a/f*\arctan(\tan(1/2*f*x+1/2*e))*B*d^2-2/a/f/(\tan(1/2*f*x+1/2*e)+1)*A*c^2+4/a/f/(\tan(1/2*f*x+1/2*e)+1)*A*c*d-2/a/f/(\tan(1/2*f*x+1/2*e)+1)*A*d^2+2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*c^2-4/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*c*d+2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*d^2 \end{aligned}$

maxima [B] time = 0.73, size = 606, normalized size = 4.24

$$Bd^2 \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{5 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{3 \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{3 \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + 4}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} + \frac{2a \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{2a \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a \sin(fx+e)^5}{(\cos(fx+e)+1)^5}} + \frac{3 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} \right) - 4 Bcd \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{a \sin(fx+e)}{\cos(fx+e)+1} + \frac{a \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a \sin(fx+e)^5}{(\cos(fx+e)+1)^5}}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} + \frac{a \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a \sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{a \sin(fx+e)^4}{(\cos(fx+e)+1)^4} + \frac{a \sin(fx+e)^5}{(\cos(fx+e)+1)^5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] $(B*d^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) + 5*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 4)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + 2*a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2*a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + a*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 3*a*\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - 4*B*c*d*((\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - 2*A*d^2*((\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) + 2*B*c^2*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) + 4*A*c*d*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) - 2*A*c^2/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1)))/f$

mupad [B] time = 16.67, size = 297, normalized size = 2.08

$$\frac{x \left(2 B c^2 - 2 A d^2 + 3 B d^2 + 4 A c d - 4 B c d \right) \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 \left(2 A d^2 - 3 B d^2 + 4 B c d \right) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 \left(2 A c^2 \right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x)),x)

[Out] $(x*(2*B*c^2 - 2*A*d^2 + 3*B*d^2 + 4*A*c*d - 4*B*c*d))/(2*a) - (\tan(e/2 + (f*x)/2))^3*(2*A*d^2 - 3*B*d^2 + 4*B*c*d) + \tan(e/2 + (f*x)/2)^4*(2*A*c^2 + 2*A*d^2 - 2*B*c^2 - 3*B*d^2 - 4*A*c*d + 4*B*c*d) + \tan(e/2 + (f*x)/2)^2*(4*A*c^2 + 6*A*d^2 - 4*B*c^2 - 5*B*d^2 - 8*A*c*d + 12*B*c*d) + 2*A*c^2 + 4*A*d^2 - 2*B*c^2 - 4*B*d^2 + \tan(e/2 + (f*x)/2)*(2*A*d^2 - B*d^2 + 4*B*c*d) - 4*A*c*d + 8*B*c*d)/(f*(a + a*\tan(e/2 + (f*x)/2) + 2*a*\tan(e/2 + (f*x)/2)^2 + 2*a*\tan(e/2 + (f*x)/2)^3 + a*\tan(e/2 + (f*x)/2)^4 + a*\tan(e/2 + (f*x)/2)^5))$

sympy [A] time = 9.44, size = 5763, normalized size = 40.30

result too large to display


```

2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 6*B*d
**2*tan(e/2 + f*x/2)**3/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)
**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2
+ f*x/2) + 2*a*f) + 10*B*d**2*tan(e/2 + f*x/2)**2/(2*a*f*tan(e/2 + f*x/2)*
*5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x/2)**3 + 4*a*f*tan(e/2
+ f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 2*B*d**2*tan(e/2 + f*x/2)/(
2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2 + f*x
/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f) + 8*B*
d**2/(2*a*f*tan(e/2 + f*x/2)**5 + 2*a*f*tan(e/2 + f*x/2)**4 + 4*a*f*tan(e/2
+ f*x/2)**3 + 4*a*f*tan(e/2 + f*x/2)**2 + 2*a*f*tan(e/2 + f*x/2) + 2*a*f),
Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2/(a*sin(e) + a), True))

```

$$3.267 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=67

$$-\frac{(A-B)(c-d) \cos(e+fx)}{af(\sin(e+fx)+1)} + \frac{x(Ad+B(c-d))}{a} - \frac{Bd \cos(e+fx)}{af}$$

[Out] (B*(c-d)+A*d)*x/a-B*d*cos(f*x+e)/a/f-(A-B)*(c-d)*cos(f*x+e)/a/f/(1+sin(f*x+e))

Rubi [A] time = 0.20, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3023, 2735, 2648}

$$-\frac{(A-B)(c-d) \cos(e+fx)}{af(\sin(e+fx)+1)} + \frac{x(Ad+B(c-d))}{a} - \frac{Bd \cos(e+fx)}{af}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]),x]

[Out] ((B*(c - d) + A*d)*x)/a - (B*d*Cos[e + f*x])/(a*f) - ((A - B)*(c - d)*Cos[e + f*x])/(a*f*(1 + Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{a + a \sin(e + fx)} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{a + a \sin(e + fx)} dx \\ &= -\frac{Bd \cos(e + fx)}{af} + \frac{\int \frac{aAc + a(Bc - d) + Ad \sin(e + fx)}{a + a \sin(e + fx)} dx}{a} \\ &= \frac{(B(c - d) + Ad)x}{a} - \frac{Bd \cos(e + fx)}{af} + ((A - B)(c - d)) \int \frac{1}{a + a \sin(e + fx)} dx \\ &= \frac{(B(c - d) + Ad)x}{a} - \frac{Bd \cos(e + fx)}{af} - \frac{(A - B)(c - d) \cos(e + fx)}{f(a + a \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.47, size = 126, normalized size = 1.88

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\left(\cos\left(\frac{1}{2}(e + fx)\right)\left((e + fx)(Ad + B(c - d)) - Bd \cos(e + fx)\right) + \sin\left(\frac{1}{2}(e + fx)\right)\right)}{af(\sin(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x]), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2]*((B*(c - d) + A*d)*(e + f*x) - B*d*Cos[e + f*x]) + (2*A*c + B*(c - d)*(-2 + e + f*x) + A*d*(-2 + e + f*x) - B*d*Cos[e + f*x])*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))

fricas [B] time = 0.47, size = 154, normalized size = 2.30

$$\frac{Bd \cos^2(fx + e) - (Bc + (A - B)d)fx + (A - B)c - (A - B)d - ((Bc + (A - B)d)fx - (A - B)c + (A - 2B)d)}{af \cos(fx + e) + af \sin(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] -(B*d*cos(f*x + e)^2 - (B*c + (A - B)*d)*f*x + (A - B)*c - (A - B)*d - ((B*c + (A - B)*d)*f*x - (A - B)*c + (A - 2*B)*d)*cos(f*x + e) - ((B*c + (A - B)*d)*f*x - B*d*cos(f*x + e) + (A - B)*c - (A - B)*d)*sin(f*x + e))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

giac [B] time = 0.14, size = 160, normalized size = 2.39

$$\frac{(Bc + Ad - Bd)(fx + e)}{a} - \frac{2\left(Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - Ad \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + Bd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + Bd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + Ac - Bc - Ad + 2Bd \right)}{f \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] ((B*c + A*d - B*d)*(f*x + e)/a - 2*(A*c*tan(1/2*f*x + 1/2*e)^2 - B*c*tan(1/2*f*x + 1/2*e)^2 - A*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e)^2 + B*d*tan(1/2*f*x + 1/2*e) + A*c - B*c - A*d + 2*B*d)/((tan(1/2*f*x + 1/2*e)^3 + tan(1/2*f*x + 1/2*e)^2 + tan(1/2*f*x + 1/2*e) + 1)*a))/f

maple [B] time = 0.36, size = 179, normalized size = 2.67

$$\frac{2Bd}{af \left(1 + \tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)} + \frac{2A \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)d}{af} + \frac{2B \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)c}{af} - \frac{2B \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)d}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] $-2/a/f*B*d/(1+\tan(1/2*f*x+1/2*e)^2)+2/a/f*A*\arctan(\tan(1/2*f*x+1/2*e))*d+2/a/f*B*\arctan(\tan(1/2*f*x+1/2*e))*c-2/a/f*B*\arctan(\tan(1/2*f*x+1/2*e))*d-2/a/f/(\tan(1/2*f*x+1/2*e)+1)*A*c+2/a/f/(\tan(1/2*f*x+1/2*e)+1)*A*d+2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*c-2/a/f/(\tan(1/2*f*x+1/2*e)+1)*B*d$

maxima [B] time = 0.49, size = 256, normalized size = 3.82

$$\frac{2 \left(Bd \left(\frac{\frac{\sin(fx+e)}{\cos(fx+e)+1} + \frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1} + \frac{a \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} \right) - Bc \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a + \frac{a \sin(fx+e)}{\cos(fx+e)+1}} \right) - Ad \left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] $-2*(B*d*((\sin(f*x + e)/(\cos(f*x + e) + 1) + \sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 2)/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1) + a*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + a*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3) + \arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a) - B*c*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) - A*d*(\arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a + 1/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))) + A*c/(a + a*\sin(f*x + e)/(\cos(f*x + e) + 1))/f$

mupad [B] time = 13.68, size = 122, normalized size = 1.82

$$\frac{x (Ad + Bc - Bd) (2Ac - 2Ad - 2Bc + 2Bd) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + 2Bd \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 2Ac - 2Ad - 2Bc + 4Bd}{a f \left(a \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + a \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + a \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x)),x)

[Out] $(x*(A*d + B*c - B*d))/a - (2*A*c - 2*A*d - 2*B*c + 4*B*d + \tan(e/2 + (f*x)/2)^2*(2*A*c - 2*A*d - 2*B*c + 2*B*d) + 2*B*d*\tan(e/2 + (f*x)/2))/(f*(a + a*\tan(e/2 + (f*x)/2) + a*\tan(e/2 + (f*x)/2)^2 + a*\tan(e/2 + (f*x)/2)^3))$

sympy [A] time = 4.18, size = 1307, normalized size = 19.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] $\text{Piecewise}((-2*A*c*\tan(e/2 + f*x/2)**2/(a*f*\tan(e/2 + f*x/2)**3 + a*f*\tan(e/2 + f*x/2)**2 + a*f*\tan(e/2 + f*x/2) + a*f) - 2*A*c/(a*f*\tan(e/2 + f*x/2)**2 + a*f*\tan(e/2 + f*x/2) + a*f))$


```

3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2
+ f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e
/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 +
a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + A*d*f*x*tan(e/2 +
f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f
*x/2) + a*f) + A*d*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 +
a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d*tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x
/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*A*d/(a*f
*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f
) + B*c*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*
x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x*tan(e/2 + f*x/2)**2/(a*f*ta
n(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) +
B*c*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**
2 + a*f*tan(e/2 + f*x/2) + a*f) + B*c*f*x/(a*f*tan(e/2 + f*x/2)**3 + a*f*ta
n(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) + 2*B*c*tan(e/2 + f*x/2)**2
/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2)
+ a*f) + 2*B*c/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan
(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)**3/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x*tan(e/2
+ f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/
2 + f*x/2) + a*f) - B*d*f*x*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3 + a*f
*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - B*d*f*x/(a*f*tan(e/2 +
f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 2*B*d*
tan(e/2 + f*x/2)**2/(a*f*tan(e/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*
f*tan(e/2 + f*x/2) + a*f) - 2*B*d*tan(e/2 + f*x/2)/(a*f*tan(e/2 + f*x/2)**3
+ a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f) - 4*B*d/(a*f*tan(e
/2 + f*x/2)**3 + a*f*tan(e/2 + f*x/2)**2 + a*f*tan(e/2 + f*x/2) + a*f), Ne(
f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a), True))

```

$$3.268 \quad \int \frac{A+B \sin(e+fx)}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=35

$$\frac{Bx}{a} - \frac{(A-B) \cos(e+fx)}{f(a \sin(e+fx) + a)}$$

[Out] B*x/a-(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2735, 2648}

$$\frac{Bx}{a} - \frac{(A-B) \cos(e+fx)}{f(a \sin(e+fx) + a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]

[Out] (B*x)/a - ((A - B)*Cos[e + f*x])/(f*(a + a*Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{a+a \sin(e+fx)} dx &= \frac{Bx}{a} - (-A+B) \int \frac{1}{a+a \sin(e+fx)} dx \\ &= \frac{Bx}{a} - \frac{(A-B) \cos(e+fx)}{f(a+a \sin(e+fx))} \end{aligned}$$

Mathematica [B] time = 0.16, size = 79, normalized size = 2.26

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right) \left(\sin\left(\frac{1}{2}(e+fx)\right) (2A+B(e+fx-2)) + B(e+fx) \cos\left(\frac{1}{2}(e+fx)\right)\right)}{af(\sin(e+fx)+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x]),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(B*(e + f*x)*Cos[(e + f*x)/2] + (2*A + B*(-2 + e + f*x))*Sin[(e + f*x)/2]))/(a*f*(1 + Sin[e + f*x]))

fricas [A] time = 0.44, size = 66, normalized size = 1.89

$$\frac{Bfx + (Bfx - A + B) \cos(fx + e) + (Bfx + A - B) \sin(fx + e) - A + B}{af \cos(fx + e) + af \sin(fx + e) + af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="fricas")

[Out] (B*f*x + (B*f*x - A + B)*cos(f*x + e) + (B*f*x + A - B)*sin(f*x + e) - A + B)/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)

giac [A] time = 0.15, size = 40, normalized size = 1.14

$$\frac{\frac{(fx+e)^B}{a} - \frac{2(A-B)}{a\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] ((f*x + e)*B/a - 2*(A - B)/(a*(tan(1/2*f*x + 1/2*e) + 1)))/f

maple [A] time = 0.23, size = 65, normalized size = 1.86

$$\frac{2B \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{af} - \frac{2A}{af\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \frac{2B}{af\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)

[Out] 2/a/f*B*arctan(tan(1/2*f*x+1/2*e))-2/a/f/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(tan(1/2*f*x+1/2*e)+1)*B

maxima [B] time = 0.52, size = 78, normalized size = 2.23

$$\frac{2\left(B\left(\frac{\arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a} + \frac{1}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right) - \frac{A}{a+\frac{a\sin(fx+e)}{\cos(fx+e)+1}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] 2*(B*(arctan(sin(f*x + e)/(cos(f*x + e) + 1)))/a + 1/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))) - A/(a + a*sin(f*x + e)/(cos(f*x + e) + 1))/f

mupad [B] time = 12.92, size = 35, normalized size = 1.00

$$\frac{Bx}{a} - \frac{2A - 2B}{af\left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x)),x)

[Out] (B*x)/a - (2*A - 2*B)/(a*f*(tan(e/2 + (f*x)/2) + 1))

sympy [A] time = 1.83, size = 109, normalized size = 3.11

$$\begin{cases} -\frac{2A}{af \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + af} + \frac{Bfx \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{af \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + af} + \frac{Bfx}{af \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + af} + \frac{2B}{af \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + af} & \text{for } f \neq 0 \\ \frac{x(A+B \sin(e))}{a \sin(e)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e)),x)
```

```
[Out] Piecewise((-2*A/(a*f*tan(e/2 + f*x/2) + a*f) + B*f*x*tan(e/2 + f*x/2)/(a*f*  
tan(e/2 + f*x/2) + a*f) + B*f*x/(a*f*tan(e/2 + f*x/2) + a*f) + 2*B/(a*f*tan  
(e/2 + f*x/2) + a*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a), True))
```

$$3.269 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=101

$$\frac{2(Bc - Ad) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{af(c-d)\sqrt{c^2 - d^2}} - \frac{(A - B) \cos(e + fx)}{f(c-d)(a \sin(e + fx) + a)}$$

[Out] $-(A-B) \cdot \cos(f \cdot x + e) / (c-d) / f / (a + a \cdot \sin(f \cdot x + e)) + 2 \cdot (-A \cdot d + B \cdot c) \cdot \arctan((d + c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) / (c^2 - d^2)^{1/2}) / a / (c-d) / f / (c^2 - d^2)^{1/2}$

Rubi [A] time = 0.17, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2(Bc - Ad) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{af(c-d)\sqrt{c^2 - d^2}} - \frac{(A - B) \cos(e + fx)}{f(c-d)(a \sin(e + fx) + a)}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])),x]`
 [Out] `(2*(B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]])/(a*(c - d)*Sqrt[c^2 - d^2]*f) - ((A - B)*Cos[e + f*x])/((c - d)*f*(a + a*Sin[e + f*x]))`

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 2660

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

Rule 2978

`Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b`

$d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)$
 $) * \text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c + d \sin(e + fx))} dx &= -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} + \frac{\int \frac{a(Bc - Ad)}{c + d \sin(e + fx)} dx}{a^2(c - d)} \\ &= -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} + \frac{(Bc - Ad) \int \frac{1}{c + d \sin(e + fx)} dx}{a(c - d)} \\ &= -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} + \frac{(2(Bc - Ad)) \text{Subst}\left(\int \frac{1}{c + 2dx + cx^2} dx\right)}{a(c - d)f} \\ &= -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} - \frac{(4(Bc - Ad)) \text{Subst}\left(\int \frac{1}{-4(c^2 - d^2) - x^2} dx\right)}{a(c - d)} \\ &= \frac{2(Bc - Ad) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{a(c - d)\sqrt{c^2 - d^2} f} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \end{aligned}$$

Mathematica [A] time = 0.33, size = 148, normalized size = 1.47

$$\frac{2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left((A - B)\sqrt{c^2 - d^2} \sin\left(\frac{1}{2}(e + fx)\right) + (Bc - Ad) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)}{af(c - d)\sqrt{c^2 - d^2}(\sin(e + fx) + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])), x]

[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((A - B)*Sqrt[c^2 - d^2]*Sin[(e + f*x)/2] + (B*c - A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(a*(c - d)*Sqrt[c^2 - d^2]*f*(1 + Sin[e + f*x]))

fricas [B] time = 0.48, size = 595, normalized size = 5.89

$$\left[\frac{2(A - B)c^2 - 2(A - B)d^2 + (Bc - Ad + (Bc - Ad) \cos(fx + e) + (Bc - Ad) \sin(fx + e))\sqrt{-c^2 + d^2} \log\left(\frac{(2c^2 - d^2)\cos(fx + e) + (Bc - Ad)\sin(fx + e)}{2((ac^3 - ac^2d - acd^2 + ad^3)f \cos(fx + e) + d)\right)}{2((ac^3 - ac^2d - acd^2 + ad^3)f \cos(fx + e) + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*(2*(A - B)*c^2 - 2*(A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d

*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - 2*((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f), -((A - B)*c^2 - (A - B)*d^2 + (B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^2 - (A - B)*d^2)*cos(f*x + e) - ((A - B)*c^2 - (A - B)*d^2)*sin(f*x + e))/((a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*cos(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f*sin(f*x + e) + (a*c^3 - a*c^2*d - a*c*d^2 + a*d^3)*f)]

giac [A] time = 0.17, size = 113, normalized size = 1.12

$$2 \frac{\left(\frac{\left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan\left(\frac{c \tan\left(\frac{1}{2} fx + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right) (Bc - Ad)}{(ac - ad)\sqrt{c^2 - d^2}} - \frac{A - B}{(ac - ad)\left(\tan\left(\frac{1}{2} fx + \frac{1}{2} e \right) + 1 \right)} \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 2*((pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))*(B*c - A*d)/((a*c - a*d)*sqrt(c^2 - d^2)) - (A - B)/((a*c - a*d)*(tan(1/2*f*x + 1/2*e) + 1)))/f

maple [A] time = 0.54, size = 176, normalized size = 1.74

$$\frac{2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) Ad}{af(c-d)\sqrt{c^2 - d^2}} + \frac{2 \arctan\left(\frac{2c \tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 2d}{2\sqrt{c^2 - d^2}}\right) Bc}{af(c-d)\sqrt{c^2 - d^2}} - \frac{2A}{af(c-d)\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \frac{2B}{af(c-d)\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] -2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*d+2/a/f/(c-d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c-2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-d)/(tan(1/2*f*x+1/2*e)+1)*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see 'assume?' for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 13.32, size = 154, normalized size = 1.52

$$\frac{2 \operatorname{atan} \left(\frac{(Ad-Bc)(2ad^2-2acd) - 2c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(Ad-Bc)(ac-ad)}{a \sqrt{c+d} (c-d)^{3/2} - a \sqrt{c+d} (c-d)^{3/2}} \right) (Ad-Bc)}{af \sqrt{c+d} (c-d)^{3/2}} - \frac{2(A-B)}{f \left(a + a \tan\left(\frac{e}{2} + \frac{fx}{2}\right) \right) (c-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))),x)`

[Out] `(2*atan((((A*d - B*c)*(2*a*d^2 - 2*a*c*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)) - (2*c*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a*c - a*d))/(a*(c + d)^(1/2)*(c - d)^(3/2)))/(2*A*d - 2*B*c))*(A*d - B*c))/(a*f*(c + d)^(1/2)*(c - d)^(3/2)) - (2*(A - B))/(f*(a + a*tan(e/2 + (f*x)/2))*(c - d))`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e)),x)`

[Out] Timed out

$$3.270 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=181

$$\frac{2 \left(Ad(2c+d) - B(c^2 + cd + d^2) \right) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e+fx) \right) + d}{\sqrt{c^2 - d^2}} \right)}{af(c-d)(c^2 - d^2)^{3/2}} + \frac{d(B(2c+d) - A(c+2d)) \cos(e+fx)}{af(c-d)^2(c+d)(c+d \sin(e+fx))} - \frac{f(c-d)(a \sin(e+fx))}{f(c-d)(a \sin(e+fx))}$$

[Out] $-2*(A*d*(2*c+d)-B*(c^2+c*d+d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/a/(c-d)/(c^2-d^2)^{(3/2)}/f+d*(B*(2*c+d)-A*(c+2*d))*\cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*\sin(f*x+e))-(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))/(c+d*\sin(f*x+e))$

Rubi [A] time = 0.35, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{2 \left(Ad(2c+d) - B(c^2 + cd + d^2) \right) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e+fx) \right) + d}{\sqrt{c^2 - d^2}} \right)}{af(c-d)(c^2 - d^2)^{3/2}} + \frac{d(B(2c+d) - A(c+2d)) \cos(e+fx)}{af(c-d)^2(c+d)(c+d \sin(e+fx))} - \frac{f(c-d)(a \sin(e+fx))}{f(c-d)(a \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2), x]

[Out] $(-2*(A*d*(2*c+d) - B*(c^2 + c*d + d^2))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/S\text{qrt}[c^2 - d^2]])/(a*(c - d)*(c^2 - d^2)^{(3/2)*f} + (d*(B*(2*c+d) - A*(c + 2*d))*\text{Cos}[e + f*x])/(a*(c - d)^2*(c + d)*f*(c + d*\text{Sin}[e + f*x])) - ((A - B)*\text{Cos}[e + f*x])/((c - d)*f*(a + a*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2754

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m, x]

```
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c + d \sin(e + fx))^2} dx = -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))(c + d \sin(e + fx))} - \frac{\int \frac{a(2Ad - B(c+d)) - a^2 \sin(e + fx)}{(c + d \sin(e + fx))^2} dx}{a^2(c - d)}$$

$$= \frac{d(B(2c + d) - A(c + 2d)) \cos(e + fx)}{a(c - d)^2(c + d)f(c + d \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))}$$

$$= \frac{d(B(2c + d) - A(c + 2d)) \cos(e + fx)}{a(c - d)^2(c + d)f(c + d \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))}$$

$$= \frac{d(B(2c + d) - A(c + 2d)) \cos(e + fx)}{a(c - d)^2(c + d)f(c + d \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))}$$

$$= \frac{d(B(2c + d) - A(c + 2d)) \cos(e + fx)}{a(c - d)^2(c + d)f(c + d \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))}$$

$$= -\frac{2(B(2c + d) - A(c + 2d)) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{a(c - d)^2(c + d)\sqrt{c^2 - d^2} f} + \frac{d(Bc - Ad) \cos(e + fx)}{a(c - d)}$$

Mathematica [A] time = 1.22, size = 209, normalized size = 1.15

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(\frac{2(B(c^2 + cd + d^2) - Ad(2c + d)) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{(c + d)\sqrt{c^2 - d^2}} + \frac{d(Bc - Ad) \cos(e + fx)}{a(c - d)} \right) / (af(c - d)^2(\sin(e + fx) + 1))$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^2), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (2*(-(A*d*(2*c + d)) + B*(c^2 + c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*Sqrt[c^2 - d^2])
```

) + (d*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x]))/(a*(c - d)^2*f*(1 + Sin[e + f*x]))

fricas [B] time = 0.51, size = 1538, normalized size = 8.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [1/2*(2*(A - B)*c^4 - 4*(A - B)*c^2*d^2 + 2*(A - B)*d^4 + 2*((A - 2*B)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e) - 2*((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e)), ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 + ((A - 2*B)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 - (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (2*A - B)*c^2*d - (A - B)*c*d^2)*cos(f*x + e) + (B*c^3 - 2*(A - B)*c^2*d - (3*A - 2*B)*c*d^2 - (A - B)*d^3 + (B*c^2*d - (2*A - B)*c*d^2 - (A - B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((A - B)*c^4 + (A - 2*B)*c^3*d + B*c^2*d^2 - (A - 2*B)*c*d^3 - A*d^4)*cos(f*x + e) - ((A - B)*c^4 - 2*(A - B)*c^2*d^2 + (A - B)*d^4 - ((A - 2*B)*c^3*d + (2*A - B)*c^2*d^2 - (A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e)^2 - (a*c^6 - a*c^5*d - 2*a*c^4*d^2 + 2*a*c^3*d^3 + a*c^2*d^4 - a*c*d^5)*f*cos(f*x + e) - (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f - ((a*c^5*d - a*c^4*d^2 - 2*a*c^3*d^3 + 2*a*c^2*d^4 + a*c*d^5 - a*d^6)*f*cos(f*x + e) + (a*c^6 - 3*a*c^4*d^2 + 3*a*c^2*d^4 - a*d^6)*f)*sin(f*x + e))]

giac [B] time = 0.19, size = 443, normalized size = 2.45

$$2 \left[\frac{(Bc^2 - 2Acd + Bcd - Ad^2 + Bd^2) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(ac^3 - ac^2d - acd^2 + ad^3) \sqrt{c^2 - d^2}} \right] - \frac{Ac^3 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - Bc^3 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + Ac^2d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)}{\sqrt{c^2 - d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] 2*((B*c^2 - 2*A*c*d + B*c*d - A*d^2 + B*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a*c^3

- a*c^2*d - a*c*d^2 + a*d^3)*sqrt(c^2 - d^2)) - (A*c^3*tan(1/2*f*x + 1/2*e)^2 - B*c^3*tan(1/2*f*x + 1/2*e)^2 + A*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c^2*d*tan(1/2*f*x + 1/2*e)^2 - B*c*d^2*tan(1/2*f*x + 1/2*e)^2 + A*d^3*tan(1/2*f*x + 1/2*e)^2 + 2*A*c^2*d*tan(1/2*f*x + 1/2*e) - 3*B*c^2*d*tan(1/2*f*x + 1/2*e) + 3*A*c*d^2*tan(1/2*f*x + 1/2*e) - 3*B*c*d^2*tan(1/2*f*x + 1/2*e) + A*d^3*tan(1/2*f*x + 1/2*e) + A*c^3 - B*c^3 + A*c^2*d - 2*B*c^2*d + A*c*d^2)/(a*c^4 - a*c^3*d - a*c^2*d^2 + a*c*d^3)*(c*tan(1/2*f*x + 1/2*e)^3 + c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e)^2 + c*tan(1/2*f*x + 1/2*e) + 2*d*tan(1/2*f*x + 1/2*e) + c))/f

maple [B] time = 0.58, size = 615, normalized size = 3.40

$$\frac{2d^3 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) A}{af(c-d)^2 \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right)d + c\right)(c+d)c} + \frac{2d^2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right) B}{af(c-d)^2 \left(\left(\tan^2\left(\frac{fx}{2} + \frac{e}{2}\right)\right)c + 2 \tan\left(\frac{fx}{2} + \frac{e}{2}\right)d + c\right)(c+d)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

[Out] -2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^3/(c+d)/c*tan(1/2*f*x+1/2*e)*A+2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c+d)*tan(1/2*f*x+1/2*e)*B-2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d^2/(c+d)*A+2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)*d/(c+d)*B*c-4/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*c*d-2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A*d^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c*d+2/a/f/(c-d)^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*d^2-2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A+2/a/f/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*B

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 15.53, size = 437, normalized size = 2.41

$$\frac{2 \operatorname{atan} \left(\frac{(2ac^3d-2ac^2d^2-2acd^3+2ad^4)(Bc^2-Ad^2+Bd^2-2Acd+Bcd)}{a(c+d)^{3/2}(c-d)^{5/2}} + \frac{2c \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(ac^3-ac^2d-acd^2+ad^3)(Bc^2-Ad^2+Bd^2-2Acd+Bcd)}{a(c+d)^{3/2}(c-d)^{5/2}} \right)}{af(c+d)^{3/2}(c-d)^{5/2}} (Bc^2 - Ad^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^2),x)

```
[Out] (2*atan((((2*a*d^4 - 2*a*c^2*d^2 - 2*a*c*d^3 + 2*a*c^3*d)*(B*c^2 - A*d^2 +
B*d^2 - 2*A*c*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)) + (2*c*tan(e/2 +
(f*x)/2)*(a*c^3 + a*d^3 - a*c*d^2 - a*c^2*d)*(B*c^2 - A*d^2 + B*d^2 - 2*A*c
*d + B*c*d))/(a*(c + d)^(3/2)*(c - d)^(5/2)))/(2*B*c^2 - 2*A*d^2 + 2*B*d^2
- 4*A*c*d + 2*B*c*d))*(B*c^2 - A*d^2 + B*d^2 - 2*A*c*d + B*c*d))/(a*f*(c +
d)^(3/2)*(c - d)^(5/2)) - ((2*(A*c^2 + A*d^2 - B*c^2 + A*c*d - 2*B*c*d))/((
c + d)*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*d^2 + 2*A*c*d - 3*B*c*d))/(c*(
c - d)^2) + (2*tan(e/2 + (f*x)/2)^2*(A*c^3 + A*d^3 - B*c^3 + A*c^2*d - B*c*
d^2 - B*c^2*d))/(c*(c + d)*(c - d)^2))/(f*(a*c + tan(e/2 + (f*x)/2)^2*(a*c
+ 2*a*d) + tan(e/2 + (f*x)/2)*(a*c + 2*a*d) + a*c*tan(e/2 + (f*x)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

$$3.271 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=283

$$\frac{d(2Ac^2 + 9Acd + 4Ad^2 - 5Bc^2 - 6Bcd - 4Bd^2) \cos(e+fx) (3Ad(2c^2 + 2cd + d^2) - B(2c^3 + 4c^2d + 7cd^2 + 2d^3))}{2af(c-d)^3(c+d)^2(c+d \sin(e+fx))} \frac{af(c-d)(c^2-d^2)^{5/2}}{af(c-d)(c^2-d^2)^{5/2}}$$

[Out] $-(3A*d*(2*c^2+2*c*d+d^2)-B*(2*c^3+4*c^2*d+7*c*d^2+2*d^3))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/a/(c-d)/(c^2-d^2)^{(5/2)}/f-1/2*d*(2*A*c+3*A*d-3*B*c-2*B*d)*\cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*\sin(f*x+e))^2-(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))/(c+d*\sin(f*x+e))^2-1/2*d*(2*A*c^2+9*A*c*d+4*A*d^2-5*B*c^2-6*B*c*d-4*B*d^2)*\cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*\sin(f*x+e))$

Rubi [A] time = 0.55, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{(3Ad(2c^2 + 2cd + d^2) - B(4c^2d + 2c^3 + 7cd^2 + 2d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2-d^2}}\right)}{af(c-d)(c^2-d^2)^{5/2}} \frac{d(2Ac^2 + 9Acd + 4Ad^2 - 5Bc^2 - 6Bcd - 4Bd^2)}{2af(c-d)^3(c+d)^2(c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^3), x]$

[Out] $-\left(\left(\left(3A*d*(2*c^2 + 2*c*d + d^2) - B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3)\right)*\text{ArcTan}\left[\frac{d + c*\text{Tan}\left[\frac{e + f*x}{2}\right]}{\sqrt{c^2 - d^2}}\right]\right)/\left(a*(c - d)*(c^2 - d^2)^{(5/2)}*f\right) - \left(d*(2*A*c - 3*B*c + 3*A*d - 2*B*d)*\text{Cos}[e + f*x]\right)/\left(2*a*(c - d)^2*(c + d)*f*(c + d*\text{Sin}[e + f*x])^2\right) - \left((A - B)*\text{Cos}[e + f*x]\right)/\left((c - d)*f*(a + a*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^2\right) - \left(d*(2*A*c^2 - 5*B*c^2 + 9*A*c*d - 6*B*c*d + 4*A*d^2 - 4*B*d^2)*\text{Cos}[e + f*x]\right)/\left(2*a*(c - d)^3*(c + d)^2*f*(c + d*\text{Sin}[e + f*x])\right)$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 204

$\text{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_*) + (b_*)*(x_) + (c_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

$\text{Int}[(a_*) + (b_*)*\sin[(c_*) + (d_*)*(x_)])^{-1}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a*e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[\dots]$

$a^2 - b^2, 0]$

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))(c + d \sin(e + fx))^3} dx = -\frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))(c + d \sin(e + fx))^2} - \frac{\int \frac{a(3Ad - B(c + d)) \cos(e + fx)}{(c + d \sin(e + fx))^3} dx}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{d(2Ac - 3Bc + 3Ad - 2Bd) \cos(e + fx)}{2a(c - d)^2(c + d)f(c + d \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{d(2Ac - 3Bc + 3Ad - 2Bd) \cos(e + fx)}{2a(c - d)^2(c + d)f(c + d \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{d(2Ac - 3Bc + 3Ad - 2Bd) \cos(e + fx)}{2a(c - d)^2(c + d)f(c + d \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{d(2Ac - 3Bc + 3Ad - 2Bd) \cos(e + fx)}{2a(c - d)^2(c + d)f(c + d \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{d(2Ac - 3Bc + 3Ad - 2Bd) \cos(e + fx)}{2a(c - d)^2(c + d)f(c + d \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{(c - d)f(a + a \sin(e + fx))} \\ = -\frac{(3Ad(2c^2 + 2cd + d^2) - B(2c^3 + 4c^2d + 7cd^2 + 2d^3)) \tan^{-1}\left(\frac{c + d \sin(e + fx)}{c - d}\right)}{a(c - d)^3(c + d)^2 \sqrt{c^2 - d^2} f}$$

Mathematica [A] time = 1.61, size = 313, normalized size = 1.11

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(\frac{d(B(3c^2 + 2cd + 2d^2) - Ad(5c + 2d)) \cos(e + fx) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)}{(c + d)^2(c + d \sin(e + fx))} + \frac{2(B(2c^3 + 4c^2d + 7cd^2 + 2d^3)) \tan^{-1}\left(\frac{c + d \sin(e + fx)}{c - d}\right)}{a(c - d)^3(c + d)^2 \sqrt{c^2 - d^2} f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])*(c + d*Sin[e + f*x])^3),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*Sin[(e + f*x)/2] + (2*(-3
*A*d*(2*c^2 + 2*c*d + d^2) + B*(2*c^3 + 4*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[
(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2]))/((c + d)^2*Sqrt[c^2 - d^2]) + ((c - d)*d*(B*c - A*d)*Cos[e + f*x]*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) + (d*(
-(A*d*(5*c + 2*d)) + B*(3*c^2 + 2*c*d + 2*d^2))*Cos[e + f*x]*(Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x])))/(2*a*(c - d)^3*
f*(1 + Sin[e + f*x]))
```

fricas [B] time = 0.61, size = 3303, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm
="fricas")
```

```
[Out] [1/4*(4*(A - B)*c^6 - 12*(A - B)*c^4*d^2 + 12*(A - B)*c^2*d^4 - 4*(A - B)*d
^6 - 2*((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3
*(3*A - 2*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^3 + 2*(4*(A - 2*B)*c^5*d +
4*(3*A - 2*B)*c^4*d^2 - (2*A - 7*B)*c^3*d^3 - 5*(3*A - 2*B)*c^2*d^4 - (2*A
- B)*c*d^5 + (3*A - 2*B)*d^6)*cos(f*x + e)^2 - (2*B*c^5 - 2*(3*A - 4*B)*c^
4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11*B)*c*d^4 -
(3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A - 7*B)*c*d^4
- (3*A - 2*B)*d^5)*cos(f*x + e)^3 - (4*B*c^4*d - 2*(6*A - 5*B)*c^3*d^2 - 1
8*(A - B)*c^2*d^3 - (12*A - 11*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 +
(2*B*c^5 - 2*(3*A - 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - 3*(3*A - 2*B)*c^2
*d^3 - (6*A - 7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e) + (2*B*c^5 - 2*(3*
A - 4*B)*c^4*d - (18*A - 17*B)*c^3*d^2 - (21*A - 20*B)*c^2*d^3 - (12*A - 11
*B)*c*d^4 - (3*A - 2*B)*d^5 - (2*B*c^3*d^2 - 2*(3*A - 2*B)*c^2*d^3 - (6*A -
7*B)*c*d^4 - (3*A - 2*B)*d^5)*cos(f*x + e)^2 + 2*(2*B*c^4*d - 2*(3*A - 2*B
)*c^3*d^2 - (6*A - 7*B)*c^2*d^3 - (3*A - 2*B)*c*d^4)*cos(f*x + e))*sin(f*x
+ e))*sqrt(-c^2 + d^2)*log(-((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x +
e) - c^2 - d^2 - 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^
2 + d^2)))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*(2*(A
- B)*c^6 + 4*(A - 2*B)*c^5*d + (8*A - 7*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (7
*A - 5*B)*c^2*d^4 - (11*A - 7*B)*c*d^5 - (3*A - 4*B)*d^6)*cos(f*x + e) - 2*
(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*d^6 - ((
2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*(3*A - 2
*B)*c*d^5 - 4*(A - B)*d^6)*cos(f*x + e)^2 - (4*(A - 2*B)*c^5*d + (14*A - 13
*B)*c^4*d^2 + (7*A + B)*c^3*d^3 - (13*A - 11*B)*c^2*d^4 - (11*A - 7*B)*c*d^
5 - (A - 2*B)*d^6)*cos(f*x + e))*sin(f*x + e))/((a*c^7*d^2 - a*c^6*d^3 - 3*
a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*co
s(f*x + e)^3 + (2*a*c^8*d - a*c^7*d^2 - 7*a*c^6*d^3 + 3*a*c^5*d^4 + 9*a*c^4
*d^5 - 3*a*c^3*d^6 - 5*a*c^2*d^7 + a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - (a*c
^9 - a*c^8*d - 2*a*c^7*d^2 + 2*a*c^6*d^3 + 2*a*c^5*d^4 + 2*a*c^4*d^5 - 2*a*c
^3*d^6 - 2*a*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3
+ 6*a*c^5*d^4 + 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*
f + ((a*c^7*d^2 - a*c^6*d^3 - 3*a*c^5*d^4 + 3*a*c^4*d^5 + 3*a*c^3*d^6 - 3*a
*c^2*d^7 - a*c*d^8 + a*d^9)*f*cos(f*x + e)^2 - 2*(a*c^8*d - a*c^7*d^2 - 3*a
*c^6*d^3 + 3*a*c^5*d^4 + 3*a*c^4*d^5 - 3*a*c^3*d^6 - a*c^2*d^7 + a*c*d^8)*f
*cos(f*x + e) - (a*c^9 + a*c^8*d - 4*a*c^7*d^2 - 4*a*c^6*d^3 + 6*a*c^5*d^4
+ 6*a*c^4*d^5 - 4*a*c^3*d^6 - 4*a*c^2*d^7 + a*c*d^8 + a*d^9)*f)*sin(f*x + e
)), 1/2*(2*(A - B)*c^6 - 6*(A - B)*c^4*d^2 + 6*(A - B)*c^2*d^4 - 2*(A - B)*
d^6 - ((2*A - 5*B)*c^4*d^2 + 3*(3*A - 2*B)*c^3*d^3 + (2*A + B)*c^2*d^4 - 3*
```


$$\frac{2/a/f/(c-d)^3/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*d^3/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*A+2/a/f/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)*B-2/a/f/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)*A}{(c+d)(c^2-d^2)(c^2-2cd+d^2)}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 17.44, size = 1076, normalized size = 3.80

$$\frac{A d^4 - 2 A c^4 + 2 B c^4 - 8 A c^2 d^2 + 4 B c^2 d^2 - 2 A c d^3 - 4 A c^3 d + B c d^3 + 8 B c^3 d}{(c+d)(c^2-d^2)(c^2-2cd+d^2)} - \frac{\tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 (2 A d^6 - 13 A c^2 d^4 - 17 A c^3 d^3 - 22 A c^4 d^2 + 4 B c^2 d^4 + 19 B c^3 d)}{c^2 (c^2 - 2 c d + d^2) (-c^3 - c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))*(c + d*sin(e + f*x))^3),x)

[Out] ((A*d^4 - 2*A*c^4 + 2*B*c^4 - 8*A*c^2*d^2 + 4*B*c^2*d^2 - 2*A*c*d^3 - 4*A*c^3*d + B*c*d^3 + 8*B*c^3*d)/((c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) - (tan(e/2 + (f*x)/2)^3*(2*A*d^6 - 13*A*c^2*d^4 - 17*A*c^3*d^3 - 22*A*c^4*d^2 + 4*B*c^2*d^4 + 19*B*c^3*d^3 + 23*B*c^4*d^2 - 2*A*c*d^5 - 8*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)^2*(2*A*d^5 - 4*A*c^5 + 4*B*c^5 - 21*A*c^2*d^3 - 14*A*c^3*d^2 + 14*B*c^2*d^3 + 17*B*c^3*d^2 - 4*A*c*d^4 - 4*A*c^4*d + 2*B*c*d^4 + 8*B*c^4*d))/(c^2*(c^2 - d^2)*(c^2 - 2*c*d + d^2)) + (tan(e/2 + (f*x)/2)^4*(2*A*c^5 - 2*A*d^5 - 2*B*c^5 + 7*A*c^2*d^3 + 2*A*c^3*d^2 - 2*B*c^2*d^3 - 7*B*c^3*d^2 + 2*A*c*d^4 + 4*A*c^4*d - 4*B*c^4*d))/(c*(c^2 - 2*c*d + d^2)*(c*d^2 - c^2*d - c^3 + d^3)) + (tan(e/2 + (f*x)/2)*(2*A*d^5 - 27*A*c^2*d^3 - 22*A*c^3*d^2 + 15*B*c^2*d^3 + 29*B*c^3*d^2 - 5*A*c*d^4 - 8*A*c^4*d + 4*B*c*d^4 + 12*B*c^4*d))/(c*(c + d)*(c^2 - d^2)*(c^2 - 2*c*d + d^2)))/(f*(tan(e/2 + (f*x)/2)^2*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^3*(2*a*c^2 + 4*a*d^2 + 4*a*c*d) + a*c^2 + tan(e/2 + (f*x)/2)*(a*c^2 + 4*a*c*d) + tan(e/2 + (f*x)/2)^4*(a*c^2 + 4*a*c*d) + a*c^2*tan(e/2 + (f*x)/2)^5)) - (atan((((2*a*d^6 - 4*a*c^2*d^4 + 4*a*c^3*d^3 + 2*a*c^4*d^2 - 2*a*c*d^5 - 2*a*c^5*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(2*a*(c + d)^(5/2)*(c - d)^(7/2)) - (c*tan(e/2 + (f*x)/2)*(a*c^5 - a*d^5 + 2*a*c^2*d^3 - 2*a*c^3*d^2 + a*c*d^4 - a*c^4*d)*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(a*(c + d)^(5/2)*(c - d)^(7/2)))/(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))*(2*B*c^3 - 3*A*d^3 + 2*B*d^3 - 6*A*c*d^2 - 6*A*c^2*d + 7*B*c*d^2 + 4*B*c^2*d))/(a*f*(c + d)^(5/2)*(c - d)^(7/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.272 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=228

$$\frac{2d(A(c^2 + 6cd - 5d^2) + B(2c^2 - 15cd + 8d^2)) \cos(e + fx)}{3a^2 f} + \frac{dx(2Ad(3c - 2d) + B(6c^2 - 12cd + 7d^2))}{2a^2} + \frac{d^2(2d^2(A(c^2 + 6cd - 5d^2) + B(2c^2 - 15cd + 8d^2)) \cos(e + fx))}{3a^2 f}$$

```
[Out] 1/2*d*(2*A*(3*c-2*d)*d+B*(6*c^2-12*c*d+7*d^2))*x/a^2+2/3*d*(A*(c^2+6*c*d-5*d^2)+B*(2*c^2-15*c*d+8*d^2))*cos(f*x+e)/a^2/f+1/6*d^2*(B*(4*c-21*d)+2*A*(c+6*d))*cos(f*x+e)*sin(f*x+e)/a^2/f-1/3*(2*B*(c-4*d)+A*(c+5*d))*cos(f*x+e)*(c+d*sin(f*x+e))^2/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^3/f/(a+a*sin(f*x+e))^2
```

Rubi [A] time = 0.52, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2977, 2734}

$$\frac{2d(A(c^2 + 6cd - 5d^2) + B(2c^2 - 15cd + 8d^2)) \cos(e + fx)}{3a^2 f} + \frac{dx(2Ad(3c - 2d) + B(6c^2 - 12cd + 7d^2))}{2a^2} + \frac{d^2(2d^2(A(c^2 + 6cd - 5d^2) + B(2c^2 - 15cd + 8d^2)) \cos(e + fx))}{3a^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^2,x]
```

```
[Out] (d*(2*A*(3*c - 2*d)*d + B*(6*c^2 - 12*c*d + 7*d^2))*x)/(2*a^2) + (2*d*(A*(c^2 + 6*c*d - 5*d^2) + B*(2*c^2 - 15*c*d + 8*d^2))*Cos[e + f*x])/(3*a^2*f) + (d^2*(B*(4*c - 21*d) + 2*A*(c + 6*d))*Cos[e + f*x]*Sin[e + f*x])/(6*a^2*f) - ((2*B*(c - 4*d) + A*(c + 5*d))*Cos[e + f*x]*(c + d*Sin[e + f*x])^2)/(3*a^2*f*(1 + Sin[e + f*x])) - ((A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(3*f*(a + a*Sin[e + f*x])^2)
```

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])*(x_), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$*B)*c*d^2 - (4*A - 7*B)*d^3)*f*x)*\cos(f*x + e))*\sin(f*x + e))/(a^2*f*\cos(f*x + e)^2 - a^2*f*\cos(f*x + e) - 2*a^2*f - (a^2*f*\cos(f*x + e) + 2*a^2*f)*\sin(f*x + e))$

giac [B] time = 0.22, size = 494, normalized size = 2.17

$$\frac{3(6Bc^2d+6Acd^2-12Bcd^2-4Ad^3+7Bd^3)(fx+e)}{a^2} + \frac{6\left(Bd^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 6Bcd^2 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2 - 2Ad^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right) + 4Bd^3 \tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{6}*(3*(6*B*c^2*d + 6*A*c*d^2 - 12*B*c*d^2 - 4*A*d^3 + 7*B*d^3)*(f*x + e)/a^2 + 6*(B*d^3*\tan(1/2*f*x + 1/2*e)^3 - 6*B*c*d^2*\tan(1/2*f*x + 1/2*e)^2 - 2*A*d^3*\tan(1/2*f*x + 1/2*e)^2 + 4*B*d^3*\tan(1/2*f*x + 1/2*e)^2 - B*d^3*\tan(1/2*f*x + 1/2*e) - 6*B*c*d^2 - 2*A*d^3 + 4*B*d^3)/((\tan(1/2*f*x + 1/2*e)^2 + 1)^2*a^2) - 4*(3*A*c^3*\tan(1/2*f*x + 1/2*e)^2 - 9*B*c^2*d*\tan(1/2*f*x + 1/2*e)^2 - 9*A*c*d^2*\tan(1/2*f*x + 1/2*e)^2 + 18*B*c*d^2*\tan(1/2*f*x + 1/2*e)^2 + 6*A*d^3*\tan(1/2*f*x + 1/2*e)^2 - 9*B*d^3*\tan(1/2*f*x + 1/2*e)^2 + 3*A*c^3*\tan(1/2*f*x + 1/2*e) + 3*B*c^3*\tan(1/2*f*x + 1/2*e) + 9*A*c^2*d*\tan(1/2*f*x + 1/2*e) - 27*B*c^2*d*\tan(1/2*f*x + 1/2*e) - 27*A*c*d^2*\tan(1/2*f*x + 1/2*e) + 45*B*c*d^2*\tan(1/2*f*x + 1/2*e) + 15*A*d^3*\tan(1/2*f*x + 1/2*e) - 21*B*d^3*\tan(1/2*f*x + 1/2*e) + 2*A*c^3 + B*c^3 + 3*A*c^2*d - 12*B*c^2*d - 12*A*c*d^2 + 21*B*c*d^2 + 7*A*d^3 - 10*B*d^3)/(a^2*(\tan(1/2*f*x + 1/2*e) + 1)^3))/f$

maple [B] time = 0.43, size = 946, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x)

[Out] $-6/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*c*d^2+6/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*A*c*d^2+6/a^2/f*d*\arctan(\tan(1/2*f*x+1/2*e))*B*c^2-12/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*B*c*d^2+1/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)^3*d^3-2/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*A*\tan(1/2*f*x+1/2*e)^2*d^3-6/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)^2*c*d^2+4/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*d^3-2/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*A*d^3-6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d+6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*c*d^2+6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*c^2*d-6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*c*d^2+4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c^2*d-4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c*d^2-4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c^2*d+4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c*d^2+4/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)^2*d^3-1/a^2/f/(1+\tan(1/2*f*x+1/2*e))^2*B*\tan(1/2*f*x+1/2*e)*d^3+6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*A*c*d^2+6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B*c^2*d-12/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B*c*d^2-2/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*d^3-2/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*c^3+2/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*d^3-4/3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c^3-4/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*A*d^3+7/a^2/f*\arctan(\tan(1/2*f*x+1/2*e))*B*d^3-2/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*A*c^3-4/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*A*d^3+6/a^2/f/(\tan(1/2*f*x+1/2*e)+1)*B*d^3+2/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*c^3+4/3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*d^3+4/3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c^3-4/3/a^2/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*d^3$

maxima [B] time = 0.57, size = 1382, normalized size = 6.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] $\frac{1}{3} \cdot (B \cdot d^3 \cdot ((75 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 97 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 126 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 98 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 63 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5 + 21 \cdot \sin(f \cdot x + e)^6 / (\cos(f \cdot x + e) + 1)^6 + 32) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 5 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 7 \cdot a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 7 \cdot a^2 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 5 \cdot a^2 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5 + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^6 / (\cos(f \cdot x + e) + 1)^6 + a^2 \cdot \sin(f \cdot x + e)^7 / (\cos(f \cdot x + e) + 1)^7) + 21 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / a^2 - 12 \cdot B \cdot c \cdot d^2 \cdot ((12 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 11 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 9 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 5) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 4 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 4 \cdot a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^2 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) + 3 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / a^2 - 4 \cdot A \cdot d^3 \cdot ((12 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 11 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 9 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + 5) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 4 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 4 \cdot a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3 + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^4 / (\cos(f \cdot x + e) + 1)^4 + a^2 \cdot \sin(f \cdot x + e)^5 / (\cos(f \cdot x + e) + 1)^5) + 3 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / a^2 + 6 \cdot B \cdot c^2 \cdot d \cdot ((9 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 4) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) + 3 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / a^2 + 6 \cdot A \cdot c \cdot d^2 \cdot ((9 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 4) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) + 3 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1))) / a^2 - 2 \cdot A \cdot c^3 \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 2) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) - 2 \cdot B \cdot c^3 \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 1) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) - 6 \cdot A \cdot c^2 \cdot d \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 1) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3)) / f$

mupad [B] time = 16.35, size = 663, normalized size = 2.91

$$\frac{d \operatorname{atan}\left(\frac{d \tan\left(\frac{e}{2} + \frac{f x}{2}\right) (6 B c^2 - 4 A d^2 + 7 B d^2 + 6 A c d - 12 B c d)}{7 B d^3 - 4 A d^3 + 6 A c d^2 - 12 B c d^2 + 6 B c^2 d}\right) (6 B c^2 - 4 A d^2 + 7 B d^2 + 6 A c d - 12 B c d) \tan\left(\frac{e}{2} + \frac{f x}{2}\right) (2 A c^3 + 16 A d^3 + 2 B c^3 - 25 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + (4 A c^3) / 3 + (20 A d^3) / 3 + (2 B c^3) / 3 - (32 B d^3) / 3 + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^6 (2 A c^3 + 4 A d^3 - 7 B d^3 - 6 A c d^2 + 12 B c d^2 - 6 B c^2 d) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 (2 A c^3 + 12 A d^3 + 2 B c^3 - 21 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^4 (2 A c^3 + 12 A d^3 + 2 B c^3 - 21 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^3 (2 A c^3 + 12 A d^3 + 2 B c^3 - 21 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 (2 A c^3 + 12 A d^3 + 2 B c^3 - 21 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + \tan\left(\frac{e}{2} + \frac{f x}{2}\right) (2 A c^3 + 12 A d^3 + 2 B c^3 - 21 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d) + 2 A c^3 + 16 A d^3 + 2 B c^3 - 25 B d^3 - 18 A c d^2 + 6 A c^2 d + 48 B c d^2 - 18 B c^2 d}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^2,x)

[Out] $(d \cdot \operatorname{atan}(((d \cdot \tan(e/2 + (f \cdot x)/2) \cdot (6 \cdot B \cdot c^2 - 4 \cdot A \cdot d^2 + 7 \cdot B \cdot d^2 + 6 \cdot A \cdot c \cdot d - 12 \cdot B \cdot c \cdot d)) / (7 \cdot B \cdot d^3 - 4 \cdot A \cdot d^3 + 6 \cdot A \cdot c \cdot d^2 - 12 \cdot B \cdot c \cdot d^2 + 6 \cdot B \cdot c^2 \cdot d)) \cdot (6 \cdot B \cdot c^2 - 4 \cdot A \cdot d^2 + 7 \cdot B \cdot d^2 + 6 \cdot A \cdot c \cdot d - 12 \cdot B \cdot c \cdot d)) / (a^2 \cdot f) - (\tan(e/2 + (f \cdot x)/2) \cdot (2 \cdot A \cdot c^3 + 16 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 25 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + (4 \cdot A \cdot c^3) / 3 + (20 \cdot A \cdot d^3) / 3 + (2 \cdot B \cdot c^3) / 3 - (32 \cdot B \cdot d^3) / 3 + \tan(e/2 + (f \cdot x)/2)^6 \cdot (2 \cdot A \cdot c^3 + 4 \cdot A \cdot d^3 - 7 \cdot B \cdot d^3 - 6 \cdot A \cdot c \cdot d^2 + 12 \cdot B \cdot c \cdot d^2 - 6 \cdot B \cdot c^2 \cdot d) + \tan(e/2 + (f \cdot x)/2)^5 \cdot (2 \cdot A \cdot c^3 + 12 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 21 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + \tan(e/2 + (f \cdot x)/2)^4 \cdot (2 \cdot A \cdot c^3 + 12 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 21 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + \tan(e/2 + (f \cdot x)/2)^3 \cdot (2 \cdot A \cdot c^3 + 12 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 21 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + \tan(e/2 + (f \cdot x)/2)^2 \cdot (2 \cdot A \cdot c^3 + 12 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 21 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + \tan(e/2 + (f \cdot x)/2) \cdot (2 \cdot A \cdot c^3 + 12 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 21 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d) + 2 \cdot A \cdot c^3 + 16 \cdot A \cdot d^3 + 2 \cdot B \cdot c^3 - 25 \cdot B \cdot d^3 - 18 \cdot A \cdot c \cdot d^2 + 6 \cdot A \cdot c^2 \cdot d + 48 \cdot B \cdot c \cdot d^2 - 18 \cdot B \cdot c^2 \cdot d)$

$$3 - 18Ac^2d + 6A^2c^2d + 36B^2cd^2 - 18B^2c^2d) + \tan(e/2 + (f*x)/2)^3(4A^3c^3 + 28A^2d^3 + 4B^2c^3 - 42B^2d^3 - 36A^2cd^2 + 12A^2c^2d + 84B^2cd^2 - 36B^2c^2d) + \tan(e/2 + (f*x)/2)^4((16A^3c^3)/3 + (56A^2d^3)/3 + (2B^2c^3)/3 - (98B^2d^3)/3 - 20A^2cd^2 + 2A^2c^2d + 56B^2cd^2 - 20B^2c^2d) + \tan(e/2 + (f*x)/2)^2((14A^3c^3)/3 + (64A^2d^3)/3 + (4B^2c^3)/3 - (97B^2d^3)/3 - 22A^2cd^2 + 4A^2c^2d + 64B^2cd^2 - 22B^2c^2d) - 8A^2cd^2 + 2A^2c^2d + 20B^2cd^2 - 8B^2c^2d)/(f*(5a^2*\tan(e/2 + (f*x)/2)^2 + 7a^2*\tan(e/2 + (f*x)/2)^3 + 7a^2*\tan(e/2 + (f*x)/2)^4 + 5a^2*\tan(e/2 + (f*x)/2)^5 + 3a^2*\tan(e/2 + (f*x)/2)^6 + a^2*\tan(e/2 + (f*x)/2)^7 + a^2 + 3a^2*\tan(e/2 + (f*x)/2)))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

$$3.273 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=132

$$\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)} + \frac{dx(Ad+2B(c-d))}{a^2} + \frac{d^2(A-4B)\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)}{3f(a \sin(e+fx))}$$

[Out] $d*(2*B*(c-d)+A*d)*x/a^2+1/3*(A-4*B)*d^2*\cos(f*x+e)/a^2/f-1/3*(c-d)*(2*B*(c-3*d)+A*(c+3*d))*\cos(f*x+e)/a^2/f/(1+\sin(f*x+e))-1/3*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/f/(a+a*\sin(f*x+e))^2$

Rubi [A] time = 0.51, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2977, 2968, 3023, 2735, 2648}

$$\frac{(c-d)(A(c+3d)+2B(c-3d))\cos(e+fx)}{3a^2f(\sin(e+fx)+1)} + \frac{dx(Ad+2B(c-d))}{a^2} + \frac{d^2(A-4B)\cos(e+fx)}{3a^2f} - \frac{(A-B)\cos(e+fx)}{3f(a \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^2,x]

[Out] $(d*(2*B*(c-d)+A*d)*x)/a^2 + ((A-4*B)*d^2*\cos[e+f*x])/(3*a^2*f) - ((c-d)*(2*B*(c-3*d)+A*(c+3*d))*\cos[e+f*x])/(3*a^2*f*(1+\sin[e+f*x])) - ((A-B)*\cos[e+f*x]*(c+d*\sin[e+f*x])^2)/(3*f*(a+a*\sin[e+f*x])^2)$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^2} dx &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{3f(a + a \sin(e + fx))^2} + \int \frac{(c + d \sin(e + fx))(a(2} \\ &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{3f(a + a \sin(e + fx))^2} + \int \frac{ac(2B(c-d) + A(c+2} \\ &= \frac{(A - 4B)d^2 \cos(e + fx)}{3a^2 f} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + f} \\ &= \frac{d(2B(c - d) + Ad)x}{a^2} + \frac{(A - 4B)d^2 \cos(e + fx)}{3a^2 f} - \frac{(A - B) \cos} \\ &= \frac{d(2B(c - d) + Ad)x}{a^2} + \frac{(A - 4B)d^2 \cos(e + fx)}{3a^2 f} - \frac{(c - d)(2B(c} \end{aligned}$$

Mathematica [B] time = 1.67, size = 338, normalized size = 2.56

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(6 \cos\left(\frac{1}{2}(e + fx)\right) (Ad(4c + d(3e + 3fx - 4)) + B(2c^2 + 2cd(3e + 3fx - 4)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x]
)^2,x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A*d*(4*c + d*(-4 + 3*e + 3*f*x))
+ B*(2*c^2 + d^2*(5 - 6*e - 6*f*x) + 2*c*d*(-4 + 3*e + 3*f*x)))*Cos[(e + f
*x)/2] - (B*(8*c^2 + d^2*(41 - 12*e - 12*f*x) + 4*c*d*(-10 + 3*e + 3*f*x))
+ 2*A*(2*c^2 + 8*c*d + d^2*(-10 + 3*e + 3*f*x)))*Cos[(3*(e + f*x))/2] + 3*B
*d^2*Cos[(5*(e + f*x))/2] + 6*(2*A*c^2 + 2*B*c^2 + 4*A*c*d - 12*B*c*d - 6*A
*d^2 + 9*B*d^2 + 8*B*c*d*e + 4*A*d^2*e - 8*B*d^2*e + 8*B*c*d*f*x + 4*A*d^2*
f*x - 8*B*d^2*f*x - 2*d*(-2*B*c*(e + f*x) - A*d*(e + f*x) + 2*B*d*(1 + e +
f*x))*Cos[e + f*x] - B*d^2*Cos[2*(e + f*x)])*Sin[(e + f*x)/2]))/(12*a^2*f*(
1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.46, size = 375, normalized size = 2.84

$$\frac{3Bd^2 \cos(fx + e)^3 - (A - B)c^2 + 2(A - B)cd - (A - B)d^2 + 6(2Bcd + (A - 2B)d^2)fx - ((A + 2B)c^2 + 2(A - B)d^2)}{12a^2f(1 + \sin(e + fx))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] -1/3*(3*B*d^2*cos(f*x + e)^3 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 +
6*(2*B*c*d + (A - 2*B)*d^2)*f*x - ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A
```

$$- 11*B*d^2 + 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*\cos(f*x + e)^2 - ((2*A + B)*c^2 + 2*(A - 4*B)*c*d - (4*A - 13*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*\cos(f*x + e) - (3*B*d^2*\cos(f*x + e)^2 - (A - B)*c^2 + 2*(A - B)*c*d - (A - B)*d^2 - 6*(2*B*c*d + (A - 2*B)*d^2)*f*x + ((A + 2*B)*c^2 + 2*(2*A - 5*B)*c*d - (5*A - 14*B)*d^2 - 3*(2*B*c*d + (A - 2*B)*d^2)*f*x)*\cos(f*x + e))*\sin(f*x + e))/(a^2*f*\cos(f*x + e)^2 - a^2*f*\cos(f*x + e) - 2*a^2*f - (a^2*f*\cos(f*x + e) + 2*a^2*f)*\sin(f*x + e))$$

giac [B] time = 0.19, size = 277, normalized size = 2.10

$$\frac{3(2Bcd+Ad^2-2Bd^2)(fx+e)}{a^2} - \frac{6Bd^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^2} - \frac{2\left(3Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out]
$$\frac{1}{3} \left(\frac{3(2Bcd+Ad^2-2Bd^2)(fx+e)}{a^2} - \frac{6Bd^2}{\left(\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+1\right)a^2} - \frac{2(3Ac^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-6Bcd\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-3Ad^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+6Bd^2\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2)}{a^2} + 3A*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right)^2 - 6*B*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right)^2 + 3*A*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right)^2 + 6*B*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right)^2 + 3*A*c^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) + 3*B*c^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) + 6*A*c*d*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) - 18*B*c*d*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) - 9*A*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) + 15*B*d^2*\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) + 2*A*c^2 + B*c^2 + 2*A*c*d - 8*B*c*d - 4*A*d^2 + 7*B*d^2) / (a^2*(\tan\left(\frac{1}{2}*f*x + \frac{1}{2}*e\right) + 1)^3) \right) / f$$

maple [B] time = 0.41, size = 489, normalized size = 3.70

$$-\frac{2Bd^2}{a^2f\left(1+\tan^2\left(\frac{fx}{2}+\frac{e}{2}\right)\right)} + \frac{2A\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d^2}{a^2f} + \frac{4dB\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)c}{a^2f} - \frac{4B\arctan\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)\right)}{a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)

[Out]
$$\frac{-2}{a^2} \frac{f*B*d^2}{(1+\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)^2)} + \frac{2}{a^2} \frac{f*A*\arctan\left(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)\right)*d^2+4}{f*d*B*\arctan\left(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)\right)*c-4}{f*B*\arctan\left(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)\right)*d^2-2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)*A*c^2+2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)*A*d^2+4}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)*B*c*d-4}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)*B*d^2+2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*A*c^2-4}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*A*c*d+2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*A*d^2-2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*B*c^2+4}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*B*c*d-2}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^2*B*d^2-4}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*A*c^2+8}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*A*c*d-4}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*A*d^2+4}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*B*c^2-8}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*B*c*d+4}{3}{f/(\tan\left(\frac{1}{2}*f*x+\frac{1}{2}*e\right)+1)^3*B*d^2}$$

maxima [B] time = 0.58, size = 831, normalized size = 6.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out]
$$-\frac{2}{3} \frac{(2*B*d^2*((12*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 9*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 3*\sin(f*x + e)^4$$

$$\frac{1}{(\cos(fx + e) + 1)^4 + 5} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 4a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + 4a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3 + 3a^2 \sin(fx + e)^4} \frac{1}{(\cos(fx + e) + 1)^4 + a^2 \sin(fx + e)^5} \frac{1}{(\cos(fx + e) + 1)^5} + 3 \arctan\left(\frac{\sin(fx + e)}{\cos(fx + e) + 1}\right) \frac{1}{a^2} - 2Bc^2 d \frac{1}{(9 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + 4} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3} + 3 \arctan\left(\frac{\sin(fx + e)}{\cos(fx + e) + 1}\right) \frac{1}{a^2} - Ad^2 \frac{1}{(9 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + 4} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3} + 3 \arctan\left(\frac{\sin(fx + e)}{\cos(fx + e) + 1}\right) \frac{1}{a^2} + Ac^2 \frac{1}{(3 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + 2} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3} + Bc^2 \frac{1}{(3 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 1} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3} + 2Ac^2 d \frac{1}{(3 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 1} \frac{1}{(a^2 + 3a^2 \sin(fx + e))} \frac{1}{(\cos(fx + e) + 1) + 3a^2 \sin(fx + e)^2} \frac{1}{(\cos(fx + e) + 1)^2 + a^2 \sin(fx + e)^3} \frac{1}{(\cos(fx + e) + 1)^3} \Big) / f$$

mupad [B] time = 16.04, size = 365, normalized size = 2.77

$$\frac{2d \operatorname{atan}\left(\frac{2d \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(Ad + 2Bc - 2Bd)}{2Ad^2 - 4Bd^2 + 4Bcd}\right)(Ad + 2Bc - 2Bd) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 (2Ac^2 - 6Ad^2 + 2Bc^2 + 12Bd^2 + 4Bc^2d - 6Ad^2c + 12Bcd^2)}{a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^2,x)
[Out] (2*d*atan((2*d*tan(e/2 + (f*x)/2)*(A*d + 2*B*c - 2*B*d))/(2*A*d^2 - 4*B*d^2 + 4*B*c*d))*(A*d + 2*B*c - 2*B*d))/(a^2*f) - (tan(e/2 + (f*x)/2)^3*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 12*B*d^2 + 4*A*c*d - 12*B*c*d) + tan(e/2 + (f*x)/2)^2*((10*A*c^2)/3 - (14*A*d^2)/3 + (2*B*c^2)/3 + (44*B*d^2)/3 + (4*A*c*d)/3 - (28*B*c*d)/3) + (4*A*c^2)/3 - (8*A*d^2)/3 + (2*B*c^2)/3 + (20*B*d^2)/3 + tan(e/2 + (f*x)/2)^4*(2*A*c^2 - 2*A*d^2 + 4*B*d^2 - 4*B*c*d) + tan(e/2 + (f*x)/2)*(2*A*c^2 - 6*A*d^2 + 2*B*c^2 + 16*B*d^2 + 4*A*c*d - 12*B*c*d) + (4*A*c*d)/3 - (16*B*c*d)/3)/(f*(4*a^2*tan(e/2 + (f*x)/2)^2 + 4*a^2*tan(e/2 + (f*x)/2)^3 + 3*a^2*tan(e/2 + (f*x)/2)^4 + a^2*tan(e/2 + (f*x)/2)^5 + a^2 + 3*a^2*tan(e/2 + (f*x)/2)))
```

sympy [A] time = 17.79, size = 5358, normalized size = 40.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^2,x)
[Out] Piecewise((-6*A*c**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 10*A*c**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f))
```



```

f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 24*B*c*d*f*x*tan(e/2 +
f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 1
2*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan
(e/2 + f*x/2) + 3*a**2*f) + 18*B*c*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2
+ f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
+ 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
6*B*c*d*f*x/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 +
12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*ta
n(e/2 + f*x/2) + 3*a**2*f) + 12*B*c*d*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2
+ f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3
+ 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) +
36*B*c*d*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e
/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)
**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 28*B*c*d*tan(e/2 + f*x/2)**2/
(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*ta
n(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/
2) + 3*a**2*f) + 36*B*c*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 16*B*c*d/(3*a**
2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2
+ f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3
*a**2*f) - 6*B*d**2*f*x*tan(e/2 + f*x/2)**5/(3*a**2*f*tan(e/2 + f*x/2)**5 +
9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*ta
n(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f*x*
tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**3/(3*a
**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/
2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) +
3*a**2*f) - 24*B*d**2*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**
5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*
f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 18*B*d**2*f
*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/
2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a
**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 12*B*d**2*tan(e/2 + f*x/2)**4/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 36*B*d**2*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 44*B*d**2*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 48*B*d**2*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 20*B*d**2/(3*a**2*f*tan(e/2 + f*x/2)**5 + 9*a**2*f*tan(e/2 + f*x/2)**4 + 12*a**2*f*tan(e/2 + f*x/2)**3 + 12*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))**2/(a*sin(e) + a)**2, True))

```

$$3.274 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=85

$$-\frac{(Ac + 2Ad + 2Bc - 5Bd) \cos(e + fx)}{3a^2 f(\sin(e + fx) + 1)} + \frac{Bdx}{a^2} - \frac{(A - B)(c - d) \cos(e + fx)}{3f(a \sin(e + fx) + a)^2}$$

[Out] B*d*x/a^2-1/3*(A*c+2*A*d+2*B*c-5*B*d)*cos(f*x+e)/a^2/f/(1+sin(f*x+e))-1/3*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2

Rubi [A] time = 0.21, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3019, 2735, 2648}

$$-\frac{(Ac + 2Ad + 2Bc - 5Bd) \cos(e + fx)}{3a^2 f(\sin(e + fx) + 1)} + \frac{Bdx}{a^2} - \frac{(A - B)(c - d) \cos(e + fx)}{3f(a \sin(e + fx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2,x]

[Out] (B*d*x)/a^2 - ((A*c + 2*B*c + 2*A*d - 5*B*d)*Cos[e + f*x])/(3*a^2*f*(1 + Sin[e + f*x])) - ((A - B)*(c - d)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2)

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3019

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)])^2), x_Symbol] := Simp[((A*b - a*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^2} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{(a + a \sin(e + fx))^2} dx \\ &= \frac{(A - B)(c - d) \cos(e + fx)}{3f(a + a \sin(e + fx))^2} - \frac{\int \frac{-a(2B(c-d) + A(c+2d)) - 3aBd \sin(e+fx)}{a+a \sin(e+fx)} dx}{3a^2} \\ &= \frac{Bdx}{a^2} - \frac{(A - B)(c - d) \cos(e + fx)}{3f(a + a \sin(e + fx))^2} + \frac{(Ac + 2Bc + 2Ad - 5Bd)}{3a} \\ &= \frac{Bdx}{a^2} - \frac{(A - B)(c - d) \cos(e + fx)}{3f(a + a \sin(e + fx))^2} - \frac{(Ac + 2Bc + 2Ad - 5Bd)}{3f(a^2 + a^2 \sin(e + fx))} \end{aligned}$$

Mathematica [B] time = 0.34, size = 180, normalized size = 2.12

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(2(A - B)(c - d) \sin\left(\frac{1}{2}(e + fx)\right) + 2(Ac + 2Ad + 2Bc - 5Bd) \sin\left(\frac{1}{2}(e + fx)\right)\right)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^2, x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2] - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*c + 2*B*c + 2*A*d - 5*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 3*B*d*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/(3*a^2*f*(1 + Sin[e + f*x])^2)

fricas [B] time = 0.43, size = 208, normalized size = 2.45

$$\frac{6Bdfx - (3Bdfx + (A + 2B)c + (2A - 5B)d) \cos(fx + e)^2 - (A - B)c + (A - B)d + (3Bdfx - (2A + B)c)}{3(a^2f \cos(fx + e))^2 - a^2f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -1/3*(6*B*d*f*x - (3*B*d*f*x + (A + 2*B)*c + (2*A - 5*B)*d)*cos(f*x + e)^2 - (A - B)*c + (A - B)*d + (3*B*d*f*x - (2*A + B)*c - (A - 4*B)*d)*cos(f*x + e) + (6*B*d*f*x + (A - B)*c - (A - B)*d + (3*B*d*f*x - (A + 2*B)*c - (2*A - 5*B)*d)*cos(f*x + e))*sin(f*x + e)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

giac [A] time = 0.15, size = 141, normalized size = 1.66

$$\frac{3(fx+e)Bd}{a^2} - \frac{2\left(3Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 3Bd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 3Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 3Ad \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 9Bd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)\right)}{a^2\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}$$

$$\frac{\dots}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] $\frac{1}{3} \cdot (3 \cdot (f \cdot x + e) \cdot B \cdot d / a^2 - 2 \cdot (3 \cdot A \cdot c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e))^2 - 3 \cdot B \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 3 \cdot A \cdot c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 3 \cdot B \cdot c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 3 \cdot A \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 9 \cdot B \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 2 \cdot A \cdot c + B \cdot c + A \cdot d - 4 \cdot B \cdot d) / (a^2 \cdot (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3) / f$

maple [B] time = 0.43, size = 252, normalized size = 2.96

$$\frac{2Bd \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{a^2 f} - \frac{2Ac}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \frac{2Bd}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \frac{2Ac}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} - \frac{2Bd}{a^2 f \left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)`

[Out] $\frac{2}{a^2} \cdot \frac{B \cdot d \cdot \arctan(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)) - 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \cdot A \cdot c + 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1) \cdot B \cdot d + 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot A \cdot c - 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot A \cdot d - 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot B \cdot c + 2/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot B \cdot d - 4/3/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot A \cdot c + 4/3/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot A \cdot d + 4/3/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot B \cdot c - 4/3/a^2/f/(\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^3 \cdot B \cdot d}{f}$

maxima [B] time = 0.50, size = 454, normalized size = 5.34

$$2 \left(Bd \left(\frac{\frac{9 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 4}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{3 \arctan\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}\right)}{a^2} \right) - \frac{Ac \left(\frac{3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2 \right)}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} - \frac{3a^2 \sin(fx+e)^3}{a^2 + \frac{3a^2 \sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2 \sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2 \sin(fx+e)^3}{(\cos(fx+e)+1)^3}} \right) / 3f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")`

[Out] $\frac{2}{3} \cdot (B \cdot d \cdot ((9 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 4) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) + 3 \cdot \arctan(\sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1)) / a^2 - A \cdot c \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + 2) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) - B \cdot c \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 1) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3) - A \cdot d \cdot (3 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 1) / (a^2 + 3 \cdot a^2 \cdot \sin(f \cdot x + e) / (\cos(f \cdot x + e) + 1) + 3 \cdot a^2 \cdot \sin(f \cdot x + e)^2 / (\cos(f \cdot x + e) + 1)^2 + a^2 \cdot \sin(f \cdot x + e)^3 / (\cos(f \cdot x + e) + 1)^3)) / f$

mupad [B] time = 13.74, size = 94, normalized size = 1.11

$$\frac{Bdx}{a^2} \frac{(2Ac - 2Bd) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^2 + (2Ac + 2Ad + 2Bc - 6Bd) \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + \frac{4Ac}{3} + \frac{2Ad}{3} + \frac{2Bc}{3} - \frac{8Bd}{3}}{a^2 f \left(\tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^2,x)`

[Out] $\frac{(B \cdot d \cdot x) / a^2 - ((4 \cdot A \cdot c) / 3 + (2 \cdot A \cdot d) / 3 + (2 \cdot B \cdot c) / 3 - (8 \cdot B \cdot d) / 3 + \tan(e/2 + (f \cdot x) / 2) \cdot (2 \cdot A \cdot c + 2 \cdot A \cdot d + 2 \cdot B \cdot c - 6 \cdot B \cdot d) + \tan(e/2 + (f \cdot x) / 2)^2 \cdot (2 \cdot A \cdot c - 2 \cdot B \cdot d)) / (a^2 \cdot f \cdot (\tan(e/2 + (f \cdot x) / 2) + 1)^3)}{f}$

sympy [A] time = 8.34, size = 1062, normalized size = 12.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**2,x)

[Out] Piecewise((-6*A*c*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*A*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*c*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B*c/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x*tan(e/2 + f*x/2)**3/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 9*B*d*f*x*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 3*B*d*f*x/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 6*B*d*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 18*B*d*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) + 8*B*d/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e))/(a*sin(e) + a)**2, True))

$$3.275 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=65

$$-\frac{(A+2B) \cos(e+fx)}{3f(a^2 \sin(e+fx)+a^2)} - \frac{(A-B) \cos(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

[Out] -1/3*(A-B)*cos(f*x+e)/f/(a+a*sin(f*x+e))^2-1/3*(A+2*B)*cos(f*x+e)/f/(a^2+a^2*sin(f*x+e))

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2750, 2648}

$$-\frac{(A+2B) \cos(e+fx)}{3f(a^2 \sin(e+fx)+a^2)} - \frac{(A-B) \cos(e+fx)}{3f(a \sin(e+fx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]

[Out] -((A - B)*Cos[e + f*x])/(3*f*(a + a*Sin[e + f*x])^2) - ((A + 2*B)*Cos[e + f*x])/(3*f*(a^2 + a^2*Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2} dx &= -\frac{(A-B) \cos(e+fx)}{3f(a+a \sin(e+fx))^2} + \frac{(A+2B) \int \frac{1}{a+a \sin(e+fx)} dx}{3a} \\ &= -\frac{(A-B) \cos(e+fx)}{3f(a+a \sin(e+fx))^2} - \frac{(A+2B) \cos(e+fx)}{3f(a^2+a^2 \sin(e+fx))} \end{aligned}$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.66

$$\frac{\cos(e+fx)((A+2B) \sin(e+fx)+2A+B)}{3a^2f(\sin(e+fx)+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^2,x]

[Out] -1/3*(Cos[e + f*x]*(2*A + B + (A + 2*B)*Sin[e + f*x]))/(a^2*f*(1 + Sin[e + f*x])^2)

fricas [A] time = 0.42, size = 117, normalized size = 1.80

$$\frac{(A + 2B) \cos(fx + e)^2 + (2A + B) \cos(fx + e) + ((A + 2B) \cos(fx + e) - A + B) \sin(fx + e) + A - B}{3(a^2 f \cos(fx + e)^2 - a^2 f \cos(fx + e) - 2a^2 f - (a^2 f \cos(fx + e) + 2a^2 f) \sin(fx + e))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="fricas")

[Out] 1/3*((A + 2*B)*cos(f*x + e)^2 + (2*A + B)*cos(f*x + e) + ((A + 2*B)*cos(f*x + e) - A + B)*sin(f*x + e) + A - B)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

giac [A] time = 0.19, size = 68, normalized size = 1.05

$$\frac{2\left(3A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 3A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 3B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 2A + B\right)}{3a^2 f \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] -2/3*(3*A*tan(1/2*f*x + 1/2*e)^2 + 3*A*tan(1/2*f*x + 1/2*e) + 3*B*tan(1/2*f*x + 1/2*e) + 2*A + B)/(a^2*f*(tan(1/2*f*x + 1/2*e) + 1)^3)

maple [A] time = 0.33, size = 70, normalized size = 1.08

$$\frac{-\frac{2(2A-2B)}{3\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^3} - \frac{2A}{\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1} - \frac{-2A+2B}{\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2}}{f a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)

[Out] 2/f/a^2*(-1/3*(2*A-2*B)/(tan(1/2*f*x+1/2*e)+1)^3-A/(tan(1/2*f*x+1/2*e)+1)-1/2*(-2*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2)

maxima [B] time = 0.35, size = 214, normalized size = 3.29

$$\frac{2\left(\frac{A\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + 2\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}} + \frac{B\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} + 1\right)}{a^2 + \frac{3a^2\sin(fx+e)}{\cos(fx+e)+1} + \frac{3a^2\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{a^2\sin(fx+e)^3}{(\cos(fx+e)+1)^3}}\right)}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] -2/3*(A*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 2)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3) + B*(3*sin(f*x + e)/(cos(f*x + e) + 1) + 1)/(a^2 + 3*a^2*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^2*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + a^2*sin(f*x + e)^3/(cos(f*x + e) + 1)^3))/f

mupad [B] time = 13.39, size = 97, normalized size = 1.49

$$\frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{5A}{2} + \frac{B}{2} - \frac{A \cos(e+fx)}{2} + \frac{B \cos(e+fx)}{2} + \frac{3A \sin(e+fx)}{2} + \frac{3B \sin(e+fx)}{2}\right)}{3a^2 f \left(\frac{3\sqrt{2} \cos\left(\frac{e}{2} - \frac{\pi}{4} + \frac{fx}{2}\right)}{2} - \frac{\sqrt{2} \cos\left(\frac{3e}{2} + \frac{\pi}{4} + \frac{3fx}{2}\right)}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^2,x)
```

```
[Out] -(2*cos(e/2 + (f*x)/2)*((5*A)/2 + B/2 - (A*cos(e + f*x))/2 + (B*cos(e + f*x))/2 + (3*A*sin(e + f*x))/2 + (3*B*sin(e + f*x))/2))/(3*a^2*f*(3*2^(1/2)*cos(e/2 - pi/4 + (f*x)/2))/2 - (2^(1/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/2)
```

sympy [A] time = 4.58, size = 372, normalized size = 5.72

$$\left\{ \begin{array}{l} \frac{6A \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2 f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2 f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 3a^2 f} - \frac{6A \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{3a^2 f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2 f \tan^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 9a^2 f \tan\left(\frac{e}{2} + \frac{fx}{2}\right) + 3a^2 f} - \frac{1}{3a^2 f \tan^3\left(\frac{e}{2} + \frac{fx}{2}\right)} \\ \frac{x(A+B \sin(e))}{(a \sin(e)+a)^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2,x)
```

```
[Out] Piecewise((-6*A*tan(e/2 + f*x/2)**2/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*A*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 4*A/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 6*B*tan(e/2 + f*x/2)/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f) - 2*B/(3*a**2*f*tan(e/2 + f*x/2)**3 + 9*a**2*f*tan(e/2 + f*x/2)**2 + 9*a**2*f*tan(e/2 + f*x/2) + 3*a**2*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**2, True))
```

$$3.276 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=152

$$\frac{2d(Bc - Ad) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{a^2 f (c - d)^2 \sqrt{c^2 - d^2}} - \frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2 f (c - d)^2 (\sin(e + fx) + 1)} - \frac{(A - B) \cos(e + fx)}{3f (c - d) (a \sin(e + fx) + a)^2}$$

[Out] $-1/3*(A*(c-4*d)+B*(2*c+d))*\cos(f*x+e)/a^2/(c-d)^2/f/(1+\sin(f*x+e))-1/3*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^2-2*d*(-A*d+B*c)*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/a^2/(c-d)^2/f/(c^2-d^2)^{(1/2)}$

Rubi [A] time = 0.42, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2d(Bc - Ad) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{a^2 f (c - d)^2 \sqrt{c^2 - d^2}} - \frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2 f (c - d)^2 (\sin(e + fx) + 1)} - \frac{(A - B) \cos(e + fx)}{3f (c - d) (a \sin(e + fx) + a)^2}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]`

[Out] $(-2*d*(B*c - A*d)*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(a^2*(c - d)^2*\text{Sqrt}[c^2 - d^2]*f) - ((A*(c - 4*d) + B*(2*c + d))*\text{Cos}[e + f*x])/(3*a^2*(c - d)^2*f*(1 + \text{Sin}[e + f*x])) - ((A - B)*\text{Cos}[e + f*x])/(3*(c - d)*f*(a + a*\text{Sin}[e + f*x])^2)$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 2660

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

Rule 2978

`Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n_)]`

$n + 1) / (a * f * (2 * m + 1) * (b * c - a * d)), x] + \text{Dist}[1 / (a * (2 * m + 1) * (b * c - a * d)),$
 $\text{Int}[(a + b * \text{Sin}[e + f * x])^{m + 1} * (c + d * \text{Sin}[e + f * x])^n * \text{Simp}[B * (a * c * m + b * d * (n + 1)) + A * (b * c * (m + 1) - a * d * (2 * m + n + 2)) + d * (A * b - a * B) * (m + n + 2) * \text{Sin}[e + f * x], x], x] / ; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b * c - a * d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2 * m] \&\& (\text{IntegerQ}[2 * n] || \text{EqQ}[c, 0])$

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))} dx = -\frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))^2} - \frac{\int \frac{-a(2Bc + A(c - 3d)) - a(A - B)d \sin(e + fx)}{(a + a \sin(e + fx))(c + d \sin(e + fx))} dx}{3a^2(c - d)}$$

$$= -\frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))}$$

$$= -\frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))}$$

$$= -\frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))}$$

$$= -\frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))}$$

$$= -\frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))} - \frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))} - \frac{2d(Bc - Ad) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{a^2(c - d)^2 \sqrt{c^2 - d^2} f} - \frac{(A(c - 4d) + B(2c + d)) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))}$$

Mathematica [A] time = 0.63, size = 229, normalized size = 1.51

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(\frac{6d(Ad - Bc) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^3 \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{\sqrt{c^2 - d^2}} + 2(A - B)(c - d) \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])),x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2] + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 4*d) + B*(2*c + d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (6*d*(-(B*c) + A*d)*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/Sqrt[c^2 - d^2])/(3*a^2*(c - d)^2*f*(1 + Sin[e + f*x])^2)

fricas [B] time = 0.50, size = 1285, normalized size = 8.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm="fricas")


```
[Out] [1/6*(2*(A - B)*c^3 - 2*(A - B)*c^2*d - 2*(A - B)*c*d^2 + 2*(A - B)*d^3 + 2
*((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x
+ e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A*d^2)*cos(f*x + e)^2 + (B*c*d -
A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 + (B*c*d - A*d^2)*cos(f*x + e))*s
in(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin
(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sq
rt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*
((2*A + B)*c^3 - (5*A - 2*B)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos
(f*x + e) - 2*((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 -
((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x
+ e))*sin(f*x + e))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos
(f*x + e)^2 - (a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e
) - 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2
*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d +
2*a^2*c*d^3 - a^2*d^4)*f)*sin(f*x + e)), 1/3*((A - B)*c^3 - (A - B)*c^2*d
- (A - B)*c*d^2 + (A - B)*d^3 + ((A + 2*B)*c^3 - (4*A - B)*c^2*d - (A + 2*B
)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e)^2 - 3*(2*B*c*d - 2*A*d^2 - (B*c*d - A
*d^2)*cos(f*x + e)^2 + (B*c*d - A*d^2)*cos(f*x + e) + (2*B*c*d - 2*A*d^2 +
(B*c*d - A*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(
f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^3 - (5*A - 2*B
)*c^2*d - (2*A + B)*c*d^2 + (5*A - 2*B)*d^3)*cos(f*x + e) - ((A - B)*c^3 -
(A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 - ((A + 2*B)*c^3 - (4*A - B)*c^
2*d - (A + 2*B)*c*d^2 + (4*A - B)*d^3)*cos(f*x + e))*sin(f*x + e))/((a^2*c^
4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e)^2 - (a^2*c^4 - 2*a^
2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f*cos(f*x + e) - 2*(a^2*c^4 - 2*a^2*c^3*d
+ 2*a^2*c*d^3 - a^2*d^4)*f - ((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^
4)*f*cos(f*x + e) + 2*(a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*f)*si
n(f*x + e))]
```

giac [A] time = 0.22, size = 259, normalized size = 1.70

$$2 \left(\frac{3(Bcd - Ad^2) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(a^2 c^2 - 2 a^2 cd + a^2 d^2) \sqrt{c^2 - d^2}} \right) + \frac{3 A c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 6 A d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 3 B d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 3 A c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)}{(a^2 c^2 - 2 a^2 cd + a^2 d^2) \sqrt{c^2 - d^2}}$$

3f

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="giac")
```

```
[Out] -2/3*(3*(B*c*d - A*d^2)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((
c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^2 - 2*a^2*c*d + a^2*d
^2)*sqrt(c^2 - d^2)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 6*A*d*tan(1/2*f*x +
1/2*e)^2 + 3*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*
c*tan(1/2*f*x + 1/2*e) - 9*A*d*tan(1/2*f*x + 1/2*e) + 3*B*d*tan(1/2*f*x + 1
/2*e) + 2*A*c + B*c - 5*A*d + 2*B*d)/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*(tan(
1/2*f*x + 1/2*e) + 1)^3))/f
```

maple [B] time = 0.56, size = 327, normalized size = 2.15

$$\frac{2d^2 \arctan \left(\frac{2c \tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 2d}{2\sqrt{c^2 - d^2}} \right) A}{f a^2 (c - d)^2 \sqrt{c^2 - d^2}} - \frac{2d \arctan \left(\frac{2c \tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 2d}{2\sqrt{c^2 - d^2}} \right) Bc}{f a^2 (c - d)^2 \sqrt{c^2 - d^2}} + \frac{2A}{f a^2 (c - d) \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 1 \right)^2} - \frac{2A}{f a^2 (c - d) \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/f/a^2*d^2/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)
/(c^2-d^2)^(1/2))*A-2/f/a^2*d/(c-d)^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1
/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1
)^2*A-2/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)/(tan(1/2*f*x
+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(c-d)^2/(
tan(1/2*f*x+1/2*e)+1)*A*c+4/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*A*d-2/f/a^
2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)*B*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x, algorithm
="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 14.78, size = 302, normalized size = 1.99

$$2d \operatorname{atan} \left(\frac{\frac{d(A d - B c)(2a^2 c^2 d - 4a^2 c d^2 + 2a^2 d^3)}{a^2 \sqrt{c+d} (c-d)^{5/2}} + \frac{2cd \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(A d - B c)(a^2 c^2 - 2a^2 c d + a^2 d^2)}{a^2 \sqrt{c+d} (c-d)^{5/2}}}{2A d^2 - 2B c d} \right) (A d - B c) \frac{2(2Ac - 5Ad + Bc + 2Bd)}{3(c-d)^2} + \frac{2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)}{f \left(a^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^3 + 3a \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))),x)
```

```
[Out] (2*d*atan(((d*(A*d - B*c)*(2*a^2*d^3 - 4*a^2*c*d^2 + 2*a^2*c^2*d))/(a^2*(c
+ d)^(1/2)*(c - d)^(5/2)) + (2*c*d*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^2*c^2
+ a^2*d^2 - 2*a^2*c*d))/(a^2*(c + d)^(1/2)*(c - d)^(5/2)))/(2*A*d^2 - 2*B*c
*d))*(A*d - B*c))/(a^2*f*(c + d)^(1/2)*(c - d)^(5/2)) - ((2*(2*A*c - 5*A*d
+ B*c + 2*B*d))/(3*(c - d)^2) + (2*tan(e/2 + (f*x)/2)*(A*c - 3*A*d + B*c +
B*d))/(c - d)^2 + (2*tan(e/2 + (f*x)/2)^2*(A*c - 2*A*d + B*d))/(c - d)^2)/(
f*(3*a^2*tan(e/2 + (f*x)/2)^2 + a^2*tan(e/2 + (f*x)/2)^3 + a^2 + 3*a^2*tan(
e/2 + (f*x)/2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

$$3.277 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=275

$$\frac{2d \left(Ad(3c + 2d) - B(2c^2 + 2cd + d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{a^2 f(c-d)^3(c+d)\sqrt{c^2 - d^2}} - \frac{d \left(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2) \right)}{3a^2 f(c-d)^3(c+d)(c+d \sin(e+fx))}$$

[Out] $-1/3*d*(A*(c^2-6*c*d-10*d^2)+B*(2*c^2+9*c*d+4*d^2))*\cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*\sin(f*x+e))-1/3*(A*c-6*A*d+2*B*c+3*B*d)*\cos(f*x+e)/a^2/(c-d)^2/f/(1+\sin(f*x+e))/(c+d*\sin(f*x+e))-1/3*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^2/(c+d*\sin(f*x+e))+2*d*(A*d*(3*c+2*d)-B*(2*c^2+2*c*d+d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^3/(c+d)/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 0.67, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{2d \left(Ad(3c + 2d) - B(2c^2 + 2cd + d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}} \right)}{a^2 f(c-d)^3(c+d)\sqrt{c^2 - d^2}} - \frac{d \left(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2) \right)}{3a^2 f(c-d)^3(c+d)(c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2), x]

[Out] $(2*d*(A*d*(3*c + 2*d) - B*(2*c^2 + 2*c*d + d^2))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(a^2*(c - d)^3*(c + d)*\text{Sqrt}[c^2 - d^2]*f) - (d*(A*(c^2 - 6*c*d - 10*d^2) + B*(2*c^2 + 9*c*d + 4*d^2))*\text{Cos}[e + f*x])/(3*a^2*(c - d)^3*(c + d)*f*(c + d*\text{Sin}[e + f*x])) - ((A*c + 2*B*c - 6*A*d + 3*B*d)*\text{Cos}[e + f*x])/(3*a^2*(c - d)^2*f*(1 + \text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])) - ((A - B)*\text{Cos}[e + f*x])/(3*(c - d)*f*(a + a*\text{Sin}[e + f*x])^2*(c + d*\text{Sin}[e + f*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2754

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e +
f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))^2} dx = -\frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))^2 (c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = -\frac{(Ac + 2Bc - 6Ad + 3Bd) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))(c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = -\frac{d(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2)) \cos(e + fx)}{3a^2(c - d)^3 (c + d)f(c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = -\frac{d(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2)) \cos(e + fx)}{3a^2(c - d)^3 (c + d)f(c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = -\frac{d(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2)) \cos(e + fx)}{3a^2(c - d)^3 (c + d)f(c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = -\frac{d(A(c^2 - 6cd - 10d^2) + B(2c^2 + 9cd + 4d^2)) \cos(e + fx)}{3a^2(c - d)^3 (c + d)f(c + d \sin(e + fx))} - \frac{\int \frac{-a(A(c-4d)+B(c-d)) \cos(e+fx)}{(a+a \sin(e+fx))^2 (c+d \sin(e+fx))^2} dx}{3(c-d)f(a + a \sin(e + fx))} \\ = \frac{2d(Ad(3c + 2d) - B(2c^2 + 2cd + d^2)) \tan^{-1}\left(\frac{d+c \tan\left(\frac{1}{2}(e+fx)\right)}{\sqrt{c^2-d^2}}\right)}{a^2(c - d)^3 (c + d)\sqrt{c^2 - d^2} f}$$

Mathematica [A] time = 2.83, size = 313, normalized size = 1.14

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(-\frac{6d(B(2c^2 + 2cd + d^2) - Ad(3c + 2d)) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^3 \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{(c + d)\sqrt{c^2 - d^2}} + \frac{3d^2}{\sqrt{c^2 - d^2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^2*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2] + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(A*(c - 7*d) + 2*B*(c + 2*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (6*d*(-(A*d*(3*c + 2*d)) + B*(2*c^2 + 2*c*d + d^2))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)*Sqrt[c^2 - d^2]) + (3*d^2*(-(B*c) + A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)/((c + d)*(c + d*Sin[e + f*x])))/(3*a^2*(c - d)^3*f*(1 + Sin[e + f*x])^2)
```

fricas [B] time = 0.59, size = 3123, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [1/6*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - 2*((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^3 + 2*((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)*sin(f*x + e)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 2*((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e) - 2*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e)*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e)), 1/3*((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^3 + ((A + 2*B)*c^5 - 5*(A - B)*c^4*d - (8*A - 5*B)*c^3*d^2 + (A - 4*B)*c^2*d^3 + 7*(A - B)*c*d^4 + (4*A - B)*d^5)*cos(f*x + e)^2 - 3*(4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^3 - (2
```

```
*B*c^3*d - 3*(A - 2*B)*c^2*d^2 - (8*A - 5*B)*c*d^3 - 2*(2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e) + (4*B*c^3*d - 2*(3*A - 4*B)*c^2*d^2 - 2*(5*A - 3*B)*c*d^3 - 2*(2*A - B)*d^4 - (2*B*c^2*d^2 - (3*A - 2*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e)^2 + (2*B*c^3*d - (3*A - 4*B)*c^2*d^2 - (5*A - 3*B)*c*d^3 - (2*A - B)*d^4)*cos(f*x + e))*sin(f*x + e)*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + ((2*A + B)*c^5 - (5*A - 8*B)*c^4*d - 16*(A - B)*c^3*d^2 - 4*(2*A + B)*c^2*d^3 + (14*A - 17*B)*c*d^4 + (13*A - 4*B)*d^5)*cos(f*x + e) - ((A - B)*c^5 - (A - B)*c^4*d - 2*(A - B)*c^3*d^2 + 2*(A - B)*c^2*d^3 + (A - B)*c*d^4 - (A - B)*d^5 - ((A + 2*B)*c^4*d - 3*(2*A - 3*B)*c^3*d^2 - (11*A - 2*B)*c^2*d^3 + 3*(2*A - 3*B)*c*d^4 + 2*(5*A - 2*B)*d^5)*cos(f*x + e)^2 - ((A + 2*B)*c^5 - (4*A - 7*B)*c^4*d - 14*(A - B)*c^3*d^2 - 2*(5*A + B)*c^2*d^3 + (13*A - 16*B)*c*d^4 + (14*A - 5*B)*d^5)*cos(f*x + e))*sin(f*x + e))/((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^3 + (a^2*c^7 - 5*a^2*c^5*d^2 + 2*a^2*c^4*d^3 + 7*a^2*c^3*d^4 - 4*a^2*c^2*d^5 - 3*a^2*c*d^6 + 2*a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f + ((a^2*c^6*d - 2*a^2*c^5*d^2 - a^2*c^4*d^3 + 4*a^2*c^3*d^4 - a^2*c^2*d^5 - 2*a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e)^2 - (a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f*cos(f*x + e) - 2*(a^2*c^7 - a^2*c^6*d - 3*a^2*c^5*d^2 + 3*a^2*c^4*d^3 + 3*a^2*c^3*d^4 - 3*a^2*c^2*d^5 - a^2*c*d^6 + a^2*d^7)*f)*sin(f*x + e))]
```

giac [A] time = 0.23, size = 425, normalized size = 1.55

$$2 \left(\frac{3 \left(2 B c^2 d - 3 A c d^2 + 2 B c d^2 - 2 A d^3 + B d^3 \right) \left(\pi \left[\frac{f x + e}{2 \pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{\left(a^2 c^4 - 2 a^2 c^3 d + 2 a^2 c d^3 - a^2 d^4 \right) \sqrt{c^2 - d^2}} + \frac{3 \left(B c d^3 \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) - A d^4 \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) + B c^2 d^2 \right)}{\left(a^2 c^5 - 2 a^2 c^4 d + 2 a^2 c^2 d^3 - a^2 c d^4 \right) \left(c \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) \right)^2} + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] -2/3*(3*(2*B*c^2*d - 3*A*c*d^2 + 2*B*c*d^2 - 2*A*d^3 + B*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^2*c^4 - 2*a^2*c^3*d + 2*a^2*c*d^3 - a^2*d^4)*sqrt(c^2 - d^2)) + 3*(B*c*d^3*tan(1/2*f*x + 1/2*e) - A*d^4*tan(1/2*f*x + 1/2*e) + B*c^2*d^2 - A*c*d^3)/((a^2*c^5 - 2*a^2*c^4*d + 2*a^2*c^2*d^3 - a^2*c*d^4)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) + (3*A*c*tan(1/2*f*x + 1/2*e)^2 - 9*A*d*tan(1/2*f*x + 1/2*e)^2 + 6*B*d*tan(1/2*f*x + 1/2*e)^2 + 3*A*c*tan(1/2*f*x + 1/2*e) + 3*B*c*tan(1/2*f*x + 1/2*e) - 15*A*d*tan(1/2*f*x + 1/2*e) + 9*B*d*tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 8*A*d + 5*B*d)/((a^2*c^3 - 3*a^2*c^2*d + 3*a^2*c*d^2 - a^2*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^3))/f

maple [B] time = 0.55, size = 770, normalized size = 2.80

$$\frac{2d^4 \tan \left(\frac{fx}{2} + \frac{e}{2} \right) A}{f a^2 (c - d)^3 \left(\left(\tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) c + 2 \tan \left(\frac{fx}{2} + \frac{e}{2} \right) d + c \right) (c + d) c} - \frac{2d^3 \tan \left(\frac{fx}{2} + \frac{e}{2} \right) B}{f a^2 (c - d)^3 \left(\left(\tan^2 \left(\frac{fx}{2} + \frac{e}{2} \right) \right) c + 2 \tan \left(\frac{fx}{2} + \frac{e}{2} \right) d + c \right) (c + d) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x)

```
[Out] 2/f/a^2/(c-d)^3*d^4/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)
/c*tan(1/2*f*x+1/2*e)*A-2/f/a^2/(c-d)^3*d^3/(tan(1/2*f*x+1/2*e)^2*c+2*tan(1
/2*f*x+1/2*e)*d+c)/(c+d)*tan(1/2*f*x+1/2*e)*B+2/f/a^2/(c-d)^3*d^3/(tan(1/2*
f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A-2/f/a^2/(c-d)^3*d^2/(tan(1
/2*f*x+1/2*e)^2*c+2*tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+6/f/a^2/(c-d)^3*d^2/(
c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2
))*A*c+4/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*
x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A-4/f/a^2/(c-d)^3*d/(c+d)/(c^2-d^2)^(1/2)*ar
ctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c^2-4/f/a^2/(c-d)^
3*d^2/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^
2)^(1/2))*B*c-2/f/a^2/(c-d)^3*d^3/(c+d)/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan
(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B+2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)
+1)^2*A-2/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)^2/(tan(1
/2*f*x+1/2*e)+1)^3*A+4/3/f/a^2/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B-2/f/a^2/(
c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c+6/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*
d-4/f/a^2/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^2,x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for
more details)Is 4*d^2-4*c^2 positive or negative?
```

mupad [B] time = 16.76, size = 844, normalized size = 3.07

$$2d \operatorname{atan} \left(\frac{d(-2a^2c^4d+4a^2c^3d^2-4a^2cd^4+2a^2d^5)(2Bc^2-2Ad^2+Bd^2-3Ac d+2Bcd) - 2cd \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(a^2c^4-2a^2c^3d+2a^2cd^3-a^2d^4)(2Bc^2-2Ad^2+Bd^2-3Ac d+2Bcd)}{a^2(c+d)^{3/2}(c-d)^{7/2}} \right) - \frac{2Bd^3-4Ad^3-6Ac d^2+4Bcd^2+4Bc^2d}{a^2(c+d)^{3/2}(c-d)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^2),x)
```

```
[Out] (2*d*atan(((d*(2*a^2*d^5 - 4*a^2*c*d^4 - 2*a^2*c^4*d + 4*a^2*c^3*d^2)*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c - d)^(7/2
)) - (2*c*d*tan(e/2 + (f*x)/2)*(a^2*c^4 - a^2*d^4 + 2*a^2*c*d^3 - 2*a^2*c^3
*d)*(2*B*c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*(c + d)^(3/2)*(c
- d)^(7/2)))/(2*B*d^3 - 4*A*d^3 - 6*A*c*d^2 + 4*B*c*d^2 + 4*B*c^2*d))*(2*B*
c^2 - 2*A*d^2 + B*d^2 - 3*A*c*d + 2*B*c*d))/(a^2*f*(c + d)^(3/2)*(c - d)^(7
/2)) - ((2*(2*A*c^3 - 3*A*d^3 + B*c^3 - 8*A*c*d^2 - 6*A*c^2*d + 8*B*c*d^2 +
6*B*c^2*d))/(3*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2
)^2*(5*A*c^3 - 9*A*d^3 + B*c^3 - 30*A*c*d^2 - 11*A*c^2*d + 27*B*c*d^2 + 17*
B*c^2*d))/(3*c*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(A*c^
4 - 3*A*d^4 + B*c^4 - 9*A*c^2*d^2 + 8*B*c^2*d^2 - 7*A*c*d^3 - 2*A*c^3*d + 7
*B*c*d^3 + 4*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2
+ (f*x)/2)*(3*A*c^4 - 3*A*d^4 + 3*B*c^4 - 27*A*c^2*d^2 + 30*B*c^2*d^2 - 25
*A*c*d^3 - 8*A*c^3*d + 13*B*c*d^3 + 14*B*c^3*d))/(3*c*(c + d)*(c - d)*(c^2
- 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^4 - A*d^4 - 3*A*c^2*d^2 + 2*
B*c^2*d^2 - 2*A*c^3*d + B*c*d^3 + 2*B*c^3*d))/(c*(c + d)*(c - d)*(c^2 - 2*c
*d + d^2)))/(f*(a^2*c + tan(e/2 + (f*x)/2)*(3*a^2*c + 2*a^2*d) + tan(e/2 +
```

```
(f*x)/2)^4*(3*a^2*c + 2*a^2*d) + tan(e/2 + (f*x)/2)^2*(4*a^2*c + 6*a^2*d) +  
tan(e/2 + (f*x)/2)^3*(4*a^2*c + 6*a^2*d) + a^2*c*tan(e/2 + (f*x)/2)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```


$$3.278 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^2(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=386

$$\frac{d \left(A \left(2c^2 - 16cd - 21d^2 \right) + B \left(4c^2 + 19cd + 12d^2 \right) \right) \cos(e+fx)}{6a^2 f(c-d)^3(c+d)(c+d \sin(e+fx))^2} + \frac{d \left(Ad \left(12c^2 + 16cd + 7d^2 \right) - B \left(6c^3 + 12c^2d \right) \right)}{a^2 f(c-d)^4(c+d)}$$

[Out] $-1/6*d*(A*(2*c^2-16*c*d-21*d^2)+B*(4*c^2+19*c*d+12*d^2))*\cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*\sin(f*x+e))^2-1/3*(A*c-8*A*d+2*B*c+5*B*d)*\cos(f*x+e)/a^2/(c-d)^2/f/(1+\sin(f*x+e))/(c+d*\sin(f*x+e))^2-1/3*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^2/(c+d*\sin(f*x+e))^2-1/6*d*(A*(2*c^3-16*c^2*d-59*c*d^2-32*d^3)+B*(4*c^3+37*c^2*d+44*c*d^2+20*d^3))*\cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*\sin(f*x+e))+d*(A*d*(12*c^2+16*c*d+7*d^2)-B*(6*c^3+12*c^2*d+13*c*d^2+4*d^3))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^2/(c-d)^4/(c+d)^2/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 0.96, antiderivative size = 386, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{d \left(Ad \left(12c^2 + 16cd + 7d^2 \right) - B \left(12c^2d + 6c^3 + 13cd^2 + 4d^3 \right) \right) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e+fx) \right) + d}{\sqrt{c^2-d^2}} \right)}{a^2 f(c-d)^4(c+d)^2 \sqrt{c^2-d^2}} + \frac{d \left(A \left(-16c^2d + 2c^3 - 59cd^2 + 4d^3 \right) \right)}{6a^2 f(c-d)^4(c+d)^2 \sqrt{c^2-d^2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + a*\text{Sin}[e + f*x])^2*(c + d*\text{Sin}[e + f*x])^3), x]$

[Out] $(d*(A*d*(12*c^2 + 16*c*d + 7*d^2) - B*(6*c^3 + 12*c^2*d + 13*c*d^2 + 4*d^3))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(a^2*(c - d)^4*(c + d)^2*\text{Sqrt}[c^2 - d^2]*f) - (d*(A*(2*c^2 - 16*c*d - 21*d^2) + B*(4*c^2 + 19*c*d + 12*d^2))*\text{Cos}[e + f*x])/(6*a^2*(c - d)^3*(c + d)*f*(c + d*\text{Sin}[e + f*x])^2) - ((A*c + 2*B*c - 8*A*d + 5*B*d)*\text{Cos}[e + f*x])/(3*a^2*(c - d)^2*f*(1 + \text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^2) - ((A - B)*\text{Cos}[e + f*x])/(3*(c - d)*f*(a + a*\text{Sin}[e + f*x])^2*(c + d*\text{Sin}[e + f*x])^2) - (d*(A*(2*c^3 - 16*c^2*d - 59*c*d^2 - 32*d^3) + B*(4*c^3 + 37*c^2*d + 44*c*d^2 + 20*d^3))*\text{Cos}[e + f*x])/(6*a^2*(c - d)^4*(c + d)^2*f*(c + d*\text{Sin}[e + f*x]))$

Rule 12

$\text{Int}[(a_*)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_*)*(v_)] /; \text{FreeQ}[b, x]$

Rule 204

$\text{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a_*) + (b_*)*(x_) + (c_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2754

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^2(c + d \sin(e + fx))^3} dx = -\frac{(A - B) \cos(e + fx)}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{(Ac + 2Bc - 8Ad + 5Bd) \cos(e + fx)}{3a^2(c - d)^2 f(1 + \sin(e + fx))(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= -\frac{d(A(2c^2 - 16cd - 21d^2) + B(4c^2 + 19cd + 12d^2)) \cos(e + fx)}{6a^2(c - d)^3(c + d)f(c + d \sin(e + fx))^2} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

$$= \frac{d(Ad(12c^2 + 16cd + 7d^2) - B(6c^3 + 12c^2d + 13cd^2 + 4d^3)) \tan(e + fx)}{a^2(c - d)^4(c + d)^2 \sqrt{c^2 - d^2} f} - \frac{\int \frac{-a(A(c-5d)}{(a+as}}{3(c - d)f(a + a \sin(e + fx))^2(c + d \sin(e + fx))^2} dx$$

Mathematica [B] time = 6.37, size = 1522, normalized size = 3.94
 result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*SIN[e + f*x])/((a + a*SIN[e + f*x])^2*(c + d*SIN[e + f*x])^3),x]
```

```
[Out] -((d*(6*B*c^3 - 12*A*c^2*d + 12*B*c^2*d - 16*A*c*d^2 + 13*B*c*d^2 - 7*A*d^3 + 4*B*d^3)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*SIN[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + SIN[(e + f*x)/2])^4)/((c - d)^4*(c + d)^2*Sqrt[c^2 - d^2]*f*(a + a*SIN[e + f*x])^2) + ((Cos[(e + f*x)/2] + SIN[(e + f*x)/2])*(48*B*c^5*Cos[(e + f*x)/2] - 96*A*c^4*d*Cos[(e + f*x)/2] + 240*B*c^4*d*Cos[(e + f*x)/2] - 524*A*c^3*d^2*Cos[(e + f*x)/2] + 536*B*c^3*d^2*Cos[(e + f*x)/2] - 776*A*c^2*d^3*Cos[(e + f*x)/2] + 701*B*c^2*d^3*Cos[(e + f*x)/2] - 487*A*c*d^4*Cos[(e + f*x)/2] + 400*B*c*d^4*Cos[(e + f*x)/2] - 112*A*d^5*Cos[(e + f*x)/2] + 70*B*d^5*Cos[(e + f*x)/2] - 16*A*c^5*Cos[(3*(e + f*x))/2] - 32*B*c^5*Cos[(3*(e + f*x))/2] + 80*A*c^4*d*Cos[(3*(e + f*x))/2] - 224*B*c^4*d*Cos[(3*(e + f*x))/2] + 536*A*c^3*d^2*Cos[(3*(e + f*x))/2] - 728*B*c^3*d^2*Cos[(3*(e + f*x))/2] + 1028*A*c^2*d^3*Cos[(3*(e + f*x))/2] - 893*B*c^2*d^3*Cos[(3*(e + f*x))/2] + 695*A*c*d^4*Cos[(3*(e + f*x))/2] - 482*B*c*d^4*Cos[(3*(e + f*x))/2] + 134*A*d^5*Cos[(3*(e + f*x))/2] - 98*B*d^5*Cos[(3*(e + f*x))/2] + 24*B*c^3*d^2*Cos[(5*(e + f*x))/2] - 12*A*c^2*d^3*Cos[(5*(e + f*x))/2] + 21*B*c^2*d^3*Cos[(5*(e + f*x))/2] - 15*A*c*d^4*Cos[(5*(e + f*x))/2] - 18*B*c*d^4*Cos[(5*(e + f*x))/2] + 6*A*d^5*Cos[(5*(e + f*x))/2] - 6*B*d^5*Cos[(5*(e + f*x))/2] + 4*A*c^3*d^2*Cos[(7*(e + f*x))/2] + 8*B*c^3*d^2*Cos[(7*(e + f*x))/2] - 32*A*c^2*d^3*Cos[(7*(e + f*x))/2] + 59*B*c^2*d^3*Cos[(7*(e + f*x))/2] - 97*A*c*d^4*Cos[(7*(e + f*x))/2] + 76*B*c*d^4*Cos[(7*(e + f*x))/2] - 52*A*d^5*Cos[(7*(e + f*x))/2] + 34*B*d^5*Cos[(7*(e + f*x))/2] + 48*A*c^5*SIN[(e + f*x)/2] + 48*B*c^5*SIN[(e + f*x)/2] - 224*A*c^4*d*SIN[(e + f*x)/2] + 416*B*c^4*d*SIN[(e + f*x)/2] - 872*A*c^3*d^2*SIN[(e + f*x)/2] + 992*B*c^3*d^2*SIN[(e + f*x)/2] - 1144*A*c^2*d^3*SIN[(e + f*x)/2] + 967*B*c^2*d^3*SIN[(e + f*x)/2] - 685*A*c*d^4*SIN[(e + f*x)/2] + 496*B*c*d^4*SIN[(e + f*x)/2] - 168*A*d^5*SIN[(e + f*x)/2] + 126*B*d^5*SIN[(e + f*x)/2] + 48*B*c^4*d*SIN[(3*(e + f*x))/2] - 132*A*c^3*d^2*SIN[(3*(e + f*x))/2] + 96*B*c^3*d^2*SIN[(3*(e + f*x))/2] - 204*A*c^2*d^3*SIN[(3*(e + f*x))/2] + 207*B*c^2*d^3*SIN[(3*(e + f*x))/2] - 165*A*c*d^4*SIN[(3*(e + f*x))/2] + 174*B*c*d^4*SIN[(3*(e + f*x))/2] - 66*A*d^5*SIN[(3*(e + f*x))/2] + 42*B*d^5*SIN[(3*(e + f*x))/2] - 16*A*c^4*d*SIN[(5*(e + f*x))/2] - 32*B*c^4*d*SIN[(5*(e + f*x))/2] + 116*A*c^3*d^2*SIN[(5*(e + f*x))/2] - 224*B*c^3*d^2*SIN[(5*(e + f*x))/2] + 412*A*c^2*d^3*SIN[(5*(e + f*x))/2] - 409*B*c^2*d^3*SIN[(5*(e + f*x))/2] + 403*A*c*d^4*SIN[(5*(e + f*x))/2] - 286*B*c*d^4*SIN[(5*(e + f*x))/2] + 114*A*d^5*SIN[(5*(e + f*x))/2] - 78*B*d^5*SIN[(5*(e + f*x))/2] + 15*B*c^2*d^3*SIN[(7*(e + f*x))/2] - 21*A*c*d^4*SIN[(7*(e + f*x))/2] + 12*B*c*d^4*SIN[(7*(e + f*x))/2] - 12*A*d^5*SIN[(7*(e + f*x))/2] + 6*B*d^5*SIN[(7*(e + f*x))/2]))/(48*(c - d)^4*(c + d)^2*f*(a + a*SIN[e + f*x])^2*(c + d*SIN[e + f*x])^2)
```

fricas [B] time = 0.70, size = 4997, normalized size = 12.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(A - B)*c^7 - 4*(A - B)*c^6*d - 12*(A - B)*c^5*d^2 + 12*(A - B)*c^4*d^3 + 12*(A - B)*c^3*d^4 - 12*(A - B)*c^2*d^5 - 4*(A - B)*c*d^6 + 4*(A - B)*d^7 - 2*(2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*cos(f*x + e)^4 - 2*(4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4*d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6 + (43*A - 28*B)*d^7)*cos(f*x + e)^3 + 2*(2*(A + 2*B)*c^7 - 6*(2*A - 3*
```

$$\begin{aligned}
& B)*c^6*d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)* \\
& c^3*d^4 + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7 \\
&)*\cos(f*x + e)^2 + 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c \\
& ^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 \\
& + (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6 \\
&)*\cos(f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B) \\
& *c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e)^3 - (6*B*c^5 \\
& *d - 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 \\
& - (76*A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*\cos(f*x + e)^2 + (6*B*c^5*d - 1 \\
& 2*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(\\
& 10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e) + (12*B*c^5*d - 24*(A - 2 \\
& *B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - \\
& 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A \\
& - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A \\
& - 3*B)*c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)* \\
& d^6)*\cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3 \\
& *d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\co \\
& s(f*x + e))*\sin(f*x + e))*\sqrt{-c^2 + d^2}*\log(-((2*c^2 - d^2)*\cos(f*x + e) \\
& ^2 - 2*c*d*\sin(f*x + e) - c^2 - d^2 - 2*(c*\cos(f*x + e))*\sin(f*x + e) + d*\co \\
& s(f*x + e))*\sqrt{-c^2 + d^2}))/((d^2*\cos(f*x + e)^2 - 2*c*d*\sin(f*x + e) - c^2 \\
& - d^2)) + 4*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 19*B)*c^5*d^2 \\
& - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - 21*B)*c^2*d \\
& ^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*\cos(f*x + e) - 2*(2*(A - B)* \\
& c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6*(A - B)*c \\
& ^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2*(A + 2*B) \\
& *c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A + 17*B)*c^2 \\
& *d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*\cos(f*x + e)^3 - (4*(A + 2 \\
& *B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(3*A + B)*c \\
& ^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - 8*B)*d^7) \\
& *\cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A - 18*B)*c \\
& ^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21*A - 22*B) \\
& *c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*\cos(f*x + e))*\sin(f*x + \\
& e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 6*a^2* \\
& c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*\cos(f*x + e)^4 - (2*a^2 \\
& *c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^5*d^5 - 1 \\
& 2*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f*x + e)^3 \\
& - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^2*c^6*d^4 \\
& + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^8 + 2*a^2* \\
& c*d^9 - 3*a^2*d^10)*f*\cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c \\
& ^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f*x + e) + 2*(a^2 \\
& *c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a \\
& ^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^5 \\
& - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*\cos(f*x + e)^3 \\
& + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6*a^2*c^5*d^5 \\
& - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a^2*d^10)*f \\
& *\cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d \\
& ^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f*x + e) - 2*(a^2*c^10 - 5*a^2*c^8*d^2 \\
& + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f)*\sin(f*x + \\
& e)), -1/6*(2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B) \\
& *c^4*d^3 + 6*(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - \\
& B)*d^7 - (2*(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3* \\
& d^4 - (16*A + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*\cos(\\
& f*x + e)^4 - (4*(A + 2*B)*c^6*d - 4*(7*A - 16*B)*c^5*d^2 - 118*(A - B)*c^4* \\
& d^3 - (106*A - 25*B)*c^3*d^4 + (71*A - 98*B)*c^2*d^5 + (134*A - 89*B)*c*d^6 \\
& + (43*A - 28*B)*d^7)*\cos(f*x + e)^3 + (2*(A + 2*B)*c^7 - 6*(2*A - 3*B)*c^6 \\
& *d - 12*(3*A - 4*B)*c^5*d^2 - 3*(18*A - 17*B)*c^4*d^3 - 3*(13*A + B)*c^3*d^4 \\
& + 3*(13*A - 17*B)*c^2*d^5 + (73*A - 49*B)*c*d^6 + 9*(3*A - 2*B)*d^7)*\cos(\\
& f*x + e)^2 - 3*(12*B*c^5*d - 24*(A - 2*B)*c^4*d^2 - 2*(40*A - 43*B)*c^3*d^3 \\
& - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)*c*d^5 - 2*(7*A - 4*B)*d^6 + (6*
\end{aligned}$$

$$\begin{aligned}
& B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos \\
& (f*x + e)^4 - (12*B*c^4*d^2 - 6*(4*A - 5*B)*c^3*d^3 - 2*(22*A - 19*B)*c^2*d \\
& ^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e)^3 - (6*B*c^5*d - \\
& 12*(A - 3*B)*c^4*d^2 - (64*A - 79*B)*c^3*d^3 - (107*A - 92*B)*c^2*d^4 - (76 \\
& *A - 55*B)*c*d^5 - 3*(7*A - 4*B)*d^6)*\cos(f*x + e)^2 + (6*B*c^5*d - 12*(A - \\
& 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - \\
& 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e) + (12*B*c^5*d - 24*(A - 2*B)*c^ \\
& 4*d^2 - 2*(40*A - 43*B)*c^3*d^3 - 6*(17*A - 14*B)*c^2*d^4 - 6*(10*A - 7*B)* \\
& c*d^5 - 2*(7*A - 4*B)*d^6 - (6*B*c^3*d^3 - 12*(A - B)*c^2*d^4 - (16*A - 13* \\
& B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x + e)^3 - 2*(6*B*c^4*d^2 - 6*(2*A - 3*B) \\
& *c^3*d^3 - (28*A - 25*B)*c^2*d^4 - (23*A - 17*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos \\
& (f*x + e)^2 + (6*B*c^5*d - 12*(A - 2*B)*c^4*d^2 - (40*A - 43*B)*c^3*d^3 - \\
& 3*(17*A - 14*B)*c^2*d^4 - 3*(10*A - 7*B)*c*d^5 - (7*A - 4*B)*d^6)*\cos(f*x \\
& + e))*\sin(f*x + e))*\sqrt{c^2 - d^2}*\arctan(-(c*\sin(f*x + e) + d)/(\sqrt{c^2 \\
& - d^2}*\cos(f*x + e))) + 2*((2*A + B)*c^7 - (5*A - 14*B)*c^6*d - 3*(12*A - 1 \\
& 9*B)*c^5*d^2 - 3*(25*A - 21*B)*c^4*d^3 - 3*(13*A + 4*B)*c^3*d^4 + 3*(20*A - \\
& 21*B)*c^2*d^5 + (73*A - 46*B)*c*d^6 + 2*(10*A - 7*B)*d^7)*\cos(f*x + e) - (\\
& 2*(A - B)*c^7 - 2*(A - B)*c^6*d - 6*(A - B)*c^5*d^2 + 6*(A - B)*c^4*d^3 + 6 \\
& *(A - B)*c^3*d^4 - 6*(A - B)*c^2*d^5 - 2*(A - B)*c*d^6 + 2*(A - B)*d^7 + (2 \\
& *(A + 2*B)*c^5*d^2 - (16*A - 37*B)*c^4*d^3 - (61*A - 40*B)*c^3*d^4 - (16*A \\
& + 17*B)*c^2*d^5 + (59*A - 44*B)*c*d^6 + 4*(8*A - 5*B)*d^7)*\cos(f*x + e)^3 - \\
& (4*(A + 2*B)*c^6*d - 30*(A - 2*B)*c^5*d^2 - 3*(34*A - 27*B)*c^4*d^3 - 15*(\\
& 3*A + B)*c^3*d^4 + 3*(29*A - 27*B)*c^2*d^5 + 15*(5*A - 3*B)*c*d^6 + (11*A - \\
& 8*B)*d^7)*\cos(f*x + e)^2 - 2*((A + 2*B)*c^7 - (4*A - 13*B)*c^6*d - 3*(11*A \\
& - 18*B)*c^5*d^2 - 6*(13*A - 11*B)*c^4*d^3 - 3*(14*A + 3*B)*c^3*d^4 + 3*(21 \\
& *A - 22*B)*c^2*d^5 + (74*A - 47*B)*c*d^6 + (19*A - 13*B)*d^7)*\cos(f*x + e)) \\
& *\sin(f*x + e))/((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^2*c^5*d^ \\
& 5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*\cos(f*x + e)^ \\
& 4 - (2*a^2*c^9*d - 3*a^2*c^8*d^2 - 6*a^2*c^7*d^3 + 10*a^2*c^6*d^4 + 6*a^2*c^ \\
& ^5*d^5 - 12*a^2*c^4*d^6 - 2*a^2*c^3*d^7 + 6*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f \\
& *x + e)^3 - (a^2*c^10 + 2*a^2*c^9*d - 7*a^2*c^8*d^2 - 8*a^2*c^7*d^3 + 18*a^ \\
& 2*c^6*d^4 + 12*a^2*c^5*d^5 - 22*a^2*c^4*d^6 - 8*a^2*c^3*d^7 + 13*a^2*c^2*d^ \\
& 8 + 2*a^2*c*d^9 - 3*a^2*d^10)*f*\cos(f*x + e)^2 + (a^2*c^10 - 5*a^2*c^8*d^2 \\
& + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f*x + e \\
&) + 2*(a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^ \\
& ^2*d^8 - a^2*d^10)*f - ((a^2*c^8*d^2 - 2*a^2*c^7*d^3 - 2*a^2*c^6*d^4 + 6*a^ \\
& 2*c^5*d^5 - 6*a^2*c^3*d^7 + 2*a^2*c^2*d^8 + 2*a^2*c*d^9 - a^2*d^10)*f*\cos(f \\
& *x + e)^3 + 2*(a^2*c^9*d - a^2*c^8*d^2 - 4*a^2*c^7*d^3 + 4*a^2*c^6*d^4 + 6* \\
& a^2*c^5*d^5 - 6*a^2*c^4*d^6 - 4*a^2*c^3*d^7 + 4*a^2*c^2*d^8 + a^2*c*d^9 - a \\
& ^2*d^10)*f*\cos(f*x + e)^2 - (a^2*c^10 - 5*a^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10 \\
& *a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f*\cos(f*x + e) - 2*(a^2*c^10 - 5*a \\
& ^2*c^8*d^2 + 10*a^2*c^6*d^4 - 10*a^2*c^4*d^6 + 5*a^2*c^2*d^8 - a^2*d^10)*f) \\
& *\sin(f*x + e))]
\end{aligned}$$

giac [B] time = 0.31, size = 944, normalized size = 2.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] $\begin{aligned}
& -1/3*(3*(6*B*c^3*d - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16*A*c*d^3 + 13*B*c*d^3 \\
& - 7*A*d^4 + 4*B*d^4)*(pi*\text{floor}(1/2*(f*x + e)/pi + 1/2)*\text{sgn}(c) + \arctan((c*\text{t} \\
& \text{an}(1/2*f*x + 1/2*e) + d)/\sqrt{c^2 - d^2}))/((a^2*c^6 - 2*a^2*c^5*d - a^2*c^ \\
& 4*d^2 + 4*a^2*c^3*d^3 - a^2*c^2*d^4 - 2*a^2*c*d^5 + a^2*d^6)*\sqrt{c^2 - d^2} \\
&)) + 3*(7*B*c^4*d^3*\tan(1/2*f*x + 1/2*e)^3 - 9*A*c^3*d^4*\tan(1/2*f*x + 1/2* \\
& e)^3 + 4*B*c^3*d^4*\tan(1/2*f*x + 1/2*e)^3 - 4*A*c^2*d^5*\tan(1/2*f*x + 1/2*e \\
&)^3 + 2*A*c*d^6*\tan(1/2*f*x + 1/2*e)^3 + 6*B*c^5*d^2*\tan(1/2*f*x + 1/2*e)^2 \\
& - 8*A*c^4*d^3*\tan(1/2*f*x + 1/2*e)^2 + 4*B*c^4*d^3*\tan(1/2*f*x + 1/2*e)^2
\end{aligned}$

$$\begin{aligned}
& - 4*A*c^3*d^4*\tan(1/2*f*x + 1/2*e)^2 + 13*B*c^3*d^4*\tan(1/2*f*x + 1/2*e)^2 \\
& - 15*A*c^2*d^5*\tan(1/2*f*x + 1/2*e)^2 + 8*B*c^2*d^5*\tan(1/2*f*x + 1/2*e)^2 \\
& - 8*A*c*d^6*\tan(1/2*f*x + 1/2*e)^2 + 2*B*c*d^6*\tan(1/2*f*x + 1/2*e)^2 + 2*A \\
& *d^7*\tan(1/2*f*x + 1/2*e)^2 + 17*B*c^4*d^3*\tan(1/2*f*x + 1/2*e) - 23*A*c^3* \\
& d^4*\tan(1/2*f*x + 1/2*e) + 12*B*c^3*d^4*\tan(1/2*f*x + 1/2*e) - 12*A*c^2*d^5 \\
& *\tan(1/2*f*x + 1/2*e) + 4*B*c^2*d^5*\tan(1/2*f*x + 1/2*e) + 2*A*c*d^6*\tan(1/ \\
& 2*f*x + 1/2*e) + 6*B*c^5*d^2 - 8*A*c^4*d^3 + 4*B*c^4*d^3 - 4*A*c^3*d^4 + B* \\
& c^3*d^4 + A*c^2*d^5)/((a^2*c^8 - 2*a^2*c^7*d - a^2*c^6*d^2 + 4*a^2*c^5*d^3 \\
& - a^2*c^4*d^4 - 2*a^2*c^3*d^5 + a^2*c^2*d^6)*(c*\tan(1/2*f*x + 1/2*e)^2 + 2* \\
& d*\tan(1/2*f*x + 1/2*e) + c)^2) + 2*(3*A*c*\tan(1/2*f*x + 1/2*e)^2 - 12*A*d*t \\
& \tan(1/2*f*x + 1/2*e)^2 + 9*B*d*\tan(1/2*f*x + 1/2*e)^2 + 3*A*c*\tan(1/2*f*x + \\
& 1/2*e) + 3*B*c*\tan(1/2*f*x + 1/2*e) - 21*A*d*\tan(1/2*f*x + 1/2*e) + 15*B*d* \\
& \tan(1/2*f*x + 1/2*e) + 2*A*c + B*c - 11*A*d + 8*B*d)/((a^2*c^4 - 4*a^2*c^3*d \\
& d + 6*a^2*c^2*d^2 - 4*a^2*c*d^3 + a^2*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^3)/f
\end{aligned}$$

maple [B] time = 0.59, size = 2641, normalized size = 6.84

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x)

[Out]
$$\begin{aligned}
& -4/f/a^2*d^4/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/ \\
& 2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B-1/f/a^2*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e \\
&)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c+4/f/a^2*d^5/(c-d)^4/(\\
& \tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2* \\
& f*x+1/2*e)^3*A+15/f/a^2*d^5/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1 \\
& /2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^2*A-8/f/a^2*d^5/(c-d)^4/(ta \\
& n(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f* \\
& x+1/2*e)^2*B+12/f/a^2*d^5/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2 \\
& *e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-4/f/a^2*d^5/(c-d)^4/(\tan(1/ \\
& 2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/ \\
& 2*e)*B+8/f/a^2*d^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c \\
&)^2/(c^2+2*c*d+d^2)*A*c^2+4/f/a^2*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan \\
& (1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A*c-6/f/a^2*d^2/(c-d)^4/(\tan(1/2*f*x \\
& +1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^3+7/f/a^2*d^4/(\\
& c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2 \\
& *d)/(c^2-d^2)^{(1/2)})*A-4/f/a^2*d^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/ \\
& 2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*B*c^2-2/f/a^2/(c-d)^4/(\tan(1/2*f*x+1/2* \\
& e)+1)*A*c+8/f/a^2/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)*A*d-6/f/a^2/(c-d)^4/(\tan(1 \\
& /2*f*x+1/2*e)+1)*B*d+2/f/a^2/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*A-2/f/a^2/(c- \\
& d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*B-4/3/f/a^2/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3* \\
& A-4/f/a^2*d^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(\\
& c^2+2*c*d+d^2)*c^2*\tan(1/2*f*x+1/2*e)^2*B-13/f/a^2*d^4/(c-d)^4/(\tan(1/2*f*x \\
& +1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e \\
&)^2*B-2/f/a^2*d^6/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c) \\
& ^2/(c^2+2*c*d+d^2)/c*\tan(1/2*f*x+1/2*e)^2*B+23/f/a^2*d^4/(c-d)^4/(\tan(1/2*f \\
& *x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2 \\
& *e)*A-13/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c* \\
& \tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c+9/f/a^2*d^4/(c-d)^4/(\tan(1/2*f \\
& *x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2 \\
& *e)^3*A-2/f/a^2*d^6/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+ \\
& c)^2/c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*A-7/f/a^2*d^3/(c-d)^4/(\tan(1/2* \\
& f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x+ \\
& 1/2*e)^3*B-4/f/a^2*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e) \\
& *d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*B+8/f/a^2*d^3/(c-d)^4/(\tan(1/ \\
& 2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*\tan(1/2*f \\
& *x+1/2*e)^2*A-2/f/a^2*d^6/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2 \\
& *e)*d+c)^2/c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A-1/f/a^2*d^5/(c-d)^4/(\tan(\\
& 1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A-17/f/a^2*d
\end{aligned}$$

$$\begin{aligned} &^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c \\ &*d+d^2)*\tan(1/2*f*x+1/2*e)*B+16/f/a^2*d^3/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2) \\ &^{(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})}*A*c+4/f/a^2 \\ &*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d \\ &+d^2)*c*\tan(1/2*f*x+1/2*e)^2*A+8/f/a^2*d^6/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+ \\ &2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*\tan(1/2*f*x+1/2*e)^2*A-2/f/a^ \\ &2*d^7/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c* \\ &d+d^2)/c^2*\tan(1/2*f*x+1/2*e)^2*A-6/f/a^2*d^2/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2 \\ &*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*\tan(1/2*f*x+1/2*e)^2*B-1 \\ &2/f/a^2*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(\\ &c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B-6/f/a^2*d/(c-d)^4/(c^2+2*c*d+d^2)/(c^2- \\ &d^2)^{(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})}*B*c^3-1 \\ &2/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)*\arctan(1/2*(2*c*\tan(1/2 \\ &*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})}*B*c^2+12/f/a^2*d^2/(c-d)^4/(c^2+2*c*d+d^2 \\ &)/(c^2-d^2)^{(1/2)*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})}* \\ &A*c^2+4/3/f/a^2/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^2/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see 'assume?' for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 17.69, size = 1686, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^3),x)

[Out]
$$\begin{aligned} &(d*\operatorname{atan}(((d*(4*a^2*c*d^6 - 2*a^2*d^7 - 2*a^2*c^6*d + 2*a^2*c^2*d^5 - 8*a^2* \\ &c^3*d^4 + 2*a^2*c^4*d^3 + 4*a^2*c^5*d^2)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16* \\ &A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(2*a^2*(c + d)^{(5/2)}*(c - \\ &d)^{(9/2)}) + (c*d*\tan(e/2 + (f*x)/2)*(2*a^2*c*d^5 - a^2*d^6 - a^2*c^6 + 2*a^ \\ &2*c^5*d + a^2*c^2*d^4 - 4*a^2*c^3*d^3 + a^2*c^4*d^2)*(6*B*c^3 - 7*A*d^3 + 4 \\ &*B*d^3 - 16*A*c*d^2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*(c + d)^{(\\ &5/2)}*(c - d)^{(9/2)})))/(4*B*d^4 - 7*A*d^4 - 12*A*c^2*d^2 + 12*B*c^2*d^2 - 16* \\ &A*c*d^3 + 13*B*c*d^3 + 6*B*c^3*d)*(6*B*c^3 - 7*A*d^3 + 4*B*d^3 - 16*A*c*d^ \\ &2 - 12*A*c^2*d + 13*B*c*d^2 + 12*B*c^2*d))/(a^2*f*(c + d)^{(5/2)}*(c - d)^{(9/ \\ &2)}) - ((\tan(e/2 + (f*x)/2)^5*(2*A*c^6 + 2*A*d^6 + 2*B*c^6 - 23*A*c^2*d^4 - \\ &40*A*c^3*d^3 - 38*A*c^4*d^2 + 6*B*c^2*d^4 + 43*B*c^3*d^3 + 40*B*c^4*d^2 - 4 \\ &*A*c*d^5 - 4*A*c^5*d + 2*B*c*d^5 + 12*B*c^5*d))/(c^2*(c^5 - 3*c^4*d - 3*c*d \\ &^4 + d^5 + 2*c^2*d^3 + 2*c^3*d^2)) + (4*A*c^5 + 3*A*d^5 + 2*B*c^5 - 46*A*c^ \\ &2*d^3 - 40*A*c^3*d^2 + 28*B*c^2*d^3 + 52*B*c^3*d^2 - 12*A*c*d^4 - 14*A*c^4* \\ &d + 3*B*c*d^4 + 20*B*c^4*d)/(3*(c + d)*(c^2 - d^2)*(3*c*d^2 - 3*c^2*d + c^3 \\ &- d^3)) + (2*\tan(e/2 + (f*x)/2)^3*(6*A*c^6 + 9*A*d^6 + 6*B*c^6 - 177*A*c^2 \\ &*d^4 - 212*A*c^3*d^3 - 102*A*c^4*d^2 + 105*B*c^2*d^4 + 215*B*c^3*d^3 + 150* \\ &B*c^4*d^2 - 33*A*c*d^5 - 16*A*c^5*d + 9*B*c*d^5 + 40*B*c^5*d))/(3*c^2*(c^2 \\ &- d^2)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (\tan(e/2 + (f*x)/2)*(6*A*c^5 + 6* \\ &A*d^5 + 6*B*c^5 - 160*A*c^2*d^3 - 114*A*c^3*d^2 + 97*B*c^2*d^3 + 156*B*c^3* \\ &d^2 - 33*A*c*d^4 - 20*A*c^4*d + 12*B*c*d^4 + 44*B*c^4*d))/(3*c*(c^2 - d^2)* \\ &(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (\tan(e/2 + (f*x)/2)^2*(14*A*c^7 + 6*A*d^ \\ &7 + 4*B*c^7 - 232*A*c^2*d^5 - 583*A*c^3*d^4 - 532*A*c^4*d^3 - 226*A*c^5*d^2 \end{aligned}$$

$$\begin{aligned}
& + 124*B*c^2*d^5 + 412*B*c^3*d^4 + 595*B*c^4*d^3 + 352*B*c^5*d^2 - 6*A*c*d^6 \\
& - 16*A*c^6*d + 6*B*c*d^6 + 82*B*c^6*d) / (3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 \\
& - 3*c^2*d + c^3 - d^3)) + (\tan(e/2 + (f*x)/2)^4*(16*A*c^7 + 18*A*d^7 + 2 \\
& *B*c^7 - 303*A*c^2*d^5 - 522*A*c^3*d^4 - 502*A*c^4*d^3 - 220*A*c^5*d^2 + 15 \\
& 6*B*c^2*d^5 + 453*B*c^3*d^4 + 538*B*c^4*d^3 + 328*B*c^5*d^2 - 48*A*c*d^6 - \\
& 14*A*c^6*d + 18*B*c*d^6 + 80*B*c^6*d) / (3*c^2*(c + d)*(c^2 - d^2)*(3*c*d^2 \\
& - 3*c^2*d + c^3 - d^3)) + (\tan(e/2 + (f*x)/2)^6*(2*A*c^6 + 2*A*d^6 - 9*A*c^2 \\
& *d^4 - 8*A*c^3*d^3 - 14*A*c^4*d^2 + 4*B*c^2*d^4 + 13*B*c^3*d^3 + 12*B*c^4*d^2 \\
& - 4*A*c*d^5 - 4*A*c^5*d + 6*B*c^5*d) / (c*(c - d)*(2*c*d + c^2 + d^2)*(3 \\
& *c*d^2 - 3*c^2*d + c^3 - d^3))) / (f*(\tan(e/2 + (f*x)/2)*(3*a^2*c^2 + 4*a^2*c \\
& *d) + \tan(e/2 + (f*x)/2)^2*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + \tan(e/2 + \\
& (f*x)/2)^5*(5*a^2*c^2 + 4*a^2*d^2 + 12*a^2*c*d) + \tan(e/2 + (f*x)/2)^3*(7* \\
& a^2*c^2 + 12*a^2*d^2 + 16*a^2*c*d) + \tan(e/2 + (f*x)/2)^4*(7*a^2*c^2 + 12*a^2 \\
& *d^2 + 16*a^2*c*d) + \tan(e/2 + (f*x)/2)^6*(3*a^2*c^2 + 4*a^2*c*d) + a^2*c \\
& ^2 + a^2*c^2*\tan(e/2 + (f*x)/2)^7)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**2/(c+d*sin(f*x+e))**3,x)

[Out] Timed out

$$3.279 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=225

$$\frac{(c-d)(A(2c^2+7cd+15d^2)+3B(c^2+6cd-15d^2))\cos(e+fx)}{15f(a^3\sin(e+fx)+a^3)} + \frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}$$

[Out] $d^2*(3*B*(c-d)+A*d)*x/a^3+1/15*d^2*(3*B*(c-9*d)+A*(2*c+7*d))*\cos(f*x+e)/a^3/f-1/15*(c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*\cos(f*x+e)/f/(a^3+a^3*\sin(f*x+e))-1/15*(3*B*(c-3*d)+2*A*(c+2*d))*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/a/f/(a+a*\sin(f*x+e))^2-1/5*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/f/(a+a*\sin(f*x+e))^3$

Rubi [A] time = 0.81, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2977, 2968, 3023, 2735, 2648}

$$\frac{(c-d)(A(2c^2+7cd+15d^2)+3B(c^2+6cd-15d^2))\cos(e+fx)}{15f(a^3\sin(e+fx)+a^3)} + \frac{d^2(A(2c+7d)+3B(c-9d))\cos(e+fx)}{15a^3f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]

[Out] $(d^2*(3*B*(c-d)+A*d)*x)/a^3+(d^2*(3*B*(c-9*d)+A*(2*c+7*d))*\text{Cos}[e+f*x])/(15*a^3*f)-((c-d)*(3*B*(c^2+6*c*d-15*d^2)+A*(2*c^2+7*c*d+15*d^2))*\text{Cos}[e+f*x])/(15*f*(a^3+a^3*\text{Sin}[e+f*x]))-((3*B*(c-3*d)+2*A*(c+2*d))*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^2)/(15*a*f*(a+a*\text{Sin}[e+f*x])^2)-((A-B)*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^3)/(5*f*(a+a*\text{Sin}[e+f*x])^3)$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)], x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&

NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^3} dx &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{5f(a + a \sin(e + fx))^3} + \int \frac{(c + d \sin(e + fx))^2(a + a \sin(e + fx))}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{(3B(c - 3d) + 2A(c + 2d)) \cos(e + fx)(c + d \sin(e + fx))^2}{15af(a + a \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{15af(a + a \sin(e + fx))^3} \\ &= -\frac{(3B(c - 3d) + 2A(c + 2d)) \cos(e + fx)(c + d \sin(e + fx))^2}{15af(a + a \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{15af(a + a \sin(e + fx))^3} \\ &= \frac{d^2(3B(c - 9d) + A(2c + 7d)) \cos(e + fx)}{15a^3f} - \frac{(3B(c - 3d) + 2A(c + 2d)) \cos(e + fx)(c + d \sin(e + fx))^2}{15af(a + a \sin(e + fx))^2} \\ &= \frac{d^2(3B(c - d) + Ad)x}{a^3} + \frac{d^2(3B(c - 9d) + A(2c + 7d)) \cos(e + fx)}{15a^3f} \\ &= \frac{d^2(3B(c - d) + Ad)x}{a^3} + \frac{d^2(3B(c - 9d) + A(2c + 7d)) \cos(e + fx)}{15a^3f} \end{aligned}$$

Mathematica [A] time = 2.97, size = 366, normalized size = 1.63

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(2(c - d) \left(A(2c^2 + 11cd + 32d^2) + 3B(c^2 + 8cd - 24d^2)\right) \sin\left(\frac{1}{2}(e + fx)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{(a + a \sin(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^3,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(6*(A - B)*(c - d)^3*Sin[(e + f*x)/2] - 3*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)^2*(3*B*(c - 6*d) + A*(2*c + 13*d))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 2*(c - d)*(3*B*(c^2 + 8*c*d - 24*d^2) + A*(2*c^2 + 11*c*d + 32*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 15*d^2*(-3*B*c - A*d + 3*B*d)*(e + f*x)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5 - 15*B*d^3*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(15*a^3*f*(1 + Sin[e + f*x])^3)

fricas [B] time = 0.48, size = 649, normalized size = 2.88

$$15Bd^3 \cos\left(fx + e\right)^4 - 3(A - B)c^3 + 9(A - B)c^2d - 9(A - B)cd^2 + 3(A - B)d^3 + \left((2A + 3B)c^3 + 3(3A + 3B)c^2d + 3(7A - 32B)c^2d^2 - (32A - 117B)d^3 - 15(3B^2cd^2 + (A - 3B)d^3)f^2\right) \cos(fx + e)^3 + 60(3B^2cd^2 + (A - 3B)d^3)f^2 \cos(fx + e)^2 - 3((2A + 3B)c^3 + 3(6A - B)c^2d - 3(A + 19B)cd^2 - (19A - 84B)d^3 + 45(3B^2cd^2 + (A - 3B)d^3)f^2) \cos(fx + e) + (15Bd^3 \cos(fx + e)^3 + 3(A - B)c^3 - 9(A - B)c^2d + 9(A - B)cd^2 - 3(A - B)d^3 + 60(3B^2cd^2 + (A - 3B)d^3)f^2) \cos(fx + e) \sin(fx + e) - ((2A + 3B)c^3 + 3(3A + 7B)c^2d + 3(7A - 32B)c^2d^2 - 2(16A - 51B)d^3 + 15(3B^2cd^2 + (A - 3B)d^3)f^2) \cos(fx + e) \sin(fx + e) - 3((2A + 3B)c^3 + 3(3A + 2B)c^2d + 3(2A - 17B)cd^2 - (17A - 62B)d^3 - 10(3B^2cd^2 + (A - 3B)d^3)f^2) \cos(fx + e) \sin(fx + e) \sin(fx + e) / (a^3 \cos(fx + e)^3 + 3a^3 \cos(fx + e)^2 - 2a^3 \cos(fx + e) - 4a^3 \cos(fx + e) \sin(fx + e) - 4a^3 \cos(fx + e) \sin(fx + e) \sin(fx + e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out]
$$-1/15*(15*B*d^3*\cos(f*x + e)^4 - 3*(A - B)*c^3 + 9*(A - B)*c^2*d - 9*(A - B)*c*d^2 + 3*(A - B)*d^3 + ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - (32*A - 117*B)*d^3 - 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2)*\cos(f*x + e)^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2*\cos(f*x + e)^2 - 3*((2*A + 3*B)*c^3 + 3*(6*A - B)*c^2*d - 3*(A + 19*B)*c*d^2 - (19*A - 84*B)*d^3 + 45*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2)*\cos(f*x + e) + (15*B*d^3*\cos(f*x + e)^3 + 3*(A - B)*c^3 - 9*(A - B)*c^2*d + 9*(A - B)*c*d^2 - 3*(A - B)*d^3 + 60*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2) *\cos(f*x + e) \sin(f*x + e) - ((2*A + 3*B)*c^3 + 3*(3*A + 7*B)*c^2*d + 3*(7*A - 32*B)*c*d^2 - 2*(16*A - 51*B)*d^3 + 15*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2)*\cos(f*x + e) \sin(f*x + e) - 3*((2*A + 3*B)*c^3 + 3*(3*A + 2*B)*c^2*d + 3*(2*A - 17*B)*c*d^2 - (17*A - 62*B)*d^3 - 10*(3*B*c*d^2 + (A - 3*B)*d^3)*f^2)*\cos(f*x + e) \sin(f*x + e) \sin(f*x + e) / (a^3*\cos(f*x + e)^3 + 3*a^3*\cos(f*x + e)^2 - 2*a^3*\cos(f*x + e) - 4*a^3*\cos(f*x + e) \sin(f*x + e) - 4*a^3*\cos(f*x + e) \sin(f*x + e) \sin(f*x + e))$$

giac [B] time = 0.22, size = 596, normalized size = 2.65

$$\frac{30Bd^3}{\left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1\right)^2} \frac{1}{a^3} - \frac{15(3Bcd^2 + Ad^3 - 3Bd^3)(fx+e)}{a^3} + \frac{2\left(15Ac^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 45Bcd^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15Ad^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 45Bd^3 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out]
$$-1/15*(30*B*d^3/((\tan(1/2*f*x + 1/2*e)^2 + 1)*a^3) - 15*(3*B*c*d^2 + A*d^3 - 3*B*d^3)*(f*x + e)/a^3 + 2*(15*A*c^3*\tan(1/2*f*x + 1/2*e)^4 - 45*B*c*d^2*\tan(1/2*f*x + 1/2*e)^4 - 15*A*d^3*\tan(1/2*f*x + 1/2*e)^4 + 45*B*d^3*\tan(1/2*f*x + 1/2*e)^4 + 30*A*c^3*\tan(1/2*f*x + 1/2*e)^3 + 15*B*c^3*\tan(1/2*f*x + 1/2*e)^3 + 45*A*c^2*d*\tan(1/2*f*x + 1/2*e)^3 - 225*B*c*d^2*\tan(1/2*f*x + 1/2*e)^3 - 75*A*d^3*\tan(1/2*f*x + 1/2*e)^3 + 210*B*d^3*\tan(1/2*f*x + 1/2*e)^3 + 40*A*c^3*\tan(1/2*f*x + 1/2*e)^2 + 15*B*c^3*\tan(1/2*f*x + 1/2*e)^2 + 45*A*c^2*d*\tan(1/2*f*x + 1/2*e)^2 + 60*B*c^2*d*\tan(1/2*f*x + 1/2*e)^2 + 60*A*c*d^2*\tan(1/2*f*x + 1/2*e)^2 - 435*B*c*d^2*\tan(1/2*f*x + 1/2*e)^2 - 145*A*d^3*\tan(1/2*f*x + 1/2*e)^2 + 360*B*d^3*\tan(1/2*f*x + 1/2*e)^2 + 20*A*c^3*\tan(1/2*f*x + 1/2*e) + 15*B*c^3*\tan(1/2*f*x + 1/2*e) + 45*A*c^2*d*\tan(1/2*f*x + 1/2*e) + 30*B*c^2*d*\tan(1/2*f*x + 1/2*e) + 30*A*c*d^2*\tan(1/2*f*x + 1/2*e) - 285*B*c*d^2*\tan(1/2*f*x + 1/2*e) - 95*A*d^3*\tan(1/2*f*x + 1/2*e) + 240*B*d^3*\tan(1/2*f*x + 1/2*e) + 7*A*c^3 + 3*B*c^3 + 9*A*c^2*d + 6*B*c^2*d + 6*A*c*d^2 - 66*B*c*d^2 - 22*A*d^3 + 57*B*d^3)/(a^3*(\tan(1/2*f*x + 1/2*e) + 1)^5))/f$$

maple [B] time = 0.43, size = 936, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)

[Out] $6/a^3/f*d^2*B*arctan(\tan(1/2*f*x+1/2*e))*c+6/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*B*c*d^2-24/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B*c^2*d+24/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B*c*d^2-6/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*c^2*d-6/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*B*d^3+4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*c^3+2/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*A*d^3-2/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*c^3-4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*d^3-16/3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c^3+4/3/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*d^3+4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c^3-4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B*c^3+4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B*d^3-8/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A*c^3+8/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A*d^3+8/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B*c^3-8/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*B*d^3-2/a^3/f*d^3*B/(1+\tan(1/2*f*x+1/2*e)^2)+2/a^3/f*d^3*A*arctan(\tan(1/2*f*x+1/2*e))+6/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2*B*c*d^2+12/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c^2*d-8/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*A*c*d^2-8/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c^2*d+4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^3*B*c*d^2-12/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A*c^2*d+2/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A*d^3+12/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A*c*d^2+12/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B*c^2*d-12/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*B*c*d^2+24/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A*c^2*d-24/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5*A*c*d^2-6/a^3/f*d^3*B*arctan(\tan(1/2*f*x+1/2*e))-2/a^3/f/(\tan(1/2*f*x+1/2*e)+1)*A*c^3+4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A*c^3-4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4*A*d^3$

maxima [B] time = 0.58, size = 1682, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out] $-2/15*(3*B*d^3*((105*\sin(f*x + e))/(\cos(f*x + e) + 1) + 189*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 200*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 160*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 75*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 15*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + 24)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 11*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 15*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 11*a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5 + 5*a^3*\sin(f*x + e)^6/(\cos(f*x + e) + 1)^6 + a^3*\sin(f*x + e)^7/(\cos(f*x + e) + 1)^7) + 15*arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^3) - 3*B*c*d^2*((95*\sin(f*x + e))/(\cos(f*x + e) + 1) + 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^3) - A*d^3*((95*\sin(f*x + e))/(\cos(f*x + e) + 1) + 145*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 75*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 22)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 15*arctan(\sin(f*x + e)/(\cos(f*x + e) + 1))/a^3) + A*c^3*(20*\sin(f*x + e)/(\cos(f*x + e) + 1) + 40*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 30*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 15*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 10*a^3*\sin(f*x + e)^3/(\cos(f*x + e) + 1)^3 + 5*a^3*\sin(f*x + e)^4/(\cos(f*x + e) + 1)^4 + a^3*\sin(f*x + e)^5/(\cos(f*x + e) + 1)^5) + 6*B*c^2*d*(5*\sin(f*x + e)/(\cos(f*x + e) + 1) + 10*\sin(f*x + e)^2/(\cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*\sin(f*x + e))/(\cos(f*x + e) + 1) + 10*a^3*\sin(f*x + e)^2/(\cos(f*x + e) + 1)$

$$\begin{aligned} &)^2 + 10a^3 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 5a^3 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + a^3 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5 + 6Ac^2 d^2 (5 \sin(fx + e) / (\cos(fx + e) + 1) + 10 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 1) / (a^3 + 5a^3 \sin(fx + e) / (\cos(fx + e) + 1) + 10a^3 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 10a^3 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 5a^3 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + a^3 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5) \\ &+ 3Bc^3 (5 \sin(fx + e) / (\cos(fx + e) + 1) + 5 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 1) / (a^3 + 5a^3 \sin(fx + e) / (\cos(fx + e) + 1) + 10a^3 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 10a^3 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 5a^3 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + a^3 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5) + 9Ac^2 d^2 (5 \sin(fx + e) / (\cos(fx + e) + 1) + 5 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 5 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 1) / (a^3 + 5a^3 \sin(fx + e) / (\cos(fx + e) + 1) + 10a^3 \sin(fx + e)^2 / (\cos(fx + e) + 1)^2 + 10a^3 \sin(fx + e)^3 / (\cos(fx + e) + 1)^3 + 5a^3 \sin(fx + e)^4 / (\cos(fx + e) + 1)^4 + a^3 \sin(fx + e)^5 / (\cos(fx + e) + 1)^5) / f \end{aligned}$$

mupad [B] time = 15.70, size = 593, normalized size = 2.64

$$\frac{2d^2 \operatorname{atan}\left(\frac{2d^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right)(Ad+3Bc-3Bd)}{2Ad^3-6Bd^3+6Bcd^2}\right)(Ad+3Bc-3Bd) \tan\left(\frac{e}{2} + \frac{fx}{2}\right)^5 (4Ac^3 - 10Ad^3 + 2Bc^3 + 30Bd^3)}{a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^3,x)

[Out] (2*d^2*atan((2*d^2*tan(e/2 + (f*x)/2)*(A*d + 3*B*c - 3*B*d))/(2*A*d^3 - 6*B*d^3 + 6*B*c*d^2))*(A*d + 3*B*c - 3*B*d))/(a^3*f) - (tan(e/2 + (f*x)/2)^5*(4*A*c^3 - 10*A*d^3 + 2*B*c^3 + 30*B*d^3 + 6*A*c^2*d - 30*B*c*d^2) + tan(e/2 + (f*x)/2)*((8*A*c^3)/3 - (38*A*d^3)/3 + 2*B*c^3 + 42*B*d^3 + 4*A*c*d^2 + 6*A*c^2*d - 38*B*c*d^2 + 4*B*c^2*d) + (14*A*c^3)/15 - (44*A*d^3)/15 + (2*B*c^3)/5 + (48*B*d^3)/5 + tan(e/2 + (f*x)/2)^4*((22*A*c^3)/3 - (64*A*d^3)/3 + 2*B*c^3 + 64*B*d^3 + 8*A*c*d^2 + 6*A*c^2*d - 64*B*c*d^2 + 8*B*c^2*d) + tan(e/2 + (f*x)/2)^3*((20*A*c^3)/3 - (68*A*d^3)/3 + 4*B*c^3 + 80*B*d^3 + 4*A*c*d^2 + 12*A*c^2*d - 68*B*c*d^2 + 4*B*c^2*d) + tan(e/2 + (f*x)/2)^2*((94*A*c^3)/15 - (334*A*d^3)/15 + (12*B*c^3)/5 + (378*B*d^3)/5 + (44*A*c*d^2)/5 + (36*A*c^2*d)/5 - (334*B*c*d^2)/5 + (44*B*c^2*d)/5) + tan(e/2 + (f*x)/2)^6*(2*A*c^3 - 2*A*d^3 + 6*B*d^3 - 6*B*c*d^2) + (4*A*c*d^2)/5 + (6*A*c^2*d)/5 - (44*B*c*d^2)/5 + (4*B*c^2*d)/5)/(f*(11*a^3*tan(e/2 + (f*x)/2)^2 + 15*a^3*tan(e/2 + (f*x)/2)^3 + 15*a^3*tan(e/2 + (f*x)/2)^4 + 11*a^3*tan(e/2 + (f*x)/2)^5 + 5*a^3*tan(e/2 + (f*x)/2)^6 + a^3*tan(e/2 + (f*x)/2)^7 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))

sympy [A] time = 56.43, size = 11456, normalized size = 50.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^3,x)

[Out] Piecewise((-30*A*c**3*tan(e/2 + f*x/2)**6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**3*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2))

$$\begin{aligned}
& + f*x/2) + 15*a**3*f) - 110*A*c**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 \\
& + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)** \\
& 5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a \\
& **3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 100*A \\
& *c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/ \\
& 2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2 \\
&)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75 \\
& *a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 94*A*c**3*tan(e/2 + f*x/2)**2/(15*a \\
& **3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(\\
& e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x \\
& /2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a \\
& **3*f) - 40*A*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a** \\
& 3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e \\
& /2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/ \\
& 2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c**3/(15*a**3*f*tan(\\
& e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/ \\
& 2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 1 \\
& 65*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 9 \\
& 0*A*c**2*d*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t \\
& an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + \\
& f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 \\
& + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)** \\
& 4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3 \\
& *f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/ \\
& 2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) \\
& + 15*a**3*f) - 180*A*c**2*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2 \\
&)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225 \\
& *a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*t \\
& an(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 108*A*c**2*d \\
& *tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f \\
& *x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 \\
& + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3 \\
& *f*tan(e/2 + f*x/2) + 15*a**3*f) - 90*A*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f* \\
& tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + \\
& f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 \\
& + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) \\
& - 18*A*c**2*d/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)* \\
& *6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225* \\
& a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan \\
& (e/2 + f*x/2) + 15*a**3*f) - 120*A*c*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*ta \\
& n(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f* \\
& x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + \\
& 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - \\
& 60*A*c*d**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f \\
& *tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 \\
& + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)* \\
& *2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 132*A*c*d**2*tan(e/2 + f*x/2 \\
&)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a \\
& **3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan \\
& (e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x \\
& /2) + 15*a**3*f) - 60*A*c*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2) \\
& **7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225* \\
& a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*ta \\
& n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*A*c*d**2/(\\
& 15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f* \\
& tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + \\
& f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + \\
& 15*a**3*f) + 15*A*d**3*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)* \\
& *7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a
\end{aligned}$$

$$\begin{aligned}
& **3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan \\
& (e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 75*A*d**3*f*x* \\
& \tan(e/2 + f*x/2)**6/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f* \\
& x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + \\
& 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3* \\
& f*\tan(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*\tan(e/2 + f*x/2)**5/(15*a* \\
& **3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e \\
& /2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/ \\
& 2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a* \\
& **3*f) + 225*A*d**3*f*x*\tan(e/2 + f*x/2)**4/(15*a**3*f*\tan(e/2 + f*x/2)**7 + \\
& 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3* \\
& f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 \\
& + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 225*A*d**3*f*x*\tan \\
& (e/2 + f*x/2)**3/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2 \\
&)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 22 \\
& 5*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*t \\
& an(e/2 + f*x/2) + 15*a**3*f) + 165*A*d**3*f*x*\tan(e/2 + f*x/2)**2/(15*a**3* \\
& f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 \\
& + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)* \\
& **3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3* \\
& f) + 75*A*d**3*f*x*\tan(e/2 + f*x/2)/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a** \\
& 3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e \\
& /2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/ \\
& 2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 15*A*d**3*f*x/(15*a**3*f* \\
& \tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + \\
& f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 \\
& + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) \\
& + 30*A*d**3*\tan(e/2 + f*x/2)**6/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f \\
& *\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 \\
& + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)* \\
& **2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 150*A*d**3*\tan(e/2 + f*x/2)* \\
& **5/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a** \\
& 3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e \\
& /2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2 \\
&) + 15*a**3*f) + 320*A*d**3*\tan(e/2 + f*x/2)**4/(15*a**3*f*\tan(e/2 + f*x/2) \\
& **7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225* \\
& a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*ta \\
& n(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 340*A*d**3*ta \\
& n(e/2 + f*x/2)**3/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/ \\
& 2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 2 \\
& 25*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f* \\
& \tan(e/2 + f*x/2) + 15*a**3*f) + 334*A*d**3*\tan(e/2 + f*x/2)**2/(15*a**3*f*t \\
& an(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f \\
& *x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 \\
& + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) \\
& + 190*A*d**3*\tan(e/2 + f*x/2)/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*ta \\
& n(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f \\
& *x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 \\
& + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) + 44*A*d**3/(15*a**3*f*\tan(e/2 + \\
& f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 \\
& + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a** \\
& 3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c* \\
& **3*\tan(e/2 + f*x/2)**5/(15*a**3*f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + \\
& f*x/2)**6 + 165*a**3*f*\tan(e/2 + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)** \\
& 4 + 225*a**3*f*\tan(e/2 + f*x/2)**3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a* \\
& **3*f*\tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**3*\tan(e/2 + f*x/2)**4/(15*a**3 \\
& *f*\tan(e/2 + f*x/2)**7 + 75*a**3*f*\tan(e/2 + f*x/2)**6 + 165*a**3*f*\tan(e/2 \\
& + f*x/2)**5 + 225*a**3*f*\tan(e/2 + f*x/2)**4 + 225*a**3*f*\tan(e/2 + f*x/2) \\
& **3 + 165*a**3*f*\tan(e/2 + f*x/2)**2 + 75*a**3*f*\tan(e/2 + f*x/2) + 15*a**3
\end{aligned}$$

$$\begin{aligned}
& *f) - 60*B*c**3*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a** \\
& 3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e \\
& /2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/ \\
& 2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 36*B*c**3*tan(e/2 + f*x/2 \\
&)**2/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a \\
& **3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan \\
& (e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x \\
& /2) + 15*a**3*f) - 30*B*c**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)** \\
& 7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a* \\
& **3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(\\
& e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**3/(15*a* \\
& **3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e \\
& /2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/ \\
& 2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a* \\
& **3*f) - 120*B*c**2*d*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 7 \\
& 5*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f* \\
& tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + \\
& f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*B*c**2*d*tan(e/2 \\
& + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 \\
& + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a** \\
& 3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/ \\
& 2 + f*x/2) + 15*a**3*f) - 132*B*c**2*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e \\
& /2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2 \\
&)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 16 \\
& 5*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60 \\
& *B*c**2*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e \\
& /2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/ \\
& 2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 7 \\
& 5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 12*B*c**2*d/(15*a**3*f*tan(e/2 + f \\
& *x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + \\
& 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3 \\
& *f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d \\
& **2*f*x*tan(e/2 + f*x/2)**7/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(\\
& e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x \\
& /2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + \\
& 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225*B*c*d**2*f*x*tan(e/2 + f*x/2) \\
& **6/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a* \\
& **3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(\\
& e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/ \\
& 2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + \\
& f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 \\
& + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a* \\
& **3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B* \\
& c*d**2*f*x*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t \\
& an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + \\
& f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 \\
& + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 675*B*c*d**2*f*x*tan(e/2 + f*x \\
& /2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165 \\
& *a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*t \\
& an(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f \\
& *x/2) + 15*a**3*f) + 495*B*c*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/ \\
& 2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2) \\
& **5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165 \\
& *a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 225 \\
& *B*c*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*t \\
& an(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + \\
& f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 \\
& + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 45*B*c*d**2*f*x/(15*a**3*f*tan \\
& (e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
\end{aligned}$$


```

75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 450*B*d**3*tan(e/2 + f*x/2)**5/(
15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*
tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 +
f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 960*B*d**3*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**7
+ 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3
*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/
2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 1200*B*d**3*tan(e
/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)*
*6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*
a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan
(e/2 + f*x/2) + 15*a**3*f) - 1134*B*d**3*tan(e/2 + f*x/2)**2/(15*a**3*f*tan
(e/2 + f*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x
/2)**5 + 225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 +
165*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) -
630*B*d**3*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**7 + 75*a**3*f*tan(
e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 + 225*a**3*f*tan(e/2 + f*x
/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 144*B*d**3/(15*a**3*f*tan(e/2 + f
*x/2)**7 + 75*a**3*f*tan(e/2 + f*x/2)**6 + 165*a**3*f*tan(e/2 + f*x/2)**5 +
225*a**3*f*tan(e/2 + f*x/2)**4 + 225*a**3*f*tan(e/2 + f*x/2)**3 + 165*a**3
*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0))
, (x*(A + B*sin(e))*(c + d*sin(e))**3/(a*sin(e) + a)**3, True))

```

$$3.280 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=164

$$\frac{(2A(c^2 + 3cd + 2d^2) + B(3c^2 + 14cd - 29d^2)) \cos(e + fx)}{15f(a^3 \sin(e + fx) + a^3)} + \frac{Bd^2x}{a^3} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{5f(a \sin(e + fx) + a)^3}$$

[Out] $B*d^2*x/a^3-1/15*(c-d)*(B*(3*c-7*d)+2*A*(c+d))*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^2-1/15*(B*(3*c^2+14*c*d-29*d^2)+2*A*(c^2+3*c*d+2*d^2))*\cos(f*x+e)/f/(a^3+a^3*\sin(f*x+e))-1/5*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/f/(a+a*\sin(f*x+e))^3$

Rubi [A] time = 0.46, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2977, 2968, 3019, 2735, 2648}

$$\frac{(2A(c^2 + 3cd + 2d^2) + B(3c^2 + 14cd - 29d^2)) \cos(e + fx)}{15f(a^3 \sin(e + fx) + a^3)} + \frac{Bd^2x}{a^3} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{5f(a \sin(e + fx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]

[Out] $(B*d^2*x)/a^3 - ((c - d)*(B*(3*c - 7*d) + 2*A*(c + d))*\text{Cos}[e + f*x])/(15*a*f*(a + a*\text{Sin}[e + f*x])^2) - ((B*(3*c^2 + 14*c*d - 29*d^2) + 2*A*(c^2 + 3*c*d + 2*d^2))*\text{Cos}[e + f*x])/(15*f*(a^3 + a^3*\text{Sin}[e + f*x])) - ((A - B)*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^2)/(5*f*(a + a*\text{Sin}[e + f*x])^3)$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] :> Simp[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 3019

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> Simp[((A*b - a*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^3} dx &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{5f(a + a \sin(e + fx))^3} + \int \frac{(c + d \sin(e + fx))(a(B(3c - 7d) + 2A(c + d)))}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{5f(a + a \sin(e + fx))^3} + \int \frac{ac(B(3c - 7d) + 2A(c + d))}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{(c - d)(B(3c - 7d) + 2A(c + d)) \cos(e + fx)}{15af(a + a \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{5f(a + a \sin(e + fx))} \\ &= \frac{Bd^2x}{a^3} - \frac{(c - d)(B(3c - 7d) + 2A(c + d)) \cos(e + fx)}{15af(a + a \sin(e + fx))^2} - \frac{(A - B) \cos(e + fx)}{5f(a + a \sin(e + fx))} \\ &= \frac{Bd^2x}{a^3} - \frac{(c - d)(B(3c - 7d) + 2A(c + d)) \cos(e + fx)}{15af(a + a \sin(e + fx))^2} - \frac{B(3c^2 + 4cd + d^2)}{15af(a + a \sin(e + fx))} \end{aligned}$$

Mathematica [B] time = 0.90, size = 514, normalized size = 3.13

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(30 \cos\left(\frac{1}{2}(e + fx)\right) \left(2Ad(c + d) + B(c^2 + 4cd + d^2(5e + 5fx - 9))\right) - 5 \cos\left(\frac{1}{2}(e + fx)\right)\right)}{(a + a \sin(e + fx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^3,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(30*(2*A*d*(c + d) + B*(c^2 + 4*c*d + d^2*(-9 + 5*e + 5*f*x)))*Cos[(e + f*x)/2] - 5*(4*A*(c^2 + 3*c*d + 2*d^2) + B*(6*c^2 + 16*c*d + d^2*(-46 + 15*e + 15*f*x)))*Cos[(3*(e + f*x))/2] - 15*B*d^2*e*Cos[(5*(e + f*x))/2] - 15*B*d^2*f*x*Cos[(5*(e + f*x))/2] + 40*A*c^2*Sin[(e + f*x)/2] + 30*B*c^2*Sin[(e + f*x)/2] + 60*A*c*d*Sin[(e + f*x)/2] + 160*B*c*d*Sin[(e + f*x)/2] + 80*A*d^2*Sin[(e + f*x)/2] - 370*B*d^2*Sin[(e + f*x)/2] + 150*B*d^2*e*Sin[(e + f*x)/2] + 150*B*d^2*f*x*Sin[(e + f*x)/2] + 60*B*c*d*Sin[(3*(e + f*x))/2] + 30*A*d^2*Sin[(3*(e + f*x))/2] - 90*B*d^2*Sin[(3*(e + f*x))/2] + 75*B*d^2*e*Sin[(3*(e + f*x))/2] + 75*B*d^2*f*x*Sin[(3*(e + f*x))/2] - 4*A*c^2*Sin[(5*(e + f*x))/2] - 6*B*c^2*Sin[(5*(e + f*x))/2] - 12*A*c*d*Sin[(5*(e + f*x))/2] - 28*B*c*d*Sin[(5*(e + f*x))/2] - 14*A*d^2*Sin[(5*(e + f*x))/2] + 64*B*d^2*Sin[(5*(e + f*x))/2] - 15*B*d^2*e*Sin[(5*(e + f*x))/2] - 15*B*d^2*f*x*Sin[(5*(e + f*x))/2]))/(60*a^3*f*(1 + Sin[e + f*x])^3)

fricas [B] time = 0.46, size = 432, normalized size = 2.63

$$\frac{60 Bd^2 fx - (15 Bd^2 fx - (2A + 3B)c^2 - 2(3A + 7B)cd - (7A - 32B)d^2) \cos(fx + e)^3 - 3(A - B)c^2 + 6(A - B)cd - 3Bd^2}{(a + a \sin(e + fx))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(60*B*d^2*f*x - (15*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 7*B)*c*d - (7*A - 32*B)*d^2)*cos(f*x + e)^3 - 3*(A - B)*c^2 + 6*(A - B)*c*d - 3*(A - B)*d^2 - (45*B*d^2*f*x + 2*(2*A + 3*B)*c^2 + 2*(6*A - B)*c*d - (A + 19*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (3*A + 2*B)*c^2 - 2*(2*A + 3*B)*c*d - 3*(A - 6*B)*d^2)*cos(f*x + e) + (60*B*d^2*f*x + 3*(A - B)*c^2 - 6*(A - B)*c*d + 3*(A - B)*d^2 - (15*B*d^2*f*x + (2*A + 3*B)*c^2 + 2*(3*A + 7*B)*c*d + (7*A - 32*B)*d^2)*cos(f*x + e)^2 + 3*(10*B*d^2*f*x - (2*A + 3*B)*c^2 - 2*(3*A + 2*B)*c*d - (2*A - 17*B)*d^2)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [B] time = 0.19, size = 382, normalized size = 2.33

$$\frac{15(f_{x+e})Bd^2}{a^3} - \frac{2\left(15Ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 - 15Bd^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 30Ac^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15Bc^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 30Acd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 - 75A^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 30A^2cd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 15B^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 30A^2cd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 40B^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 20A^2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 - 145B^2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 20A^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 15B^2c^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 30A^2cd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 20B^2cd \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 10A^2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) - 95B^2d^2 \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 7A^2c^2 + 3B^2c^2 + 6A^2cd + 4B^2cd + 2A^2d^2 - 22B^2d^2\right)}{a^3(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1)^5}/f$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

```
[Out] 1/15*(15*(f*x + e)*B*d^2/a^3 - 2*(15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 75*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 + 30*A*c*d*tan(1/2*f*x + 1/2*e)^2 + 40*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 145*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) + 30*A*c*d*tan(1/2*f*x + 1/2*e) + 20*B*c*d*tan(1/2*f*x + 1/2*e) + 10*A*d^2*tan(1/2*f*x + 1/2*e) - 95*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 + 6*A*c*d + 4*B*c*d + 2*A*d^2 - 22*B*d^2)/(a^3*(tan(1/2*f*x + 1/2*e) + 1)^5)/f
```

maple [B] time = 0.42, size = 617, normalized size = 3.76

$$-\frac{2Bc^2}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} + \frac{2Bd^2}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)^2} + \frac{2Bd^2 \arctan\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right)\right)}{a^3f} - \frac{2Ac^2}{a^3f\left(\tan\left(\frac{fx}{2} + \frac{e}{2}\right) + 1\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x)
```

```
[Out] -2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*c^2+2/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*B*d^2+2/a^3/f*B*d^2*arctan(tan(1/2*f*x+1/2*e))-2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*A*c^2+2/a^3/f/(tan(1/2*f*x+1/2*e)+1)*B*d^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^2*A*c^2-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*d^2-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A*c^2-8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*A*d^2+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*c^2+8/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*d^2-16/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c^2-8/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*c^2+4/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*B*d^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*c^2+4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*A*d^2-16/5/a^3/f/(tan(1/2*f*x+1/2*e)+1)^5*B*c*d-4/a^3/f/(tan(1/2*f*x+1/2*e)+1)^4*B*c^2+8/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*c*d-16/3/a^3/f/(tan(1/2*f*x+1/2*e)+1)^3*A*d^2
```

$e+1)^3 B c d - 4/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^2 A c d - 8/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4 A c d + 8/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^4 B c d + 16/5/a^3/f/(\tan(1/2*f*x+1/2*e)+1)^5 A c d$

maxima [B] time = 0.55, size = 1132, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^3,x, algorithm="maxima")

[Out]
$$\frac{2}{15} (B d^2 \left(\frac{95 \sin(f x + e)}{\cos(f x + e) + 1} + 145 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 75 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 15 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + 22 \right) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) + 15 \arctan(\sin(f x + e) / (\cos(f x + e) + 1)) / a^3 - A c^2 (20 \sin(f x + e) / (\cos(f x + e) + 1) + 40 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 30 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 15 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + 7) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) - 4 B c d (5 \sin(f x + e) / (\cos(f x + e) + 1) + 10 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 1) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) - 2 A d^2 (5 \sin(f x + e) / (\cos(f x + e) + 1) + 10 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 1) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) - 3 B c^2 (5 \sin(f x + e) / (\cos(f x + e) + 1) + 5 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 5 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 1) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 1) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) - 6 A c d (5 \sin(f x + e) / (\cos(f x + e) + 1) + 5 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 5 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 1) / (a^3 + 5 a^3 \sin(f x + e) / (\cos(f x + e) + 1) + 10 a^3 \sin(f x + e)^2 / (\cos(f x + e) + 1)^2 + 10 a^3 \sin(f x + e)^3 / (\cos(f x + e) + 1)^3 + 5 a^3 \sin(f x + e)^4 / (\cos(f x + e) + 1)^4 + a^3 \sin(f x + e)^5 / (\cos(f x + e) + 1)^5) / f$$

mupad [B] time = 16.54, size = 286, normalized size = 1.74

$$\frac{B d^2 x \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^2 \left(\frac{16 A c^2}{3} + \frac{8 A d^2}{3} + 2 B c^2 - \frac{58 B d^2}{3} + 4 A c d + \frac{16 B c d}{3}\right) + \frac{14 A c^2}{15} + \frac{4 A d^2}{15} + \frac{2 B c^2}{5} - \frac{44 B d^2}{15} + \tan\left(\frac{e}{2} + \frac{f x}{2}\right)}{a^3} \frac{f \left(a^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 + 5 a^3\right)}{f \left(a^3 \tan\left(\frac{e}{2} + \frac{f x}{2}\right)^5 + 5 a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^3,x)

[Out]
$$\frac{(B d^2 x) / a^3 - (\tan(e/2 + (f x) / 2)^2 \left(\frac{16 A c^2}{3} + \frac{8 A d^2}{3} + 2 B c^2 - \frac{58 B d^2}{3} + 4 A c d + \frac{16 B c d}{3} \right) + \frac{14 A c^2}{15} + \frac{4 A d^2}{15} + \frac{2 B c^2}{5} - \frac{44 B d^2}{15} + \tan(e/2 + (f x) / 2)^3 (4 A c^2 + 2 B c^2 - 10 B d^2 + 4 A c d) + \tan(e/2 + (f x) / 2)^4 (2 A c^2 - 2 B d^2) + \tan(e/2 + (f x) / 2) \left(\frac{8 A c^2}{3} + \frac{4 A d^2}{3} + 2 B c^2 - \frac{38 B d^2}{3} + 4 A c d + \frac{8 B c d}{3} \right) + \frac{4 A c d}{5} + \frac{8 B c d}{15}}{f \left(10 a^3 \tan(e/2 + (f x) / 2)^2 + 10 a^3 \right)}$$

$$a^3 \tan(e/2 + (f*x)/2)^3 + 5*a^3 \tan(e/2 + (f*x)/2)^4 + a^3 \tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3 \tan(e/2 + (f*x)/2))$$

sympy [A] time = 33.40, size = 3468, normalized size = 21.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**3,x)

[Out] Piecewise((-30*A*c**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*A*c**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*A*d**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*A*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 4*A*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c**2*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*B*c*d*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*c*d*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 8*B*c*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d

```

**2*f*x**tan(e/2 + f*x/2)**5/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(
e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x
/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 75*B*d**2*f*x*tan(e/2 +
f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 +
150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f
*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**3/(15*a**
3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/
2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) + 150*B*d**2*f*x*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x
/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 1
50*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 7
5*B*d**2*f*x*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*ta
n(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f
*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 15*B*d**2*f*x/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) + 30*B*d**2*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5
+ 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**
3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 150*B*d
**2*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2
+ f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)*
**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) + 290*B*d**2*tan(e/2 + f*x/2)*
**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**
3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/
2 + f*x/2) + 15*a**3*f) + 190*B*d**2*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 +
f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3
+ 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f)
+ 44*B*d**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4
+ 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3
*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c + d*sin(e)
)**2/(a*sin(e) + a)**3, True))

```


$$3.281 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=127

$$\frac{(2Ac + 3Ad + 3Bc + 7Bd) \cos(e + fx)}{15f(a^3 \sin(e + fx) + a^3)} - \frac{(2Ac + 3Ad + 3Bc - 8Bd) \cos(e + fx)}{15af(a \sin(e + fx) + a)^2} - \frac{(A - B)(c - d) \cos(e + fx)}{5f(a \sin(e + fx) + a)^3}$$

[Out] -1/5*(A-B)*(c-d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^3-1/15*(2*A*c+3*A*d+3*B*c-8*B*d)*cos(f*x+e)/a/f/(a+a*sin(f*x+e))^2-1/15*(2*A*c+3*A*d+3*B*c+7*B*d)*cos(f*x+e)/f/(a^3+a^3*sin(f*x+e))

Rubi [A] time = 0.23, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2968, 3019, 2750, 2648}

$$\frac{(2Ac + 3Ad + 3Bc + 7Bd) \cos(e + fx)}{15f(a^3 \sin(e + fx) + a^3)} - \frac{(2Ac + 3Ad + 3Bc - 8Bd) \cos(e + fx)}{15af(a \sin(e + fx) + a)^2} - \frac{(A - B)(c - d) \cos(e + fx)}{5f(a \sin(e + fx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]

[Out] -((A - B)*(c - d)*Cos[e + f*x])/(5*f*(a + a*Sin[e + f*x])^3) - ((2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x])/(15*a*f*(a + a*Sin[e + f*x])^2) - ((2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[e + f*x])/(15*f*(a^3 + a^3*Sin[e + f*x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3019

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[((A*b - a*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^3} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{(a + a \sin(e + fx))^3} dx \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{5f(a + a \sin(e + fx))^3} - \frac{\int \frac{-a(2Ac + 3Bc + 3Ad - 3Bd) - 5aBd \sin(e + fx)}{(a + a \sin(e + fx))^2} dx}{5a^2} \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{5f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc + 3Ad - 8Bd) \cos(e + fx)}{15af(a + a \sin(e + fx))^2} \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{5f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc + 3Ad - 8Bd) \cos(e + fx)}{15af(a + a \sin(e + fx))^2} \end{aligned}$$

Mathematica [A] time = 0.69, size = 176, normalized size = 1.39

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(15(Ad + B(c + 2d)) \cos\left(\frac{1}{2}(e + fx)\right) - 5(2Ac + 3Ad + 3Bc + 4Bd) \cos\left(\frac{3}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^3,x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*(A*d + B*(c + 2*d))*Cos[(e + f*x)/2] - 5*(2*A*c + 3*B*c + 3*A*d + 4*B*d)*Cos[(3*(e + f*x))/2] - 2*(-3*(3*A*c + 2*B*c + 2*A*d + 8*B*d) + (2*A*c + 3*B*c + 3*A*d - 8*B*d)*Cos[e + f*x] + (2*A*c + 3*B*c + 3*A*d + 7*B*d)*Cos[2*(e + f*x)])*Sin[(e + f*x)/2))/(30*a^3*f*(1 + Sin[e + f*x])^3)

fricas [B] time = 0.43, size = 271, normalized size = 2.13

$$\frac{((2A + 3B)c + (3A + 7B)d) \cos(fx + e)^3 - (2(2A + 3B)c + (6A - B)d) \cos(fx + e)^2 - 3(A - B)c + 3(A - B)d}{15(a^3 f \cos(fx + e))^3 + 3a^3 f \cos(fx + e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*(((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^3 - (2*(2*A + 3*B)*c + (6*A - B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d - 3*((3*A + 2*B)*c + (2*A + 3*B)*d)*cos(f*x + e) - (((2*A + 3*B)*c + (3*A + 7*B)*d)*cos(f*x + e)^2 - 3*(A - B)*c + 3*(A - B)*d + 3*((2*A + 3*B)*c + (3*A + 2*B)*d)*cos(f*x + e))*sin(f*x + e)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e))^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e)

giac [A] time = 0.18, size = 223, normalized size = 1.76

$$2 \left(15Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 30Ac \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15Bc \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15Ad \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 40A \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")

```
[Out] -2/15*(15*A*c*tan(1/2*f*x + 1/2*e)^4 + 30*A*c*tan(1/2*f*x + 1/2*e)^3 + 15*B*c*tan(1/2*f*x + 1/2*e)^3 + 15*A*d*tan(1/2*f*x + 1/2*e)^3 + 40*A*c*tan(1/2*f*x + 1/2*e)^2 + 15*B*c*tan(1/2*f*x + 1/2*e)^2 + 15*A*d*tan(1/2*f*x + 1/2*e)^2 + 20*B*d*tan(1/2*f*x + 1/2*e)^2 + 20*A*c*tan(1/2*f*x + 1/2*e) + 15*B*c*tan(1/2*f*x + 1/2*e) + 15*A*d*tan(1/2*f*x + 1/2*e) + 10*B*d*tan(1/2*f*x + 1/2*e) + 7*A*c + 3*B*c + 3*A*d + 2*B*d)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)
```

maple [A] time = 0.44, size = 151, normalized size = 1.19

$$\frac{\frac{2(8Ac-6Ad-6Bc+4Bd)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3} - \frac{-8Ac+8Ad+8Bc-8Bd}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4} - \frac{-4Ac+2Ad+2Bc}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} - \frac{2(4Ac-4Ad-4Bc+4Bd)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5} - \frac{2Ac}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1}}{fa^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)
```

```
[Out] 2/f/a^3*(-1/3*(8*A*c-6*A*d-6*B*c+4*B*d)/(tan(1/2*f*x+1/2*e)+1)^3-1/4*(-8*A*c+8*A*d+8*B*c-8*B*d)/(tan(1/2*f*x+1/2*e)+1)^4-1/2*(-4*A*c+2*A*d+2*B*c)/(tan(1/2*f*x+1/2*e)+1)^2-1/5*(4*A*c-4*A*d-4*B*c+4*B*d)/(tan(1/2*f*x+1/2*e)+1)^5-A*c/(tan(1/2*f*x+1/2*e)+1))
```

maxima [B] time = 0.46, size = 733, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -2/15*(A*c*(20*sin(f*x + e)/(cos(f*x + e) + 1) + 40*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 30*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 15*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + 7)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 2*B*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 10*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*B*c*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5) + 3*A*d*(5*sin(f*x + e)/(cos(f*x + e) + 1) + 5*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 5*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 1)/(a^3 + 5*a^3*sin(f*x + e)/(cos(f*x + e) + 1) + 10*a^3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 10*a^3*sin(f*x + e)^3/(cos(f*x + e) + 1)^3 + 5*a^3*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 + a^3*sin(f*x + e)^5/(cos(f*x + e) + 1)^5))/f
```

mupad [B] time = 14.26, size = 245, normalized size = 1.93

$$\frac{2 \cos\left(\frac{e}{2} + \frac{fx}{2}\right) \left(\frac{53Ac}{4} + 3Ad + 3Bc + \frac{13Bd}{4} - 4Ac \cos(e + fx) + \frac{3Ad \cos(e+fx)}{2} + \frac{3Bc \cos(e+fx)}{2} + Bd \cos(e + fx)\right)}{a^3 + 5a^3 \sin\left(\frac{e}{2} + \frac{fx}{2}\right) + 10a^3 \sin^2\left(\frac{e}{2} + \frac{fx}{2}\right) + 10a^3 \sin^3\left(\frac{e}{2} + \frac{fx}{2}\right) + 5a^3 \sin^4\left(\frac{e}{2} + \frac{fx}{2}\right) + a^3 \sin^5\left(\frac{e}{2} + \frac{fx}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^3,x)
```

```
[Out] (2*cos(e/2 + (f*x)/2)*((53*A*c)/4 + 3*A*d + 3*B*c + (13*B*d)/4 - 4*A*c*cos(
e + f*x) + (3*A*d*cos(e + f*x))/2 + (3*B*c*cos(e + f*x))/2 + B*d*cos(e + f*
x) + (25*A*c*sin(e + f*x))/2 + (15*A*d*sin(e + f*x))/2 + (15*B*c*sin(e + f*
x))/2 + (5*B*d*sin(e + f*x))/2 - (9*A*c*cos(2*e + 2*f*x))/4 - (3*A*d*cos(2*
e + 2*f*x))/2 - (3*B*c*cos(2*e + 2*f*x))/2 - (9*B*d*cos(2*e + 2*f*x))/4 - (
5*A*c*sin(2*e + 2*f*x))/4 + (5*B*d*sin(2*e + 2*f*x))/4)/(15*a^3*f*((5*2^(1
/2)*cos((3*e)/2 + pi/4 + (3*f*x)/2))/4 - (5*2^(1/2)*cos(e/2 - pi/4 + (f*x)/
2))/2 + (2^(1/2)*cos((5*e)/2 - pi/4 + (5*f*x)/2))/4))
```

sympy [A] time = 15.10, size = 1819, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**3,x)
```

```
[Out] Piecewise((-30*A*c*tan(e/2 + f*x/2)**4/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*
a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*ta
n(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 60*A*c*tan(e/
2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 80*A*c*tan(e/2 + f*x/2)**2/(15*a**3*f*
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 1
5*a**3*f) - 40*A*c*tan(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**
3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e
/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 14*A*c/(15*a**3*
f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)**3/(15*a**3*f*tan(e/2 + f*x/2)**5 + 7
5*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(
e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*
a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*A*d*tan(e/2 + f*x/2)/(15*a**3*f*t
an(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f
*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15
*a**3*f) - 6*A*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)
)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75
*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**3/(15*a**3
*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2
+ f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2)
+ 15*a**3*f) - 30*B*c*tan(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f
tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 30*B*c*tan
(e/2 + f*x/2)/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**
4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a*
**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 6*B*c/(15*a**3*f*tan(e/2 + f*x/2)**5 +
75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*
f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 40*B*d*ta
n(e/2 + f*x/2)**2/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/
2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 7
5*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f) - 20*B*d*tan(e/2 + f*x/2)/(15*a**3*f
tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*tan(e/2 +
f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 + f*x/2) +
15*a**3*f) - 4*B*d/(15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x
/2)**4 + 150*a**3*f*tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 +
75*a**3*f*tan(e/2 + f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))*(c +
d*sin(e))/(a*sin(e) + a)**3, True))
```

$$3.282 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3} dx$$

Optimal. Leaf size=102

$$\frac{(2A+3B) \cos(e+fx)}{15f(a^3 \sin(e+fx)+a^3)} - \frac{(2A+3B) \cos(e+fx)}{15af(a \sin(e+fx)+a)^2} - \frac{(A-B) \cos(e+fx)}{5f(a \sin(e+fx)+a)^3}$$

[Out] $-1/5*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^3-1/15*(2*A+3*B)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^2-1/15*(2*A+3*B)*\cos(f*x+e)/f/(a^3+a^3*\sin(f*x+e))$

Rubi [A] time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2750, 2650, 2648}

$$\frac{(2A+3B) \cos(e+fx)}{15f(a^3 \sin(e+fx)+a^3)} - \frac{(2A+3B) \cos(e+fx)}{15af(a \sin(e+fx)+a)^2} - \frac{(A-B) \cos(e+fx)}{5f(a \sin(e+fx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]

[Out] $-((A-B)*\text{Cos}[e+f*x])/((5*f*(a+a*\text{Sin}[e+f*x])^3)-((2*A+3*B)*\text{Cos}[e+f*x]))/(15*a*f*(a+a*\text{Sin}[e+f*x])^2)-((2*A+3*B)*\text{Cos}[e+f*x])/((15*f*(a^3+a^3*\text{Sin}[e+f*x])))$

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3} dx &= -\frac{(A-B) \cos(e+fx)}{5f(a+a \sin(e+fx))^3} + \frac{(2A+3B) \int \frac{1}{(a+a \sin(e+fx))^2} dx}{5a} \\ &= -\frac{(A-B) \cos(e+fx)}{5f(a+a \sin(e+fx))^3} - \frac{(2A+3B) \cos(e+fx)}{15af(a+a \sin(e+fx))^2} + \frac{(2A+3B) \int \frac{1}{a+a \sin(e+fx)} dx}{15a^2} \\ &= -\frac{(A-B) \cos(e+fx)}{5f(a+a \sin(e+fx))^3} - \frac{(2A+3B) \cos(e+fx)}{15af(a+a \sin(e+fx))^2} - \frac{(2A+3B) \cos(e+fx)}{15f(a^3+a^3 \sin(e+fx))} \end{aligned}$$

Mathematica [A] time = 0.09, size = 63, normalized size = 0.62

$$\frac{\cos(e + fx) \left((2A + 3B) \sin^2(e + fx) + (6A + 9B) \sin(e + fx) + 7A + 3B \right)}{15a^3 f (\sin(e + fx) + 1)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^3,x]

[Out] -1/15*(Cos[e + f*x]*(7*A + 3*B + (6*A + 9*B)*Sin[e + f*x] + (2*A + 3*B)*Sin[e + f*x]^2))/(a^3*f*(1 + Sin[e + f*x])^3)

fricas [A] time = 0.42, size = 190, normalized size = 1.86

$$\frac{(2A + 3B) \cos^3(fx + e) - 2(2A + 3B) \cos^2(fx + e) - 3(3A + 2B) \cos(fx + e) - \left((2A + 3B) \cos(fx + e) \right)^2}{15 \left(a^3 f \cos^3(fx + e) + 3a^3 f \cos^2(fx + e) - 2a^3 f \cos(fx + e) - 4a^3 f + \left(a^3 f \cos(fx + e) \right)^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -1/15*((2*A + 3*B)*cos(f*x + e)^3 - 2*(2*A + 3*B)*cos(f*x + e)^2 - 3*(3*A + 2*B)*cos(f*x + e) - ((2*A + 3*B)*cos(f*x + e)^2 + 3*(2*A + 3*B)*cos(f*x + e) - 3*A + 3*B)*sin(f*x + e) - 3*A + 3*B)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e))^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))

giac [A] time = 0.17, size = 130, normalized size = 1.27

$$\frac{2 \left(15A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^4 + 30A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 15B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^3 + 40A \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 + 15B \tan\left(\frac{1}{2}fx + \frac{1}{2}e\right)^2 \right)}{15a^3 f \left(\tan\left(\frac{1}{2}fx + \frac{1}{2}e\right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x, algorithm="giac")

[Out] -2/15*(15*A*tan(1/2*f*x + 1/2*e)^4 + 30*A*tan(1/2*f*x + 1/2*e)^3 + 15*B*tan(1/2*f*x + 1/2*e)^3 + 40*A*tan(1/2*f*x + 1/2*e)^2 + 15*B*tan(1/2*f*x + 1/2*e)^2 + 20*A*tan(1/2*f*x + 1/2*e) + 15*B*tan(1/2*f*x + 1/2*e) + 7*A + 3*B)/(a^3*f*(tan(1/2*f*x + 1/2*e) + 1)^5)

maple [A] time = 0.32, size = 114, normalized size = 1.12

$$\frac{\frac{2(8A-6B)}{3\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^3} - \frac{2(4A-4B)}{5\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^5} - \frac{2A}{\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1} - \frac{-4A+2B}{\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^2} - \frac{-8A+8B}{2\left(\tan\left(\frac{fx}{2}+\frac{e}{2}\right)+1\right)^4}}{f a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3,x)

[Out] 2/f/a^3*(-1/3*(8*A-6*B)/(tan(1/2*f*x+1/2*e)+1)^3-1/5*(4*A-4*B)/(tan(1/2*f*x+1/2*e)+1)^5-A/(tan(1/2*f*x+1/2*e)+1)-1/2*(-4*A+2*B)/(tan(1/2*f*x+1/2*e)+1)^2-1/4*(-8*A+8*B)/(tan(1/2*f*x+1/2*e)+1)^4)

maxima [B] time = 0.39, size = 387, normalized size = 3.79

$$\frac{2 \left(\frac{A \left(\frac{20 \sin^2(fx+e)}{\cos(fx+e)+1} + \frac{40 \sin(fx+e)}{(\cos(fx+e)+1)^2} + \frac{30 \sin^2(fx+e)}{(\cos(fx+e)+1)^3} + \frac{15 \sin^3(fx+e)}{(\cos(fx+e)+1)^4} + 7 \right)}{a^3 + \frac{5a^3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{10a^3 \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{5a^3 \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{a^3 \sin^5(fx+e)}{(\cos(fx+e)+1)^5}} + \frac{3B \left(\frac{5 \sin(fx+e)}{\cos(fx+e)+1} + \frac{5 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{5 \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{5 \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{5 \sin^5(fx+e)}{(\cos(fx+e)+1)^5} \right)}{a^3 + \frac{5a^3 \sin(fx+e)}{\cos(fx+e)+1} + \frac{10a^3 \sin^2(fx+e)}{(\cos(fx+e)+1)^2} + \frac{10a^3 \sin^3(fx+e)}{(\cos(fx+e)+1)^3} + \frac{5a^3 \sin^4(fx+e)}{(\cos(fx+e)+1)^4} + \frac{a^3 \sin^5(fx+e)}{(\cos(fx+e)+1)^5}} \right)}{15f}$$


```
15*a**3*f*tan(e/2 + f*x/2)**5 + 75*a**3*f*tan(e/2 + f*x/2)**4 + 150*a**3*f*  
tan(e/2 + f*x/2)**3 + 150*a**3*f*tan(e/2 + f*x/2)**2 + 75*a**3*f*tan(e/2 +  
f*x/2) + 15*a**3*f), Ne(f, 0)), (x*(A + B*sin(e))/(a*sin(e) + a)**3, True))
```


$$3.283 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=229

$$\frac{2d^2(Bc - Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3 f(c-d)^3 \sqrt{c^2-d^2}} - \frac{(A(2c^2 - 9cd + 22d^2) + B(3c^2 - 16cd - 2d^2)) \cos(e+fx)}{15f(c-d)^3 (a^3 \sin(e+fx) + a^3)} - \frac{(2Ac - 7Ad)}{15af(c-d)}$$

[Out] $-1/5*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^3-1/15*(2*A*c-7*A*d+3*B*c+2*B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^2-1/15*(B*(3*c^2-16*c*d-2*d^2)+A*(2*c^2-9*c*d+22*d^2))*\cos(f*x+e)/(c-d)^3/f/(a^3+a^3*\sin(f*x+e))+2*d^2*(-A*d+B*c)*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^3/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 0.72, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2978, 12, 2660, 618, 204}

$$\frac{2d^2(Bc - Ad) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{a^3 f(c-d)^3 \sqrt{c^2-d^2}} - \frac{(A(2c^2 - 9cd + 22d^2) + B(3c^2 - 16cd - 2d^2)) \cos(e+fx)}{15f(c-d)^3 (a^3 \sin(e+fx) + a^3)} - \frac{(2Ac - 7Ad)}{15af(c-d)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])),x]

[Out] $(2*d^2*(B*c - A*d)*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2]]/\text{Sqrt}[c^2 - d^2])/(a^3*(c - d)^3*\text{Sqrt}[c^2 - d^2]*f) - ((A - B)*\text{Cos}[e + f*x])/(5*(c - d)*f*(a + a*\text{Sin}[e + f*x])^3) - ((2*A*c + 3*B*c - 7*A*d + 2*B*d)*\text{Cos}[e + f*x])/(15*a*(c - d)^2*f*(a + a*\text{Sin}[e + f*x])^2) - ((B*(3*c^2 - 16*c*d - 2*d^2) + A*(2*c^2 - 9*c*d + 22*d^2))*\text{Cos}[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*\text{Sin}[e + f*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2978

```

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&& !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))} dx &= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{\int \frac{-a(2Ac + 3Bc - 5Ad) - 2a(A - B)d \sin(e + fx)}{(a + a \sin(e + fx))^2 (c + d \sin(e + fx))} dx}{5a^2(c - d)} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3} - \frac{(2Ac + 3Bc - 7Ad + 2Bd) \cos(e + fx)}{15a(c - d)^2 f(a + a \sin(e + fx))} \\
&= \frac{2d^2(Bc - Ad) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{a^3(c - d)^3 \sqrt{c^2 - d^2} f} - \frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))}
\end{aligned}$$

Mathematica [B] time = 1.25, size = 502, normalized size = 2.19

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(-\frac{60d^2(Ad - Bc) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)^5 \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e + fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{\sqrt{c^2 - d^2}} + 20Ac^2 \sin\left(\frac{1}{2}(e + fx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])
), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(15*B*c^2*Cos[(e + f*x)/2] - 15*A*c*
d*Cos[(e + f*x)/2] - 75*B*c*d*Cos[(e + f*x)/2] + 75*A*d^2*Cos[(e + f*x)/2]
- 10*A*c^2*Cos[(3*(e + f*x))/2] - 15*B*c^2*Cos[(3*(e + f*x))/2] + 45*A*c*d*
Cos[(3*(e + f*x))/2] + 65*B*c*d*Cos[(3*(e + f*x))/2] - 95*A*d^2*Cos[(3*(e +
f*x))/2] + 10*B*d^2*Cos[(3*(e + f*x))/2] + 20*A*c^2*Sin[(e + f*x)/2] + 15*
B*c^2*Sin[(e + f*x)/2] - 75*A*c*d*Sin[(e + f*x)/2] - 85*B*c*d*Sin[(e + f*x)

```

$$\begin{aligned} & /2] + 145*A*d^2*\sin[(e + f*x)/2] - 20*B*d^2*\sin[(e + f*x)/2] - (60*d^2*(-(B \\ & *c) + A*d)*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/\text{Sqrt}[c^2 - d^2]]*(\text{Cos}[(e + f*x)/ \\ & 2] + \text{Sin}[(e + f*x)/2])^5)/\text{Sqrt}[c^2 - d^2] - 15*B*c*d*\sin[(3*(e + f*x))/2] + \\ & 15*A*d^2*\sin[(3*(e + f*x))/2] - 2*A*c^2*\sin[(5*(e + f*x))/2] - 3*B*c^2*\sin \\ & [(5*(e + f*x))/2] + 9*A*c*d*\sin[(5*(e + f*x))/2] + 16*B*c*d*\sin[(5*(e + f*x) \\ &)/2] - 22*A*d^2*\sin[(5*(e + f*x))/2] + 2*B*d^2*\sin[(5*(e + f*x))/2]))/(30* \\ & a^3*(c - d)^3*f*(1 + \text{Sin}[e + f*x])^3 \end{aligned}$$

fricas [B] time = 0.54, size = 2292, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [1/30*(6*(A - B)*c^4 - 12*(A - B)*c^3*d + 12*(A - B)*c*d^3 - 6*(A - B)*d^4 - 2*((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) + d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2)) + 6*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - 2*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f + ((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*sin(f*x + e)), 1/15*(3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^3 + (2*(2*A + 3*B)*c^4 - (18*A + 17*B)*c^3*d + 5*(5*A - 2*B)*c^2*d^2 + (18*A + 17*B)*c*d^3 - (29*A - 4*B)*d^4)*cos(f*x + e)^2 + 15*(4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^3 - 3*(B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x + e) + (4*B*c*d^2 - 4*A*d^3 - (B*c*d^2 - A*d^3)*cos(f*x + e)^2 + 2*(B*c*d^2 - A*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(c^2 - d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + 3*((3*A + 2*B)*c^4 - (11*A + 9*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (11*A + 9*B)*c*d^3 - 3*(6*A - B)*d^4)*cos(f*x + e) - (3*(A - B)*c^4 - 6*(A - B)*c^3*d + 6*(A - B)*c*d^3 - 3*(A - B)*d^4 - ((2*A + 3*B)*c^4 - (9*A + 16*B)*c^3*d + 5*(4*A - B)*c^2*d^2 + (9*A + 16*B)*c*d^3 - 2*(11*A - B)*d^4)*cos(f*x + e)^2 - 3*((2*A + 3*B)*c^4 - (9*A + 11*B)*c^3*d + 5*(3*A - B)*c^2*d^2 + (9*A + 11*B)*c*d^3 - (17*A - 2*B)*d^4)*cos(f*x + e))*sin(f*x + e))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^3 + 3*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e)^2 - 2*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*cos(f*x + e) - 4*(a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*f*sin(f*x + e))

$$d^2 + 2a^3c^2d^3 - 3a^3c^2d^4 + a^3d^5) * f + ((a^3c^5 - 3a^3c^4d + 2a^3c^3d^2 + 2a^3c^2d^3 - 3a^3c^2d^4 + a^3d^5) * f * \cos(f*x + e)^2 - 2 * (a^3c^5 - 3a^3c^4d + 2a^3c^3d^2 + 2a^3c^2d^3 - 3a^3c^2d^4 + a^3d^5) * f * \cos(f*x + e) - 4 * (a^3c^5 - 3a^3c^4d + 2a^3c^3d^2 + 2a^3c^2d^3 - 3a^3c^2d^4 + a^3d^5) * f) * \sin(f*x + e)]$$

giac [B] time = 0.26, size = 578, normalized size = 2.52

$$2 \left(\frac{15 (Bcd^2 - Ad^3) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(a^3c^3 - 3a^3c^2d + 3a^3cd^2 - a^3d^3) \sqrt{c^2 - d^2}} - \frac{15 Ac^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 - 45 Acd \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 + 45 Ad^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 - 15 Bd^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 + 30 A^2 c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 15 B^2 c \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 105 A^2 d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 45 B^2 d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 + 135 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 30 B^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 + 40 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 15 B^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 135 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 65 B^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 185 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 40 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 20 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 15 B^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) - 75 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 115 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) - 20 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 7 A^2 c^2 + 3 B^2 c^2 - 24 A^2 c d - 11 B^2 c d + 32 A^2 d^2 - 7 B^2 d^2}{(a^3c^3 - 3a^3c^2d + 3a^3cd^2 - a^3d^3) * (\tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 1)^5} \right) / f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] 2/15*(15*(B*c*d^2 - A*d^3)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*sqrt(c^2 - d^2)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 45*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 45*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 15*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 105*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 45*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 135*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 30*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 135*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 65*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 185*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 40*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 75*A*c*d*tan(1/2*f*x + 1/2*e) - 55*B*c*d*tan(1/2*f*x + 1/2*e) + 115*A*d^2*tan(1/2*f*x + 1/2*e) - 20*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 24*A*c*d - 11*B*c*d + 32*A*d^2 - 7*B*d^2)/((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f

maple [B] time = 0.54, size = 606, normalized size = 2.65

$$\frac{2d^3 \arctan \left(\frac{2c \tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 2d}{2\sqrt{c^2 - d^2}} \right) A}{f a^3 (c - d)^3 \sqrt{c^2 - d^2}} + \frac{2d^2 \arctan \left(\frac{2c \tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 2d}{2\sqrt{c^2 - d^2}} \right) Bc}{f a^3 (c - d)^3 \sqrt{c^2 - d^2}} + \frac{4A}{f a^3 (c - d) \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 1 \right)^4} - \frac{15 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 - 45 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 + 45 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 - 15 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^4 + 30 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 15 B^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 105 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 45 B^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 + 135 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 - 30 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^3 + 40 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 15 B^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 135 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 65 B^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 185 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 - 40 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right)^2 + 20 A^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 15 B^2 c^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) - 75 A^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) - 55 B^2 c d \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 115 A^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) - 20 B^2 d^2 \tan \left(\frac{1}{2} fx + \frac{1}{2} e \right) + 7 A^2 c^2 + 3 B^2 c^2 - 24 A^2 c d - 11 B^2 c d + 32 A^2 d^2 - 7 B^2 d^2}{f a^3 (c - d) \left(\tan \left(\frac{fx}{2} + \frac{e}{2} \right) + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)

[Out] -2/f/a^3*d^3/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*A+2/f/a^3*d^2/(c-d)^3/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^(1/2))*B*c+4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4*A-4/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d)/(tan(1/2*f*x+1/2*e)+1)^5*B+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*A*c-6/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*A*d-2/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B*c+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^2*B*d-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*c+20/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B*c-16/3/f/a^3/(c-d)^2/(tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c^2+6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*c*d-6/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*A*d^2+2/f/a^3/(c-d)^3/(tan(1/2*f*x+1/2*e)+1)*B*d^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see 'assume?' for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 17.18, size = 591, normalized size = 2.58

$$2d^2 \operatorname{atan} \left(\frac{\frac{d^2(A d - B c) (-2a^3 c^3 d + 6a^3 c^2 d^2 - 6a^3 c d^3 + 2a^3 d^4) - 2cd^2 \tan\left(\frac{e}{2} + \frac{fx}{2}\right) (A d - B c) (a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3)}{a^3 \sqrt{c+d} (c-d)^{7/2}}}{2Ad^3 - 2Bcd^2} \right) (Ad - Bc) \frac{2(7Ac^2 + 32c^2 d - 7Ad^2)}{a^3 f \sqrt{c+d} (c-d)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^3*(c + d*sin(e + f*x))),x)

[Out] (2*d^2*atan(((d^2*(A*d - B*c))*(2*a^3*d^4 - 6*a^3*c*d^3 - 2*a^3*c^3*d + 6*a^3*c^2*d^2))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)) - (2*c*d^2*tan(e/2 + (f*x)/2)*(A*d - B*c)*(a^3*c^3 - a^3*d^3 + 3*a^3*c*d^2 - 3*a^3*c^2*d))/(a^3*(c + d)^(1/2)*(c - d)^(7/2)))/(2*A*d^3 - 2*B*c*d^2)*(A*d - B*c))/(a^3*f*(c + d)^(1/2)*(c - d)^(7/2)) - ((2*(7*A*c^2 + 32*A*d^2 + 3*B*c^2 - 7*B*d^2 - 24*A*c*d - 11*B*c*d))/(15*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)*(4*A*c^2 + 23*A*d^2 + 3*B*c^2 - 4*B*d^2 - 15*A*c*d - 11*B*c*d))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^3*(2*A*c^2 + 9*A*d^2 + B*c^2 - 2*B*d^2 - 7*A*c*d - 3*B*c*d))/((c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^2*(8*A*c^2 + 37*A*d^2 + 3*B*c^2 - 8*B*d^2 - 27*A*c*d - 13*B*c*d))/(3*(c - d)*(c^2 - 2*c*d + d^2)) + (2*tan(e/2 + (f*x)/2)^4*(A*c^2 + 3*A*d^2 - B*d^2 - 3*A*c*d))/((c - d)*(c^2 - 2*c*d + d^2)))/(f*(10*a^3*tan(e/2 + (f*x)/2)^2 + 10*a^3*tan(e/2 + (f*x)/2)^3 + 5*a^3*tan(e/2 + (f*x)/2)^4 + a^3*tan(e/2 + (f*x)/2)^5 + a^3 + 5*a^3*tan(e/2 + (f*x)/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.284 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=381

$$\frac{2d^2 \left(Ad(4c+3d) - B(3c^2+3cd+d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}} \right)}{a^3 f(c-d)^4(c+d)\sqrt{c^2-d^2}} \frac{\left(A(2c^2-12cd+45d^2) + B(3c^2-23cd-15d^2) \right)}{15f(c-d)^3 \left(a^3 \sin(e+fx) + a^3 \right) (c+d \sin(e+fx))}$$

[Out] $-1/15*d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d^3))*\cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*\sin(f*x+e))-1/5*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^3/(c+d*\sin(f*x+e))-1/15*(2*A*c-9*A*d+3*B*c+4*B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^2/(c+d*\sin(f*x+e))-1/15*(B*(3*c^2-23*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*\cos(f*x+e)/(c-d)^3/f/(a^3+a^3*\sin(f*x+e))/(c+d*\sin(f*x+e))-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^4/(c+d)/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 1.08, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{2d^2 \left(Ad(4c+3d) - B(3c^2+3cd+d^2) \right) \tan^{-1} \left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}} \right)}{a^3 f(c-d)^4(c+d)\sqrt{c^2-d^2}} \frac{d \left(A(-12c^2d+2c^3+43cd^2+72d^3) + B(-23cd^2-15d^3) \right)}{15a^3 f(c-d)^4(c+d)(c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]

[Out] $(-2*d^2*(A*d*(4*c+3*d)-B*(3*c^2+3*c*d+d^2))*\text{ArcTan}[(d+c*\text{Tan}[(e+f*x)/2])/ \text{Sqrt}[c^2-d^2]])/(a^3*(c-d)^4*(c+d)*\text{Sqrt}[c^2-d^2]*f) - (d*(B*(3*c^3-23*c^2*d-63*c*d^2-22*d^3)+A*(2*c^3-12*c^2*d+43*c*d^2+72*d^3))*\text{Cos}[e+f*x])/(15*a^3*(c-d)^4*(c+d)*f*(c+d*\text{Sin}[e+f*x])) - ((A-B)*\text{Cos}[e+f*x])/(5*(c-d)*f*(a+a*\text{Sin}[e+f*x])^3*(c+d*\text{Sin}[e+f*x])) - ((2*A*c+3*B*c-9*A*d+4*B*d)*\text{Cos}[e+f*x])/(15*a*(c-d)^2*f*(a+a*\text{Sin}[e+f*x])^2*(c+d*\text{Sin}[e+f*x])) - ((B*(3*c^2-23*c*d-15*d^2)+A*(2*c^2-12*c*d+45*d^2))*\text{Cos}[e+f*x])/(15*(c-d)^3*f*(a^3+a^3*\text{Sin}[e+f*x])*(c+d*\text{Sin}[e+f*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

$e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 2754

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x)))^m \cdot (c + (d \cdot \sin(e + f \cdot x)) + (f \cdot x))], x_Symbol] \rightarrow -\text{Simp}[(b \cdot c - a \cdot d) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} / (f \cdot (m+1) \cdot (a^2 - b^2)), x] + \text{Dist}[1 / ((m+1) \cdot (a^2 - b^2)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot \text{Simp}[(a \cdot c - b \cdot d) \cdot (m+1) - (b \cdot c - a \cdot d) \cdot (m+2) \cdot \text{Sin}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntegerQ}[2 \cdot m]$

Rule 2978

$\text{Int}[(a + (b \cdot \sin(e + f \cdot x)))^m \cdot (A + (B \cdot \sin(e + f \cdot x)) + (f \cdot x)) \cdot (c + (d \cdot \sin(e + f \cdot x)) + (f \cdot x))^n], x_Symbol] \rightarrow \text{Simp}[(b \cdot (A \cdot b - a \cdot B) \cdot \text{Cos}[e + f \cdot x] \cdot (a + b \cdot \text{Sin}[e + f \cdot x])^m \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^{n+1} / (a \cdot f \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), x] + \text{Dist}[1 / (a \cdot (2 \cdot m + 1) \cdot (b \cdot c - a \cdot d)), \text{Int}[(a + b \cdot \text{Sin}[e + f \cdot x])^{m+1} \cdot (c + d \cdot \text{Sin}[e + f \cdot x])^n \cdot \text{Simp}[B \cdot (a \cdot c \cdot m + b \cdot d \cdot (n+1)) + A \cdot (b \cdot c \cdot (m+1) - a \cdot d \cdot (2 \cdot m + n + 2)) + d \cdot (A \cdot b - a \cdot B) \cdot (m + n + 2) \cdot \text{Sin}[e + f \cdot x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{!GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2 \cdot m] \ \&\& \ (\text{IntegerQ}[2 \cdot n] \ || \ \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^2} dx &= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))} - \frac{\int \frac{-a(2A(c - d) + B(a^2 - c^2)) \cos(e + fx)}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^2} dx}{15a(c - d)f(c + d \sin(e + fx))} \\ &= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))} - \frac{(2A - B) \cos(e + fx)}{15a(c - d)f(c + d \sin(e + fx))} \\ &= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))} - \frac{(2A - B) \cos(e + fx)}{15a(c - d)f(c + d \sin(e + fx))} \\ &= -\frac{d(B(3c^3 - 23c^2d - 63cd^2 - 22d^3) + A(2c^3 - 12c^2d + 43cd^2))}{15a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\ &= -\frac{d(B(3c^3 - 23c^2d - 63cd^2 - 22d^3) + A(2c^3 - 12c^2d + 43cd^2))}{15a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\ &= -\frac{d(B(3c^3 - 23c^2d - 63cd^2 - 22d^3) + A(2c^3 - 12c^2d + 43cd^2))}{15a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\ &= -\frac{d(B(3c^3 - 23c^2d - 63cd^2 - 22d^3) + A(2c^3 - 12c^2d + 43cd^2))}{15a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\ &= -\frac{2d^2(Ad(4c + 3d) - B(3c^2 + 3cd + d^2)) \tan^{-1}\left(\frac{d + c \tan\left(\frac{1}{2}(e + fx)\right)}{\sqrt{c^2 - d^2}}\right)}{a^3(c - d)^4(c + d)\sqrt{c^2 - d^2} f} \end{aligned}$$

Mathematica [B] time = 6.38, size = 1253, normalized size = 3.29

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] (2*d^2*(3*B*c^2 - 4*A*c*d + 3*B*c*d - 3*A*d^2 + B*d^2)*ArcTan[(Sec[(e + f*x)/2]*(d*Cos[(e + f*x)/2] + c*Sin[(e + f*x)/2]))/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)/((c - d)^4*(c + d)*Sqrt[c^2 - d^2]*f*(a + a*Sin[e + f*x])^3) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(60*B*c^4*Cos[(e + f*x)/2] - 80*A*c^3*d*Cos[(e + f*x)/2] - 390*B*c^3*d*Cos[(e + f*x)/2] + 540*A*c^2*d^2*Cos[(e + f*x)/2] - 1090*B*c^2*d^2*Cos[(e + f*x)/2] + 1430*A*c*d^3*Cos[(e + f*x)/2] - 885*B*c*d^3*Cos[(e + f*x)/2] + 735*A*d^4*Cos[(e + f*x)/2] - 320*B*d^4*Cos[(e + f*x)/2] - 40*A*c^4*Cos[(3*(e + f*x))/2] - 60*B*c^4*Cos[(3*(e + f*x))/2] + 196*A*c^3*d*Cos[(3*(e + f*x))/2] + 304*B*c^3*d*Cos[(3*(e + f*x))/2] - 476*A*c^2*d^2*Cos[(3*(e + f*x))/2] + 1076*B*c^2*d^2*Cos[(3*(e + f*x))/2] - 1546*A*c*d^3*Cos[(3*(e + f*x))/2] + 1181*B*c*d^3*Cos[(3*(e + f*x))/2] - 969*A*d^4*Cos[(3*(e + f*x))/2] + 334*B*d^4*Cos[(3*(e + f*x))/2] + 60*B*c^2*d^2*Cos[(5*(e + f*x))/2] - 90*A*c*d^3*Cos[(5*(e + f*x))/2] + 15*B*c*d^3*Cos[(5*(e + f*x))/2] - 15*A*d^4*Cos[(5*(e + f*x))/2] + 30*B*d^4*Cos[(5*(e + f*x))/2] + 4*A*c^3*d*Cos[(7*(e + f*x))/2] + 6*B*c^3*d*Cos[(7*(e + f*x))/2] - 24*A*c^2*d^2*Cos[(7*(e + f*x))/2] - 46*B*c^2*d^2*Cos[(7*(e + f*x))/2] + 86*A*c*d^3*Cos[(7*(e + f*x))/2] - 111*B*c*d^3*Cos[(7*(e + f*x))/2] + 129*A*d^4*Cos[(7*(e + f*x))/2] - 44*B*d^4*Cos[(7*(e + f*x))/2] + 80*A*c^4*Sin[(e + f*x)/2] + 60*B*c^4*Sin[(e + f*x)/2] - 340*A*c^3*d*Sin[(e + f*x)/2] - 440*B*c^3*d*Sin[(e + f*x)/2] + 820*A*c^2*d^2*Sin[(e + f*x)/2] - 1520*B*c^2*d^2*Sin[(e + f*x)/2] + 2140*A*c*d^3*Sin[(e + f*x)/2] - 1435*B*c*d^3*Sin[(e + f*x)/2] + 975*A*d^4*Sin[(e + f*x)/2] - 340*B*d^4*Sin[(e + f*x)/2] - 90*B*c^3*d*Sin[(3*(e + f*x))/2] + 120*A*c^2*d^2*Sin[(3*(e + f*x))/2] - 390*B*c^2*d^2*Sin[(3*(e + f*x))/2] + 540*A*c*d^3*Sin[(3*(e + f*x))/2] - 315*B*c*d^3*Sin[(3*(e + f*x))/2] + 285*A*d^4*Sin[(3*(e + f*x))/2] - 150*B*d^4*Sin[(3*(e + f*x))/2] - 8*A*c^4*Sin[(5*(e + f*x))/2] - 12*B*c^4*Sin[(5*(e + f*x))/2] + 28*A*c^3*d*Sin[(5*(e + f*x))/2] + 62*B*c^3*d*Sin[(5*(e + f*x))/2] - 52*A*c^2*d^2*Sin[(5*(e + f*x))/2] + 362*B*c^2*d^2*Sin[(5*(e + f*x))/2] - 568*A*c*d^3*Sin[(5*(e + f*x))/2] + 553*B*c*d^3*Sin[(5*(e + f*x))/2] - 555*A*d^4*Sin[(5*(e + f*x))/2] + 190*B*d^4*Sin[(5*(e + f*x))/2] - 15*B*c*d^3*Sin[(7*(e + f*x))/2] + 15*A*d^4*Sin[(7*(e + f*x))/2]))/(120*(c - d)^4*(c + d)*f*(a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x]))
```

fricas [B] time = 0.66, size = 4486, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(A - B)*c^6 - 12*(A - B)*c^5*d - 6*(A - B)*c^4*d^2 + 24*(A - B)*c^3*d^3 - 6*(A - B)*c^2*d^4 - 12*(A - B)*c*d^5 + 6*(A - B)*d^6 - 2*((2*A + 3*B)*c^5*d - (12*A + 23*B)*c^4*d^2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*cos(f*x + e)^4 - 2*((2*A + 3*B)*c^6 - 2*(3*A + 7*B)*c^5*d + 5*(A - 18*B)*c^4*d^2 + (147*A - 152*B)*c^3*d^3 + 4*(41*A + 9*B)*c^2*d^4 - (141*A - 166*B)*c*d^5 - 3*(57*A - 17*B)*d^6)*cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^6 - (19*A + 16*B)*c^5*d + 11*(2*A - 7*B)*c^4*d^2 + 8*(16*A - 11*B)*c^3*d^3 + 2*(32*A + 23*B)*c^2*d^4 - (109*A - 104*B)*c*d^5 - 5*(18*A - 5*B)*d^6)*cos(f*x + e)^2 + 15*(12*B*c^3*d^2 - 8*(2*A - 3*B)*c^2*d^3 - 4*(7*A - 4*B)*c*d^4 - 4*(3*A - B)*d^5 + (3*B*c^2*d^3 - (4*A - 3*B)*c*d^4 - (3*A - B)*d^5)*cos(f*x + e)^4 - (3*B*c^3*d^2 - (4*A - 9*B)*c^2*d^3 - (11*A - 7*B)*c*d^4 - 2*(3*A - B)*d^5)*cos(f*x + e)^3 - (9*B*c^3*d^2 - 12*(A - 2*B)*c^2*d^3 - (29*A - 18*B)*c*d^4 - 5*(3*A - B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^3*d^2 - 2*(2*A - 3*B)*c^2*d^3 - (7*A - 4*B)*c*d^4 -
```


$$\begin{aligned}
& (3A - B)d^5 \cos(fx + e) + (12Bc^3d^2 - 8(2A - 3B)c^2d^3 - 4(7A - 4B)c^2d^4 - 4(3A - B)d^5 - (3Bc^2d^3 - (4A - 3B)c^2d^4 - (3A - B)d^5) \cos(fx + e)^3 - (3Bc^3d^2 - 4(A - 3B)c^2d^3 - 5(3A - 2B)c^2d^4 - 3(3A - B)d^5) \cos(fx + e)^2 + 2(3Bc^3d^2 - 2(2A - 3B)c^2d^3 - (7A - 4B)c^2d^4 - (3A - B)d^5) \cos(fx + e) \sin(fx + e)) \sqrt{-c^2 + d^2} \log(((2c^2 - d^2) \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2 + 2(c \cos(fx + e) \sin(fx + e) + d \cos(fx + e)) \sqrt{-c^2 + d^2}) / (d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2)) + 6((3A + 2B)c^6 - (11A + 9B)c^5d + (12A - 47B)c^4d^2 + 2(41A - 31B)c^3d^3 + (47A + 28B)c^2d^4 - 71(A - B)c^2d^5 - (62A - 17B)d^6) \cos(fx + e) - 2(3(A - B)c^6 - 6(A - B)c^5d - 3(A - B)c^4d^2 + 12(A - B)c^3d^3 - 3(A - B)c^2d^4 - 6(A - B)c^2d^5 + 3(A - B)d^6 + ((2A + 3B)c^5d - (12A + 23B)c^4d^2 + (41A - 66B)c^3d^3 + (84A + B)c^2d^4 - (43A - 63B)c^2d^5 - 2(36A - 11B)d^6) \cos(fx + e)^3 - ((2A + 3B)c^6 - (8A + 17B)c^5d + (17A - 67B)c^4d^2 + 2(53A - 43B)c^3d^3 + 5(16A + 7B)c^2d^4 - (98A - 103B)c^2d^5 - (99A - 29B)d^6) \cos(fx + e)^2 - 3((2A + 3B)c^6 - (9A + 11B)c^5d + (13A - 48B)c^4d^2 + 2(39A - 29B)c^3d^3 + 3(16A + 9B)c^2d^4 - 69(A - B)c^2d^5 - 9(7A - 2B)d^6) \cos(fx + e) \sin(fx + e)) / ((a^3c^7d - 3a^3c^6d^2 + a^3c^5d^3 + 5a^3c^4d^4 - 5a^3c^3d^5 - a^3c^2d^6 + 3a^3c^2d^7 - a^3d^8) f \cos(fx + e)^4 - (a^3c^8 - a^3c^7d - 5a^3c^6d^2 + 7a^3c^5d^3 + 5a^3c^4d^4 - 11a^3c^3d^5 + a^3c^2d^6 + 5a^3c^2d^7 - 2a^3d^8) f \cos(fx + e)^3 - (3a^3c^8 - 4a^3c^7d - 12a^3c^6d^2 + 20a^3c^5d^3 + 10a^3c^4d^4 - 28a^3c^3d^5 + 4a^3c^2d^6 + 12a^3c^2d^7 - 5a^3d^8) f \cos(fx + e)^2 + 2(a^3c^8 - 2a^3c^7d - 2a^3c^6d^2 + 6a^3c^5d^3 - 6a^3c^3d^5 + 2a^3c^2d^6 + 2a^3c^2d^7 - a^3d^8) f \cos(fx + e) + 4(a^3c^8 - 2a^3c^7d - 2a^3c^6d^2 + 6a^3c^5d^3 - 6a^3c^3d^5 + 2a^3c^2d^6 + 2a^3c^2d^7 - a^3d^8) f - ((a^3c^7d - 3a^3c^6d^2 + a^3c^5d^3 + 5a^3c^4d^4 - 5a^3c^3d^5 - a^3c^2d^6 + 3a^3c^2d^7 - a^3d^8) f \cos(fx + e)^3 + (a^3c^8 - 8a^3c^6d^2 + 8a^3c^5d^3 + 10a^3c^4d^4 - 16a^3c^3d^5 + 8a^3c^2d^7 - 3a^3d^8) f \cos(fx + e)^2 - 2(a^3c^8 - 2a^3c^7d - 2a^3c^6d^2 + 6a^3c^5d^3 - 6a^3c^3d^5 + 2a^3c^2d^6 + 2a^3c^2d^7 - a^3d^8) f \cos(fx + e) - 4(a^3c^8 - 2a^3c^7d - 2a^3c^6d^2 + 6a^3c^5d^3 - 6a^3c^3d^5 + 2a^3c^2d^6 + 2a^3c^2d^7 - a^3d^8) f) \sin(fx + e)), -1/15(3(A - B)c^6 - 6(A - B)c^5d - 3(A - B)c^4d^2 + 12(A - B)c^3d^3 - 3(A - B)c^2d^4 - 6(A - B)c^2d^5 + 3(A - B)d^6 - ((2A + 3B)c^5d - (12A + 23B)c^4d^2 + (41A - 66B)c^3d^3 + (84A + B)c^2d^4 - (43A - 63B)c^2d^5 - 2(36A - 11B)d^6) \cos(fx + e)^4 - ((2A + 3B)c^6 - 2(3A + 7B)c^5d + 5(A - 18B)c^4d^2 + (147A - 152B)c^3d^3 + 4(41A + 9B)c^2d^4 - (141A - 166B)c^2d^5 - 3(57A - 17B)d^6) \cos(fx + e)^3 + (2(2A + 3B)c^6 - (19A + 16B)c^5d + 11(2A - 7B)c^4d^2 + 8(16A - 11B)c^3d^3 + 2(32A + 23B)c^2d^4 - (109A - 104B)c^2d^5 - 5(18A - 5B)d^6) \cos(fx + e)^2 + 15(12Bc^3d^2 - 8(2A - 3B)c^2d^3 - 4(7A - 4B)c^2d^4 - 4(3A - B)d^5 + (3Bc^2d^3 - (4A - 3B)c^2d^4 - (3A - B)d^5) \cos(fx + e)^4 - (3Bc^3d^2 - (4A - 9B)c^2d^3 - (11A - 7B)c^2d^4 - 2(3A - B)d^5) \cos(fx + e)^3 - (9Bc^3d^2 - 12(A - 2B)c^2d^3 - (29A - 18B)c^2d^4 - 5(3A - B)d^5) \cos(fx + e)^2 + 2(3Bc^3d^2 - 2(2A - 3B)c^2d^3 - (7A - 4B)c^2d^4 - (3A - B)d^5) \cos(fx + e) + (12Bc^3d^2 - 8(2A - 3B)c^2d^3 - 4(7A - 4B)c^2d^4 - 4(3A - B)d^5 - (3Bc^2d^3 - (4A - 3B)c^2d^4 - (3A - B)d^5) \cos(fx + e)^3 - (3Bc^3d^2 - 4(A - 3B)c^2d^3 - 5(3A - 2B)c^2d^4 - 3(3A - B)d^5) \cos(fx + e)^2 + 2(3Bc^3d^2 - 2(2A - 3B)c^2d^3 - (7A - 4B)c^2d^4 - (3A - B)d^5) \cos(fx + e) \sin(fx + e)) \sqrt{c^2 - d^2} \arctan(-(c \sin(fx + e) + d) / (\sqrt{c^2 - d^2} \cos(fx + e))) + 3((3A + 2B)c^6 - (11A + 9B)c^5d + (12A - 47B)c^4d^2 + 2(41A - 31B)c^3d^3 + (47A + 28B)c^2d^4 - 71(A - B)c^2d^5 - (62A - 17B)d^6) \cos(fx + e) - (3(A - B)c^6 - 6(A - B)c^5d - 3(A - B)c^4d^2 + 12(A - B)c^3d^3 - 3(A - B)c^2d^4 - 6(A - B)c^2d^5 + 3(A - B)d^6 + ((2A + 3B)c^5d - (12A + 23B)c^4d^2
\end{aligned}$$

$$2 + (41*A - 66*B)*c^3*d^3 + (84*A + B)*c^2*d^4 - (43*A - 63*B)*c*d^5 - 2*(36*A - 11*B)*d^6)*\cos(f*x + e)^3 - ((2*A + 3*B)*c^6 - (8*A + 17*B)*c^5*d + (17*A - 67*B)*c^4*d^2 + 2*(53*A - 43*B)*c^3*d^3 + 5*(16*A + 7*B)*c^2*d^4 - (98*A - 103*B)*c*d^5 - (99*A - 29*B)*d^6)*\cos(f*x + e)^2 - 3*((2*A + 3*B)*c^6 - (9*A + 11*B)*c^5*d + (13*A - 48*B)*c^4*d^2 + 2*(39*A - 29*B)*c^3*d^3 + 3*(16*A + 9*B)*c^2*d^4 - 69*(A - B)*c*d^5 - 9*(7*A - 2*B)*d^6)*\cos(f*x + e)*\sin(f*x + e))/((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*\cos(f*x + e)^4 - (a^3*c^8 - a^3*c^7*d - 5*a^3*c^6*d^2 + 7*a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 11*a^3*c^3*d^5 + a^3*c^2*d^6 + 5*a^3*c*d^7 - 2*a^3*d^8)*f*\cos(f*x + e)^3 - (3*a^3*c^8 - 4*a^3*c^7*d - 12*a^3*c^6*d^2 + 20*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 28*a^3*c^3*d^5 + 4*a^3*c^2*d^6 + 12*a^3*c*d^7 - 5*a^3*d^8)*f*\cos(f*x + e)^2 + 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*\cos(f*x + e) + 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f - ((a^3*c^7*d - 3*a^3*c^6*d^2 + a^3*c^5*d^3 + 5*a^3*c^4*d^4 - 5*a^3*c^3*d^5 - a^3*c^2*d^6 + 3*a^3*c*d^7 - a^3*d^8)*f*\cos(f*x + e))^3 + (a^3*c^8 - 8*a^3*c^6*d^2 + 8*a^3*c^5*d^3 + 10*a^3*c^4*d^4 - 16*a^3*c^3*d^5 + 8*a^3*c*d^7 - 3*a^3*d^8)*f*\cos(f*x + e)^2 - 2*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f*\cos(f*x + e) - 4*(a^3*c^8 - 2*a^3*c^7*d - 2*a^3*c^6*d^2 + 6*a^3*c^5*d^3 - 6*a^3*c^3*d^5 + 2*a^3*c^2*d^6 + 2*a^3*c*d^7 - a^3*d^8)*f)*\sin(f*x + e)]$$

giac [B] time = 0.30, size = 772, normalized size = 2.03

$$2 \left(\frac{15 \left(3 B c^2 d^2 - 4 A c d^3 + 3 B c d^3 - 3 A d^4 + B d^4 \right) \left(\pi \left\lfloor \frac{f x + e}{2 \pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{\left(a^3 c^5 - 3 a^3 c^4 d + 2 a^3 c^3 d^2 + 2 a^3 c^2 d^3 - 3 a^3 c d^4 + a^3 d^5 \right) \sqrt{c^2 - d^2}} \right) + \frac{15 \left(B c d^4 \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) - A d^5 \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) \right)}{\left(a^3 c^6 - 3 a^3 c^5 d + 2 a^3 c^4 d^2 + 2 a^3 c^3 d^3 - 3 a^3 c^2 d^4 + a^3 c d^5 \right) \left(\frac{c \tan \left(\frac{1}{2} f x + \frac{1}{2} e \right) + d}{\sqrt{c^2 - d^2}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] 2/15*(15*(3*B*c^2*d^2 - 4*A*c*d^3 + 3*B*c*d^3 - 3*A*d^4 + B*d^4)*(pi*floor(1/2*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2 - d^2)))/((a^3*c^5 - 3*a^3*c^4*d + 2*a^3*c^3*d^2 + 2*a^3*c^2*d^3 - 3*a^3*c*d^4 + a^3*d^5)*sqrt(c^2 - d^2)) + 15*(B*c*d^4*tan(1/2*f*x + 1/2*e) - A*d^5*tan(1/2*f*x + 1/2*e) + B*c^2*d^3 - A*c*d^4)/((a^3*c^6 - 3*a^3*c^5*d + 2*a^3*c^4*d^2 + 2*a^3*c^3*d^3 - 3*a^3*c^2*d^4 + a^3*c*d^5)*(c*tan(1/2*f*x + 1/2*e)^2 + 2*d*tan(1/2*f*x + 1/2*e) + c)) - (15*A*c^2*tan(1/2*f*x + 1/2*e)^4 - 60*A*c*d*tan(1/2*f*x + 1/2*e)^4 + 90*A*d^2*tan(1/2*f*x + 1/2*e)^4 - 45*B*d^2*tan(1/2*f*x + 1/2*e)^4 + 30*A*c^2*tan(1/2*f*x + 1/2*e)^3 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^3 - 150*A*c*d*tan(1/2*f*x + 1/2*e)^3 - 60*B*c*d*tan(1/2*f*x + 1/2*e)^3 + 300*A*d^2*tan(1/2*f*x + 1/2*e)^3 - 135*B*d^2*tan(1/2*f*x + 1/2*e)^3 + 40*A*c^2*tan(1/2*f*x + 1/2*e)^2 + 15*B*c^2*tan(1/2*f*x + 1/2*e)^2 - 190*A*c*d*tan(1/2*f*x + 1/2*e)^2 - 100*B*c*d*tan(1/2*f*x + 1/2*e)^2 + 420*A*d^2*tan(1/2*f*x + 1/2*e)^2 - 185*B*d^2*tan(1/2*f*x + 1/2*e)^2 + 20*A*c^2*tan(1/2*f*x + 1/2*e) + 15*B*c^2*tan(1/2*f*x + 1/2*e) - 110*A*c*d*tan(1/2*f*x + 1/2*e) - 80*B*c*d*tan(1/2*f*x + 1/2*e) + 270*A*d^2*tan(1/2*f*x + 1/2*e) - 115*B*d^2*tan(1/2*f*x + 1/2*e) + 7*A*c^2 + 3*B*c^2 - 34*A*c*d - 16*B*c*d + 72*A*d^2 - 32*B*d^2)/((a^3*c^4 - 4*a^3*c^3*d + 6*a^3*c^2*d^2 - 4*a^3*c*d^3 + a^3*d^4)*(tan(1/2*f*x + 1/2*e) + 1)^5))/f

maple [B] time = 0.57, size = 1049, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sin(f*x+e))/(a+a*\sin(f*x+e))^3/(c+d*\sin(f*x+e))^2,x)$

[Out]
$$\frac{2/f/a^3*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*\tan(1/2*f*x+1/2*e)*B-6/f/a^3*d^4/(c-d)^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A+2/f/a^3*d^4/(c-d)^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B+2/f/a^3*d^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*B*c+6/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)*B*d^2+4/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*A*c-8/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*A*d-2/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*B*c+6/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^2*B*d-2/f/a^3*d^4/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)*A+8/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)*A*c*d-16/3/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3*A*c+8/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3*A*d+4/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3*B*c-20/3/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)*A*c^2-12/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)*A*d^2+4/f/a^3/(c-d)^2/(\tan(1/2*f*x+1/2*e)+1)^4*A-2/f/a^3*d^5/(c-d)^4/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c+d)/c*\tan(1/2*f*x+1/2*e)*A-8/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c+6/f/a^3*d^2/(c-d)^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c^2+6/f/a^3*d^3/(c-d)^4/(c+d)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c-8/5/f/a^3/(c-d)^2/(\tan(1/2*f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d)^2/(\tan(1/2*f*x+1/2*e)+1)^5*B-4/f/a^3/(c-d)^2/(\tan(1/2*f*x+1/2*e)+1)^4*B$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\sin(f*x+e))/(a+a*\sin(f*x+e))^3/(c+d*\sin(f*x+e))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details) Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 17.67, size = 1349, normalized size = 3.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B*\sin(e + f*x))/((a + a*\sin(e + f*x))^3*(c + d*\sin(e + f*x))^2),x)$

[Out]
$$\frac{(2*d^2*\text{atan}(((d^2*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))*(2*a^3*d^6 - 6*a^3*c*d^5 + 2*a^3*c^5*d + 4*a^3*c^2*d^4 + 4*a^3*c^3*d^3 - 6*a^3*c^4*d^2)))/(a^3*(c + d)^{(3/2)}*(c - d)^{(9/2)}) + (2*c*d^2*\tan(e/2 + (f*x)/2)*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d)*(a^3*c^5 + a^3*d^5 - 3*a^3*c*d^4 - 3*a^3*c^4*d + 2*a^3*c^2*d^3 + 2*a^3*c^3*d^2))/(a^3*(c + d)^{(3/2)}*(c - d)^{(9/2)})/(2*B*d^4 - 6*A*d^4 + 6*B*c^2*d^2 - 8*A*c*d^3 + 6*B*c*d^3))*(3*B*c^2 - 3*A*d^2 + B*d^2 - 4*A*c*d + 3*B*c*d))/(a^3*f*(c + d)^{(3/2)}*(c - d)^{(9/2)}) - ((2*(7*A*c^4 + 15*A*d^4 + 3*B*c^4 + 38*A*c^2*d^2 - 48*B*c^2*d^2 + 72*A*c*d^3 - 27*A*c^3*d - 47*B*c*d^3 - 13*B*c^3*d))/(15*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (4*\tan(e/2 + (f*x)/2)^3*(5*A*c^4 + 15*A*d^4 + 3*B*c^4 + 19*A*c^2*d^2 - 45*B*c^2*d^2 + 84*A*c*d^3 - 18*A*c^3*d - 52*B*c*d^3 - 11*B*c^3*d))/(3*c*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*\tan(e/2 + (f*x)/2)*(20*A*c^5 + 15*A*d^5 + 15*B*c^5 + 346*A*c^2*d^3 + 106*A*c^3*d^2 -$$

$$\begin{aligned}
& (286*B*c^2*d^3 - 221*B*c^3*d^2 + 219*A*c*d^4 - 76*A*c^4*d - 79*B*c*d^4 - 59 \\
& *B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*\tan(\\
& e/2 + (f*x)/2)^5*(2*A*c^5 + 5*A*d^5 + B*c^5 + 24*A*c^2*d^3 + 4*A*c^3*d^2 - \\
& 16*B*c^2*d^3 - 13*B*c^3*d^2 + 13*A*c*d^4 - 6*A*c^4*d - 11*B*c*d^4 - 3*B*c^4 \\
& *d))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*\tan(e/2 + (f* \\
& x)/2)^4*(11*A*c^5 + 30*A*d^5 + 3*B*c^5 + 162*A*c^2*d^3 + 4*A*c^3*d^2 - 139* \\
& B*c^2*d^3 - 84*B*c^3*d^2 + 135*A*c*d^4 - 27*A*c^4*d - 84*B*c*d^4 - 11*B*c^4 \\
& *d))/(3*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*\tan(e/2 + (\\
& f*x)/2)^2*(47*A*c^5 + 75*A*d^5 + 18*B*c^5 + 812*A*c^2*d^3 + 88*A*c^3*d^2 - \\
& 757*B*c^2*d^3 - 463*B*c^3*d^2 + 690*A*c*d^4 - 137*A*c^4*d - 305*B*c*d^4 - 6 \\
& 8*B*c^4*d))/(15*c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d + c^3 - d^3)) + (2*\tan \\
& (e/2 + (f*x)/2)^6*(A*c^5 + A*d^5 + 6*A*c^2*d^3 + 2*A*c^3*d^2 - 3*B*c^2*d^3 \\
& - 3*B*c^3*d^2 - 3*A*c^4*d - B*c*d^4))/(c*(c + d)*(c - d)*(3*c*d^2 - 3*c^2*d \\
& + c^3 - d^3)))/(f*(a^3*c + \tan(e/2 + (f*x)/2)*(5*a^3*c + 2*a^3*d) + \tan(e/ \\
& 2 + (f*x)/2)^6*(5*a^3*c + 2*a^3*d) + \tan(e/2 + (f*x)/2)^2*(11*a^3*c + 10*a^ \\
& 3*d) + \tan(e/2 + (f*x)/2)^5*(11*a^3*c + 10*a^3*d) + \tan(e/2 + (f*x)/2)^3*(1 \\
& 5*a^3*c + 20*a^3*d) + \tan(e/2 + (f*x)/2)^4*(15*a^3*c + 20*a^3*d) + a^3*c*\tan \\
& (e/2 + (f*x)/2)^7))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**2,x)

[Out] Timed out

$$3.285 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^3(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=508

$$\frac{\left(A(2c^2 - 15cd + 76d^2) + 3B(c^2 - 10cd - 12d^2)\right) \cos(e+fx)}{15f(c-d)^3(a^3 \sin(e+fx) + a^3)(c+d \sin(e+fx))^2} \frac{d^2(Ad(20c^2 + 30cd + 13d^2) - 3B(4c^3 + 8c^2d + 4cd^2 + 2d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{a^3 f(c-d)^5(c+d)^2 \sqrt{c^2 - d^2}}$$

[Out] $-1/30*d*(3*B*(2*c^3-20*c^2*d-57*c*d^2-30*d^3)+A*(4*c^3-30*c^2*d+146*c*d^2+195*d^3))*\cos(f*x+e)/a^3/(c-d)^4/(c+d)/f/(c+d*\sin(f*x+e))^2-1/5*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^3/(c+d*\sin(f*x+e))^2-1/15*(2*A*c-11*A*d+3*B*c+6*B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^2/(c+d*\sin(f*x+e))^2-1/15*(3*B*(c^2-10*c*d-12*d^2)+A*(2*c^2-15*c*d+76*d^2))*\cos(f*x+e)/(c-d)^3/f/(a^3+a^3*\sin(f*x+e))/(c+d*\sin(f*x+e))^2-1/30*d*(3*B*(2*c^4-20*c^3*d-119*c^2*d^2-130*c*d^3-48*d^4)+A*(4*c^4-30*c^3*d+142*c^2*d^2+525*c*d^3+304*d^4))*\cos(f*x+e)/a^3/(c-d)^5/(c+d)^2/f/(c+d*\sin(f*x+e))-d^2*(A*d*(20*c^2+30*c*d+13*d^2)-3*B*(4*c^3+8*c^2*d+7*c*d^2+2*d^3))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^(1/2))/a^3/(c-d)^5/(c+d)^2/f/(c^2-d^2)^(1/2)$

Rubi [A] time = 1.45, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2978, 2754, 12, 2660, 618, 204}

$$\frac{d^2(Ad(20c^2 + 30cd + 13d^2) - 3B(8c^2d + 4c^3 + 7cd^2 + 2d^3)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right) + d}{\sqrt{c^2 - d^2}}\right)}{a^3 f(c-d)^5(c+d)^2 \sqrt{c^2 - d^2}} d(A(142c^2d^2 - 30c^3d + 4cd^3 + 2d^4))$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3), x]

[Out] $-((d^2*(A*d*(20*c^2 + 30*c*d + 13*d^2) - 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(a^3*(c - d)^5*(c + d)^2*\text{Sqrt}[c^2 - d^2]*f) - (d*(3*B*(2*c^3 - 20*c^2*d - 57*c*d^2 - 30*d^3) + A*(4*c^3 - 30*c^2*d + 146*c*d^2 + 195*d^3))*\text{Cos}[e + f*x])/(30*a^3*(c - d)^4*(c + d)*f*(c + d*\text{Sin}[e + f*x])^2) - ((A - B)*\text{Cos}[e + f*x])/(5*(c - d)*f*(a + a*\text{Sin}[e + f*x])^3*(c + d*\text{Sin}[e + f*x])^2) - ((2*A*c + 3*B*c - 11*A*d + 6*B*d)*\text{Cos}[e + f*x])/(15*a*(c - d)^2*f*(a + a*\text{Sin}[e + f*x])^2*(c + d*\text{Sin}[e + f*x])^2) - ((3*B*(c^2 - 10*c*d - 12*d^2) + A*(2*c^2 - 15*c*d + 76*d^2))*\text{Cos}[e + f*x])/(15*(c - d)^3*f*(a^3 + a^3*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^2) - (d*(3*B*(2*c^4 - 20*c^3*d - 119*c^2*d^2 - 130*c*d^3 - 48*d^4) + A*(4*c^4 - 30*c^3*d + 142*c^2*d^2 + 525*c*d^3 + 304*d^4))*\text{Cos}[e + f*x])/(30*a^3*(c - d)^5*(c + d)^2*f*(c + d*\text{Sin}[e + f*x]))$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

$\text{Int}[(a_ + (b_.)\sin[(c_.) + (d_.)x])^{-1}, x_Symbol] \rightarrow \text{With}[\{e = \text{FreeFactors}[\text{Tan}[(c + dx)/2], x]\}, \text{Dist}[(2e)/d, \text{Subst}[\text{Int}[1/(a + 2be*x + ae^2x^2), x], x, \text{Tan}[(c + dx)/2]/e], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2754

$\text{Int}[(a_ + (b_.)\sin[(e_.) + (f_.)x])^{(m_)}((c_.) + (d_.)\sin[(e_.) + (f_.)x])], x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)\text{Cos}[e + fx]*(a + b\text{Sin}[e + fx])^{(m+1)} / (f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/((m+1)*(a^2 - b^2)), \text{Int}[(a + b\text{Sin}[e + fx])^{(m+1)}\text{Simp}[(a*c - b*d)*(m+1) - (b*c - a*d)*(m+2)*\text{Sin}[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1] \&\& \text{IntegerQ}[2*m]$

Rule 2978

$\text{Int}[(a_ + (b_.)\sin[(e_.) + (f_.)x])^{(m_)}((A_.) + (B_.)\sin[(e_.) + (f_.)x])^{(n_)}], x_Symbol] \rightarrow \text{Simp}[(b*(A*b - a*B)\text{Cos}[e + fx]*(a + b\text{Sin}[e + fx])^m*(c + d\text{Sin}[e + fx])^{(n+1)}) / (a*f*(2*m+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*(2*m+1)*(b*c - a*d)), \text{Int}[(a + b\text{Sin}[e + fx])^{(m+1)}*(c + d\text{Sin}[e + fx])^n*\text{Simp}[B*(a*c*m + b*d*(n+1)) + A*(b*c*(m+1) - a*d*(2*m+n+2)) + d*(A*b - a*B)*(m+n+2)*\text{Sin}[e + fx], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{LtQ}[m, -2^{(-1)}] \&\& !\text{GtQ}[n, 0] \&\& \text{IntegerQ}[2*m] \&\& (\text{IntegerQ}[2*n] || \text{EqQ}[c, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^3} dx &= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^2} - \int \frac{-a(2Ac - a^2)}{(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^3} dx \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^2} - \frac{(2)}{15a(c - d)} \\
&= -\frac{(A - B) \cos(e + fx)}{5(c - d)f(a + a \sin(e + fx))^3 (c + d \sin(e + fx))^2} - \frac{(2)}{15a(c - d)} \\
&= -\frac{d(3B(2c^3 - 20c^2d - 57cd^2 - 30d^3) + A(4c^3 - 30c^2d + 146cd^2 - 30d^3))}{30a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\
&= -\frac{d(3B(2c^3 - 20c^2d - 57cd^2 - 30d^3) + A(4c^3 - 30c^2d + 146cd^2 - 30d^3))}{30a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\
&= -\frac{d(3B(2c^3 - 20c^2d - 57cd^2 - 30d^3) + A(4c^3 - 30c^2d + 146cd^2 - 30d^3))}{30a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\
&= -\frac{d(3B(2c^3 - 20c^2d - 57cd^2 - 30d^3) + A(4c^3 - 30c^2d + 146cd^2 - 30d^3))}{30a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\
&= -\frac{d(3B(2c^3 - 20c^2d - 57cd^2 - 30d^3) + A(4c^3 - 30c^2d + 146cd^2 - 30d^3))}{30a^3(c - d)^4(c + d)f(c + d \sin(e + fx))} \\
&= -\frac{d^2(Ad(20c^2 + 30cd + 13d^2) - 3B(4c^3 + 8c^2d + 7cd^2 + 2d^3))}{a^3(c - d)^5(c + d)^2\sqrt{c^2 - d^2}f}
\end{aligned}$$

Mathematica [A] time = 4.96, size = 548, normalized size = 1.08

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(4(A(2c^2 - 19cd + 107d^2) + 3B(c^2 - 12cd - 19d^2)) \sin\left(\frac{1}{2}(e + fx)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^3*(c + d*Sin[e + f*x])^3), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(12*(A - B)*(c - d)^2*Sin[(e + f*x)/2] + 6*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 4*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 2*(c - d)*(A*(2*c - 17*d) + 3*B*(c + 4*d))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + 4*(3*B*(c^2 - 12*c*d - 19*d^2) + A*(2*c^2 - 19*c*d + 107*d^2))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (30*d^2*(-A*d*(20*c^2 + 30*c*d + 13*d^2)) + 3*B*(4*c^3 + 8*c^2*d + 7*c*d^2 + 2*d^3))*ArcTan[(d + c*Tan[(e + f*x)/2])/Sqrt[c^2 - d^2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)^2*Sqrt[c^2 - d^2]) + (15*(c - d)*d^3*(B*c - A*d)*Cos[e + f*x]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/((c + d)*(c + d*Sin[e + f*x])^2) + (15*d^3*(-3*A*d*(3*c + 2*d) + B*(7*c^2 + 6*c*d + 2*d^3)))/((c + d)^2*Sqrt[c^2 - d^2])

$$\begin{aligned} & \cdot \cos[e + f*x] * (\cos[(e + f*x)/2] + \sin[(e + f*x)/2])^5 / ((c + d)^2 * (c + \\ & d * \sin[e + f*x])) / (30 * a^3 * (c - d)^5 * f * (1 + \sin[e + f*x])^3) \end{aligned}$$

fricas [B] time = 0.85, size = 7283, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/60 * (12 * (A - B) * c^8 - 24 * (A - B) * c^7 * d - 24 * (A - B) * c^6 * d^2 + 72 * (A - B) * \\ & c^5 * d^3 - 72 * (A - B) * c^3 * d^5 + 24 * (A - B) * c^2 * d^6 + 24 * (A - B) * c * d^7 - 12 * \\ & (A - B) * d^8 + 2 * (2 * (2 * A + 3 * B) * c^6 * d^2 - 30 * (A + 2 * B) * c^5 * d^3 + 3 * (46 * A - 1 \\ & 21 * B) * c^4 * d^4 + 15 * (37 * A - 22 * B) * c^3 * d^5 + 3 * (54 * A + 71 * B) * c^2 * d^6 - 15 * (35 \\ & * A - 26 * B) * c * d^7 - 16 * (19 * A - 9 * B) * d^8) * \cos(f * x + e)^5 - 2 * (4 * (2 * A + 3 * B) * c \\ & ^7 * d - 4 * (13 * A + 27 * B) * c^6 * d^2 + 18 * (12 * A - 37 * B) * c^5 * d^3 + 6 * (181 * A - 171 * \\ & B) * c^4 * d^4 + 3 * (328 * A - 33 * B) * c^3 * d^5 - 9 * (69 * A - 104 * B) * c^2 * d^6 - (1208 * A \\ & - 753 * B) * c * d^7 - (413 * A - 198 * B) * d^8) * \cos(f * x + e)^4 - 2 * (2 * (2 * A + 3 * B) * c^8 \\ & - 6 * (A + 4 * B) * c^7 * d - 20 * (A + 21 * B) * c^6 * d^2 + 6 * (128 * A - 293 * B) * c^5 * d^3 + \\ & 3 * (892 * A - 827 * B) * c^4 * d^4 + 3 * (769 * A - 49 * B) * c^3 * d^5 - (1573 * A - 2373 * B) * c^2 \\ & * d^6 - 3 * (1023 * A - 643 * B) * c * d^7 - (1087 * A - 522 * B) * d^8) * \cos(f * x + e)^3 + 4 \\ & * (2 * (2 * A + 3 * B) * c^8 - 5 * (4 * A + 3 * B) * c^7 * d + (19 * A - 174 * B) * c^6 * d^2 + 15 * (22 \\ & * A - 35 * B) * c^5 * d^3 + 3 * (233 * A - 173 * B) * c^4 * d^4 + 15 * (23 * A + 10 * B) * c^3 * d^5 - \\ & (526 * A - 591 * B) * c^2 * d^6 - 5 * (131 * A - 78 * B) * c * d^7 - 4 * (49 * A - 24 * B) * d^8) * \cos \\ & (f * x + e)^2 - 15 * (48 * B * c^5 * d^2 - 16 * (5 * A - 12 * B) * c^4 * d^3 - 4 * (70 * A - 81 * B) \\ & * c^3 * d^4 - 12 * (31 * A - 24 * B) * c^2 * d^5 - 4 * (56 * A - 33 * B) * c * d^6 - 4 * (13 * A - 6 * B) \\ &) * d^7 + (12 * B * c^3 * d^4 - 4 * (5 * A - 6 * B) * c^2 * d^5 - 3 * (10 * A - 7 * B) * c * d^6 - (13 * \\ & A - 6 * B) * d^7) * \cos(f * x + e)^5 + (24 * B * c^4 * d^3 - 4 * (10 * A - 21 * B) * c^3 * d^4 - 6 * \\ & (20 * A - 19 * B) * c^2 * d^5 - (116 * A - 75 * B) * c * d^6 - 3 * (13 * A - 6 * B) * d^7) * \cos(f * x \\ & + e)^4 - (12 * B * c^5 * d^2 - 4 * (5 * A - 18 * B) * c^4 * d^3 - (110 * A - 153 * B) * c^3 * d^4 - \\ & (193 * A - 162 * B) * c^2 * d^5 - (142 * A - 87 * B) * c * d^6 - 3 * (13 * A - 6 * B) * d^7) * \cos(f \\ & * x + e)^3 - (36 * B * c^5 * d^2 - 12 * (5 * A - 16 * B) * c^4 * d^3 - (290 * A - 387 * B) * c^3 * d \\ & ^4 - (479 * A - 396 * B) * c^2 * d^5 - (340 * A - 207 * B) * c * d^6 - 7 * (13 * A - 6 * B) * d^7) * \\ & \cos(f * x + e)^2 + 2 * (12 * B * c^5 * d^2 - 4 * (5 * A - 12 * B) * c^4 * d^3 - (70 * A - 81 * B) * c \\ & ^3 * d^4 - 3 * (31 * A - 24 * B) * c^2 * d^5 - (56 * A - 33 * B) * c * d^6 - (13 * A - 6 * B) * d^7) * \\ & \cos(f * x + e) + (48 * B * c^5 * d^2 - 16 * (5 * A - 12 * B) * c^4 * d^3 - 4 * (70 * A - 81 * B) * c \\ & ^3 * d^4 - 12 * (31 * A - 24 * B) * c^2 * d^5 - 4 * (56 * A - 33 * B) * c * d^6 - 4 * (13 * A - 6 * B) * d \\ & ^7 + (12 * B * c^3 * d^4 - 4 * (5 * A - 6 * B) * c^2 * d^5 - 3 * (10 * A - 7 * B) * c * d^6 - (13 * A - \\ & 6 * B) * d^7) * \cos(f * x + e)^4 - 2 * (12 * B * c^4 * d^3 - 4 * (5 * A - 9 * B) * c^3 * d^4 - 5 * (10 \\ & * A - 9 * B) * c^2 * d^5 - (43 * A - 27 * B) * c * d^6 - (13 * A - 6 * B) * d^7) * \cos(f * x + e)^3 \\ & - (12 * B * c^5 * d^2 - 4 * (5 * A - 24 * B) * c^4 * d^3 - 75 * (2 * A - 3 * B) * c^3 * d^4 - (293 * A \\ & - 252 * B) * c^2 * d^5 - 3 * (76 * A - 47 * B) * c * d^6 - 5 * (13 * A - 6 * B) * d^7) * \cos(f * x + e) \\ & ^2 + 2 * (12 * B * c^5 * d^2 - 4 * (5 * A - 12 * B) * c^4 * d^3 - (70 * A - 81 * B) * c^3 * d^4 - 3 * (\\ & 31 * A - 24 * B) * c^2 * d^5 - (56 * A - 33 * B) * c * d^6 - (13 * A - 6 * B) * d^7) * \cos(f * x + e) \\ &) * \sin(f * x + e) * \sqrt{-c^2 + d^2} * \log(-((2 * c^2 - d^2) * \cos(f * x + e)^2 - 2 * c * d \\ & * \sin(f * x + e) - c^2 - d^2 - 2 * (c * \cos(f * x + e) * \sin(f * x + e) + d * \cos(f * x + e) \\ &) * \sqrt{-c^2 + d^2})) / (d^2 * \cos(f * x + e)^2 - 2 * c * d * \sin(f * x + e) - c^2 - d^2)) \\ & + 12 * ((3 * A + 2 * B) * c^8 - (11 * A + 9 * B) * c^7 * d + (9 * A - 109 * B) * c^6 * d^2 + (213 * A \\ & - 353 * B) * c^5 * d^3 + 5 * (95 * A - 71 * B) * c^4 * d^4 + (237 * A + 103 * B) * c^3 * d^5 - (35 \\ & 9 * A - 399 * B) * c^2 * d^6 - (439 * A - 259 * B) * c * d^7 - (128 * A - 63 * B) * d^8) * \cos(f * x \\ & + e) - 2 * (6 * (A - B) * c^8 - 12 * (A - B) * c^7 * d - 12 * (A - B) * c^6 * d^2 + 36 * (A - B) \\ &) * c^5 * d^3 - 36 * (A - B) * c^3 * d^5 + 12 * (A - B) * c^2 * d^6 + 12 * (A - B) * c * d^7 - 6 * \\ & (A - B) * d^8 + (2 * (2 * A + 3 * B) * c^6 * d^2 - 30 * (A + 2 * B) * c^5 * d^3 + 3 * (46 * A - 121 \\ & * B) * c^4 * d^4 + 15 * (37 * A - 22 * B) * c^3 * d^5 + 3 * (54 * A + 71 * B) * c^2 * d^6 - 15 * (35 * A \\ & - 26 * B) * c * d^7 - 16 * (19 * A - 9 * B) * d^8) * \cos(f * x + e)^4 + (4 * (2 * A + 3 * B) * c^7 * d \\ & - 6 * (8 * A + 17 * B) * c^6 * d^2 + 6 * (31 * A - 121 * B) * c^5 * d^3 + 3 * (408 * A - 463 * B) * c^ \\ & 4 * d^4 + 3 * (513 * A - 143 * B) * c^3 * d^5 - 3 * (153 * A - 383 * B) * c^2 * d^6 - (1733 * A - 1 \\ & 143 * B) * c * d^7 - 3 * (239 * A - 114 * B) * d^8) * \cos(f * x + e)^3 - 2 * ((2 * A + 3 * B) * c^8 - \\ & (7 * A + 18 * B) * c^7 * d + (14 * A - 159 * B) * c^6 * d^2 + 3 * (97 * A - 172 * B) * c^5 * d^3 + 6 \end{aligned}$$

$$\begin{aligned}
&*(121*A - 91*B)*c^4*d^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^6 - (668*A - 393*B)*c*d^7 - 5*(37*A - 18*B)*d^8)*\cos(f*x + e)^2 - 6*((2*A + 3*B)*c^8 - (9*A + 11*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c^5*d^3 + 5*(95*A - 71*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)*c^2*d^6 - 9*(49*A - 29*B)*c*d^7 - (127*A - 62*B)*d^8)*\sin(f*x + e))/((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - 2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11)*f*\cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + 26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*f*\cos(f*x + e)^3 - (3*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - 22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*\cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e)^4 - 2*(a^3*c^10*d - 2*a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8*d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d^11)*f*\cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f)*\sin(f*x + e)), -1/30*(6*(A - B)*c^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A + 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37*A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(19*A - 9*B)*d^8)*\cos(f*x + e)^5 - (4*(2*A + 3*B)*c^7*d - 4*(13*A + 27*B)*c^6*d^2 + 18*(12*A - 37*B)*c^5*d^3 + 6*(181*A - 171*B)*c^4*d^4 + 3*(328*A - 33*B)*c^3*d^5 - 9*(69*A - 104*B)*c^2*d^6 - (1208*A - 753*B)*c*d^7 - (413*A - 198*B)*d^8)*\cos(f*x + e)^4 - (2*(2*A + 3*B)*c^8 - 6*(A + 4*B)*c^7*d - 20*(A + 21*B)*c^6*d^2 + 6*(128*A - 293*B)*c^5*d^3 + 3*(892*A - 827*B)*c^4*d^4 + 3*(769*A - 49*B)*c^3*d^5 - (1573*A - 2373*B)*c^2*d^6 - 3*(1023*A - 643*B)*c*d^7 - (1087*A - 522*B)*d^8)*\cos(f*x + e)^3 + 2*(2*(2*A + 3*B)*c^8 - 5*(4*A + 3*B)*c^7*d + (19*A - 174*B)*c^6*d^2 + 15*(22*A - 35*B)*c^5*d^3 + 3*(233*A - 173*B)*c^4*d^4 + 15*(23*A + 10*B)*c^3*d^5 - (526*A - 591*B)*c^2*d^6 - 5*(131*A - 78*B)*c*d^7 - 4*(49*A - 24*B)*d^8)*\cos(f*x + e)^2 + 15*(48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*\cos(f*x + e)^5 + (24*B*c^4*d^3 - 4*(10*A - 21*B)*c^3*d^4 - 6*(20*A - 19*B)*c^2*d^5 - (116*A - 75*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*\cos(f*x + e)^4 - (12*B*c^5*d^2 - 4*(5*A - 18*B)*c^4*d^3 - (110*A - 153*B)*c^3*d^4 - (193*A - 162*B)*c^2*d^5 - (142*A - 87*B)*c*d^6 - 3*(13*A - 6*B)*d^7)*\cos(f*x + e)^3 - (36*B*c^5*d^2 - 12*(5*A - 16*B)*c^4*d^3 - (290*A - 387*B)*c^3*d^4 - (479*A - 396*B)*c^2*d^5 - (340*A - 207*B)*c*d^6 - 7*(13*A - 6*B)*d^7)*\cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B)*c*d^6 - (13*A - 6*B)*d^7)*\cos(f*x + e) + (48*B*c^5*d^2 - 16*(5*A - 12*B)*c^4*d^3 - 4*(70*A - 81*B)*c^3*d^4 - 12*(31*A - 24*B)*c^2*d^5 - 4
\end{aligned}$$

$$\begin{aligned}
&*(56*A - 33*B)*c*d^6 - 4*(13*A - 6*B)*d^7 + (12*B*c^3*d^4 - 4*(5*A - 6*B)*c \\
&^2*d^5 - 3*(10*A - 7*B)*c*d^6 - (13*A - 6*B)*d^7)*\cos(f*x + e)^4 - 2*(12*B* \\
&c^4*d^3 - 4*(5*A - 9*B)*c^3*d^4 - 5*(10*A - 9*B)*c^2*d^5 - (43*A - 27*B)*c* \\
&d^6 - (13*A - 6*B)*d^7)*\cos(f*x + e)^3 - (12*B*c^5*d^2 - 4*(5*A - 24*B)*c^4 \\
&*d^3 - 75*(2*A - 3*B)*c^3*d^4 - (293*A - 252*B)*c^2*d^5 - 3*(76*A - 47*B)*c \\
&*d^6 - 5*(13*A - 6*B)*d^7)*\cos(f*x + e)^2 + 2*(12*B*c^5*d^2 - 4*(5*A - 12*B) \\
&)*c^4*d^3 - (70*A - 81*B)*c^3*d^4 - 3*(31*A - 24*B)*c^2*d^5 - (56*A - 33*B) \\
&*c*d^6 - (13*A - 6*B)*d^7)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{c^2 - d^2}*\arct \\
&\text{an}(-(c*\sin(f*x + e) + d)/(\sqrt{c^2 - d^2}*\cos(f*x + e))) + 6*((3*A + 2*B)*c \\
&^8 - (11*A + 9*B)*c^7*d + (9*A - 109*B)*c^6*d^2 + (213*A - 353*B)*c^5*d^3 + \\
&5*(95*A - 71*B)*c^4*d^4 + (237*A + 103*B)*c^3*d^5 - (359*A - 399*B)*c^2*d^ \\
&6 - (439*A - 259*B)*c*d^7 - (128*A - 63*B)*d^8)*\cos(f*x + e) - (6*(A - B)*c \\
&^8 - 12*(A - B)*c^7*d - 12*(A - B)*c^6*d^2 + 36*(A - B)*c^5*d^3 - 36*(A - B) \\
&)*c^3*d^5 + 12*(A - B)*c^2*d^6 + 12*(A - B)*c*d^7 - 6*(A - B)*d^8 + (2*(2*A \\
&+ 3*B)*c^6*d^2 - 30*(A + 2*B)*c^5*d^3 + 3*(46*A - 121*B)*c^4*d^4 + 15*(37* \\
&A - 22*B)*c^3*d^5 + 3*(54*A + 71*B)*c^2*d^6 - 15*(35*A - 26*B)*c*d^7 - 16*(\\
&19*A - 9*B)*d^8)*\cos(f*x + e)^4 + (4*(2*A + 3*B)*c^7*d - 6*(8*A + 17*B)*c^6 \\
&*d^2 + 6*(31*A - 121*B)*c^5*d^3 + 3*(408*A - 463*B)*c^4*d^4 + 3*(513*A - 14 \\
&3*B)*c^3*d^5 - 3*(153*A - 383*B)*c^2*d^6 - (1733*A - 1143*B)*c*d^7 - 3*(239 \\
&*A - 114*B)*d^8)*\cos(f*x + e)^3 - 2*((2*A + 3*B)*c^8 - (7*A + 18*B)*c^7*d + \\
&(14*A - 159*B)*c^6*d^2 + 3*(97*A - 172*B)*c^5*d^3 + 6*(121*A - 91*B)*c^4*d \\
&^4 + 3*(128*A + 47*B)*c^3*d^5 - (557*A - 612*B)*c^2*d^6 - (668*A - 393*B)*c \\
&*d^7 - 5*(37*A - 18*B)*d^8)*\cos(f*x + e)^2 - 6*((2*A + 3*B)*c^8 - (9*A + 11 \\
&*B)*c^7*d + (11*A - 111*B)*c^6*d^2 + (207*A - 347*B)*c^5*d^3 + 5*(95*A - 71 \\
&*B)*c^4*d^4 + (243*A + 97*B)*c^3*d^5 - (361*A - 401*B)*c^2*d^6 - 9*(49*A - \\
&29*B)*c*d^7 - (127*A - 62*B)*d^8)*\cos(f*x + e))*\sin(f*x + e))/((a^3*c^9*d^2 \\
&- 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a^3*c^ \\
&3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e)^5 + (2*a^3*c^10*d - 3*a^3*c \\
&^9*d^2 - 9*a^3*c^8*d^3 + 16*a^3*c^7*d^4 + 12*a^3*c^6*d^5 - 30*a^3*c^5*d^6 - \\
&2*a^3*c^4*d^7 + 24*a^3*c^3*d^8 - 6*a^3*c^2*d^9 - 7*a^3*c*d^10 + 3*a^3*d^11) \\
&)*f*\cos(f*x + e)^4 - (a^3*c^11 + a^3*c^10*d - 9*a^3*c^9*d^2 - a^3*c^8*d^3 + \\
&26*a^3*c^7*d^4 - 6*a^3*c^6*d^5 - 34*a^3*c^5*d^6 + 14*a^3*c^4*d^7 + 21*a^3*c \\
&^3*d^8 - 11*a^3*c^2*d^9 - 5*a^3*c*d^10 + 3*a^3*d^11)*f*\cos(f*x + e)^3 - (3 \\
&*a^3*c^11 + a^3*c^10*d - 23*a^3*c^9*d^2 + 3*a^3*c^8*d^3 + 62*a^3*c^7*d^4 - \\
&22*a^3*c^6*d^5 - 78*a^3*c^5*d^6 + 38*a^3*c^4*d^7 + 47*a^3*c^3*d^8 - 27*a^3*c \\
&^2*d^9 - 11*a^3*c*d^10 + 7*a^3*d^11)*f*\cos(f*x + e)^2 + 2*(a^3*c^11 - a^3*c \\
&^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - \\
&10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^1 \\
&0 + a^3*d^11)*f*\cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5 \\
&*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^ \\
&4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f + ((a^3*c^ \\
&9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^6*d^5 - 6*a^3*c^5*d^6 - 6*a^3*c^4*d^7 + 8*a \\
&^3*c^3*d^8 - 3*a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e)^4 - 2*(a^3*c^10*d - 2* \\
&a^3*c^9*d^2 - 3*a^3*c^8*d^3 + 8*a^3*c^7*d^4 + 2*a^3*c^6*d^5 - 12*a^3*c^5*d^ \\
&6 + 2*a^3*c^4*d^7 + 8*a^3*c^3*d^8 - 3*a^3*c^2*d^9 - 2*a^3*c*d^10 + a^3*d^11) \\
&)*f*\cos(f*x + e)^3 - (a^3*c^11 + 3*a^3*c^10*d - 13*a^3*c^9*d^2 - 7*a^3*c^8* \\
&d^3 + 42*a^3*c^7*d^4 - 2*a^3*c^6*d^5 - 58*a^3*c^5*d^6 + 18*a^3*c^4*d^7 + 37 \\
&*a^3*c^3*d^8 - 17*a^3*c^2*d^9 - 9*a^3*c*d^10 + 5*a^3*d^11)*f*\cos(f*x + e)^2 \\
&+ 2*(a^3*c^11 - a^3*c^10*d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^ \\
&4 - 10*a^3*c^6*d^5 - 10*a^3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^ \\
&3*c^2*d^9 - a^3*c*d^10 + a^3*d^11)*f*\cos(f*x + e) + 4*(a^3*c^11 - a^3*c^10* \\
&d - 5*a^3*c^9*d^2 + 5*a^3*c^8*d^3 + 10*a^3*c^7*d^4 - 10*a^3*c^6*d^5 - 10*a^ \\
&3*c^5*d^6 + 10*a^3*c^4*d^7 + 5*a^3*c^3*d^8 - 5*a^3*c^2*d^9 - a^3*c*d^10 + a \\
&^3*d^11)*f)*\sin(f*x + e))]
\end{aligned}$$

giac [B] time = 0.59, size = 1272, normalized size = 2.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out]
$$\frac{1}{15} \cdot (15 \cdot (12 \cdot B \cdot c^3 \cdot d^2 - 20 \cdot A \cdot c^2 \cdot d^3 + 24 \cdot B \cdot c^2 \cdot d^3 - 30 \cdot A \cdot c \cdot d^4 + 21 \cdot B \cdot c \cdot d^4 - 13 \cdot A \cdot d^5 + 6 \cdot B \cdot d^5) \cdot (\pi \cdot \text{floor}(1/2 \cdot (f \cdot x + e) / \pi + 1/2) \cdot \text{sgn}(c) + \arctan((c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + d) / \sqrt{c^2 - d^2}))) / ((a^3 \cdot c^7 - 3 \cdot a^3 \cdot c^6 \cdot d + a^3 \cdot c^5 \cdot d^2 + 5 \cdot a^3 \cdot c^4 \cdot d^3 - 5 \cdot a^3 \cdot c^3 \cdot d^4 - a^3 \cdot c^2 \cdot d^5 + 3 \cdot a^3 \cdot c \cdot d^6 - a^3 \cdot d^7) \cdot \sqrt{c^2 - d^2}) + 15 \cdot (9 \cdot B \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 11 \cdot A \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 6 \cdot B \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 6 \cdot A \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 2 \cdot A \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 8 \cdot B \cdot c^5 \cdot d^3 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 10 \cdot A \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 6 \cdot B \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 6 \cdot A \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 17 \cdot B \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 19 \cdot A \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 12 \cdot B \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 12 \cdot A \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 2 \cdot B \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 2 \cdot A \cdot d^8 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 23 \cdot B \cdot c^4 \cdot d^4 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 29 \cdot A \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 18 \cdot B \cdot c^3 \cdot d^5 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 18 \cdot A \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 4 \cdot B \cdot c^2 \cdot d^6 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 2 \cdot A \cdot c \cdot d^7 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 8 \cdot B \cdot c^5 \cdot d^3 - 10 \cdot A \cdot c^4 \cdot d^4 + 6 \cdot B \cdot c^4 \cdot d^4 - 6 \cdot A \cdot c^3 \cdot d^5 + B \cdot c^3 \cdot d^5 + A \cdot c^2 \cdot d^6) / ((a^3 \cdot c^9 - 3 \cdot a^3 \cdot c^8 \cdot d + a^3 \cdot c^7 \cdot d^2 + 5 \cdot a^3 \cdot c^6 \cdot d^3 - 5 \cdot a^3 \cdot c^5 \cdot d^4 - a^3 \cdot c^4 \cdot d^5 + 3 \cdot a^3 \cdot c^3 \cdot d^6 - a^3 \cdot c^2 \cdot d^7) \cdot (c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 2 \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + c)^2) - 2 \cdot (15 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 - 75 \cdot A \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 + 150 \cdot A \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 - 90 \cdot B \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^4 + 30 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 15 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 195 \cdot A \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 75 \cdot B \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 525 \cdot A \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 - 300 \cdot B \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 + 40 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 15 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 245 \cdot A \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 135 \cdot B \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 745 \cdot A \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 - 420 \cdot B \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 + 20 \cdot A \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 15 \cdot B \cdot c^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 145 \cdot A \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 105 \cdot B \cdot c \cdot d \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 485 \cdot A \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) - 270 \cdot B \cdot d^2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 7 \cdot A \cdot c^2 + 3 \cdot B \cdot c^2 - 44 \cdot A \cdot c \cdot d - 21 \cdot B \cdot c \cdot d + 127 \cdot A \cdot d^2 - 72 \cdot B \cdot d^2) / ((a^3 \cdot c^5 - 5 \cdot a^3 \cdot c^4 \cdot d + 10 \cdot a^3 \cdot c^3 \cdot d^2 - 10 \cdot a^3 \cdot c^2 \cdot d^3 + 5 \cdot a^3 \cdot c \cdot d^4 - a^3 \cdot d^5) \cdot (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^5) / f$$

maple [B] time = 0.65, size = 2918, normalized size = 5.74

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x)

[Out]
$$\begin{aligned} & -10/f/a^3 \cdot d^4 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot A \cdot c^2 - 19/f/a^3 \cdot d^6 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot A - 6/f/a^3 \cdot d^5 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot A \cdot c - 6/f/a^3 \cdot d^6 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^3 \cdot A + 8/f/a^3 \cdot d^3 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot B \cdot c^3 + 6/f/a^3 \cdot d^4 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot B \cdot c^2 - 18/f/a^3 \cdot d^6 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot A + 4/f/a^3 \cdot d^6 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot B - 13/f/a^3 \cdot d^5 / (c-d)^5 / (c^2 + 2 \cdot c \cdot d + d^2) / (c^2 - d^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 2 \cdot d) / (c^2 - d^2)^{(1/2)}) \cdot A + 6/f/a^3 \cdot d^5 / (c-d)^5 / (c^2 + 2 \cdot c \cdot d + d^2) / (c^2 - d^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot c \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 2 \cdot d) / (c^2 - d^2)^{(1/2)}) \cdot B + 12/f/a^3 \cdot d^6 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot B + 1/f/a^3 \cdot d^5 / (c-d)^5 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e)^2 \cdot c + 2 \cdot \tan(1/2 \cdot f \cdot x + 1/2 \cdot e) \cdot d + c)^2 / (c^2 + 2 \cdot c \cdot d + d^2) \cdot B \cdot c - 2/f/a^3 / (c-d)^4 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot B \cdot c + 8/f/a^3 / (c-d)^4 / (\tan(1/2 \cdot f \cdot x + 1/2 \cdot e) + 1)^2 \cdot B \cdot d - 16/3/f/a^3 / (c-d) \end{aligned}$$

$$\begin{aligned} &)^4/(\tan(1/2*f*x+1/2*e)+1)^3*A*c+28/3/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)^3 \\ &3*A*d+4/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)^3*B*c-8/f/a^3/(c-d)^4/(\tan(1/2 \\ &*f*x+1/2*e)+1)^3*B*d-2/f/a^3/(c-d)^5/(\tan(1/2*f*x+1/2*e)+1)*A*c^2-20/f/a^3/ \\ &(c-d)^5/(\tan(1/2*f*x+1/2*e)+1)*A*d^2+2/f/a^3*d^7/(c-d)^5/(\tan(1/2*f*x+1/2*e \\ &)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*A+23 \\ &/f/a^3*d^4/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/ \\ &(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B+18/f/a^3*d^5/(c-d)^5/(\tan(1/2*f*x+1/2* \\ &e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)*B+2 \\ &4/f/a^3*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2 \\ &*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c^2+1/f/a^3*d^6/(c-d)^5/(\tan(1/2*f*x+1/ \\ &2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*A+10/f/a^3/(c-d)^5/(ta \\ &n(1/2*f*x+1/2*e)+1)*A*c*d+2/f/a^3*d^7/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan \\ &(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*\tan(1/2*f*x+1/2*e)^2*B-29/f/a^3*d^ \\ &5/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+ \\ &d^2)*\tan(1/2*f*x+1/2*e)*A+12/f/a^3*d^2/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1 \\ &/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c^3-20/f/a^3 \\ &*d^3/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/ \\ &2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c^2-12/f/a^3*d^7/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2 \\ &*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)/c*\tan(1/2*f*x+1/2*e)^2*A-30/ \\ &f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f \\ &*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*c+21/f/a^3*d^4/(c-d)^5/(c^2+2*c*d+d^2)/(c \\ &^2-d^2)^{(1/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*c+ \\ &2/f/a^3*d^8/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^ \\ &2+2*c*d+d^2)/c^2*\tan(1/2*f*x+1/2*e)^2*A+8/f/a^3*d^3/(c-d)^5/(\tan(1/2*f*x+1/ \\ &2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^3*\tan(1/2*f*x+1/2*e) \\ &^2*B+6/f/a^3*d^4/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^ \\ &2/(c^2+2*c*d+d^2)*c^2*\tan(1/2*f*x+1/2*e)^2*B-11/f/a^3*d^5/(c-d)^5/(\tan(1/2* \\ &f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/ \\ &2*e)^3*A+2/f/a^3*d^7/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d \\ &+c)^2/c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*A+9/f/a^3*d^4/(c-d)^5/(\tan(1/2 \\ &*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2*c^2/(c^2+2*c*d+d^2)*\tan(1/2*f*x \\ &+1/2*e)^3*B+6/f/a^3*d^5/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e \\ &)*d+c)^2*c/(c^2+2*c*d+d^2)*\tan(1/2*f*x+1/2*e)^3*B-10/f/a^3*d^4/(c-d)^5/(\tan \\ &(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c^2*\tan(1/2 \\ &*f*x+1/2*e)^2*A-6/f/a^3*d^5/(c-d)^5/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1 \\ &/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1/2*f*x+1/2*e)^2*A+17/f/a^3*d^5/(c-d)^5/ \\ &(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)^2/(c^2+2*c*d+d^2)*c*\tan(1 \\ &/2*f*x+1/2*e)^2*B+12/f/a^3/(c-d)^5/(\tan(1/2*f*x+1/2*e)+1)*B*d^2+4/f/a^3/(c- \\ &d)^4/(\tan(1/2*f*x+1/2*e)+1)^2*A*c-10/f/a^3/(c-d)^4/(\tan(1/2*f*x+1/2*e)+1)^2 \\ &*A*d+4/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^4*A-4/f/a^3/(c-d)^3/(\tan(1/2*f* \\ &x+1/2*e)+1)^4*B-8/5/f/a^3/(c-d)^3/(\tan(1/2*f*x+1/2*e)+1)^5*A+8/5/f/a^3/(c-d \\ &)^3/(\tan(1/2*f*x+1/2*e)+1)^5*B \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^3/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 19.88, size = 2387, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A + B\sin(e + f*x))/((a + a*\sin(e + f*x))^3*(c + d*\sin(e + f*x))^3),x)$

[Out] $((15*A*d^6 - 14*A*c^6 - 6*B*c^6 - 404*A*c^2*d^4 - 420*A*c^3*d^3 - 92*A*c^4*d^2 + 234*B*c^2*d^4 + 450*B*c^3*d^3 + 222*B*c^4*d^2 - 90*A*c*d^5 + 60*A*c^5*d + 15*B*c*d^5 + 30*B*c^5*d)/(15*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^7*(2*A*d^8 - 4*A*c^8 - 2*B*c^8 - 49*A*c^2*d^6 - 141*A*c^3*d^5 - 200*A*c^4*d^4 - 122*A*c^5*d^3 + 2*A*c^6*d^2 + 12*B*c^2*d^6 + 95*B*c^3*d^5 + 187*B*c^4*d^4 + 146*B*c^5*d^3 + 58*B*c^6*d^2 - 2*A*c*d^7 + 10*A*c^7*d + 2*B*c*d^7 + 6*B*c^7*d))/(c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^6*(30*A*d^8 - 28*A*c^8 - 6*B*c^8 - 759*A*c^2*d^6 - 1707*A*c^3*d^5 - 1960*A*c^4*d^4 - 870*A*c^5*d^3 + 62*A*c^6*d^2 + 336*B*c^2*d^6 + 1257*B*c^3*d^5 + 1893*B*c^4*d^4 + 1350*B*c^5*d^3 + 414*B*c^6*d^2 - 114*A*c*d^7 + 54*A*c^7*d + 30*B*c*d^7 + 18*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^5*(60*A*d^8 - 32*A*c^8 - 18*B*c^8 - 1857*A*c^2*d^6 - 3763*A*c^3*d^5 - 3560*A*c^4*d^4 - 1294*A*c^5*d^3 + 70*A*c^6*d^2 + 900*B*c^2*d^6 + 2859*B*c^3*d^5 + 3705*B*c^4*d^4 + 2358*B*c^5*d^3 + 678*B*c^6*d^2 - 270*A*c*d^7 + 62*A*c^7*d + 60*B*c*d^7 + 42*B*c^7*d))/(3*c^2*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^2*(30*A*d^8 - 108*A*c^8 - 42*B*c^8 - 2501*A*c^2*d^6 - 8725*A*c^3*d^5 - 10616*A*c^4*d^4 - 4810*A*c^5*d^3 + 10*A*c^6*d^2 + 1056*B*c^2*d^6 + 5235*B*c^3*d^5 + 9891*B*c^4*d^4 + 7770*B*c^5*d^3 + 2370*B*c^6*d^2 - 30*A*c*d^7 + 290*A*c^7*d + 30*B*c*d^7 + 150*B*c^7*d))/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^3*(150*A*d^8 - 140*A*c^8 - 90*B*c^8 - 7945*A*c^2*d^6 - 19441*A*c^3*d^5 - 18600*A*c^4*d^4 - 6898*A*c^5*d^3 + 210*A*c^6*d^2 + 3660*B*c^2*d^6 + 13311*B*c^3*d^5 + 19455*B*c^4*d^4 + 12618*B*c^5*d^3 + 3570*B*c^6*d^2 - 570*A*c*d^7 + 314*A*c^7*d + 150*B*c*d^7 + 246*B*c^7*d))/(15*c^2*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)*(30*A*d^7 - 40*A*c^7 - 30*B*c^7 - 1901*A*c^2*d^5 - 3400*A*c^3*d^4 - 2018*A*c^4*d^3 - 190*A*c^5*d^2 + 921*B*c^2*d^5 + 2655*B*c^3*d^4 + 2778*B*c^4*d^3 + 1050*B*c^5*d^2 - 195*A*c*d^6 + 154*A*c^6*d + 60*B*c*d^6 + 126*B*c^6*d))/(15*c*(c + d)^2*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) - (\tan(e/2 + (f*x)/2)^8*(2*A*c^7 - 2*A*d^7 + 11*A*c^2*d^5 + 20*A*c^3*d^4 + 30*A*c^4*d^3 + 2*A*c^5*d^2 - 6*B*c^2*d^5 - 21*B*c^3*d^4 - 24*B*c^4*d^3 - 12*B*c^5*d^2 + 6*A*c*d^6 - 6*A*c^6*d))/(c*(c - d)*(2*c*d + c^2 + d^2)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)) + (\tan(e/2 + (f*x)/2)^4*(300*A*d^7 - 204*A*c^7 - 66*B*c^7 - 10235*A*c^2*d^5 - 14330*A*c^3*d^4 - 7254*A*c^4*d^3 - 316*A*c^5*d^2 + 5460*B*c^2*d^5 + 12675*B*c^3*d^4 + 10764*B*c^4*d^3 + 3666*B*c^5*d^2 - 1650*A*c*d^6 + 614*A*c^6*d + 300*B*c*d^6 + 276*B*c^6*d))/(15*c^2*(c + d)*(c - d)*(c^4 - 4*c^3*d - 4*c*d^3 + d^4 + 6*c^2*d^2)))/(f*(\tan(e/2 + (f*x)/2)*(5*a^3*c^2 + 4*a^3*c*d) + \tan(e/2 + (f*x)/2)^2*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*c*d) + \tan(e/2 + (f*x)/2)^7*(12*a^3*c^2 + 4*a^3*d^2 + 20*a^3*c*d) + \tan(e/2 + (f*x)/2)^3*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + \tan(e/2 + (f*x)/2)^6*(20*a^3*c^2 + 20*a^3*d^2 + 44*a^3*c*d) + \tan(e/2 + (f*x)/2)^4*(26*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + \tan(e/2 + (f*x)/2)^5*(26*a^3*c^2 + 40*a^3*d^2 + 60*a^3*c*d) + \tan(e/2 + (f*x)/2)^8*(5*a^3*c^2 + 4*a^3*c*d) + a^3*c^2 + a^3*c^2*\tan(e/2 + (f*x)/2)^9)) - (d^2*\text{atan}(((d^2*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(2*a^3*d^8 - 6*a^3*c*d^7 - 2*a^3*c^7*d + 2*a^3*c^2*d^6 + 10*a^3*c^3*d^5 - 10*a^3*c^4*d^4 - 2*a^3*c^5*d^3 + 6*a^3*c^6*d^2))/(2*a^3*(c + d)^(5/2)*(c - d)^(11/2)) - (c*d^2*\tan(e/2 + (f*x)/2)*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d)*(a^3*c^7 - a^3*d^7 + 3*a^3*c*d^6 - 3*a^3*c^6*d - a^3*c^2*d^5 - 5*a^3*c^3*d^4 + 5*a^3*c^4*d^3 + a^3*c^5*d^2)))/(a^3*(c + d)^(5/2)*(c - d)^(11/2)))/(6*B*d^5 - 13*A*d^5 - 20*A*c^2*d^3 + 24*B*c^2*d^3 + 12*B*c^3*d^2 - 30*A*c*d^4 + 21*B*c*d^4))*(12*B*c^3 - 13*A*d^3 + 6*B*d^3 - 30*A*c*d^2 - 20*A*c^2*d + 21*B*c*d^2 + 24*B*c^2*d))/(a^3*f*(c + d)^(5/2)*(c - d)^(11/2))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**3/(c+d*sin(f*x+e))**3,x)

[Out] Timed out

$$3.286 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$$

Optimal. Leaf size=256

$$\frac{4a(c+d)(15c^2+10cd+7d^2)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}} + \frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))}{63df\sqrt{a\sin(e+fx)+a}}$$

[Out] $4/105*d*(c+d)*(-9*A*d+B*c-8*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/a/f+4/315*a*(c+d)*(-9*A*d+B*c-8*B*d)*(15*c^2+10*c*d+7*d^2)*\cos(f*x+e)/d/f/(a+a*\sin(f*x+e))^{(1/2)}+2/63*a*(-9*A*d+B*c-8*B*d)*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/d/f/(a+a*\sin(f*x+e))^{(1/2)}-2/9*a*B*\cos(f*x+e)*(c+d*\sin(f*x+e))^4/d/f/(a+a*\sin(f*x+e))^{(1/2)}+8/315*(5*c-d)*(c+d)*(-9*A*d+B*c-8*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.46, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2981, 2770, 2761, 2751, 2646}

$$\frac{4a(c+d)(15c^2+10cd+7d^2)(-9Ad+Bc-8Bd)\cos(e+fx)}{315df\sqrt{a\sin(e+fx)+a}} + \frac{2a(-9Ad+Bc-8Bd)\cos(e+fx)(c+d\sin(e+fx))}{63df\sqrt{a\sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
 [Out] $(4*a*(c+d)*(B*c-9*A*d-8*B*d)*(15*c^2+10*c*d+7*d^2)*\text{Cos}[e+f*x])/(315*d*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])+(8*(5*c-d)*(c+d)*(B*c-9*A*d-8*B*d)*\text{Cos}[e+f*x]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])/(315*f)+(4*d*(c+d)*(B*c-9*A*d-8*B*d)*\text{Cos}[e+f*x]*(a+a*\text{Sin}[e+f*x])^{(3/2)})/(105*a*f)+(2*a*(B*c-9*A*d-8*B*d)*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^3)/(63*d*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])-(2*a*B*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^4)/(9*d*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])$

Rule 2646

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2761

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 2770

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Sin[e + f*x])^n)/(f*(2*n + 1)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*(2*n + 1)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^4}{9df \sqrt{a + a \sin(e + fx)}} + \frac{(9aA}{9df \sqrt{a + a \sin(e + fx)}} \\ &= \frac{2a(Bc - 9Ad - 8Bd) \cos(e + fx)(c + d \sin(e + fx))^3}{63df \sqrt{a + a \sin(e + fx)}} \\ &= \frac{4d(c + d)(Bc - 9Ad - 8Bd) \cos(e + fx)(a + a \sin(e + fx))^2}{105af} \\ &= \frac{8(5c - d)(c + d)(Bc - 9Ad - 8Bd) \cos(e + fx)(a + a \sin(e + fx))}{315f} \\ &= \frac{4a(c + d)(Bc - 9Ad - 8Bd) (15c^2 + 10cd + 7d^2)}{315df \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.25, size = 305, normalized size = 1.19

$$\frac{\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-4d(27Ad(7c + 2d) + B(189c^2 + 162cd + 83d^2)) \cos\left(\frac{1}{2}(e + fx)\right) + \dots \right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]
```

```
[Out] -1/1260*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])*(2520*A*c^3 + 1680*B*c^3 + 5040*A*c^2*d + 4788*B*c^2*d + 4788*A*c*d^2 + 4104*B*c*d^2 + 1368*A*d^3 + 1321*B*d^3 - 4*d*(27*A*d*(7*c + 2*d) + B*(189*c^2 + 162*c*d + 83*d^2))*Cos[2*(e + f*x)] + 35*B*d^3*Cos[4*(e + f*x)] + 840*B*c^3*Sin[e + f*x] + 2520*A*c^2*d*Sin[e + f*x] + 2016*B*c^2*d*Sin[e + f*x] + 2016*A*c*d^2*Sin[e + f*x] + 2538*B*c*d^2*Sin[e + f*x] + 846*A*d^3*Sin[e + f*x] + 752*B*d^3*Sin[e + f*x] - 270*B*c*d^2*Sin[3*(e + f*x)] - 90*A*d^3*Sin[3*(e + f*x)] - 80*B*d^3*Sin[3*(e + f*x)])/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```



```

os(1/2*(f*x+exp(1))-1/4*pi))+18*B*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(16*A*c^3*sign(cos(1/2*(f*x+exp(
1))-1/4*pi))+6*B*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*c*d^2*sign(cos
(1/2*(f*x+exp(1))-1/4*pi))+24*B*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*s
in(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2+224*B*d^3*f*sign(cos(1/2*(f*x+exp(1)
-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)+pi))/(112*f)^2+288*B*d^3*f*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))*sin(1/4*(18*f*x+18*exp(1)-pi))/(144*f)^2)

```

maple [A] time = 1.40, size = 242, normalized size = 0.95

$$\frac{2(1 + \sin(fx + e))a(\sin(fx + e) - 1)((-45Ad^3 - 135Bcd^2 - 40Bd^3)\sin(fx + e)(\cos^2(fx + e)) + (315Ac^2d^3 - 189Acd^2 - 162Bcd^2 - 118Bd^3)\cos(fx + e)^2 + 315A^2c^2d^3 + 630A^2cd^2 + 693A^2cd^2 + 198Ad^3 + 210B^2c^3 + 693B^2cd^2 + 594B^2cd^2 + 211B^2d^3)}{\cos(fx + e)(a + a\sin(fx + e))^{1/2}/f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x)

[Out] $\frac{2/315*(1+\sin(f*x+e))*a*(\sin(f*x+e)-1)*((-45*A*d^3-135*B*c*d^2-40*B*d^3)*\sin(f*x+e)*\cos(f*x+e)^2+(315*A*c^2*d+252*A*c*d^2+117*A*d^3+105*B*c^3+252*B*c^2*d+351*B*c*d^2+104*B*d^3)*\sin(f*x+e)+35*B*\cos(f*x+e)^4*d^3+(-189*A*c*d^2-54*A*d^3-189*B*c^2*d-162*B*c*d^2-118*B*d^3)*\cos(f*x+e)^2+315*A*c^3+630*A*c^2*d+693*A*c*d^2+198*A*d^3+210*B*c^3+693*B*c^2*d+594*B*c*d^2+211*B*d^3)}{\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3, x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3, x)

$$3.287 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal. Leaf size=192

$$\frac{2a(15c^2 + 10cd + 7d^2)(-7Ad + Bc - 6Bd) \cos(e + fx)}{105df\sqrt{a \sin(e + fx) + a}} + \frac{2d(-7Ad + Bc - 6Bd) \cos(e + fx)(a \sin(e + fx) + a)}{35af}$$

[Out] $2/35*d*(-7*A*d+B*c-6*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/a/f+2/105*a*(-7*A*d+B*c-6*B*d)*(15*c^2+10*c*d+7*d^2)*\cos(f*x+e)/d/f/(a+a*\sin(f*x+e))^{(1/2)}-2/7*a*B*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/d/f/(a+a*\sin(f*x+e))^{(1/2)}+4/105*(5*c-d)*(-7*A*d+B*c-6*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.34, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2981, 2761, 2751, 2646}

$$\frac{2a(15c^2 + 10cd + 7d^2)(-7Ad + Bc - 6Bd) \cos(e + fx)}{105df\sqrt{a \sin(e + fx) + a}} + \frac{2d(-7Ad + Bc - 6Bd) \cos(e + fx)(a \sin(e + fx) + a)}{35af}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]

[Out] $(2*a*(B*c - 7*A*d - 6*B*d)*(15*c^2 + 10*c*d + 7*d^2)*\text{Cos}[e + f*x])/(105*d*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (4*(5*c - d)*(B*c - 7*A*d - 6*B*d)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(105*f) + (2*d*(B*c - 7*A*d - 6*B*d)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(35*a*f) - (2*a*B*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^3)/(7*d*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2761

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 2981

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^n, x_Symbol] := Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b

```
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^3}{7df\sqrt{a + a \sin(e + fx)}} + \frac{(7aA + 7aB \sin(e + fx)) \cos(e + fx)}{35af} \\ &= \frac{2d(Bc - 7Ad - 6Bd) \cos(e + fx)(a + a \sin(e + fx))}{35af} \\ &= \frac{4(5c - d)(Bc - 7Ad - 6Bd) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{105f} \\ &= \frac{2a(Bc - 7Ad - 6Bd) (15c^2 + 10cd + 7d^2) \cos(e + fx)}{105df\sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.74, size = 176, normalized size = 0.92

$$\frac{\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left((56Ad(5c + 2d) + B(140c^2 + 224cd + 141d^2)) \sin(e + fx) - 210f \left(\sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{210f \left(\sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] -1/210*((Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(4
20*A*c^2 + 280*B*c^2 + 560*A*c*d + 532*B*c*d + 266*A*d^2 + 228*B*d^2 - 6*d*
(14*B*c + 7*A*d + 6*B*d)*Cos[2*(e + f*x)] + (56*A*d*(5*c + 2*d) + B*(140*c^
2 + 224*c*d + 141*d^2))*Sin[e + f*x] - 15*B*d^2*Sin[3*(e + f*x)]))/(f*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.43, size = 306, normalized size = 1.59

$$\frac{2 \left(15 B d^2 \cos^4(fx + e) + 3 \left(14 B c d + (7 A + 6 B) d^2 \right) \cos^3(fx + e) - 35 (3 A + B) c^2 - 14 (5 A + 7 B) c d - (49 A + 27 B) d^2 \right) \sqrt{a \sin(fx + e) + a}}{210 f \left(\sin\left(\frac{1}{2}(e + fx)\right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2*(a+a*sin(f*x+e))^(1/2),x, alg
orithm="fricas")
```

```
[Out] 2/105*(15*B*d^2*cos(f*x + e)^4 + 3*(14*B*c*d + (7*A + 6*B)*d^2)*cos(f*x + e
)^3 - 35*(3*A + B)*c^2 - 14*(5*A + 7*B)*c*d - (49*A + 27*B)*d^2 - (35*B*c^2
+ 14*(5*A + B)*c*d + (7*A + 36*B)*d^2)*cos(f*x + e)^2 - (35*(3*A + 2*B)*c^
2 + 14*(10*A + 11*B)*c*d + 11*(7*A + 6*B)*d^2)*cos(f*x + e) + (15*B*d^2*cos
(f*x + e)^3 + 35*(3*A + B)*c^2 + 14*(5*A + 7*B)*c*d + (49*A + 27*B)*d^2 - 3
*(14*B*c*d + (7*A + B)*d^2)*cos(f*x + e)^2 - (35*B*c^2 + 14*(5*A + 4*B)*c*d
+ (28*A + 39*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/
(f*cos(f*x + e) + f*sin(f*x + e) + f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError


```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2,
x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx)) (c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2*(a+a*sin(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2, x)
```

$$3.288 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$$

Optimal. Leaf size=118

$$\frac{2(5Ad + 5Bc - 2Bd) \cos(e + fx) \sqrt{a \sin(e + fx) + a}}{15f} - \frac{2a(15Ac + 5Ad + 5Bc + 7Bd) \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}} - \frac{2Bd \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}}$$

[Out] $-2/5*B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-2/15*a*(15*A*c+5*A*d+5*B*c+7*B*d)*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/15*(5*A*d+5*B*c-2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f$

Rubi [A] time = 0.25, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2968, 3023, 2751, 2646}

$$\frac{2(5Ad + 5Bc - 2Bd) \cos(e + fx) \sqrt{a \sin(e + fx) + a}}{15f} - \frac{2a(15Ac + 5Ad + 5Bc + 7Bd) \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}} - \frac{2Bd \cos(e + fx)}{15f \sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] $(-2*a*(15*A*c + 5*B*c + 5*A*d + 7*B*d)*Cos[e + f*x])/(15*f*Sqrt[a + a*Sin[e + f*x]]) - (2*(5*B*c + 5*A*d - 2*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(15*f) - (2*B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*a*f)$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\cos(1/2*(f*x+\exp(1))-1/4*\pi))+2*B*c*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi)))*\cos(1/4*(2*f*x+2*\exp(1)+\pi))/(2*f)^2-6*f*(2*A*d*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi))+2*B*c*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi)))*\cos(1/4*(6*f*x+6*\exp(1)-\pi))/(6*f)^2+2*f*(4*A*c*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi))+2*B*d*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi)))*\sin(1/4*(2*f*x-\pi)+1/2*\exp(1))/(2*f)^2-24*B*d*f*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi))*\sin(1/4*(6*f*x+6*\exp(1)+\pi))/(12*f)^2-40*B*d*f*\text{sign}(\cos(1/2*(f*x+\exp(1))-1/4*\pi))*\sin(1/4*(10*f*x+10*\exp(1)-\pi))/(20*f)^2)$$

maple [A] time = 1.36, size = 102, normalized size = 0.86

$$\frac{2(1 + \sin(fx + e))a(\sin(fx + e) - 1)(3B(\sin^2(fx + e))d + 5A\sin(fx + e)d + 5B\sin(fx + e)c + 4B\sin(fx + e)a)}{15\cos(fx + e)\sqrt{a + a\sin(fx + e)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)

[Out] 2/15*(1+sin(f*x+e))*a*(sin(f*x+e)-1)*(3*B*sin(f*x+e)^2*d+5*A*sin(f*x+e)*d+5*B*sin(f*x+e)*c+4*B*sin(f*x+e)*d+15*A*c+10*A*d+10*B*c+8*B*d)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx)) (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)), x)

$$3.289 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx)) dx$$

Optimal. Leaf size=62

$$-\frac{2a(3A + B) \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{2B \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f}$$

[Out] $-2/3*a*(3*A+B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(1/2)}-2/3*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2751, 2646}

$$-\frac{2a(3A + B) \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{2B \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{3f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]

[Out] $(-2*a*(3*A + B)*\cos[e + f*x])/(3*f*\text{Sqrt}[a + a*\sin[e + f*x]]) - (2*B*\cos[e + f*x]*\text{Sqrt}[a + a*\sin[e + f*x]])/(3*f)$

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m]/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx)) dx &= -\frac{2B \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{3f} + \frac{1}{3}(3A + B) \int \sqrt{a + a \sin(e + fx)} dx \\ &= -\frac{2a(3A + B) \cos(e + fx)}{3f\sqrt{a + a \sin(e + fx)}} - \frac{2B \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{3f} \end{aligned}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 1.32

$$\frac{2\sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (3A + B \sin(e + fx) + 2B)}{3f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]),x]

[Out] $(-2*(\cos[(e + f*x)/2] - \sin[(e + f*x)/2])*sqrt[a*(1 + \sin[e + f*x])]*(3*A + 2*B + B*\sin[e + f*x]))/(3*f*(\cos[(e + f*x)/2] + \sin[(e + f*x)/2]))$

fricas [A] time = 0.44, size = 85, normalized size = 1.37

$$\frac{2\left(B\cos\left(fx+e\right)^2+(3A+2B)\cos\left(fx+e\right)+\left(B\cos\left(fx+e\right)-3A-B\right)\sin\left(fx+e\right)+3A+B\right)\sqrt{a\sin\left(fx+e\right)}}{3\left(f\cos\left(fx+e\right)+f\sin\left(fx+e\right)+f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] $-2/3*(B*\cos(f*x + e)^2 + (3*A + 2*B)*\cos(f*x + e) + (B*\cos(f*x + e) - 3*A - B)*\sin(f*x + e) + 3*A + B)*sqrt(a*\sin(f*x + e) + a)/(f*\cos(f*x + e) + f*\sin(f*x + e) + f)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)sqrt(2*a)*(2*A*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1)))/f-4*B*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(2*f)^2-12*B*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(6*f)^2

maple [A] time = 1.02, size = 58, normalized size = 0.94

$$\frac{2\left(1+\sin\left(fx+e\right)\right)a\left(\sin\left(fx+e\right)-1\right)\left(B\sin\left(fx+e\right)+3A+2B\right)}{3\cos\left(fx+e\right)\sqrt{a+a\sin\left(fx+e\right)}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x)

[Out] $2/3*(1+\sin(f*x+e))*a*(\sin(f*x+e)-1)*(B*\sin(f*x+e)+3*A+2*B)/\cos(f*x+e)/(a+a*\sin(f*x+e))^(1/2)/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B\sin(fx+e) + A)\sqrt{a\sin(fx+e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (A + B\sin(e + fx))\sqrt{a + a\sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2),x)

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2),x)`

[Out] `Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x)), x)`

$$3.290 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=100

$$\frac{2\sqrt{a}(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d} \cos(e+fx)}{\sqrt{c+d}\sqrt{a \sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}} - \frac{2aB \cos(e+fx)}{df\sqrt{a \sin(e+fx)+a}}$$

[Out] $2*(-A*d+B*c)*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}/d^{(3/2)}/f/(c+d)^{(1/2)}-2*a*B*\cos(f*x+e)/d/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2981, 2773, 208}

$$\frac{2\sqrt{a}(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d} \cos(e+fx)}{\sqrt{c+d}\sqrt{a \sin(e+fx)+a}}\right)}{d^{3/2}f\sqrt{c+d}} - \frac{2aB \cos(e+fx)}{df\sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*(A + B*\operatorname{Sin}[e + f*x]))/(c + d*\operatorname{Sin}[e + f*x]), x]$
 [Out] $(2*\operatorname{Sqrt}[a]*(B*c - A*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(d^{(3/2)}*\operatorname{Sqrt}[c + d]*f) - (2*a*B*\operatorname{Cos}[e + f*x])/(d*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])]/((c_.) + (d_)*\sin[(e_.) + (f_)*(x_)]), x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2981

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])]*((A_.) + (B_)*\sin[(e_.) + (f_)*(x_)])*((c_.) + (d_)*\sin[(e_.) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow \operatorname{Simp}[(-2*b*B*\operatorname{Cos}[e + f*x]*(c + d*\operatorname{Sin}[e + f*x])^{(n+1)})/(d*f*(2*n+3)*\operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]]), x] + \operatorname{Dist}[(A*b*d*(2*n+3) - B*(b*c - 2*a*d*(n+1)))/(b*d*(2*n+3)), \operatorname{Int}[\operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]]*(c + d*\operatorname{Sin}[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \operatorname{!LtQ}[n, -1]$

Rubi steps

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx = -\frac{2aB \cos(e + fx)}{df \sqrt{a + a \sin(e + fx)}} + \frac{(-aBc + aAd) \int \frac{\sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx}{ad}$$

$$= -\frac{2aB \cos(e + fx)}{df \sqrt{a + a \sin(e + fx)}} + \frac{(2a(Bc - Ad)) \text{Subst}\left(\int \frac{1}{ac + ad - dx^2} dx, x\right)}{df}$$

$$= \frac{2\sqrt{a} (Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c + d} \sqrt{a + a \sin(e + fx)}}\right)}{d^{3/2} \sqrt{c + d} f} - \frac{2aB \cos(e + fx)}{df \sqrt{a + a \sin(e + fx)}}$$

Mathematica [C] time = 9.03, size = 903, normalized size = 9.03

$$\left(\frac{1}{2} + \frac{i}{2}\right) \frac{(2-2i)B\sqrt{d} \cos\left(\frac{fx}{2}\right) \left(\cos\left(\frac{e}{2}\right) - \sin\left(\frac{e}{2}\right)\right)}{f} + \frac{(Ad-Bc)\left(\cos\left(\frac{e}{2}\right) + i \sin\left(\frac{e}{2}\right)\right) (-1+i)x \cos(e) + (1+i)x \sin(e) + \frac{\text{RootSum}\left[d e^{2ie} \#1^4 + 2icce^{ie} \#1^2 - d\right]}{\dots}}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]),x]

[Out] ((1/2 + I/2)*(((-2 + 2*I)*B*Sqrt[d]*Cos[(f*x)/2]*(Cos[e/2] - Sin[e/2]))/f + ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2]))*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) &]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]/(4*f) + (1 + I)*x*Sin[e])/((Sqrt[c + d]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((-(B*c) + A*d)*(Cos[e/2] + I*Sin[e/2]))*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - #1] + S

$$\frac{\sqrt{d} \sqrt{c+d} f x \sqrt{1 + (2I) \sqrt{d} \sqrt{c+d} \log[E^{((I/2) f x)} - \#1] \sqrt{1 - ((1+I) c f x \sqrt{1^2}) / \sqrt{E^{((-I) e}}]} + ((2 - 2I) c \log[E^{((I/2) f x)} - \#1] \sqrt{1^2}) / \sqrt{E^{((-I) e}}]} - I \sqrt{d} \sqrt{c+d} E^{(I e)} f x \sqrt{1^3 + 2 \sqrt{d} \sqrt{c+d} E^{(I e)} \log[E^{((I/2) f x)} - \#1] \sqrt{1^3}} / (d - I c E^{(I e)} \sqrt{1^2}) \&] \sqrt{\cos[e] - I \sin[e]} (-1 - I \cos[e] + \sin[e]) / (4 f)} / (\sqrt{c+d} (\cos[e] + I (-1 + \sin[e])) \sqrt{\cos[e] - I \sin[e]}) + ((2 - 2I) B \sqrt{d} (\cos[e/2] + \sin[e/2]) \sin[(f x) / 2]) / f \sqrt{a (1 + \sin[e + f x])}} / (d^{(3/2)} (\cos[(e + f x) / 2] + \sin[(e + f x) / 2]))$$

fricas [A] time = 1.26, size = 651, normalized size = 6.51

$$\left[\frac{(Bc - Ad + (Bc - Ad) \cos(fx + e) + (Bc - Ad) \sin(fx + e)) \sqrt{\frac{a}{cd+d^2}} \log\left(\frac{ad^2 \cos(fx+e)^3 - ac^2 - 2acd - ad^2 - (6acd+7a^2)}{\dots}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [-1/2*((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e)) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f), ((B*c - A*d + (B*c - A*d)*cos(f*x + e) + (B*c - A*d)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) + a))/(d*f*cos(f*x + e) + d*f*sin(f*x + e) + d*f)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [A] time = 1.79, size = 139, normalized size = 1.39

$$\frac{2(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(A \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)} d}{\sqrt{a(c+d)d}}\right) ad - B \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a(c+d)d}}\right) \right)}{d \sqrt{a(c+d)d} \cos(fx + e) \sqrt{a + a \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x)

[Out] $-2*(1+\sin(f*x+e))*(-a*(\sin(f*x+e)-1))^{(1/2)}*(A*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{(1/2)})*d/(a*(c+d)*d)^{(1/2)}*a*d-B*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{(1/2)})*d/(a*(c+d)*d)^{(1/2)})*a*c+B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}/d/(a*(c+d)*d)^{(1/2)}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a}}{d \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)), x)`

[Out] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e)),x)`

[Out] Timed out

$$3.291 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=126

$$\frac{a(Bc - Ad) \cos(e + fx)}{df(c + d)\sqrt{a \sin(e + fx) + a} (c + d \sin(e + fx))} - \frac{\sqrt{a} (Ad + B(c + 2d)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{3/2} f (c + d)^{3/2}}$$

[Out] $-(A*d+B*(c+2*d))*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}/d^{(3/2)}/(c+d)^{(3/2)}/f+a*(-A*d+B*c)*\cos(f*x+e)/d/(c+d)/f/(c+d*\sin(f*x+e))/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {2980, 2773, 208}

$$\frac{a(Bc - Ad) \cos(e + fx)}{df(c + d)\sqrt{a \sin(e + fx) + a} (c + d \sin(e + fx))} - \frac{\sqrt{a} (Ad + B(c + 2d)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{3/2} f (c + d)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2, x]`

[Out] $-\left(\frac{\operatorname{Sqrt}[a]*(A*d + B*(c + 2*d))*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x]}{\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]}\right]}{d^{(3/2)}*(c + d)^{(3/2)}*f}\right) + (a*(B*c - A*d)*\operatorname{Cos}[e + f*x])/(d*(c + d)*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*(c + d*\operatorname{Sin}[e + f*x]))$

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 2773

`Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]`

Rule 2980

`Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]`

Rubi steps

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx = \frac{a(Bc - Ad) \cos(e + fx)}{d(c + d)f\sqrt{a + a \sin(e + fx)}(c + d \sin(e + fx))} + \frac{(-aAd - B(a^2 + d^2)) \cos(e + fx)}{d^2(c + d)f\sqrt{a + a \sin(e + fx)}} - \frac{a(Ad + B(c + 2d)) \cos(e + fx)}{d^2(c + d)f\sqrt{a + a \sin(e + fx)}} + \frac{\sqrt{a} (Ad + B(c + 2d)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c + d} \sqrt{a + a \sin(e + fx)}} \right)}{d^{3/2}(c + d)^{3/2}f} + \frac{(-\sqrt{d} \sqrt{c + d} e^{ie} f x \#1)}{d(c + d)f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [C] time = 8.80, size = 901, normalized size = 7.15

$\left(\frac{1}{4} + \frac{i}{4}\right) \sqrt{a(\sin(e + fx) + 1)}$

$(Ad + B(c + 2d)) \left(\cos\left(\frac{e}{2}\right) + i \sin\left(\frac{e}{2}\right) \right) (-1 + i)x \cos(e) + (1 + i)x \sin(e) + \frac{\text{RootSum}\left[d e^{2ie} \#1^4 + 2icce^{ie} \#1^2 - d\&, \frac{-\sqrt{d} \sqrt{c + d} e^{ie} f x \#1}{d(c + d)f\sqrt{a + a \sin(e + fx)}}\right]}{d^2(c + d)f\sqrt{a + a \sin(e + fx)}}$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]
[Out] ((1/4 + I/4)*Sqrt[a*(1 + Sin[e + f*x])]*(((A*d + B*(c + 2*d))*(Cos[e/2] + I*Sin[e/2]))*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 &, ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^((-I)*e)]]*Log[E^((I/2)*f*x) - #1] - I*Sqrt[d]*Sqrt[c + d]*f*x*#1 + 2*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 + ((1 - I)*c*f*x*#1^2)/Sqrt[E^((-I)*e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3)/(d - I*c*E^(I*e)*#1^2) & ]*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(3/2)*(Cos[e] + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((A*d + B*(c + 2*d))*(Cos[e/2] + I*Sin[e/2]))*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2*I)*c*E^(I*e)*#1^2 + d*E^((2*I)*e)*#1^4 &, ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x + (2 + 2*I)*d*Sqrt[E^((-I)*e)]]*Log[E^((I/2)*f*x) - #1] + Sqrt[d]*Sqrt[c + d]*f*x
```

```
#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - #1]*#1 - ((1 + I)*c*f*x*
#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - #1]*#1^2)/Sqrt[E^
((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*#1^3 + 2*Sqrt[d]*Sqrt[c + d]*
E^(I*e)*Log[E^((I/2)*f*x) - #1]*#1^3/(d - I*c*E^(I*e)*#1^2) & ]*Sqrt[Cos[e
] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e])/(4*f))/((c + d)^(3/2)*(Cos[e] + I*
(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((2 - 2*I)*Sqrt[d]*(-(B*c) + A*d)
*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x]))/
(d^(3/2)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [B] time = 1.45, size = 1012, normalized size = 8.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [-1/4*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d^2)*cos
(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3*B)*c*d
+ (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*
c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*
cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a
)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*c
os(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e
))*sin(f*x + e)/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 -
2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f
*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c - A*d + (B*c - A*d)*co
s(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^2 +
d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*c*d^2 +
d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)*sin(f*
x + e)), 1/2*((B*c^2 + (A + 3*B)*c*d + (A + 2*B)*d^2 - (B*c*d + (A + 2*B)*d
^2)*cos(f*x + e)^2 + (B*c^2 + (A + 2*B)*c*d)*cos(f*x + e) + (B*c^2 + (A + 3
*B)*c*d + (A + 2*B)*d^2 + (B*c*d + (A + 2*B)*d^2)*cos(f*x + e))*sin(f*x + e
))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e)
- c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c - A*d + (B*c -
A*d)*cos(f*x + e) - (B*c - A*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c
*d^2 + d^3)*f*cos(f*x + e)^2 - (c^2*d + c*d^2)*f*cos(f*x + e) - (c^2*d + 2*
c*d^2 + d^3)*f - ((c*d^2 + d^3)*f*cos(f*x + e) + (c^2*d + 2*c*d^2 + d^3)*f)
*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 2.02, size = 274, normalized size = 2.17

$$\frac{(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(\sin(fx + e) \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}d}{\sqrt{acd+ad^2}}\right) ad (Ad + Bc + 2Bd) + Aa \right)}{d(c \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)

[Out] $-(1+\sin(f*x+e))*(-a*(\sin(f*x+e)-1))^{1/2}*(\sin(f*x+e)*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2})*d/(a*c*d+a*d^2)^{1/2})*a*d*(A*d+B*c+2*B*d)+A*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2})*d/(a*c*d+a*d^2)^{1/2})*a*c*d+B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2})*d/(a*c*d+a*d^2)^{1/2})*a*c^2+2*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2})*d/(a*c*d+a*d^2)^{1/2})*a*c*d+A*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*d-B*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*c)/d/(c+d)/(c+d*\sin(f*x+e))/(a*(c+d)*d)^{1/2}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a}}{(d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^2,x)

[Out] Timed out

$$3.292 \quad \int \frac{\sqrt{a+a \sin(e+fx)} (A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=192

$$\frac{\sqrt{a}(3Ad + B(c + 4d)) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{4d^{3/2} f (c+d)^{5/2}} - \frac{a(3Ad + B(c + 4d)) \cos(e+fx)}{4df(c+d)^2 \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))} + \frac{2df}{2df}$$

[Out] $-1/4*(3*A*d+B*(c+4*d))*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})*a^{(1/2)}/d^{(3/2)}/(c+d)^{(5/2)}/f+1/2*a*(-A*d+B*c)*\cos(f*x+e)/d/(c+d)/f/(c+d*\sin(f*x+e))^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}-1/4*a*(3*A*d+B*(c+4*d))*\cos(f*x+e)/d/(c+d)^2/f/(c+d*\sin(f*x+e))/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.37, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2980, 2772, 2773, 208}

$$\frac{\sqrt{a}(3Ad + B(c + 4d)) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{4d^{3/2} f (c+d)^{5/2}} - \frac{a(3Ad + B(c + 4d)) \cos(e+fx)}{4df(c+d)^2 \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))} + \frac{2df}{2df}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3, x]

[Out] $-(\operatorname{Sqrt}[a]*(3*A*d + B*(c + 4*d))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(4*d^{(3/2)}*(c + d)^{(5/2)}*f) + (a*(B*c - A*d)*\operatorname{Cos}[e + f*x])/(2*d*(c + d)*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*(c + d*\operatorname{Sin}[e + f*x])^2) - (a*(3*A*d + B*(c + 4*d))*\operatorname{Cos}[e + f*x])/(4*d*(c + d)^2*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*(c + d*\operatorname{Sin}[e + f*x]))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2772

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[((2*n + 3)*(b*c - a*d))/(2*b*(n + 1)*(c^2 - d^2)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2980

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*

$(b*c + a*d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1))]/(2*d*(n + 1)*(b*c + a*d)), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^(n + 1), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \& \& \text{NeQ}[b*c - a*d, 0] \& \& \text{EqQ}[a^2 - b^2, 0] \& \& \text{NeQ}[c^2 - d^2, 0] \& \& \text{LtQ}[n, -1]$

Rubi steps

$$\int \frac{\sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx = \frac{a(Bc - Ad) \cos(e + fx)}{2d(c + d)f\sqrt{a + a \sin(e + fx)}(c + d \sin(e + fx))^2} + \frac{(-3aAd - a^3)}{4d(c + d)^2 f} - \frac{a(3Ad + B(c + 4d)) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c+d} \sqrt{a+a \sin(e+fx)}}\right)}{4d^{3/2}(c + d)^{5/2} f} + \frac{a^3}{2d(c + d)f}$$

Mathematica [C] time = 10.27, size = 967, normalized size = 5.04

$$\left(\frac{1}{16} + \frac{i}{16} \right) \sqrt{a(\sin(e + fx) + 1)} \left((3Ad + B(c + 4d)) \left(\cos\left(\frac{e}{2}\right) + i \sin\left(\frac{e}{2}\right) \right) (-1 + i)x \cos(e) + (1 + i)x \sin(e) + \frac{\text{RootSum}\left[de^{2ie}\#1^4 + 2ice^{ie}\#1^2 - d\&, -\sqrt{d} \sqrt{c+d} e^{ie}\right]}{\dots} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
[Out] ((1/16 + I/16)*Sqrt[a*(1 + Sin[e + f*x]))*(((3*A*d + B*(c + 4*d))*(Cos[e/2] + I*Sin[e/2]))*((-1 + I)*x*Cos[e] + (RootSum[-d + (2*I)*c*E^(I*e)]*#1^2 + d*E^((2*I)*e)*#1^4 & , ((1 + I)*d*Sqrt[E^((-I)*e)]*f*x - (2 - 2*I)*d*Sqrt[E^
```

$$\begin{aligned}
& (-I)*e)]*Log[E^((I/2)*f*x) - \#1] - I*Sqrt[d]*Sqrt[c + d]*f*x*\#1 + 2*Sqrt[d] \\
& *Sqrt[c + d]*Log[E^((I/2)*f*x) - \#1]*\#1 + ((1 - I)*c*f*x*\#1^2)/Sqrt[E^((-I) \\
& *e)] + ((2 + 2*I)*c*Log[E^((I/2)*f*x) - \#1]*\#1^2)/Sqrt[E^((-I)*e)] - Sqrt[d] \\
& *Sqrt[c + d]*E^(I*e)*f*x*\#1^3 - (2*I)*Sqrt[d]*Sqrt[c + d]*E^(I*e)*Log[E^((\\
& I/2)*f*x) - \#1]*\#1^3)/(d - I*c*E^(I*e)*\#1^2) &]*(Cos[e] + I*(-1 + Sin[e])) \\
& *Sqrt[Cos[e] - I*Sin[e]])/(4*f) + (1 + I)*x*Sin[e]))/((c + d)^(5/2)*(Cos[e] \\
& + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) + ((3*A*d + B*(c + 4*d))*(Cos[\\
& e/2] + I*Sin[e/2]))*((1 - I)*x*Cos[e] - (1 + I)*x*Sin[e] + (RootSum[-d + (2* \\
& I)*c*E^(I*e)*\#1^2 + d*E^((2*I)*e)*\#1^4 & , ((1 - I)*d*Sqrt[E^((-I)*e)]*f*x \\
& + (2 + 2*I)*d*Sqrt[E^((-I)*e)]*Log[E^((I/2)*f*x) - \#1] + Sqrt[d]*Sqrt[c + d] \\
& *f*x*\#1 + (2*I)*Sqrt[d]*Sqrt[c + d]*Log[E^((I/2)*f*x) - \#1]*\#1 - ((1 + I)* \\
& c*f*x*\#1^2)/Sqrt[E^((-I)*e)] + ((2 - 2*I)*c*Log[E^((I/2)*f*x) - \#1]*\#1^2)/S \\
& qrt[E^((-I)*e)] - I*Sqrt[d]*Sqrt[c + d]*E^(I*e)*f*x*\#1^3 + 2*Sqrt[d]*Sqrt[c \\
& + d]*E^(I*e)*Log[E^((I/2)*f*x) - \#1]*\#1^3)/(d - I*c*E^(I*e)*\#1^2) &]*Sqrt \\
& [Cos[e] - I*Sin[e]]*(-1 - I*Cos[e] + Sin[e]))/(4*f)))/((c + d)^(5/2)*(Cos[e] \\
& + I*(-1 + Sin[e]))*Sqrt[Cos[e] - I*Sin[e]]) - ((4 - 4*I)*Sqrt[d]*(-(B*c) \\
& + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*f*(c + d*Sin[e + f*x] \\
&))^2) - ((2 - 2*I)*Sqrt[d]*(3*A*d + B*(c + 4*d))*(Cos[(e + f*x)/2] - Sin[(e \\
& + f*x)/2]))/((c + d)^2*f*(c + d*Sin[e + f*x])))/(d^(3/2)*(Cos[(e + f*x)/2] \\
& + Sin[(e + f*x)/2]))
\end{aligned}$$

fricas [B] time = 2.28, size = 1750, normalized size = 9.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, alg
 orithm="fricas")

[Out] [-1/16*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3
 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c
 *d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d
 ^2 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A +
 3*B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d +
 d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*
 d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x +
 e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 +
 (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))*sqrt(a/(
 c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e
)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x
 + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d
 ^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) -
 c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2
 - (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2
 *A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3
 *A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*
 d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 +
 d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*c
 os(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3
 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(
 f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))
 , 1/8*((B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3*B)*c*d^2 + (3*A + 4*B)*d^3 -
 (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^3 - (2*B*c^2*d + 3*(2*A + 3*B)*c*
 d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 + (B*c^3 + (3*A + 4*B)*c^2*d + B*c*d^2
 + (3*A + 4*B)*d^3)*cos(f*x + e) + (B*c^3 + 3*(A + 2*B)*c^2*d + 3*(2*A + 3
 *B)*c*d^2 + (3*A + 4*B)*d^3 - (B*c*d^2 + (3*A + 4*B)*d^3)*cos(f*x + e)^2 +
 2*(B*c^2*d + (3*A + 4*B)*c*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d +
 d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-
 a/(c*d + d^2))/(a*cos(f*x + e))) - 2*(B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2

- (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e)^2 + (B*c^2 - (5*A + 2*B)*c*d - 2*A*d^2)*cos(f*x + e) - (B*c^2 - (5*A + B)*c*d + (A + 4*B)*d^2 + (B*c*d + (3*A + 4*B)*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/((c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^3 + (2*c^3*d^2 + 5*c^2*d^3 + 4*c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^4*d + 2*c^3*d^2 + 2*c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f + ((c^2*d^3 + 2*c*d^4 + d^5)*f*cos(f*x + e)^2 - 2*(c^3*d^2 + 2*c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^4*d + 4*c^3*d^2 + 6*c^2*d^3 + 4*c*d^4 + d^5)*f)*sin(f*x + e))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.43, size = 628, normalized size = 3.27

$$\left(-2 \sin (f x+e) \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} d}{\sqrt{a c d+a d^2}}\right) a^2 c d(3 A d+B c+4 B d)+\operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} d}{\sqrt{a c d+a d^2}}\right) a^2 d^2(3 A d+B c+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x)

[Out] 1/4/a*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d*(3*A*d+B*c+4*B*d)+arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(3*A*d+B*c+4*B*d)*cos(f*x+e)^2+3*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+4*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-4*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-5*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-3*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin (f x+e)+A) \sqrt{a \sin (f x+e)+a}}{(d \sin (f x+e)+c)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)/(d*sin(f*x + e) + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^3, x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2))/(c + d*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(a+a*sin(f*x+e))**(1/2)/(c+d*sin(f*x+e))**3, x)

[Out] Timed out

$$3.293 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx$$

Optimal. Leaf size=374

$$\frac{2a^2 (11Ad(c - 17d) - 3B(c^2 - 9cd + 56d^2)) \cos(e + fx)(c + d \sin(e + fx))^3}{693d^2 f \sqrt{a \sin(e + fx) + a}} + \frac{4a^2(c + d)(15c^2 + 10cd + 7d^2)(11A - 3B)}{3465d^2 f}$$

[Out] 4/1155*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f+4/3465*a^2*(c+d)*(15*c^2+10*c*d+7*d^2)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^2*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/99*a^2*(3*B*(c-4*d)-11*A*d)*cos(f*x+e)*(c+d*sin(f*x+e))^4/d^2/f/(a+a*sin(f*x+e))^(1/2)+8/3465*a*(5*c-d)*(c+d)*(11*A*(c-17*d)*d-3*B*(c^2-9*c*d+56*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/11*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^4*(a+a*sin(f*x+e))^(1/2)/d/f

Rubi [A] time = 0.92, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$\frac{2a^2 (11Ad(c - 17d) - 3B(c^2 - 9cd + 56d^2)) \cos(e + fx)(c + d \sin(e + fx))^3}{693d^2 f \sqrt{a \sin(e + fx) + a}} + \frac{4a^2(c + d)(15c^2 + 10cd + 7d^2)(11A - 3B)}{3465d^2 f}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3, x]

[Out] (4*a^2*(c + d)*(15*c^2 + 10*c*d + 7*d^2)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]/(3465*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (8*a*(5*c - d)*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]/(3465*d*f) + (4*(c + d)*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*f) + (2*a^2*(11*A*(c - 17*d)*d - 3*B*(c^2 - 9*c*d + 56*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(693*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(3*B*(c - 4*d) - 11*A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^4)/(99*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^4)/(11*d*f)

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2761

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*S

```
imp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^
2 - b^2, 0] && !LtQ[m, -1]
```

Rule 2770

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Simp[(-2*b*Cos[e + f*x]*(c + d*Ssin[e + f*x])
^n)/(f*(2*n + 1)*Sqrt[a + b*Ssin[e + f*x]]), x] + Dist[(2*n*(b*c + a*d))/(b*
(2*n + 1)), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^(n - 1), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0
] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && IntegerQ[2*n]
```

Rule 2976

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Si
mp[(b*B*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m - 1)*(c + d*Ssin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Ssin[e + f*x
])^(m - 1)*(c + d*Ssin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (
f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Ssin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Ssin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx = -\frac{2aB \cos(e + fx) \sqrt{a + a \sin(e + fx)} (c + a \sin(e + fx))}{11df}$$

$$= \frac{2a^2(3B(c - 4d) - 11Ad) \cos(e + fx)(c + a \sin(e + fx))}{99d^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{2a^2 (11A(c - 17d)d - 3B(c^2 - 9cd + 56d^2))}{693d^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{4(c + d) (11A(c - 17d)d - 3B(c^2 - 9cd + 56d^2))}{115d^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{8a(5c - d)(c + d) (11A(c - 17d)d - 3B(c^2 - 9cd + 56d^2))}{3465d^2 f \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 4.61, size = 390, normalized size = 1.04

$$a\sqrt{a(\sin(e+fx)+1)}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(-8(11Ad(189c^2+351cd+137d^2))+3B(231c^3+12\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]

[Out] -1/27720*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(92400*A*c^3 + 72072*B*c^3 + 216216*A*c^2*d + 195624*B*c^2*d + 195624*A*c*d^2 + 177474*B*c*d^2 + 59158*A*d^3 + 55482*B*d^3 - 8*(11*A*d*(189*c^2 + 351*c*d + 137*d^2) + 3*B*(231*c^3 + 1287*c^2*d + 1507*c*d^2 + 581*d^3))*Cos[2*(e + f*x)] + 70*d^2*(33*B*c + 11*A*d + 21*B*d)*Cos[4*(e + f*x)] + 18480*A*c^3*Sin[e + f*x] + 33264*B*c^3*Sin[e + f*x] + 99792*A*c^2*d*Sin[e + f*x] + 100188*B*c^2*d*Sin[e + f*x] + 100188*A*c*d^2*Sin[e + f*x] + 105468*B*c*d^2*Sin[e + f*x] + 35156*A*d^3*Sin[e + f*x] + 34734*B*d^3*Sin[e + f*x] - 5940*B*c^2*d*Sin[3*(e + f*x)] - 5940*A*c*d^2*Sin[3*(e + f*x)] - 11220*B*c*d^2*Sin[3*(e + f*x)] - 3740*A*d^3*Sin[3*(e + f*x)] - 4935*B*d^3*Sin[3*(e + f*x)] + 315*B*d^3*Sin[5*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))

fricas [A] time = 0.47, size = 637, normalized size = 1.70

$$2\left(315Bad^3\cos(fx+e)^6+35(33Bacd^2+(11A+21B)ad^3)\cos(fx+e)^5+924(5A+3B)ac^3+396(21A+\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] -2/3465*(315*B*a*d^3*cos(f*x + e)^6 + 35*(33*B*a*c*d^2 + (11*A + 21*B)*a*d^3)*cos(f*x + e)^5 + 924*(5*A + 3*B)*a*c^3 + 396*(21*A + 19*B)*a*c^2*d + 132*(57*A + 47*B)*a*c*d^2 + 4*(517*A + 483*B)*a*d^3 - 5*(297*B*a*c^2*d + 33*(9*A + 10*B)*a*c*d^2 + 10*(11*A + 21*B)*a*d^3)*cos(f*x + e)^4 - (693*B*a*c^3 + 297*(7*A + 13*B)*a*c^2*d + 33*(117*A + 172*B)*a*c*d^2 + 2*(946*A + 1239*B)*a*d^3)*cos(f*x + e)^3 + (231*(5*A + 6*B)*a*c^3 + 99*(42*A + 43*B)*a*c^2*d + 33*(129*A + 134*B)*a*c*d^2 + (1474*A + 1491*B)*a*d^3)*cos(f*x + e)^2 + (231*(25*A + 21*B)*a*c^3 + 99*(147*A + 143*B)*a*c^2*d + 33*(429*A + 409*B)*a*c*d^2 + (4499*A + 4431*B)*a*d^3)*cos(f*x + e) + (315*B*a*d^3*cos(f*x + e)^5 - 924*(5*A + 3*B)*a*c^3 - 396*(21*A + 19*B)*a*c^2*d - 132*(57*A + 47*B)*a*c*d^2 - 4*(517*A + 483*B)*a*d^3 - 35*(33*B*a*c*d^2 + (11*A + 12*B)*a*d^3)*cos(f*x + e)^4 - 5*(297*B*a*c^2*d + 33*(9*A + 17*B)*a*c*d^2 + (187*A + 294*B)*a*d^3)*cos(f*x + e)^3 + 3*(231*B*a*c^3 + 99*(7*A + 8*B)*a*c^2*d + 33*(24*A + 29*B)*a*c*d^2 + (319*A + 336*B)*a*d^3)*cos(f*x + e)^2 + (231*(5*A + 9*B)*a*c^3 + 99*(63*A + 67*B)*a*c^2*d + 33*(201*A + 221*B)*a*c*d^2 + 17*(143*A + 147*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="giac")


```
*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+18*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+24*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2-16*f*(16*A*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+10*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+48*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f*(16*A*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*A*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+10*B*a*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*A*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+48*A*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+36*B*a*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(6*f*x+6*exp(1)-pi))/(48*f)^2-576*B*a*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+18*exp(1)+pi))/(288*f)^2-704*B*a*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2)
```

maple [A] time = 1.37, size = 312, normalized size = 0.83

$$\frac{2(1 + \sin(fx + e))a^2(\sin(fx + e) - 1)(315B(\cos^4(fx + e))\sin(fx + e)d^3 + (-1485Ac d^2 - 935A d^3 - 1485A^2 c d))}{(a + a \sin(fx + e))^{3/2}(A + B \sin(fx + e))(c + d \sin(fx + e))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)

[Out] 2/3465*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*(315*B*cos(f*x+e)^4*sin(f*x+e)*d^3+(-1485*A*c*d^2-935*A*d^3-1485*B*c^2*d-2805*B*c*d^2-1470*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(1155*A*c^3+6237*A*c^2*d+6633*A*c*d^2+2431*A*d^3+2079*B*c^3+6633*B*c^2*d+7293*B*c*d^2+2499*B*d^3)*sin(f*x+e)+(385*A*d^3+1155*B*c*d^2+735*B*d^3)*cos(f*x+e)^4+(-2079*A*c^2*d-3861*A*c*d^2-1892*A*d^3-693*B*c^3-3861*B*c^2*d-5676*B*c*d^2-2478*B*d^3)*cos(f*x+e)^2+5775*A*c^3+14553*A*c^2*d+14157*A*c*d^2+4499*A*d^3+4851*B*c^3+14157*B*c^2*d+13497*B*c*d^2+4431*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{3/2} (d \sin(fx + e) + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3, x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)

[Out] Timed out

$$3.294 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal. Leaf size=294

$$\frac{2a^2(15c^2 + 10cd + 7d^2)(3Ad(c - 13d) - B(c^2 - 7cd + 34d^2)) \cos(e + fx)}{315d^2 f \sqrt{a \sin(e + fx) + a}} + \frac{2a^2(-9Ad + 3Bc - 10Bd) \cos(e + fx)}{63d^2 f \sqrt{a \sin(e + fx)}}$$

[Out] 2/105*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f+2/315*a^2*(15*c^2+10*c*d+7*d^2)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)/d^2/f/(a+a*sin(f*x+e))^(1/2)+2/63*a^2*(-9*A*d+3*B*c-10*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^2/f/(a+a*sin(f*x+e))^(1/2)+4/315*a*(5*c-d)*(3*A*(c-13*d)*d-B*(c^2-7*c*d+34*d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d/f-2/9*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d/f

Rubi [A] time = 0.71, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^2(15c^2 + 10cd + 7d^2)(3Ad(c - 13d) - B(c^2 - 7cd + 34d^2)) \cos(e + fx)}{315d^2 f \sqrt{a \sin(e + fx) + a}} + \frac{2a^2(-9Ad + 3Bc - 10Bd) \cos(e + fx)}{63d^2 f \sqrt{a \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2, x]

[Out] (2*a^2*(15*c^2 + 10*c*d + 7*d^2)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]/(315*d^2*f*Sqrt[a + a*Sin[e + f*x]]) + (4*a*(5*c - d)*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]/(315*d*f) + (2*(3*A*(c - 13*d)*d - B*(c^2 - 7*c*d + 34*d^2))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(105*f) + (2*a^2*(3*B*c - 9*A*d - 10*B*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(63*d^2*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(9*d*f)

Rule 2646

Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2761

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -1]

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)])^(m_)*((A_) + (B_.)*sin[(e_) +
(f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := -Simp[
(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]*((A_) + (B_.)*sin[(e_) + (
f_.)*(x_)])*((c_) + (d_.)*sin[(e_) + (f_.)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^2 dx = -\frac{2aB \cos(e + fx) \sqrt{a + a \sin(e + fx)} (c + a \sin(e + fx))}{9df}$$

$$= \frac{2a^2(3Bc - 9Ad - 10Bd) \cos(e + fx)(c + a \sin(e + fx))}{63d^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{2(3A(c - 13d)d - B(c^2 - 7cd + 34d^2)) \cos(e + fx)(c + a \sin(e + fx))}{105f}$$

$$= \frac{4a(5c - d)(3A(c - 13d)d - B(c^2 - 7cd + 34d^2)) \cos(e + fx)(c + a \sin(e + fx))}{315a}$$

$$= \frac{2a^2(15c^2 + 10cd + 7d^2)(3A(c - 13d)d - B(c^2 - 7cd + 34d^2)) \cos(e + fx)(c + a \sin(e + fx))}{315d^2 f \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 2.26, size = 267, normalized size = 0.91

$$\frac{a \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) \left(-4(9Ad(14c + 13d) + B(63c^2 + 234cd + 137d^2)) \right)}{315d^2 f \sqrt{a + a \sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*
x])^2,x]
```

```
[Out] -1/1260*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])])
*(4200*A*c^2 + 3276*B*c^2 + 6552*A*c*d + 5928*B*c*d + 2964*A*d^2 + 2689*B*d
^2 - 4*(9*A*d*(14*c + 13*d) + B*(63*c^2 + 234*c*d + 137*d^2))*Cos[2*(e + f*
x)] + 35*B*d^2*Cos[4*(e + f*x)] + 840*A*c^2*Sin[e + f*x] + 1512*B*c^2*Sin[e
+ f*x] + 3024*A*c*d*Sin[e + f*x] + 3036*B*c*d*Sin[e + f*x] + 1518*A*d^2*Si
n[e + f*x] + 1598*B*d^2*Sin[e + f*x] - 180*B*c*d*Sin[3*(e + f*x)] - 90*A*d^
```



```

a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(14*f*x+14*exp(1)-pi))/(5
6*f)^2+12*f*(-2*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(c
os(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4
*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a*c*d*sign(cos(1/2*(f*x+exp
(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a*d^2*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))-2*B*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-
2*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*A*a*c*d*sign(cos(1/2*(f*x+ex
p(1))-1/4*pi))-4*B*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*
x+10*exp(1)-pi))/(20*f)^2-8*f*(8*A*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
+6*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c^2*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*A*a*c*d*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+12*B*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi
)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(8*A*a*c^2*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))+6*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*B*a*c^2*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)+16*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+12*B*a*c*d*sign(cos(1/2*(f*
x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+8*f*(16*A*a*c^2*s
ign(cos(1/2*(f*x+exp(1))-1/4*pi))+8*A*a*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))+8*B*a*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+6*B*a*d^2*sign(cos(1/2*(f*
x+exp(1))-1/4*pi))+16*A*a*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+16*B*a*c*d
*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(8*f)^2
+224*B*a*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(14*f*x+14*exp(1)
+pi))/(112*f)^2+288*B*a*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(1
8*f*x+18*exp(1)-pi))/(144*f)^2)

```

maple [A] time = 1.44, size = 207, normalized size = 0.70

$$\frac{2(1 + \sin(fx + e))a^2(\sin(fx + e) - 1)((-45Ad^2 - 90Bcd - 85Bd^2)\sin(fx + e)(\cos^2(fx + e)) + (105Acd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)

[Out] 2/315*(1+sin(f*x+e))*a^2*(sin(f*x+e)-1)*((-45*A*d^2-90*B*c*d-85*B*d^2)*sin(f*x+e)*cos(f*x+e)^2+(105*A*c^2+378*A*c*d+201*A*d^2+189*B*c^2+402*B*c*d+221*B*d^2)*sin(f*x+e)+35*B*cos(f*x+e)^4*d^2+(-126*A*c*d-117*A*d^2-63*B*c^2-234*B*c*d-172*B*d^2)*cos(f*x+e)^2+525*A*c^2+882*A*c*d+429*A*d^2+441*B*c^2+858*B*c*d+409*B*d^2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}(d \sin(fx + e) + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{3}{2}} (c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2, x)

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^{\frac{3}{2}} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)`

[Out] `Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2, x)`

$$3.295 \quad \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$$

Optimal. Leaf size=165

$$\frac{8a^2(35Ac + 21Ad + 21Bc + 19Bd) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}} - \frac{2(7Ad + 7Bc - 2Bd) \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{35f} - \frac{2a(35A^2c + 21A^2d + 21B^2c + 19B^2d) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}}$$

[Out] $-2/35*(7*A*d+7*B*c-2*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{3/2}/f-2/7*B*d*\cos(f*x+e)*(a+a*\sin(f*x+e))^{5/2}/a/f-8/105*a^2*(35*A*c+21*A*d+21*B*c+19*B*d)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{1/2}-2/105*a*(35*A*c+21*A*d+21*B*c+19*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{1/2}/f$

Rubi [A] time = 0.32, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{8a^2(35Ac + 21Ad + 21Bc + 19Bd) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}} - \frac{2(7Ad + 7Bc - 2Bd) \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{35f} - \frac{2a(35A^2c + 21A^2d + 21B^2c + 19B^2d) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{3/2}*(A + B*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x]), x]$

[Out] $(-8*a^2*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*\text{Cos}[e + f*x])/(105*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*a*(35*A*c + 21*B*c + 21*A*d + 19*B*d)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(105*f) - (2*(7*B*c + 7*A*d - 2*B*d)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2})/(35*f) - (2*B*d*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{5/2})/(7*a*f)$

Rule 2646

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] := \text{Simp}[(-2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2647

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]^{(n_)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rule 2751

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)]), x_Symbol] := -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rule 2968

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]^{(m_)*((A_) + (B_)*\sin[(e_) + (f_)*(x_)]), x_Symbol] := \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx))^{3/2} (Ac + (Bc + Ad) \sin(e + fx) \\ &= -\frac{2Bd \cos(e + fx)(a + a \sin(e + fx))^{5/2}}{7af} + \frac{2 \int (a + a \sin(e + fx))^{3/2} (Bc + Ad) \sin(e + fx) dx}{7af} \\ &= -\frac{2(7Bc + 7Ad - 2Bd) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{35f} \\ &= -\frac{2a(35Ac + 21Bc + 21Ad + 19Bd) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{105f} \\ &= -\frac{8a^2(35Ac + 21Bc + 21Ad + 19Bd) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{105f \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [A] time = 1.07, size = 144, normalized size = 0.87

$$\frac{a \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) ((140Ac + 252Ad + 252Bc + 253Bd) \sin(e + fx) - 60a \cos(e + fx))}{210f \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]), x]
```

```
[Out] -1/210*(a*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(700*A*c + 546*B*c + 546*A*d + 494*B*d - 6*(7*B*c + 7*A*d + 13*B*d)*Cos[2*(e + f*x)] + (140*A*c + 252*B*c + 252*A*d + 253*B*d)*Sin[e + f*x] - 15*B*d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

fricas [A] time = 0.45, size = 257, normalized size = 1.56

$$\frac{2 \left(15Bad \cos(fx + e)^4 + 3(7Bac + (7A + 13B)ad) \cos(fx + e)^3 - 28(5A + 3B)ac - 4(21A + 19B)ad - (7(5A + 3B)ac + 4(21A + 19B)ad - (7(5A + 6B)ac + (42A + 43B)ad) \cos(fx + e)^2 - (7(25A + 21B)ac + (147A + 143B)ad) \cos(fx + e) + (15B*ad \cos(fx + e)^3 + 28(5A + 3B)ac + 4(21A + 19B)ad - 3(7B*ac + (7A + 8B)ad) \cos(fx + e)^2 - (7(5A + 9B)ac + (63A + 67B)ad) \cos(fx + e)) \sin(fx + e) \right) \sqrt{a \sin(fx + e) + a}}{(f \cos(fx + e) + f \sin(fx + e) + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2/105*(15*B*a*d*cos(f*x + e)^4 + 3*(7*B*a*c + (7*A + 13*B)*a*d)*cos(f*x + e)^3 - 28*(5*A + 3*B)*a*c - 4*(21*A + 19*B)*a*d - (7*(5*A + 6*B)*a*c + (42*A + 43*B)*a*d)*cos(f*x + e)^2 - (7*(25*A + 21*B)*a*c + (147*A + 143*B)*a*d)*cos(f*x + e) + (15*B*a*d*cos(f*x + e)^3 + 28*(5*A + 3*B)*a*c + 4*(21*A + 19*B)*a*d - 3*(7*B*a*c + (7*A + 8*B)*a*d)*cos(f*x + e)^2 - (7*(5*A + 9*B)*a*c + (63*A + 67*B)*a*d)*cos(f*x + e))*sin(f*x + e))/sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)
```


[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)), x)

3.296 $\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A + 3B) \cos(e + fx)}{15f\sqrt{a \sin(e + fx) + a}} - \frac{2a(5A + 3B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} - \frac{2B \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{5f}$$

[Out] $-2/5*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/f-8/15*a^2*(5*A+3*B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(1/2)}-2/15*a*(5*A+3*B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/f$

Rubi [A] time = 0.09, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2751, 2647, 2646}

$$\frac{8a^2(5A + 3B) \cos(e + fx)}{15f\sqrt{a \sin(e + fx) + a}} - \frac{2a(5A + 3B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{15f} - \frac{2B \cos(e + fx)(a \sin(e + fx) + a)^{3/2}}{5f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{(3/2)}*(A + B*\text{Sin}[e + f*x]), x]$

[Out] $(-8*a^2*(5*A + 3*B)*\text{Cos}[e + f*x])/(15*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*a*(5*A + 3*B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(15*f) - (2*B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{(3/2)})/(5*f)$

Rule 2646

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2647

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rule 2751

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) dx &= -\frac{2B \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} + \frac{1}{5}(5A + 3B) \int (a + a \sin(e + fx))^{3/2} dx \\ &= -\frac{2a(5A + 3B) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} - \frac{2B \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5f} \\ &= -\frac{8a^2(5A + 3B) \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{2a(5A + 3B) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{15f} \end{aligned}$$

[Out] $2/15*(1+\sin(f*x+e))*a^2*(\sin(f*x+e)-1)*(\sin(f*x+e)*(5*A+9*B)-3*B*\cos(f*x+e))^2+25*A+21*B)/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2),x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^{\frac{3}{2}} (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e)),x)`

[Out] `Integral((a*(sin(e + f*x) + 1))**(3/2)*(A + B*sin(e + f*x)), x)`

$$3.297 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=153

$$\frac{2a^{3/2}(c-d)(Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d} \cos(e+fx)}{\sqrt{c+d}\sqrt{a \sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}} + \frac{2a^2(-3Ad+3Bc-4Bd) \cos(e+fx)}{3d^2f\sqrt{a \sin(e+fx)+a}} - \frac{2aB \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{3df}$$

[Out] $-2*a^{(3/2)}*(c-d)*(-A*d+B*c)*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/d^{(5/2)}/f/(c+d)^{(1/2)}+2/3*a^2*(-3*A*d+3*B*c-4*B*d)*\cos(f*x+e)/d^2/f/(a+a*\sin(f*x+e))^{(1/2)}-2/3*a*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/d/f$

Rubi [A] time = 0.50, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^2(-3Ad+3Bc-4Bd) \cos(e+fx)}{3d^2f\sqrt{a \sin(e+fx)+a}} - \frac{2a^{3/2}(c-d)(Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d} \cos(e+fx)}{\sqrt{c+d}\sqrt{a \sin(e+fx)+a}}\right)}{d^{5/2}f\sqrt{c+d}} - \frac{2aB \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{3df}$$

Antiderivative was successfully verified.

[In] `Int[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]), x]`

[Out] $(-2*a^{(3/2)}*(c-d)*(B*c-A*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\cos[e+f*x])/(\operatorname{Sqrt}[c+d]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])]/(d^{(5/2)}*\operatorname{Sqrt}[c+d]*f) + (2*a^2*(3*B*c-3*A*d-4*B*d)*\cos[e+f*x])/(3*d^2*f*\operatorname{Sqrt}[a+a*\sin[e+f*x]]) - (2*a*B*\cos[e+f*x]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])/(3*d*f)$

Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 2773

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]`

Rule 2976

`Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^(n+1))/(d*f*(m+n+1)), x] + Dist[1/(d*(m+n+1)), Int[(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m+n+1) + B*(a*c*(m-1) + b*d*(n+1)) + (A*b*d*(m+n+1) - B*(b*c*m - a*d*(2*m+n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])`

Rule 2981

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp`

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2}(A + B \sin(e + fx))}{c + d \sin(e + fx)} dx = -\frac{2aB \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{3df} + \frac{2 \int \frac{\sqrt{a+a \sin(e+fx)} \left(\frac{1}{2}a\right)}{c + d \sin(e + fx)} dx}{3df}$$

$$= \frac{2a^2(3Bc - 3Ad - 4Bd) \cos(e + fx)}{3d^2 f \sqrt{a + a \sin(e + fx)}} - \frac{2aB \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{3df}$$

$$= \frac{2a^2(3Bc - 3Ad - 4Bd) \cos(e + fx)}{3d^2 f \sqrt{a + a \sin(e + fx)}} - \frac{2aB \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{3df}$$

$$= -\frac{2a^{3/2}(c - d)(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a+a \sin(e+fx)}}\right)}{d^{5/2} \sqrt{c + d} f} + \frac{2a^2(3Bc - 3Ad - 4Bd) \cos(e + fx)}{3d^2 f \sqrt{a + a \sin(e + fx)}}$$

Mathematica [B] time = 3.54, size = 356, normalized size = 2.33

$$(a(\sin(e + fx) + 1))^{3/2} \left(6\sqrt{d} (2Ad - 2Bc + 3Bd) \sin\left(\frac{1}{2}(e + fx)\right) - 6\sqrt{d} (2Ad - 2Bc + 3Bd) \cos\left(\frac{1}{2}(e + fx)\right) + \dots \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x]), x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-6*Sqrt[d]*(-2*B*c + 2*A*d + 3*B*d)*Cos[(e +
f*x)/2] - 2*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(B*c - A*d)*(e + f
*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d
]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2])))]/
Sqrt[c + d] + (3*(c - d)*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*
x)/4] + Tan[(e + f*x)/4]^2)))/Sqrt[c + d] + 6*Sqrt[d]*(-2*B*c + 2*A*d + 3*
B*d)*Sin[(e + f*x)/2] - 2*B*d^(3/2)*Sin[(3*(e + f*x))/2]))/(6*d^(5/2)*f*(Co
s[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 1.47, size = 880, normalized size = 5.75

$$\left[\frac{3(Bac^2 - (A + B)acd + Aad^2 + (Bac^2 - (A + B)acd + Aad^2) \cos(fx + e) + (Bac^2 - (A + B)acd + Aad^2) \sin(fx + e)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*(B*a*c^2 - (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*
a*d^2)*cos(f*x + e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt
(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a
*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)
*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2
+ 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) +
a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*
cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x +
e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2
- 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(
f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))] + 4*(B*a*d*cos(f*x + e)^2 - 3*
B*a*c + (3*A + 4*B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d
*cos(f*x + e) + 3*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e
) + a))/(d^2*f*cos(f*x + e) + d^2*f*sin(f*x + e) + d^2*f), -1/3*(3*(B*a*c^2
- (A + B)*a*c*d + A*a*d^2 + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*cos(f*x +
e) + (B*a*c^2 - (A + B)*a*c*d + A*a*d^2)*sin(f*x + e))*sqrt(-a/(c*d + d^2))
*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*
d + d^2)))/(a*cos(f*x + e))] + 2*(B*a*d*cos(f*x + e)^2 - 3*B*a*c + (3*A + 4*
B)*a*d - (3*B*a*c - (3*A + 5*B)*a*d)*cos(f*x + e) + (B*a*d*cos(f*x + e) + 3
*B*a*c - (3*A + 4*B)*a*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^2*f*co
s(f*x + e) + d^2*f*sin(f*x + e) + d^2*f)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

[Out] Timed out

maple [B] time = 1.73, size = 291, normalized size = 1.90

$$2(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(3A \operatorname{arctanh} \left(\frac{\sqrt{-a(\sin(fx + e) - 1)} d}{\sqrt{a(c + d)d}} \right) a^2 cd - 3A \operatorname{arctanh} \left(\frac{\sqrt{-a(\sin(fx + e) - 1)}}{\sqrt{a(c + d)d}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
```

```
[Out] 2/3*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d-3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*d^2-3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2+3*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c*d+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d+3*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c-6*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d)/d^2/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{d \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{3/2}}{c + d \sin(e + f x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.298 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=191

$$\frac{a^{3/2} (Ad(c+3d) - B(3c^2 + 3cd - 2d^2)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{5/2} f (c+d)^{3/2}} - \frac{a^2 (-Ad + 3Bc + 2Bd) \cos(e+fx)}{d^2 f (c+d) \sqrt{a \sin(e+fx)+a}} + \frac{a(Bc - Ad)}{d^2 f (c+d)}$$

[Out] $-a^{(3/2)}*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)/(c+d)^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)/d^{(5/2)/(c+d)^{(3/2)/f-a^2*(-A*d+3*B*c+2*B*d)*\cos(f*x+e)/d^2/(c+d)/f/(a+a*\sin(f*x+e))^{(1/2)+a*(-A*d+B*c)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)/d/(c+d)/f/(c+d*\sin(f*x+e))}}$

Rubi [A] time = 0.55, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2975, 2981, 2773, 208}

$$\frac{a^{3/2} (Ad(c+3d) - B(3c^2 + 3cd - 2d^2)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{5/2} f (c+d)^{3/2}} - \frac{a^2 (-Ad + 3Bc + 2Bd) \cos(e+fx)}{d^2 f (c+d) \sqrt{a \sin(e+fx)+a}} + \frac{a(Bc - Ad)}{d^2 f (c+d)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\frac{(a + a*\operatorname{Sin}[e + f*x])^{(3/2)}*(A + B*\operatorname{Sin}[e + f*x])}{(c + d*\operatorname{Sin}[e + f*x])^2}, x]$

[Out] $-\left(\frac{a^{(3/2)}*(A*d*(c+3*d)-B*(3*c^2+3*c*d-2*d^2))*\operatorname{ArcTanh}[\frac{\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e+f*x]}{\operatorname{Sqrt}[c+d]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]}]}{d^{(5/2)}*(c+d)^{(3/2)}*f}\right) - \frac{a^2*(3*B*c-A*d+2*B*d)*\operatorname{Cos}[e+f*x]}{d^2*(c+d)*f*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]} + \frac{a*(B*c-A*d)*\operatorname{Cos}[e+f*x]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]]}{d*(c+d)*f*(c+d*\operatorname{Sin}[e+f*x])}$

Rule 208

$\operatorname{Int}[\frac{(a_ + (b_)*(x_)^2)^{-1}}{x_Symbol}] \rightarrow \operatorname{Simp}[\frac{\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]]}{a, x}] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 2773

$\operatorname{Int}[\frac{\operatorname{Sqrt}[(a_ + (b_)*\operatorname{sin}[(e_ + (f_)*(x_))]/((c_ + (d_)*\operatorname{sin}[(e_ + (f_)*(x_))])]}{x_Symbol}] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\operatorname{Cos}[e + f*x])/ \operatorname{Sqrt}[a + b*\operatorname{Sin}[e + f*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2975

$\operatorname{Int}[\frac{(a_ + (b_)*\operatorname{sin}[(e_ + (f_)*(x_))]^m * ((A_ + (B_)*\operatorname{sin}[(e_ + (f_)*(x_))])^n}{x_Symbol}] \rightarrow -\operatorname{Simp}[\frac{b^2*(B*c - A*d)*\operatorname{Cos}[e + f*x]*(a + b*\operatorname{Sin}[e + f*x])^{m-1}*(c + d*\operatorname{Sin}[e + f*x])^{n+1}}{d*f*(n+1)*(b*c + a*d)}, x] - \operatorname{Dist}[b/(d*(n+1)*(b*c + a*d)), \operatorname{Int}[(a + b*\operatorname{Sin}[e + f*x])^{m-1}*(c + d*\operatorname{Sin}[e + f*x])^{n+1}*\operatorname{Simp}[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1))]*\operatorname{Sin}[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \operatorname{GtQ}[m, 1/2] \ \&\& \operatorname{LtQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& (\operatorname{IntegerQ}[2*n] \ \|\ \operatorname{EqQ}[c, 0])]$

Rule 2981

$\operatorname{Int}[\frac{\operatorname{Sqrt}[(a_ + (b_)*\operatorname{sin}[(e_ + (f_)*(x_))]]*(A_ + (B_)*\operatorname{sin}[(e_ + (f_)*(x_))])^n}{x_Symbol}] \rightarrow \operatorname{Simp}$


```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2}(A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx = \frac{a(Bc - Ad) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{d(c + d)f(c + d \sin(e + fx))} + \int \frac{\sqrt{a + a \sin(e + fx)}}{\dots}$$

$$= -\frac{a^2(3Bc - Ad + 2Bd) \cos(e + fx)}{d^2(c + d)f \sqrt{a + a \sin(e + fx)}} + \frac{a(Bc - Ad) \cos(e + fx)}{d(c + d)f(c + d)}$$

$$= -\frac{a^2(3Bc - Ad + 2Bd) \cos(e + fx)}{d^2(c + d)f \sqrt{a + a \sin(e + fx)}} + \frac{a(Bc - Ad) \cos(e + fx)}{d(c + d)f(c + d)}$$

$$= -\frac{a^{3/2} (Ad(c + 3d) - B(3c^2 + 3cd - 2d^2)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c+d} \sqrt{a+a \sin(e+fx)}} \right)}{d^{5/2}(c + d)^{3/2} f}$$

Mathematica [A] time = 5.01, size = 381, normalized size = 1.99

$$(a(\sin(e + fx) + 1))^{3/2} \left(\frac{(Ad(c+3d)+B(-3c^2-3cd+2d^2)) \left(2 \log \left(\sqrt{d} \sqrt{c+d} \left(\tan^2 \left(\frac{1}{4}(e+fx) \right) + 2 \tan \left(\frac{1}{4}(e+fx) \right) - 1 \right) + (c+d) \sec^2 \left(\frac{1}{4}(e+fx) \right) \right) - 2 \log \dots}{(c+d)^{3/2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-8*B*Sqrt[d]*Cos[(e + f*x)/2] + ((-(A*d*(c +
3*d)) + B*(3*c^2 + 3*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] +
2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] -
Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2])))))/(c + d)^(3/2) + ((A*d*(c + 3*d) +
B*(-3*c^2 - 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c
+ d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] +
Tan[(e + f*x)/4]^2)))/(c + d)^(3/2) + 8*B*Sqrt[d]*Sin[(e + f*x)/2] - (4*Sqr
rt[d]*(-c + d)*(-B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c +
d)*(c + d*Sin[e + f*x])))/(4*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^3)
```

fricas [B] time = 1.68, size = 1428, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, alg
orithm="fricas")
```

```
[Out] [1/4*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^
3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 +
```

```
(3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e)), -1/2*((3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 - (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (A - 3*B)*a*c^2*d - (3*A + 2*B)*a*c*d^2)*cos(f*x + e) + (3*B*a*c^3 - (A - 6*B)*a*c^2*d - (4*A - B)*a*c*d^2 - (3*A + 2*B)*a*d^3 + (3*B*a*c^2*d - (A - 3*B)*a*c*d^2 - (3*A + 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2)))/(a*cos(f*x + e))) - 2*(3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 + 2*(B*a*c*d + B*a*d^2)*cos(f*x + e)^2 + (3*B*a*c^2 - (A - B)*a*c*d + A*a*d^2)*cos(f*x + e) - (3*B*a*c^2 - (A + B)*a*c*d + (A - 2*B)*a*d^2 - 2*(B*a*c*d + B*a*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^3 + d^4)*f*cos(f*x + e)^2 - (c^2*d^2 + c*d^3)*f*cos(f*x + e) - (c^2*d^2 + 2*c*d^3 + d^4)*f - ((c*d^3 + d^4)*f*cos(f*x + e) + (c^2*d^2 + 2*c*d^3 + d^4)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.32, size = 592, normalized size = 3.10

$$a(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(-\sin(fx + e) d \left(A \operatorname{arctanh} \left(\frac{\sqrt{a-a \sin(fx+e)} d}{\sqrt{acd+ad^2}} \right) acd + 3A \operatorname{arctanh} \left(\frac{\sqrt{a}}{\sqrt{acd+ad^2}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

```
[Out] a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-sin(f*x+e)*d*(A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2-3*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d+2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*d^2+2*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*c+2*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*d-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c^2*d-3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a*c*d
```

$2+3*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2)^{1/2})*a*c^3+3*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2)^{1/2})*a*c^2*d-2*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2)^{1/2})*a*c*d^2+A*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*c*d-A*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*d^2-3*B*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*c^2-B*(a-a*\sin(f*x+e))^{1/2}*(a*(c+d)*d)^{1/2}*c*d/d^2/(c+d)/(c+d*\sin(f*x+e))/(a*(c+d)*d)^{1/2}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}}{(d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{3}{2}}}{(c + d \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out

$$3.299 \quad \int \frac{(a+a \sin(e+fx))^{3/2}(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=221

$$\frac{a^{3/2} \left(Ad(c+7d) + 3B(c^2 + 3cd + 4d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{4d^{5/2} f(c+d)^{5/2}} + \frac{a^2 \left(Ad(c-5d) + B(3c^2 + 5cd - 4d^2) \right) \cos(e+fx)}{4d^2 f(c+d)^2 \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))}$$

[Out] $-1/4*a^{(3/2)}*(A*d*(c+7*d)+3*B*(c^2+3*c*d+4*d^2))*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)/(c+d)^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})}/d^{(5/2)/(c+d)^{(5/2)/f+1/4*a^2*(A*(c-5*d)*d+B*(3*c^2+5*c*d-4*d^2))*\cos(f*x+e)/d^2/(c+d)^2/f/(c+d*\sin(f*x+e)))/(a+a*\sin(f*x+e))^{(1/2)+1/2*a*(-A*d+B*c)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)})}/d/(c+d)/f/(c+d*\sin(f*x+e))^2$

Rubi [A] time = 0.61, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2975, 2980, 2773, 208}

$$\frac{a^2 \left(Ad(c-5d) + B(3c^2 + 5cd - 4d^2) \right) \cos(e+fx)}{4d^2 f(c+d)^2 \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))} - \frac{a^{3/2} \left(Ad(c+7d) + 3B(c^2 + 3cd + 4d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{4d^{5/2} f(c+d)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\sin[e + f*x])^{(3/2)}*(A + B*\sin[e + f*x])]/(c + d*\sin[e + f*x])^3, x]$

[Out] $-(a^{(3/2)}*(A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])]/(4*d^{(5/2)}*(c + d)^{(5/2)*f} + (a*(B*c - A*d)*\operatorname{Cos}[e + f*x]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])/(2*d*(c + d)*f*(c + d*\sin[e + f*x])^2) + (a^2*(A*(c - 5*d)*d + B*(3*c^2 + 5*c*d - 4*d^2))*\operatorname{Cos}[e + f*x])/4*d^2*(c + d)^2*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]]*(c + d*\sin[e + f*x]))$

Rule 208

$\operatorname{Int}[(a + (b_*)*(x_*)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])]/((c + (d_*)*\sin[(e_*) + (f_*)*(x_*)])], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[a + b*\sin[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2975

$\operatorname{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)])^{(m_*)}*((A + (B_*)*\sin[(e_*) + (f_*)*(x_*)])^{(n_*)})], x_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(B*c - A*d)*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m-1)}*(c + d*\sin[e + f*x])^{(n+1)})/(d*f*(n+1)*(b*c + a*d)), x] - \operatorname{Dist}[b/(d*(n+1)*(b*c + a*d)), \operatorname{Int}[(a + b*\sin[e + f*x])^{(m-1)}*(c + d*\sin[e + f*x])^{(n+1)}*\operatorname{Simp}[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1))]*\sin[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \ \operatorname{NeQ}[c^2 - d^2, 0] \ \&\& \ \operatorname{GtQ}[m, 1/2] \ \&\& \ \operatorname{LtQ}[n, -1] \ \&\& \ \operatorname{IntegerQ}[2*m] \ \&\& \ (\operatorname{IntegerQ}[2*n] \ || \ \operatorname{EqQ}[c, 0])$

Rule 2980

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]]], x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{3/2}(A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx = \frac{a(Bc - Ad) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{\int \frac{\sqrt{a + a \sin(e + fx)}}{(c + d \sin(e + fx))^2} dx}{1}$$

$$= \frac{a(Bc - Ad) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{a^2 (A(c - 5d))}{4d^2(c + d)^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{a(Bc - Ad) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{a^2 (A(c - 5d))}{4d^2(c + d)^2 f \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{a^{3/2} (Ad(c + 7d) + 3B(c^2 + 3cd + 4d^2)) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c+d} \sqrt{a+a \sin(e+fx)}} \right)}{4d^{5/2}(c + d)^{5/2} f}$$

Mathematica [A] time = 5.34, size = 416, normalized size = 1.88

$$(a(\sin(e + fx) + 1))^{3/2} \left(-\frac{4\sqrt{d}(Ad(c+7d)+B(-5c^2-7cd+4d^2))(\cos(\frac{1}{2}(e+fx))-\sin(\frac{1}{2}(e+fx)))}{(c+d)^2(c+d \sin(e+fx))} + \frac{(Ad(c+7d)+3B(c^2+3cd+4d^2))(2 \log(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a+a \sin(e+fx)}}))}{(c+d)^2(c+d \sin(e+fx))} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3, x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(3/2)*(-(((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2])])))/(c + d)^(5/2)) + ((A*d*(c + 7*d) + 3*B*(c^2 + 3*c*d + 4*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2])))/(c + d)^(5/2) - (8*Sqrt[d]*(-c + d)*(-B*c) + A*d*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*Sin[e + f*x])^2) - (4*Sqrt[d]*(A*d*(c + 7*d) + B*(-5*c^2 - 7*c*d + 4*d^2))*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)^2*(c + d*Sin[e + f*x])))/(16*d^(5/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3)
```

fricas [B] time = 2.59, size = 2208, normalized size = 9.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] [-1/16*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A
+ 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3
+ (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3*d + (2*A + 21*B)*a*c^2*d^
2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + (3*B*a*c^
4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A +
12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A + 11*
B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^2*d^2
+ (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^2 + 2*(3*B*a*c^3*d
+ (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos(f*x + e))*sin(f*x + e))*s
qrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6
*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^
3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d
^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^
2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x
+ e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^
2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*co
s(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a*c^3 + (A + 2*B)*a
*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 + (5*B*a*c^2*d - (A - 7*
B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 + (A + 7*B)*a*c
^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x + e) - (3*B*a*c^3 + (A + 2*
B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^3 - (5*B*a*c^2*d - (A
- 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(
f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^3 + (2*c^3*d^3 + 5
*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4*d^2 + 2*c^3*d^3 + 2*c^2*d
^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c
*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e)^2 - 2*(c^3*d^3 +
2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*c^3*d^3 + 6*c^2*d^4 + 4*c*
d^5 + d^6)*f)*sin(f*x + e)), 1/8*((3*B*a*c^4 + (A + 15*B)*a*c^3*d + 3*(3*A
+ 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4 - (3*B*a*c^
2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e)^3 - (6*B*a*c^3
*d + (2*A + 21*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*
cos(f*x + e)^2 + (3*B*a*c^4 + (A + 9*B)*a*c^3*d + (7*A + 15*B)*a*c^2*d^2 +
(A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*x + e) + (3*B*a*c^4 + (A + 15
*B)*a*c^3*d + 3*(3*A + 11*B)*a*c^2*d^2 + 3*(5*A + 11*B)*a*c*d^3 + (7*A + 12
*B)*a*d^4 - (3*B*a*c^2*d^2 + (A + 9*B)*a*c*d^3 + (7*A + 12*B)*a*d^4)*cos(f*
x + e)^2 + 2*(3*B*a*c^3*d + (A + 9*B)*a*c^2*d^2 + (7*A + 12*B)*a*c*d^3)*cos
(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e)
+ a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) -
2*(3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*a*d^
3 + (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e)^2 +
(3*B*a*c^3 + (A + 7*B)*a*c^2*d - (7*A + 2*B)*a*c*d^2 - 2*A*a*d^3)*cos(f*x +
e) - (3*B*a*c^3 + (A + 2*B)*a*c^2*d - 3*(2*A + 3*B)*a*c*d^2 + (5*A + 4*B)*
a*d^3 - (5*B*a*c^2*d - (A - 7*B)*a*c*d^2 - (7*A + 4*B)*a*d^3)*cos(f*x + e))
*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f
*x + e)^3 + (2*c^3*d^3 + 5*c^2*d^4 + 4*c*d^5 + d^6)*f*cos(f*x + e)^2 - (c^4
*d^2 + 2*c^3*d^3 + 2*c^2*d^4 + 2*c*d^5 + d^6)*f*cos(f*x + e) - (c^4*d^2 + 4
*c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f + ((c^2*d^4 + 2*c*d^5 + d^6)*f*cos(
f*x + e)^2 - 2*(c^3*d^3 + 2*c^2*d^4 + c*d^5)*f*cos(f*x + e) - (c^4*d^2 + 4*
c^3*d^3 + 6*c^2*d^4 + 4*c*d^5 + d^6)*f)*sin(f*x + e))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

[Out] Timed out

maple [B] time = 2.82, size = 895, normalized size = 4.05

$$\left(-2 \sin (f x+e) \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} d}{\sqrt{a c d+a d^2}}\right) a^2 c d\left(A c d+7 A d^2+3 B c^2+9 B c d+12 B d^2\right)+\operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} d}{\sqrt{a c d+a d^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)

[Out] 1/4*(-2*sin(f*x+e)*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)+arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^2*(A*c*d+7*A*d^2+3*B*c^2+9*B*c*d+12*B*d^2)*cos(f*x+e)^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-7*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4+A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+7*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3-3*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4-9*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d-15*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-9*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-12*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^4-5*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d-7*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2+4*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^3+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d-8*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-9*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3+3*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d+5*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2-4*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin (f x+e)+A)(a \sin (f x+e)+a)^{\frac{3}{2}}}{(d \sin (f x+e)+c)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)/(d*sin(f*x + e) + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A+B \sin (e+f x))(a+a \sin (e+f x))^{\frac{3}{2}}}{(c+d \sin (e+f x))^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^3,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2))/(c + d*sin(e + f*x))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)

[Out] Timed out

$$3.300 \quad \int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^3 dx$$

Optimal. Leaf size=534

$$\frac{2a^3 (-39Acd + 299Ad^2 + 15Bc^2 - 75Bcd + 280Bd^2) \cos(e + fx)(c + d \sin(e + fx))^4}{1287d^3 f \sqrt{a \sin(e + fx) + a}} - \frac{2a^3 (13Ad(3c^2 - 38ca$$

[Out] $-4/15015*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/d/f-2/13*a*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)*(c+d*\sin(f*x+e))^4/d/f-4/45045*a^3*(c+d)*(15*c^2+10*c*d+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos(f*x+e)/d^3/f/(a+a*\sin(f*x+e))^{(1/2)}-2/9009*a^3*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/d^3/f/(a+a*\sin(f*x+e))^{(1/2)}-2/1287*a^3*(-39*A*c*d+299*A*d^2+15*B*c^2-75*B*c*d+280*B*d^2)*\cos(f*x+e)*(c+d*\sin(f*x+e))^4/d^3/f/(a+a*\sin(f*x+e))^{(1/2)}-8/45045*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/d^2/f+2/143*a^2*(-13*A*d+5*B*c-16*B*d)*\cos(f*x+e)*(c+d*\sin(f*x+e))^4*(a+a*\sin(f*x+e))^{(1/2)}/d^2/f$

Rubi [A] time = 1.20, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2976, 2981, 2770, 2761, 2751, 2646}

$$\frac{2a^3 (-39Acd + 299Ad^2 + 15Bc^2 - 75Bcd + 280Bd^2) \cos(e + fx)(c + d \sin(e + fx))^4}{1287d^3 f \sqrt{a \sin(e + fx) + a}} - \frac{2a^3 (13Ad(3c^2 - 38ca$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3, x]

[Out] $(-4*a^3*(c+d)*(15*c^2+10*c*d+7*d^2)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos[e+f*x]/(45045*d^3*f*\sqrt{a+a*\sin[e+f*x]})-(8*a^2*(5*c-d)*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos[e+f*x]*\sqrt{a+a*\sin[e+f*x]}/(45045*d^2*f)-(4*a*(c+d)*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos[e+f*x]*(a+a*\sin[e+f*x])^{(3/2)}/(15015*d*f)-(2*a^3*(13*A*d*(3*c^2-38*c*d+355*d^2)-B*(15*c^3-150*c^2*d+799*c*d^2-4184*d^3))*\cos[e+f*x]*(c+d*\sin[e+f*x])^3/(9009*d^3*f*\sqrt{a+a*\sin[e+f*x]})-(2*a^3*(15*B*c^2-39*A*c*d-75*B*c*d+299*A*d^2+280*B*d^2))*\cos[e+f*x]*(c+d*\sin[e+f*x])^4/(1287*d^3*f*\sqrt{a+a*\sin[e+f*x]})+(2*a^2*(5*B*c-13*A*d-16*B*d))*\cos[e+f*x]*\sqrt{a+a*\sin[e+f*x]}*(c+d*\sin[e+f*x])^4/(143*d^2*f)-(2*a*B*\cos[e+f*x]*(a+a*\sin[e+f*x])^{(3/2)}*(c+d*\sin[e+f*x])^4)/(13*d*f)$

Rule 2646

Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e +

$f*x])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rule 2761

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))^m]*((c_ + (d_)*\sin[(e_ + (f_)*(x_)]))^2, x_Symbol] \text{:>} -\text{Simp}[(d^2*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m+1})/(b*f*(m + 2)), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -1]$

Rule 2770

$\text{Int}[\text{Sqrt}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))*(c_ + (d_)*\sin[(e_ + (f_)*(x_)]))^n], x_Symbol] \text{:>} \text{Simp}[(-2*b*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*(2*n + 1)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(2*n*(b*c + a*d))/(b*(2*n + 1)), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{n-1}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2976

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))^m]*((A_ + (B_)*\sin[(e_ + (f_)*(x_)]))^n), x_Symbol] \text{:>} -\text{Simp}[(b*B*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{m-1}*(c + d*\text{Sin}[e + f*x])^{n+1})/(d*f*(m + n + 1)), x] + \text{Dist}[1/(d*(m + n + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{m-1}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{GtQ}[m, 1/2] \ \&\& \ !\text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rule 2981

$\text{Int}[\text{Sqrt}[(a_ + (b_)*\sin[(e_ + (f_)*(x_)]))*(A_ + (B_)*\sin[(e_ + (f_)*(x_)]))^n], x_Symbol] \text{:>} \text{Simp}[(-2*b*B*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{n+1})/(d*f*(2*n + 3)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]), x] + \text{Dist}[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), \text{Int}[\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ !\text{LtQ}[n, -1]$

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^3 dx &= -\frac{2aB \cos(e + fx) (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^3}{13df} \\
&= \frac{2a^2(5Bc - 13Ad - 16Bd) \cos(e + fx) \sqrt{a - d \sin(e + fx)}}{143d^2 f} \\
&= -\frac{2a^3 (15Bc^2 - 39Acd - 75Bcd + 299Ad^2)}{1287d^3 f \sqrt{a - d \sin(e + fx)}} \\
&= -\frac{2a^3 (13Ad (3c^2 - 38cd + 355d^2) - B (15c^2 + 10cd + 7d^2))}{9} \\
&= -\frac{4a(c + d) (13Ad (3c^2 - 38cd + 355d^2) - B (15c^2 + 10cd + 7d^2))}{8a^2(5c - d)(c + d) (13Ad (3c^2 - 38cd + 355d^2) - B (15c^2 + 10cd + 7d^2))} \\
&= -\frac{4a^3(c + d) (15c^2 + 10cd + 7d^2) (13Ad (3c^2 - 38cd + 355d^2) - B (15c^2 + 10cd + 7d^2))}{8a^2(5c - d)(c + d) (13Ad (3c^2 - 38cd + 355d^2) - B (15c^2 + 10cd + 7d^2))}
\end{aligned}$$

Mathematica [C] time = 6.87, size = 1565, normalized size = 2.93

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3,x]

[Out] (B*d^3*Cos[(13*(e + f*x))/2]*(a*(1 + Sin[e + f*x]))^(5/2))/(416*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 - I/16)*Cos[(e + f*x)/2] + (1/16 - I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((40*A*c^3 + 30*B*c^3 + 90*A*c^2*d + 78*B*c^2*d + 78*A*c*d^2 + 69*B*c*d^2 + 23*A*d^3 + 21*B*d^3)*((-1/16 + I/16)*Cos[(e + f*x)/2] + (1/16 + I/16)*Sin[(e + f*x)/2])*(a*(1 + Sin[e + f*x]))^(5/2))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 + I/192)*Cos[(3*(e + f*x))/2] - (1/192 + I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((80*A*c^3 + 88*B*c^3 + 264*A*c^2*d + 240*B*c^2*d + 240*A*c*d^2 + 228*B*c*d^2 + 76*A*d^3 + 71*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((-1/192 - I/192)*Cos[(3*(e + f*x))/2] - (1/192 - I/192)*Sin[(3*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 - I/320)*Cos[(5*(e + f*x))/2] - (1/320 + I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((16*A*c^3 + 40*B*c^3 + 120*A*c^2*d + 144*B*c^2*d + 144*A*c*d^2 + 150*B*c*d^2 + 50*A*d^3 + 51*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/320 + I/320)*Cos[(5*(e + f*x))/2] - (1/320 - I/320)*Sin[(5*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/224 + I/224)*Cos[(7*(e + f*x))/2] + (1/224 - I/224)*Sin[(7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5) + ((4*B*c^3 + 12*A*c^2*d + 30*B*c^2*d + 30*A*c*d^2 + 39*B*c*d^2 + 13*A*d^3 + 15*B*d^3)*(a*(1 + Sin[e + f*x]))^(5/2)*((1/224 - I/224)*Cos[(7*(e + f*x))/2] - (1/224 + I/224)*Sin[(7*(e + f*x))/2]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)


```

xp(1))-1/4*pi))-24*A*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-54*B*a^2*
c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-48*B*a^2*c^2*d*sign(cos(1/2*(f*x+e
xp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)-pi))/(224*f)^2-16*f*(32*A*a^2*c^
3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+22*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))
-1/4*pi))+28*B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+20*B*a^2*d^3*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))+72*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))+84*A*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+66*B*a^2*c*d^2*sign
(cos(1/2*(f*x+exp(1))-1/4*pi))+72*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*cos(1/4*(2*f*x+2*exp(1)+pi))/(16*f)^2-48*f*(32*A*a^2*c^3*sign(cos(1/
2*(f*x+exp(1))-1/4*pi))+22*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+28*
B*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+20*B*a^2*d^3*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+72*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+84*A*a^
2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+66*B*a^2*c*d^2*sign(cos(1/2*(f*x
+exp(1))-1/4*pi))+72*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/
4*(6*f*x+6*exp(1)-pi))/(48*f)^2+16*f*(48*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1)
)-1/4*pi))+24*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+32*B*a^2*c^3*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))+22*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+84*A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+96*A*a^2*c^2*d*sign(
cos(1/2*(f*x+exp(1))-1/4*pi))+72*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*
pi))+84*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(2*f*x-pi)+
1/2*exp(1))/(16*f)^2+192*f*(-32*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi)
)-64*A*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*B*a^2*c^3*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-62*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*
A*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*c^2*d*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-192*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-1
92*B*a^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(6*f*x+6*exp(1)+
pi))/(192*f)^2+320*f*(-32*A*a^2*c^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*A
*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-64*B*a^2*c^3*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))-62*B*a^2*d^3*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*
c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*A*a^2*c^2*d*sign(cos(1/2*(f*x+
exp(1))-1/4*pi))-192*B*a^2*c*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-192*B*a
^2*c^2*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(10*f*x+10*exp(1)-pi))
/(320*f)^2-1408*B*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*sin(1/4*(22*
f*x+22*exp(1)+pi))/(704*f)^2-1664*B*a^2*d^3*f*sign(cos(1/2*(f*x+exp(1))-1/4
*pi))*sin(1/4*(26*f*x+26*exp(1)-pi))/(832*f)^2

```

maple [A] time = 1.40, size = 374, normalized size = 0.70

$$\frac{2(1 + \sin(fx + e))a^3(\sin(fx + e) - 1)((4095Ad^3 + 12285Bcd^2 + 11970Bd^3)\sin(fx + e)(\cos^4(fx + e)) + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x)

```

[Out] 2/45045*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*((4095*A*d^3+12285*B*c*d^2+11970*
B*d^3)*sin(f*x+e)*cos(f*x+e)^4+(-19305*A*c^2*d-55770*A*c*d^2-31265*A*d^3-64
35*B*c^3-55770*B*c^2*d-93795*B*c*d^2-44860*B*d^3)*cos(f*x+e)^2*sin(f*x+e)+(
42042*A*c^3+167310*A*c^2*d+181038*A*c*d^2+64090*A*d^3+55770*B*c^3+181038*B*
c^2*d+192270*B*c*d^2+66362*B*d^3)*sin(f*x+e)-3465*B*d^3*cos(f*x+e)^6+(15015
*A*c*d^2+14560*A*d^3+15015*B*c^2*d+43680*B*c*d^2+28700*B*d^3)*cos(f*x+e)^4+
(-9009*A*c^3-77220*A*c^2*d-123981*A*c*d^2-56810*A*d^3-25740*B*c^3-123981*B*
c^2*d-170430*B*c*d^2-72109*B*d^3)*cos(f*x+e)^2+138138*A*c^3+373230*A*c^2*d+
359502*A*c*d^2+116090*A*d^3+124410*B*c^3+359502*B*c^2*d+348270*B*c*d^2+1138
18*B*d^3)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (d \sin(fx + e) + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3, x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

3.301 $\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$

Optimal. Leaf size=429

$$\frac{2a^3 (11Ad(3c - 19d) - B(15c^2 - 65cd + 194d^2)) \cos(e + fx)(c + d \sin(e + fx))^3 - 2a^3 (15c^2 + 10cd + 7d^2) (11Ad - 693d^3 f \sqrt{a \sin(e + fx) + a})}{693d^3 f \sqrt{a \sin(e + fx) + a}}$$

```
[Out] -2/1155*a*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/f-2/11*a*B*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)*(c+d*sin(f*x+e))^3/d/f-2/3465*a^3*(15*c^2+10*c*d+7*d^2)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)/d^3/f/(a+a*sin(f*x+e))^(1/2)+2/693*a^3*(11*A*(3*c-19*d)*d-B*(15*c^2-65*c*d+194*d^2))*cos(f*x+e)*(c+d*sin(f*x+e))^3/d^3/f/(a+a*sin(f*x+e))^(1/2)-4/3465*a^2*(5*c-d)*(11*A*d*(c^2-10*c*d+73*d^2)-B*(5*c^3-40*c^2*d+169*c*d^2-710*d^3))*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/f+2/99*a^2*(-11*A*d+5*B*c-14*B*d)*cos(f*x+e)*(c+d*sin(f*x+e))^3*(a+a*sin(f*x+e))^(1/2)/d^2/f
```

Rubi [A] time = 1.07, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2976, 2981, 2761, 2751, 2646}

$$\frac{2a^3 (11Ad(3c - 19d) - B(15c^2 - 65cd + 194d^2)) \cos(e + fx)(c + d \sin(e + fx))^3 - 2a^3 (15c^2 + 10cd + 7d^2) (11Ad - 693d^3 f \sqrt{a \sin(e + fx) + a})}{693d^3 f \sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2, x]
```

```
[Out] (-2*a^3*(15*c^2 + 10*c*d + 7*d^2)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x])/((3465*d^3*f*Sqrt[a + a*Sin[e + f*x]]) - (4*a^2*(5*c - d)*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3465*d^2*f) - (2*a*(11*A*d*(c^2 - 10*c*d + 73*d^2) - B*(5*c^3 - 40*c^2*d + 169*c*d^2 - 710*d^3))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(1155*d*f) + (2*a^3*(11*A*(3*c - 19*d)*d - B*(15*c^2 - 65*c*d + 194*d^2))*Cos[e + f*x]*(c + d*Sin[e + f*x])^3)/(693*d^3*f*Sqrt[a + a*Sin[e + f*x]]) + (2*a^2*(5*B*c - 11*A*d - 14*B*d)*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^3)/(99*d^2*f) - (2*a*B*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3)/(11*d*f)
```

Rule 2646

```
Int[Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(-2*b*Cos[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2751

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2761

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(d^2*Cos[e + f*x]*(a + b*Sin[e + f*x])^m
```



```
+ 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[b*(d^2*(m + 1) + c^2*(m + 2)) - d*(a*d - 2*b*c*(m + 2))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rule 2976

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx = -\frac{2aB \cos(e + fx)(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))}{11df}$$

$$= \frac{2a^2(5Bc - 11Ad - 14Bd) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{99d^2 f}$$

$$= \frac{2a^3 (11A(3c - 19d)d - B(15c^2 - 65cd + 15d^2)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{693d^3 f \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{2a (11Ad(c^2 - 10cd + 73d^2) - B(5c^3 - 15cd^2 + 7d^3)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{346d^3 \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{2a^3 (15c^2 + 10cd + 7d^2) (11Ad(c^2 - 10cd + 73d^2) - B(5c^3 - 15cd^2 + 7d^3)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{346d^3 \sqrt{a + a \sin(e + fx)}}$$

Mathematica [B] time = 6.61, size = 891, normalized size = 2.08

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(-277200A \cos\left(\frac{1}{2}(e + fx)\right) c^2 - 207900B \cos\left(\frac{1}{2}(e + fx)\right) c^2 - 46200A \cos\left(\frac{3}{2}(e + fx)\right) c^2 \right)}{346d^3 \sqrt{a + a \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]

[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-277200*A*c^2*Cos[(e + f*x)/2] - 207900*B*c^2*Cos[(e + f*x)/2] - 415800*A*c*d*Cos[(e + f*x)/2] - 360360*B*c*d*Cos[(e + f*x)/2] - 180180*A*d^2*Cos[(e + f*x)/2] - 159390*B*d^2*Cos[(e + f*x)/2] - 46200*A*c^2*Cos[(3*(e + f*x))/2] - 50820*B*c^2*Cos[(3*(e + f*x))/2] - 101640*A*c*d*Cos[(3*(e + f*x))/2] - 92400*B*c*d*Cos[(3*(e + f*x))/2] - 46200*A*d^2*Cos[(3*(e + f*x))/2] - 43890*B*d^2*Cos[(3*(e + f*x))/2] + 5544*A*c^2*Cos[(5*(e + f*x))/2] + 13860*B*c^2*Cos[(5*(e + f*x))/2] + 27720*A*c*d*Cos[(5*(e + f*x))/2] + 33264*B*c*d*Cos[(5*(e + f*x))/2] + 16632*A*d^2*Cos[(5*(e + f*x))/2] + 17325*B*d^2*Cos[(5*(e + f*x))/2] + 1980*B*c^2*Cos[(7*(e + f*x))/2] + 3960*A*c*d*Cos[(7*(e + f*x))/2] + 9900*B*c*d*Cos[(7*(e + f*x))/2] + 4950*A*d^2*Cos[(7*(e + f*x))/2] + 6435*B*d^2*Cos[(7*(e + f*x))/2] - 1540*B*c*d*Cos[(9*(e + f*x))/2] - 770*A*d^2*Cos[(9*(e + f*x))/2] - 1925*B*d^2*Cos[(9*(e + f*x))/2] - 315*B*d^2*Cos[(11*(e + f*x))/2] + 277200*A*c^2*Sin[(e + f*x)/2] + 207900*B*c^2*Sin[(e + f*x)/2] + 415800*A*c*d*Sin[(e + f*x)/2] + 360360*B*c*d*Sin[(e + f*x)/2] + 180180*A*d^2*Sin[(e + f*x)/2] + 159390*B*d^2*Sin[(e + f*x)/2] - 46200*A*c^2*Sin[(3*(e + f*x))/2] - 50820*B*c^2*Sin[(3*(e + f*x))/2] - 101640*A*c*d*Sin[(3*(e + f*x))/2] - 92400*B*c*d*Sin[(3*(e + f*x))/2] - 46200*A*d^2*Sin[(3*(e + f*x))/2] - 43890*B*d^2*Sin[(3*(e + f*x))/2] - 5544*A*c^2*Sin[(5*(e + f*x))/2] - 13860*B*c^2*Sin[(5*(e + f*x))/2] - 27720*A*c*d*Sin[(5*(e + f*x))/2] - 33264*B*c*d*Sin[(5*(e + f*x))/2] - 16632*A*d^2*Sin[(5*(e + f*x))/2] - 17325*B*d^2*Sin[(5*(e + f*x))/2] + 1980*B*c^2*Sin[(7*(e + f*x))/2] + 3960*A*c*d*Sin[(7*(e + f*x))/2] + 9900*B*c*d*Sin[(7*(e + f*x))/2] + 4950*A*d^2*Sin[(7*(e + f*x))/2] + 6435*B*d^2*Sin[(7*(e + f*x))/2] + 1540*B*c*d*Sin[(9*(e + f*x))/2] + 770*A*d^2*Sin[(9*(e + f*x))/2] + 1925*B*d^2*Sin[(9*(e + f*x))/2] - 315*B*d^2*Sin[(11*(e + f*x))/2]))/(55440*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)

fricas [A] time = 0.47, size = 593, normalized size = 1.38

$$\frac{2 \left(315 B a^2 d^2 \cos(fx + e)^6 + 35 \left(22 B a^2 c d + (11 A + 32 B) a^2 d^2 \right) \cos(fx + e)^5 + 1056 (7 A + 5 B) a^2 c^2 + 704 (15 A + 13 B) a^2 c d + 32 (143 A + 125 B) a^2 d^2 - 5 (99 B a^2 c^2 + 22 (9 A + 19 B) a^2 c d + (209 A + 320 B) a^2 d^2) \cos(fx + e)^4 - (99 (7 A + 20 B) a^2 c^2 + 22 (180 A + 289 B) a^2 c d + (3179 A + 4370 B) a^2 d^2) \cos(fx + e)^3 + (33 (7 A + 85 B) a^2 c^2 + 22 (255 A + 263 B) a^2 c d + (2893 A + 2965 B) a^2 d^2) \cos(fx + e)^2 + 2 (33 (161 A + 145 B) a^2 c^2 + 22 (435 A + 419 B) a^2 c d + (4609 A + 4465 B) a^2 d^2) \cos(fx + e) + (315 B a^2 d^2 \cos(fx + e)^5 - 1056 (7 A + 5 B) a^2 c^2 - 704 (15 A + 13 B) a^2 c d - 32 (143 A + 125 B) a^2 d^2 - 35 (22 B a^2 c d + (11 A + 23 B) a^2 d^2) \cos(fx + e)^4 - 5 (99 B a^2 c^2 + 22 (9 A + 26 B) a^2 c d + 13 (22 A + 37 B) a^2 d^2) \cos(fx + e)^3 + 3 (33 (7 A + 15 B) a^2 c^2 + 22 (45 A + 53 B) a^2 c d + (583 A + 655 B) a^2 d^2) \cos(fx + e)^2 + 2 (33 (49 A + 65 B) a^2 c^2 + 22 (195 A + 211 B) a^2 c d + (2321 A + 2465 B) a^2 d^2) \cos(fx + e) \right) \sin(fx + e) \sqrt{a \sin(fx + e) + a}}{(f \cos(fx + e) + f \sin(fx + e) + f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] -2/3465*(315*B*a^2*d^2*cos(f*x + e)^6 + 35*(22*B*a^2*c*d + (11*A + 32*B)*a^2*d^2)*cos(f*x + e)^5 + 1056*(7*A + 5*B)*a^2*c^2 + 704*(15*A + 13*B)*a^2*c*d + 32*(143*A + 125*B)*a^2*d^2 - 5*(99*B*a^2*c^2 + 22*(9*A + 19*B)*a^2*c*d + (209*A + 320*B)*a^2*d^2)*cos(f*x + e)^4 - (99*(7*A + 20*B)*a^2*c^2 + 22*(180*A + 289*B)*a^2*c*d + (3179*A + 4370*B)*a^2*d^2)*cos(f*x + e)^3 + (33*(7*A + 85*B)*a^2*c^2 + 22*(255*A + 263*B)*a^2*c*d + (2893*A + 2965*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(161*A + 145*B)*a^2*c^2 + 22*(435*A + 419*B)*a^2*c*d + (4609*A + 4465*B)*a^2*d^2)*cos(f*x + e) + (315*B*a^2*d^2*cos(f*x + e)^5 - 1056*(7*A + 5*B)*a^2*c^2 - 704*(15*A + 13*B)*a^2*c*d - 32*(143*A + 125*B)*a^2*d^2 - 35*(22*B*a^2*c*d + (11*A + 23*B)*a^2*d^2)*cos(f*x + e)^4 - 5*(99*B*a^2*c^2 + 22*(9*A + 26*B)*a^2*c*d + 13*(22*A + 37*B)*a^2*d^2)*cos(f*x + e)^3 + 3*(33*(7*A + 15*B)*a^2*c^2 + 22*(45*A + 53*B)*a^2*c*d + (583*A + 655*B)*a^2*d^2)*cos(f*x + e)^2 + 2*(33*(49*A + 65*B)*a^2*c^2 + 22*(195*A + 211*B)*a^2*c*d + (2321*A + 2465*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e)*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e) + f*sin(f*x + e) + f)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.


```

*(f*x+exp(1))-1/4*pi))-4*B*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*A*a
^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*B*a^2*c*d*sign(cos(1/2*(f*x+exp
(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a^2*c^2*sig
n(cos(1/2*(f*x+exp(1))-1/4*pi))-4*A*a^2*d^2*sign(cos(1/2*(f*x+exp(1))-1/4*p
i))-4*B*a^2*c^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*d^2*sign(cos(1/2
*(f*x+exp(1))-1/4*pi))-8*A*a^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-8*B*a
^2*c*d*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(
20*f)^2-576*B*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(18*f*x+
18*exp(1)+pi))/(288*f)^2-704*B*a^2*d^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))
*cos(1/4*(22*f*x+22*exp(1)-pi))/(352*f)^2

```

maple [A] time = 1.43, size = 257, normalized size = 0.60

$$\frac{2(1 + \sin(fx + e))a^3(\sin(fx + e) - 1)(315Bd^2 \sin(fx + e)(\cos^4(fx + e)) + (-990Acd - 1430Ad^2 - 495Bc^2))}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] 2/3465*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(315*B*d^2*sin(f*x+e)*cos(f*x+e)^4
+(-990*A*c*d-1430*A*d^2-495*B*c^2-2860*B*c*d-2405*B*d^2)*cos(f*x+e)^2*sin(f
*x+e)+(3234*A*c^2+8580*A*c*d+4642*A*d^2+4290*B*c^2+9284*B*c*d+4930*B*d^2)*s
in(f*x+e)+(385*A*d^2+770*B*c*d+1120*B*d^2)*cos(f*x+e)^4+(-693*A*c^2-3960*A*
c*d-3179*A*d^2-1980*B*c^2-6358*B*c*d-4370*B*d^2)*cos(f*x+e)^2+10626*A*c^2+1
9140*A*c*d+9218*A*d^2+9570*B*c^2+18436*B*c*d+8930*B*d^2)/cos(f*x+e)/(a+a*si
n(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}(d \sin(fx + e) + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) +
c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{5}{2}} (c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2,
x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

3.302 $\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$

Optimal. Leaf size=212

$$\frac{64a^3(21Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)}{315f\sqrt{a \sin(e + fx) + a}} - \frac{16a^2(21Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)\sqrt{a \sin(e + fx)}}{315f}$$

[Out] $-2/105*a*(21*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{3/2}/f-2/63*(9*A*d+9*B*c-2*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{5/2}/f-2/9*B*d*\cos(f*x+e)*(a+a*\sin(f*x+e))^{7/2}/a/f-64/315*a^3*(21*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{1/2}-16/315*a^2*(21*A*c+15*A*d+15*B*c+13*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{1/2}/f$

Rubi [A] time = 0.37, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2968, 3023, 2751, 2647, 2646}

$$\frac{16a^2(21Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{315f} - \frac{64a^3(21Ac + 15Ad + 15Bc + 13Bd) \cos(e + fx)}{315f\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{5/2}*(A + B*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x]),x]$

[Out] $(-64*a^3*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*\text{Cos}[e + f*x])/(315*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (16*a^2*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(315*f) - (2*a*(21*A*c + 15*B*c + 15*A*d + 13*B*d)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2})/(105*f) - (2*(9*B*c + 9*A*d - 2*B*d)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{5/2})/(63*f) - (2*B*d*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{7/2})/(9*a*f)$

Rule 2646

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{EqQ}[a^2 - b^2, 0]$

Rule 2647

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[n - 1/2, 0]$

Rule 2751

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{LtQ}[m, -2^{(-1)}]$

Rule 2968

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x]$

x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx))^{5/2} (Ac + (Bc + Ad) \sin(e + fx) \\ &= -\frac{2Bd \cos(e + fx)(a + a \sin(e + fx))^{7/2}}{9af} + \frac{2 \int (a + a \sin(e + fx))^{3/2} (Bc + Ad) \sin(e + fx) dx}{9af} \\ &= -\frac{2(9Bc + 9Ad - 2Bd) \cos(e + fx)(a + a \sin(e + fx))^{5/2}}{63f} + \frac{2 \int (a + a \sin(e + fx))^{1/2} (Bc + Ad) \sin(e + fx) dx}{9af} \\ &= -\frac{2a(21Ac + 15Bc + 15Ad + 13Bd) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{105f} + \frac{2 \int (a + a \sin(e + fx))^{1/2} (Bc + Ad) \sin(e + fx) dx}{9af} \\ &= -\frac{16a^2(21Ac + 15Bc + 15Ad + 13Bd) \cos(e + fx)(a + a \sin(e + fx))^{1/2}}{315f} + \frac{2 \int (a + a \sin(e + fx))^{1/2} (Bc + Ad) \sin(e + fx) dx}{9af} \\ &= -\frac{64a^3(21Ac + 15Bc + 15Ad + 13Bd) \cos(e + fx)(a + a \sin(e + fx))^{1/2}}{315f \sqrt{a + a \sin(e + fx)}} + \frac{2 \int (a + a \sin(e + fx))^{1/2} (Bc + Ad) \sin(e + fx) dx}{9af} \end{aligned}$$

Mathematica [A] time = 4.24, size = 202, normalized size = 0.95

$$\frac{a^2 \sqrt{a(\sin(e + fx) + 1)} \left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right) \right) (-4(63Ac + 180Ad + 180Bc + 254Bd) \cos(2(e + fx)) + \dots)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]), x]

[Out] -1/1260*(a^2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*Sqrt[a*(1 + Sin[e + f*x])]*(7476*A*c + 6240*B*c + 6240*A*d + 5653*B*d - 4*(63*A*c + 180*B*c + 180*A*d + 254*B*d)*Cos[2*(e + f*x)] + 35*B*d*Cos[4*(e + f*x)] + 2352*A*c*Sin[e + f*x] + 3030*B*c*Sin[e + f*x] + 3030*A*d*Sin[e + f*x] + 3116*B*d*Sin[e + f*x] - 90*B*c*Sin[3*(e + f*x)] - 90*A*d*Sin[3*(e + f*x)] - 260*B*d*Sin[3*(e + f*x)]))/(f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))

fricas [A] time = 0.46, size = 361, normalized size = 1.70

$$\frac{2 \left(35 Ba^2 d \cos(fx + e)^5 - 5 (9 Ba^2 c + (9 A + 19 B) a^2 d) \cos(fx + e)^4 + 96 (7 A + 5 B) a^2 c + 32 (15 A + 13 B) a^2 d \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)), x, algorithm="fricas")

maple [A] time = 1.39, size = 152, normalized size = 0.72

$$\frac{2(1 + \sin(fx + e))a^3(\sin(fx + e) - 1)((-45Ad - 45Bc - 130Bd)\sin(fx + e)(\cos^2(fx + e)) + (294Ac + 390$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)`

[Out] $\frac{2}{315}(1 + \sin(fx + e))a^3(\sin(fx + e) - 1)((-45Ad - 45Bc - 130Bd)\sin(fx + e)\cos^2(fx + e) + (294Ac + 390Ad + 390Bc + 422Bd)\sin(fx + e) + 35Bd\cos(fx + e)^4 + (-63Ac - 180Ad - 180Bc - 289Bd)\cos^2(fx + e) + 966Ac + 870Ad + 870Bc + 838Bd)/\cos(fx + e)/(a + a\sin(fx + e))^{1/2}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}(d \sin(fx + e) + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)*(d*sin(f*x + e) + c), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx))(a + a \sin(e + fx))^{\frac{5}{2}}(c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)),x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x)), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)`

[Out] Timed out

3.303 $\int (a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx)) dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A + 5B) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}} - \frac{16a^2(7A + 5B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{105f} - \frac{2a(7A + 5B) \cos(e + fx)(a \sin(e + fx))^{5/2}}{35f}$$

[Out] $-2/35*a*(7*A+5*B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{3/2}/f-2/7*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{5/2}/f-64/105*a^3*(7*A+5*B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{1/2}-16/105*a^2*(7*A+5*B)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{1/2}/f$

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2751, 2647, 2646}

$$\frac{64a^3(7A + 5B) \cos(e + fx)}{105f\sqrt{a \sin(e + fx) + a}} - \frac{16a^2(7A + 5B) \cos(e + fx)\sqrt{a \sin(e + fx) + a}}{105f} - \frac{2a(7A + 5B) \cos(e + fx)(a \sin(e + fx))^{5/2}}{35f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^{5/2}*(A + B*\text{Sin}[e + f*x]),x]$

[Out] $(-64*a^3*(7*A + 5*B)*\text{Cos}[e + f*x])/(105*f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (16*a^2*(7*A + 5*B)*\text{Cos}[e + f*x]*\text{Sqrt}[a + a*\text{Sin}[e + f*x]])/(105*f) - (2*a*(7*A + 5*B)*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{3/2})/(35*f) - (2*B*\text{Cos}[e + f*x]*(a + a*\text{Sin}[e + f*x])^{5/2})/(7*f)$

Rule 2646

$\text{Int}[\text{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(-2*b*\text{Cos}[c + d*x])/(d*\text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2647

$\text{Int}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(a + b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(a*(2*n-1))/n, \text{Int}[(a + b*\text{Sin}[c + d*x])^{(n-1)}, x], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[n - 1/2, 0]$

Rule 2751

$\text{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rubi steps


```
rt(2*a)*(2*f*(6*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+4*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(2*f*x-pi)+1/2*exp(1))/(2*f)^2-8*f*(16*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(2*f*x+2*exp(1)+pi))/(8*f)^2-24*f*(16*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))+14*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*cos(1/4*(6*f*x+6*exp(1)-pi))/(24*f)^2+12*f*(-2*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(6*f*x+6*exp(1)+pi))/(12*f)^2+20*f*(-2*A*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi))-4*B*a^2*sign(cos(1/2*(f*x+exp(1))-1/4*pi)))*sin(1/4*(10*f*x+10*exp(1)-pi))/(20*f)^2+80*B*a^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(10*f*x+10*exp(1)+pi))/(40*f)^2+112*B*a^2*f*sign(cos(1/2*(f*x+exp(1))-1/4*pi))*cos(1/4*(14*f*x+14*exp(1)-pi))/(56*f)^2)
```

maple [A] time = 1.24, size = 99, normalized size = 0.72

$$\frac{2(1 + \sin(fx + e))a^3(\sin(fx + e) - 1)(-15B(\cos^2(fx + e))\sin(fx + e) + (98A + 130B)\sin(fx + e) + (-105 \cos(fx + e)\sqrt{a + a \sin(fx + e)}f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x)

[Out] 2/105*(1+sin(f*x+e))*a^3*(sin(f*x+e)-1)*(-15*B*cos(f*x+e)^2*sin(f*x+e)+(98*A+130*B)*sin(f*x+e)+(-21*A-60*B)*cos(f*x+e)^2+322*A+290*B)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^{\frac{5}{2}} (A + B \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e)),x)

[Out] Integral((a*(sin(e + f*x) + 1))**(5/2)*(A + B*sin(e + f*x)), x)

$$3.304 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=218

$$\frac{2a^{5/2}(c-d)^2(Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{d} \cos(e+fx)}{\sqrt{c+d}\sqrt{a \sin(e+fx)+a}}\right)}{d^{7/2}f\sqrt{c+d}} + \frac{2a^3(5Ad(3c-7d)-B(15c^2-35cd+32d^2)) \cos(e+fx)}{15d^3f\sqrt{a \sin(e+fx)+a}} + \dots$$

[Out] $-2/5*a*B*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/d/f+2*a^{(5/2)}*(c-d)^2*(-A*d+B*c)*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/d^{(7/2)}/f/(c+d)^{(1/2)}+2/15*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*\cos(f*x+e)/d^3/f/(a+a*\sin(f*x+e))^{(1/2)}+2/15*a^2*(-5*A*d+5*B*c-8*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/d^2/f$

Rubi [A] time = 0.88, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2976, 2981, 2773, 208}

$$\frac{2a^3(5Ad(3c-7d)-B(15c^2-35cd+32d^2)) \cos(e+fx)}{15d^3f\sqrt{a \sin(e+fx)+a}} + \frac{2a^2(-5Ad+5Bc-8Bd) \cos(e+fx)\sqrt{a \sin(e+fx)+a}}{15d^2f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+a*\sin[e+f*x])^{(5/2)}*(A+B*\sin[e+f*x])]/(c+d*\sin[e+f*x]), x]$

[Out] $(2*a^{(5/2)}*(c-d)^2*(B*c-A*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\cos[e+f*x])]/(\operatorname{Sqrt}[c+d]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])]/(d^{(7/2)}*\operatorname{Sqrt}[c+d]*f) + (2*a^3*(5*A*(3*c-7*d)*d-B*(15*c^2-35*c*d+32*d^2))*\cos[e+f*x])/(15*d^3*f*\operatorname{Sqrt}[a+a*\sin[e+f*x]]) + (2*a^2*(5*B*c-5*A*d-8*B*d)*\cos[e+f*x]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])/(15*d^2*f) - (2*a*B*\cos[e+f*x]*(a+a*\sin[e+f*x])^{(3/2)})/(5*d*f)$

Rule 208

$\operatorname{Int}[(a_+)+(b_+)*(x_+)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a_+)+(b_+)*\sin[(e_+)+(f_+)*(x_+)]]/((c_+)+(d_+)*\sin[(e_+)+(f_+)*(x_+)]), x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c+a*d-d*x^2), x], x, (b*\cos[e+f*x])/(\operatorname{Sqrt}[a+b*\sin[e+f*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \operatorname{NeQ}[c^2-d^2, 0]$

Rule 2976

$\operatorname{Int}[(a_+)+(b_+)*\sin[(e_+)+(f_+)*(x_+)]]^{(m_+)}*((A_+)+(B_+)*\sin[(e_+)+(f_+)*(x_+)])^{(n_+)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*B*\cos[e+f*x]*(a+b*\sin[e+f*x])^{(m-1)}*(c+d*\sin[e+f*x])^{(n+1)})/(d*f*(m+n+1)), x] + \operatorname{Dist}[1/(d*(m+n+1)), \operatorname{Int}[(a+b*\sin[e+f*x])^{(m-1)}*(c+d*\sin[e+f*x])^n*\operatorname{Simp}[a*A*d*(m+n+1)+B*(a*c*(m-1)+b*d*(n+1)+(A*b*d*(m+n+1)-B*(b*c*m-a*d*(2*m+n)))*\sin[e+f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, n\}, x \ \&\& \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \operatorname{EqQ}[a^2-b^2, 0] \ \&\& \operatorname{NeQ}[c^2-d^2, 0] \ \&\& \operatorname{GtQ}[m, 1/2] \ \&\& \operatorname{!LtQ}[n, -1] \ \&\& \operatorname{IntegerQ}[2*m] \ \&\& (\operatorname{IntegerQ}[2*n] \ || \ \operatorname{EqQ}[c, 0])]$

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{5/2}(A + B \sin(e + fx))}{c + d \sin(e + fx)} dx = -\frac{2aB \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{5df} + \frac{2 \int \frac{(a + a \sin(e + fx))^{3/2}}{c + d \sin(e + fx)} dx}{15d^2f} - \frac{2a^2(5Bc - 5Ad - 8Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15d^2f} - \frac{2a^3(5A(3c - 7d)d - B(15c^2 - 35cd + 32d^2)) \cos(e + fx)}{15d^3f \sqrt{a + a \sin(e + fx)}} + \frac{2a^3(5A(3c - 7d)d - B(15c^2 - 35cd + 32d^2)) \cos(e + fx)}{15d^3f \sqrt{a + a \sin(e + fx)}} + \frac{2a^{5/2}(c - d)^2(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c + d} \sqrt{a + a \sin(e + fx)}}\right)}{d^{7/2} \sqrt{c + d} f} + \frac{2a^3(5A(3c - 7d)d - B(15c^2 - 35cd + 32d^2)) \cos(e + fx)}{15d^3f \sqrt{a + a \sin(e + fx)}}$$

Mathematica [B] time = 5.82, size = 450, normalized size = 2.06

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(30\sqrt{d} (Ad(5d - 2c) + B(2c^2 - 5cd + 5d^2)) \sin\left(\frac{1}{2}(e + fx)\right) - 30\sqrt{d} (Ad(5d - 2c) + B(2c^2 - 5cd + 5d^2)) \cos\left(\frac{1}{2}(e + fx)\right) \right)}{d^{7/2} \sqrt{c + d} f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]), x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Cos[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Cos[(3*(e + f*x))/2] + 3*B*d^(5/2)*Cos[(5*(e + f*x))/2] + (15*(c - d)^2*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2]))))/Sqrt[c + d] - (15*(c - d)^2*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)))/Sqrt[c + d] + 30*Sqrt[d]*(A*d*(-2*c + 5*d) + B*(2*c^2 - 5*c*d + 5*d^2))*Sin[(e + f*x)/2] - 5*d^(3/2)*(-2*B*c + 2*A*d + 5*B*d)*Sin[(3*(e + f*x))/2] - 3*B*d^(5/2)*Sin[(5*(e + f*x))/2])/(30*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)
```

fricas [B] time = 2.27, size = 1314, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] [1/30*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) + d^3*f), 1/15*(15*(B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3 + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*cos(f*x + e) + (B*a^2*c^3 - (A + 2*B)*a^2*c^2*d + (2*A + B)*a^2*c*d^2 - A*a^2*d^3)*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e))) + 2*(3*B*a^2*d^2*cos(f*x + e)^3 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32*B)*a^2*d^2 + (5*B*a^2*c*d - (5*A + 11*B)*a^2*d^2)*cos(f*x + e)^2 - (15*B*a^2*c^2 - 5*(3*A + 8*B)*a^2*c*d + 2*(20*A + 23*B)*a^2*d^2)*cos(f*x + e) - (3*B*a^2*d^2*cos(f*x + e)^2 - 15*B*a^2*c^2 + 5*(3*A + 7*B)*a^2*c*d - (35*A + 32*B)*a^2*d^2 - (5*B*a^2*c*d - (5*A + 14*B)*a^2*d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(d^3*f*cos(f*x + e) + d^3*f*sin(f*x + e) + d^3*f)]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.05, size = 543, normalized size = 2.49

$$2(1 + \sin(fx + e))\sqrt{-a(\sin(fx + e) - 1)} \left(-3B(a - a\sin(fx + e))^{\frac{5}{2}}\sqrt{a(c+d)d}d^2 + 5A(a - a\sin(fx + e))^{\frac{5}{2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] 2/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(-3*B*(a-a*sin(f*x+e))^(5/2)*(a*(c+d)*d)^(1/2)*d^2+5*A*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*a*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c^2*d+30*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*c*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^3*d^3-5*B*(a-a*sin(f*x+e))^(

$$\frac{3}{2} * (a * (c+d) * d)^{(1/2)} * a * c * d + 20 * B * (a - a * \sin(f * x + e))^{(3/2)} * (a * (c+d) * d)^{(1/2)} * a * d^2 + 15 * B * \operatorname{arctanh}((a - a * \sin(f * x + e))^{(1/2)} * d / (a * c * d + a * d^2)^{(1/2)}) * a^3 * c^3 - 30 * B * \operatorname{arctanh}((a - a * \sin(f * x + e))^{(1/2)} * d / (a * c * d + a * d^2)^{(1/2)}) * a^3 * c^2 * d + 15 * B * \operatorname{arctanh}((a - a * \sin(f * x + e))^{(1/2)} * d / (a * c * d + a * d^2)^{(1/2)}) * a^3 * c * d^2 + 15 * A * (a - a * \sin(f * x + e))^{(1/2)} * (a * (c+d) * d)^{(1/2)} * a^2 * c * d - 45 * A * (a - a * \sin(f * x + e))^{(1/2)} * (a * (c+d) * d)^{(1/2)} * a^2 * d^2 - 15 * B * (a - a * \sin(f * x + e))^{(1/2)} * (a * (c+d) * d)^{(1/2)} * a^2 * c^2 + 45 * B * (a - a * \sin(f * x + e))^{(1/2)} * (a * (c+d) * d)^{(1/2)} * a^2 * c * d - 60 * B * (a - a * \sin(f * x + e))^{(1/2)} * (a * (c+d) * d)^{(1/2)} * a^2 * d^2 / d^3 / (a * (c+d) * d)^{(1/2)} / \cos(f * x + e) / (a + a * \sin(f * x + e))^{(1/2)} / f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{5/2}}{d \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{5/2}}{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)), x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] Timed out

$$3.305 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=265

$$\frac{a^{5/2}(c-d) \left(Ad(3c+5d) - B(5c^2+5cd-2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{7/2} f (c+d)^{3/2}} - \frac{a^3 \left(3Ad(3c+d) - B(15c^2-5cd-14d^2) \right) \cos(e+fx)}{3d^3 f (c+d) \sqrt{a \sin(e+fx)+a}}$$

[Out] a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))/d^(7/2)/(c+d)^(3/2)/f+a*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^(3/2)/d/(c+d)/f/(c+d*sin(f*x+e))-1/3*a^3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*cos(f*x+e)/d^3/(c+d)/f/(a+a*sin(f*x+e))^(1/2)-1/3*a^2*(-3*A*d+5*B*c+2*B*d)*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/d^2/(c+d)/f

Rubi [A] time = 0.94, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2975, 2976, 2981, 2773, 208}

$$\frac{a^3 \left(3Ad(3c+d) - B(15c^2-5cd-14d^2) \right) \cos(e+fx)}{3d^3 f (c+d) \sqrt{a \sin(e+fx)+a}} + \frac{a^{5/2}(c-d) \left(Ad(3c+5d) - B(5c^2+5cd-2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{d^{7/2} f (c+d)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2, x]

[Out] (a^(5/2)*(c-d)*(A*d*(3*c+5*d)-B*(5*c^2+5*c*d-2*d^2))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e+f*x])/(Sqrt[c+d]*Sqrt[a+a*Sin[e+f*x]])]/(d^(7/2)*(c+d)^(3/2)*f) - (a^3*(3*A*d*(3*c+d)-B*(15*c^2-5*c*d-14*d^2))*Cos[e+f*x]/(3*d^3*(c+d)*f*Sqrt[a+a*Sin[e+f*x]]) - (a^2*(5*B*c-3*A*d+2*B*d)*Cos[e+f*x]*Sqrt[a+a*Sin[e+f*x]])/(3*d^2*(c+d)*f) + (a*(B*c-A*d)*Cos[e+f*x]*(a+a*Sin[e+f*x])^(3/2))/(d*(c+d)*f*(c+d*Sin[e+f*x]))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2773

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2975

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^(n+1))/(d*f*(n+1)*(b*c + a*d)), x] - Dist[b/(d*(n+1)*(b*c + a*d)), Int[(a + b*Sin[e + f*x])^(m-1)*(c + d*Sin[e + f*x])^(n+1)*Simp[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1)))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&

GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2976

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -Si
mp[(b*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n +
1))/(d*f*(m + n + 1)), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f*x
])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1) +
b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f*x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1] &
& IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{5/2} (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx = \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{d(c + d)f(c + d \sin(e + fx))} + \frac{\int \frac{(a + a \sin(e + fx))^{5/2}}{(c + d \sin(e + fx))^2} dx}{d(c + d)f(c + d \sin(e + fx))}$$

$$= -\frac{a^2(5Bc - 3Ad + 2Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3d^2(c + d)f} + \frac{a^2}{3d^2(c + d)f \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{a^3 (3Ad(3c + d) - B(15c^2 - 5cd - 14d^2)) \cos(e + fx)}{3d^3(c + d)f \sqrt{a + a \sin(e + fx)}} - \frac{a^2}{3d^3(c + d)f \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{a^3 (3Ad(3c + d) - B(15c^2 - 5cd - 14d^2)) \cos(e + fx)}{3d^3(c + d)f \sqrt{a + a \sin(e + fx)}} - \frac{a^2}{3d^3(c + d)f \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{a^{5/2}(c - d) (Ad(3c + 5d) - B(5c^2 + 5cd - 2d^2)) \tanh^{-1} \left(\frac{\sqrt{a + a \sin(e + fx)}}{\sqrt{c + d}} \right)}{d^{7/2}(c + d)^{3/2} f}$$

Mathematica [A] time = 5.97, size = 460, normalized size = 1.74

$$\frac{(a(\sin(e + fx) + 1))^{5/2} \left(\frac{3(c-d)(B(5c^2+5cd-2d^2)-Ad(3c+5d)) \left(2 \log \left(\sqrt{d} \sqrt{c+d} \left(\tan^2 \left(\frac{1}{4}(e+fx) \right) + 2 \tan \left(\frac{1}{4}(e+fx) \right) - 1 \right) + (c+d) \sec^2 \left(\frac{1}{4}(e+fx) \right) \right)}{(c+d)^{3/2}} \right)}{d^{7/2}(c+d)^{3/2} f} \right)}{d^{7/2}(c+d)^{3/2} f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e +
f*x])^2,x]
```

```
[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(-12*sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Cos[(e + f*x)/2] - 4*B*d^(3/2)*Cos[(3*(e + f*x))/2] - (3*(c - d)*(-(A*d*(3*c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + sqrt[d]*sqrt[c + d]*Cos[(e + f*x)/2] - sqrt[d]*sqrt[c + d]*Sin[(e + f*x)/2])))))/(c + d)^(3/2) + (3*(c - d)*(-(A*d*(3*c + 5*d)) + B*(5*c^2 + 5*c*d - 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + sqrt[d]*sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)))/(c + d)^(3/2) + 12*sqrt[d]*(-4*B*c + 2*A*d + 5*B*d)*Sin[(e + f*x)/2] - (12*(c - d)^2*sqrt[d]*(-(B*c) + A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/((c + d)*(c + d*sin[e + f*x])) - 4*B*d^(3/2)*Sin[(3*(e + f*x))/2])/((12*d^(7/2)*f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))^5)
```

fricas [B] time = 2.59, size = 2046, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a/(c*d + d^2))*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*(c^2*d + 4*c*d^2 + 3*d^3 - (c*d^2 + d^3)*cos(f*x + e)^2 + (c^2*d + 3*c*d^2 + 2*d^3)*cos(f*x + e) - (c^2*d + 4*c*d^2 + 3*d^3 + (c*d^2 + d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(a/(c*d + d^2)) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e)), 1/6*(3*(5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 - (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e)^2 + (5*B*a^2*c^4 - 3*A*a^2*c^3*d - (2*A + 7*B)*a^2*c^2*d^2 + (5*A + 2*B)*a^2*c*d^3)*cos(f*x + e) + (5*B*a^2*c^4 - (3*A - 5*B)*a^2*c^3*d - (5*A + 7*B)*a^2*c^2*d^2 + (3*A - 5*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4 + (5*B*a^2*c^3*d - 3*A*a^2*c^2*d^2 - (2*A + 7*B)*a^2*c*d^3 + (5*A + 2*B)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2)))/(a*cos(f*x + e))) - 2*(15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d + 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^3 + 2*(5*B*a^2*c^2*d - (3*A + 2*B)*a^2*c*d^2 - (3*A + 7*B)*a^2*d^3)*cos(f*x + e)^2 + (15*B*a^2*c^3 - (9*A + 10*B)*a^2*c^2*d - 15*B*a^2*c*d^2 - (3*A + 2*B)*a^2*d^3)*cos(f*x + e) - (15*B*a^2*c^3 - (9*A + 20*B)*a^2*c^2*d
```

+ 3*(2*A - 3*B)*a^2*c*d^2 + (3*A + 14*B)*a^2*d^3 + 2*(B*a^2*c*d^2 + B*a^2*d^3)*cos(f*x + e)^2 - 2*(5*B*a^2*c^2*d - 3*(A + B)*a^2*c*d^2 - (3*A + 8*B)*a^2*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/((c*d^4 + d^5)*f*cos(f*x + e)^2 - (c^2*d^3 + c*d^4)*f*cos(f*x + e) - (c^2*d^3 + 2*c*d^4 + d^5)*f - ((c*d^4 + d^5)*f*cos(f*x + e) + (c^2*d^3 + 2*c*d^4 + d^5)*f)*sin(f*x + e))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.54, size = 932, normalized size = 3.52

$$a(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(\sin(fx + e) d \left(9A \operatorname{arctanh} \left(\frac{\sqrt{a - a \sin(fx + e)} d}{\sqrt{acd + ad^2}} \right) a^2 c^2 d + 6A \operatorname{arctanh} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

[Out] 1/3*a*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*d*(9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d+6*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3-15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^3+21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^2-6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*d^3+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*d^2-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-6*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2-6*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d-18*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^2)+9*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^3*d+6*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-15*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d+2*B*(a-a*sin(f*x+e))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^2-15*a^2*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*B*c^4+21*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c^2*d^2-6*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^2*c*d^3-9*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d-3*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^3+15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3-12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d-15*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a*c*d^2)/d^3/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{5}{2}}}{(d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(5/2)/(d*sin(f*x + e) + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + f x)) (a + a \sin(e + f x))^{5/2}}{(c + d \sin(e + f x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)

[Out] Timed out

$$3.306 \quad \int \frac{(a+a \sin(e+fx))^{5/2}(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=308

$$\frac{a^{5/2} \left(Ad \left(3c^2 + 10cd + 19d^2 \right) - B \left(15c^3 + 30c^2d + 7cd^2 - 20d^3 \right) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{4d^{7/2} f (c+d)^{5/2}} + \frac{a^3 \left(3Ad(c+3d) - B(15c^2 + 25cd + 4d^2) \right) \cos(e+fx) \sqrt{a}}{4d^3 f (c+d)^2 \sqrt{a \sin(e+fx)+a}}$$

[Out] $-1/4*a^{(5/2)}*(A*d*(3*c^2+10*c*d+19*d^2)-B*(15*c^3+30*c^2*d+7*c*d^2-20*d^3))$
 $*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)}/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/d^{(7/2)}/(c+d)^{(5/2)}/f+1/2*a*(-A*d+B*c)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(3/2)}/d/(c+d)/f/(c+d*\sin(f*x+e))^{(1/2)}+1/4*a^3*(3*A*d*(c+3*d)-B*(15*c^2+25*c*d+4*d^2))*\cos(f*x+e)/d^3/(c+d)^2/f/(a+a*\sin(f*x+e))^{(1/2)}-1/4*a^2*(A*d*(c+7*d)-B*(5*c^2+7*c*d-4*d^2))*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/d^2/(c+d)^2/f/(c+d*\sin(f*x+e))$

Rubi [A] time = 0.97, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2975, 2981, 2773, 208}

$$\frac{a^3 \left(3Ad(c+3d) - B(15c^2 + 25cd + 4d^2) \right) \cos(e+fx)}{4d^3 f (c+d)^2 \sqrt{a \sin(e+fx)+a}} - \frac{a^2 \left(Ad(c+7d) - B(5c^2 + 7cd - 4d^2) \right) \cos(e+fx) \sqrt{a}}{4d^2 f (c+d)^2 (c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + a*\sin[e + f*x])^{(5/2)}*(A + B*\sin[e + f*x])]/(c + d*\sin[e + f*x])^3, x]$

[Out] $-(a^{(5/2)}*(A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])]/(4*d^{(7/2)}*(c + d)^{(5/2)}*f) + (a^3*(3*A*d*(c + 3*d) - B*(15*c^2 + 25*c*d + 4*d^2))*\operatorname{Cos}[e + f*x])/((4*d^3*(c + d)^2*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]]) + (a*(B*c - A*d)*\operatorname{Cos}[e + f*x]*(a + a*\sin[e + f*x])^{(3/2)})/(2*d*(c + d)*f*(c + d*\sin[e + f*x])^2) - (a^2*(A*d*(c + 7*d) - B*(5*c^2 + 7*c*d - 4*d^2))*\operatorname{Cos}[e + f*x]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])/(4*d^2*(c + d)^2*f*(c + d*\sin[e + f*x])))$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])/((c_.) + (d_)*\sin[(e_.) + (f_)*(x_)])], x_Symbol] \rightarrow \operatorname{Dist}[(-2*b)/f, \operatorname{Subst}[\operatorname{Int}[1/(b*c + a*d - d*x^2), x], x, (b*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[a + b*\sin[e + f*x]])], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{NeQ}[c^2 - d^2, 0]$

Rule 2975

$\operatorname{Int}[(a_ + (b_)*\sin[(e_.) + (f_)*(x_)])^{(m_)}*((A_.) + (B_)*\sin[(e_.) + (f_)*(x_)])^{(n_)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(B*c - A*d)*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{(m-1)}*(c + d*\sin[e + f*x])^{(n+1)})/(d*f*(n+1)*(b*c + a*d)), x] - \operatorname{Dist}[b/(d*(n+1)*(b*c + a*d)), \operatorname{Int}[(a + b*\sin[e + f*x])^{(m-1)}*(c + d*\sin[e + f*x])^{(n+1)}*\operatorname{Simp}[a*A*d*(m-n-2) - B*(a*c*(m-1) + b*d*(n+1)) - (A*b*d*(m+n+1) - B*(b*c*m - a*d*(n+1))]*\sin[e + f*x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A$

, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])

Rule 2981

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp [(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^{5/2}(A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx = \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{2d(c + d)f(c + d \sin(e + fx))^2} + \frac{\int \frac{(a + a \sin(e + fx))^3}{(c + d \sin(e + fx))^3} dx}{1}$$

$$= \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{2d(c + d)f(c + d \sin(e + fx))^2} - \frac{a^2 (Ad(c + 7d))}{2d(c + d)f(c + d \sin(e + fx))^2}$$

$$= \frac{a^3 (3Ad(c + 3d) - B(15c^2 + 25cd + 4d^2)) \cos(e + fx)}{4d^3(c + d)^2 f \sqrt{a + a \sin(e + fx)}} + \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{2d(c + d)f(c + d \sin(e + fx))^2}$$

$$= \frac{a^3 (3Ad(c + 3d) - B(15c^2 + 25cd + 4d^2)) \cos(e + fx)}{4d^3(c + d)^2 f \sqrt{a + a \sin(e + fx)}} + \frac{a(Bc - Ad) \cos(e + fx)(a + a \sin(e + fx))^{3/2}}{2d(c + d)f(c + d \sin(e + fx))^2}$$

$$= -\frac{a^{5/2} (Ad(3c^2 + 10cd + 19d^2) - B(15c^3 + 30c^2d + 7cd^2 - 20d^3))}{4d^{7/2}(c + d)^{5/2} f}$$

Mathematica [A] time = 8.24, size = 504, normalized size = 1.64

$$(a(\sin(e + fx) + 1))^{5/2} \left(\frac{(Ad(3c^2 + 10cd + 19d^2) - B(15c^3 + 30c^2d + 7cd^2 - 20d^3)) \left(2 \log \left(\sqrt{d} \sqrt{c+d} \left(\tan^2 \left(\frac{1}{4}(e+fx) \right) + 2 \tan \left(\frac{1}{4}(e+fx) \right) - 1 \right) + (c+d) \sec^2 \left(\frac{1}{4}(e+fx) \right) \right)}{(c+d)^{5/2}} \right)}{1} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^(5/2)*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] ((a*(1 + Sin[e + f*x]))^(5/2)*(((-(A*d*(3*c^2 + 10*c*d + 19*d^2)) + B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[-(Sec[(e + f*x)/4]^2*(c + d + Sqrt[d]*Sqrt[c + d]*Cos[(e + f*x)/2] - Sqrt[d]*Sqrt[c + d]*Sin[(e + f*x)/2])))]/(c + d)^(5/2) + ((A*d*(3*c^2 + 10*c*d + 19*d^2) - B*(15*c^3 + 30*c^2*d + 7*c*d^2 - 20*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[(c + d)*Sec[(e + f*x)/4]^2 + Sqrt[d]*Sqrt[c + d]*(-1 + 2*Tan[(e + f*x)/4] + Tan[(e + f*x)/4]^2)]))/(c + d)^(5/2) - (4*Sqrt[d]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(15*B*c^4 - 3*A*c^3*d + 20*B*c^3*d - 8*A*c^2*d^2 - B*c^2*d^2 + 9*A*c*d^3 + 10*B*c*d^3 + 2*A*d^4 + 4*B*d^4 - 4*B*d^2*(c + d)^2*Cos[2*(e + f*x)] + d*(A*d*(-5*c^2 - 6*c*d + 11*d^2) + B*(


```

os(f*x + e))*sin(f*x + e))*sqrt(-a/(c*d + d^2))*arctan(1/2*sqrt(a*sin(f*x +
e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-a/(c*d + d^2))/(a*cos(f*x + e)))
- 2*(15*B*a^2*c^4 - (3*A + 5*B)*a^2*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*
A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^
3 + B*a^2*d^4)*cos(f*x + e)^3 + (25*B*a^2*c^3*d - (5*A - 26*B)*a^2*c^2*d^2
- 3*(2*A + 5*B)*a^2*c*d^3 + (11*A - 4*B)*a^2*d^4)*cos(f*x + e)^2 + (15*B*a^
2*c^4 - (3*A - 20*B)*a^2*c^3*d - (8*A - 3*B)*a^2*c^2*d^2 + 9*(A + 2*B)*a^2*
c*d^3 + 2*(A + 4*B)*a^2*d^4)*cos(f*x + e) - (15*B*a^2*c^4 - (3*A + 5*B)*a^2
*c^3*d - (3*A + 31*B)*a^2*c^2*d^2 + (15*A + 17*B)*a^2*c*d^3 - (9*A - 4*B)*a
^2*d^4 - 8*(B*a^2*c^2*d^2 + 2*B*a^2*c*d^3 + B*a^2*d^4)*cos(f*x + e)^2 - (25
*B*a^2*c^3*d - (5*A - 34*B)*a^2*c^2*d^2 - (6*A - B)*a^2*c*d^3 + (11*A + 4*B
)*a^2*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((c^2*d^5
+ 2*c*d^6 + d^7)*f*cos(f*x + e)^3 + (2*c^3*d^4 + 5*c^2*d^5 + 4*c*d^6 + d^7)
*f*cos(f*x + e)^2 - (c^4*d^3 + 2*c^3*d^4 + 2*c^2*d^5 + 2*c*d^6 + d^7)*f*cos
(f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f + ((c^2*d^5
+ 2*c*d^6 + d^7)*f*cos(f*x + e)^2 - 2*(c^3*d^4 + 2*c^2*d^5 + c*d^6)*f*cos(
f*x + e) - (c^4*d^3 + 4*c^3*d^4 + 6*c^2*d^5 + 4*c*d^6 + d^7)*f)*sin(f*x + e
))]

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

[Out] Timed out

maple [B] time = 2.97, size = 1587, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] -1/4*a*(16*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^3*d
+32*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c^2*d^2+16*B
*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)*a*c*d^3+8*B*(-a*(si
n(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c^2*d^2+16*B*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*c*d^3-3*B*(-a*(sin(f*x+e)-
1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2-11*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+
d)*d)^(1/2)*d^4-4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*d^4-15*a^2*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*B*c^5-7*B*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+20*B*arctanh((-a*(
sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-2*B*(-a*(sin(f*x+e)-1
))^3/2*(a*(c+d)*d)^(1/2)*c^2*d^2-13*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*
d)^(1/2)*a*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*sin(f*x+e)^2*a^2*c^2*d^3+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)
*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4-15*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^3*d^2-30*B*arctanh((-a*(sin(f*x+e)-1)
)^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c^2*d^3-7*B*arctanh((-a*(sin(
f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*c*d^4+6*A*arctanh((-
a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2+20*A*ar
ctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d^3
+38*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2
*c*d^4+8*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*sin(f*x+e)^2*a*d^4+2
9*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-30*B*arctanh((-a*(s
in(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^4*d-60*B*arctanh(
```



```
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^3*d^2-14*B*
arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a^2*c^2*d
^3+40*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)*a
^2*c*d^4-3*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^3*d-13*A*(-a*(
sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*c^2*d^2+3*A*(-a*(sin(f*x+e)-1))^(1
/2)*(a*(c+d)*d)^(1/2)*a*c*d^3+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*c*d^3+3*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^4
*d+10*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^3*d^2+
19*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^2*c^2*d^3-9*B
*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^3*d+13*A*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+15*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(
1/2)*a*c^4+4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a*d^4+19*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^5+20*B
*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*sin(f*x+e)^2*a^2*d^
5+5*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c^2*d^2+15*B*(-a*(sin(f*x
+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*c*d^3-30*B*arctanh((-a*(sin(f*x+e)-1))^(1/2
))*d/(a*(c+d)*d)^(1/2))*a^2*c^4*d*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/
(a*(c+d)*d)^(1/2)/(c+d*sin(f*x+e))^2/(c+d)^2/d^3/cos(f*x+e)/(a+a*sin(f*x+e)
)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^{5/2}}{(c + d \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3,x)
```

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(5/2))/(c + d*sin(e + f*x))^
3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**(5/2)*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.307 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=284

$$\frac{2d(7Ad(9c-d) + B(24c^2 - 15cd + 31d^2)) \cos(e+fx) \sqrt{a \sin(e+fx) + a}}{105af} - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 37d^3)) \cos(e+fx) \sqrt{a \sin(e+fx) + a}}{105f \sqrt{a}}$$

[Out] $-(A-B)*(c-d)^3*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)}})*2^{(1/2)/f/a^{(1/2)}}-4/105*(7*A*d*(21*c^2-12*c*d+7*d^2)+B*(36*c^3-63*c^2*d+144*c*d^2-37*d^3))*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(1/2)}-2/35*(7*A*d+6*B*c-B*d)*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/f/(a+a*\sin(f*x+e))^{(1/2)}-2/7*B*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/f/(a+a*\sin(f*x+e))^{(1/2)}-2/105*d*(7*A*(9*c-d)*d+B*(24*c^2-15*c*d+31*d^2))*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/a/f$

Rubi [A] time = 1.00, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{2d(7Ad(9c-d) + B(24c^2 - 15cd + 31d^2)) \cos(e+fx) \sqrt{a \sin(e+fx) + a}}{105af} - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 37d^3)) \cos(e+fx) \sqrt{a \sin(e+fx) + a}}{105f \sqrt{a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sin}[e + f*x])*(c + d*\operatorname{Sin}[e + f*x])^3/\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]], x]$

[Out] $-(\operatorname{Sqrt}[2]*(A - B)*(c - d)^3*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(\operatorname{Sqrt}[a]*f) - (4*(7*A*d*(21*c^2 - 12*c*d + 7*d^2) + B*(36*c^3 - 63*c^2*d + 144*c*d^2 - 37*d^3))*\operatorname{Cos}[e + f*x])/((105*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]) - (2*d*(7*A*(9*c - d)*d + B*(24*c^2 - 15*c*d + 31*d^2))*\operatorname{Cos}[e + f*x]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])/(105*a*f) - (2*(6*B*c + 7*A*d - B*d)*\operatorname{Cos}[e + f*x]*(c + d*\operatorname{Sin}[e + f*x])^2)/(35*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]) - (2*B*\operatorname{Cos}[e + f*x]*(c + d*\operatorname{Sin}[e + f*x])^3)/(7*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*\operatorname{sin}[(c_ + (d_)*(x_)])], x_Symbol] :> \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/ \operatorname{Sqrt}[a + b*\operatorname{Sin}[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2751

$\operatorname{Int}[(a_ + (b_)*\operatorname{sin}[(e_ + (f_)*(x_)]])^{(m_)}*((c_ + (d_)*\operatorname{sin}[(e_ + (f_)*(x_)])), x_Symbol] :> -\operatorname{Simp}[(d*\operatorname{Cos}[e + f*x]*(a + b*\operatorname{Sin}[e + f*x])^m)/(f*(m + 1)), x] + \operatorname{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \operatorname{Int}[(a + b*\operatorname{Sin}[e + f*x])^m, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, x\} \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{!LtQ}[m, -2^{-1}]$

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{\sqrt{a + a \sin(e + fx)}} dx = -\frac{2B \cos(e + fx)(c + d \sin(e + fx))^3}{7f\sqrt{a + a \sin(e + fx)}} + \frac{2 \int \frac{(c + d \sin(e + fx))^2 \left(\frac{1}{2}a(7A + B \sin(e + fx))\right)}{\sqrt{a + a \sin(e + fx)}} dx}{7f\sqrt{a + a \sin(e + fx)}} - \frac{2(6Bc + 7Ad - Bd) \cos(e + fx)(c + d \sin(e + fx))^2}{35f\sqrt{a + a \sin(e + fx)}} - \frac{2B \cos(e + fx)(c + d \sin(e + fx))}{35f\sqrt{a + a \sin(e + fx)}} - \frac{2(6Bc + 7Ad - Bd) \cos(e + fx)(c + d \sin(e + fx))^2}{35f\sqrt{a + a \sin(e + fx)}} - \frac{2B \cos(e + fx)(c + d \sin(e + fx))}{35f\sqrt{a + a \sin(e + fx)}} - \frac{2d(7A(9c - d)d + B(24c^2 - 15cd + 31d^2)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{105af} - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 3d^3)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{105f\sqrt{a + a \sin(e + fx)}} - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 3d^3)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{105f\sqrt{a + a \sin(e + fx)}} - \frac{\sqrt{2}(A - B)(c - d)^3 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} f} - \frac{4(7Ad(21c^2 - 12cd + 7d^2) + B(36c^3 - 63c^2d + 144cd^2 - 3d^3)) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{105f\sqrt{a + a \sin(e + fx)}}$$

Mathematica [C] time = 0.86, size = 375, normalized size = 1.32

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(-35d(2Ad(6c - d) + B(12c^2 - 6cd + 5d^2)) \sin\left(\frac{3}{2}(e + fx)\right) - 35d(2Ad(6c - d) + B(12c^2 - 6cd + 5d^2)) \cos\left(\frac{3}{2}(e + fx)\right)\right) \sqrt{a + a \sin(e + fx)}$$


```
*sign(tan((f*x+exp(1))/2)+1))-1/2822400*(-1411200*A*a^3*d^3*sign(tan((f*x+
exp(1))/2)+1)-1411200*B*a^3*c^3*sign(tan((f*x+exp(1))/2)+1)+1411200*B*a^3*d
^3*sign(tan((f*x+exp(1))/2)+1)+4233600*A*a^3*c^2*d*sign(tan((f*x+exp(1))/2)
+1)-4233600*A*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)-4233600*B*a^3*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)+4233600*B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))-
1/2822400*(1223040*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+1411200*B*a^3*c^3*
sign(tan((f*x+exp(1))/2)+1)-577920*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-14
11200*A*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)+4233600*A*a^3*c^2*d*sign(tan(
(f*x+exp(1))/2)+1)+3669120*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1)-1411200*
B*a^3*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+sqrt(2)*(A*c^3-A*d^3-B*c^3+B*d^3+
3*A*c*d^2-3*A*c^2*d-3*B*c*d^2+3*B*c^2*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)
-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign(t
an((f*x+exp(1))/2)+1)+(-105*A*a*c^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+315*A*a*
c^2*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))-315*A*a*c*d^2*sqrt(2)*atan(sqrt(a)/sqr
t(-a))+105*A*a*d^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+315*A*c^2*d*sqrt(-a)*sqrt
(2)*sqrt(a)-210*A*c*d^2*sqrt(-a)*sqrt(2)*sqrt(a)+119*A*d^3*sqrt(-a)*sqrt(2)
*sqrt(a)+105*B*a*c^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))-315*B*a*c^2*d*sqrt(2)*a
tan(sqrt(a)/sqrt(-a))+315*B*a*c*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))-105*B*a*
d^3*sqrt(2)*atan(sqrt(a)/sqrt(-a))+105*B*c^3*sqrt(-a)*sqrt(2)*sqrt(a)-210*B
*c^2*d*sqrt(-a)*sqrt(2)*sqrt(a)+357*B*c*d^2*sqrt(-a)*sqrt(2)*sqrt(a)-92*B*d
^3*sqrt(-a)*sqrt(2)*sqrt(a))/105/a/sqrt(-a)*sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.95, size = 610, normalized size = 2.15

$$(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(105A a^{\frac{7}{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a-a\sin(fx+e)} \sqrt{2}}{2\sqrt{a}} \right) c^3 - 315A a^{\frac{7}{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a-a\sin(fx+e)} \sqrt{2}}{2\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)

```
[Out] -1/105*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(105*A*a^(7/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3-315*A*a^(7/2)*2^(1/2)*a
rctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d+315*A*a^(7/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d^2-105*A*a^(7/2)
*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^3-105*B*a^(7
/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^3+315*B*a
^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2*d-31
5*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d
^2+105*B*a^(7/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
)*d^3-30*B*d^3*(a-a*sin(f*x+e))^(7/2)+42*A*(a-a*sin(f*x+e))^(5/2)*a*d^3+126
*B*(a-a*sin(f*x+e))^(5/2)*a*c*d^2+84*B*(a-a*sin(f*x+e))^(5/2)*a*d^3-210*A*(
a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-70*A*(a-a*sin(f*x+e))^(3/2)*a^2*d^3-210*B*(
a-a*sin(f*x+e))^(3/2)*a^2*c^2*d-210*B*(a-a*sin(f*x+e))^(3/2)*a^2*c*d^2-140*
B*(a-a*sin(f*x+e))^(3/2)*a^2*d^3+630*A*c^2*d*a^3*(a-a*sin(f*x+e))^(1/2)+210
*A*a^3*d^3*(a-a*sin(f*x+e))^(1/2)+210*B*c^3*a^3*(a-a*sin(f*x+e))^(1/2)+630*
B*a^3*c*d^2*(a-a*sin(f*x+e))^(1/2))/a^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^3}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^3}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^3}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2), x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.308 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=200

$$\frac{4(5Ad(3c-d) + B(6c^2 - 7cd + 7d^2)) \cos(e+fx)}{15f\sqrt{a \sin(e+fx) + a}} - \frac{2d(5Ad + 4Bc - Bd) \cos(e+fx)\sqrt{a \sin(e+fx) + a}}{15af} \sqrt{2}$$

[Out] $-(A-B)*(c-d)^2*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)}})*2^{(1/2)/f/a^{(1/2)}-4/15*(5*A*(3*c-d)*d+B*(6*c^2-7*c*d+7*d^2))*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(1/2)}-2/5*B*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/f/(a+a*\sin(f*x+e))^{(1/2)}-2/15*d*(5*A*d+4*B*c-B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/a/f$

Rubi [A] time = 0.58, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2983, 2968, 3023, 2751, 2649, 206}

$$\frac{4(5Ad(3c-d) + B(6c^2 - 7cd + 7d^2)) \cos(e+fx)}{15f\sqrt{a \sin(e+fx) + a}} - \frac{2d(5Ad + 4Bc - Bd) \cos(e+fx)\sqrt{a \sin(e+fx) + a}}{15af} \sqrt{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\sin[e + f*x])*(c + d*\sin[e + f*x])^2/\operatorname{Sqrt}[a + a*\sin[e + f*x]], x]$

[Out] $-\left(\frac{\operatorname{Sqrt}[2]*(A - B)*(c - d)^2*\operatorname{ArcTanh}[\frac{\operatorname{Sqrt}[a]*\cos[e + f*x]}{\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]]}]}{\operatorname{Sqrt}[a]*f} - \frac{4*(5*A*(3*c - d)*d + B*(6*c^2 - 7*c*d + 7*d^2))*\cos[e + f*x]}{(15*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]])} - \frac{(2*d*(4*B*c + 5*A*d - B*d))*\cos[e + f*x]*\operatorname{Sqrt}[a + a*\sin[e + f*x]]}{(15*a*f)} - \frac{(2*B*\cos[e + f*x])*(c + d*\sin[e + f*x])^2}{(5*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]])}\right)$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2]]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}Q[a, 0] \ \|\ \operatorname{Lt}Q[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\cos[c + d*x])/\operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2751

$\operatorname{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]^{(m_)*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow -\operatorname{Simp}[(d*\cos[e + f*x]*(a + b*\sin[e + f*x])^m)/(f*(m + 1)), x] + \operatorname{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \operatorname{Int}[(a + b*\sin[e + f*x])^m, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{!Lt}Q[m, -2^{(-1)}]$

Rule 2968

$\operatorname{Int}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)]]^{(m_)*((A_) + (B_)*\sin[(e_) + (f_)*(x_)])*((c_) + (d_)*\sin[(e_) + (f_)*(x_)])}, x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := -Simp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{\sqrt{a + a \sin(e + fx)}} dx = -\frac{2B \cos(e + fx)(c + d \sin(e + fx))^2}{5f\sqrt{a + a \sin(e + fx)}} + \frac{2 \int \frac{(c + d \sin(e + fx))\left(\frac{1}{2}a(5Ac + 5Ad - Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}\right)}{\sqrt{a + a \sin(e + fx)}} dx}{5f\sqrt{a + a \sin(e + fx)}} + \frac{2 \int \frac{\frac{1}{2}ac(5Ac - Bc + 4Bd) + \left(\frac{1}{2}a(5A(3c - d)d + B(6c^2 - 7cd + 7d^2)) \cos(e + fx)\right)}{\sqrt{a + a \sin(e + fx)}} dx}{5f\sqrt{a + a \sin(e + fx)}} - \frac{2d(4Bc + 5Ad - Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{15af} - \frac{2B \cos(e + fx)(c + d \sin(e + fx))^2}{5f\sqrt{a + a \sin(e + fx)}} - \frac{4(5A(3c - d)d + B(6c^2 - 7cd + 7d^2)) \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{2d(4Bc + 5Ad - Bd) \cos(e + fx)}{15f\sqrt{a + a \sin(e + fx)}} - \frac{\sqrt{2}(A - B)(c - d)^2 \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} f} - \frac{4(5A(3c - d)d + B(6c^2 - 7cd + 7d^2)) \cos(e + fx)}{\sqrt{a} f}$$

Mathematica [C] time = 0.54, size = 246, normalized size = 1.23

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(30(Ad(4c - d) + 2B(c^2 - cd + d^2)) \sin\left(\frac{1}{2}(e + fx)\right) - 30(Ad(4c - d) + 2B(c^2 - cd + d^2)) \cos\left(\frac{1}{2}(e + fx)\right)\right)}{\sqrt{a + a \sin(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/Sqrt[a + a*Sin[e + f*x]], x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((60 + 60*I)*(-1)^(3/4)*(A - B)*(c - d)^2*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]) - 30*(A*(4*c
```



```
+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^2*(tan((f*x+exp(1))/2)*(tan((f
*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(1/3600*tan((f*x+ex
p(1))/2)*(-600*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+1800*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)+1560*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)-1200*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
+1/3600*(1800*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-1800*B*a^2*c^2*sign(tan
((f*x+exp(1))/2)+1)-1800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*A*a^2*c
*d*sign(tan((f*x+exp(1))/2)+1)+3600*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1)))
+1/3600*(-2400*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)+4800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+7200*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)-4800*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+1/3600*(2400*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)-4800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-7200*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)+4800*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+1/3600*(-1800*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+1800*B*a^2*c^2*sign(t
an((f*x+exp(1))/2)+1)+1800*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)+3600*A*a^2
*c*d*sign(tan((f*x+exp(1))/2)+1)-3600*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1)
))+1/3600*(600*A*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-1800*B*a^2*c^2*sign(ta
n((f*x+exp(1))/2)+1)-1560*B*a^2*d^2*sign(tan((f*x+exp(1))/2)+1)-3600*A*a^2*
c*d*sign(tan((f*x+exp(1))/2)+1)+1200*B*a^2*c*d*sign(tan((f*x+exp(1))/2)+1))
)+sqrt(2)*(A*c^2+A*d^2-B*c^2-B*d^2-2*A*c*d+2*B*c*d)*atan((-sqrt(a)*tan((f*x
+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt
(-a)/sign(tan((f*x+exp(1))/2)+1)+(-15*A*a*c^2*sqrt(2)*atan(sqrt(a)/sqrt(-a)
)+30*A*a*c*d*sqrt(2)*atan(sqrt(a)/sqrt(-a))-15*A*a*d^2*sqrt(2)*atan(sqrt(a)
/sqrt(-a))+30*A*c*d*sqrt(-a)*sqrt(2)*sqrt(a)-10*A*d^2*sqrt(-a)*sqrt(2)*sqrt
(a)+15*B*a*c^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))-30*B*a*c*d*sqrt(2)*atan(sqrt(
a)/sqrt(-a))+15*B*a*d^2*sqrt(2)*atan(sqrt(a)/sqrt(-a))+15*B*c^2*sqrt(-a)*sq
rt(2)*sqrt(a)-20*B*c*d*sqrt(-a)*sqrt(2)*sqrt(a)+17*B*d^2*sqrt(-a)*sqrt(2)*s
qrt(a))/15/a/sqrt(-a)*sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.76, size = 396, normalized size = 1.98

$$(1 + \sin(fx + e)) \sqrt{-a(\sin(fx + e) - 1)} \left(15A a^{\frac{5}{2}} \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{2\sqrt{a}} \right) c^2 - 30A a^{\frac{5}{2}} \sqrt{2} \operatorname{arctanh} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)

```
[Out] -1/15*(1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)*(15*A*a^(5/2)*2^(1/2)*arctan
h(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2-30*A*a^(5/2)*2^(1/2)*arct
anh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d+15*A*a^(5/2)*2^(1/2)*ar
ctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2-15*B*a^(5/2)*2^(1/2)*
arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c^2+30*B*a^(5/2)*2^(1/2)
)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*c*d-15*B*a^(5/2)*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*d^2+6*B*(a-a*sin(f*
x+e))^(5/2)*d^2-10*A*(a-a*sin(f*x+e))^(3/2)*a*d^2-20*B*(a-a*sin(f*x+e))^(3/
2)*a*c*d-10*B*(a-a*sin(f*x+e))^(3/2)*a*d^2+60*A*a^2*c*d*(a-a*sin(f*x+e))^(1
/2)+30*B*a^2*c^2*(a-a*sin(f*x+e))^(1/2)+30*B*a^2*d^2*(a-a*sin(f*x+e))^(1/2)
)/a^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^2}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^2}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^2}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.309 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=130

$$\frac{2(3Ad + 3Bc - 2Bd) \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{\sqrt{2}(A - B)(c - d) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{\sqrt{a}f} - \frac{2Bd \cos(e + fx)\sqrt{a \sin(e + fx)}}{3af}$$

[Out] $-(A-B)*(c-d)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})$
 $*2^{(1/2)}/f/a^{(1/2)}-2/3*(3*A*d+3*B*c-2*B*d)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(1$
 $/2)-2/3*B*d*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/a/f$

Rubi [A] time = 0.27, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2968, 3023, 2751, 2649, 206}

$$\frac{2(3Ad + 3Bc - 2Bd) \cos(e + fx)}{3f\sqrt{a \sin(e + fx) + a}} - \frac{\sqrt{2}(A - B)(c - d) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a \sin(e + fx) + a}}\right)}{\sqrt{a}f} - \frac{2Bd \cos(e + fx)\sqrt{a \sin(e + fx)}}{3af}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\sin[e + f*x])*(c + d*\sin[e + f*x])/Sqrt[a + a*\sin[e + f*x]], x]$

[Out] $-(\operatorname{Sqrt}[2]*(A - B)*(c - d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])]/(\operatorname{Sqrt}[a]*f)) - (2*(3*B*c + 3*A*d - 2*B*d)*\operatorname{Cos}[e + f*x])$
 $/((3*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]]) - (2*B*d*\operatorname{Cos}[e + f*x]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])) / (3*a*f)$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt}Q[a, 0] \mid \mid \operatorname{Lt}Q[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*\sin[(c_ + (d_)*(x_))]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/Sqrt[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2751

$\operatorname{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*((c_ + (d_)*\sin[(e_ + (f_)*(x_))]), x_Symbol] \rightarrow -\operatorname{Simp}[(d*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m)/(f*(m + 1)), x] + \operatorname{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \operatorname{Int}[(a + b*\sin[e + f*x])^m, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& !\operatorname{Lt}Q[m, -2^{(-1)}]$

Rule 2968

$\operatorname{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*((A_ + (B_)*\sin[(e_ + (f_)*(x_))])*(c_ + (d_)*\sin[(e_ + (f_)*(x_))]), x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 3023

$\operatorname{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))]^{(m_)}*((A_ + (B_)*\sin[(e_ + (f_)*(x_))]) + (C_)*\sin[(e_ + (f_)*(x_))]^2), x_Symbol] \rightarrow -\operatorname{Simp}[(C*\operatorname{Cos}$

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{\sqrt{a + a \sin(e + fx)}} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx \\ &= -\frac{2Bd \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3af} + \frac{2 \int \frac{\frac{1}{2}a(3Ac+Bd) + \frac{1}{2}a(3Bc+3Ad)}{\sqrt{a+a \sin(e+fx)}}}{3a} \\ &= -\frac{2(3Bc + 3Ad - 2Bd) \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2Bd \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3af} \\ &= -\frac{2(3Bc + 3Ad - 2Bd) \cos(e + fx)}{3f \sqrt{a + a \sin(e + fx)}} - \frac{2Bd \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3af} \\ &= -\frac{\sqrt{2}(A - B)(c - d) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} f} - \frac{2(3Bc + 3Ad - 2Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{3f \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.48, size = 135, normalized size = 1.04

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(2\left(\cos\left(\frac{1}{2}(e + fx)\right) - \sin\left(\frac{1}{2}(e + fx)\right)\right) (3Ad + 3Bc + Bd \sin(e + fx) - Bd)\right)}{3f \sqrt{a(\sin(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/Sqrt[a + a*Sin[e + f*x]], x]
```

```
[Out] -1/3*((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((-6 - 6*I)*(-1)^(3/4)*(A - B)*(c - d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + 2*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(3*B*c + 3*A*d - B*d + B*d*Sin[e + f*x]))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.47, size = 303, normalized size = 2.33

$$3 \sqrt{2} \left((A-B)ac - (A-B)ad + ((A-B)ac - (A-B)ad) \cos(fx+e) + ((A-B)ac - (A-B)ad) \sin(fx+e) \right) \log \left(\frac{\cos(fx+e)^2 - (\cos(fx+e)-2) \sin(fx+e) - \frac{2\sqrt{2}\sqrt{a \sin(fx+e)+a}}{\sqrt{a}}}{\cos(fx+e)^2 - (\cos(fx+e)+2) \sin(fx+e)} \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2), x, algorith="fricas")
```

```
[Out] 1/6*(3*sqrt(2)*((A - B)*a*c - (A - B)*a*d + ((A - B)*a*c - (A - B)*a*d)*cos(f*x + e) + ((A - B)*a*c - (A - B)*a*d)*sin(f*x + e))*log(-(cos(f*x + e))^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e))^2
```


Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.310 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=79

$$-\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{2B \cos(e+fx)}{f \sqrt{a \sin(e+fx)+a}}$$

[Out] $-(A-B) \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot \cos(f \cdot x+e) \cdot a^{(1/2)} \cdot 2^{(1/2)} / (a+a \cdot \sin(f \cdot x+e))^{(1/2)}\right) \cdot 2^{(1/2)} / f / a^{(1/2)} - 2 \cdot B \cdot \cos(f \cdot x+e) / f / (a+a \cdot \sin(f \cdot x+e))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2751, 2649, 206}

$$-\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} f} - \frac{2B \cos(e+fx)}{f \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]

[Out] $-\left(\frac{\operatorname{Sqrt}[2] \cdot (A - B) \cdot \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a] \cdot \operatorname{Cos}[e + f \cdot x]}{\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[a + a \cdot \operatorname{Sin}[e + f \cdot x]]}\right]}{\operatorname{Sqrt}[a] \cdot f}\right) - \frac{2 \cdot B \cdot \operatorname{Cos}[e + f \cdot x]}{f \cdot \operatorname{Sqrt}[a + a \cdot \operatorname{Sin}[e + f \cdot x]]}$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx &= -\frac{2B \cos(e+fx)}{f \sqrt{a+a \sin(e+fx)}} + (A-B) \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx \\ &= -\frac{2B \cos(e+fx)}{f \sqrt{a+a \sin(e+fx)}} - \frac{(2(A-B)) \operatorname{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a \cos(e+fx)}{\sqrt{a+a \sin(e+fx)}}\right)}{f} \\ &= -\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a \sin(e+fx)}}\right)}{\sqrt{a} f} - \frac{2B \cos(e+fx)}{f \sqrt{a+a \sin(e+fx)}} \end{aligned}$$

Mathematica [C] time = 0.22, size = 106, normalized size = 1.34

$$\frac{2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(B \left(\sin\left(\frac{1}{2}(e + fx)\right) - \cos\left(\frac{1}{2}(e + fx)\right) \right) + (1 + i)(-1)^{3/4}(A - B) \tanh^{-1}\left(\frac{1}{2}\right) \right)}{f \sqrt{a(\sin(e + fx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/Sqrt[a + a*Sin[e + f*x]],x]
```

```
[Out] (2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((1 + I)*(-1)^(3/4)*(A - B)*ArcTan
h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] + B*(-Cos[(e + f*x)/2] +
Sin[(e + f*x)/2])))/(f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 0.45, size = 210, normalized size = 2.66

$$\frac{\sqrt{2}((A-B)a \cos(fx+e)+(A-B)a \sin(fx+e)+(A-B)a) \log\left(\frac{\cos(fx+e)^2 - (\cos(fx+e)-2) \sin(fx+e) + \frac{2\sqrt{2}\sqrt{a \sin(fx+e)+a}(\cos(fx+e)-\sin(fx+e)+1)}{\sqrt{a}} + 3 \cos(fx+e)}{\cos(fx+e)^2 - (\cos(fx+e)+2) \sin(fx+e) - \cos(fx+e) - 2}\right)}{\sqrt{a} \cdot 2 \left(af \cos(fx + e) + af \sin(fx + e) + af \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*((A - B)*a*cos(f*x + e) + (A - B)*a*sin(f*x + e) + (A - B)*a)
*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*
sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e
) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2
))/sqrt(a) + 4*(B*cos(f*x + e) - B*sin(f*x + e) + B)*sqrt(a*sin(f*x + e) +
a))/(a*f*cos(f*x + e) + a*f*sin(f*x + e) + a*f)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B/sign(ta
n((f*x+exp(1))/2)+1)+1/2*B*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)+1))
/(a*tan((f*x+exp(1))/2)^2+a)+sqrt(2)*(A-B)*atan((-sqrt(a)*tan((f*x+exp(1))/
2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(-a)/sign
(tan((f*x+exp(1))/2)+1)+(-A*a*sqrt(2)*atan(sqrt(a)/sqrt(-a))+B*a*sqrt(2)*at
an(sqrt(a)/sqrt(-a))+B*sqrt(-a)*sqrt(2)*sqrt(a))/a/sqrt(-a)*sign(tan((f*x+
exp(1))/2)+1))
```

maple [A] time = 1.29, size = 128, normalized size = 1.62

$$\frac{\left(1 + \sin(fx + e)\right) \sqrt{-a(\sin(fx + e) - 1)} \left(\sqrt{a} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin(fx+e)} \sqrt{2}}{2\sqrt{a}}\right) A - \sqrt{a} \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin(fx+e)} \sqrt{2}}{2\sqrt{a}}\right) \right)}{a \cos(fx + e) \sqrt{a + a \sin(fx + e)} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

[Out] $-(1+\sin(f*x+e))*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)}))*A-a^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*B+2*(a-a*\sin(f*x+e))^{(1/2)}*B/a/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/sqrt(a*sin(f*x + e) + a), x)

mupad [B] time = 1.06, size = 151, normalized size = 1.91

$$\frac{A F\left(\frac{\pi}{4} - \frac{e}{2} - \frac{fx}{2} \middle| 1\right) \sqrt{\frac{2(a+a \sin(e+fx))}{a}}}{f \sqrt{a+a \sin(e+fx)}} - \frac{B \left(4 E\left(\operatorname{asin}\left(\frac{\sqrt{2} \sqrt{1-\sin(e+fx)}}{2}\right) \middle| 1\right) - 2 F\left(\operatorname{asin}\left(\frac{\sqrt{2} \sqrt{1-\sin(e+fx)}}{2}\right) \middle| 1\right)\right)}{f \cos(e+fx) \sqrt{a+a \sin(e+fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(1/2),x)

[Out] $-(A*\operatorname{ellipticF}(\pi/4 - e/2 - (f*x)/2, 1)*((2*(a + a*\sin(e + f*x)))/a)^{(1/2)})/(f*(a + a*\sin(e + f*x))^{(1/2)}) - (B*(4*\operatorname{ellipticE}(\operatorname{asin}((2^{(1/2)}*(1 - \sin(e + f*x))^{(1/2)}))/2), 1) - 2*\operatorname{ellipticF}(\operatorname{asin}((2^{(1/2)}*(1 - \sin(e + f*x))^{(1/2)}))/2), 1))*(\cos(e + f*x)^2)^{(1/2)}*((a + a*\sin(e + f*x))/(2*a))^{(1/2)})/(f*\cos(e + f*x)*(a + a*\sin(e + f*x))^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x)

[Out] Integral((A + B*sin(e + f*x))/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.311 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=136

$$-\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} f(c-d)} - \frac{2(Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} \sqrt{d} f(c-d) \sqrt{c+d}}$$

[Out] $-(A-B) \operatorname{arctanh}\left(\frac{1/2 \cos(f*x+e) * a^{(1/2)} * 2^{(1/2)}}{(a+a \sin(f*x+e))^{(1/2)}}\right) * 2^{(1/2)} / (c-d) / f / a^{(1/2)} - 2 * (-A*d+B*c) \operatorname{arctanh}\left(\frac{\cos(f*x+e) * a^{(1/2)} * d^{(1/2)}}{(c+d)^{(1/2)} / (a+a \sin(f*x+e))^{(1/2)}}\right) / (c-d) / f / a^{(1/2)} / d^{(1/2)} / (c+d)^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2985, 2649, 206, 2773, 208}

$$-\frac{\sqrt{2}(A-B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} f(c-d)} - \frac{2(Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{\sqrt{a} \sqrt{d} f(c-d) \sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])),x]

[Out] $-\left(\frac{\sqrt{2} * (A - B) * \operatorname{ArcTanh}\left[\frac{\sqrt{a} * \cos[e + f*x]}{\sqrt{2} * \sqrt{a + a \sin[e + f*x]}}\right]}{\sqrt{a} * (c - d) * f}\right) - \frac{2 * (B * c - A * d) * \operatorname{ArcTanh}\left[\frac{\sqrt{a} * \sqrt{d} * \cos[e + f*x]}{\sqrt{c + d} * \sqrt{a + a \sin[e + f*x]}}\right]}{\sqrt{a} * (c - d) * \sqrt{d} * \sqrt{c + d} * f}$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2985

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x

] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} dx = \frac{(A - B) \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx}{c - d} + \frac{(Bc - Ad) \int \frac{\sqrt{a+a \sin(e+fx)}}{c+d \sin(e+fx)} dx}{a(c - d)}$$

$$= -\frac{(2(A - B)) \text{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a \cos(e+fx)}{\sqrt{a+a \sin(e+fx)}}\right)}{(c - d)f} - \frac{(2(Bc - Ad))}{(c - d)f}$$

$$= -\frac{\sqrt{2}(A - B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a \sin(e+fx)}}\right)}{\sqrt{a}(c - d)f} - \frac{2(Bc - Ad) \tanh^{-1}\left(\frac{c + d \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}}\right)}{\sqrt{a}(c - d)\sqrt{a}}$$

Mathematica [C] time = 3.34, size = 238, normalized size = 1.75

$$\frac{(-1)^{3/4} \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(\sqrt[4]{-1} (Bc - Ad) \left(\log\left(\sec^2\left(\frac{1}{4}(e + fx)\right)\right) \left(\sqrt{c + d} - \sqrt{d} \sin\left(\frac{1}{2}(e + fx)\right) \right) \right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x]
```

```
[Out] ((-1)^(3/4)*((2 + 2*I)*(A - B)*Sqrt[d]*Sqrt[c + d]*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]) + (-1)^(1/4)*(B*c - A*d)*(Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])] - Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])/((c - d)*Sqrt[d]*Sqrt[c + d]*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 1.49, size = 744, normalized size = 5.47

$$\sqrt{acd + ad^2} (Bc - Ad) \log \left(\frac{ad^2 \cos^3(fx+e) - ac^2 - 2acd - ad^2 - (6acd + 7ad^2) \cos^2(fx+e) - 4\sqrt{acd+ad^2} (d \cos(fx+e)^2 - (c+2d) \cos(fx+e) + c) \sin(fx+e) - (a^2c^2 + 8acd + 9ad^2) \cos(fx+e) + (ad^2 \cos^2(fx+e) - a^2c^2 - 2acd - ad^2 + 2(3acd + 4ad^2) \cos(fx+e)) \sin(fx+e)}{d^2 \cos^3(fx+e) + (2cd + d^2) \cos^2(fx+e) + (c + 2d) \cos(fx+e) + c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2), x, algorith="fricas")
```

```
[Out] [1/2*(sqrt(a*c*d + a*d^2)*(B*c - A*d)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos^2(f*x + e) + (c + 2*d)*cos(f*x + e) + c), x]
```

```

2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos
(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + sqrt
(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x + e) -
2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*
x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) +
2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f), -1/2*
(2*sqrt(-a*c*d - a*d^2)*(B*c - A*d)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*
sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))
) - sqrt(2)*((A - B)*a*c*d + (A - B)*a*d^2)*log(-(cos(f*x + e)^2 - (cos(f*x
+ e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e)
- sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*
x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a))/((a*c^2*d - a*d^3)*f
)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2),x, algo
rithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*
pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(co
s((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Dis
continuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not checkedU
nable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nos
tep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
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x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi
/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to chec
k sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)U
nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + f x)}{\sqrt{a + a \sin(e + f x)} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)

[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(1/2),x)

[Out] Timed out

$$3.312 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=207

$$-\frac{(Bc - Ad) \cos(e + fx)}{f(c^2 - d^2) \sqrt{a \sin(e + fx) + a(c + d \sin(e + fx))}} + \frac{(Ad(3c + d) - B(c^2 + cd + 2d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c+d} \sqrt{a \sin(e + fx)}}\right)}{\sqrt{a} \sqrt{d} f(c - d)^2 (c + d)^{3/2}}$$

[Out] $-(A-B) \cdot \operatorname{arctanh}\left(\frac{1/2 \cdot \cos(f \cdot x + e) \cdot a^{1/2} \cdot 2^{1/2}}{(a + a \cdot \sin(f \cdot x + e))^{1/2}}\right) \cdot 2^{1/2} / (c-d)^{2/2} / f / a^{1/2} + (A \cdot d \cdot (3 \cdot c + d) - B \cdot (c^2 + c \cdot d + 2 \cdot d^2)) \cdot \operatorname{arctanh}\left(\frac{\cos(f \cdot x + e) \cdot a^{1/2} \cdot d^{1/2}}{(c+d)^{1/2} \cdot (a + a \cdot \sin(f \cdot x + e))^{1/2}}\right) / (c-d)^{2/2} / (c+d)^{3/2} / f / a^{1/2} / d^{1/2} - (-A \cdot d + B \cdot c) \cdot \cos(f \cdot x + e) / (c^2 - d^2) / f / (c + d \cdot \sin(f \cdot x + e)) / (a + a \cdot \sin(f \cdot x + e))^{1/2}$

Rubi [A] time = 0.62, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$-\frac{(Bc - Ad) \cos(e + fx)}{f(c^2 - d^2) \sqrt{a \sin(e + fx) + a(c + d \sin(e + fx))}} + \frac{(Ad(3c + d) - B(c^2 + cd + 2d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e + fx)}{\sqrt{c+d} \sqrt{a \sin(e + fx)}}\right)}{\sqrt{a} \sqrt{d} f(c - d)^2 (c + d)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B \cdot \sin[e + f \cdot x]) / (\operatorname{Sqrt}[a + a \cdot \sin[e + f \cdot x]] \cdot (c + d \cdot \sin[e + f \cdot x])^2), x]$

[Out] $-\left(\frac{\operatorname{Sqrt}[2] \cdot (A - B) \cdot \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a] \cdot \cos[e + f \cdot x]}{\operatorname{Sqrt}[2] \cdot \operatorname{Sqrt}[a + a \cdot \sin[e + f \cdot x]]}\right]}{\operatorname{Sqrt}[a] \cdot (c - d)^2 \cdot f}\right) + \left(\frac{(A \cdot d \cdot (3 \cdot c + d) - B \cdot (c^2 + c \cdot d + 2 \cdot d^2)) \cdot \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a] \cdot \operatorname{Sqrt}[d] \cdot \cos[e + f \cdot x]}{\operatorname{Sqrt}[c + d] \cdot \operatorname{Sqrt}[a + a \cdot \sin[e + f \cdot x]]}\right]}{\operatorname{Sqrt}[a] \cdot (c - d)^2 \cdot \operatorname{Sqrt}[d] \cdot (c + d)^{3/2} \cdot f} - \frac{(B \cdot c - A \cdot d) \cdot \cos[e + f \cdot x]}{(c^2 - d^2) \cdot f \cdot \operatorname{Sqrt}[a + a \cdot \sin[e + f \cdot x]] \cdot (c + d \cdot \sin[e + f \cdot x])}\right)$

Rule 206

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 \cdot \operatorname{ArcTanh}[\operatorname{Rt}[-b, 2] \cdot x] / \operatorname{Rt}[a, 2]) / (\operatorname{Rt}[a, 2] \cdot \operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\operatorname{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2] \cdot \operatorname{ArcTanh}[x / \operatorname{Rt}[-(a/b), 2]]) / a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

$\operatorname{Int}[1 / \operatorname{Sqrt}[(a + (b \cdot x) \cdot \sin[(c \cdot x) + (d \cdot x) \cdot (x \cdot x)])], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1 / (2 \cdot a - x^2), x], x, (b \cdot \cos[c + d \cdot x]) / \operatorname{Sqrt}[a + b \cdot \sin[c + d \cdot x]]], x] /;$ FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

$\operatorname{Int}[\operatorname{Sqrt}[(a + (b \cdot x) \cdot \sin[(e \cdot x) + (f \cdot x) \cdot (x \cdot x)])] / ((c \cdot x) + (d \cdot x) \cdot \sin[(e \cdot x) + (f \cdot x) \cdot (x \cdot x)]), x_Symbol] \rightarrow \operatorname{Dist}[(-2 \cdot b) / f, \operatorname{Subst}[\operatorname{Int}[1 / (b \cdot c + a \cdot d - d \cdot x^2), x], x, (b \cdot \cos[e + f \cdot x]) / \operatorname{Sqrt}[a + b \cdot \sin[e + f \cdot x]]], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[b \cdot c - a \cdot d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} dx = -\frac{(Bc - Ad) \cos(e + fx)}{(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} - \frac{\int \frac{-\frac{1}{2}a(A(2c+d) + B(c^2-d^2)) \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}} dx}{(c^2-d^2) f \sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))} + \frac{(A-B) \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx}{(c^2-d^2) f \sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))} - \frac{(2(A-B)S)}{(c^2-d^2) f \sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))} = -\frac{(Bc - Ad) \cos(e + fx)}{(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} - \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} + \frac{(A - B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}} dx}{(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} - \frac{(2(A - B)S)}{(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))} = -\frac{\sqrt{2} (A - B) \tanh^{-1} \left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}} \right)}{\sqrt{a} (c - d)^2 f} + \frac{(Ad(3c + d) - B(c^2 + d^2)) \cos(e + fx)}{\sqrt{a} (c + d)^{3/2}}$$

Mathematica [C] time = 6.97, size = 374, normalized size = 1.81

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \left(-\frac{(B(c^2 + cd + 2d^2) - Ad(3c + d)) \left(2 \log\left(\sec^2\left(\frac{1}{4}(e + fx)\right)\right) \left(\sqrt{c + d} - \sqrt{d} \sin\left(\frac{1}{2}(e + fx)\right) + \sqrt{d} \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)}{\sqrt{d} (c + d)^{3/2}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/(Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2),x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*((8 + 8*I)*(-1)^(3/4)*(A - B)*ArcTan[h[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])]/(Sqrt[d]*(c + d)^(3/2)) + ((-(A*d*(3*c + d)) + B*(c^2 + c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])]/(Sqrt[d]*(c + d)^(3/2)) - (4*(c - d)*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(c + d)*(c + d*Sin[e + f*x])))/(4*(c - d)^2*f*Sqrt[a*(1 + Sin[e + f*x])])
```

fricas [B] time = 3.77, size = 2159, normalized size = 10.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [-1/4*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 - 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e)) - 2*sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e))^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) - 4*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x + e)), 1/2*((B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 - (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e)^2 + (B*c^3 - (3*A - B)*c^2*d - (A - 2*B)*c*d^2)*cos(f*x + e) + (B*c^3 - (3*A - 2*B)*c^2*d - (4*A - 3*B)*c*d^2 - (A - 2*B)*d^3 + (B*c^2*d - (3*A - B)*c*d^2 - (A - 2*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(-a*c*d - a*d^2)*arctan(1/2*sqrt(-a*c*d - a*d^2)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*cos(f*x + e))) + sqrt(2)*((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 - ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e)^2 + ((A - B)*a*c^3*d + 2*(A - B)*a*c^2*d^2 + (A - B)*a*c*d^3)*cos(f*x + e) + ((A - B)*a*c^3*d + 3*(A - B)*a*c^2*d^2 + 3*(A - B)*a*c*d^3 + (A - B)*a*d^4 + ((A - B)*a*c^2*d^2 + 2*(A - B)*a*c*d^3 + (A - B)*a*d^4)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e))^2 - (cos(f*x + e) - 2)*sin(f*x + e) + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 2*(B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4 + (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*cos(f*x + e) - (B*c^3*d - A*c^2*d^2 - B*c*d^3 + A*d^4)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e)^2 - (a*c^5*d - 2*a*c^3*d^3 + a*c*d^5)*f*cos(f*x + e) - (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f - ((a*c^4*d^2 - 2*a*c^2*d^4 + a*d^6)*f*cos(f*x + e) + (a*c^5*d + a*c^4*d^2 - 2*a*c^3*d^3 - 2*a*c^2*d^4 + a*c*d^5 + a*d^6)*f)*sin(f*x + e))]

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] Timed out

maple [B] time = 2.72, size = 899, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x)

[Out] (1+sin(f*x+e))*(-a*(sin(f*x+e)-1))^(1/2)/a^(5/2)*(sin(f*x+e)*d*(3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d-2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d+3*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d+A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c^2*d-2*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2+B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d)/(c-d)^2/(c+d)/(c+d*sin(f*x+e))/(a*(c+d)*d)^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{\sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/(sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2), x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(1/2), x)
```

```
[Out] Timed out
```

$$3.313 \quad \int \frac{A+B \sin(e+fx)}{\sqrt{a+a \sin(e+fx)} (c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=309

$$\frac{(Ad(7c+d) - B(3c^2 + cd + 4d^2)) \cos(e+fx)}{4f(c^2 - d^2)^2 \sqrt{a \sin(e+fx) + a} (c+d \sin(e+fx))} - \frac{(Bc - Ad) \cos(e+fx)}{2f(c^2 - d^2) \sqrt{a \sin(e+fx) + a} (c+d \sin(e+fx))^2} + \frac{(Ad(1$$

[Out] $-(A-B) \operatorname{arctanh}\left(\frac{1}{2} \cos(fx+e) a^{1/2} 2^{1/2} / (a+a \sin(fx+e))^{1/2}\right) 2^{1/2} / (c-d)^3 / f / a^{1/2} + 1/4 (A d (15 c^2 + 10 c d + 7 d^2) - B (3 c^3 + 6 c^2 d + 19 c d^2 + 4 d^3)) \operatorname{arctanh}\left(\frac{\cos(fx+e) a^{1/2} d^{1/2}}{(c+d)^{1/2} (a+a \sin(fx+e))^{1/2}}\right) / (c-d)^3 / (c+d)^{5/2} / f / a^{1/2} / d^{1/2} - 1/2 (-A d + B c) \cos(fx+e) / (c^2 - d^2) / f / (c+d \sin(fx+e))^2 / (a+a \sin(fx+e))^{1/2} + 1/4 (A d (7 c + d) - B (3 c^2 + c d + 4 d^2)) \cos(fx+e) / (c^2 - d^2)^2 / f / (c+d \sin(fx+e)) / (a+a \sin(fx+e))^{1/2}$

Rubi [A] time = 1.05, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2984, 2985, 2649, 206, 2773, 208}

$$\frac{(Ad(7c+d) - B(3c^2 + cd + 4d^2)) \cos(e+fx)}{4f(c^2 - d^2)^2 \sqrt{a \sin(e+fx) + a} (c+d \sin(e+fx))} - \frac{(Bc - Ad) \cos(e+fx)}{2f(c^2 - d^2) \sqrt{a \sin(e+fx) + a} (c+d \sin(e+fx))^2} + \frac{(Ad(1$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B \sin[e + f*x]) / (\text{Sqrt}[a + a \sin[e + f*x]] * (c + d \sin[e + f*x])^3), x]$

[Out] $-\left(\frac{\text{Sqrt}[2] * (A - B) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a] * \text{Cos}[e + f*x]}{\text{Sqrt}[2] * \text{Sqrt}[a + a \sin[e + f*x]]}\right]}{\text{Sqrt}[a] * (c - d)^3 * f}\right) + \left(\frac{(A d (15 c^2 + 10 c d + 7 d^2) - B (3 c^3 + 6 c^2 d + 19 c d^2 + 4 d^3)) * \text{ArcTanh}\left[\frac{\text{Sqrt}[a] * \text{Sqrt}[d] * \text{Cos}[e + f*x]}{\text{Sqrt}[c + d] * \text{Sqrt}[a + a \sin[e + f*x]]}\right]}{4 * \text{Sqrt}[a] * (c - d)^3 * \text{Sqrt}[d] * (c + d)^{5/2} * f}\right) - \left(\frac{(B * c - A * d) * \text{Cos}[e + f*x]}{2 * (c^2 - d^2) * f * \text{Sqrt}[a + a \sin[e + f*x]] * (c + d \sin[e + f*x])^2}\right) + \left(\frac{(A d (7 c + d) - B (3 c^2 + c d + 4 d^2)) * \text{Cos}[e + f*x]}{4 * (c^2 - d^2)^2 * f * \text{Sqrt}[a + a \sin[e + f*x]] * (c + d \sin[e + f*x])}\right)$

Rule 206

$\text{Int}[(a_) + (b_) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 * \text{ArcTanh}[\text{Rt}[-b, 2] * x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] * \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_) + (b_) * (x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b]$

Rule 2649

$\text{Int}[1 / \text{Sqrt}[(a_) + (b_) * \sin[(c_) + (d_) * (x_)]], x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1 / (2 * a - x^2), x], x, (b * \text{Cos}[c + d * x]) / \text{Sqrt}[a + b * \sin[c + d * x]]], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2773

$\text{Int}[\text{Sqrt}[(a_) + (b_) * \sin[(e_) + (f_) * (x_)]] / ((c_) + (d_) * \sin[(e_) + (f_) * (x_)]), x_Symbol] \rightarrow \text{Dist}[(-2 * b) / f, \text{Subst}[\text{Int}[1 / (b * c + a * d - d * x^2), x$

], x, (b*cos[e + f*x])/sqrt[a + b*sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2984

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 2985

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])]*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/sqrt[a + b*sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[sqrt[a + b*sin[e + f*x]]/(c + d*sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + B \sin(e + fx)}{\sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^3} dx &= -\frac{(Bc - Ad) \cos(e + fx)}{2(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} - \int \frac{-\frac{1}{2} a}{\dots} \\ &= -\frac{(Bc - Ad) \cos(e + fx)}{2(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} + \frac{(Aa - Bc)}{4(c^2 - d^2)} \\ &= -\frac{(Bc - Ad) \cos(e + fx)}{2(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} + \frac{(Aa - Bc)}{4(c^2 - d^2)} \\ &= -\frac{(Bc - Ad) \cos(e + fx)}{2(c^2 - d^2) f \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^2} + \frac{(Aa - Bc)}{4(c^2 - d^2)} \\ &= -\frac{\sqrt{2}(A - B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{\sqrt{a} (c - d)^3 f} + \frac{(Ad(15c^2 + 10cd + 5d^2) - Aa^2)}{4(c^2 - d^2)} \end{aligned}$$

Mathematica [C] time = 10.78, size = 847, normalized size = 2.74

$$\frac{(2 + 2i)(A - B) \tanh^{-1}\left(\left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \sec\left(\frac{1}{4}(e + fx)\right) \left(\cos\left(\frac{1}{4}(e + fx)\right) - \sin\left(\frac{1}{4}(e + fx)\right)\right)\right) \left(\cos\left(\frac{1}{2}(e + fx)\right)\right)}{\left(\sqrt[4]{-1} c^3 - 3\sqrt[4]{-1} dc^2 + 3\sqrt[4]{-1} d^2c - \sqrt[4]{-1} d^3\right) f \sqrt{a} (\sin(e + fx) + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*sin[e + f*x])/(sqrt[a + a*sin[e + f*x]]*(c + d*sin[e + f*x])^3), x]

```
[Out] ((2 + 2*I)*(A - B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(e + f*x)/4]*(Cos[(e + f*x)/4] - Sin[(e + f*x)/4])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))/(((-1)^(1/4)*c^3 - 3*(-1)^(1/4)*c^2*d + 3*(-1)^(1/4)*c*d^2 - (-1)^(1/4)*d^3)*f*Sqrt[a*(1 + Sin[e + f*x])] - ((-A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])/(16*(c - d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((-A*d*(15*c^2 + 10*c*d + 7*d^2)) + B*(3*c^3 + 6*c^2*d + 19*c*d^2 + 4*d^3))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])/(16*(c - d)^3*Sqrt[d]*(c + d)^(5/2)*f*Sqrt[a*(1 + Sin[e + f*x])]) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-B*c*Cos[(e + f*x)/2]) + A*d*Cos[(e + f*x)/2] + B*c*Sin[(e + f*x)/2] - A*d*Sin[(e + f*x)/2]))/(2*(c - d)*(c + d)*f*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^2) + ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-3*B*c^2*Cos[(e + f*x)/2] + 7*A*c*d*Cos[(e + f*x)/2] - B*c*d*Cos[(e + f*x)/2] + A*d^2*Cos[(e + f*x)/2] - 4*B*d^2*Cos[(e + f*x)/2] + 3*B*c^2*Sin[(e + f*x)/2] - 7*A*c*d*Sin[(e + f*x)/2] + B*c*d*Sin[(e + f*x)/2] - A*d^2*Sin[(e + f*x)/2] + 4*B*d^2*Sin[(e + f*x)/2]))/(4*(c - d)^2*(c + d)^2*f*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x]))
```

fricas [B] time = 7.78, size = 4180, normalized size = 13.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (35*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e) + (3*B*c^5 - 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*cos(f*x + e)^2 + 2*(3*B*c^4*d - 3*(5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*cos(f*x + e))*sin(f*x + e))*sqrt(a*c*d + a*d^2)*log((a*d^2*cos(f*x + e)^3 - a*c^2 - 2*a*c*d - a*d^2 - (6*a*c*d + 7*a*d^2)*cos(f*x + e)^2 + 4*sqrt(a*c*d + a*d^2)*(d*cos(f*x + e)^2 - (c + 2*d)*cos(f*x + e) + (d*cos(f*x + e) + c + 3*d)*sin(f*x + e) - c - 3*d)*sqrt(a*sin(f*x + e) + a) - (a*c^2 + 8*a*c*d + 9*a*d^2)*cos(f*x + e) + (a*d^2*cos(f*x + e)^2 - a*c^2 - 2*a*c*d - a*d^2 + 2*(3*a*c*d + 4*a*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) - 8*sqrt(2)*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3*(A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e) + ((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*cos(f*x + e)^2 + 2*((A - B)*a*c^4*d^2 + 3*(A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*cos(f*x + e))*sin(f*x + e))*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a))*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt
```


$$\begin{aligned}
& t(a) + 3\cos(f*x + e) + 2)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)\sin(f*x + e) \\
& - \cos(f*x + e) - 2))/\sqrt{a} + 4*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A \\
& - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 \\
& + (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A \\
& - 4*B)*d^6)*\cos(f*x + e)^2 + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^3 \\
& *d^3 + (11*A - B)*c^2*d^4 + (A - 2*B)*c*d^5 - 2*A*d^6)*\cos(f*x + e) - (5*B \\
& *c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 \\
& - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A \\
& - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*\cos(f*x + e))*\sin(f*x + e) \\
&)*\sqrt{a*\sin(f*x + e) + a)} / ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9) \\
&)*f*\cos(f*x + e)^3 + (2*a*c^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 + \\
& 6*a*c^3*d^6 + 3*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f*\cos(f*x + e)^2 - (a*c^8*d \\
& - 2*a*c^6*d^3 + 2*a*c^2*d^7 - a*d^9)*f*\cos(f*x + e) - (a*c^8*d + 2*a*c^7*d \\
& ^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a \\
& *d^9)*f + ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*\cos(f*x + e)^2 \\
& - 2*(a*c^7*d^2 - 3*a*c^5*d^4 + 3*a*c^3*d^6 - a*c*d^8)*f*\cos(f*x + e) - (a*c \\
& ^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 \\
& - 2*a*c*d^8 - a*d^9)*f)*\sin(f*x + e)), 1/8*((3*B*c^5 - 3*(5*A - 4*B)*c^4*d \\
& - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (\\
& 7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 - (10*A - 19*B)*c*d^4 \\
& - (7*A - 4*B)*d^5)*\cos(f*x + e)^3 - (6*B*c^4*d - 15*(2*A - B)*c^3*d^2 - (3 \\
& 5*A - 44*B)*c^2*d^3 - 3*(8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5)*\cos(f*x + e)^2 \\
& + (3*B*c^5 - 3*(5*A - 2*B)*c^4*d - 2*(5*A - 11*B)*c^3*d^2 - 2*(11*A - 5*B) \\
& *c^2*d^3 - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*\cos(f*x + e) + (3*B*c^5 - \\
& 3*(5*A - 4*B)*c^4*d - 2*(20*A - 17*B)*c^3*d^2 - 6*(7*A - 8*B)*c^2*d^3 - 3* \\
& (8*A - 9*B)*c*d^4 - (7*A - 4*B)*d^5 - (3*B*c^3*d^2 - 3*(5*A - 2*B)*c^2*d^3 \\
& - (10*A - 19*B)*c*d^4 - (7*A - 4*B)*d^5)*\cos(f*x + e)^2 + 2*(3*B*c^4*d - 3* \\
& (5*A - 2*B)*c^3*d^2 - (10*A - 19*B)*c^2*d^3 - (7*A - 4*B)*c*d^4)*\cos(f*x + \\
& e))*\sin(f*x + e))*\sqrt{-a*c*d - a*d^2}*\arctan(1/2*\sqrt{-a*c*d - a*d^2}*\sqrt{ \\
& a*\sin(f*x + e) + a}*(d*\sin(f*x + e) - c - 2*d)/((a*c*d + a*d^2)*\cos(f*x + \\
& e))) - 4*\sqrt{2}*((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d^2 + 10*(A - B)*a*c^3* \\
& d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6 - ((A - B)*a \\
& *c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + (A - B)*a*d^6)*\cos(f*x \\
& + e)^3 - (2*(A - B)*a*c^4*d^2 + 7*(A - B)*a*c^3*d^3 + 9*(A - B)*a*c^2*d^4 \\
& + 5*(A - B)*a*c*d^5 + (A - B)*a*d^6)*\cos(f*x + e)^2 + ((A - B)*a*c^5*d + 3* \\
& (A - B)*a*c^4*d^2 + 4*(A - B)*a*c^3*d^3 + 4*(A - B)*a*c^2*d^4 + 3*(A - B)*a \\
& *c*d^5 + (A - B)*a*d^6)*\cos(f*x + e) + ((A - B)*a*c^5*d + 5*(A - B)*a*c^4*d \\
& ^2 + 10*(A - B)*a*c^3*d^3 + 10*(A - B)*a*c^2*d^4 + 5*(A - B)*a*c*d^5 + (A - \\
& B)*a*d^6 - ((A - B)*a*c^3*d^3 + 3*(A - B)*a*c^2*d^4 + 3*(A - B)*a*c*d^5 + \\
& (A - B)*a*d^6)*\cos(f*x + e)^2 + 2*((A - B)*a*c^4*d^2 + 3*(A - B)*a*c^3*d^3 \\
& + 3*(A - B)*a*c^2*d^4 + (A - B)*a*c*d^5)*\cos(f*x + e))*\sin(f*x + e))*\log(- \\
& (\cos(f*x + e)^2 - (\cos(f*x + e) - 2)\sin(f*x + e) - 2*\sqrt{2}*\sqrt{a*\sin(f*x \\
& + e) + a}*(\cos(f*x + e) - \sin(f*x + e) + 1))/\sqrt{a} + 3*\cos(f*x + e) + 2)/ \\
& (\cos(f*x + e)^2 - (\cos(f*x + e) + 2)\sin(f*x + e) - \cos(f*x + e) - 2))/\sqrt{ \\
& a} + 2*(5*B*c^5*d - (9*A + 2*B)*c^4*d^2 + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - \\
& B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - 4*B)*d^6 + (3*B*c^4*d^2 - (7*A - B)* \\
& c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c*d^5 + (A - 4*B)*d^6)*\cos(f*x + e)^2 \\
& + (5*B*c^5*d - (9*A - B)*c^4*d^2 - (A + 3*B)*c^3*d^3 + (11*A - B)*c^2*d^4 \\
& + (A - 2*B)*c*d^5 - 2*A*d^6)*\cos(f*x + e) - (5*B*c^5*d - (9*A + 2*B)*c^4*d^2 \\
& + 2*(3*A - 2*B)*c^3*d^3 + 2*(6*A - B)*c^2*d^4 - (6*A + B)*c*d^5 - (3*A - \\
& 4*B)*d^6 - (3*B*c^4*d^2 - (7*A - B)*c^3*d^3 - (A - B)*c^2*d^4 + (7*A - B)*c \\
& *d^5 + (A - 4*B)*d^6)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a*\sin(f*x + e) + a)} \\
& / ((a*c^6*d^3 - 3*a*c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*\cos(f*x + e)^3 + (2*a*c \\
& ^7*d^2 + a*c^6*d^3 - 6*a*c^5*d^4 - 3*a*c^4*d^5 + 6*a*c^3*d^6 + 3*a*c^2*d^7 \\
& - 2*a*c*d^8 - a*d^9)*f*\cos(f*x + e)^2 - (a*c^8*d - 2*a*c^6*d^3 + 2*a*c^2*d^7 \\
& - a*d^9)*f*\cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^6*d^3 - 6*a*c^5* \\
& d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f + ((a*c^6*d^3 - 3*a* \\
& c^4*d^5 + 3*a*c^2*d^7 - a*d^9)*f*\cos(f*x + e)^2 - 2*(a*c^7*d^2 - 3*a*c^5*d^4 \\
& + 3*a*c^3*d^6 - a*c*d^8)*f*\cos(f*x + e) - (a*c^8*d + 2*a*c^7*d^2 - 2*a*c^
\end{aligned}$$

```
6*d^3 - 6*a*c^5*d^4 + 6*a*c^3*d^6 + 2*a*c^2*d^7 - 2*a*c*d^8 - a*d^9)*f)*sin
(f*x + e))]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (4*pi/x/2
)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check si
gn: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)
Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integrat
ion of abs or sign assumes constant sign by intervals (correct if the argum
ent is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check si
gn: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_noste
p/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1)
)/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_n
ostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to
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i/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
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i/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*
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nable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)
>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sig
n: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*
pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to c
heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/
2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x
/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check
sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(
-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign:
```



```

0,9,1,1]%%}+%%{%%{3,[6]%%},[0,8,2,1]%%}+%%{%%{-1,[6]%%},[0,7,3,1]%%}
% / %%{%%{-1,[3]%%},[12,3,0,0]%%}+%%{%%{poly1[%%{12,[3]%%},0]:[1,0,
%%{-1,[1]%%}]%%},[11,2,1,0]%%}+%%{%%{-6,[4]%%},[10,3,0,0]%%}+%%{%%
{-48,[4]%%},[10,1,2,0]%%}+%%{%%{poly1[%%{36,[4]%%},0]:[1,0,%%{-1,[1]
%%}]%%},[9,2,1,0]%%}+%%{%%{poly1[%%{64,[4]%%},0]:[1,0,%%{-1,[1]%%}]%%
},[9,0,3,0]%%}+%%{%%{-15,[5]%%},[8,3,0,0]%%}+%%{%%{poly1[%%{24,[5]
%%},0]:[1,0,%%{-1,[1]%%}]%%},[7,2,1,0]%%}+%%{%%{poly1[%%{-192,[5]%%},0
]:[1,0,%%{-1,[1]%%}]%%},[7,0,3,0]%%}+%%{%%{-20,[6]%%},[6,3,0,0]%%}+
%%{%%{96,[6]%%},[6,1,2,0]%%}+%%{%%{poly1[%%{-24,[6]%%},0]:[1,0,%%{-1
,[1]%%}]%%},[5,2,1,0]%%}+%%{%%{poly1[%%{192,[6]%%},0]:[1,0,%%{-1,[1]
%%}]%%},[5,0,3,0]%%}+%%{%%{-15,[7]%%},[4,3,0,0]%%}+%%{%%{poly1[%%{-3
6,[7]%%},0]:[1,0,%%{-1,[1]%%}]%%},[3,2,1,0]%%}+%%{%%{poly1[%%{-64,[7]
%%},0]:[1,0,%%{-1,[1]%%}]%%},[3,0,3,0]%%}+%%{%%{-6,[8]%%},[2,3,0,0]%%}
%%}+%%{%%{-48,[8]%%},[2,1,2,0]%%}+%%{%%{poly1[%%{-12,[8]%%},0]:[1,0,
%%{-1,[1]%%}]%%},[1,2,1,0]%%}+%%{%%{-1,[9]%%},[0,3,0,0]%%} Error: Ba
d Argument Value

```

maple [B] time = 3.58, size = 2277, normalized size = 7.37

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \frac{(A+B \sin(f*x+e))}{(c+d \sin(f*x+e))^3 (a+a \sin(f*x+e))^{1/2}} dx$

[Out]
$$\begin{aligned}
 & -1/4*(8*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c*d^3+16*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c^2*d^2+6*B*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{1/2})*d/(a*(c+d)*d)^{1/2})*a^{9/2}*\sin(f*x+e)*c^4*d-8*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*a^4*c^3*d-4*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*a^4*c^2*d^2+3*B*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{1/2})*d/(a*(c+d)*d)^{1/2})*a^{9/2}*c^5-8*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c^3*d-16*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c^2*d^2-8*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c*d^3+4*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*c^2*d^2+8*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*c*d^3-4*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*c^2*d^2-8*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*c*d^3+8*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)*a^4*c^3*d+7*A*(-a*(\sin(f*x+e)-1))^{3/2}*(a*(c+d)*d)^{1/2})*a^{5/2}*c^2*d^2-3*B*(-a*(\sin(f*x+e)-1))^{3/2}*(a*(c+d)*d)^{1/2})*a^{5/2}*c^3*d-6*A*(-a*(\sin(f*x+e)-1))^{3/2}*(a*(c+d)*d)^{1/2})*a^{5/2}*c*d^3+2*B*(-a*(\sin(f*x+e)-1))^{3/2}*(a*(c+d)*d)^{1/2})*a^{5/2}*c^2*d^2-3*B*(-a*(\sin(f*x+e)-1))^{3/2}*(a*(c+d)*d)^{1/2})*a^{5/2}*c*d^3+4*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*a^4*c^4+4*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*d^4-4*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*\sin(f*x+e)^2*a^4*d^4+8*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*a^4*c^3*d+4*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{1/2})*2^{1/2}/a^{1/2})*a^4*c^2*d^2-B*(-a*(\sin(f*x+e)-1))^{1/2}*(a*(c+d)*d)^{1/2})*a^{7/2}*c^2*d^2+B*(-a*(\sin(f*x+e)-1))^{1/2}*(a*(c+d)*d)^{1/2})*a^{7/2}*c*d^3-15*A*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{1/2})*d/(a*(c+d)*d)^{1/2})*a^{9/2}*\sin(f*x+e)^2*c^2*d^3-10*A*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{1/2})*d/(a*(c+d)*d)^{1/2})*a^{9/2}*\sin(f*x+e)^2*c^3*d^2+6*B*\operatorname{arctanh}((-a*(\sin(f*x+e)-1))^{1/2})*d/(a*(c+d)*d)^{1/2})*a^{9/2}*\sin(f
 \end{aligned}$$

```

*x+e)^2*c^2*d^3+19*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(9/2)*sin(f*x+e)^2*c*d^4-30*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+
d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2-20*A*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3-14*A*arctanh((-a*(sin(f
*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4-9*A*(-a*(sin(
f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+12*B*arctanh((-a*(sin(f*x+
e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^3*d^2+5*B*(-a*(sin(f
*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c^4-10*A*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+38*B*arctanh((-a*(sin(f*x+e)
-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c^2*d^3+9*A*(-a*(sin(f*x
+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*c*d^3-B*(-a*(sin(f*x+e)-1))^(1/2)*(
a*(c+d)*d)^(1/2)*a^(7/2)*c^3*d+8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(
c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)*c*d^4+19*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^3*d^2+4*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^2*d^3-A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+
d)*d)^(1/2)*a^(7/2)*d^4+A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/
2)*c^2*d^2-7*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/
2)*sin(f*x+e)^2*d^5-4*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+
e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^4*c^4+4*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(9/2)*sin(f*x+e)^2*d^5-7*A*arctanh((-a*(sin(f*x+e)-
1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(9/2)*c^2*d^3-A*(-a*(sin(f*x+e)-1))^(3/2)*
(a*(c+d)*d)^(1/2)*a^(5/2)*d^4+4*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/
2)*a^(5/2)*d^4-15*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*
a^(9/2)*c^4*d+6*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(9/2)*c^4*d-4*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(7/2)*d^4/a^
(9/2)*(-a*(sin(f*x+e)-1))^(1/2)*(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin(f
*x+e))^2/(c+d)^2/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x, alg
orithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{\sqrt{a + a \sin(e + f x)} (c + d \sin(e + f x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^3
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(1/2),x)
```

```
[Out] Timed out
```

$$3.314 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=283

$$\frac{(c-d)^2(A(c+11d)+3B(c-5d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} + \frac{d^2(15Ac-35Ad-51Bc+39Bd) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{30a^2 f}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/f/(a+a*\sin(f*x+e))^{3/2}-1/4*(c-d)^2*(3*B*(c-5*d)+A*(c+11*d))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{1/2}*2^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{3/2}/f*2^{1/2}+1/15*d*(15*A*c^2-120*A*c*d+65*A*d^2-99*B*c^2+168*B*c*d-93*B*d^2)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{1/2}+1/10*(5*A-9*B)*d*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/a/f/(a+a*\sin(f*x+e))^{1/2}+1/30*d^2*(15*A*c-35*A*d-51*B*c+39*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{1/2}/a^2/f$

Rubi [A] time = 1.00, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2977, 2983, 2968, 3023, 2751, 2649, 206}

$$\frac{d^2(15Ac-35Ad-51Bc+39Bd) \cos(e+fx) \sqrt{a \sin(e+fx)+a}}{30a^2 f} - \frac{(c-d)^2(A(c+11d)+3B(c-5d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $-((c-d)^2*(3*B*(c-5*d)+A*(c+11*d))*\operatorname{ArcTanh}[\frac{\sqrt{a}*\cos[e+f*x]}{\sqrt{2}*\sqrt{a+a*\sin[e+f*x]}}])/(2*\sqrt{2}*a^{3/2}*f)+(d*(15*A*c^2-99*B*c^2-120*A*c*d+168*B*c*d+65*A*d^2-93*B*d^2)*\cos[e+f*x])/(15*a*f*\sqrt{a+a*\sin[e+f*x]})+(d^2*(15*A*c-51*B*c-35*A*d+39*B*d)*\cos[e+f*x]*\sqrt{a+a*\sin[e+f*x]})/(30*a^2*f)+((5*A-9*B)*d*\cos[e+f*x]*(c+d*\sin[e+f*x])^2)/(10*a*f*\sqrt{a+a*\sin[e+f*x]})-((A-B)*\cos[e+f*x]*(c+d*\sin[e+f*x])^3)/(2*f*(a+a*\sin[e+f*x])^{3/2})$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a

+ b*Sin[e + f*x]^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2977

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])

Rule 2983

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Si
mp[(B*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(m + n
+ 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[
e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m +
n + 1) + B*(a*d*m + b*c*n))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d,
e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{3/2}} dx &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{2f(a + a \sin(e + fx))^{3/2}} + \int \frac{(c + d \sin(e + fx))^2 \left(\frac{1}{2}a(3B\right)}{\dots} \\
&= \frac{(5A - 9B)d \cos(e + fx)(c + d \sin(e + fx))^2}{10af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))} \\
&= \frac{(5A - 9B)d \cos(e + fx)(c + d \sin(e + fx))^2}{10af\sqrt{a + a \sin(e + fx)}} - \frac{(A - B) \cos(e + fx)}{2f(a + a \sin(e + fx))} \\
&= \frac{d^2(15Ac - 51Bc - 35Ad + 39Bd) \cos(e + fx)\sqrt{a + a \sin(e + fx)}}{30a^2f} \\
&= \frac{d(15Ac^2 - 99Bc^2 - 120Acd + 168Bcd + 65Ad^2 - 93Bd^2) \cos(e + fx)}{15af\sqrt{a + a \sin(e + fx)}} \\
&= \frac{d(15Ac^2 - 99Bc^2 - 120Acd + 168Bcd + 65Ad^2 - 93Bd^2) \cos(e + fx)}{15af\sqrt{a + a \sin(e + fx)}} \\
&= -\frac{(c - d)^2(3B(c - 5d) + A(c + 11d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{2\sqrt{2}a^{3/2}f} + \dots
\end{aligned}$$

Mathematica [C] time = 1.07, size = 684, normalized size = 2.42

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left((30 + 30i)(-1)^{3/4}(c - d)^2(A(c + 11d) + 3B(c - 5d)) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(3/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-30*A*c^3*Cos[(e + f*x)/2] + 30*B*c^3*Cos[(e + f*x)/2] + 90*A*c^2*d*Cos[(e + f*x)/2] - 270*B*c^2*d*Cos[(e + f*x)/2] - 270*A*c*d^2*Cos[(e + f*x)/2] + 330*B*c*d^2*Cos[(e + f*x)/2] + 110*A*d^3*Cos[(e + f*x)/2] - 165*B*d^3*Cos[(e + f*x)/2] - 180*B*c^2*d*Cos[(3*(e + f*x))/2] - 180*A*c*d^2*Cos[(3*(e + f*x))/2] + 210*B*c*d^2*Cos[(3*(e + f*x))/2] + 70*A*d^3*Cos[(3*(e + f*x))/2] - 123*B*d^3*Cos[(3*(e + f*x))/2] + 30*B*c*d^2*Cos[(5*(e + f*x))/2] + 10*A*d^3*Cos[(5*(e + f*x))/2] - 9*B*d^3*Cos[(5*(e + f*x))/2] + 3*B*d^3*Cos[(7*(e + f*x))/2] + 30*A*c^3*Sin[(e + f*x)/2] - 30*B*c^3*Sin[(e + f*x)/2] - 90*A*c^2*d*Sin[(e + f*x)/2] + 270*B*c^2*d*Sin[(e + f*x)/2] + 270*A*c*d^2*Sin[(e + f*x)/2] - 330*B*c*d^2*Sin[(e + f*x)/2] - 110*A*d^3*Sin[(e + f*x)/2] + 165*B*d^3*Sin[(e + f*x)/2] + (30 + 30*I)*(-1)^(3/4)*(c - d)^2*(3*B*(c - 5*d) + A*(c + 11*d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 180*B*c^2*d*Sin[(3*(e + f*x))/2] - 180*A*c*d^2*Sin[(3*(e + f*x))/2] + 210*B*c*d^2*Sin[(3*(e + f*x))/2] + 70*A*d^3*Sin[(3*(e + f*x))/2] - 123*B*d^3*Sin[(3*(e + f*x))/2] - 30*B*c*d^2*Sin[(5*(e + f*x))/2] - 10*A*d^3*Sin[(5*(e + f*x))/2] + 9*B*d^3*Sin[(5*(e + f*x))/2] + 3*B*d^3*Sin[(7*(e + f*x))/2]))/(60*f*(a*(1 + Sin[e + f*x]))^(3/2))


```

+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+a)^2*(tan((f*x+exp(1))/2)*(tan((f
*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(1/3600*tan((f*x+ex
p(1))/2)*(-2400*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+3960*B*a*d^3*sign(tan((
f*x+exp(1))/2)+1)+5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-7200*B*a*c*d^2
*sign(tan((f*x+exp(1))/2)+1)+5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1))+1/
3600*(3600*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-5400*B*a*d^3*sign(tan((f*x+e
xp(1))/2)+1)-5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+10800*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)-5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/360
0*(-6000*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+10800*B*a*d^3*sign(tan((f*x+ex
p(1))/2)+1)+10800*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-18000*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)+10800*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/36
00*(6000*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-10800*B*a*d^3*sign(tan((f*x+ex
p(1))/2)+1)-10800*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+18000*B*a*c*d^2*sig
n(tan((f*x+exp(1))/2)+1)-10800*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/36
00*(-3600*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)+5400*B*a*d^3*sign(tan((f*x+ex
p(1))/2)+1)+5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)-10800*B*a*c*d^2*sign
(tan((f*x+exp(1))/2)+1)+5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+1/3600
*(2400*A*a*d^3*sign(tan((f*x+exp(1))/2)+1)-3960*B*a*d^3*sign(tan((f*x+exp(1
))/2)+1)-5400*A*a*c*d^2*sign(tan((f*x+exp(1))/2)+1)+7200*B*a*c*d^2*sign(tan
((f*x+exp(1))/2)+1)-5400*B*a*c^2*d*sign(tan((f*x+exp(1))/2)+1)))+2*(1/4*(-3
*A*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*A
*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*B*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*A*c*d^2
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+9*A*c^2*d
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+9*B*c*d^2
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*c^2*d
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+A*a*c^3*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-A*a*d^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+A*sqrt(a)*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-A*sqrt(a)*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-B*a*c^3*(
-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+B*a*d^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*sqrt(a)*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+B*sqrt(a)*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+3*A*a*c*d
^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-3*A*a*c^2
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+3*A*sqrt(
a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-3
*A*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2
+a))^2-3*B*a*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))+3*B*a*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)
^2+a))-3*B*sqrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^2+3*B*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan(
(f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*a*c^3-A*sqrt(a)*a*d^3-B*sqrt(a)*a*c^3+B*sq
rt(a)*a*d^3+3*A*sqrt(a)*a*c*d^2-3*A*sqrt(a)*a*c^2*d-3*B*sqrt(a)*a*c*d^2+3*B
*sqrt(a)*a*c^2*d)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2
)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(A*c^3+11*A*d^3+3*B*c^3-15*B
*d^3-21*A*c*d^2+9*A*c^2*d+33*B*c*d^2-21*B*c^2*d)*atan((-sqrt(a)*tan((f*x+ex
p(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)
/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))

```

maple [B] time = 1.87, size = 1030, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/60*(sin(f*x+e)*(-40*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d^3+360*A*c*d^2*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-120*A*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-120*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^2+360*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-360*B*a^(5/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)+240*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)-40*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*d^3+30*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^3-90*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^2*d+450*A*c*d^2*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-150*A*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+24*B*d^3*(a-a*sin(f*x+e))^(5/2)*a^(1/2)-120*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)*c*d^2-30*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)*c^3+450*B*c^2*d*a^(5/2)*(a-a*sin(f*x+e))^(1/2)-450*B*a^(5/2)*c*d^2*(a-a*sin(f*x+e))^(1/2)+270*B*a^(5/2)*d^3*(a-a*sin(f*x+e))^(1/2)+15*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3+135*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d-315*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2+165*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3+45*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^3-315*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c^2*d+495*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*c*d^2-225*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*d^3)*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^3}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**3/(a+a*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```


x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 2977

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Sim
p[((A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/
(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{3/2}} dx = -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{2f(a + a \sin(e + fx))^{3/2}} + \frac{\int \frac{(c + d \sin(e + fx)) \left(\frac{1}{2}a(Ac - Bc) + \frac{1}{2}a(Ac + 3Bc + 4Ad - 4Bd) \sin(e + fx)\right)}{(a + a \sin(e + fx))^{3/2}} dx}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{2f(a + a \sin(e + fx))^{3/2}} + \frac{\int \frac{\frac{1}{2}ac(Ac + 3Bc + 4Ad - 4Bd) \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= \frac{(3A - 7B)d^2 \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{6a^2 f} - \frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= \frac{d(3Ac - 15Bc - 9Ad + 13Bd) \cos(e + fx)}{3af \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)d^2 \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= \frac{d(3Ac - 15Bc - 9Ad + 13Bd) \cos(e + fx)}{3af \sqrt{a + a \sin(e + fx)}} + \frac{(3A - 7B)d^2 \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}}$$

$$= -\frac{(c - d)(Ac + 3Bc + 7Ad - 11Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{2\sqrt{2} a^{3/2} f} + \frac{(3A - 7B)d^2 \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}}$$

Mathematica [C] time = 0.74, size = 357, normalized size = 1.76

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(6(A - B)(c - d)^2 \sin\left(\frac{1}{2}(e + fx)\right) - 3(A - B)(c - d)^2 \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{(a + a \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])
]^(3/2), x]
```



```

)/2)+1))) - 1/144*(-72*A*d^2*sign(tan((f*x+exp(1))/2)+1)+144*B*d^2*sign(tan((f*x+exp(1))/2)+1)-144*B*c*d*sign(tan((f*x+exp(1))/2)+1))-1/144*(72*A*d^2*sign(tan((f*x+exp(1))/2)+1)-96*B*d^2*sign(tan((f*x+exp(1))/2)+1)+144*B*c*d*sign(tan((f*x+exp(1))/2)+1)))+2*(1/4*(-3*A*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*A*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+6*A*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-6*B*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+A*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+A*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+A*sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-B*a*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*a*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*sqrt(a)*c^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-B*sqrt(a)*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-2*A*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-2*A*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*B*a*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+2*B*sqrt(a)*c*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*a*c^2+A*sqrt(a)*a*d^2-B*sqrt(a)*a*c^2-B*sqrt(a)*a*d^2-2*A*sqrt(a)*a*c*d+2*B*sqrt(a)*a*c*d)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(A*c^2-7*A*d^2+3*B*c^2+11*B*d^2+6*A*c*d-14*B*c*d)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))

```

maple [B] time = 1.59, size = 694, normalized size = 3.42

$$\frac{\left(\sin(fx + e)\left(3A\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\right)a^2c^2 + 18A\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\right)a^2cd - 21A\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)a^2cd}{(-(-\sqrt{a}\tan\left(\frac{f*x+\exp(1)}{2}\right)+\sqrt{a\tan\left(\frac{f*x+\exp(1)}{2}\right)^2+a})^2+2\sqrt{a}\left(-\sqrt{a}\tan\left(\frac{f*x+\exp(1)}{2}\right)+\sqrt{a\tan\left(\frac{f*x+\exp(1)}{2}\right)^2+a}\right)+a)^2/a\operatorname{sign}\left(\tan\left(\frac{f*x+\exp(1)}{2}\right)+1\right)+1/4\left(Ac^2-7Ad^2+3Bc^2+11Bd^2+6Acd-14Bcd\right)\operatorname{atan}\left(\frac{-\sqrt{a}\tan\left(\frac{f*x+\exp(1)}{2}\right)-\sqrt{a}+\sqrt{a\tan\left(\frac{f*x+\exp(1)}{2}\right)^2+a}}{\sqrt{2}\sqrt{-a}}\right)/\sqrt{2}\sqrt{-a}}{a\operatorname{sign}\left(\tan\left(\frac{f*x+\exp(1)}{2}\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2), x)

```

[Out] -1/12/a^(7/2)*(sin(f*x+e)*(3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*d^2+24*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e))^(3/2)*a^(1/2)+48*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-24*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c^2+18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c*d-21*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*d^2+6*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-12*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d+30*A*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+9*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c^2-42*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*c*d+33*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2))*2^(1/2)/a^(1/2))*a^2*d^2-8*B*d^2*(a-a*sin(f*x+e))^(3/2)*a^(1/2)-6*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2+60*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)-30*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))*(-a*(sin(f*x+e)-1))^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```


maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^2}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(3/2),x)

[Out] Timed out

$$3.316 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=133

$$\frac{(Ac + 3Ad + 3Bc - 7Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A-B)(c-d) \cos(e+fx)}{2f(a \sin(e+fx) + a)^{3/2}} - \frac{2Bd \cos(e+fx)}{af \sqrt{a \sin(e+fx) + a}}$$

[Out] $-1/2*(A-B)*(c-d)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(3/2)}-1/4*(A*c+3*A*d+3*B*c-7*B*d)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})/a^{(3/2)})/f*2^{(1/2)}-2*B*d*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2968, 3019, 2751, 2649, 206}

$$\frac{(Ac + 3Ad + 3Bc - 7Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A-B)(c-d) \cos(e+fx)}{2f(a \sin(e+fx) + a)^{3/2}} - \frac{2Bd \cos(e+fx)}{af \sqrt{a \sin(e+fx) + a}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $-((A*c + 3*B*c + 3*A*d - 7*B*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*f) - ((A - B)*(c - d)*\operatorname{Cos}[e + f*x])/(2*f*(a + a*\operatorname{Sin}[e + f*x])^{(3/2)}) - (2*B*d*\operatorname{Cos}[e + f*x])/(a*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3019

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^{3/2}} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3B(c-d) + A(c+3d)) - 2aBd \sin(e+fx)}{\sqrt{a+a \sin(e+fx)}}}{2a^2} \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}} - \frac{2Bd \cos(e + fx)}{af\sqrt{a + a \sin(e + fx)}} + \frac{(Ac + 3Bc + 3Ad - 7Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a \sin(e+fx)}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A - B)(c - d) \cos(e + fx)}{2f(a + a \sin(e + fx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.46, size = 246, normalized size = 1.85

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(2(A - B)(c - d) \sin\left(\frac{1}{2}(e + fx)\right) - (A - B)(c - d) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)}{2f(a + a \sin(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(3/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2] - (A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A*c + 3*B*c + 3*A*d - 7*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 4*B*d*Cos[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 4*B*d*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 0.48, size = 407, normalized size = 3.06

$$\sqrt{2} \left(((A + 3B)c + (3A - 7B)d) \cos(fx + e)^2 - 2(A + 3B)c - 2(3A - 7B)d - ((A + 3B)c + (3A - 7B)d) \cos(fx + e) \right) / (2f(a + a \sin(e + fx))^{3/2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2), x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*(((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c - 2*(3*A - 7*B)*d - ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e) - (2*(A + 3*B)*c + (3*A - 7*B)*d)*sin(f*x + e)) / (2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

```
*B)*c + 2*(3*A - 7*B)*d + ((A + 3*B)*c + (3*A - 7*B)*d)*cos(f*x + e))*sin(f
*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
))*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x
+ e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f
*x + e) - cos(f*x + e) - 2)) - 4*(4*B*d*cos(f*x + e)^2 + (A - B)*c - (A - B
)*d + ((A - B)*c - (A - 5*B)*d)*cos(f*x + e) + (4*B*d*cos(f*x + e) - (A - B
)*c + (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e
)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x
+ e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B*d/a/sig
n(tan((f*x+exp(1))/2)+1)+1/2*B*d*tan((f*x+exp(1))/2)/a/sign(tan((f*x+exp(1)
)/2)+1)))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/4*(-3*A*c*(-sqrt(a)*tan((f*x+exp(
1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*A*d*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*c*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^3-3*B*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(
a*tan((f*x+exp(1))/2)^2+a))^3+A*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))-A*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))+A*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x
+exp(1))/2)^2+a))^2-A*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^2-B*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))+B*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))-B*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^2+B*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^2+A*sqrt(a)*a*c-A*sqrt(a)*a*d-B*sqrt(a)*a*c+B*sqrt(a)*a*d)/((-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(ta
n((f*x+exp(1))/2)+1)+1/4*(A*c+3*A*d+3*B*c-7*B*d)*atan((-sqrt(a)*tan((f*x+ex
p(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)
/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1)))
```

maple [B] time = 1.32, size = 389, normalized size = 2.92

$$\left(\sin(fx + e) \left(A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a-a\sin(fx+e)} \sqrt{2}}{2\sqrt{a}} \right) ac + 3A\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a-a\sin(fx+e)} \sqrt{2}}{2\sqrt{a}} \right) ad + 3B\sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a-a\sin(fx+e)} \sqrt{2}}{2\sqrt{a}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)
```

```
[Out] -1/4/a^(5/2)*(sin(f*x+e)*(A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
*c-7*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a*d+8*B*
(a-a*sin(f*x+e))^(1/2)*a^(1/2)*d)+A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1
/2)*2^(1/2)/a^(1/2))*a*c+3*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a*d+3*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^
```

$(1/2)) * a * c - 7 * B * 2^{(1/2)} * \operatorname{arctanh}(1/2 * (a - a * \sin(f * x + e))^{(1/2)} * 2^{(1/2)} / a^{(1/2)}) * a * d + 2 * A * (a - a * \sin(f * x + e))^{(1/2)} * a^{(1/2)} * c - 2 * A * (a - a * \sin(f * x + e))^{(1/2)} * a^{(1/2)} * d - 2 * B * (a - a * \sin(f * x + e))^{(1/2)} * a^{(1/2)} * c + 10 * B * (a - a * \sin(f * x + e))^{(1/2)} * a^{(1/2)} * d * (-a * (\sin(f * x + e) - 1))^{(1/2)} / \cos(f * x + e) / (a + a * \sin(f * x + e))^{(1/2)} / f$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)

$$3.317 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=87

$$-\frac{(A+3B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(3/2)}-1/4*(A+3*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})}/a^{(3/2)}/f*2^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2750, 2649, 206}

$$-\frac{(A+3B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A-B) \cos(e+fx)}{2f(a \sin(e+fx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2), x]

[Out] $-((A+3*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])])/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*f) - ((A-B)*\operatorname{Cos}[e+f*x])/(2*f*(a+a*\sin[e+f*x])^{(3/2)})$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}} dx &= -\frac{(A-B) \cos(e+fx)}{2f(a+a \sin(e+fx))^{3/2}} + \frac{(A+3B) \int \frac{1}{\sqrt{a+a \sin(e+fx)}} dx}{4a} \\ &= -\frac{(A-B) \cos(e+fx)}{2f(a+a \sin(e+fx))^{3/2}} - \frac{(A+3B) \operatorname{Subst}\left(\int \frac{1}{2a-x^2} dx, x, \frac{a \cos(e+fx)}{\sqrt{a+a \sin(e+fx)}}\right)}{2af} \\ &= -\frac{(A+3B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a+a \sin(e+fx)}}\right)}{2\sqrt{2} a^{3/2} f} - \frac{(A-B) \cos(e+fx)}{2f(a+a \sin(e+fx))^{3/2}} \end{aligned}$$

Mathematica [C] time = 0.20, size = 150, normalized size = 1.72

$$\frac{\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\left(2(A-B)\sin\left(\frac{1}{2}(e+fx)\right) + (B-A)\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{2f(a(\sin(e+fx) + \cos(e+fx)))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(3/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*Sin[(e + f*x)/2] + (-A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A + 3*B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(2*f*(a*(1 + Sin[e + f*x]))^(3/2))

fricas [B] time = 0.44, size = 293, normalized size = 3.37

$$\frac{\sqrt{2}\left((A+3B)\cos^2(fx+e) - (A+3B)\cos(fx+e) - ((A+3B)\cos(fx+e) + 2A+6B)\sin(fx+e) - 2A\right)}{8\left(a^2f\cos^2(fx+e) - a^2f\cos(fx+e) - (a^2f\cos(fx+e) + 2A+6B)\sin(fx+e) - 2A\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] 1/8*(sqrt(2)*((A + 3*B)*cos(f*x + e)^2 - (A + 3*B)*cos(f*x + e) - ((A + 3*B)*cos(f*x + e) + 2*A + 6*B)*sin(f*x + e) - 2*A - 6*B)*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*((A - B)*cos(f*x + e) - (A - B)*sin(f*x + e) + A - B)*sqrt(a*sin(f*x + e) + a)/(a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - 2*a^2*f - (a^2*f*cos(f*x + e) + 2*a^2*f)*sin(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/4*(-3*A*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+3*B*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+A*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-B*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-B*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+A*sqrt(a)*a-B*sqrt(a)*a)/(-(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^2/a/sign(tan((f*x+exp(1))/2)+1)+1/4*(A+3*B)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/sqrt(-a)/a/sign(tan((f*x+exp(1))/2)+1))

maple [B] time = 1.09, size = 176, normalized size = 2.02

$$\frac{\left(\sin(fx+e)\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)a(A+3B) + A\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}a + 3B\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}a\right)}{4a^{\frac{5}{2}}\cos(fx+e)\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x)`

[Out]
$$-1/4/a^{5/2}*(\sin(f*x+e)*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2})/a^{1/2))*a*(A+3*B)+A*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2})/a^{1/2))*2^{1/2}*a+3*B*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2})/a^{1/2))*2^{1/2}*a+2*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*A-2*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*B)*(-a*(\sin(f*x+e)-1))^{1/2}/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2),x)`

[Out] `int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2),x)`

[Out] `Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(3/2), x)`

$$3.318 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=187

$$\frac{(A(c-5d) + B(3c+d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f(c-d)^2} + \frac{2\sqrt{d} (Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{a^{3/2} f(c-d)^2 \sqrt{c+d}} - \frac{(A-B)}{2f(c-d)(a)}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^{(3/2)}-1/4*(A*(c-5*d)+B*(3*c+d))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})/a^{(3/2)/(c-d)^2/f*2^{(1/2)}+2*(-A*d+B*c)*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)/(c+d)^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})}*d^{(1/2)/a^{(3/2)/(c-d)^2/f/(c+d)^{(1/2)}}$

Rubi [A] time = 0.59, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$\frac{(A(c-5d) + B(3c+d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{2\sqrt{2} a^{3/2} f(c-d)^2} + \frac{2\sqrt{d} (Bc-Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}}\right)}{a^{3/2} f(c-d)^2 \sqrt{c+d}} - \frac{(A-B)}{2f(c-d)(a)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])), x]

[Out] $-((A*(c-5*d) + B*(3*c+d))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])]/(2*\operatorname{Sqrt}[2]*a^{(3/2)}*(c-d)^2*f) + (2*\operatorname{Sqrt}[d]*(B*c-A*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[c+d]*\operatorname{Sqrt}[a+a*\sin[e+f*x]])]/(a^{(3/2)}*(c-d)^2*\operatorname{Sqrt}[c+d]*f) - ((A-B)*\operatorname{Cos}[e+f*x])/((2*(c-d)*f*(a+a*\sin[e+f*x])^{(3/2)}))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2978

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim

```
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]]*(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} dx = -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}} - \frac{\int \frac{-\frac{1}{2}a(3Bc + A(c - 4d)) - \frac{1}{2}a(A - B)d \sin(e + fx)}{\sqrt{a + a \sin(e + fx)}(c + d \sin(e + fx))} dx}{2a^2(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}} - \frac{(d(Bc - Ad)) \int \frac{\sqrt{a + a \sin(e + fx)}}{c + d \sin(e + fx)} dx}{a^2(c - d)^2}$$

$$= -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}} + \frac{(2d(Bc - Ad)) \text{Subst}\left(\int \frac{1}{ac + ad + a^2 \sin^2(u)} du\right)}{a(c - d)}$$

$$= -\frac{(A(c - 5d) + B(3c + d)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{2\sqrt{2} a^{3/2}(c - d)^2 f} + \frac{2\sqrt{d}(Bc - Ad)}{a(c - d)}$$

Mathematica [C] time = 3.14, size = 419, normalized size = 2.24

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(2(A - B)(c - d) \sin\left(\frac{1}{2}(e + fx)\right) + (B - A)(c - d) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(2*(A - B)*(c - d)*Sin[(e + f*x)/2] + (-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (1 + I)*(-1)^(3/4)*(A*(c - 5*d) + B*(3*c + d))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(B*c - A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d] + (Sqrt[d]*(-B*c) + A*d)*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/Sqrt[c + d))/(2*(c - d)^2*f*(a*(1 + Sin[e + f*x]))^(3/2))
```

fricas [B] time = 3.29, size = 1561, normalized size = 8.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorith
m="fricas")
```

```
[Out] [1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e)^2 - 2*(A + 3*B)*c +
2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e) - (2*(A + 3*B)*c
- 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x + e))*sin(f*x + e))*s
qrt(a)*log(-(a*cos(f*x + e))^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*
(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*
a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) -
cos(f*x + e) - 2)) + 4*(2*B*a*c - 2*A*a*d - (B*a*c - A*a*d)*cos(f*x + e)^2
+ (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d + (B*a*c - A*a*d)*cos(f
*x + e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d
+ 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^
2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d
+ 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)
*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x +
e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d
^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2
+ d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*d*cos(f*x + e) - c^2 - 2*c
*d - d^2)*sin(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*
d)*cos(f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e)
+ a))/((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2
*c*d + a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^
2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*
d^2)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(((A + 3*B)*c - (5*A - B)*d)*cos(f*x +
e)^2 - 2*(A + 3*B)*c + 2*(5*A - B)*d - ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e) - (2*(A + 3*B)*c - 2*(5*A - B)*d + ((A + 3*B)*c - (5*A - B)*d)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e))^2 - 2*sqrt(2)*sqrt(a*sin(
f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e)
- (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e)
+ 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 8*(2*B*a*c - 2*A*a*d - (B*a*c -
A*a*d)*cos(f*x + e)^2 + (B*a*c - A*a*d)*cos(f*x + e) + (2*B*a*c - 2*A*a*d +
(B*a*c - A*a*d)*cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/
2*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d)))/
(d*cos(f*x + e))) + 4*((A - B)*c - (A - B)*d + ((A - B)*c - (A - B)*d)*cos(
f*x + e) - ((A - B)*c - (A - B)*d)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/
((a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f*cos(f*x + e)^2 - (a^2*c^2 - 2*a^2*c*d +
a^2*d^2)*f*cos(f*x + e) - 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f - ((a^2*c^2 -
2*a^2*c*d + a^2*d^2)*f*cos(f*x + e) + 2*(a^2*c^2 - 2*a^2*c*d + a^2*d^2)*f)
*sin(f*x + e))]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorith
m="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
```


abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep+1)]Warning, assuming $-c^3d^5-c^2d^6+cd^7+d^8-cd^7-d^8$ is positive. Hint: run assume to make assumptions on a variableWarning, assuming $-a^3c^3d^5-a^2c^2d^6+a^2cd^7+a^2d^8-a^2cd^7-a^2d^8$ is positive. Hint: run assume to make assumptions on a variableWarning, need to choose a branch for the root of a polynomial with parameters. This might be wrong. Non regular value [0] was discarded and replaced randomly by 0=[-90]Evaluation time: 67.84sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

maple [B] time = 2.01, size = 624, normalized size = 3.34

$$\left(\sin(fx + e) \left(8A \operatorname{arctanh} \left(\frac{\sqrt{a-a \sin(fx+e)} d}{\sqrt{acd+ad^2}} \right) a^{\frac{3}{2}} d^2 - 8B \operatorname{arctanh} \left(\frac{\sqrt{a-a \sin(fx+e)} d}{\sqrt{acd+ad^2}} \right) a^{\frac{3}{2}} cd + A \sqrt{a(c+d)} d \sqrt{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x)

[Out]
$$-1/4/a^{5/2}*(\sin(f*x+e))*(8*A*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2))^{1/2})*a^{3/2}*d^2-8*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2))^{1/2})*a^{3/2}*c*d+A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*c-5*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*d+3*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*c+B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*d+8*A*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2))^{1/2})*a^{3/2}*d^2-8*B*\operatorname{arctanh}((a-a*\sin(f*x+e))^{1/2}*d/(a*c*d+a*d^2))^{1/2})*a^{3/2}*c*d+A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*c-5*A*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*d+3*B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*c+B*(a*(c+d)*d)^{1/2}*2^{1/2}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2}*2^{1/2}/a^{1/2})*a*d+2*A*(a*(c+d)*d)^{1/2}*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*c-2*A*(a*(c+d)*d)^{1/2}*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*d-2*B*(a*(c+d)*d)^{1/2}*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*c+2*B*(a*(c+d)*d)^{1/2}*(a-a*\sin(f*x+e))^{1/2}*a^{1/2}*d*(-a*(\sin(f*x+e)-1))^{1/2}/(a*(c+d)*d)^{1/2}/(c-d)^2/\cos(f*x+e)/(a+a*\sin(f*x+e))^{1/2}/f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(a \sin(fx + e) + a)^{\frac{3}{2}} (d \sin(fx + e) + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e)),x, algorith="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{\frac{3}{2}} (c + d \sin(e + fx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),  
x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))),  
x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

$$3.319 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=292

$$\frac{\sqrt{d} \left(Ad(5c + 3d) - B(3c^2 + 3cd + 2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right) (Ac - 9Ad + 3Bc + 5Bd) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{a^{3/2} f (c-d)^3 (c+d)^{3/2} \cdot 2\sqrt{2} a^{3/2} f (c-d)^3}$$

[Out] $-1/2*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^{3/2}/(c+d*\sin(f*x+e))^{-1/4}*(A*c-9*A*d+3*B*c+5*B*d)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{1/2}*2^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{3/2}/(c-d)^3/f*2^{1/2}-(A*d*(5*c+3*d)-B*(3*c^2+3*c*d+2*d^2))*\operatorname{arctanh}(\cos(f*x+e)*a^{1/2}*d^{1/2}/(c+d)^{1/2}/(a+a*\sin(f*x+e))^{1/2})*d^{1/2}/a^{3/2}/(c-d)^3/(c+d)^{3/2}/f+1/2*d*(B*(3*c+d)-A*(c+3*d))*\cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*\sin(f*x+e))/(a+a*\sin(f*x+e))^{1/2}$

Rubi [A] time = 1.02, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{\sqrt{d} \left(Ad(5c + 3d) - B(3c^2 + 3cd + 2d^2) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right) (Ac - 9Ad + 3Bc + 5Bd) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right)}{a^{3/2} f (c-d)^3 (c+d)^{3/2} \cdot 2\sqrt{2} a^{3/2} f (c-d)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2), x]

[Out] $-((A*c + 3*B*c - 9*A*d + 5*B*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(2*\operatorname{Sqrt}[2]*a^{3/2}*(c-d)^3*f) - (\operatorname{Sqrt}[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(a^{3/2}*(c-d)^3*(c+d)^{3/2}*f) - ((A - B)*\operatorname{Cos}[e + f*x])/(2*(c-d)*f*(a + a*\sin[e + f*x])^{3/2}*(c + d*\sin[e + f*x])) + (d*(B*(3*c + d) - A*(c + 3*d))*\operatorname{Cos}[e + f*x])/(2*a*(c-d)^2*(c+d)*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]]*(c + d*\sin[e + f*x]))$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/((c_) + (d_.)*sin[(e_) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))^2} dx = -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} - \frac{\int \frac{-\frac{1}{2}a(Ac + Bc) \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx}{2a(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} + \frac{\int \frac{a(Ac + Bc) \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx}{2a(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} + \frac{\int \frac{a(Ac + Bc) \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx}{2a(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{2(c - d)f(a + a \sin(e + fx))^{3/2}(c + d \sin(e + fx))} + \frac{\int \frac{a(Ac + Bc) \cos(e + fx)}{\sqrt{a + a \sin(e + fx)}} dx}{2a(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{2\sqrt{2} a^{3/2}(c - d)^3 f} - \frac{(Ac + 3Bc - 9Ad + 5Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right) + \sqrt{d} (Ac + Bc)}{2\sqrt{2} a^{3/2}(c - d)^3 f}$$

Mathematica [C] time = 9.54, size = 542, normalized size = 1.86

$$\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right) \left(\frac{\sqrt{d}(B(3c^2+3cd+2d^2)-Ad(5c+3d))\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)^2 \left(2\log\left(\sec^2\left(\frac{1}{4}(e+fx)\right)\right)\left(\sqrt{c+d}\right)\right)}{(c+d)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(4*(A - B)*(c - d)*Sin[(e + f*x)/2] + 2*(-A + B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + (2 + 2*I)*(-1)^(3/4)*(A*c + 3*B*c - 9*A*d + 5*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (Sqrt[d]*(-(A*d*(5*c + 3*d)) + B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] + Sqrt[d]*Cos[(e + f*x)/2] - Sqrt[d]*Sin[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)^(3/2) + (Sqrt[d]*(A*d*(5*c + 3*d) - B*(3*c^2 + 3*c*d + 2*d^2))*(e + f*x - 2*Log[Sec[(e + f*x)/4]^2] + 2*Log[Sec[(e + f*x)/4]^2*(Sqrt[c + d] - Sqrt[d]*Cos[(e + f*x)/2] + Sqrt[d]*Sin[(e + f*x)/2]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(c + d)^(3/2) + (4*(c - d)*d*(B*c - A*d)*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/((c + d)*(c + d*Sin[e + f*x])))/(4*(c - d)^3*f*(a*(1 + Sin[e + f*x]))^(3/2))

fricas [B] time = 7.67, size = 3403, normalized size = 11.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] [1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3 - 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e)*sqrt(a)*log(-(a*cos(f*x + e))^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 2*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*sin(f*x + e)*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d + 7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sqrt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos(f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2

```

- c^2 - 2*c*d - d^2 - (c^2 + d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2 - 2*c*
d*cos(f*x + e) - c^2 - 2*c*d - d^2)*sin(f*x + e))) + 4*((A - B)*c^3 - (A -
B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2
- (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A + 3*B)*c*d
^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A - B)*c*d^2 +
(A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^3)*cos(f*x
+ e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^2*c^3*d^2 +
2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*d^2 + 2*a^2
*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5 - a^2*c^4*d
- 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e) - 2*
(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)
*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^2
- (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^
5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3
+ a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e)), 1/8*(sqrt(2)*(2*(A + 3*B)*c^3 - 2*
(7*A - 11*B)*c^2*d - 2*(17*A - 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)
*c^2*d - 8*(A - B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e)^3 - ((A + 3*B)*c^3
- 2*(3*A - 7*B)*c^2*d - (25*A - 21*B)*c*d^2 - 2*(9*A - 5*B)*d^3)*cos(f*x +
e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A - 13*B)*c*d^2 - (9*A -
5*B)*d^3)*cos(f*x + e) + (2*(A + 3*B)*c^3 - 2*(7*A - 11*B)*c^2*d - 2*(17*A
- 13*B)*c*d^2 - 2*(9*A - 5*B)*d^3 - ((A + 3*B)*c^2*d - 8*(A - B)*c*d^2 - (9
*A - 5*B)*d^3)*cos(f*x + e)^2 + ((A + 3*B)*c^3 - (7*A - 11*B)*c^2*d - (17*A
- 13*B)*c*d^2 - (9*A - 5*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-
(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e
) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e)
- 2)) - 4*(6*B*a*c^3 - 2*(5*A - 6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(
3*A - 2*B)*a*d^3 - (3*B*a*c^2*d - (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*
cos(f*x + e)^3 - (3*B*a*c^3 - (5*A - 9*B)*a*c^2*d - (13*A - 8*B)*a*c*d^2 -
2*(3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5*A - 6*B)*a*c^2*d - (8
*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e) + (6*B*a*c^3 - 2*(5*A -
6*B)*a*c^2*d - 2*(8*A - 5*B)*a*c*d^2 - 2*(3*A - 2*B)*a*d^3 - (3*B*a*c^2*d
- (5*A - 3*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e)^2 + (3*B*a*c^3 - (5
*A - 6*B)*a*c^2*d - (8*A - 5*B)*a*c*d^2 - (3*A - 2*B)*a*d^3)*cos(f*x + e))*
sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x + e) + a)*(d*s
in(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d)))/(d*cos(f*x + e))) + 4*((A - B)*
c^3 - (A - B)*c^2*d - (A - B)*c*d^2 + (A - B)*d^3 + ((A - 3*B)*c^2*d + 2*(A
+ B)*c*d^2 - (3*A - B)*d^3)*cos(f*x + e)^2 + ((A - B)*c^3 - 2*B*c^2*d + (A
+ 3*B)*c*d^2 - 2*A*d^3)*cos(f*x + e) - ((A - B)*c^3 - (A - B)*c^2*d - (A -
B)*c*d^2 + (A - B)*d^3 - ((A - 3*B)*c^2*d + 2*(A + B)*c*d^2 - (3*A - B)*d^
3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^2*c^4*d - 2*a^
2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(f*x + e)^3 + (a^2*c^5 - 4*a^2*c^3*
d^2 + 2*a^2*c^2*d^3 + 3*a^2*c*d^4 - 2*a^2*d^5)*f*cos(f*x + e)^2 - (a^2*c^5
- a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f*cos(f*
x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d^4
- a^2*d^5)*f + ((a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c*d^4 - a^2*d^5)*f*cos(
f*x + e)^2 - (a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*a^2*c^2*d^3 + a^2*c*d
^4 - a^2*d^5)*f*cos(f*x + e) - 2*(a^2*c^5 - a^2*c^4*d - 2*a^2*c^3*d^2 + 2*
a^2*c^2*d^3 + a^2*c*d^4 - a^2*d^5)*f)*sin(f*x + e))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
```



```

)*d^(1/2)*a^(1/2)*c*d^2+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(
(1/2)*c*d^2-6*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x
+e)*d^3+A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*a*c^3-24*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(3/2)*sin(f*x+e)*c^2*d^2+20*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/
(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2+2*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*c^3-4*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)
*d^3-2*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3+12*A*arcta
nh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3-12*B*arctan
h((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^3*d-12*B*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c^2*d^2-8*B*arctanh
((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*c*d^3+12*A*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^2*d^4-8*B
*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*sin(f*x+e)^
2*d^4+12*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(3/2)*s
in(f*x+e)*d^4-8*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^
(3/2)*sin(f*x+e)*d^4*(-a*(sin(f*x+e)-1))^(1/2)/(a*(c+d)*d)^(1/2)/(c+d*sin(
f*x+e))/(c+d)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^{3/2} (c + d \sin(e + f x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**2,x)
```

[Out] Timed out

$$3.320 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=402

$$\frac{\sqrt{d} \left(Ad(35c^2 + 42cd + 19d^2) - 3B(5c^3 + 10c^2d + 13cd^2 + 4d^3) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right) (A(c-13d) + 3B(c+3d))}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}$$

[Out] -1/2*(A-B)*cos(f*x+e)/(c-d)/f/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^2-1/4*(A*(c-13*d)+3*B*(c+3*d))*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/(c-d)^4/f*2^(1/2)-1/4*(A*d*(35*c^2+42*c*d+19*d^2)-3*B*(5*c^3+10*c^2*d+13*c*d^2+4*d^3))*arctanh(cos(f*x+e)*a^(1/2)*d^(1/2)/(c+d)^(1/2)/(a+a*sin(f*x+e))^(1/2))*d^(1/2)/a^(3/2)/(c-d)^4/(c+d)^(5/2)/f+1/2*d*(B*(2*c+d)-A*(c+2*d))*cos(f*x+e)/a/(c-d)^2/(c+d)/f/(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(1/2)+1/4*d*(3*B*(3*c^2+3*c*d+2*d^2)-A*(2*c^2+15*c*d+7*d^2))*cos(f*x+e)/a/(c-d)^3/(c+d)^2/f/(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 1.56, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{\sqrt{d} \left(Ad(35c^2 + 42cd + 19d^2) - 3B(10c^2d + 5c^3 + 13cd^2 + 4d^3) \right) \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{d} \cos(e+fx)}{\sqrt{c+d} \sqrt{a \sin(e+fx)+a}} \right) (A(c-13d) + 3B(c+3d))}{4a^{3/2}f(c-d)^4(c+d)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^3), x]

[Out] -((A*(c - 13*d) + 3*B*(c + 3*d))*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(2*Sqrt[2]*a^(3/2)*(c - d)^4*f) - (Sqrt[d]*(A*d*(35*c^2 + 42*c*d + 19*d^2) - 3*B*(5*c^3 + 10*c^2*d + 13*c*d^2 + 4*d^3))*ArcTanh[(Sqrt[a]*Sqrt[d]*Cos[e + f*x])/(Sqrt[c + d]*Sqrt[a + a*Sin[e + f*x]])])/(4*a^(3/2)*(c - d)^4*(c + d)^(5/2)*f) - ((A - B)*Cos[e + f*x])/(2*(c - d)*f*(a + a*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^2) + (d*(B*(2*c + d) - A*(c + 2*d))*Cos[e + f*x])/(2*a*(c - d)^2*(c + d)*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^2) + (d*(3*B*(3*c^2 + 3*c*d + 2*d^2) - A*(2*c^2 + 15*c*d + 7*d^2))*Cos[e + f*x])/(4*a*(c - d)^3*(c + d)^2*f*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$+ f*x)/2] - 4*B*d^4*\sin[(e + f*x)/2] - 8*A*c^3*d*\sin[(3*(e + f*x))/2] + 26$$

$$*B*c^3*d*\sin[(3*(e + f*x))/2] - 40*A*c^2*d^2*\sin[(3*(e + f*x))/2] + 31*B*c^2$$

$$*d^2*\sin[(3*(e + f*x))/2] - 25*A*c*d^3*\sin[(3*(e + f*x))/2] + 13*B*c*d^3*\sin$$

$$[(3*(e + f*x))/2] + A*d^4*\sin[(3*(e + f*x))/2] + 2*B*d^4*\sin[(3*(e + f*x))$$

$$)/2] - 2*A*c^2*d^2*\sin[(5*(e + f*x))/2] + 9*B*c^2*d^2*\sin[(5*(e + f*x))/2]$$

$$- 15*A*c*d^3*\sin[(5*(e + f*x))/2] + 9*B*c*d^3*\sin[(5*(e + f*x))/2] - 7*A*d^4$$

$$*sin[(5*(e + f*x))/2] + 6*B*d^4*\sin[(5*(e + f*x))/2])/(16*(c - d)^3*(c +$$

$$d)^2*f*(a*(1 + \sin[e + f*x]))^(3/2)*(c + d*\sin[e + f*x])^2)$$

fricas [B] time = 15.27, size = 5864, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="fricas")

[Out] [1/16*(2*sqrt(2)*(2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c
^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d
^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13
*A - 9*B)*d^5)*cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - 11*B)*c^3*d^2
- (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15*B)*c^3*d^2 -
2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B)*d^5)*cos(f*
x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 -
2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*
x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27*B)*c^3*d^2 -
4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9*B)*d^5 - ((A
+ 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 - (13*A - 9*B
)d^5)*cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3*d^2 - 36*(A
- B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e)^2 + (
A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 - 2*(37*A - 3
3*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5)*cos(f*x + e))*sin(
f*x + e))*sqrt(a)*log(-(a*cos(f*x + e))^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) +
a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*
x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(
f*x + e) - cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(
56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d
^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(
14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^4 - (30*B*a*c^4*d
- 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80*A - 63*B)*a*
c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^3 - (15*B*a*c^5 - 5*(7*A - 18*B)*
a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2*d^3 - (202*A
- 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5)*cos(f*x + e)^2 + (15*B*a*c^5 - 5
*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A - 20*B)*a*c^2*d
^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e) + (30*B*a*c^5
- 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*
a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 - (15*B*a*c^3*d
^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5
)*cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2 - (77*A - 69*
B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x + e)^2
+ (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*
A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5)*cos(f*x
+ e))*sin(f*x + e))*sqrt(d/(a*c + a*d))*log((d^2*cos(f*x + e)^3 - (6*c*d +
7*d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2)*cos(f*x + e)^2 -
c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*cos(f*x + e) + (c^2 + 4*c*d +
3*d^2 + (c*d + d^2)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)*sq
rt(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*cos(f*x + e) + (d^2*cos(f*x + e)^2
- c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*cos(f*x + e))*sin(f*x + e))/(d^2*cos
f*x + e)^3 + (2*c*d + d^2)*cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 +

$$\begin{aligned}
& d^2) * \cos(f*x + e) + (d^2 * \cos(f*x + e)^2 - 2*c*d * \cos(f*x + e) - c^2 - 2*c*d \\
& - d^2) * \sin(f*x + e)) - 4*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3* \\
& d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^ \\
& 3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5) * \cos(f*x + e)^3 \\
& + ((4*A - 13*B)*c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A \\
& - 4*B)*c*d^4 + 2*(2*A - B)*d^5) * \cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11 \\
& *B)*c^4*d + (13*A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4 \\
& - (5*A - 6*B)*d^5) * \cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - \\
& B)*c^3*d^2 + 4*(A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - \\
& 9*B)*c^3*d^2 + 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5) * \cos(f*x \\
& + e)^2 - ((4*A - 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 - \\
& (17*A - 7*B)*c*d^4 - (3*A - 4*B)*d^5) * \cos(f*x + e)) * \sin(f*x + e)) * \sqrt{a * \sin \\
& (f*x + e) + a) / ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^ \\
& 5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8) * f * \cos(f*x + e)^4 - (2*a^2*c^7*d - \\
& 3*a^2*c^6*d^2 - 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^ \\
& 6 + a^2*d^8) * f * \cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6 \\
& *a^2*c^5*d^3 + 12*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^ \\
& 7 + 3*a^2*d^8) * f * \cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 \\
& - 4*a^2*c^2*d^6 + a^2*d^8) * f * \cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6 \\
& a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8) * f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - \\
& a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8) * f * \cos(f* \\
& x + e)^3 + 2*(a^2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a \\
& ^2*c^3*d^5 - 3*a^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8) * f * \cos(f*x + e)^2 - (a^2*c \\
& ^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8) * f * \cos(f*x + e \\
&) - 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8) * f \\
&) * \sin(f*x + e)), 1/8 * (\sqrt{2}) * (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(2 \\
& 3*A - 27*B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(\\
& 13*A - 9*B)*d^5 + ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B \\
&) * c*d^4 - (13*A - 9*B)*d^5) * \cos(f*x + e)^4 - (2*(A + 3*B)*c^4*d - 3*(7*A - \\
& 11*B)*c^3*d^2 - (61*A - 57*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B \\
&) * d^5) * \cos(f*x + e)^3 - ((A + 3*B)*c^5 - (7*A - 27*B)*c^4*d - 6*(11*A - 15* \\
& B)*c^3*d^2 - 2*(73*A - 69*B)*c^2*d^3 - (127*A - 99*B)*c*d^4 - 3*(13*A - 9*B \\
&) * d^5) * \cos(f*x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27 \\
& *B)*c^3*d^2 - 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B \\
&) * d^5) * \cos(f*x + e) + (2*(A + 3*B)*c^5 - 6*(3*A - 7*B)*c^4*d - 4*(23*A - 27 \\
& *B)*c^3*d^2 - 4*(37*A - 33*B)*c^2*d^3 - 6*(17*A - 13*B)*c*d^4 - 2*(13*A - 9 \\
& *B)*d^5 - ((A + 3*B)*c^3*d^2 - (11*A - 15*B)*c^2*d^3 - (25*A - 21*B)*c*d^4 \\
& - (13*A - 9*B)*d^5) * \cos(f*x + e)^3 - 2*((A + 3*B)*c^4*d - 2*(5*A - 9*B)*c^3 \\
& *d^2 - 36*(A - B)*c^2*d^3 - 2*(19*A - 15*B)*c*d^4 - (13*A - 9*B)*d^5) * \cos(f \\
& *x + e)^2 + ((A + 3*B)*c^5 - 3*(3*A - 7*B)*c^4*d - 2*(23*A - 27*B)*c^3*d^2 \\
& - 2*(37*A - 33*B)*c^2*d^3 - 3*(17*A - 13*B)*c*d^4 - (13*A - 9*B)*d^5) * \cos(f \\
& *x + e)) * \sin(f*x + e)) * \sqrt{a} * \log(- (a * \cos(f*x + e))^2 - 2 * \sqrt{2}) * \sqrt{a * \sin \\
& (f*x + e) + a} * \sqrt{a} * (\cos(f*x + e) - \sin(f*x + e) + 1) + 3*a * \cos(f*x + e \\
&) - (a * \cos(f*x + e) - 2*a) * \sin(f*x + e) + 2*a) / (\cos(f*x + e)^2 - (\cos(f*x + \\
& e) + 2) * \sin(f*x + e) - \cos(f*x + e) - 2)) + (30*B*a*c^5 - 10*(7*A - 12*B)* \\
& a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(23*A - 20*B)*a*c^2*d^3 - 2*(80*A \\
& - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 + (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a \\
& *c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A - 12*B)*a*d^5) * \cos(f*x + e)^4 - \\
& (30*B*a*c^4*d - 5*(14*A - 15*B)*a*c^3*d^2 - (119*A - 108*B)*a*c^2*d^3 - (80 \\
& *A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5) * \cos(f*x + e)^3 - (15*B*a*c^5 - 5* \\
& (7*A - 18*B)*a*c^4*d - 2*(91*A - 102*B)*a*c^3*d^2 - 2*(146*A - 129*B)*a*c^2 \\
& *d^3 - (202*A - 165*B)*a*c*d^4 - 3*(19*A - 12*B)*a*d^5) * \cos(f*x + e)^2 + (1 \\
& 5*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3*d^2 - 6*(23*A - \\
& 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a*d^5) * \cos(f*x + e) \\
& + (30*B*a*c^5 - 10*(7*A - 12*B)*a*c^4*d - 4*(56*A - 57*B)*a*c^3*d^2 - 12*(\\
& 23*A - 20*B)*a*c^2*d^3 - 2*(80*A - 63*B)*a*c*d^4 - 2*(19*A - 12*B)*a*d^5 - \\
& (15*B*a*c^3*d^2 - 5*(7*A - 6*B)*a*c^2*d^3 - 3*(14*A - 13*B)*a*c*d^4 - (19*A \\
& - 12*B)*a*d^5) * \cos(f*x + e)^3 - 2*(15*B*a*c^4*d - 5*(7*A - 9*B)*a*c^3*d^2 \\
& - (77*A - 69*B)*a*c^2*d^3 - (61*A - 51*B)*a*c*d^4 - (19*A - 12*B)*a*d^5) * \cos
\end{aligned}$$

$$\begin{aligned}
& s(f*x + e)^2 + (15*B*a*c^5 - 5*(7*A - 12*B)*a*c^4*d - 2*(56*A - 57*B)*a*c^3 \\
& *d^2 - 6*(23*A - 20*B)*a*c^2*d^3 - (80*A - 63*B)*a*c*d^4 - (19*A - 12*B)*a* \\
& d^5)*\cos(f*x + e)*\sin(f*x + e)*\sqrt{-d/(a*c + a*d)}*\arctan(1/2*\sqrt{a*\sin \\
& (f*x + e) + a}*(d*\sin(f*x + e) - c - 2*d)*\sqrt{-d/(a*c + a*d)})/(d*\cos(f*x + \\
& e)) - 2*(2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4*(A - B)* \\
& c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 + 13*A*c^2 \\
& *d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*\cos(f*x + e)^3 + ((4*A - 13*B)* \\
& c^4*d + (15*A + 2*B)*c^3*d^2 - (14*A - 9*B)*c^2*d^3 - (9*A - 4*B)*c*d^4 + 2 \\
& *(2*A - B)*d^5)*\cos(f*x + e)^2 + (2*(A - B)*c^5 + (2*A - 11*B)*c^4*d + (13* \\
& A - 3*B)*c^3*d^2 + (3*A + 5*B)*c^2*d^3 - 5*(3*A - B)*c*d^4 - (5*A - 6*B)*d^5 \\
&)*\cos(f*x + e) - (2*(A - B)*c^5 - 2*(A - B)*c^4*d - 4*(A - B)*c^3*d^2 + 4* \\
& (A - B)*c^2*d^3 + 2*(A - B)*c*d^4 - 2*(A - B)*d^5 - ((2*A - 9*B)*c^3*d^2 + \\
& 13*A*c^2*d^3 - (8*A - 3*B)*c*d^4 - (7*A - 6*B)*d^5)*\cos(f*x + e)^2 - ((4*A \\
& - 13*B)*c^4*d + (17*A - 7*B)*c^3*d^2 - (A - 9*B)*c^2*d^3 - (17*A - 7*B)*c*d^4 \\
& - (3*A - 4*B)*d^5)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a*\sin(f*x + e) + a)} \\
& /((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4*a^2*c^3*d^5 - a^2*c^2*d^6 \\
& - 2*a^2*c*d^7 + a^2*d^8)*f*\cos(f*x + e)^4 - (2*a^2*c^7*d - 3*a^2*c^6*d^2 - \\
& 4*a^2*c^5*d^3 + 7*a^2*c^4*d^4 + 2*a^2*c^3*d^5 - 5*a^2*c^2*d^6 + a^2*d^8)*f* \\
& \cos(f*x + e)^3 - (a^2*c^8 + 2*a^2*c^7*d - 6*a^2*c^6*d^2 - 6*a^2*c^5*d^3 + 1 \\
& 2*a^2*c^4*d^4 + 6*a^2*c^3*d^5 - 10*a^2*c^2*d^6 - 2*a^2*c*d^7 + 3*a^2*d^8)*f* \\
& \cos(f*x + e)^2 + (a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 \\
& + a^2*d^8)*f*\cos(f*x + e) + 2*(a^2*c^8 - 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4* \\
& a^2*c^2*d^6 + a^2*d^8)*f - ((a^2*c^6*d^2 - 2*a^2*c^5*d^3 - a^2*c^4*d^4 + 4* \\
& a^2*c^3*d^5 - a^2*c^2*d^6 - 2*a^2*c*d^7 + a^2*d^8)*f*\cos(f*x + e)^3 + 2*(a^ \\
& 2*c^7*d - a^2*c^6*d^2 - 3*a^2*c^5*d^3 + 3*a^2*c^4*d^4 + 3*a^2*c^3*d^5 - 3*a \\
& ^2*c^2*d^6 - a^2*c*d^7 + a^2*d^8)*f*\cos(f*x + e)^2 - (a^2*c^8 - 4*a^2*c^6*d \\
& ^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f*\cos(f*x + e) - 2*(a^2*c^8 - \\
& 4*a^2*c^6*d^2 + 6*a^2*c^4*d^4 - 4*a^2*c^2*d^6 + a^2*d^8)*f)*\sin(f*x + e))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
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*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep
/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integ
ration of abs or sign assumes constant sign by intervals (correct if the ar
gument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*t_nostep+exp(
1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/

```

t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
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pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to ch
eck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4
*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sign assumes con
stant sign by intervals (correct if the argument is real):Check [abs(t_nost
ep+1)]Evaluation time: 3.24Error: Bad Argument Type

```

maple [B] time = 3.97, size = 4707, normalized size = 11.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x)
```



```
(-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c^3*d^3+9
0*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)*c^2*d^4+24*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5
/2)*sin(f*x+e)*c*d^5+B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*
c*d^4-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c^5-70*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d
)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^3*d^3-119*A*arctanh((-a*(sin(f*x+e)-1))^(1/
2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^4+17*A*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c*d^4-9*B*(-a*(sin(f*x+e)-1))
^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(f*x+e)*c^2*d^3)/(a*(c+d)*d)^(1/2)/(c+d
*sin(f*x+e))^2/(c+d)^2/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^{3/2} (c + d \sin(e + f x))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^3
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**3,x)
```

[Out] Timed out

$$3.321 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^3}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=308

$$\frac{(c-d)(3A(c^2+6cd+25d^2)+B(5c^2+62cd-163d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right) + d^2(9Ac+39Ad+15Bc-9Bd)}{16\sqrt{2} a^{5/2} f}$$

[Out] $-1/16*(3*A*c+9*A*d+5*B*c-17*B*d)*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/a/f/(a+a*\sin(f*x+e))^{(3/2)}-1/4*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^3/f/(a+a*\sin(f*x+e))^{(5/2)}-1/32*(c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)/(a+a*\sin(f*x+e))^{(1/2)})/a^{(5/2)}/f*2^{(1/2)}+1/24*d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*\cos(f*x+e)/a^2/f/(a+a*\sin(f*x+e))^{(1/2)}+1/48*d^2*(9*A*c+39*A*d+15*B*c-95*B*d)*\cos(f*x+e)*(a+a*\sin(f*x+e))^{(1/2)}/a^3/f$

Rubi [A] time = 1.06, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2977, 2968, 3023, 2751, 2649, 206}

$$\frac{d(A(9c^2+36cd-93d^2)+B(15c^2-228cd+197d^2)) \cos(e+fx)}{24a^2 f \sqrt{a \sin(e+fx)+a}} \frac{(c-d)(3A(c^2+6cd+25d^2)+B(5c^2+62cd-163d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right) + d^2(9Ac+39Ad+15Bc-9Bd)}{16\sqrt{2} a^{5/2} f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+B*\text{Sin}[e+f*x])*(c+d*\text{Sin}[e+f*x])^3/(a+a*\text{Sin}[e+f*x])^{(5/2)}], x]$

[Out] $-((c-d)*(B*(5*c^2+62*c*d-163*d^2)+3*A*(c^2+6*c*d+25*d^2))*\text{ArcTanh}[(\text{Sqrt}[a]*\text{Cos}[e+f*x])/(\text{Sqrt}[2]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])])/(16*\text{Sqrt}[2]*a^{(5/2)}*f)+(d*(A*(9*c^2+36*c*d-93*d^2)+B*(15*c^2-228*c*d+197*d^2))*\text{Cos}[e+f*x]/(24*a^2*f*\text{Sqrt}[a+a*\text{Sin}[e+f*x]])+(d^2*(9*A*c+15*B*c+39*A*d-95*B*d))*\text{Cos}[e+f*x]*\text{Sqrt}[a+a*\text{Sin}[e+f*x]]/(48*a^3*f)-((3*A*c+5*B*c+9*A*d-17*B*d)*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^2)/(16*a*f*(a+a*\text{Sin}[e+f*x])^{(3/2)})-((A-B)*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^3)/(4*f*(a+a*\text{Sin}[e+f*x])^{(5/2)})$

Rule 206

$\text{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 2649

$\text{Int}[1/\text{Sqrt}[(a_+ + (b_+)*\sin[(c_+ + (d_+)*(x_+)])], x_Symbol] \rightarrow \text{Dist}[-2/d, \text{Subst}[\text{Int}[1/(2*a - x^2), x], x, (b*\text{Cos}[c + d*x])/(\text{Sqrt}[a + b*\text{Sin}[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 2751

$\text{Int}[(a_+ + (b_+)*\sin[(e_+ + (f_+)*(x_+)])^{(m_+)}*((c_+ + (d_+)*\sin[(e_+ + (f_+)*(x_+)])), x_Symbol] \rightarrow -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m + 1)), x] + \text{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ !\text{LtQ}[m, -2^{(-1)}]$

Rule 2968


```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2977

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n/(a*f*(2*m + 1)), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{5/2}} dx = -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^3}{4f(a + a \sin(e + fx))^{5/2}} + \frac{\int \frac{(c + d \sin(e + fx))^2 \left(\frac{1}{2}\right)}{a + a \sin(e + fx)} dx}{16a^2 f \sqrt{a + a \sin(e + fx)}} = -\frac{(3Ac + 5Bc + 9Ad - 17Bd) \cos(e + fx)(c + d \sin(e + fx))^2}{16af(a + a \sin(e + fx))^{3/2}} = -\frac{(3Ac + 5Bc + 9Ad - 17Bd) \cos(e + fx)(c + d \sin(e + fx))^2}{16af(a + a \sin(e + fx))^{3/2}} = \frac{d^2(9Ac + 15Bc + 39Ad - 95Bd) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{48a^3 f} = \frac{d(A(9c^2 + 36cd - 93d^2) + B(15c^2 - 228cd + 197d^2)) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{24a^2 f \sqrt{a + a \sin(e + fx)}} = \frac{d(A(9c^2 + 36cd - 93d^2) + B(15c^2 - 228cd + 197d^2)) \cos(e + fx) \sqrt{a + a \sin(e + fx)}}{24a^2 f \sqrt{a + a \sin(e + fx)}} = -\frac{(c - d)(B(5c^2 + 62cd - 163d^2) + 3A(c^2 + 6cd + 25d^2)) \tan(e + fx)}{16\sqrt{2} a^{5/2} f}$$

Mathematica [C] time = 1.79, size = 523, normalized size = 1.70

$$\left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right) \left((3+3i)(-1)^{3/4}(c-d) \left(3A(c^2+6cd+25d^2) + B(5c^2+62cd-163d^2)\right)\right) \left(\sin\left(\frac{1}{2}(e+fx)\right) + \cos\left(\frac{1}{2}(e+fx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^3)/(a + a*Sin[e + f*x])^(5/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(24*(A - B)*(c - d)^3*Sin[(e + f*x)/2] - 12*(A - B)*(c - d)^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 6*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 3*(c - d)^2*(B*(5*c - 29*d) + 3*A*(c + 7*d))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (3 + 3*I)*(-1)^(3/4)*(c - d)*(B*(5*c^2 + 62*c*d - 163*d^2) + 3*A*(c^2 + 6*c*d + 25*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d^3*Cos[(3*(e + f*x))/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(-6*B*c - 2*A*d + 5*B*d)*(Cos[(e + f*x)/2] + I*Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (24 + 24*I)*d^2*(6*B*c + 2*A*d - 5*B*d)*(I*Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 16*B*d^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4*Sin[(3*(e + f*x))/2]))/(48*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.50, size = 980, normalized size = 3.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")
```

```
[Out] -1/192*(3*sqrt(2)*(4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e) + (4*(3*A + 5*B)*c^3 + 12*(5*A + 19*B)*c^2*d + 12*(19*A - 75*B)*c*d^2 - 4*(75*A - 163*B)*d^3 - ((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^3 + 3*(5*A + 19*B)*c^2*d + 3*(19*A - 75*B)*c*d^2 - (75*A - 163*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e))^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^3*cos(f*x + e)^4 - 12*(A - B)*c^3 + 36*(A - B)*c^2*d - 36*(A - B)*c*d^2 + 12*(A - B)*d^3 + 32*(9*B*c*d^2 + (3*A - 5*B)*d^3)*cos(f*x + e)^3 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 53*B)*c*d^2 + (53*A - 93*B)*d^3)*cos(f*x + e)^2 - 3*((7*A + B)*c^3 + 3*(A - 9*B)*c^2*d - 27*(A - 9*B)*c*d^2 + 9*(9*A - 17*B)*d^3)*cos(f*x + e) + (32*B*d^3*cos(f*x + e)^3 + 12*(A - B)*c^3 - 36*(A - B)*c^2*d + 36*(A - B)*c*d^2 - 12*(A - B)*d^3 - 96*(3*B*c*d^2 + (A - 2*B)*d^3)*cos(f*x + e)^2 - 3*((3*A + 5*B)*c^3 + 3*(5*A - 13*B)*c^2*d - 3*(13*A - 85*B)*c*d^2 + (85*A - 157*B)*d^3)*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/
x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Un
able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)2/f*(2/sqrt(a*tan((f*x+exp(1))/2)^2+a)/(a*tan((f*x+exp(1))/2)^2+
a)*(tan((f*x+exp(1))/2)*(tan((f*x+exp(1))/2)*(-1/144*tan((f*x+exp(1))/2)*(-
72*A*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+168*B*a^3*d^3*sign(tan((f*x+exp(1)
)/2)+1)-216*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1))/a^4-1/144*(72*A*a^3*d^
3*sign(tan((f*x+exp(1))/2)+1)-216*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+216
*B*a^3*c*d^2*sign(tan((f*x+exp(1))/2)+1))/a^4)-1/144*(-72*A*a^3*d^3*sign(ta
n((f*x+exp(1))/2)+1)+216*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)-216*B*a^3*c*
d^2*sign(tan((f*x+exp(1))/2)+1))/a^4)-1/144*(72*A*a^3*d^3*sign(tan((f*x+exp
(1))/2)+1)-168*B*a^3*d^3*sign(tan((f*x+exp(1))/2)+1)+216*B*a^3*c*d^2*sign(t
an((f*x+exp(1))/2)+1))/a^4)+2*(1/32*(-29*A*c^3*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-43*A*d^3*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+5*B*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^7+67*B*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^7+57*A*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+s
qrt(a*tan((f*x+exp(1))/2)^2+a))^7+15*A*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^7-129*B*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+57*B*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+75*A*sqrt(a)*c^3*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+237*A*sqrt(a)*d^3*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+29*B*sqrt(a)*c^3*(-sqr
t(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-341*B*sqrt(a)*d
^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-399*A*s
qrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))
^6+87*A*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^6+711*B*sqrt(a)*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f
*x+exp(1))/2)^2+a))^6-399*B*sqrt(a)*c^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqr
t(a*tan((f*x+exp(1))/2)^2+a))^6-55*A*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^5-161*A*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+
sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-17*B*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2
)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+233*B*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/
2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+267*A*a*c*d^2*(-sqrt(a)*tan((f*x+exp(
1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-51*A*a*c^2*d*(-sqrt(a)*tan((f*x+e
xp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-483*B*a*c*d^2*(-sqrt(a)*tan((f
*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5+267*B*a*c^2*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*c^3*(-s
qrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-221*A*sqrt(a)
*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*
B*sqrt(a)*a*c^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+
a))^4+325*B*sqrt(a)*a*d^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp
(1))/2)^2+a))^4+351*A*sqrt(a)*a*c*d^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^4-39*A*sqrt(a)*a*c^2*d*(-sqrt(a)*tan((f*x+exp(1)
)/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-663*B*sqrt(a)*a*c*d^2*(-sqrt(a)*tan(
(f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+351*B*sqrt(a)*a*c^2*d*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+A*a^2*c^3*(-
sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-25*A*a^2*d^3
*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*a^2*c
```

$$\begin{aligned} &^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3+33*B*a^2*d^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3+51*A*a^2*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3-27*A*a^2*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3-75*B*a^2*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3+51*B*a^2*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^3+27*A*a^3*c^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+93*A*a^3*d^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+65*A*\sqrt{a}*a^2*c^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+103*A*\sqrt{a}*a^2*d^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+13*B*a^3*c^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-133*B*a^3*d^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-9*B*\sqrt{a}*a^2*c^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2-159*B*\sqrt{a}*a^2*d^3*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2-159*A*a^3*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+39*A*a^3*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-141*A*\sqrt{a}*a^2*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2-27*A*\sqrt{a}*a^2*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+279*B*a^3*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-159*B*a^3*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+309*B*\sqrt{a}*a^2*c*d^2*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2-141*B*\sqrt{a}*a^2*c^2*d*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+7*A*\sqrt{a}*a^3*c^3+17*A*\sqrt{a}*a^3*d^3+B*\sqrt{a}*a^3*c^3-25*B*\sqrt{a}*a^3*d^3-27*A*\sqrt{a}*a^3*c*d^2+3*A*\sqrt{a}*a^3*c^2*d+51*B*\sqrt{a}*a^3*c*d^2-27*B*\sqrt{a}*a^3*c^2*d)/a^2/(-(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+2*\sqrt{a}*(-\sqrt{a})*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+a)^4/\text{sign}(\tan((f*x+\exp(1))/2)+1)+1/32*(3*A*c^3-75*A*d^3+5*B*c^3+163*B*d^3+57*A*c*d^2+15*A*c^2*d-225*B*c*d^2+57*B*c^2*d)*\text{atan}((-\sqrt{a})*\tan((f*x+\exp(1))/2)-\sqrt{a})+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})/\sqrt{2}/\sqrt{-a})/\sqrt{2}/a^2/\sqrt{-a})/\text{sign}(\tan((f*x+\exp(1))/2)+1))) \end{aligned}$$

maple [B] time = 2.47, size = 1438, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sin(f*x+e))*(c+d*\sin(f*x+e))^3/(a+a*\sin(f*x+e))^{5/2}, x)$

[Out] $\frac{1}{96}a^{9/2}*((9A^2)^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^3+45A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^2*d+171A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c*d^2-225A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*d^3+192A*(a-a*\sin(f*x+e))^{1/2})^{3/2}*d^3+15B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^3+171B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^2*d-675B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c*d^2+489B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*d^3+576B*(a-a*\sin(f*x+e))^{1/2})^{3/2}*c*d^2-384B*(a-a*\sin(f*x+e))^{1/2})^{3/2}*d^3-64B*(a-a*\sin(f*x+e))^{3/2})^{1/2}*d^3*\cos(f*x+e)^2-2*\sin(f*x+e)*(9A^2)^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^3+45A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^2*d+171A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c*d^2-225A^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*d^3+192A*(a-a*\sin(f*x+e))^{1/2})^{3/2}*d^3+15B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^3+171B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c^2*d-675B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*c*d^2+489B^2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})^{2^{1/2}}/a^{1/2})^2*a^2*d^3+576B*(a-a*\sin(f*x+e))^{1/2})^{3/2}*c*d^2-384B*(a-a*\sin(f$

```

*x+e))^(1/2)*a^(3/2)*d^3-64*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3-90*A*2^(1
/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2*d+18*A*(a-a
*sin(f*x+e))^(3/2)*a^(1/2)*c^3+126*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^3+30*
B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^3-60*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^
3-36*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^3-612*A*(a-a*sin(f*x+e))^(1/2)*a^(3
/2)*d^3+1092*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^3-46*B*(a-a*sin(f*x+e))^(3/
2)*a^(1/2)*d^3-342*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c*d^2-342*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^2*d+1350*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c*d^2-18*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/
a^(1/2))*a^2*c^3+450*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*d^3-30*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*c^3-978*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1
/2))*a^2*d^3-1836*B*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2+396*B*(a-a*sin(f*x
+e))^(1/2)*a^(3/2)*c^2*d-108*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2*d+396*A*(
a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d^2-234*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^
2*d+378*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2+90*A*(a-a*sin(f*x+e))^(3/2)*
a^(1/2)*c^2*d-234*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d^2*(-a*(sin(f*x+e)-1
))^1/2/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^3}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^3/(a*sin(f*x + e) + a)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^3}{(a + a \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^3)/(a + a*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^3/(a+a*sin(f*x+e))^(5/2),x)
```

```
[Out] Timed out
```

$$3.322 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^2}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=219

$$\frac{(A(3c^2 + 10cd + 19d^2) + B(5c^2 + 38cd - 75d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} + \frac{d^2(A-9B) \cos(e+fx)}{4a^2 f \sqrt{a \sin(e+fx)+a}} \frac{(A-B)}{a}$$

[Out] $-1/16*(c-d)*(3*A*c+5*A*d+5*B*c-13*B*d)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{3/2}-1/4*(A-B)*\cos(f*x+e)*(c+d*\sin(f*x+e))^2/f/(a+a*\sin(f*x+e))^{5/2}-1/32*(B*(5*c^2+38*c*d-75*d^2)+A*(3*c^2+10*c*d+19*d^2))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{1/2})*2^{1/2}/(a+a*\sin(f*x+e))^{1/2}/a^{5/2}/f*2^{1/2}+1/4*(A-9*B)*d^2*\cos(f*x+e)/a^2/f/(a+a*\sin(f*x+e))^{1/2}$

Rubi [A] time = 0.58, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2977, 2968, 3019, 2751, 2649, 206}

$$\frac{(A(3c^2 + 10cd + 19d^2) + B(5c^2 + 38cd - 75d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} + \frac{d^2(A-9B) \cos(e+fx)}{4a^2 f \sqrt{a \sin(e+fx)+a}} \frac{(A-B)}{a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\sin[e + f*x])*(c + d*\sin[e + f*x])^2/(a + a*\sin[e + f*x])^{5/2}, x]$

[Out] $-((B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(16*\operatorname{Sqrt}[2]*a^{5/2}*f) - ((c - d)*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*\operatorname{Cos}[e + f*x])/(16*a*f*(a + a*\sin[e + f*x])^{3/2}) + ((A - 9*B)*d^2*\operatorname{Cos}[e + f*x])/(4*a^2*f*\operatorname{Sqrt}[a + a*\sin[e + f*x]]) - ((A - B)*\operatorname{Cos}[e + f*x]*(c + d*\sin[e + f*x])^2)/(4*f*(a + a*\sin[e + f*x])^{5/2})$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/\operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2751

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow -\operatorname{Simp}[(d*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m)/(f*(m + 1)), x] + \operatorname{Dist}[(a*d*m + b*c*(m + 1))/(b*(m + 1)), \operatorname{Int}[(a + b*\sin[e + f*x])^m, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{EqQ}[a^2 - b^2, 0] \ \&\& \operatorname{!LtQ}[m, -2^{-1}]$

Rule 2968

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]]^{(m_.)}*((A_.) + (B_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2),$

$x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 2977

$\text{Int}[(a_ + (b_.)\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((A_.) + (B_.)\sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)\sin[(e_.) + (f_.)*(x_)])^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(A*b - a*B)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^n/(a*f*(2*m + 1)), x] - \text{Dist}[1/(a*b*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*(c + d*\text{Sin}[e + f*x])^{(n - 1)}*\text{Simp}[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m + b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{LtQ}[m, -2^{(-1)}] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[2*m] \ \&\& \ (\text{IntegerQ}[2*n] \ || \ \text{EqQ}[c, 0])$

Rule 3019

$\text{Int}[(a_ + (b_.)\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((A_.) + (B_.)\sin[(e_.) + (f_.)*(x_)] + (C_.)\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] \rightarrow \text{Simp}[(A*b - a*B + b*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m/(a*f*(2*m + 1)), x] + \text{Dist}[1/(a^2*(2*m + 1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*\text{Simp}[a*A*(m + 1) + m*(b*B - a*C) + b*C*(2*m + 1)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{5/2}} dx &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{4f(a + a \sin(e + fx))^{5/2}} + \int \frac{(c + d \sin(e + fx))\left(\frac{1}{2}a\right)}{(a + a \sin(e + fx))^{5/2}} dx \\ &= -\frac{(A - B) \cos(e + fx)(c + d \sin(e + fx))^2}{4f(a + a \sin(e + fx))^{5/2}} + \int \frac{\frac{1}{2}ac(3Ac + 5Bc + 4Ad)}{(a + a \sin(e + fx))^{5/2}} dx \\ &= -\frac{(c - d)(3Ac + 5Bc + 5Ad - 13Bd) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} - \frac{(A - B) \cos(e + fx)}{4f} \\ &= -\frac{(c - d)(3Ac + 5Bc + 5Ad - 13Bd) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} + \frac{(A - 9B)d}{4a^2 f \sqrt{a + a \sin(e + fx)}} \\ &= -\frac{(c - d)(3Ac + 5Bc + 5Ad - 13Bd) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} + \frac{(A - 9B)d}{4a^2 f \sqrt{a + a \sin(e + fx)}} \\ &= -\frac{(B(5c^2 + 38cd - 75d^2) + A(3c^2 + 10cd + 19d^2)) \tanh^{-1}\left(\frac{c + d \sin(e + fx)}{\sqrt{a + a \sin(e + fx)}}\right)}{16\sqrt{2} a^{5/2} f} \end{aligned}$$

Mathematica [C] time = 1.14, size = 544, normalized size = 2.48

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left((2 + 2i)(-1)^{3/4} (A(3c^2 + 10cd + 19d^2) + B(5c^2 + 38cd - 75d^2))\right) \left(\sin\left(\frac{1}{2}(e + fx)\right)\right)}{16\sqrt{2} a^{5/2} f}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2)/(a + a*Sin[e + f*x])^(5/2), x]

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(-11*A*c^2*cos[(e + f*x)/2] + 3*B*c^2*cos[(e + f*x)/2] + 6*A*c*d*cos[(e + f*x)/2] + 10*B*c*d*cos[(e + f*x)/2] + 5*A*d^2*cos[(e + f*x)/2] - 45*B*d^2*cos[(e + f*x)/2] - 3*A*c^2*cos[(3*(e + f*x))/2] - 5*B*c^2*cos[(3*(e + f*x))/2] - 10*A*c*d*cos[(3*(e + f*x))/2] + 26*B*c*d*cos[(3*(e + f*x))/2] + 13*A*d^2*cos[(3*(e + f*x))/2] - 69*B*d^2*cos[(3*(e + f*x))/2] + 16*B*d^2*cos[(5*(e + f*x))/2] + 11*A*c^2*sin[(e + f*x)/2] - 3*B*c^2*sin[(e + f*x)/2] - 6*A*c*d*sin[(e + f*x)/2] - 10*B*c*d*sin[(e + f*x)/2] - 5*A*d^2*sin[(e + f*x)/2] + 45*B*d^2*sin[(e + f*x)/2] + (2 + 2*I)*(-1)^(3/4)*(B*(5*c^2 + 38*c*d - 75*d^2) + A*(3*c^2 + 10*c*d + 19*d^2))*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 3*A*c^2*sin[(3*(e + f*x))/2] - 5*B*c^2*sin[(3*(e + f*x))/2] - 10*A*c*d*sin[(3*(e + f*x))/2] + 26*B*c*d*sin[(3*(e + f*x))/2] + 13*A*d^2*sin[(3*(e + f*x))/2] - 69*B*d^2*sin[(3*(e + f*x))/2] - 16*B*d^2*sin[(5*(e + f*x))/2]))/(32*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.48, size = 744, normalized size = 3.40

$$\sqrt{2} \left(\left((3A + 5B)c^2 + 2(5A + 19B)cd + (19A - 75B)d^2 \right) \cos(fx + e)^3 - 4(3A + 5B)c^2 - 8(5A + 19B)cd - 4(19A - 75B)d^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/64*(sqrt(2)*(((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^3 - 4*(3*A + 5*B)*c^2 - 8*(5*A + 19*B)*c*d - 4*(19*A - 75*B)*d^2 + 3*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 - 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e) - (4*(3*A + 5*B)*c^2 + 8*(5*A + 19*B)*c*d + 4*(19*A - 75*B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e)^2 + 2*((3*A + 5*B)*c^2 + 2*(5*A + 19*B)*c*d + (19*A - 75*B)*d^2)*cos(f*x + e))*sin(f*x + e)*sqrt(a)*log(-(a*cos(f*x + e))^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(32*B*d^2*cos(f*x + e)^3 - 4*(A - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A - 53*B)*d^2)*cos(f*x + e)^2 - ((7*A + B)*c^2 + 2*(A - 9*B)*c*d - 9*(A - 9*B)*d^2)*cos(f*x + e) - (32*B*d^2*cos(f*x + e)^2 - 4*(A - B)*c^2 + 8*(A - B)*c*d - 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 + 2*(5*A - 13*B)*c*d - (13*A - 85*B)*d^2)*cos(f*x + e))*sin(f*x + e)*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*(2*sqrt(a*tan((f*x+exp(1))/2)^2+a)*(-1/2*B*d^2/a^2/sign(tan((f*x+exp(1))/2)+1)+1/2*B*d^2*tan((f*x+exp(1))/2)/a^2/sign(tan((f*x+exp(1))/2)+1)))/(a*tan((f*x+exp(1))/2)^2+a)+2*(1/32*(-29*A*c^2*(-sqrt(a)*t
```


$$\begin{aligned} & \text{an}((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7+19*A*d^2*(-\sqrt{a}*\tan \\ & \text{an}((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7+5*B*c^2*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7-43*B*d^2*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7+10*A*c*d*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7+38*B*c*d*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^7+75*A*\sqrt{a}*c^2*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^6-133*A*\sqrt{a}*d^2*(-\sqrt{a}*\tan \\ & ((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^6+29*B*\sqrt{a} \\ & *c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^6+237*B \\ & *\sqrt{a}*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a}) \\ & ^6+58*A*\sqrt{a}*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2) \\ &)^2+a})^6-266*B*\sqrt{a}*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+e \\ & xp(1))/2)^2+a})^6-55*A*a*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+ \\ & exp(1))/2)^2+a})^5+89*A*a*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x \\ & +exp(1))/2)^2+a})^5-17*B*a*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x \\ & x+\exp(1))/2)^2+a})^5-161*B*a*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((\\ & f*x+\exp(1))/2)^2+a})^5-34*A*a*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan(\\ & (f*x+\exp(1))/2)^2+a})^5+178*B*a*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan \\ & n((f*x+\exp(1))/2)^2+a})^5-91*A*\sqrt{a}*a*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+ \\ & \sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^4+117*A*\sqrt{a}*a*d^2*(-\sqrt{a}*\tan((f*x+e \\ & xp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^4-13*B*\sqrt{a}*a*c^2*(-\sqrt{a}*\tan \\ & an((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^4-221*B*\sqrt{a}*a*d^2*(- \\ & -\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^4-26*A*\sqrt{a} \\ &)*a*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^4+23 \\ & 4*B*\sqrt{a}*a*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^ \\ & 2+a})^4+A*a^2*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^ \\ & 2+a})^3+17*A*a^2*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/ \\ & 2)^2+a})^3-9*B*a^2*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1) \\ &)/2)^2+a})^3-25*B*a^2*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp \\ & (1))/2)^2+a})^3-18*A*a^2*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+ \\ & exp(1))/2)^2+a})^3+34*B*a^2*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f \\ & *x+\exp(1))/2)^2+a})^3+27*A*a^3*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan \\ & ((f*x+\exp(1))/2)^2+a})-53*A*a^3*d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan \\ & n((f*x+\exp(1))/2)^2+a})+65*A*\sqrt{a}*a^2*c^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+ \\ & \sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2-47*A*\sqrt{a}*a^2*d^2*(-\sqrt{a}*\tan((f*x+ \\ & exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+13*B*a^3*c^2*(-\sqrt{a}*\tan((f \\ & *x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+93*B*a^3*d^2*(-\sqrt{a}*\tan((\\ & f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-9*B*\sqrt{a}*a^2*c^2*(-\sqrt{a} \\ &)*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+103*B*\sqrt{a}*a^2 \\ & *d^2*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+26*A* \\ & a^3*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})-18*A \\ & *\sqrt{a}*a^2*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2 \\ & +a})^2-106*B*a^3*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/ \\ & 2)^2+a})-94*B*\sqrt{a}*a^2*c*d*(-\sqrt{a}*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x \\ & +exp(1))/2)^2+a})^2+7*A*\sqrt{a}*a^3*c^2-9*A*\sqrt{a}*a^3*d^2+B*\sqrt{a}*a^3*c \\ & ^2+17*B*\sqrt{a}*a^3*d^2+2*A*\sqrt{a}*a^3*c*d-18*B*\sqrt{a}*a^3*c*d)/a^2/(-(-\sqrt{a} \\ &)*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})^2+2*\sqrt{a}*(-\sqrt{a} \\ &)*\tan((f*x+\exp(1))/2)+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a})+a)^4/\text{sign}(\tan((\\ & f*x+\exp(1))/2)+1)+1/32*(3*A*c^2+19*A*d^2+5*B*c^2-75*B*d^2+10*A*c*d+38*B*c*d \\ &)*\text{atan}((-\sqrt{a}*\tan((f*x+\exp(1))/2)-\sqrt{a}+\sqrt{a*\tan((f*x+\exp(1))/2)^2+a} \\ &))/\sqrt{2}/\sqrt{-a})/\sqrt{2}/a^2/\sqrt{-a}/\text{sign}(\tan((f*x+\exp(1))/2)+1))) \end{aligned}$$

maple [B] time = 2.04, size = 982, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\sin(f*x+e))*(c+d*\sin(f*x+e))^2/(a+a*\sin(f*x+e))^{5/2}, x)$

[Out] $-1/32*(2*\sin(f*x+e)*(3*A*2^{1/2}*\text{arctanh}(1/2*(a-a*\sin(f*x+e))^{1/2})*2^{1/2})$

```

/a^(1/2))*a^2*c^2+10*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a
^(1/2))*a^2*c*d+19*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*a^2*d^2+5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2)
))*a^2*c^2+38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*a^2*c*d-75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a
^2*d^2+64*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2))+(-3*A*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-10*A*2^(1/2)*arctanh(1/2*(a-
a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-19*A*2^(1/2)*arctanh(1/2*(a-a*
sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-64*B*d^2*a^(3/2)*(a-a*sin(f*x+e)
)^(1/2)-5*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2
*c^2-38*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
*d+75*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2
)*cos(f*x+e)^2-6*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2-20*A*(a-a*sin(f*x+e)
)^(3/2)*a^(1/2)*c*d+26*A*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+20*A*(a-a*sin(f*
x+e))^(1/2)*a^(3/2)*c^2+24*A*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c*d-44*A*d^2*a^
(3/2)*(a-a*sin(f*x+e))^(1/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)
)*2^(1/2)/a^(1/2))*a^2*c^2+20*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*a^2*c*d+38*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(
1/2)/a^(1/2))*a^2*d^2-10*B*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+52*B*(a-a*sin
(f*x+e))^(3/2)*a^(1/2)*c*d-42*B*d^2*(a-a*sin(f*x+e))^(3/2)*a^(1/2)+12*B*(a-
a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-88*B*c*d*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+204
*B*d^2*a^(3/2)*(a-a*sin(f*x+e))^(1/2)+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x
+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+76*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e
))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-150*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(
f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^2}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2/(a+a*sin(f*x+e))^(5/2),x, alg
orithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2/(a*sin(f*x + e) + a)^(
5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^2}{(a + a \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^2)/(a + a*sin(e + f*x))^(5/2
), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2/(a+a*sin(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

$$3.323 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=151

$$\frac{(3Ac + 5Ad + 5Bc + 19Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(3Ac + 5Ad + 5Bc - 13Bd) \cos(e+fx)}{16af(a \sin(e+fx) + a)^{3/2}} - \frac{(A-B)(c-d)}{4f(a \sin(e+fx) + a)^{3/2}}$$

[Out] $-1/4*(A-B)*(c-d)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(5/2)}-1/16*(3*A*c+5*A*d+5*B*c-13*B*d)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(3/2)}-1/32*(3*A*c+5*A*d+5*B*c+19*B*d)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)})/a^{(5/2)}/f*2^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2968, 3019, 2750, 2649, 206}

$$\frac{(3Ac + 5Ad + 5Bc + 19Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(3Ac + 5Ad + 5Bc - 13Bd) \cos(e+fx)}{16af(a \sin(e+fx) + a)^{3/2}} - \frac{(A-B)(c-d)}{4f(a \sin(e+fx) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\sin[e + f*x])*(c + d*\sin[e + f*x])/(a + a*\sin[e + f*x])^{(5/2)}, x]$

[Out] $-((3*A*c + 5*B*c + 5*A*d + 19*B*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*f) - ((A - B)*(c - d)*\operatorname{Cos}[e + f*x])/(4*f*(a + a*\sin[e + f*x])^{(5/2)}) - ((3*A*c + 5*B*c + 5*A*d - 13*B*d)*\operatorname{Cos}[e + f*x])/(16*a*f*(a + a*\sin[e + f*x])^{(3/2)})$

Rule 206

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt}Q[a, 0] \mid \mid \operatorname{Lt}Q[b, 0])$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_ + (b_)*\sin[(c_ + (d_)*(x_))]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/ \operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2750

$\operatorname{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))]^{(m_)*((c_ + (d_)*\sin[(e_ + (f_)*(x_))])}, x_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^m]/(a*f*(2*m + 1)), x] + \operatorname{Dist}[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), \operatorname{Int}[(a + b*\sin[e + f*x])^{(m + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{EqQ}[a^2 - b^2, 0] \&\& \operatorname{Lt}Q[m, -2^{(-1)}]$

Rule 2968

$\operatorname{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))]^{(m_)*((A_ + (B_)*\sin[(e_ + (f_)*(x_))])}, x_Symbol] \rightarrow \operatorname{Int}[(a + b*\sin[e + f*x])^m*(A*c + (B*c + A*d)*\sin[e + f*x] + B*d*\sin[e + f*x]^2), x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 3019

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))}{(a + a \sin(e + fx))^{5/2}} dx &= \int \frac{Ac + (Bc + Ad) \sin(e + fx) + Bd \sin^2(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac + 5Bc + 5Ad - 5Bd) - 4aBd \sin(e + fx)}{(a + a \sin(e + fx))^{3/2}} dx}{4a^2} \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3Ac + 5Bc + 5Ad - 13Bd) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} \\ &= -\frac{(A - B)(c - d) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3Ac + 5Bc + 5Ad - 13Bd) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} \\ &\quad - \frac{(3Ac + 5Bc + 5Ad + 19Bd) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(A - B)}{4f(a + a \sin(e + fx))^{5/2}} \end{aligned}$$

Mathematica [C] time = 0.77, size = 267, normalized size = 1.77

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(8(A - B)(c - d) \sin\left(\frac{1}{2}(e + fx)\right) - (3Ac + 5Ad + 5Bc - 13Bd)\right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)}{16\sqrt{2} a^{5/2} f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]))/(a + a*Sin[e + f*x])^(5/2), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)*Sin[(e + f*x)/2] - 4*(A - B)*(c - d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(3*A*c + 5*B*c + 5*A*d - 13*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A*c + 5*B*c + 5*A*d - 13*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A*c + 5*B*c + 5*A*d + 19*B*d)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

fricas [B] time = 0.48, size = 536, normalized size = 3.55

$$\sqrt{2} \left(((3A + 5B)c + (5A + 19B)d) \cos^3(fx + e) + 3((3A + 5B)c + (5A + 19B)d) \cos^2(fx + e) - 4(3A + 5B) \cos(fx + e) + 4(3A + 5B) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2), x, algor
ithm="fricas")
```

```
[Out] 1/64*(sqrt(2)*(((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^3 + 3*((3*A +
5*B)*c + (5*A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*
d - 2*((3*A + 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e) + (((3*A + 5*B)*c + (5*
A + 19*B)*d)*cos(f*x + e)^2 - 4*(3*A + 5*B)*c - 4*(5*A + 19*B)*d - 2*((3*A
+ 5*B)*c + (5*A + 19*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a)*log(-(a*cos(
f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin
(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2
*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))
+ 4*((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e)^2 + 4*(A - B)*c - 4*(A -
B)*d + ((7*A + B)*c + (A - 9*B)*d)*cos(f*x + e) - (4*(A - B)*c - 4*(A - B)
*d - ((3*A + 5*B)*c + (5*A - 13*B)*d)*cos(f*x + e))*sin(f*x + e))*sqrt(a*si
n(f*x + e) + a)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*c
os(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^
3*f)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algor
ithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*2*(1/32*(-29*A*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+
exp(1))/2)^2+a))^7+5*A*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(
1))/2)^2+a))^7+5*B*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/
2)^2+a))^7+19*B*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^7+75*A*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1)
)/2)^2+a))^6+29*A*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+e
xp(1))/2)^2+a))^6+29*B*sqrt(a)*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((
f*x+exp(1))/2)^2+a))^6-133*B*sqrt(a)*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a
*tan((f*x+exp(1))/2)^2+a))^6-55*A*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*
tan((f*x+exp(1))/2)^2+a))^5-17*A*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*t
an((f*x+exp(1))/2)^2+a))^5-17*B*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*ta
n((f*x+exp(1))/2)^2+a))^5+89*B*a*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan
((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sq
rt(a*tan((f*x+exp(1))/2)^2+a))^4-13*A*sqrt(a)*a*d*(-sqrt(a)*tan((f*x+exp(1)
)/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*B*sqrt(a)*a*c*(-sqrt(a)*tan((f*x
+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+117*B*sqrt(a)*a*d*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+A*a^2*c*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*A*a^2*d*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*a^2*c*(-sqrt(a)*tan
((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+17*B*a^2*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*A*a^3*c*(-sqrt(a)*t
an((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+13*A*a^3*d*(-sqrt(a)*ta
n((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+65*A*sqrt(a)*a^2*c*(-sq
rt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-9*A*sqrt(a)*a^2
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+13*B*a^
3*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-53*B*a^3
*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))-9*B*sqrt(
a)*a^2*c*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-4
7*B*sqrt(a)*a^2*d*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^
2+a))^2+7*A*sqrt(a)*a^3*c+A*sqrt(a)*a^3*d+B*sqrt(a)*a^3*c-9*B*sqrt(a)*a^3*d
)/a^2/(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*
sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a)^4
/sign(tan((f*x+exp(1))/2)+1)+1/32*(3*A*c+5*A*d+5*B*c+19*B*d)*atan((-sqrt(a)
*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(
-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))
```

maple [B] time = 1.64, size = 449, normalized size = 2.97

$$\left(2 \sin(fx + e) \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{2\sqrt{a}} \right) \right) a^2 (3Ac + 5Ad + 5Bc + 19Bd) - \sqrt{2} \operatorname{arctanh} \left(\frac{\sqrt{a - a \sin(fx + e)} \sqrt{2}}{2\sqrt{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2), x)

[Out]
$$-1/32*(2*\sin(f*x+e)*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)-2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*(3*A*c+5*A*d+5*B*c+19*B*d)*\cos(f*x+e)^2+6*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c+10*A*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*d+20*A*(a-a*\sin(f*x+e))^{(1/2)}*a^{(3/2)}*c+12*A*(a-a*\sin(f*x+e))^{(1/2)}*a^{(3/2)}*d-6*A*(a-a*\sin(f*x+e))^{(3/2)}*a^{(1/2)}*c-10*A*(a-a*\sin(f*x+e))^{(3/2)}*a^{(1/2)}*d+10*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c+38*B*2^{(1/2)}*\operatorname{arctanh}(1/2*(a-a*\sin(f*x+e))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*d+12*B*(a-a*\sin(f*x+e))^{(1/2)}*a^{(3/2)}*c-44*B*(a-a*\sin(f*x+e))^{(1/2)}*a^{(3/2)}*d-10*B*(a-a*\sin(f*x+e))^{(3/2)}*a^{(1/2)}*c+26*B*(a-a*\sin(f*x+e))^{(3/2)}*a^{(1/2)}*d)*(-a*(\sin(f*x+e)-1))^{(1/2)}/a^{(9/2)}/(1+\sin(f*x+e))/\cos(f*x+e)/(a+a*\sin(f*x+e))^{(1/2)}/f$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)}{(a \sin(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2), x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)/(a*sin(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))}{(a + a \sin(e + fx))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2), x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x)))/(a + a*sin(e + f*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2), x)

[Out] Timed out

$$3.324 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=126

$$-\frac{(3A+5B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(3A+5B) \cos(e+fx)}{16af(a \sin(e+fx)+a)^{3/2}} - \frac{(A-B) \cos(e+fx)}{4f(a \sin(e+fx)+a)^{5/2}}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)/f/(a+a*\sin(f*x+e))^{(5/2)}-1/16*(3*A+5*B)*\cos(f*x+e)/a/f/(a+a*\sin(f*x+e))^{(3/2)}-1/32*(3*A+5*B)*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)})/(a+a*\sin(f*x+e))^{(1/2)}/a^{(5/2)}/f*2^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2750, 2650, 2649, 206}

$$-\frac{(3A+5B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(3A+5B) \cos(e+fx)}{16af(a \sin(e+fx)+a)^{3/2}} - \frac{(A-B) \cos(e+fx)}{4f(a \sin(e+fx)+a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2), x]

[Out] $-((3*A+5*B)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e+f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a+a*\operatorname{Sin}[e+f*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*f) - ((A-B)*\operatorname{Cos}[e+f*x])/(4*f*(a+a*\operatorname{Sin}[e+f*x])^{(5/2)}) - ((3*A+5*B)*\operatorname{Cos}[e+f*x])/(16*a*f*(a+a*\operatorname{Sin}[e+f*x])^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2650

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n + 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2750

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2}} dx &= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} + \frac{(3A + 5B) \int \frac{1}{(a + a \sin(e + fx))^{3/2}} dx}{8a} \\
&= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3A + 5B) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} + \frac{(3A + 5B) \int \frac{1}{\sqrt{a + a \sin(e + fx)}}}{32a^2} \\
&= -\frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3A + 5B) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}} - \frac{(3A + 5B) \operatorname{Subst}\left(\int \frac{1}{2a - x^2}\right)}{16a^2} \\
&= -\frac{(3A + 5B) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e + fx)}{\sqrt{2} \sqrt{a + a \sin(e + fx)}}\right)}{16\sqrt{2} a^{5/2} f} - \frac{(A - B) \cos(e + fx)}{4f(a + a \sin(e + fx))^{5/2}} - \frac{(3A + 5B) \cos(e + fx)}{16af(a + a \sin(e + fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.37, size = 227, normalized size = 1.80

$$\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left(8(A - B) \sin\left(\frac{1}{2}(e + fx)\right) - (3A + 5B) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right)\right)^3$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sin[e + f*x])/(a + a*Sin[e + f*x])^(5/2), x]

[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*Sin[(e + f*x)/2] + 4*(-A + B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])) + 2*(3*A + 5*B)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (3*A + 5*B)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(3*A + 5*B)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4)/(16*f*(a*(1 + Sin[e + f*x]))^(5/2))

fricas [B] time = 0.46, size = 392, normalized size = 3.11

$$\sqrt{2} \left((3A + 5B) \cos(fx + e)^3 + 3(3A + 5B) \cos(fx + e)^2 - 2(3A + 5B) \cos(fx + e) + (3A + 5B) \cos(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] 1/64*(sqrt(2)*((3*A + 5*B)*cos(f*x + e)^3 + 3*(3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e)^2 - 2*(3*A + 5*B)*cos(f*x + e) - 12*A - 20*B)*sin(f*x + e) - 12*A - 20*B)*sqrt(a)*log(-(a*cos(f*x + e))^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*((3*A + 5*B)*cos(f*x + e)^2 + (7*A + B)*cos(f*x + e) + ((3*A + 5*B)*cos(f*x + e) - 4*A + 4*B)*sin(f*x + e) + 4*A - 4*B)*sqrt(a*sin(f*x + e) + a)/(a^3*f*cos(f*x + e)^3 + 3*a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f + (a^3*f*cos(f*x + e)^2 - 2*a^3*f*cos(f*x + e) - 4*a^3*f)*sin(f*x + e))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*(1/32*(-29*A*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+5*B*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^7+75*A*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6+29*B*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^6-55*A*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-17*B*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^5-91*A*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4-13*B*sqrt(a)*a*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^4+A*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3-9*B*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+27*A*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^3+65*A*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+13*B*a^3*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2-9*B*sqrt(a)*a^2*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+7*A*sqrt(a)*a^3+B*sqrt(a)*a^3/a^2/(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))^2+2*sqrt(a)*(-sqrt(a)*tan((f*x+exp(1))/2)+sqrt(a*tan((f*x+exp(1))/2)^2+a))+a^4/sign(tan((f*x+exp(1))/2)+1)+1/32*(3*A+5*B)*atan((-sqrt(a)*tan((f*x+exp(1))/2)-sqrt(a)+sqrt(a*tan((f*x+exp(1))/2)^2+a))/sqrt(2)/sqrt(-a))/sqrt(2)/a^2/sqrt(-a)/sign(tan((f*x+exp(1))/2)+1))

maple [B] time = 1.70, size = 279, normalized size = 2.21

$$\frac{\left(2 \sin (f x+e) \sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} \sqrt{2}}{2 \sqrt{a}}\right) a^3(3 A+5 B)-\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{a-a \sin (f x+e)} \sqrt{2}}{2 \sqrt{a}}\right) a^3(3 A+5 B)\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x)

[Out] -1/32*(2*sin(f*x+e)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(3*A+5*B)-2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(3*A+5*B)*cos(f*x+e)^2+20*A*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-6*A*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+12*B*(a-a*sin(f*x+e))^(1/2)*a^(5/2)-10*B*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+6*A*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3+10*B*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^3*(-a*(sin(f*x+e)-1))^(1/2)/a^(11/2)/(1+sin(f*x+e))/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin (f x+e)+A}{\left(a \sin (f x+e)+a\right)^{\frac{5}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/(a*sin(f*x + e) + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B \sin (e+f x)}{\left(a+a \sin (e+f x)\right)^{\frac{5}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)`

[Out] `int((A + B*sin(e + f*x))/(a + a*sin(e + f*x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a(\sin(e + fx) + 1))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2), x)`

[Out] `Integral((A + B*sin(e + f*x))/(a*(sin(e + f*x) + 1))**(5/2), x)`

$$3.325 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))} dx$$

Optimal. Leaf size=261

$$\frac{\left(A(3c^2 - 14cd + 43d^2) + B(5c^2 - 34cd - 3d^2)\right) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right) + 2d^{3/2}(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} c}{\sqrt{c+d} \sqrt{as}}\right)}{16\sqrt{2} a^{5/2} f(c-d)^3 + a^{5/2} f(c-d)^3 \sqrt{c+d}}$$

[Out] $-1/4*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^{(5/2)}-1/16*(3*A*c-11*A*d+5*B*c+3*B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^{(3/2)}-1/32*(B*(5*c^2-34*c*d-3*d^2)+A*(3*c^2-14*c*d+43*d^2))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{(1/2)}*2^{(1/2)})/(a+a*\sin(f*x+e))^{(1/2)}/a^{(5/2)}/(c-d)^3/f*2^{(1/2)}-2*d^{(3/2)}*(-A*d+B*c)*\operatorname{arctanh}(\cos(f*x+e)*a^{(1/2)}*d^{(1/2)})/(c+d)^{(1/2)}/(a+a*\sin(f*x+e))^{(1/2)}/a^{(5/2)}/(c-d)^3/f/(c+d)^{(1/2)}$

Rubi [A] time = 0.98, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {2978, 2985, 2649, 206, 2773, 208}

$$\frac{\left(A(3c^2 - 14cd + 43d^2) + B(5c^2 - 34cd - 3d^2)\right) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right) + 2d^{3/2}(Bc - Ad) \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{d} c}{\sqrt{c+d} \sqrt{as}}\right)}{16\sqrt{2} a^{5/2} f(c-d)^3 + a^{5/2} f(c-d)^3 \sqrt{c+d}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])), x]

[Out] $-((B*(5*c^2 - 34*c*d - 3*d^2) + A*(3*c^2 - 14*c*d + 43*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(16*\operatorname{Sqrt}[2]*a^{(5/2)}*(c - d)^3*f) - (2*d^{(3/2)}*(B*c - A*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\sin[e + f*x]])])/(a^{(5/2)}*(c - d)^3*\operatorname{Sqrt}[c + d]*f) - ((A - B)*\operatorname{Cos}[e + f*x])/(4*(c - d)*f*(a + a*\sin[e + f*x])^{(5/2)}) - ((3*A*c + 5*B*c - 11*A*d + 3*B*d)*\operatorname{Cos}[e + f*x])/((16*a*(c - d)^2*f*(a + a*\sin[e + f*x])^{(3/2)})$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2649

Int[1/Sqrt[(a_) + (b_.)*sin[(c_) + (d_.)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 2773

Int[Sqrt[(a_) + (b_.)*sin[(e_) + (f_.)*(x_)]]/((c_) + (d_.)*sin[(e_) + (f_.)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + a \sin(e + fx))^{5/2}(c + d \sin(e + fx))} dx = -\frac{(A - B) \cos(e + fx)}{4(c - d)f(a + a \sin(e + fx))^{5/2}} - \frac{\int \frac{-\frac{1}{2}a(3Ac+5Bc-8Ad)-\frac{3}{2}a(A-B)d \sin(e+fx)}{(a+a \sin(e+fx))^{3/2}(c+d \sin(e+fx))} dx}{4a^2(c - d)}$$

$$= -\frac{(A - B) \cos(e + fx)}{4(c - d)f(a + a \sin(e + fx))^{5/2}} - \frac{(3Ac + 5Bc - 11Ad + 3Bd) \cos(e + fx)}{16a(c - d)^2 f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{(A - B) \cos(e + fx)}{4(c - d)f(a + a \sin(e + fx))^{5/2}} - \frac{(3Ac + 5Bc - 11Ad + 3Bd) \cos(e + fx)}{16a(c - d)^2 f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{(A - B) \cos(e + fx)}{4(c - d)f(a + a \sin(e + fx))^{5/2}} - \frac{(3Ac + 5Bc - 11Ad + 3Bd) \cos(e + fx)}{16a(c - d)^2 f(a + a \sin(e + fx))^{5/2}}$$

$$= -\frac{(B(5c^2 - 34cd - 3d^2) + A(3c^2 - 14cd + 43d^2)) \tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}\sqrt{a + a \sin(e + fx)}}\right)}{16\sqrt{2} a^{5/2}(c - d)^3 f}$$

Mathematica [C] time = 5.61, size = 550, normalized size = 2.11

$$\frac{\left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right)\right) \left((1 + i)(-1)^{3/4} \left(A(3c^2 - 14cd + 43d^2) + B(5c^2 - 34cd - 3d^2) \right) \left(\sin\left(\frac{1}{2}(e + fx)\right) + \cos\left(\frac{1}{2}(e + fx)\right) \right) \right)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + a*Sin[e + f*x])^(5/2)*(c + d*Sin[e + f*x])), x]
```

```
[Out] ((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(8*(A - B)*(c - d)^2*Sin[(e + f*x)/2] + 4*(-A + B)*(c - d)^2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 2*(c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (c - d)*(3*A*c + 5*B*c - 11*A*d + 3*B*d)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (1 + I)*(-1)^(3/4)*(B*(5*c^2 - 34*c*d - 3*d^2) + A
```

$$\frac{(3c^2 - 14cd + 43d^2) \operatorname{ArcTanh}\left[\frac{1}{2} + \frac{1}{2}(-1)^{3/4}(-1 + \tan\left(\frac{e + fx}{4}\right))\right] (\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right))^4 + (8d^{3/2}(-Bc) + Ad)(e + fx - 2\log[\sec\left(\frac{e + fx}{4}\right)^2] + 2\log[\sec\left(\frac{e + fx}{4}\right)^2(\sqrt{c + d} + \sqrt{d}\cos\left(\frac{e + fx}{2}\right) - \sqrt{d}\sin\left(\frac{e + fx}{2}\right))]}{(\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right))^4}\right) + (8d^{3/2}(Bc - Ad)(e + fx - 2\log[\sec\left(\frac{e + fx}{4}\right)^2] + 2\log[\sec\left(\frac{e + fx}{4}\right)^2(\sqrt{c + d} - \sqrt{d}\cos\left(\frac{e + fx}{2}\right) + \sqrt{d}\sin\left(\frac{e + fx}{2}\right))]}{(\cos\left(\frac{e + fx}{2}\right) + \sin\left(\frac{e + fx}{2}\right))^4}\right)}{\sqrt{c + d}}$$

fricas [B] time = 6.57, size = 2577, normalized size = 9.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] $\frac{1}{64} \sqrt{2} \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos^3(fx + e) - 4(3A + 5B)c^2 + 8(7A + 17B)cd - 4(43A - 3B)d^2 + 3 \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos^2(fx + e) - 2 \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos(fx + e) - (4(3A + 5B)c^2 - 8(7A + 17B)cd + 4(43A - 3B)d^2 - ((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2) \cos^2(fx + e) + 2((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2) \cos(fx + e)) \sin(fx + e) \sqrt{a} \log(-a \cos^2(fx + e) - 2\sqrt{2} \sqrt{a \sin(fx + e) + a} \sqrt{a} (\cos(fx + e) - \sin(fx + e) + 1) + 3a \cos(fx + e) - (a \cos(fx + e) - 2a) \sin(fx + e) + 2a) / (\cos^2(fx + e) - (\cos(fx + e) + 2) \sin(fx + e) - \cos(fx + e) - 2)) - 32(4Bac d - 4Aa d^2 - (Bac d - Aa d^2) \cos^3(fx + e) - 3(Bac d - Aa d^2) \cos^2(fx + e) + 2(Bac d - Aa d^2) \cos(fx + e) + (4Bac d - 4Aa d^2 - (Bac d - Aa d^2) \cos^2(fx + e) + 2(Bac d - Aa d^2) \cos(fx + e)) \sin(fx + e)) \sqrt{d/(ac + ad)} \log((d^2 \cos^3(fx + e) - (6cd + 7d^2) \cos^2(fx + e) - c^2 - 2cd - d^2 - 4((cd + d^2) \cos^2(fx + e) - c^2 - 4cd - 3d^2 - (c^2 + 3cd + 2d^2) \cos(fx + e) + (c^2 + 4cd + 3d^2 + (cd + d^2) \cos(fx + e)) \sin(fx + e)) \sqrt{a \sin(fx + e) + a} \sqrt{d/(ac + ad)} - (c^2 + 8cd + 9d^2) \cos^2(fx + e) + (d^2 \cos^2(fx + e) - c^2 - 2cd - d^2 + 2(3cd + 4d^2) \cos(fx + e)) \sin(fx + e)) / (d^2 \cos^3(fx + e) + (2cd + d^2) \cos^2(fx + e) - c^2 - 2cd - d^2 - (c^2 + d^2) \cos(fx + e) + (d^2 \cos^2(fx + e) - 2cd \cos(fx + e) - c^2 - 2cd - d^2) \sin(fx + e))) + 4(4(A - B)c^2 - 8(A - B)cd + 4(A - B)d^2 + ((3A + 5B)c^2 - 2(7A + 17B)cd + (11A - 3B)d^2) \cos^2(fx + e) + ((7A + 17B)c^2 - 2(11A - 3B)cd + (15A - 7B)d^2) \cos(fx + e) - (4(A - B)c^2 - 8(A - B)cd + 4(A - B)d^2 - ((3A + 5B)c^2 - 2(7A + 17B)cd + (11A - 3B)d^2) \cos(fx + e)) \sin(fx + e)) \sqrt{a \sin(fx + e) + a} / ((a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f \cos^3(fx + e) + 3(a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f \cos^2(fx + e) - 2(a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f \cos(fx + e) - 4(a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f + ((a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f \cos^2(fx + e) - 2(a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f \cos(fx + e) - 4(a^3 c^3 - 3a^3 c^2 d + 3a^3 c d^2 - a^3 d^3) f) \sin(fx + e)), \frac{1}{64} \sqrt{2} \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos^3(fx + e) - 4(3A + 5B)c^2 + 8(7A + 17B)cd - 4(43A - 3B)d^2 + 3 \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos^2(fx + e) - 2 \left((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2 \right) \cos(fx + e) - (4(3A + 5B)c^2 - 8(7A + 17B)cd + 4(43A - 3B)d^2 - ((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2) \cos^2(fx + e) + 2((3A + 5B)c^2 - 2(7A + 17B)cd + (43A - 3B)d^2) \cos(fx + e)) \sin(fx + e) \sqrt{a} \log(-a \cos^2(fx + e) - 2\sqrt{2} \sqrt{a \sin(fx + e) + a} \sqrt{a} (\cos(fx + e) - \sin(fx + e) + 1) + 3a \cos(fx + e) - (a \cos(fx + e) - 2a) \sin(fx + e) + 2a) / (\cos^2(fx + e) - (\cos(fx + e) + 2) \sin(fx + e) - \cos(fx + e) - 2)) +$

```

64*(4*B*a*c*d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^3 - 3*(B*a*c*d
- A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*cos(f*x + e) + (4*B*a*c*
d - 4*A*a*d^2 - (B*a*c*d - A*a*d^2)*cos(f*x + e)^2 + 2*(B*a*c*d - A*a*d^2)*
cos(f*x + e))*sin(f*x + e))*sqrt(-d/(a*c + a*d))*arctan(1/2*sqrt(a*sin(f*x
+ e) + a)*(d*sin(f*x + e) - c - 2*d)*sqrt(-d/(a*c + a*d))/(d*cos(f*x + e)))
+ 4*(4*(A - B)*c^2 - 8*(A - B)*c*d + 4*(A - B)*d^2 + ((3*A + 5*B)*c^2 - 2*
(7*A + B)*c*d + (11*A - 3*B)*d^2)*cos(f*x + e)^2 + ((7*A + B)*c^2 - 2*(11*A
- 3*B)*c*d + (15*A - 7*B)*d^2)*cos(f*x + e) - (4*(A - B)*c^2 - 8*(A - B)*c
*d + 4*(A - B)*d^2 - ((3*A + 5*B)*c^2 - 2*(7*A + B)*c*d + (11*A - 3*B)*d^2)
*cos(f*x + e))*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/((a^3*c^3 - 3*a^3*c^
2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^3 + 3*(a^3*c^3 - 3*a^3*c^2*d +
3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*
c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 -
a^3*d^3)*f + ((a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e
)^2 - 2*(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f*cos(f*x + e) - 4*
(a^3*c^3 - 3*a^3*c^2*d + 3*a^3*c*d^2 - a^3*d^3)*f)*sin(f*x + e))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algor
ithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check si
gn: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unabl
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*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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able to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>
(-4*pi/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable
to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (
2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign assu
mes constant sign by intervals (correct if the argument is real):Check [abs
(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2)>(
-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)
Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not check
edUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check
sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_no
step/2)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_
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to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4p
i/x/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/
2)Warning, integration of abs or sign assumes constant sign by intervals (c
orrect if the argument is real):Check [abs(t_nostep+1)]Warning, assuming -c
^3*d^7-c^2*d^8+c*d^9+d^10-c*d^9-d^10 is positive. Hint: run assume to make
assumptions on a variableWarning, assuming -a*c^3*d^7-a*c^2*d^8+a*c*d^9+a*d
^10-a*c*d^9-a*d^10 is positive. Hint: run assume to make assumptions on a v
ariableWarning, need to choose a branch for the root of a polynomial with p
arameters. This might be wrong.Non regular value [0] was discarded and repl
aced randomly by 0=[37]Evaluation time: 76.21sym2poly/r2sym(const gen & e,c
onst index_m & i,const vecteur & l) Error: Bad Argument Value

```

maple [B] time = 2.66, size = 1418, normalized size = 5.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] -1/32*(2*sin(f*x+e)*(-64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2))^(
1/2))*a^(5/2)*d^3+64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2))^(1/2)
)*a^(5/2)*c*d^2+3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(
1/2)*2^(1/2)/a^(1/2))*a^2*c^2-14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+43*A*(a*(c+d)*d)^(1/2)*2^(1/
2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2+5*B*(a*(c+d)
*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
^2-34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)

```

```

)/a^(1/2))*a^2*c*d-3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)+(64*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3-64*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+14*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-43*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-5*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2+34*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+3*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2)*cos(f*x+e)^2-128*A*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*d^3+128*B*arctanh((a-a*sin(f*x+e))^(1/2)*d/(a*c*d+a*d^2)^(1/2))*a^(5/2)*c*d^2+6*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-28*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d+86*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-6*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+28*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d-22*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+20*A*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*c^2-72*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d+52*A*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^2+10*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c^2-68*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*c*d-6*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2*d^2-10*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c^2+4*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*c*d+6*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(3/2)*a^(1/2)*d^2+12*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2+8*B*(a-a*sin(f*x+e))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d-20*B*(a*(c+d)*d)^(1/2)*(a-a*sin(f*x+e))^(1/2)*a^(3/2)*d^2*(-a*(sin(f*x+e)-1))^(1/2)/a^(9/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c-d)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e)),x, algorithm="maxima")
```

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))), x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e)),x)
```

```
[Out] Timed out
```

$$3.326 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=395

$$\frac{(A(3c^2 - 22cd + 115d^2) + B(5c^2 - 58cd - 43d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{16\sqrt{2} a^{5/2} f(c-d)^4} + \frac{d^{3/2} (Ad(7c + 5d) - B(5c^2 + 5cd + 2d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f(c-d)^4(c+d)^{3/2}}$$

[Out] $d^{3/2}*(A*d*(7*c+5*d)-B*(5*c^2+5*c*d+2*d^2))*\operatorname{arctanh}(\cos(f*x+e)*a^{1/2}*d^{1/2}/(c+d)^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{5/2}/(c-d)^4/(c+d)^{3/2}/f-1/4*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^{5/2}/(c+d*\sin(f*x+e))-1/16*(3*A*c-15*A*d+5*B*c+7*B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^{3/2}/(c+d*\sin(f*x+e))-1/32*(B*(5*c^2-58*c*d-43*d^2)+A*(3*c^2-22*c*d+115*d^2))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{1/2}*2^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{5/2}/(c-d)^4/f*2^{1/2}-1/16*d*(A*(3*c^2-16*c*d-35*d^2)+B*(5*c^2+32*c*d+11*d^2))*\cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*\sin(f*x+e))/(a+a*\sin(f*x+e))^{1/2}$

Rubi [A] time = 1.54, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{d(A(3c^2 - 16cd - 35d^2) + B(5c^2 + 32cd + 11d^2)) \cos(e+fx)}{16a^2 f(c-d)^3(c+d) \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))} + \frac{d^{3/2} (Ad(7c + 5d) - B(5c^2 + 5cd + 2d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right)}{a^{5/2} f(c-d)^4(c+d)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\operatorname{Sin}[e + f*x])/((a + a*\operatorname{Sin}[e + f*x])^{5/2}*(c + d*\operatorname{Sin}[e + f*x])^2), x]$

[Out] $-((B*(5*c^2 - 58*c*d - 43*d^2) + A*(3*c^2 - 22*c*d + 115*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(16*\operatorname{Sqrt}[2]*a^{5/2}*(c - d)^4*f) + (d^{3/2}*(A*d*(7*c + 5*d) - B*(5*c^2 + 5*c*d + 2*d^2))*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[d]*\operatorname{Cos}[e + f*x])/(\operatorname{Sqrt}[c + d]*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]])])/(a^{5/2}*(c - d)^4*(c + d)^{3/2}*f) - ((A - B)*\operatorname{Cos}[e + f*x])/(4*(c - d)*f*(a + a*\operatorname{Sin}[e + f*x])^{5/2}*(c + d*\operatorname{Sin}[e + f*x])) - ((3*A*c + 5*B*c - 15*A*d + 7*B*d)*\operatorname{Cos}[e + f*x])/(16*a*(c - d)^2*f*(a + a*\operatorname{Sin}[e + f*x])^{3/2}*(c + d*\operatorname{Sin}[e + f*x])) - (d*(A*(3*c^2 - 16*c*d - 35*d^2) + B*(5*c^2 + 32*c*d + 11*d^2))*\operatorname{Cos}[e + f*x])/(16*a^2*(c - d)^3*(c + d)*f*\operatorname{Sqrt}[a + a*\operatorname{Sin}[e + f*x]]*(c + d*\operatorname{Sin}[e + f*x]))$

Rule 206

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ \|\ \operatorname{LtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b]$

Rule 2649

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_) + (b_)*\sin[(c_) + (d_)*(x_)]], x_Symbol] \rightarrow \operatorname{Dist}[-2/d, \operatorname{Subst}[\operatorname{Int}[1/(2*a - x^2), x], x, (b*\operatorname{Cos}[c + d*x])/ \operatorname{Sqrt}[a + b*\sin[c + d*x]]], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{EqQ}[a^2 - b^2, 0]$

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$2] + 21*A*c^2*d*\sin[(3*(e + f*x))/2] - 29*B*c^2*d*\sin[(3*(e + f*x))/2] + 54 *A*c*d^2*\sin[(3*(e + f*x))/2] - 86*B*c*d^2*\sin[(3*(e + f*x))/2] + 75*A*d^3* \sin[(3*(e + f*x))/2] - 19*B*d^3*\sin[(3*(e + f*x))/2] - 3*A*c^2*d*\sin[(5*(e + f*x))/2] - 5*B*c^2*d*\sin[(5*(e + f*x))/2] + 16*A*c*d^2*\sin[(5*(e + f*x))/2] - 32*B*c*d^2*\sin[(5*(e + f*x))/2] + 35*A*d^3*\sin[(5*(e + f*x))/2] - 11*B *d^3*\sin[(5*(e + f*x))/2]))/(64*(c - d)^3*(c + d)*f*(a*(1 + \sin[e + f*x]))^(5/2)*(c + d*\sin[e + f*x]))$$

fricas [B] time = 14.83, size = 5151, normalized size = 13.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg orithm="fricas")

[Out]
$$\begin{aligned} & [-1/64*(\sqrt{2}*(4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 + ((3*A + 5*B)*c^3*d \\ & - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*\cos(f*x + e)^4 - ((3*A + 5*B)*c^4 - (13*A + 43*B)*c^3*d + (55*A - 207*B)*c^2*d^2 \\ & + 7*(43*A - 35*B)*c*d^3 + 2*(115*A - 43*B)*d^4)*\cos(f*x + e)^3 - (3*(3*A + 5*B)*c^4 - 2*(21*A + 67*B)*c^3*d + 8*(23*A - 71*B)*c^2*d^2 + 2*(405*A - 317 \\ & *B)*c*d^3 + 5*(115*A - 43*B)*d^4)*\cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16*(13*A - 9*B)*c*d^3 + (115*A \\ & - 43*B)*d^4)*\cos(f*x + e) + (4*(3*A + 5*B)*c^4 - 64*(A + 3*B)*c^3*d + 8*(37*A - 77*B)*c^2*d^2 + 64*(13*A - 9*B)*c*d^3 + 4*(115*A - 43*B)*d^4 - ((3*A + \\ & 5*B)*c^3*d - (19*A + 53*B)*c^2*d^2 + (93*A - 101*B)*c*d^3 + (115*A - 43*B)*d^4)*\cos(f*x + e)^3 - ((3*A + 5*B)*c^4 - 2*(5*A + 19*B)*c^3*d + 4*(9*A - 6 \\ & 5*B)*c^2*d^2 + 2*(197*A - 173*B)*c*d^3 + 3*(115*A - 43*B)*d^4)*\cos(f*x + e)^2 + 2*((3*A + 5*B)*c^4 - 16*(A + 3*B)*c^3*d + 2*(37*A - 77*B)*c^2*d^2 + 16 \\ & *(13*A - 9*B)*c*d^3 + (115*A - 43*B)*d^4)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a}*\log(-(\cos(f*x + e))^2 + 2*\sqrt{2}*\sqrt{a*\sin(f*x + e) + a}*\sqrt{a}*(\cos \\ & (f*x + e) - \sin(f*x + e) + 1) + 3*a*\cos(f*x + e) - (a*\cos(f*x + e) - 2*a)*\sin(f*x + e) + 2*a)/(\cos(f*x + e))^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \cos(f \\ & *x + e) - 2)) - 16*(20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 + (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (\\ & 5*A - 2*B)*a*d^4)*\cos(f*x + e)^4 - (5*B*a*c^3*d - (7*A - 15*B)*a*c^2*d^2 - (19*A - 12*B)*a*c*d^3 - 2*(5*A - 2*B)*a*d^4)*\cos(f*x + e)^3 - (15*B*a*c^3*d \\ & - (21*A - 40*B)*a*c^2*d^2 - (50*A - 31*B)*a*c*d^3 - 5*(5*A - 2*B)*a*d^4)*\cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*\cos(f*x + e) + (20*B*a*c^3*d - 4*(7*A - 10*B)*a*c^2*d^2 - 4*(12*A - 7*B)*a*c*d^3 - 4*(5*A - 2*B)*a*d^4 - (5*B*a*c^2*d^2 - (7*A - 5*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*\cos(f*x + e)^3 - (5*B*a*c^3*d - (7*A - 20*B)*a*c^2*d^2 - (26*A - 17*B)*a*c*d^3 - 3*(5*A - 2*B)*a*d^4)*\cos(f*x + e)^2 + 2*(5*B*a*c^3*d - (7*A - 10*B)*a*c^2*d^2 - (12*A - 7*B)*a*c*d^3 - (5*A - 2*B)*a*d^4)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{d/(a*c + a*d)}*\log((d^2*\cos(f*x + e))^3 - (6*c*d + 7*d^2)*\cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - 4*((c*d + d^2)*\cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*\cos(f*x + e) + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*\cos(f*x + e))*\sin(f*x + e))*\sqrt{a*\sin(f*x + e) + a}*\sqrt{d/(a*c + a*d)} - (c^2 + 8*c*d + 9*d^2)*\cos(f*x + e) + (d^2*\cos(f*x + e))^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*\cos(f*x + e))*\sin(f*x + e))/(d^2*\cos(f*x + e)^3 + (2*c*d + d^2)*\cos(f*x + e)^2 - c^2 - 2*c*d - d^2 - (c^2 + d^2)*\cos(f*x + e) + (d^2*\cos(f*x + e))^2 - 2*c*d*\cos(f*x + e) - c^2 - 2*c*d - d^2)*\sin(f*x + e))) + 4*(4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c^2*d^2 - (19*A - 27*B)*c^2*d^2 - (19*A + 21*B)*c*d^3 + (35*A - 11*B)*d^4)*\cos(f*x + e)^3 + ((3*A + 5*B)*c^4 - (15*A - 7*B)*c^3*d - (7*A - 15*B)*c^2*d^2 - (A + 23*B)*c*d^3 + 4*(5*A - B)*d^4)*\cos(f*x + e)^2 + ((7*A + B)*c^4 - 20*(A - B)*c^3*d - 2*(13*A - 21*B)*c^2*d^2 - 4*(3*A + 13*B)*c*d^3 + (51*A - 11*B)*d^4)*\cos(f*x + e) - (4*(A - B)*c^4 - 8*(A - B)*c^3*d + 8*(A - B)*c^2*d^2 - 4*(A - B)*d^4 -$$

$$4 - (a^3c^6 - a^3c^5d - 4a^3c^4d^2 + 6a^3c^3d^3 + a^3c^2d^4 - 5a^3c^d^5 + 2a^3d^6)*f*\cos(f*x + e)^3 - (3a^3c^6 - 4a^3c^5d - 9a^3c^4d^2 + 16a^3c^3d^3 + a^3c^2d^4 - 12a^3c^d^5 + 5a^3d^6)*f*\cos(f*x + e)^2 + 2*(a^3c^6 - 2a^3c^5d - a^3c^4d^2 + 4a^3c^3d^3 - a^3c^2d^4 - 2a^3c^d^5 + a^3d^6)*f*\cos(f*x + e) + 4*(a^3c^6 - 2a^3c^5d - a^3c^4d^2 + 4a^3c^3d^3 - a^3c^2d^4 - 2a^3c^d^5 + a^3d^6)*f - ((a^3c^5d - 3a^3c^4d^2 + 2a^3c^3d^3 + 2a^3c^2d^4 - 3a^3c^d^5 + a^3d^6)*f*\cos(f*x + e)^3 + (a^3c^6 - 7a^3c^4d^2 + 8a^3c^3d^3 + 3a^3c^2d^4 - 8a^3c^d^5 + 3a^3d^6)*f*\cos(f*x + e)^2 - 2*(a^3c^6 - 2a^3c^5d - a^3c^4d^2 + 4a^3c^3d^3 - a^3c^2d^4 - 2a^3c^d^5 + a^3d^6)*f*\cos(f*x + e) - 4*(a^3c^6 - 2a^3c^5d - a^3c^4d^2 + 4a^3c^3d^3 - a^3c^2d^4 - 2a^3c^d^5 + a^3d^6)*f)*\sin(f*x + e)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
 Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
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 arning, integration of abs or sign assumes constant sign by intervals (corr
 ect if the argument is real):Check [abs(cos((f*t_nostep+exp(1))/2-pi/4))]Un
 able to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to check sig
 n: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Discontinuities at zeroes of cos((f*
 t_nostep+exp(1))/2-pi/4) were not checkedUnable to check sign: (4*pi/t_nost
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nostep/2)>(-4*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Una
ble to check sign: (2*pi/x/2)>(-2*pi/x/2)Warning, integration of abs or sig
n assumes constant sign by intervals (correct if the argument is real):Chec
k [abs(t_nostep+1)]Evaluation time: 1.57index.cc index_m i_lex_is_greater E
rror: Bad Argument Value

```

maple [B] time = 4.20, size = 4092, normalized size = 10.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x)
```

```

[Out] 1/32/a^(9/2)*(-323*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-
1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d^3+13*A*(a*(c+d)*d)^(1/2)*2^(1
/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2
*c^3*d-320*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)
*sin(f*x+e)*c^3*d^2+19*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x
+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^3*a^2*c^2*d^2-55*A*(a*(c+d)*d)^(1
/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+
e)^2*a^2*c^2*d^2-301*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e
)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)^2*a^2*c*d^3+101*B*(a*(c+d)*d)^(1/2)
*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*
a^2*c^3*d+255*B*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(
1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c^2*d^2+187*B*(a*(c+d)*d)^(1/2)*2^(1/2
)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^2*c*d
^3+35*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(

```


$$\begin{aligned}
& 1/2)/a^{(1/2)})*\sin(f*x+e)*a^2*c^3*d-167*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(\\
& 1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)*a^2*c^2*d^2+207*B \\
& *(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)}) \\
& *\sin(f*x+e)^2*a^2*c^2*d^2+245*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2 \\
& *(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a^2*c*d^3+53*B*(a \\
& (c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)} \\
&))*\sin(f*x+e)^3*a^2*c^2*d^2-5*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\\
& \sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^2*c^3*d+101*B*(a*(c+d) \\
& *d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin \\
& (f*x+e)^3*a^2*c*d^3-93*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f* \\
& x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^2*c*d^3+43*B*(a*(c+d)*d)^{(1/ \\
& 2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e \\
&)^2*a^2*c^3*d-3*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1)) \\
& ^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^2*c^3*d-480*B*\operatorname{arctanh}((-a*(\sin(f*x+e) \\
&)-1))^{(1/2)}*d/(a*(c+d)*d)^{(1/2)}*a^{(5/2)}*\sin(f*x+e)*c^2*d^3-160*B*\operatorname{arctanh}((\\
& -a*(\sin(f*x+e)-1))^{(1/2)}*d/(a*(c+d)*d)^{(1/2)}*a^{(5/2)}*\sin(f*x+e)^3*c*d^4+22 \\
& *B*(-a*(\sin(f*x+e)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*c^3*d-288*B*\operatorname{arctanh} \\
& (-a*(\sin(f*x+e)-1))^{(1/2)}*d/(a*(c+d)*d)^{(1/2)}*a^{(5/2)}*\sin(f*x+e)*c*d^4-148 \\
& *A*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*\sin(f*x+e)*d^4+52*B* \\
& (-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*\sin(f*x+e)*d^4+84*A*(-a \\
& *(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*c^3*d+20*A*(-a*(\sin(f*x+e) \\
& -1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*c^2*d^2-52*A*(-a*(\sin(f*x+e)-1))^{(1/2)} \\
& *(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*c*d^3-52*B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d) \\
& ^{(1/2)}*a^{(3/2)}*c^3*d-32*B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/ \\
& 2)}*\sin(f*x+e)^2*c^2*d^2+32*B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(\\
& 3/2)}*\sin(f*x+e)^2*c*d^3-20*A*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a \\
& ^{(3/2)}*\sin(f*x+e)*c^3*d+84*A*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(\\
& 3/2)}*\sin(f*x+e)*c^2*d^2-12*B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a \\
& ^{(3/2)}*\sin(f*x+e)*c^3*d-116*B*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a \\
& ^{(3/2)}*\sin(f*x+e)*c^2*d^2+84*A*(-a*(\sin(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}* \\
& a^{(3/2)}*\sin(f*x+e)*c*d^3-115*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(s \\
& in(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)*a^2*d^4+10*B*(-a*(\sin(f*x+e) \\
&)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c^3*d+22*B*(-a*(\sin(f*x+e) \\
& -1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c^2*d^2-10*B*(-a*(\sin(f*x+e) \\
&)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c*d^3-10*B*(a*(c+d)*d)^{(1/ \\
& 2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e \\
&)*a^2*c^4-5*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/ \\
& 2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a^2*c^4+86*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arc} \\
& \operatorname{tanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a^2*d^4-11 \\
& 5*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2) \\
& }/a^{(1/2)})*\sin(f*x+e)^3*a^2*d^4+43*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(\\
& -a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^3*a^2*d^4+76*B*(-a*(si \\
& n(f*x+e)-1))^{(1/2)}*(a*(c+d)*d)^{(1/2)}*a^{(3/2)}*\sin(f*x+e)*c*d^3+43*B*(a*(c+d) \\
& *d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin \\
& (f*x+e)*a^2*d^4+19*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e) \\
& -1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c^3*d-93*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctan} \\
& h(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c^2*d^2-3*A*(a*(c+d)*d \\
&)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(\\
& f*x+e)^2*a^2*c^4-230*A*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e) \\
&)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f*x+e)^2*a^2*d^4-115*A*(a*(c+d)*d)^{(1/2)}*2 \\
& ^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c*d^3+53* \\
& B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a \\
& ^{(1/2)})*a^2*c^3*d+101*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+ \\
& e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c^2*d^2+43*B*(a*(c+d)*d)^{(1/2)}*2^{(1/2)}*\operatorname{ar} \\
& \operatorname{ctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*a^2*c*d^3+6*A*(-a*(\sin \\
& (f*x+e)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c^3*d-38*A*(-a*(\sin(\\
& f*x+e)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c^2*d^2-6*A*(-a*(\sin(\\
& f*x+e)-1))^{(3/2)}*(a*(c+d)*d)^{(1/2)}*a^{(1/2)}*\sin(f*x+e)*c*d^3-6*A*(a*(c+d)*d) \\
& ^{(1/2)}*2^{(1/2)}*\operatorname{arctanh}(1/2*(-a*(\sin(f*x+e)-1))^{(1/2)}*2^{(1/2)}/a^{(1/2)})*\sin(f
\end{aligned}$$

```

*x+e)*a^2*c^4+32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*sin(
f*x+e)^2*c*d^3+448*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)*c^2*d^3+544*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c
+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*c*d^4+224*A*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3+608*A*arctanh((-a*(si
n(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4-160*B*ar
ctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c
^3*d^2-10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^2*d^2-22*
B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c*d^3-5*B*(a*(c+d)*d)
^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*a^2*c
^4-20*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^2*d^2+84*B*(-
a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c*d^3+38*A*(-a*(sin(f*x+e
)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4-22*B*(-a*(sin(f*x+e)-1
))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*sin(f*x+e)*d^4-38*A*(-a*(sin(f*x+e)-1))^(
3/2)*(a*(c+d)*d)^(1/2)*a^(1/2)*c^3*d-6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d
)*d)^(1/2)*a^(1/2)*c^2*d^2+38*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)
*a^(1/2)*c*d^3-3*A*(a*(c+d)*d)^(1/2)*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1)
)^(1/2)*2^(1/2)/a^(1/2))*a^2*c^4-32*A*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)
^(1/2)*a^(3/2)*sin(f*x+e)^2*d^4-480*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(
a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c^2*d^3-384*B*arctanh((-a*(sin(f*x+e
)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*c*d^4+224*A*arctanh((
-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^3*c*d^4-16
0*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+
e)^3*c^2*d^3-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a
^(5/2)*c^3*d^2+160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))
*a^(5/2)*sin(f*x+e)^3*d^5+160*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*c*d^4+320*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d
)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5+224*A*arctanh((-a*(sin(f*x+e)-1))^(1/2)
*d/(a*(c+d)*d)^(1/2))*a^(5/2)*c^2*d^3-128*B*arctanh((-a*(sin(f*x+e)-1))^(1
/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)^2*d^5+160*A*arctanh((-a*(sin(f*
x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5-64*B*arctanh((-a
*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(1/2))*a^(5/2)*sin(f*x+e)*d^5-20*A*(-a
*(sin(f*x+e)-1))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*c^4-32*A*(-a*(sin(f*x+e)-1
))^(1/2)*(a*(c+d)*d)^(1/2)*a^(3/2)*d^4-12*B*(-a*(sin(f*x+e)-1))^(1/2)*(a*(c
+d)*d)^(1/2)*a^(3/2)*c^4-64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*
d)^(1/2))*a^(5/2)*sin(f*x+e)^3*d^5+6*A*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d
)^(1/2)*a^(1/2)*c^4-160*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*c^2*d^3-64*B*arctanh((-a*(sin(f*x+e)-1))^(1/2)*d/(a*(c+d)*d)^(
1/2))*a^(5/2)*c*d^4+10*B*(-a*(sin(f*x+e)-1))^(3/2)*(a*(c+d)*d)^(1/2)*a^(1/
2)*c^4*(-a*(sin(f*x+e)-1))^(1/2)/(1+sin(f*x+e))/(a*(c+d)*d)^(1/2)/(c+d*sin
(f*x+e))/(c+d)/(c-d)^4/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f

```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^2,x, alg
orithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + f x)}{(a + a \sin(e + f x))^{5/2} (c + d \sin(e + f x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2),x)
```

```
[Out] int((A + B*sin(e + f*x))/((a + a*sin(e + f*x))^(5/2)*(c + d*sin(e + f*x))^2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**2,x)
```

```
[Out] Timed out
```

$$3.327 \quad \int \frac{A+B \sin(e+fx)}{(a+a \sin(e+fx))^{5/2}(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=519

$$\frac{(3A(c^2 - 10cd + 73d^2) + B(5c^2 - 82cd - 115d^2)) \tanh^{-1}\left(\frac{\sqrt{a} \cos(e+fx)}{\sqrt{2} \sqrt{a \sin(e+fx)+a}}\right) d^{3/2} (3Ad(21c^2 + 30cd + 13d^2) - 16\sqrt{2} a^{5/2} f(c-d)^5)}{16\sqrt{2} a^{5/2} f(c-d)^5} + \dots$$

[Out] $\frac{1}{4}d^{3/2}*(3A*d*(21*c^2+30*c*d+13*d^2)-B*(35*c^3+70*c^2*d+67*c*d^2+20*d^3))*\operatorname{arctanh}(\cos(f*x+e)*a^{1/2}*d^{1/2}/(c+d)^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{5/2}/(c-d)^5/(c+d)^{5/2}/f-1/4*(A-B)*\cos(f*x+e)/(c-d)/f/(a+a*\sin(f*x+e))^{5/2}/(c+d*\sin(f*x+e))^2-1/16*(3A*c-19A*d+5B*c+11B*d)*\cos(f*x+e)/a/(c-d)^2/f/(a+a*\sin(f*x+e))^{3/2}/(c+d*\sin(f*x+e))^2-1/32*(B*(5*c^2-82*c*d-115*d^2)+3A*(c^2-10*c*d+73*d^2))*\operatorname{arctanh}(1/2*\cos(f*x+e)*a^{1/2}*2^{1/2}/(a+a*\sin(f*x+e))^{1/2})/a^{5/2}/(c-d)^5/f*2^{1/2}-1/16*d*(A*(3*c^2-20*c*d-31*d^2)+B*(5*c^2+28*c*d+15*d^2))*\cos(f*x+e)/a^2/(c-d)^3/(c+d)/f/(c+d*\sin(f*x+e))^2/(a+a*\sin(f*x+e))^{1/2}-1/16*d*(3A*(c^3-7*c^2*d-37*c*d^2-21*d^3)+B*(5*c^3+73*c^2*d+79*c*d^2+35*d^3))*\cos(f*x+e)/a^2/(c-d)^4/(c+d)^2/f/(c+d*\sin(f*x+e))/(a+a*\sin(f*x+e))^{1/2}$

Rubi [A] time = 2.15, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2978, 2984, 2985, 2649, 206, 2773, 208}

$$\frac{d(3A(-7c^2d + c^3 - 37cd^2 - 21d^3) + B(73c^2d + 5c^3 + 79cd^2 + 35d^3)) \cos(e+fx)}{16a^2 f(c-d)^4(c+d)^2 \sqrt{a \sin(e+fx)+a} (c+d \sin(e+fx))} - \frac{d(A(3c^2 - 20cd - 31d^2) + B(5c^3 + 73c^2d + 79cd^2 + 35d^3)) \cos(e+fx)}{16a^2 f(c-d)^3(c+d) \sqrt{a \sin(e+fx)+a}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(A + B*\sin[e + f*x])/((a + a*\sin[e + f*x])^{5/2}*(c + d*\sin[e + f*x])^3), x]$

[Out] $-\left(\left(B*(5*c^2 - 82*c*d - 115*d^2) + 3A*(c^2 - 10*c*d + 73*d^2)\right)*\operatorname{ArcTanh}\left[\frac{\sqrt{a}*\cos[e + f*x]}{\sqrt{2}*\sqrt{a + a*\sin[e + f*x]}}\right]\right)/(16*\sqrt{2}*a^{5/2}*(c - d)^5*f) + (d^{3/2}*(3A*d*(21*c^2 + 30*c*d + 13*d^2) - B*(35*c^3 + 70*c^2*d + 67*c*d^2 + 20*d^3))*\operatorname{ArcTanh}\left[\frac{\sqrt{a}*\sqrt{d}*\cos[e + f*x]}{\sqrt{c + d}*\sqrt{a + a*\sin[e + f*x]}}\right])/(4*a^{5/2}*(c - d)^5*(c + d)^{5/2}*f) - \left(\frac{(A - B)*\cos[e + f*x]}{4*(c - d)*f*(a + a*\sin[e + f*x])^{5/2}*(c + d*\sin[e + f*x])^2} - \left(\frac{(3A*c + 5B*c - 19A*d + 11B*d)*\cos[e + f*x]}{16*a*(c - d)^2*f*(a + a*\sin[e + f*x])^{3/2}*(c + d*\sin[e + f*x])^2} - \frac{d*(A*(3*c^2 - 20*c*d - 31*d^2) + B*(5*c^2 + 28*c*d + 15*d^2))*\cos[e + f*x]}{16*a^2*(c - d)^3*(c + d)*f*\sqrt{a + a*\sin[e + f*x]}*(c + d*\sin[e + f*x])^2} - \frac{d*(3A*(c^3 - 7*c^2*d - 37*c*d^2 - 21*d^3) + B*(5*c^3 + 73*c^2*d + 79*c*d^2 + 35*d^3))*\cos[e + f*x]}{16*a^2*(c - d)^4*(c + d)^2*f*\sqrt{a + a*\sin[e + f*x]}*(c + d*\sin[e + f*x])}\right)$

Rule 206

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_0 + (b_0*x)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}[\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 2649

```
Int[1/Sqrt[(a_) + (b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[-2/d, Subst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(-2*b)/f, Subst[Int[1/(b*c + a*d - d*x^2), x], x, (b*Cos[e + f*x])/Sqrt[a + b*Sin[e + f*x]]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2978

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Simp[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)]^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 2985

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*(c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} & \cos\left(\frac{3(e+fx)}{2}\right) - 299Bcd^4\cos\left(\frac{3(e+fx)}{2}\right) + 79A^5d^5\cos\left(\frac{3(e+fx)}{2}\right) - 59Bd^5\cos\left(\frac{3(e+fx)}{2}\right) + 12A^4cd\cos\left(\frac{5(e+fx)}{2}\right) \\ & + 20Bc^4d\cos\left(\frac{5(e+fx)}{2}\right) - 73A^3d^2\cos\left(\frac{5(e+fx)}{2}\right) + 217Bc^3d^2\cos\left(\frac{5(e+fx)}{2}\right) - 353A^2d^3\cos\left(\frac{5(e+fx)}{2}\right) + 397Bc^2d^3\cos\left(\frac{5(e+fx)}{2}\right) \\ & - 419Acd^4\cos\left(\frac{5(e+fx)}{2}\right) + 251Bcd^4\cos\left(\frac{5(e+fx)}{2}\right) - 127A^5d^5\cos\left(\frac{5(e+fx)}{2}\right) + 75Bd^5\cos\left(\frac{5(e+fx)}{2}\right) \\ & + 3A^3d^2\cos\left(\frac{7(e+fx)}{2}\right) + 5Bc^3d^2\cos\left(\frac{7(e+fx)}{2}\right) - 21A^2d^3\cos\left(\frac{7(e+fx)}{2}\right) + 73Bc^2d^3\cos\left(\frac{7(e+fx)}{2}\right) \\ & - 111Acd^4\cos\left(\frac{7(e+fx)}{2}\right) + 79Bcd^4\cos\left(\frac{7(e+fx)}{2}\right) - 63A^5d^5\cos\left(\frac{7(e+fx)}{2}\right) + 35Bd^5\cos\left(\frac{7(e+fx)}{2}\right) + 44A^5\sin\left(\frac{e+fx}{2}\right) \\ & - 12Bc^5\sin\left(\frac{e+fx}{2}\right) - 84A^4d\sin\left(\frac{e+fx}{2}\right) + 116Bc^4d\sin\left(\frac{e+fx}{2}\right) - 249A^3d^2\sin\left(\frac{e+fx}{2}\right) + 433Bc^3d^2\sin\left(\frac{e+fx}{2}\right) \\ & - 385A^2d^3\sin\left(\frac{e+fx}{2}\right) + 277Bc^2d^3\sin\left(\frac{e+fx}{2}\right) - 239Acd^4\sin\left(\frac{e+fx}{2}\right) + 95Bcd^4\sin\left(\frac{e+fx}{2}\right) - 47A^5d^5\sin\left(\frac{e+fx}{2}\right) \\ & + 51Bd^5\sin\left(\frac{e+fx}{2}\right) - 12A^5\sin\left(\frac{3(e+fx)}{2}\right) - 20Bc^5\sin\left(\frac{3(e+fx)}{2}\right) + 40A^4d\sin\left(\frac{3(e+fx)}{2}\right) - 104Bc^4d\sin\left(\frac{3(e+fx)}{2}\right) \\ & + 261A^3d^2\sin\left(\frac{3(e+fx)}{2}\right) - 581Bc^3d^2\sin\left(\frac{3(e+fx)}{2}\right) + 781A^2d^3\sin\left(\frac{3(e+fx)}{2}\right) - 665Bc^2d^3\sin\left(\frac{3(e+fx)}{2}\right) \\ & + 579Acd^4\sin\left(\frac{3(e+fx)}{2}\right) - 299Bcd^4\sin\left(\frac{3(e+fx)}{2}\right) + 79A^5d^5\sin\left(\frac{3(e+fx)}{2}\right) - 59Bd^5\sin\left(\frac{3(e+fx)}{2}\right) \\ & - 12A^4d\sin\left(\frac{5(e+fx)}{2}\right) - 20Bc^4d\sin\left(\frac{5(e+fx)}{2}\right) + 73A^3d^2\sin\left(\frac{5(e+fx)}{2}\right) - 217Bc^3d^2\sin\left(\frac{5(e+fx)}{2}\right) \\ & + 353A^2d^3\sin\left(\frac{5(e+fx)}{2}\right) - 397Bc^2d^3\sin\left(\frac{5(e+fx)}{2}\right) + 419Acd^4\sin\left(\frac{5(e+fx)}{2}\right) - 251Bcd^4\sin\left(\frac{5(e+fx)}{2}\right) \\ & + 127A^5d^5\sin\left(\frac{5(e+fx)}{2}\right) - 75Bd^5\sin\left(\frac{5(e+fx)}{2}\right) + 3A^3d^2\sin\left(\frac{7(e+fx)}{2}\right) + 5Bc^3d^2\sin\left(\frac{7(e+fx)}{2}\right) \\ & - 21A^2d^3\sin\left(\frac{7(e+fx)}{2}\right) + 73Bc^2d^3\sin\left(\frac{7(e+fx)}{2}\right) - 111Acd^4\sin\left(\frac{7(e+fx)}{2}\right) + 79Bcd^4\sin\left(\frac{7(e+fx)}{2}\right) \\ & - 63A^5d^5\sin\left(\frac{7(e+fx)}{2}\right) + 35Bd^5\sin\left(\frac{7(e+fx)}{2}\right) \Big) / (128(c-d)^4(c+d)^2 f(a(1+\sin[e+fx]))^{5/2} (c+d\sin[e+fx])^2 \end{aligned}$$

fricas [B] time = 45.09, size = 8555, normalized size = 16.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] $\frac{1}{64}(\sqrt{2})(4(3A+5B)c^6 - 8(9A+31B)c^5d + 4(117A - 413B)c^4d^2 + 16(177A - 233B)c^3d^3 + 4(1197A - 1013B)c^2d^4 + 8(423A - 271B)c*d^5 + 4(219A - 115B)d^6 + ((3A+5B)c^4d^2 - 24(A+3B)c^3d^3 + 2(81A - 137B)c^2d^4 + 24(17A - 13B)c*d^5 + (219A - 115B)d^6)\cos(f*x+e)^5 + (2(3A+5B)c^5d - 3(13A+43B)c^4d^2 + 4(63A - 191B)c^3d^3 + 6(217A - 241B)c^2d^4 + 2(831A - 583B)c*d^5 + 3(219A - 115B)d^6)\cos(f*x+e)^4 - ((3A+5B)c^6 - 4(3A+13B)c^5d + (75A - 547B)c^4d^2 + 8(123A - 203B)c^3d^3 + 19(123A - 115B)c^2d^4 + 4(525A - 349B)c*d^5 + 3(219A - 115B)d^6)\cos(f*x+e)^3 - (3(3A+5B)c^6 - 2(21A+83B)c^5d + (267A - 1507B)c^4d^2 + 4(669A - 1045B)c^3d^3 + (5871A - 5383B)c^2d^4 + 2(2523A - 1667B)c*d^5 + 7(219A - 115B)d^6)\cos(f*x+e)^2 + 2((3A+5B)c^6 - 2(9A+31B)c^5d + (117A - 413B)c^4d^2 + 4(177A - 233B)c^3d^3 + (1197A - 1013B)c^2d^4 + 2(423A - 271B)c*d^5 + (219A - 115B)d^6)\cos(f*x+e) + (4(3A+5B)c^6 - 8(9A+31B)c^5d + 4(117A - 413B)c^4d^2 + 16(177A - 233B)c^3d^3 + 4(1197A - 1013B)c^2d^4 + 8(423A - 271B)c*d^5 + 4(219A - 115B)d^6 + ((3A+5B)c^4d^2 - 24(A+3B)c^3d^3 + 2(81A - 137B)c^2d^4 + 24(17A - 13B)c*d^5 + (219A - 115B)d^6)\cos(f*x+e)^4 - 2((3A+5B)c^5d - (21A+67B)c^4d^2 + 2(69A - 173B)c^3d^3 + 2(285A - 293B)c^2d^4 + (627A - 427B)c*d^5 + (219A - 115B)d^6)\cos(f*x+e)^3 - ((3A+5B)c^6$

$$\begin{aligned}
& - 6*(A + 7*B)*c^5*d + 3*(11*A - 227*B)*c^4*d^2 + 12*(105*A - 193*B)*c^3*d^3 \\
& + 3*(1159*A - 1119*B)*c^2*d^4 + 6*(559*A - 375*B)*c*d^5 + 5*(219*A - 115*B) \\
& *d^6)*\cos(f*x + e)^2 + 2*((3*A + 5*B)*c^6 - 2*(9*A + 31*B)*c^5*d + (117*A \\
& - 413*B)*c^4*d^2 + 4*(177*A - 233*B)*c^3*d^3 + (1197*A - 1013*B)*c^2*d^4 + \\
& 2*(423*A - 271*B)*c*d^5 + (219*A - 115*B)*d^6)*\cos(f*x + e))*\sin(f*x + e))* \\
& \text{sqrt}(a)*\log(-(a*\cos(f*x + e))^2 - 2*\text{sqrt}(2)*\text{sqrt}(a*\sin(f*x + e) + a)*\text{sqrt}(a) \\
& *(\cos(f*x + e) - \sin(f*x + e) + 1) + 3*a*\cos(f*x + e) - (a*\cos(f*x + e) - 2 \\
& *a)*\sin(f*x + e) + 2*a)/(\cos(f*x + e)^2 - (\cos(f*x + e) + 2)*\sin(f*x + e) - \\
& \cos(f*x + e) - 2)) - 4*(140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108 \\
& *A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c \\
& *d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - \\
& (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x + e)^5 + (70*B*a*c^4* \\
& d^2 - 7*(18*A - 35*B)*a*c^3*d^3 - (369*A - 344*B)*a*c^2*d^4 - (348*A - 241* \\
& B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*\cos(f*x + e)^4 - (35*B*a*c^5*d - 21*(3* \\
& A - 10*B)*a*c^4*d^2 - 2*(171*A - 226*B)*a*c^3*d^3 - 6*(98*A - 83*B)*a*c^2*d \\
& ^4 - (426*A - 281*B)*a*c*d^5 - 3*(39*A - 20*B)*a*d^6)*\cos(f*x + e)^3 - (105 \\
& *B*a*c^5*d - 7*(27*A - 80*B)*a*c^4*d^2 - 6*(150*A - 191*B)*a*c^3*d^3 - 2*(7 \\
& 29*A - 610*B)*a*c^2*d^4 - 3*(340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6) \\
& *\cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - \\
& 121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - \\
& (39*A - 20*B)*a*d^6)*\cos(f*x + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4* \\
& d^2 - 8*(108*A - 121*B)*a*c^3*d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A \\
& - 107*B)*a*c*d^5 - 4*(39*A - 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B) \\
& *a*c^2*d^4 - (90*A - 67*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x + e)^4 - \\
& 2*(35*B*a*c^4*d^2 - 21*(3*A - 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 - \\
& 3*(43*A - 29*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x + e)^3 - (35*B*a*c^5 \\
& *d - 7*(9*A - 40*B)*a*c^4*d^2 - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 38 \\
& 6*B)*a*c^2*d^4 - (684*A - 455*B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*\cos(f*x + \\
& e)^2 + 2*(35*B*a*c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^ \\
& 3*d^3 - 2*(141*A - 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20* \\
& B)*a*d^6)*\cos(f*x + e))*\sin(f*x + e))*\text{sqrt}(d/(a*c + a*d))*\log((d^2*\cos(f*x \\
& + e)^3 - (6*c*d + 7*d^2)*\cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 4*((c*d + d^2) \\
&)*\cos(f*x + e)^2 - c^2 - 4*c*d - 3*d^2 - (c^2 + 3*c*d + 2*d^2)*\cos(f*x + e) \\
& + (c^2 + 4*c*d + 3*d^2 + (c*d + d^2)*\cos(f*x + e))*\sin(f*x + e))*\text{sqrt}(a*\text{si} \\
& \text{n}(f*x + e) + a)*\text{sqrt}(d/(a*c + a*d)) - (c^2 + 8*c*d + 9*d^2)*\cos(f*x + e) + \\
& (d^2*\cos(f*x + e)^2 - c^2 - 2*c*d - d^2 + 2*(3*c*d + 4*d^2)*\cos(f*x + e))*\text{s} \\
& \text{in}(f*x + e))/((d^2*\cos(f*x + e)^3 + (2*c*d + d^2)*\cos(f*x + e)^2 - c^2 - 2*c \\
& *d - d^2 - (c^2 + d^2)*\cos(f*x + e) + (d^2*\cos(f*x + e)^2 - 2*c*d*\cos(f*x + \\
& e) - c^2 - 2*c*d - d^2)*\sin(f*x + e))) - 4*(4*(A - B)*c^6 - 8*(A - B)*c^5* \\
& d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)* \\
& c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(\\
& 15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)*d^6)*\cos(f*x + e) \\
& ^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38*A - 31*B)*c^3*d^ \\
& 3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(19*A - 11*B)*d^6) \\
& *\cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (31*A - 75*B)*c^4*d \\
& ^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20*(5*A - 3*B)*c*d^5 \\
& + (51*A - 31*B)*d^6)*\cos(f*x + e)^2 + ((7*A + B)*c^6 - 2*(9*A - 17*B)*c^5* \\
& d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15*A + 121*B)*c^2*d^ \\
& 4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*\cos(f*x + e) - (4*(A - B)*c \\
& ^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c \\
& ^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^4*d^2 - 4*(6*A - \\
& 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B) \\
&)*d^6)*\cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 24*B)*c^4*d^2 - 4* \\
& (16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33*B)*c*d^5 + 2*(8* \\
& A - 5*B)*d^6)*\cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A - 13*B)*c^5*d - 3* \\
& (25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A + 125*B)*c^2*d^4 + \\
& 6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*\cos(f*x + e))*\sin(f*x + e))*\text{sqrt} \\
& (a*\sin(f*x + e) + a))/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c \\
& ^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d^9)*f*\cos(f*x + e
\end{aligned}$$

$$\begin{aligned}
&)^5 + (2a^3c^8d - 3a^3c^7d^2 - 7a^3c^6d^3 + 13a^3c^5d^4 + 5a^3c^4d^5 - 17a^3c^3d^6 + 3a^3c^2d^7 + 7a^3c^1d^8 - 3a^3d^9) * f * \cos(f * x + e)^4 - (a^3c^9 + a^3c^8d - 8a^3c^7d^2 + 18a^3c^6d^3 - 6a^3c^5d^4 - 16a^3c^4d^5 - 16a^3c^3d^6 + 8a^3c^2d^7 + 5a^3c^1d^8 - 3a^3d^9) * f * \cos(f * x + e)^3 - (3a^3c^9 + a^3c^8d - 20a^3c^7d^2 + 4a^3c^6d^3 + 42a^3c^5d^4 - 18a^3c^4d^5 - 36a^3c^3d^6 + 20a^3c^2d^7 + 11a^3c^1d^8 - 7a^3d^9) * f * \cos(f * x + e)^2 + 2 * (a^3c^9 - a^3c^8d - 4a^3c^7d^2 + 4a^3c^6d^3 + 6a^3c^5d^4 - 6a^3c^4d^5 - 4a^3c^3d^6 + 4a^3c^2d^7 + a^3c^1d^8 - a^3d^9) * f * \cos(f * x + e) + 4 * (a^3c^9 - a^3c^8d - 4a^3c^7d^2 + 4a^3c^6d^3 + 6a^3c^5d^4 - 6a^3c^4d^5 - 4a^3c^3d^6 + 4a^3c^2d^7 + a^3c^1d^8 - a^3d^9) * f + ((a^3c^7d^2 - 3a^3c^6d^3 + a^3c^5d^4 + 5a^3c^4d^5 - 5a^3c^3d^6 - a^3c^2d^7 + 3a^3c^1d^8 - a^3d^9) * f * \cos(f * x + e)^4 - 2 * (a^3c^8d - 2a^3c^7d^2 - 2a^3c^6d^3 + 6a^3c^5d^4 - 6a^3c^4d^5 + 2a^3c^3d^6 + 2a^3c^2d^7 + 2a^3c^1d^8 - a^3d^9) * f * \cos(f * x + e)^3 - (a^3c^9 + 3a^3c^8d - 12a^3c^7d^2 - 4a^3c^6d^3 + 30a^3c^5d^4 - 6a^3c^4d^5 - 28a^3c^3d^6 + 12a^3c^2d^7 + 9a^3c^1d^8 - 5a^3d^9) * f * \cos(f * x + e)^2 + 2 * (a^3c^9 - a^3c^8d - 4a^3c^7d^2 + 4a^3c^6d^3 + 6a^3c^5d^4 - 6a^3c^4d^5 - 4a^3c^3d^6 + 4a^3c^2d^7 + a^3c^1d^8 - a^3d^9) * f * \cos(f * x + e) + 4 * (a^3c^9 - a^3c^8d - 4a^3c^7d^2 + 4a^3c^6d^3 + 6a^3c^5d^4 - 6a^3c^4d^5 - 4a^3c^3d^6 + 4a^3c^2d^7 + a^3c^1d^8 - a^3d^9) * f) * \sin(f * x + e)), 1/64 * (\sqrt{2}) * (4 * (3A + 5B) * c^6 - 8 * (9A + 31B) * c^5 * d + 4 * (117A - 413B) * c^4 * d^2 + 16 * (177A - 233B) * c^3 * d^3 + 4 * (1197A - 1013B) * c^2 * d^4 + 8 * (423A - 271B) * c * d^5 + 4 * (219A - 115B) * d^6 + ((3A + 5B) * c^4 * d^2 - 24 * (A + 3B) * c^3 * d^3 + 2 * (81A - 137B) * c^2 * d^4 + 24 * (17A - 13B) * c * d^5 + (219A - 115B) * d^6) * \cos(f * x + e)^5 + (2 * (3A + 5B) * c^5 * d - 3 * (13A + 43B) * c^4 * d^2 + 4 * (63A - 191B) * c^3 * d^3 + 6 * (217A - 241B) * c^2 * d^4 + 2 * (831A - 583B) * c * d^5 + 3 * (219A - 115B) * d^6) * \cos(f * x + e)^4 - ((3A + 5B) * c^6 - 4 * (3A + 13B) * c^5 * d + (75A - 547B) * c^4 * d^2 + 8 * (123A - 203B) * c^3 * d^3 + 19 * (123A - 115B) * c^2 * d^4 + 4 * (525A - 349B) * c * d^5 + 3 * (219A - 115B) * d^6) * \cos(f * x + e)^3 - (3 * (3A + 5B) * c^6 - 2 * (21A + 83B) * c^5 * d + (267A - 1507B) * c^4 * d^2 + 4 * (669A - 1045B) * c^3 * d^3 + (5871A - 5383B) * c^2 * d^4 + 2 * (2523A - 1667B) * c * d^5 + 7 * (219A - 115B) * d^6) * \cos(f * x + e)^2 + 2 * ((3A + 5B) * c^6 - 2 * (9A + 31B) * c^5 * d + (117A - 413B) * c^4 * d^2 + 4 * (177A - 233B) * c^3 * d^3 + (1197A - 1013B) * c^2 * d^4 + 2 * (423A - 271B) * c * d^5 + (219A - 115B) * d^6) * \cos(f * x + e) + (4 * (3A + 5B) * c^6 - 8 * (9A + 31B) * c^5 * d + 4 * (117A - 413B) * c^4 * d^2 + 16 * (177A - 233B) * c^3 * d^3 + 4 * (1197A - 1013B) * c^2 * d^4 + 8 * (423A - 271B) * c * d^5 + 4 * (219A - 115B) * d^6 + ((3A + 5B) * c^4 * d^2 - 24 * (A + 3B) * c^3 * d^3 + 2 * (81A - 137B) * c^2 * d^4 + 24 * (17A - 13B) * c * d^5 + (219A - 115B) * d^6) * \cos(f * x + e)^4 - 2 * ((3A + 5B) * c^5 * d - (21A + 67B) * c^4 * d^2 + 2 * (69A - 173B) * c^3 * d^3 + 2 * (285A - 293B) * c^2 * d^4 + (627A - 427B) * c * d^5 + (219A - 115B) * d^6) * \cos(f * x + e)^3 - ((3A + 5B) * c^6 - 6 * (A + 7B) * c^5 * d + 3 * (11A - 227B) * c^4 * d^2 + 12 * (105A - 193B) * c^3 * d^3 + 3 * (1159A - 1119B) * c^2 * d^4 + 6 * (559A - 375B) * c * d^5 + 5 * (219A - 115B) * d^6) * \cos(f * x + e)^2 + 2 * ((3A + 5B) * c^6 - 2 * (9A + 31B) * c^5 * d + (117A - 413B) * c^4 * d^2 + 4 * (177A - 233B) * c^3 * d^3 + (1197A - 1013B) * c^2 * d^4 + 2 * (423A - 271B) * c * d^5 + (219A - 115B) * d^6) * \cos(f * x + e)) * \sin(f * x + e)) * \sqrt{a} * \log(-(a * \cos(f * x + e))^2 - 2 * \sqrt{2} * \sqrt{a * \sin(f * x + e) + a} * \sqrt{a} * (\cos(f * x + e) - \sin(f * x + e) + 1) + 3 * a * \cos(f * x + e) - (a * \cos(f * x + e) - 2 * a) * \sin(f * x + e) + 2 * a) / (\cos(f * x + e)^2 - (\cos(f * x + e) + 2) * \sin(f * x + e) - \cos(f * x + e) - 2)) - 8 * (140B * a * c^5 * d - 28 * (9A - 20B) * a * c^4 * d^2 - 8 * (108A - 121B) * a * c^3 * d^3 - 8 * (141A - 112B) * a * c^2 * d^4 - 4 * (168A - 107B) * a * c * d^5 - 4 * (39A - 20B) * a * d^6 + (35B * a * c^3 * d^3 - 7 * (9A - 10B) * a * c^2 * d^4 - (90A - 67B) * a * c * d^5 - (39A - 20B) * a * d^6) * \cos(f * x + e)^5 + (70B * a * c^4 * d^2 - 7 * (18A - 35B) * a * c^3 * d^3 - (369A - 344B) * a * c^2 * d^4 - (348A - 241B) * a * c * d^5 - 3 * (39A - 20B) * a * d^6) * \cos(f * x + e)^4 - (35B * a * c^5 * d - 21 * (3A - 10B) * a * c^4 * d^2 - 2 * (171A - 226B) * a * c^3 * d^3 - 6 * (98A - 83B) * a * c^2 * d^4 - (426A - 281B) * a * c * d^5 - 3 * (39A - 20B) * a * d^6) * \cos(f * x + e)^3 - (105B * a * c^5 * d - 7 * (27A - 80B) * a * c^4 * d^2 - 6 * (150A - 191B) * a * c^3 * d^3 - 2 * (729A - 610B) * a * c^2 * d^4 - 3 * (
\end{aligned}$$

$$\begin{aligned}
& 340*A - 223*B)*a*c*d^5 - 7*(39*A - 20*B)*a*d^6)*\cos(f*x + e)^2 + 2*(35*B*a* \\
& c^5*d - 7*(9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A - \\
& 112*B)*a*c^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x \\
& + e) + (140*B*a*c^5*d - 28*(9*A - 20*B)*a*c^4*d^2 - 8*(108*A - 121*B)*a*c^3 \\
& *d^3 - 8*(141*A - 112*B)*a*c^2*d^4 - 4*(168*A - 107*B)*a*c*d^5 - 4*(39*A - \\
& 20*B)*a*d^6 + (35*B*a*c^3*d^3 - 7*(9*A - 10*B)*a*c^2*d^4 - (90*A - 67*B)*a* \\
& c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x + e)^4 - 2*(35*B*a*c^4*d^2 - 21*(3*A - \\
& 5*B)*a*c^3*d^3 - (153*A - 137*B)*a*c^2*d^4 - 3*(43*A - 29*B)*a*c*d^5 - (39 \\
& *A - 20*B)*a*d^6)*\cos(f*x + e)^3 - (35*B*a*c^5*d - 7*(9*A - 40*B)*a*c^4*d^2 \\
& - 2*(234*A - 331*B)*a*c^3*d^3 - 2*(447*A - 386*B)*a*c^2*d^4 - (684*A - 455 \\
& *B)*a*c*d^5 - 5*(39*A - 20*B)*a*d^6)*\cos(f*x + e)^2 + 2*(35*B*a*c^5*d - 7*(\\
& 9*A - 20*B)*a*c^4*d^2 - 2*(108*A - 121*B)*a*c^3*d^3 - 2*(141*A - 112*B)*a*c \\
& ^2*d^4 - (168*A - 107*B)*a*c*d^5 - (39*A - 20*B)*a*d^6)*\cos(f*x + e))*\sin(f \\
& *x + e))*\sqrt{-d/(a*c + a*d))*\arctan(1/2*\sqrt{a*\sin(f*x + e) + a}*(d*\sin(f* \\
& x + e) - c - 2*d)*\sqrt{-d/(a*c + a*d)})/(d*\cos(f*x + e))) - 4*(4*(A - B)*c^6 \\
& - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3*d^3 - 4*(A - B)*c^2 \\
& *d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 - ((3*A + 5*B)*c^4*d^2 - 4*(6*A - 17 \\
& *B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d^5 + 7*(9*A - 5*B)* \\
& d^6)*\cos(f*x + e)^4 - (2*(3*A + 5*B)*c^5*d - (41*A - 101*B)*c^4*d^2 - 4*(38 \\
& *A - 31*B)*c^3*d^3 - 2*(39*A + 35*B)*c^2*d^4 + 10*(17*A - 11*B)*c*d^5 + 5*(\\
& 19*A - 11*B)*d^6)*\cos(f*x + e)^3 + ((3*A + 5*B)*c^6 - 16*(A - B)*c^5*d - (3 \\
& 1*A - 75*B)*c^4*d^2 - 4*(21*A - 11*B)*c^3*d^3 - (23*A + 49*B)*c^2*d^4 + 20* \\
& (5*A - 3*B)*c*d^5 + (51*A - 31*B)*d^6)*\cos(f*x + e)^2 + ((7*A + B)*c^6 - 2* \\
& (9*A - 17*B)*c^5*d - (79*A - 175*B)*c^4*d^2 - 28*(7*A - 3*B)*c^3*d^3 - (15* \\
& A + 121*B)*c^2*d^4 + 2*(107*A - 59*B)*c*d^5 + (87*A - 55*B)*d^6)*\cos(f*x + \\
& e) - (4*(A - B)*c^6 - 8*(A - B)*c^5*d - 4*(A - B)*c^4*d^2 + 16*(A - B)*c^3* \\
& d^3 - 4*(A - B)*c^2*d^4 - 8*(A - B)*c*d^5 + 4*(A - B)*d^6 + ((3*A + 5*B)*c^ \\
& 4*d^2 - 4*(6*A - 17*B)*c^3*d^3 - 6*(15*A - B)*c^2*d^4 + 4*(12*A - 11*B)*c*d \\
& ^5 + 7*(9*A - 5*B)*d^6)*\cos(f*x + e)^3 - 2*((3*A + 5*B)*c^5*d - 2*(11*A - 2 \\
& 4*B)*c^4*d^2 - 4*(16*A - 7*B)*c^3*d^3 + 2*(3*A - 19*B)*c^2*d^4 + (61*A - 33 \\
& *B)*c*d^5 + 2*(8*A - 5*B)*d^6)*\cos(f*x + e)^2 - ((3*A + 5*B)*c^6 - 2*(5*A - \\
& 13*B)*c^5*d - 3*(25*A - 57*B)*c^4*d^2 - 4*(53*A - 25*B)*c^3*d^3 - (11*A + \\
& 125*B)*c^2*d^4 + 6*(37*A - 21*B)*c*d^5 + (83*A - 51*B)*d^6)*\cos(f*x + e))*s \\
& \sin(f*x + e))*\sqrt{a*\sin(f*x + e) + a)}/((a^3*c^7*d^2 - 3*a^3*c^6*d^3 + a^3* \\
& c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a^3*c*d^8 - a^3*d \\
& ^9)*f*\cos(f*x + e)^5 + (2*a^3*c^8*d - 3*a^3*c^7*d^2 - 7*a^3*c^6*d^3 + 13*a^ \\
& 3*c^5*d^4 + 5*a^3*c^4*d^5 - 17*a^3*c^3*d^6 + 3*a^3*c^2*d^7 + 7*a^3*c*d^8 - \\
& 3*a^3*d^9)*f*\cos(f*x + e)^4 - (a^3*c^9 + a^3*c^8*d - 8*a^3*c^7*d^2 + 18*a^3 \\
& *c^5*d^4 - 6*a^3*c^4*d^5 - 16*a^3*c^3*d^6 + 8*a^3*c^2*d^7 + 5*a^3*c*d^8 - 3 \\
& *a^3*d^9)*f*\cos(f*x + e)^3 - (3*a^3*c^9 + a^3*c^8*d - 20*a^3*c^7*d^2 + 4*a^ \\
& 3*c^6*d^3 + 42*a^3*c^5*d^4 - 18*a^3*c^4*d^5 - 36*a^3*c^3*d^6 + 20*a^3*c^2*d \\
& ^7 + 11*a^3*c*d^8 - 7*a^3*d^9)*f*\cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - \\
& 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d \\
& ^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*\cos(f*x + e) + 4*(a^3*c^9 - a^3 \\
& *c^8*d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4* \\
& a^3*c^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f + ((a^3*c^7*d^2 - 3*a^ \\
& 3*c^6*d^3 + a^3*c^5*d^4 + 5*a^3*c^4*d^5 - 5*a^3*c^3*d^6 - a^3*c^2*d^7 + 3*a \\
& ^3*c*d^8 - a^3*d^9)*f*\cos(f*x + e)^4 - 2*(a^3*c^8*d - 2*a^3*c^7*d^2 - 2*a^3 \\
& *c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^3*d^6 + 2*a^3*c^2*d^7 + 2*a^3*c*d^8 - a^ \\
& 3*d^9)*f*\cos(f*x + e)^3 - (a^3*c^9 + 3*a^3*c^8*d - 12*a^3*c^7*d^2 - 4*a^3*c \\
& ^6*d^3 + 30*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 28*a^3*c^3*d^6 + 12*a^3*c^2*d^7 + \\
& 9*a^3*c*d^8 - 5*a^3*d^9)*f*\cos(f*x + e)^2 + 2*(a^3*c^9 - a^3*c^8*d - 4*a^3 \\
& *c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c^3*d^6 + \\
& 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f*\cos(f*x + e) + 4*(a^3*c^9 - a^3*c^8* \\
& d - 4*a^3*c^7*d^2 + 4*a^3*c^6*d^3 + 6*a^3*c^5*d^4 - 6*a^3*c^4*d^5 - 4*a^3*c \\
& ^3*d^6 + 4*a^3*c^2*d^7 + a^3*c*d^8 - a^3*d^9)*f)*\sin(f*x + e))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))^(5/2)/(c+d*sin(f*x+e))^3,x, alg
orithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)
Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2
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ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(cos((f*t_nostep+exp(1))/2-pi/4))]Unable to check sign: (4*pi/t_nostep/2
)>(-4*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep
/2)Discontinuities at zeroes of cos((f*t_nostep+exp(1))/2-pi/4) were not ch
eckedUnable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to ch
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```


Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+a*sin(f*x+e))**(5/2)/(c+d*sin(f*x+e))**3,x)
```

```
[Out] Timed out
```

$$3.328 \quad \int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=221

$$\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

[Out] $-8*a^2*B*AppellF1(1/2,-n,-5/2,3/2,d*(1-\sin(f*x+e))/(c+d),1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n*2^{(1/2)}/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(1+\sin(f*x+e))^{(1/2)}-4*a^2*(A-B)*AppellF1(1/2,-n,-3/2,3/2,d*(1-\sin(f*x+e))/(c+d),1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n*2^{(1/2)}/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(1+\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2987, 2784, 139, 138}

$$\frac{4\sqrt{2}a^2(A-B)\cos(e+fx)(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n}F_1\left(\frac{1}{2};-\frac{3}{2},-n;\frac{3}{2};\frac{1}{2}(1-\sin(e+fx)),\frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] $(-8*\text{Sqrt}[2]*a^2*B*AppellF1[1/2,-5/2,-n,3/2,(1-\text{Sin}[e+f*x])/2,(d*(1-\text{Sin}[e+f*x]))/(c+d)]*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^n)/(f*\text{Sqrt}[1+\text{Sin}[e+f*x]]*((c+d*\text{Sin}[e+f*x])/(c+d))^n)-(4*\text{Sqrt}[2]*a^2*(A-B)*AppellF1[1/2,-3/2,-n,3/2,(1-\text{Sin}[e+f*x])/2,(d*(1-\text{Sin}[e+f*x]))/(c+d)]*\text{Cos}[e+f*x]*(c+d*\text{Sin}[e+f*x])^n)/(f*\text{Sqrt}[1+\text{Sin}[e+f*x]]*((c+d*\text{Sin}[e+f*x])/(c+d))^n)$

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2784

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*x)^n]/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,

f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m]

Rule 2987

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]

Rubi steps

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx = (A - B) \int (a + a \sin(e + fx))^2 (c + d \sin(e + fx))^n dx - \frac{(a^2(A - B) \cos(e + fx)) \text{Subst}\left(\int \frac{(1+x)^{3/2}(c+dx)^n}{\sqrt{1-x}} dx\right)}{f \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} = \frac{(a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^n)}{f \sqrt{1 - \sin(e + fx)}} = -\frac{8\sqrt{2} a^2 BF_1\left(\frac{1}{2}; -\frac{5}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f \sqrt{1 - \sin(e + fx)}}$$

Mathematica [F] time = 26.08, size = 0, normalized size = 0.00

$$\int (a + a \sin(e + fx))^2 (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

[Out] Integrate[(a + a*Sin[e + f*x])^2*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left((A + 2B)a^2 \cos(fx + e)^2 - 2(A + B)a^2 + \left(Ba^2 \cos(fx + e)^2 - 2(A + B)a^2\right) \sin(fx + e)\right)(d \sin(fx + e) + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral(-((A + 2*B)*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2 + (B*a^2*cos(f*x + e)^2 - 2*(A + B)*a^2)*sin(f*x + e))*(d*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 3.87, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^2 (A + B \sin(fx + e)) (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^2 (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^2*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^2 (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^2*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^2*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] Timed out

$$3.329 \quad \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=217

$$\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; -\frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

[Out] $-4*a*B*AppellF1(1/2, -n, -3/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n*2^{(1/2)}/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(1+\sin(f*x+e))^{(1/2)}-2*a*(A-B)*AppellF1(1/2, -n, -1/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n*2^{(1/2)}/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(1+\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2968, 3017, 2755, 139, 138, 2784}

$$\frac{2\sqrt{2}a(A-B)\cos(e+fx)(c+d\sin(e+fx))^n \left(\frac{c+d\sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; -\frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{f\sqrt{\sin(e+fx)+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

[Out] $(-4*\text{Sqrt}[2]*a*B*AppellF1[1/2, -3/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*\text{Sqrt}[1 + \text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n - (2*\text{Sqrt}[2]*a*(A - B)*AppellF1[1/2, -1/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(f*\text{Sqrt}[1 + \text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n)$

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2755

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] := Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 - (d*x)/c], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Ne

$Q[b*c - a*d, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{!IntegerQ}[2*m] \ \&\& \ \text{EqQ}[c^2 - d^2, 0]$
 $]$

Rule 2784

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}*((c_ + (d_)*\sin[(e_ + (f_)*(x_))])^{(n_)}), x_Symbol] \ :> \ \text{Dist}[(a^m*\text{Cos}[e + f*x])/(f*\text{Sqrt}[1 + \text{Sin}[e + f*x]]*\text{Sqrt}[1 - \text{Sin}[e + f*x]]), \text{Subst}[\text{Int}[(1 + (b*x)/a)^{(m - 1/2)}*(c + d*x)^n/\text{Sqrt}[1 - (b*x)/a], x], x, \text{Sin}[e + f*x]], x] \ /; \ \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{NeQ}[c^2 - d^2, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 2968

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}*((A_ + (B_)*\sin[(e_ + (f_)*(x_))])^{(n_)}*((c_ + (d_)*\sin[(e_ + (f_)*(x_))])^{(p_)}), x_Symbol] \ :> \ \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(A*c + (B*c + A*d)*\text{Sin}[e + f*x] + B*d*\text{Sin}[e + f*x]^2), x] \ /; \ \text{FreeQ}\{a, b, c, d, e, f, A, B, m\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 3017

$\text{Int}[(a_ + (b_)*\sin[(e_ + (f_)*(x_))])^{(m_)}*((A_ + (B_)*\sin[(e_ + (f_)*(x_))])^{(n_)} + (C_)*\sin[(e_ + (f_)*(x_))]^2), x_Symbol] \ :> \ \text{Dist}[A - C, \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(1 + \text{Sin}[e + f*x]), x], x] + \text{Dist}[C, \text{Int}[(a + b*\text{Sin}[e + f*x])^m*(1 + \text{Sin}[e + f*x])^2, x], x] \ /; \ \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x] \ \&\& \ \text{EqQ}[A - B + C, 0] \ \&\& \ \text{!IntegerQ}[2*m]$

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^n dx &= \int (c + d \sin(e + fx))^n (aA + (aA + aB) \sin(e + fx) \\ &+ (aA - B) \sin^2(e + fx)) dx \\ &= (a(A - B)) \int (1 + \sin(e + fx))(c + d \sin(e + fx))^n dx \\ &+ \frac{(a(A - B) \cos(e + fx)) \text{Subst}\left(\int \frac{\sqrt{1+x}(c+dx)^n dx}{\sqrt{1-x}}\right)}{f \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} \\ &= \frac{(a(A - B) \cos(e + fx)(c + d \sin(e + fx))^n \left(-\frac{c+dx}{\sqrt{1-x}}\right)}{f \sqrt{1 - \sin(e + fx)}} \\ &= -\frac{4\sqrt{2} aBF_1\left(\frac{1}{2}; -\frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right), \frac{d}{f}}{f \sqrt{1 - \sin(e + fx)}} \end{aligned}$$

Mathematica [F] time = 9.07, size = 0, normalized size = 0.00

$$\int (a + a \sin(e + fx))(A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

[Out] Integrate[(a + a*Sin[e + f*x])*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Ba \cos(fx + e)^2 - (A + B)a \sin(fx + e) - (A + B)a\right)(d \sin(fx + e) + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral(-(B*a*cos(f*x + e)^2 - (A + B)*a*sin(f*x + e) - (A + B)*a)*(d*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)(d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 3.37, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))(A + B \sin(fx + e))(c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)(d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx))(a + a \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] Timed out

$$3.330 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{a+a \sin(e+fx)} dx$$

Optimal. Leaf size=221

$$\frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; \frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2} a f \sqrt{\sin(e+fx)+1}} \sqrt{2} B c$$

[Out] $-1/2*(A-B)*\text{AppellF1}(1/2, -n, 3/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n/a/f/(((c+d*\sin(f*x+e))/(c+d))^n)*2^{(1/2)/(1+\sin(f*x+e))^{(1/2)}-B*\text{AppellF1}(1/2, -n, 1/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n*2^{(1/2)}/a/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(1+\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2987, 2784, 139, 138, 2665}

$$\frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; \frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{\sqrt{2} a f \sqrt{\sin(e+fx)+1}} \sqrt{2} B c$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^n/(a + a*\text{Sin}[e + f*x]), x]$

[Out] $-((\text{Sqrt}[2]*B*\text{AppellF1}[1/2, 1/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(a*f*\text{Sqrt}[1 + \text{Sin}[e + f*x]])*((c + d*\text{Sin}[e + f*x])/(c + d))^n) - ((A - B)*\text{AppellF1}[1/2, 3/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(\text{Sqrt}[2]*a*f*\text{Sqrt}[1 + \text{Sin}[e + f*x]])*((c + d*\text{Sin}[e + f*x])/(c + d))^n$

Rule 138

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x_Symbol] :> \text{Simp}[(a + b*x)^{(m + 1)}*\text{AppellF1}[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& \text{GtQ}[b/(b*e - a*f), 0] \&\& !(\text{GtQ}[d/(d*a - c*b), 0] \&\& \text{GtQ}[d/(d*e - c*f), 0]) \&\& \text{SimplerQ}[c + d*x, a + b*x] \&\& !(\text{GtQ}[f/(f*a - e*b), 0] \&\& \text{GtQ}[f/(f*c - e*d), 0]) \&\& \text{SimplerQ}[e + f*x, a + b*x]$

Rule 139

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x_Symbol] :> \text{Dist}[(e + f*x)^{\text{FracPart}[p]}/((b/(b*e - a*f))^{\text{IntPart}[p]}*((b*(e + f*x))/(b*e - a*f))^{\text{FracPart}[p]}), \text{Int}[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& !\text{GtQ}[b/(b*e - a*f), 0]$

Rule 2665

$\text{Int}[(a_ + (b_)*\sin[(c_ + (d_)*(x_))]^{(n_)}), x_Symbol] :> \text{Dist}[\text{Cos}[c + d*x]/(d*\text{Sqrt}[1 + \text{Sin}[c + d*x]]*\text{Sqrt}[1 - \text{Sin}[c + d*x]]), \text{Subst}[\text{Int}[(a + b*x)^n/(\text{Sqrt}[1 + x]*\text{Sqrt}[1 - x]), x], x, \text{Sin}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n]$

Rule 2784

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e
+ f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*
x)^n]/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e,
f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{a + a \sin(e + fx)} dx &= (A - B) \int \frac{(c + d \sin(e + fx))^n}{a + a \sin(e + fx)} dx + \frac{B \int (c + d \sin(e + fx))^n dx}{a} \\ &= \frac{((A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{1-x}(1+x)^{3/2}} dx, x, \sin(e + fx)\right)}{af \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} \\ &= \frac{\left((A - B) \cos(e + fx)(c + d \sin(e + fx))^n \left(-\frac{c+d \sin(e+fx)}{-c-d}\right)^{-n}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} dx, x, \sin(e + fx)\right)}{af \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} \\ &= -\frac{\sqrt{2} BF_1\left(\frac{1}{2}; \frac{1}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx)), \frac{d(1 - \sin(e+fx))}{c+d}\right) \cos(e + fx)}{af \sqrt{1 + \sin(e + fx)}} \end{aligned}$$

Mathematica [F] time = 4.65, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{a + a \sin(e + fx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]), x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]), x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{a \sin(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm
="fricas")
```

[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(fx + e))(c + d \sin(fx + e))^n}{a + a \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)

[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{a \sin(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{a + a \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)),x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e)),x)

[Out] Timed out

$$3.331 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{(a+a \sin(e+fx))^2} dx$$

Optimal. Leaf size=223

$$\frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; \frac{5}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2} a^2 f \sqrt{\sin(e+fx)+1}} B c$$

[Out] $-1/2*B*AppellF1(1/2, -n, 3/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n/a^2/f/(((c+d*\sin(f*x+e))/(c+d))^n)*2^{(1/2)/(1+\sin(f*x+e))^{(1/2)}-1/4*(A-B)*AppellF1(1/2, -n, 5/2, 3/2, d*(1-\sin(f*x+e))/(c+d), 1/2-1/2*\sin(f*x+e))*\cos(f*x+e)*(c+d*\sin(f*x+e))^n/a^2/f/(((c+d*\sin(f*x+e))/(c+d))^n)*2^{(1/2)/(1+\sin(f*x+e))^{(1/2)}}$

Rubi [A] time = 0.34, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2987, 2784, 139, 138}

$$\frac{(A-B) \cos(e+fx)(c+d \sin(e+fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} F_1\left(\frac{1}{2}; \frac{5}{2}, -n; \frac{3}{2}; \frac{1}{2}(1-\sin(e+fx)), \frac{d(1-\sin(e+fx))}{c+d}\right)}{2\sqrt{2} a^2 f \sqrt{\sin(e+fx)+1}} B c$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^2,x]

[Out] $-((B*AppellF1[1/2, 3/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(\text{Sqrt}[2]*a^2*f*\text{Sqrt}[1 + \text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n) - ((A - B)*AppellF1[1/2, 5/2, -n, 3/2, (1 - \text{Sin}[e + f*x])/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^n)/(2*\text{Sqrt}[2]*a^2*f*\text{Sqrt}[1 + \text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n)$

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 2784

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*x)^n]/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&

IntegerQ[m]

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x]
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^2} dx = (A - B) \int \frac{(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^2} dx + \frac{B \int \frac{(c + d \sin(e + fx))^n}{a + a \sin(e + fx)} dx}{a}$$

$$= \frac{((A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(c + dx)^n}{\sqrt{1-x}(1+x)^{5/2}} dx, x, \sin(e + fx)\right)}{a^2 f \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} +$$

$$= \frac{\left((A - B) \cos(e + fx)(c + d \sin(e + fx))^n \left(-\frac{c + d \sin(e + fx)}{-c - d}\right)^{-n}\right) \operatorname{Subst}\left(\int \frac{(c + dx)^n}{\sqrt{1-x}(1+x)^{5/2}} dx, x, \sin(e + fx)\right)}{a^2 f \sqrt{1 - \sin(e + fx)} \sqrt{1 + \sin(e + fx)}} +$$

$$= -\frac{BF_1\left(\frac{1}{2}; \frac{3}{2}, -n; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx)), \frac{d(1 - \sin(e + fx))}{c + d}\right) \cos(e + fx)(c + d \sin(e + fx))^n}{\sqrt{2} a^2 f \sqrt{1 + \sin(e + fx)}}$$

Mathematica [F] time = 28.14, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

```
[Out] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x]
)^2, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] integral(-(B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 -
2*a^2*sin(f*x + e) - 2*a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)

maple [F] time = 10.29, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(fx + e))(c + d \sin(fx + e))^n}{(a + a \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)

[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{(a \sin(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^2,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2,x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n/(a+a*sin(f*x+e))**2,x)

[Out] Timed out

3.332 $\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$

Optimal. Leaf size=427

$$\frac{2a^2(A - B)(c - d(4n + 5)) \cos(e + fx)(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}} - \frac{2a^2(A - B)(c - d(4n + 5)) \cos(e + fx)(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}}$$

```
[Out] -2*a^2*(A-B)*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d/f/(3+2*n)/(a+a*sin(f*x+e))
^(1/2)+2*a^2*B*(3*c-d*(11+4*n))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)/d^2/f/(4*
n^2+16*n+15)/(a+a*sin(f*x+e))^(1/2)+2*a^2*(A-B)*(c-d*(5+4*n))*cos(f*x+e)*hy
pergeom([1/2, -n], [3/2], d*(1-sin(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d/f/(3+2
*n)/(((c+d*sin(f*x+e))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a^2*B*(3*c^2-2*c*
d*(7+4*n)+d^2*(16*n^2+56*n+43))*cos(f*x+e)*hypergeom([1/2, -n], [3/2], d*(1-s
in(f*x+e))/(c+d))*(c+d*sin(f*x+e))^n/d^2/f/(4*n^2+16*n+15)/(((c+d*sin(f*x+e
))/(c+d))^n)/(a+a*sin(f*x+e))^(1/2)-2*a*B*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)
*(a+a*sin(f*x+e))^(1/2)/d/f/(5+2*n)
```

Rubi [A] time = 0.91, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2987, 2763, 21, 2776, 70, 69, 2981}

$$\frac{2a^2(A - B)(c - d(4n + 5)) \cos(e + fx)(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}} - \frac{2a^2(A - B)(c - d(4n + 5)) \cos(e + fx)(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,
x]
```

```
[Out] (-2*a^2*(A - B)*Cos[e + f*x]*(c + d*Sin[e + f*x])^(1 + n))/(d*f*(3 + 2*n)*S
qrt[a + a*Sin[e + f*x]]) + (2*a^2*B*(3*c - d*(11 + 4*n))*Cos[e + f*x]*(c +
d*Sin[e + f*x])^(1 + n))/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]
]) - (2*a*B*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]]*(c + d*Sin[e + f*x])^(1 +
n))/(d*f*(5 + 2*n)) + (2*a^2*(A - B)*(c - d*(5 + 4*n))*Cos[e + f*x]*Hyperg
eometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c + d*Sin[e + f*
x])^n)/(d*f*(3 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((c + d*Sin[e + f*x])/(c + d
))^n) - (2*a^2*B*(3*c^2 - 2*c*d*(7 + 4*n) + d^2*(43 + 56*n + 16*n^2))*Cos[e
+ f*x]*Hypergeometric2F1[1/2, -n, 3/2, (d*(1 - Sin[e + f*x]))/(c + d)]*(c
+ d*Sin[e + f*x])^n)/(d^2*f*(3 + 2*n)*(5 + 2*n)*Sqrt[a + a*Sin[e + f*x]]*((
c + d*Sin[e + f*x])/(c + d))^n)
```

Rule 21

```
Int[(u_.)*((a_.) + (b_.)*(v_.))^(m_.)*((c_.) + (d_.)*(v_.))^(n_.), x_Symbol] :=
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 69

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Simp[(((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c -
a*d)]))/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c
+ d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))
^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d)
, x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !In
tegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])
```

Rule 2763

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] := -Simp[(b^2*Cos[e + f*x]*(a + b*Ssin[e + f*x]
)^(m - 2)*(c + d*Ssin[e + f*x])^(n + 1))/(d*f*(m + n)), x] + Dist[1/(d*(m +
n)), Int[(a + b*Ssin[e + f*x])^(m - 2)*(c + d*Ssin[e + f*x])^n*Simp[a*b*c*(m
- 2) + b^2*d*(n + 1) + a^2*d*(m + n) - b*(b*c*(m - 1) - a*d*(3*m + 2*n - 2)
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c -
a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && !LtQ[n,
-1] && (IntegersQ[2*m, 2*n] || IntegerQ[m + 1/2] || (IntegerQ[m] && EqQ[c,
0]))
```

Rule 2776

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Ssin[e
+ f*x]]*Sqrt[a - b*Ssin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 2981

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp
[(-2*b*B*Cos[e + f*x]*(c + d*Ssin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Ssin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Ssin[e + f*x]]*(c + d*Ssin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rule 2987

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Ssin[e + f*x])^m*(c + d*Ssin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Ssin[e + f*x])^(m + 1)*(c + d*Ssin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + a \sin(e + fx))^{3/2} (A + B \sin(e + fx)) (c + d \sin(e + fx))^n dx &= (A - B) \int (a + a \sin(e + fx))^{3/2} (c + d \sin(e + fx))^n dx \\
&= -\frac{2a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^1}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} \\
&= -\frac{2a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^1}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} \\
&= -\frac{2a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^1}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} \\
&= -\frac{2a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^1}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} \\
&= -\frac{2a^2(A - B) \cos(e + fx)(c + d \sin(e + fx))^1}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 26.65, size = 245, normalized size = 0.57

$$a^2 \cos(e + fx)(c + d \sin(e + fx))^n \left(\frac{c + d \sin(e + fx)}{c + d} \right)^{-n} \left(-30(A + B)(c - d(4n + 5)) {}_2F_1 \left(\frac{1}{2}, -n; \frac{3}{2}; -\frac{d(\sin(e + fx) - 1)}{c + d} \right) + 30 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sin[e + f*x])^(3/2)*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] -1/15*(a^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^n*(-30*(A + B)*(c - d*(5 + 4*n))*Hypergeometric2F1[1/2, -n, 3/2, -((d*(-1 + Sin[e + f*x]))/(c + d))] + 6*B*d*(3 + 2*n)*Hypergeometric2F1[5/2, -n, 7/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^4 + 20*B*d*(3 + 2*n)*Hypergeometric2F1[3/2, -n, 5/2, -((d*(-1 + Sin[e + f*x]))/(c + d))]*(-1 + Sin[e + f*x]) + 30*(A + B)*(c + d)*((c + d*Sin[e + f*x])/(c + d))^(1 + n)))/(d*f*(3 + 2*n)*Sqrt[a*(1 + Sin[e + f*x])]*((c + d*Sin[e + f*x])/(c + d))^n)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(-\left(Ba \cos(fx + e) \right)^2 - (A + B)a \sin(fx + e) - (A + B)a \sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c)^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral(-(B*a*cos(f*x + e))^2 - (A + B)*a*sin(f*x + e) - (A + B)*a*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}} (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 1.28, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^{\frac{3}{2}} (A + B \sin(fx + e)) (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^{\frac{3}{2}}(d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^{\frac{3}{2}} (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n, x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(3/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)

[Out] Timed out

$$3.333 \quad \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=167

$$\frac{2a \cos(e + fx)(Ad(2n + 3) - B(c - 2d(n + 1)))(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}}$$

[Out] $-2*a*B*\cos(f*x+e)*(c+d*\sin(f*x+e))^{(1+n)}/d/f/(3+2*n)/(a+a*\sin(f*x+e))^{(1/2)}$
 $-2*a*(A*d*(3+2*n)-B*(c-2*d*(1+n)))*\cos(f*x+e)*\text{hypergeom}([1/2, -n], [3/2], d*(1-\sin(f*x+e))/(c+d))*(c+d*\sin(f*x+e))^n/d/f/(3+2*n)/(((c+d*\sin(f*x+e))/(c+d))^n)/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.26, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2981, 2776, 70, 69}

$$\frac{2a \cos(e + fx)(-Ad(2n + 3) + Bc - 2Bd(n + 1))(c + d \sin(e + fx))^n \left(\frac{c+d \sin(e+fx)}{c+d}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{d(1-\sin(e+fx))}{c+d}\right)}{df(2n + 3)\sqrt{a \sin(e + fx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]`

[Out] $(-2*a*B*\text{Cos}[e + f*x]*(c + d*\text{Sin}[e + f*x])^{(1 + n)})/(d*f*(3 + 2*n)*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) + (2*a*(B*c - 2*B*d*(1 + n) - A*d*(3 + 2*n))*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, -n, 3/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*(c + d*\text{Sin}[e + f*x])^n)/(d*f*(3 + 2*n)*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n)$

Rule 69

`Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))]/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))`

Rule 70

`Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*Simp[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && (RationalQ[m] || !SimplerQ[n + 1, m + 1])`

Rule 2776

`Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]`

Rule 2981

`Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp`

```
[(-2*b*B*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]]), x] + Dist[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3)), Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
/; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
b^2, 0] && NeQ[c^2 - d^2, 0] && !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^{1+n}}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} + \\ &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^{1+n}}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} + \\ &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^{1+n}}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} + \\ &= -\frac{2aB \cos(e + fx)(c + d \sin(e + fx))^{1+n}}{df(3 + 2n)\sqrt{a + a \sin(e + fx)}} + \end{aligned}$$

Mathematica [F] time = 8.25, size = 0, normalized size = 0.00

$$\int \sqrt{a + a \sin(e + fx)} (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n,x]
```

```
[Out] Integrate[Sqrt[a + a*Sin[e + f*x]]*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]
)^n, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\sqrt{a \sin(fx + e) + a}\left(d \sin(fx + e) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)\sqrt{a \sin(fx + e) + a}(d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, alg
orithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)
^n, x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + a \sin(fx + e)} (A + B \sin(fx + e)) (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n, x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^(1/2)*(c + d*sin(e + f*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\sin(e + fx) + 1)} (A + B \sin(e + fx)) (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**(1/2)*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**n,x)

[Out] Integral(sqrt(a*(sin(e + f*x) + 1))*(A + B*sin(e + f*x))*(c + d*sin(e + f*x))**n, x)

$$3.334 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{\sqrt{a+a \sin(e+fx)}} dx$$

Optimal. Leaf size=220

$$\frac{(A-B) \cos(e+fx) \sqrt{\frac{d(1-\sin(e+fx))}{c+d}} (c+d \sin(e+fx))^{n+1} F_1\left(n+1; \frac{1}{2}, 1; n+2; \frac{c+d \sin(e+fx)}{c+d}, \frac{c+d \sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx)) \sqrt{a \sin(e+fx)+a}} \quad 2B c$$

[Out] $-2*B*\cos(f*x+e)*\text{hypergeom}([1/2, -n], [3/2], d*(1-\sin(f*x+e))/(c+d))*(c+d*\sin(f*x+e))^n/f/(((c+d*\sin(f*x+e))/(c+d))^n)/(a+a*\sin(f*x+e))^{(1/2)}-(A-B)*\text{Appell}1F1(1+n, 1, 1/2, 2+n, (c+d*\sin(f*x+e))/(c-d), (c+d*\sin(f*x+e))/(c+d))*\cos(f*x+e)*(c+d*\sin(f*x+e))^{(1+n)}*(d*(1-\sin(f*x+e))/(c+d))^{(1/2)}/(c-d)/f/(1+n)/(1-\sin(f*x+e))/(a+a*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.40, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.189$, Rules used = {2987, 2788, 137, 136, 2776, 70, 69}

$$\frac{(A-B) \cos(e+fx) \sqrt{\frac{d(1-\sin(e+fx))}{c+d}} (c+d \sin(e+fx))^{n+1} F_1\left(n+1; \frac{1}{2}, 1; n+2; \frac{c+d \sin(e+fx)}{c+d}, \frac{c+d \sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)(1-\sin(e+fx)) \sqrt{a \sin(e+fx)+a}} \quad 2B c$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^n/\text{Sqrt}[a + a*\text{Sin}[e + f*x]], x]$

[Out] $-(((A - B)*\text{Appell}1F1[1 + n, 1/2, 1, 2 + n, (c + d*\text{Sin}[e + f*x])/(c + d), (c + d*\text{Sin}[e + f*x])/(c - d)]*\text{Cos}[e + f*x]*\text{Sqrt}[(d*(1 - \text{Sin}[e + f*x]))/(c + d)]*(c + d*\text{Sin}[e + f*x])^{(1 + n)})/((c - d)*f*(1 + n)*(1 - \text{Sin}[e + f*x])*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]) - (2*B*\text{Cos}[e + f*x]*\text{Hypergeometric}2F1[1/2, -n, 3/2, (d*(1 - \text{Sin}[e + f*x]))/(c + d)]*(c + d*\text{Sin}[e + f*x])^n)/(f*\text{Sqrt}[a + a*\text{Sin}[e + f*x]]*((c + d*\text{Sin}[e + f*x])/(c + d))^n)$

Rule 69

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*\text{Hypergeometric}2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))]/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& (\text{RationalQ}[m] || !(\text{RationalQ}[n] \&\& \text{GtQ}[-(d/(b*c - a*d)), 0])]$

Rule 70

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Dist}[(c + d*x)^{\text{FracPart}[n]}/((b/(b*c - a*d))^{\text{IntPart}[n]}*((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}], \text{Int}[(a + b*x)^m*\text{Simp}[(b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d), x]^n, x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& (\text{RationalQ}[m] || !\text{SimplerQ}[n + 1, m + 1])]$

Rule 136

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x_Symbol] \rightarrow \text{Simp}[(b*e - a*f)^p*(a + b*x)^{(m + 1)}*\text{Appell}1F1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x))/(b*e - a*f)]]/(b^{(p + 1)}*(m + 1)*(b/(b*c - a*d))^n), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& !(\text{GtQ}[d/(d*a - c*b), 0] \&\& \text{SimplerQ}[c + d*x, a + b*x])]$

Rule 137

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c
- a*d), 0] && !SimplerQ[c + d*x, a + b*x]
```

Rule 2776

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]]*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*cos[e + f*x])/(f*Sqrt[a + b*sin[e
+ f*x]])*Sqrt[a - b*sin[e + f*x]]], Subst[Int[(c + d*x)^n/Sqrt[a - b*x], x],
x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[2*n]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*cos[e + f*x])/(f*Sqrt[a + b*sin[e
+ f*x]])*Sqrt[a - b*sin[e + f*x]]], Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)
)^n/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{\sqrt{a + a \sin(e + fx)}} dx = (A - B) \int \frac{(c + d \sin(e + fx))^n}{\sqrt{a + a \sin(e + fx)}} dx + \frac{B \int \sqrt{a + a \sin(e + fx)} (c + d \sin(e + fx))^n dx}{a}$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \text{Subst}\left(\int \frac{(c+dx)^n}{\sqrt{a-ax}(a+ax)} dx, x, \sin(e + fx)\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{d(a - a \sin(e + fx))}{ac + ad}}\right) \text{Subst}\left(\int \frac{(c+dx)^n}{(a+ax) \sqrt{\frac{ad}{ac+ad} - \frac{a}{ac}}} dx, x, \sin(e + fx)\right)}{f(a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{(A - B) F_1\left(1 + n; \frac{1}{2}, 1; 2 + n; \frac{c+d \sin(e+fx)}{c+d}, \frac{c+d \sin(e+fx)}{c-d}\right) \cos(e + fx)}{(c - d) f (1 + n) (1 - \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

Mathematica [A] time = 5.57, size = 244, normalized size = 1.11

$$\cos(e + fx) \sqrt{a(\sin(e + fx) + 1)} (c + d \sin(e + fx))^n \frac{4(A-B) \sqrt{\frac{\sin(e+fx)-1}{\sin(e+fx)+1}} \left(\frac{c-d}{d \sin(e+fx)+d} + 1\right)^{-n} F_1\left(-n-\frac{1}{2}; -\frac{1}{2}, -n; \frac{1}{2}, -n; \frac{2}{\sin(e+fx)+1}, \frac{c+d \sin(e+fx)}{c-d}\right)}{2n+1}$$

4af(sin(e + fx))

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/Sqrt[a + a*Sin[e + f*x]],x]

[Out] (Cos[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(c + d*Sin[e + f*x])^n*(-(((A + B)*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]))/(-c + d)]*Sqrt[2 - 2*Sin[e + f*x]])/((c + d*Sin[e + f*x])/(c - d))^n + (4*(A - B)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2/(1 + Sin[e + f*x]), (-c + d)/(d + d*Sin[e + f*x])]*Sqrt[(-1 + Sin[e + f*x])/(1 + Sin[e + f*x])])/((1 + 2*n)*(1 + (c - d)/(d + d*Sin[e + f*x]))^n)))/(4*a*f*(-1 + Sin[e + f*x]))

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{\sqrt{a \sin(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(fx + e))(c + d \sin(fx + e))^n}{\sqrt{a + a \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)

[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{\sqrt{a \sin(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/sqrt(a*sin(f*x + e) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^n}{\sqrt{a + a \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2), x)

[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx)) (c + d \sin(e + fx))^n}{\sqrt{a (\sin(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(1/2), x)

[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n/sqrt(a*(sin(e + f*x) + 1)), x)

$$3.335 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^n}{(a+a \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=269

$$\frac{d(A-B) \cos(e+fx) \sqrt{\frac{d(1-\sin(e+fx))}{c+d}} (c+d \sin(e+fx))^{n+1} F_1\left(n+1; \frac{1}{2}, 2; n+2; \frac{c+d \sin(e+fx)}{c+d}, \frac{c+d \sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a \sin(e+fx)) \sqrt{a \sin(e+fx)+a}} B \cos$$

[Out] -B*AppellF1(1+n,1,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/a/(c-d)/f/(1+n)/(1-sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)+(A-B)*d*AppellF1(1+n,2,1/2,2+n,(c+d*sin(f*x+e))/(c-d),(c+d*sin(f*x+e))/(c+d))*cos(f*x+e)*(c+d*sin(f*x+e))^(1+n)*(d*(1-sin(f*x+e))/(c+d))^(1/2)/(c-d)^2/f/(1+n)/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(1/2)

Rubi [A] time = 0.47, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {2987, 2788, 137, 136}

$$\frac{d(A-B) \cos(e+fx) \sqrt{\frac{d(1-\sin(e+fx))}{c+d}} (c+d \sin(e+fx))^{n+1} F_1\left(n+1; \frac{1}{2}, 2; n+2; \frac{c+d \sin(e+fx)}{c+d}, \frac{c+d \sin(e+fx)}{c-d}\right)}{f(n+1)(c-d)^2(a-a \sin(e+fx)) \sqrt{a \sin(e+fx)+a}} B \cos$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2), x]

[Out] -((B*AppellF1[1 + n, 1/2, 1, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/(a*(c - d)*f*(1 + n)*(1 - Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])) + ((A - B)*d*AppellF1[1 + n, 1/2, 2, 2 + n, (c + d*Sin[e + f*x])/(c + d), (c + d*Sin[e + f*x])/(c - d)]*Cos[e + f*x]*Sqrt[(d*(1 - Sin[e + f*x])/(c + d)]*(c + d*Sin[e + f*x])^(1 + n))/((c - d)^2*f*(1 + n)*(a - a*Sin[e + f*x])*Sqrt[a + a*Sin[e + f*x]])

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 137

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/(b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n], Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 2788

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m,

n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[m]

Rule 2987

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^{3/2}} dx &= (A - B) \int \frac{(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^{3/2}} dx + \frac{B \int \frac{(c + d \sin(e + fx))^n}{\sqrt{a + a \sin(e + fx)}} dx}{a} \\ &= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(c + dx)^n}{\sqrt{a - ax}(a + ax)^2} dx, x, \sin(e + fx)\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} \\ &= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{d(a - a \sin(e + fx))}{ac + ad}}\right) \operatorname{Subst}\left(\int \frac{(c + dx)^n}{(a + ax)^2 \sqrt{\frac{ad}{ac + ad} - a}} dx, x, \frac{d(a - a \sin(e + fx))}{ac + ad}\right)}{f(a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}} \\ &= -\frac{BF_1\left(1 + n; \frac{1}{2}, 1; 2 + n; \frac{c + d \sin(e + fx)}{c + d}, \frac{c + d \sin(e + fx)}{c - d}\right) \cos(e + fx) \sqrt{\frac{d(a - a \sin(e + fx))}{ac + ad}}}{(c - d)f(1 + n)(a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}} \end{aligned}$$

Mathematica [B] time = 17.23, size = 603, normalized size = 2.24

$$\sec(e + fx)(c + d \sin(e + fx))^n \left(aA(\sin(e + fx) + 1) \left(a\sqrt{2 - 2 \sin(e + fx)}(\sin(e + fx) + 1) \left(\frac{c + d \sin(e + fx)}{c - d} \right)^{-n} F_1\left(1, \frac{1}{2}, -n, 2, (1 + \sin(e + fx))/2, (d*(1 + \sin(e + fx)))/(-c + d)\right) \sqrt{2 - 2 \sin(e + fx)} * (1 + \sin(e + fx)) / ((c + d \sin(e + fx)) / (c - d))^n - (4 * \sqrt{(-1 + \sin(e + fx)) / (1 + \sin(e + fx))} * (-2 * a * (1 + 2 * n) * \operatorname{AppellF1}[1/2 - n, -1/2, -n, 3/2 - n, 2 / (1 + \sin(e + fx)), (-c + d) / (d + d \sin(e + fx))] + a * (-1 + 2 * n) * \operatorname{AppellF1}[-1/2 - n, -1/2, -n, 1/2 - n, 2 / (1 + \sin(e + fx)), (-c + d) / (d + d \sin(e + fx))] * (1 + \sin(e + fx))) / ((-1 + 4 * n^2) * (1 + (c - d) / (d + d \sin(e + fx)))^n) + a * A * (1 + \sin(e + fx)) * ((a * \operatorname{AppellF1}[1, 1/2, -n, 2, (1 + \sin(e + fx)) / 2, (d * (1 + \sin(e + fx)) / (-c + d)) * \sqrt{2 - 2 \sin(e + fx)} * (1 + \sin(e + fx)) / ((c + d \sin(e + fx)) / (c - d))^n - (4 * \sqrt{(-1 + \sin(e + fx)) / (1 + \sin(e + fx))} * (2 * a * (1 + 2 * n) * \operatorname{AppellF1}[1/2 - n, -1/2, -n, 3/2 - n, 2 / (1 + \sin(e + fx)), (-c + d) / (d + d \sin(e + fx))] + a * (-1 + 2 * n) * \operatorname{AppellF1}[-1/2 - n, -1/2, -n, 1/2 - n, 2 / (1 + \sin(e + fx)), (-c + d) / (d + d \sin(e + fx))] * (1 + \sin(e + fx))) / ((-1 + 4 * n^2) * (1 + (c - d) / (d + d \sin(e + fx)))^n)) / (8 * a^3 * f * \sqrt{a * (1 + \sin(e + fx))}) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n)/(a + a*Sin[e + f*x])^(3/2), x]

[Out] (Sec[e + f*x]*(c + d*Sin[e + f*x])^n*(a*B*(1 + Sin[e + f*x])*((a*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x])/2, (d*(1 + Sin[e + f*x]) / (-c + d)) * Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]) / ((c + d*Sin[e + f*x]) / (c - d))^n - (4*Sqrt[(-1 + Sin[e + f*x]) / (1 + Sin[e + f*x])] * (-2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2 / (1 + Sin[e + f*x]), (-c + d) / (d + d*Sin[e + f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2 / (1 + Sin[e + f*x]), (-c + d) / (d + d*Sin[e + f*x])] * (1 + Sin[e + f*x])) / ((-1 + 4*n^2) * (1 + (c - d) / (d + d*Sin[e + f*x]))^n) + a*A*(1 + Sin[e + f*x]) * ((a*AppellF1[1, 1/2, -n, 2, (1 + Sin[e + f*x]) / 2, (d*(1 + Sin[e + f*x]) / (-c + d)) * Sqrt[2 - 2*Sin[e + f*x]]*(1 + Sin[e + f*x]) / ((c + d*Sin[e + f*x]) / (c - d))^n - (4*Sqrt[(-1 + Sin[e + f*x]) / (1 + Sin[e + f*x])] * (2*a*(1 + 2*n)*AppellF1[1/2 - n, -1/2, -n, 3/2 - n, 2 / (1 + Sin[e + f*x]), (-c + d) / (d + d*Sin[e + f*x])] + a*(-1 + 2*n)*AppellF1[-1/2 - n, -1/2, -n, 1/2 - n, 2 / (1 + Sin[e + f*x]), (-c + d) / (d + d*Sin[e + f*x])] * (1 + Sin[e + f*x])) / ((-1 + 4*n^2) * (1 + (c - d) / (d + d*Sin[e + f*x]))^n)) / (8*a^3*f*Sqrt[a*(1 + Sin[e + f*x])])

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{a \sin(fx + e) + a} (d \sin(fx + e) + c)^n}{a^2 \cos(fx + e)^2 - 2a^2 \sin(fx + e) - 2a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(a*sin(f*x + e) + a)*(d*sin(f*x + e) + c)^n/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)

maple [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(fx + e))(c + d \sin(fx + e))^n}{(a + a \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)

[Out] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^n}{(a \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^n/(a*sin(f*x + e) + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a + a \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2), x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n)/(a + a*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^n}{(a(\sin(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n/(a+a*sin(f*x+e))^(3/2), x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^n/(a*(sin(e + f*x) + 1))^(3/2), x)
```


$$3.336 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Optimal. Leaf size=351

$$2^{m+\frac{1}{2}} \cos(e + fx) \left(A(m+3) \left(c^2(m^2 + 3m + 2) + 2cdm(m+2) + d^2(m^2 + m + 1) \right) + B \left(c^2m(m^2 + 5m + 6) \right) \right)$$

```
[Out] (d*(A*d*(3+m)+B*(d*m+2*c))-2*(2+m)*(A*c*d*(3+m)+B*(c*d*m+c^2+d^2))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)/(3+m)-2^(1/2+m)*(A*(3+m)*(2*c*d*m*(2+m)+d^2*(m^2+m+1)+c^2*(m^2+3*m+2))+B*(d^2*m*(m^2+3*m+5)+c^2*m*(m^2+5*m+6)+2*c*d*(m^3+4*m^2+4*m+3))*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(3+m)/(m^2+3*m+2)-d*(A*d*(3+m)+B*(d*m+2*c))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)/(3+m)-B*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^2/f/(3+m)
```

Rubi [A] time = 0.99, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2983, 2968, 3023, 2751, 2652, 2651}

$$2^{m+\frac{1}{2}} \cos(e + fx) \left(A(m+3) \left(c^2(m^2 + 3m + 2) + 2cdm(m+2) + d^2(m^2 + m + 1) \right) + B \left(c^2m(m^2 + 5m + 6) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^2,x]
```

```
[Out] ((d*(A*d*(3 + m) + B*(2*c + d*m)) - 2*(2 + m)*(A*c*d*(3 + m) + B*(c^2 + d^2 + c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)*(3 + m)) - (2^(1/2 + m)*(A*(3 + m)*(2*c*d*m*(2 + m) + d^2*(1 + m + m^2) + c^2*(2 + 3*m + m^2)) + B*(d^2*m*(5 + 3*m + m^2) + c^2*m*(6 + 5*m + m^2) + 2*c*d*(3 + 4*m + 4*m^2 + m^3)))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m/(f*(1 + m)*(2 + m)*(3 + m)) - (d*(A*d*(3 + m) + B*(2*c + d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m)*(3 + m)) - (B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^2)/(f*(3 + m))
```

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2])/(d*sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2751

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := -Simp[(d*cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Int[(a + b*SIN[e + f*x])^m*(A*c + (B*c + A*d)*SIN[e + f*x] + B*d*SIN[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 2983

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol] := -Simp[(B*Cos[e + f*x]*(a + b*SIN[e + f*x])^m*(c + d*SIN[e + f*x])^n)/(f*(m + n + 1)), x] + Dist[1/(b*(m + n + 1)), Int[(a + b*SIN[e + f*x])^m*(c + d*SIN[e + f*x])^(n - 1)*Simp[A*b*c*(m + n + 1) + B*(a*c*m + b*d*n) + (A*b*d*(m + n + 1) + B*(a*d*m + b*c*n))*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 0] && (IntegerQ[n] || EqQ[m + 1/2, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*SIN[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c + d \sin(e + fx))^2}{f(3 + m)} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m (c + d \sin(e + fx))^2}{f(3 + m)} \\ &= -\frac{d(Ad(3 + m) + B(2c + dm)) \cos(e + fx)(a + a \sin(e + fx))^m}{af(2 + m)(3 + m)} \\ &= \frac{(d(Ad(3 + m) + B(2c + dm)) - 2(2 + m)(Ac + Bc)) \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)(3 + m)} \\ &= \frac{(d(Ad(3 + m) + B(2c + dm)) - 2(2 + m)(Ac + Bc)) \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)(3 + m)} \\ &= \frac{(d(Ad(3 + m) + B(2c + dm)) - 2(2 + m)(Ac + Bc)) \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)(3 + m)} \end{aligned}$$

Mathematica [A] time = 7.73, size = 300, normalized size = 0.85

$$\csc^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^m (a(\sin(e + fx) + 1))^m \left(-2(A + B)(c + d)^2 \tan\left(\frac{1}{4}(2e + 2fx - \pi)\right) {}_2F_1\left(\frac{1}{2}, m + 4; \frac{3}{2}; -\tan\left(\frac{1}{4}(2e + 2fx - \pi)\right)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*SIN[e + f*x])^m*(A + B*SIN[e + f*x])*(c + d*SIN[e + f*x])^2, x]
```

```
[Out] -(((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c + d)^2*Hypergeometric2F1[1/2, 4 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4] - (2*(c + d)*(3*A*c + B*c - A*d - 3*B*d)*Hypergeometric2F1[3/2, 4 + m, 5/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^3)/3 - (2*(c - d)*(A*(3*c + d) - B*(c + 3*d))*Hypergeometric2F1[5/2, 4 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^5)/5 - (2*(A - B)*(c - d)^2*Hypergeometric2F1[7/2, 4 + m, 9/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^7)/7))/f)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(Ac^2 + 2Bcd + Ad^2 - (2Bcd + Ad^2)\cos(fx + e)\right)^2 - \left(Bd^2\cos(fx + e)\right)^2 - Bc^2 - 2Acd - Bd^2\right)\sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral((A*c^2 + 2*B*c*d + A*d^2 - (2*B*c*d + A*d^2)*cos(f*x + e))^2 - (B*d^2*cos(f*x + e)^2 - B*c^2 - 2*A*c*d - B*d^2)*sin(f*x + e))*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)^2 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 9.53, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)^2 (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^2*(a*sin(f*x + e) + a)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2,x)
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^2, x)
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**2,x)
[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x))**2, x)
```

3.337 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx)) dx$

Optimal. Leaf size=199

$$\frac{2^{m+\frac{1}{2}} \cos(e + fx) \left(A(m+2)(cm + c + dm) + B \left(cm(m+2) + d(m^2 + m + 1) \right) \right) (\sin(e + fx) + 1)^{-m-\frac{1}{2}} (a \sin(e + fx) + a)}{f(m+1)(m+2)}$$

[Out] (B*d-(A*d+B*c)*(2+m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)/(2+m)-2^(1/2+m)* (A*(2+m)*(c*m+d*m+c)+B*(c*m*(2+m)+d*(m^2+m+1)))*cos(f*x+e)*hypergeom([1/2, 1/2-m], [3/2], 1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(m^2+3*m+2)-B*d*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/(2+m)

Rubi [A] time = 0.36, antiderivative size = 198, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {2968, 3023, 2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}} \cos(e + fx) \left(A(m+2)(cm + c + dm) + Bcm(m+2) + Bd(m^2 + m + 1) \right) (\sin(e + fx) + 1)^{-m-\frac{1}{2}} (a \sin(e + fx) + a)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),x]

[Out] ((B*d - (B*c + A*d)*(2 + m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) - (2^(1/2 + m)*(B*c*m*(2 + m) + A*(2 + m)*(c + c*m + d*m) + B*d*(1 + m + m^2))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m)*(2 + m)) - (B*d*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*(2 + m))

Rule 2651

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2968

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2), x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx)) dx &= \int (a + a \sin(e + fx))^m (Ac + (Bc + Ad) \sin(e + fx) \\ &+ Bd \cos(e + fx)(a + a \sin(e + fx))^{1+m}) dx \\ &= -\frac{Bd \cos(e + fx)(a + a \sin(e + fx))^{1+m}}{af(2 + m)} + \int (a + a \sin(e + fx))^m (Ac + (Bc + Ad) \sin(e + fx)) dx \\ &= \frac{(Bd - (Bc + Ad)(2 + m)) \cos(e + fx)(a + a \sin(e + fx))^{1+m}}{f(1 + m)(2 + m)} \\ &= \frac{(Bd - (Bc + Ad)(2 + m)) \cos(e + fx)(a + a \sin(e + fx))^{1+m}}{f(1 + m)(2 + m)} \\ &= \frac{(Bd - (Bc + Ad)(2 + m)) \cos(e + fx)(a + a \sin(e + fx))^{1+m}}{f(1 + m)(2 + m)} \end{aligned}$$

Mathematica [A] time = 3.59, size = 212, normalized size = 1.07

$$\frac{\csc^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^m (a(\sin(e + fx) + 1))^m \left(-2(A + B)(c + d) \tan\left(\frac{1}{4}(2e + 2fx - \pi)\right) {}_2F_1\left(\frac{1}{2}, m + 3; \frac{3}{2}; -\tan\left(\frac{1}{4}(2e + 2fx - \pi)\right)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x]),
x]
```

```
[Out] -((((Csc[(2*e + Pi + 2*f*x)/4]^2)^m*(a*(1 + Sin[e + f*x]))^m*(-2*(A + B)*(c
+ d)*Hypergeometric2F1[1/2, 3 + m, 3/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2
e - Pi + 2*f*x)/4] - (4*(A*c - B*d)*Hypergeometric2F1[3/2, 3 + m, 5/2, -T
an[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2*e - Pi + 2*f*x)/4]^3)/3 - (2*(A - B)*(c
- d)*Hypergeometric2F1[5/2, 3 + m, 7/2, -Tan[(2*e - Pi + 2*f*x)/4]^2]*Tan[(2
e - Pi + 2*f*x)/4]^5)/5))/f)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(Bd \cos(fx + e)^2 - Ac - Bd - (Bc + Ad) \sin(fx + e)\right)(a \sin(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*(a*si
n(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 7.61, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)),x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\sin(e + fx) + 1))^m (A + B \sin(e + fx)) (c + d \sin(e + fx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e)),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*(c + d*sin(e + f*x)), x)

3.338 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx$

Optimal. Leaf size=117

$$\frac{2^{m+\frac{1}{2}}(Am + A + Bm) \cos(e + fx)(\sin(e + fx) + 1)^{-m-\frac{1}{2}}(a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(m+1)}$$

[Out] -B*cos(f*x+e)*(a+a*sin(f*x+e))^m/f/(1+m)-2^(1/2+m)*(A*m+B*m+A)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f/(1+m)

Rubi [A] time = 0.08, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}}(Am + A + Bm) \cos(e + fx)(\sin(e + fx) + 1)^{-m-\frac{1}{2}}(a \sin(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right)}{f(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]

[Out] -((B*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(f*(1 + m))) - (2^(1/2 + m)*(A + A*m + B*m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 + m))

Rule 2651

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rubi steps

$$\begin{aligned} \int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) dx &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} + \frac{(A + Am + Bm) \int (a + a \sin(e + fx))^m dx}{1 + m} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} + \frac{((A + Am + Bm)(1 + \sin(e + fx)) \int (a + a \sin(e + fx))^m dx)}{1 + m} \\ &= -\frac{B \cos(e + fx)(a + a \sin(e + fx))^m}{f(1 + m)} - \frac{2^{\frac{1}{2}+m}(A + Am + Bm) \cos(e + fx)}{f(1 + m)} \end{aligned}$$

Mathematica [C] time = 1.82, size = 275, normalized size = 2.35

$$\sin^{-2m}\left(\frac{1}{4}(2e + 2fx + \pi)\right) (a(\sin(e + fx) + 1))^m \left(\frac{2\sqrt{2} A \sin\left(\frac{1}{4}(2e + 2fx - \pi)\right) \cos^{2m+1}\left(\frac{1}{4}(2e + 2fx - \pi)\right) {}_2F_1\left(\frac{1}{2}, m + \frac{1}{2}; m + \frac{3}{2}; \sin^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)\right)}{(2m+1)\sqrt{1 - \sin(e + fx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]),x]

[Out] -(((a*(1 + Sin[e + f*x]))^m*(((−1)^(1/4)*2^(−1 − 2*m)*B*(−(((−1)^(3/4)*(I + E^(I*(e + f*x))))/E^((I/2)*(e + f*x))))^(1 + 2*m)*(E^((2*I)*(e + f*x))^(−1 + m)*Hypergeometric2F1[1, m, −m, (−I)/E^(I*(e + f*x))] − (1 + m)*Hypergeometric2F1[1, 2 + m, 2 − m, (−I)/E^(I*(e + f*x))]))/(E^(((3*I)/2)*(e + f*x))^(−1 + m^2)) + (2*sqrt[2]*A*cos[(2*e − Pi + 2*f*x)/4]^(1 + 2*m)*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, Sin[(2*e + Pi + 2*f*x)/4]^2]*Sin[(2*e − Pi + 2*f*x)/4])/((1 + 2*m)*sqrt[1 − Sin[e + f*x]])))/(f*Sin[(2*e + Pi + 2*f*x)/4]^(2*m)))

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m,x)`

[Out] `int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + f x) + 1))^m (A + B \sin(e + f x)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e)),x)`

[Out] `Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x)), x)`

$$3.339 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{c+d \sin(e+fx)} dx$$

Optimal. Leaf size=191

$$\frac{\sqrt{2}(Bc - Ad) \cos(e + fx)(a \sin(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}} \quad B2^n$$

[Out] $-2^{(1/2+m)} * B * \cos(f*x+e) * \text{hypergeom}([1/2, 1/2-m], [3/2], 1/2-1/2*\sin(f*x+e)) * (1 + \sin(f*x+e))^{(-1/2-m)} * (a+a*\sin(f*x+e))^m / d / f - (-A*d+B*c) * \text{AppellF1}(1/2+m, 1, 1/2, 3/2+m, -d*(1+\sin(f*x+e))/(c-d), 1/2+1/2*\sin(f*x+e)) * \cos(f*x+e) * (a+a*\sin(f*x+e))^m * 2^{(1/2)} / (c-d) / d / f / (1+2*m) / (1-\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.29, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2986, 2652, 2651, 2788, 137, 136}

$$\frac{\sqrt{2}(Bc - Ad) \cos(e + fx)(a \sin(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d(\sin(e+fx)+1)}{c-d}\right)}{df(2m+1)(c-d)\sqrt{1-\sin(e+fx)}} \quad B2^n$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m * (A + B*\text{Sin}[e + f*x]) / (c + d*\text{Sin}[e + f*x]), x]$

[Out] $-((\text{Sqrt}[2] * (B*c - A*d) * \text{AppellF1}[1/2 + m, 1/2, 1, 3/2 + m, (1 + \text{Sin}[e + f*x]) / 2, -((d*(1 + \text{Sin}[e + f*x])) / (c - d))] * \text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m) / ((c - d) * d * f * (1 + 2*m) * \text{Sqrt}[1 - \text{Sin}[e + f*x]]) - (2^{(1/2 + m)} * B * \text{Cos}[e + f*x] * \text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Sin}[e + f*x]) / 2] * (1 + \text{Sin}[e + f*x])^{(-1/2 - m)} * (a + a*\text{Sin}[e + f*x])^m) / (d*f)$

Rule 136

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x] := \text{Simp}[(b*e - a*f)^p * (a + b*x)^{m+1} * \text{AppellF1}[m+1, -n, -p, m+2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^{(p+1)} * (m+1) * (b/(b*c - a*d))^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& \text{GtQ}[d/(d*a - c*b), 0] \&\& \text{SimplerQ}[c + d*x, a + b*x]$

Rule 137

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x] := \text{Dist}[(c + d*x)^{\text{FracPart}[n]} / ((b/(b*c - a*d))^{\text{IntPart}[n]} * ((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m * ((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n * (e + f*x)^p, x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IntegerQ}[m] \&\& \text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& \text{SimplerQ}[c + d*x, a + b*x]$

Rule 2651

$\text{Int}[(a + b*\text{sin}[c + d*x])^n, x] := -\text{Simp}[(2^{(n + 1/2)} * a^{(n - 1/2)} * b * \text{Cos}[c + d*x] * \text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1 - (b*\text{Sin}[c + d*x])/a)] / 2) / (d * \text{Sqrt}[a + b*\text{Sin}[c + d*x]]), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rule 2652

$\text{Int}[(a + b*\text{sin}[c + d*x])^n, x] := \text{Dist}[(a^{\text{IntPart}[n]} * (a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]}) / (1 + (b*\text{Sin}[c + d*x])/a)^{\text{FracPart}[n]}$

], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2788

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[m]

Rule 2986

Int((((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[m + 1/2, 0]

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx = \frac{B \int (a + a \sin(e + fx))^m dx}{d} - \frac{(Bc - Ad) \int \frac{(a + a \sin(e + fx))^m}{c + d \sin(e + fx)} dx}{d}$$

$$= \frac{(a^2(Bc - Ad) \cos(e + fx)) \text{Subst} \left(\int \frac{(a + ax)^{-\frac{1}{2} + m}}{\sqrt{a - ax}(c + dx)} dx, x, \sin(e + fx) \right)}{df \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{2^{2 + m} B \cos(e + fx) {}_2F_1 \left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx)) \right) (1 + \sin(e + fx))}{df}$$

$$= \frac{\sqrt{2} (Bc - Ad) F_1 \left(\frac{1}{2} + m; \frac{1}{2}, 1; \frac{3}{2} + m; \frac{1}{2}(1 + \sin(e + fx)) \right) - \frac{d(1 + \sin(e + fx))}{c + d}}{(c - d)df(1 + 2m)\sqrt{1 - \sin(e + fx)}}$$

Mathematica [B] time = 7.08, size = 473, normalized size = 2.48

$$(a(\sin(e + fx) + 1))^m \left(\frac{6(c+d)(Bc-Ad) \cot\left(\frac{1}{4}(2e+2fx+\pi)\right) \sec^2\left(\frac{1}{4}(2e+2fx-\pi)\right) \sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m}}{d(c+d \sin(e+fx)) \left(\sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right) \left(4dF_1\left(\frac{3}{2}; \frac{1}{2}-m, 2; \frac{5}{2}; \cos^2\left(\frac{1}{4}(2e+2fx+\pi)\right), \frac{2d \sin^2\left(\frac{1}{4}(2e+2fx-\pi)\right)}{c+d}\right) - (2m-1)(c+d)F_1\left(\frac{3}{2}\right) \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x]), x]

[Out] ((a*(1 + Sin[e + f*x]))^m*((Sqrt[2]*B*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 + m, 3/2 + m, (Cos[e + f*x]^2*Csc[(2*e - Pi + 2*f*x)/4]^2)/4])/((d + 2*d*m)*Sqrt[1 - Sin[e + f*x]]) + (6*(c + d)*(B*c - A*d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*Sec

$$\frac{[(2e - \pi + 2fx)/4]^2 (\sin[(2e + \pi + 2fx)/4]^2)^{(1/2 - m)}}{d(c + d \sin[e + fx])} (3(c + d) \operatorname{AppellF1}[1/2, 1/2 - m, 1, 3/2, \cos[(2e + \pi + 2fx)/4]^2, (2d \sin[(2e - \pi + 2fx)/4]^2)/(c + d)] + (4d \operatorname{AppellF1}[3/2, 1/2 - m, 2, 5/2, \cos[(2e + \pi + 2fx)/4]^2, (2d \sin[(2e - \pi + 2fx)/4]^2)/(c + d)] - (c + d)(-1 + 2m) \operatorname{AppellF1}[3/2, 3/2 - m, 1, 5/2, \cos[(2e + \pi + 2fx)/4]^2, (2d \sin[(2e - \pi + 2fx)/4]^2)/(c + d)]) \sin[(2e - \pi + 2fx)/4]^2) / f$$

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{d \sin(fx + e) + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{d \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)

maple [F] time = 4.05, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{c + d \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{d \sin(fx + e) + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)),x)
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x)), x)
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e)),x)
[Out] Timed out
```

$$3.340 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=293

$$\frac{\sqrt{2} \cos(e+fx) \left(Ad(c(1-m) - dm) - B(c^2(-m) - cdm + d^2) \right) (a \sin(e+fx) + a)^m F_1 \left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2} \right)}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

[Out] $2^{(1/2+m)} * (-A*d+B*c) * m * \cos(f*x+e) * \text{hypergeom}([1/2, 1/2-m], [3/2], 1/2-1/2*\sin(f*x+e)) * (1+\sin(f*x+e))^{(-1/2-m)} * (a+a*\sin(f*x+e))^m / d / (c^2-d^2) / f - (-A*d+B*c) * \cos(f*x+e) * (a+a*\sin(f*x+e))^m / (c^2-d^2) / f / (c+d*\sin(f*x+e)) + (A*d*(c*(1-m)-d*m) - B*(-c^2*m-c*d*m+d^2)) * \text{AppellF1}(1/2+m, 1, 1/2, 3/2+m, -d*(1+\sin(f*x+e)) / (c-d), 1/2+1/2*\sin(f*x+e)) * \cos(f*x+e) * (a+a*\sin(f*x+e))^m * 2^{(1/2)} / (c-d)^2 / d / (c+d) / f / (1+2*m) / (1-\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.62, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\sqrt{2} \cos(e+fx) \left(Ad(c(1-m) - dm) - B(c^2(-m) - cdm + d^2) \right) (a \sin(e+fx) + a)^m F_1 \left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2} \right)}{df(2m+1)(c-d)^2(c+d)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] $(\text{Sqrt}[2] * (A*d*(c*(1-m) - d*m) - B*(d^2 - c^2*m - c*d*m)) * \text{AppellF1}[1/2 + m, 1/2, 1, 3/2 + m, (1 + \text{Sin}[e + f*x])/2, -((d*(1 + \text{Sin}[e + f*x]))/(c - d))] * \text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m) / ((c - d)^2*d*(c + d)*f*(1 + 2*m)*\text{Sqrt}[1 - \text{Sin}[e + f*x]]) + (2^{(1/2 + m)} * (B*c - A*d)*m*\text{Cos}[e + f*x]*\text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Sin}[e + f*x])/2] * (1 + \text{Sin}[e + f*x])^{(-1/2 - m)} * (a + a*\text{Sin}[e + f*x])^m) / (d*(c^2 - d^2)*f) - ((B*c - A*d)*\text{Cos}[e + f*x] * (a + a*\text{Sin}[e + f*x])^m) / ((c^2 - d^2)*f*(c + d*\text{Sin}[e + f*x]))$

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]) / (b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 137

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n] / ((b/(b*c - a*d))^IntPart[n] * ((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m * ((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n * (e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x]

Rule 2651

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2]) / (d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2788

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[m]

Rule 2984

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m + 1/2, 0])

Rule 2986

Int((((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[m + 1/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx &= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{(c^2 - d^2) f (c + d \sin(e + fx))} - \frac{\int \frac{(a + a \sin(e + fx))^{m-1} (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx}{d} \\ &= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{(c^2 - d^2) f (c + d \sin(e + fx))} - \frac{((Bc - Ad)m) \int \frac{(a + a \sin(e + fx))^{m-1} (A + B \sin(e + fx))}{c + d \sin(e + fx)} dx}{d} \\ &= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{(c^2 - d^2) f (c + d \sin(e + fx))} + \frac{a^2 (Ad(c(1 - m) - dm) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \sin(e + fx))\right) - B(d^2 - c^2 m - cdm)) F_1\left(\frac{1}{2} + m; \frac{1}{2}, 1; \frac{1}{2}(1 - \sin(e + fx))\right)}{d(c^2 - d^2) f} \\ &= \frac{\sqrt{2} (Ad(c(1 - m) - dm) - B(d^2 - c^2 m - cdm)) F_1\left(\frac{1}{2} + m; \frac{1}{2}, 1; \frac{1}{2}(1 - \sin(e + fx))\right)}{(c - d)^2 d (c + d) f} \end{aligned}$$

Mathematica [B] time = 2.50, size = 651, normalized size = 2.22

$$6(c+d) \cot\left(\frac{1}{4}(2e+2fx+\pi)\right) \sin^2\left(\frac{1}{4}(2e+2fx+\pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e+2fx-\pi)\right)^{m-\frac{1}{2}} (a(\sin(e+fx)+1))^m$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(((B*c) + A*d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(-3*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2) + (B*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*AppellF1[1/2, 1/2 - m, 1, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*AppellF1[3/2, 1/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^2)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)

maple [F] time = 14.32, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c + d \sin(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)`

[Out] `int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c + d \sin(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2,x)`

[Out] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^2, x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**2,x)`

[Out] Timed out

3.341
$$\int \frac{(a+a \sin(e+fx))^m(A+B \sin(e+fx))}{(c+d \sin(e+fx))^3} dx$$

Optimal. Leaf size=467

$$\frac{\cos(e+fx)\left(B\left(c^3(1-m)m+2c^2d(1-m)m-cd^2(m^2-3m+3)+2d^3m\right)-Ad\left(-\left(c^2(m^2-3m+2)\right)\right)+2cd\right)}{\sqrt{2}df(2m+1)(c-d)^3(c-d)}$$

```
[Out] -2^(-1/2+m)*m*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*hypergeom([1/2, 1/2-m], [3/2], 1/2-1/2*sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f*x+e))^m/d/(c^2-d^2)^2/f-1/2*(-A*d+B*c)*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)/f/(c+d*sin(f*x+e))^2+1/2*(A*d*(c*(3-m)-d*m)-B*(2*d^2+c^2*(1-m)-c*d*m))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c^2-d^2)^2/f/(c+d*sin(f*x+e))+1/2*(B*(2*d^3*m+c^3*(1-m)*m+2*c^2*d*(1-m)*m-c*d^2*(m^2-3*m+3))-A*d*(2*c*d*(2-m)*m-c^2*(m^2-3*m+2)-d^2*(m^2-m+1))*AppellF1(1/2+m,1,1/2,3/2+m,-d*(1+sin(f*x+e)))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m/(c-d)^3/d/(c+d)^2/f/(1+2*m)*2^(1/2)/(1-sin(f*x+e))^(1/2)
```

Rubi [A] time = 1.35, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2984, 2986, 2652, 2651, 2788, 137, 136}

$$\frac{\cos(e+fx)\left(B\left(2c^2d(1-m)m+c^3(1-m)m-cd^2(m^2-3m+3)+2d^3m\right)-Ad\left(-\left(c^2(m^2-3m+2)\right)\right)+2cd\right)}{\sqrt{2}df(2m+1)(c-d)^3(c-d)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]
[Out] ((B*(2*d^3*m + c^3*(1 - m)*m + 2*c^2*d*(1 - m)*m - c*d^2*(3 - 3*m + m^2)) - A*d*(2*c*d*(2 - m)*m - c^2*(2 - 3*m + m^2) - d^2*(1 - m + m^2))*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m/(Sqrt[2]*(c - d)^3*d*(c + d)^2*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]) - (2^(-1/2 + m)*m*(A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(d*(c^2 - d^2)^2*f) - ((B*c - A*d)*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*(c^2 - d^2)*f*(c + d*Sin[e + f*x])^2) + ((A*d*(c*(3 - m) - d*m) - B*(2*d^2 + c^2*(1 - m) - c*d*m))*Cos[e + f*x]*(a + a*Sin[e + f*x])^m)/(2*(c^2 - d^2)^2*f*(c + d*Sin[e + f*x]))
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplifierQ[c + d*x, a + b*x])
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplifierQ[c + d*x, a + b*x]
```

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n +
1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 2652

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[(a^IntPa
rt[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n
], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2984

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Sim
p[((B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n
+ 1))/(f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(b*(n + 1)*(c^2 - d^2)), Int[(a
+ b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n + 1
)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && EqQ
[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
+ 1/2, 0])
```

Rule 2986

```
Int((((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)]))/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[B
/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Sin
[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B,
m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && N
eQ[m + 1/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c + d \sin(e + fx))^3} dx &= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} - \int \frac{(a + a \sin(e + fx))}{(c + d \sin(e + fx))^3} dx \\
&= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} + \frac{(Ad(c(3 - m)))}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} \\
&= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} + \frac{(Ad(c(3 - m)))}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} \\
&= -\frac{(Bc - Ad) \cos(e + fx) (a + a \sin(e + fx))^m}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} + \frac{(Ad(c(3 - m)))}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} \\
&= -\frac{2^{-\frac{1}{2}+m} m (Ad(c(3 - m) - dm) - B(2d^2 + c^2(1 - m) - cdm))}{2(c^2 - d^2) f (c + d \sin(e + fx))^2} \\
&= \frac{(B(2d^3 m + c^3(1 - m)m + 2c^2 d(1 - m)m - cd^2(3 - 3m + m^2))}{2(c^2 - d^2) f (c + d \sin(e + fx))^2}
\end{aligned}$$

Mathematica [A] time = 2.53, size = 651, normalized size = 1.39

$$6(c + d) \cot\left(\frac{1}{4}(2e + 2fx + \pi)\right) \sin^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^3,x]

[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(((-(B*c) + A*d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(-3*(c + d)*AppellF1[1/2, 1/2 - m, 3, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-12*d*AppellF1[3/2, 1/2 - m, 4, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2) + (B*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(-3*(c + d)*AppellF1[1/2, 1/2 - m, 2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-8*d*AppellF1[3/2, 1/2 - m, 3, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^3)

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{3cd^2 \cos(fx + e)^2 - c^3 - 3cd^2 + (d^3 \cos(fx + e)^2 - 3c^2d - d^3) \sin(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(3*c*d^2*cos(f*x + e)^2 - c^3 - 3*c*d^2 + (d^3*cos(f*x + e)^2 - 3*c^2*d - d^3)*sin(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)

maple [F] time = 12.57, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c + d \sin(fx + e))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c + d \sin(e + fx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3,x)

```
[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^3,x)
```

```
[Out] Timed out
```

$$3.342 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^{3/2} dx$$

Optimal. Leaf size=284

$$\frac{\sqrt{2}(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

[Out] (A-B)*(c-d)*AppellF1(1/2+m,-3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*(c-d)*AppellF1(3/2+m,-3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)

Rubi [A] time = 0.63, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)(c-d)\cos(e+fx)(a\sin(e+fx)+a)^m\sqrt{c+d\sin(e+fx)}F_1\left(m+\frac{1}{2};\frac{1}{2},-\frac{3}{2};m+\frac{3}{2};\frac{1}{2}(\sin(e+fx)+1)\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}\sqrt{\frac{c+d\sin(e+fx)}{c-d}}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(3/2), x]

[Out] (Sqrt[2]*(A - B)*(c - d)*AppellF1[1/2 + m, 1/2, -3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)]) + (Sqrt[2]*B*(c - d)*AppellF1[3/2 + m, 1/2, -3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/(c - d)])

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140


```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*cos[e + f*x])/(f*Sqrt[a + b*sin[e
+ f*x]]*Sqrt[a - b*sin[e + f*x]]), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*sin[e + f*x])^(m + 1)*(c + d*sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^{3/2} dx = (A - B) \int (a + a \sin(e + fx))^m (c + d \sin(e + fx))^{3/2} dx$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^{-1/2}}{\sqrt{a - a \sin(e + fx)}} dx\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) \operatorname{Subst}\left(\int \frac{(a+ax)^{-1/2}}{\sqrt{a - a \sin(e + fx)}} dx\right)}{\sqrt{2} f (a - a \sin(e + fx))}$$

$$= \frac{\left(a(A - B)(ac - ad) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) \operatorname{Subst}\left(\int \frac{(a+ax)^{-1/2}}{\sqrt{a - a \sin(e + fx)}} dx\right)}{\sqrt{2} f (a - a \sin(e + fx))}$$

$$= \frac{\sqrt{2} (A - B)(c - d) F_1\left(\frac{1}{2} + m; \frac{1}{2}, -\frac{3}{2}; \frac{3}{2} + m\right)}{\sqrt{2} f (a - a \sin(e + fx))}$$

Mathematica [B] time = 8.14, size = 3281, normalized size = 11.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(
3/2), x]
```

```

[Out] -(((((-2*B*c*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^
2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 +
2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3
*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/2)*Sqrt[c + d -
2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)
]/2]^2)/(c + d))) - (2*A*d*AppellF1[3/2, (1 - 2*m)/2, -1/2, 5/2, Sin[(-e +
Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2
- f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2)*Sin[(-e + P
i/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2 + (-1 + 2*m)/
2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(3*Sqrt[(c + d - 2*d*Sin[(
-e + Pi/2 - f*x)/2]^2)/(c + d)]) + (B*d*AppellF1[5/2, (1 - 2*m)/2, -1/2, 7/
2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Co
s[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
)*Sin[(-e + Pi/2 - f*x)/2]^5*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^((1 - 2*m)/2
+ (-1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(5*Sqrt[(c +
d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]) - (2*B*d*AppellF1[3/2, (-1 -
2*m)/2, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2
]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]^3*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^((-1 - 2*m)/2 + (1 + 2*m)/2)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2
])/Sqrt[(c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (3*B*d*(c + d)*
AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e
+ Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(3 + 2*m)*(Cos[(-e +
Pi/2 - f*x)/2]^2)^(1/2 + (-4 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(
-e + Pi/2 - f*x)/2]^2)^(3/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^
2])/(-3*(c + d)*AppellF1[1/2, -3/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]
^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -3/2 - m
, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c
+ d)] + (c + d)*(3 + 2*m)*AppellF1[3/2, -1/2 - m, -1/2, 5/2, Sin[(-e + Pi/
2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 -
f*x)/2]^2 - (6*B*c*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi
/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 -
f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(1/2 + (-2 - 2*m)/2)*Sin[(-e
+ Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^2)^(1/2 + m)*Sqrt[c + d - 2
*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*AppellF1[1/2, -1/2 - m, -1/2, 3
/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] +
(2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Si
n[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1 + 2*m)*AppellF1[3/2, 1/2 -
m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(
c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2 - (6*A*d*(c + d)*AppellF1[1/2, -1/2 -
m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)
/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(1 + 2*m)*(Cos[(-e + Pi/2 - f*x)/2]^2)^(
1/2 + (-2 - 2*m)/2)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e + Pi/2 - f*x)/2]^
2)^(1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])/(-3*(c + d)*Appe
llF1[1/2, -1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + P
i/2 - f*x)/2]^2)/(c + d)] + (2*d*AppellF1[3/2, -1/2 - m, 1/2, 5/2, Sin[(-e
+ Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] + (c + d)*(1
+ 2*m)*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2]^2 + (6*A*c*(
c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*S
in[(-e + Pi/2 - f*x)/2]^2)/(c + d)]*Cos[(-e + Pi/2 - f*x)/2]^(-1 + 2*m)*(Co
s[(-e + Pi/2 - f*x)/2]^2)^(1/2 - m)*Sin[(-e + Pi/2 - f*x)/2]*(1 - Sin[(-e +
Pi/2 - f*x)/2]^2)^(-1/2 + m)*Sqrt[c + d - 2*d*Sin[(-e + Pi/2 - f*x)/2]^2])
/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*x)/2]^2, (
2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)] - (2*d*AppellF1[3/2, 1/2 - m, 1/2,
5/2, Sin[(-e + Pi/2 - f*x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)]
+ (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Sin[(-e + Pi/2 - f*
x)/2]^2, (2*d*Sin[(-e + Pi/2 - f*x)/2]^2)/(c + d)])*Sin[(-e + Pi/2 - f*x)/2
]^2 + (3*B*d*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Sin[(-e + Pi/2 - f*

```

$x)/2]^2, (2*d*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)/(c + d)]*\cos[(-e + \text{Pi}/2 - f*x)/2]$
 $\wedge(-1 + 2*m)*(\cos[(-e + \text{Pi}/2 - f*x)/2]^2)^{(1/2 - m)}*\sin[(-e + \text{Pi}/2 - f*x)/2]$
 $*(1 - \sin[(-e + \text{Pi}/2 - f*x)/2]^2)^{(-1/2 + m)}*\text{Sqrt}[c + d - 2*d*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)]/(3*(c + d)*\text{AppellF1}[1/2, 1/2 - m, -1/2, 3/2, \sin[(-e + \text{Pi}/2 - f*x)/2]^2, (2*d*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)/(c + d)] - (2*d*\text{AppellF1}[3/2, 1/2 - m, 1/2, 5/2, \sin[(-e + \text{Pi}/2 - f*x)/2]^2, (2*d*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)/(c + d)] + (c + d)*(-1 + 2*m)*\text{AppellF1}[3/2, 3/2 - m, -1/2, 5/2, \sin[(-e + \text{Pi}/2 - f*x)/2]^2, (2*d*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)/(c + d)])*\sin[(-e + \text{Pi}/2 - f*x)/2]^2)*(a + a*\sin[e + f*x])^m)/(f*\cos[(-e + \text{Pi}/2 - f*x)/2]^(2*m))$

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$\text{integral}\left(-\left(Bd \cos(fx + e)^2 - Ac - Bd - (Bc + Ad) \sin(fx + e)\right)\sqrt{d \sin(fx + e) + c} (a \sin(fx + e) + a)^m, x\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="fricas")`

[Out] `integral(-(B*d*cos(f*x + e)^2 - A*c - B*d - (B*c + A*d)*sin(f*x + e))*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)^{\frac{3}{2}}(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")`

[Out] `integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)`

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)`

[Out] `int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(d \sin(fx + e) + c)^{\frac{3}{2}}(a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)*(a*sin(f*x + e) + a)^m, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m (c + d \sin(e + fx))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),  
x)
```

```
[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(3/2),  
x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.343 $\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) \sqrt{c + d \sin(e + fx)}$

Optimal. Leaf size=274

$$\frac{\sqrt{2}(A - B) \cos(e + fx)(a \sin(e + fx) + a)^m \sqrt{c + d \sin(e + fx)} F_1\left(m + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d}{c-d}\right)}{f(2m + 1) \sqrt{1 - \sin(e + fx)} \sqrt{\frac{c + d \sin(e + fx)}{c - d}}}$$

```
[Out] (A-B)*AppellF1(1/2+m,-1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)+B*AppellF1(3/2+m,-1/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*(c+d*sin(f*x+e))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2)/((c+d*sin(f*x+e))/(c-d))^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A - B) \cos(e + fx)(a \sin(e + fx) + a)^m \sqrt{c + d \sin(e + fx)} F_1\left(m + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d}{c-d}\right)}{f(2m + 1) \sqrt{1 - \sin(e + fx)} \sqrt{\frac{c + d \sin(e + fx)}{c - d}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[c + d*Sin[e + f*x]])/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/c - d]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[c + d*Sin[e + f*x]])/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[(c + d*Sin[e + f*x])/c - d])
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x)/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*(c + d*x)/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
```

```
) + (b*d*x)/(b*c - a*d)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a + b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && !IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Dist[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x] + Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx)) \sqrt{c + d \sin(e + fx)} dx = (A - B) \int (a + a \sin(e + fx))^m \sqrt{c + d \sin(e + fx)} dx + \dots$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m} \sqrt{a-ax}}{\sqrt{a-ax}} dx\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}} + \dots$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) \operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m} \sqrt{a-ax}}{\sqrt{a-ax}} dx\right)}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} + \dots$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) \sqrt{c + d \sin(e + fx)}}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} + \dots$$

$$= \frac{\sqrt{2} (A - B) F_1\left(\frac{1}{2} + m; \frac{1}{2}, -\frac{1}{2}; \frac{3}{2} + m; \frac{1}{2}(1 + \sin(e + fx))\right)}{f(1 + \sin(e + fx)) \sqrt{c + d \sin(e + fx)}} + \dots$$

Mathematica [B] time = 11.89, size = 672, normalized size = 2.45

$$6(c + d) \cot\left(\frac{1}{4}(2e + 2fx + \pi)\right) \sin^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m \sqrt{c + d \sin(e + fx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]],x]
```

```
[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*Sqrt[c + d*Sin[e + f*x]]*(((B*c - A*d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)))*Cos[(2*e + Pi + 2*f*x)/4]^2 - (B*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-6*d*AppellF1[3/2, 1/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\sqrt{d \sin(fx + e) + c} \left(a \sin(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)\sqrt{d \sin(fx + e) + c} (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) \sqrt{c + d \sin(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

```
[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)\sqrt{d \sin(fx + e) + c} (a \sin(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + fx)) (a + a \sin(e + fx))^m \sqrt{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sin(e + fx) + 1))^m (A + B \sin(e + fx)) \sqrt{c + d \sin(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x)), x)

$$3.344 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{\sqrt{c+d \sin(e+fx)}} dx$$

Optimal. Leaf size=274

$$\frac{\sqrt{2} (A - B) \cos(e + fx) (a \sin(e + fx) + a)^m \sqrt{\frac{c+d \sin(e+fx)}{c-d}} F_1\left(m + \frac{1}{2}; \frac{1}{2}, \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d(\sin(e+fx)}{c-d}\right)}{f(2m + 1) \sqrt{1 - \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}$$

[Out] (A-B)*AppellF1(1/2+m,1/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,1/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/f/(3+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)

Rubi [A] time = 0.54, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2} (A - B) \cos(e + fx) (a \sin(e + fx) + a)^m \sqrt{\frac{c+d \sin(e+fx)}{c-d}} F_1\left(m + \frac{1}{2}; \frac{1}{2}, \frac{1}{2}; m + \frac{3}{2}; \frac{1}{2}(\sin(e + fx) + 1), -\frac{d(\sin(e+fx)}{c-d}\right)}{f(2m + 1) \sqrt{1 - \sin(e + fx)} \sqrt{c + d \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]], x]

[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 1/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)]/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 1/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)]/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]])*Sqrt[a - b*Sin[e + f*x]], Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{\sqrt{c + d \sin(e + fx)}} dx = (A - B) \int \frac{(a + a \sin(e + fx))^m}{\sqrt{c + d \sin(e + fx)}} dx + \frac{B \int \frac{(a + a \sin(e + fx))^{1+m}}{\sqrt{c + d \sin(e + fx)}} dx}{a}$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \text{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax} \sqrt{c+dx}} dx, x, \sin(e + fx) \right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}} \right) \text{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2} - \frac{x}{2}} \sqrt{c+dx}} dx, \frac{1}{2} - \frac{x}{2}, \sin(e + fx) \right)}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}} \sqrt{\frac{a(c+d \sin(e + fx))}{ac - ad}} \right) \text{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{1 - \frac{x}{2}} \sqrt{c+dx}} dx, \frac{1}{2} - \frac{x}{2}, \sin(e + fx) \right)}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)} \sqrt{c}}$$

$$= \frac{\sqrt{2} (A - B) F_1 \left(\frac{1}{2} + m; \frac{1}{2}, \frac{1}{2}; \frac{3}{2} + m; \frac{1}{2} (1 + \sin(e + fx)), -\frac{d(1 + \sin(e + fx))}{c - d} \right)}{f (1 + 2m) \sqrt{1 - \sin(e + fx)} \sqrt{c}}$$

Mathematica [B] time = 5.71, size = 672, normalized size = 2.45

$$6(c + d) \cot \left(\frac{1}{4} (2e + 2fx + \pi) \right) \sin^2 \left(\frac{1}{4} (2e + 2fx + \pi) \right)^{\frac{1}{2}-m} \cos^2 \left(\frac{1}{4} (2e + 2fx - \pi) \right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m \left(\frac{1}{\cos^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/Sqrt[c + d*Sin[e + f*x]],x]

[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*((B*c - A*d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]/(3*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2 - (B*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-2*d*AppellF1[3/2, 1/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2))*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*Sqrt[c + d*Sin[e + f*x]])

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{d \sin(fx + e) + c}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{d \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{\sqrt{c + d \sin(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{\sqrt{d \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/sqrt(d*sin(f*x + e) + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{\sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{\sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(1/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/sqrt(c + d*sin(e + f*x)), x)

$$3.345 \quad \int \frac{(a+a \sin(e+fx))^m (A+B \sin(e+fx))}{(c+d \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=288

$$\frac{\sqrt{2}(A-B) \cos(e+fx)(a \sin(e+fx)+a)^m \sqrt{\frac{c+d \sin(e+fx)}{c-d}} F_1\left(m+\frac{1}{2}; \frac{1}{2}, \frac{3}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1), -\frac{d(\sin(e+fx))}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d \sin(e+fx)}}$$

[Out] (A-B)*AppellF1(1/2+m,3/2,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/(c-d)/f/(1+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,3/2,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*2^(1/2)*((c+d*sin(f*x+e))/(c-d))^(1/2)/a/(c-d)/f/(3+2*m)/(1-sin(f*x+e))^(1/2)/(c+d*sin(f*x+e))^(1/2)

Rubi [A] time = 0.55, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B) \cos(e+fx)(a \sin(e+fx)+a)^m \sqrt{\frac{c+d \sin(e+fx)}{c-d}} F_1\left(m+\frac{1}{2}; \frac{1}{2}, \frac{3}{2}; m+\frac{3}{2}; \frac{1}{2}(\sin(e+fx)+1), -\frac{d(\sin(e+fx))}{c-d}\right)}{f(2m+1)(c-d)\sqrt{1-\sin(e+fx)}\sqrt{c+d \sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2), x]

[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, 3/2, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*Sqrt[(c + d*Sin[e + f*x])/(c - d)]/((c - d)*f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, 3/2, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*Sqrt[(c + d*Sin[e + f*x])/(c - d)]/(a*(c - d)*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int \frac{(a + a \sin(e + fx))^m (A + B \sin(e + fx))}{(c + d \sin(e + fx))^{3/2}} dx = (A - B) \int \frac{(a + a \sin(e + fx))^m}{(c + d \sin(e + fx))^{3/2}} dx + \frac{B \int \frac{(a + a \sin(e + fx))^{1+m}}{(c + d \sin(e + fx))^{3/2}} dx}{a}$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{a-ax}(c+dx)^{3/2}} dx, x, \sin(e + fx) \right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a-a \sin(e+fx)}{a}} \right) \operatorname{Subst} \left(\int \frac{(a+ax)^{-\frac{1}{2}+m}}{\sqrt{\frac{1}{2}-\frac{x}{2}}(c+dx)^{3/2}} dx \right)}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^3(A - B) \cos(e + fx) \sqrt{\frac{a-a \sin(e+fx)}{a}} \sqrt{\frac{a(c+d \sin(e+fx))}{ac-ad}} \right) \operatorname{Subst} \left(\int \dots \right)}{\sqrt{2} (ac - ad) f (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\sqrt{2} (A - B) F_1 \left(\frac{1}{2} + m; \frac{1}{2}, \frac{3}{2}; \frac{3}{2} + m; \frac{1}{2} (1 + \sin(e + fx)) \right), -\frac{d(1+\sin(e+fx))}{c-d}}{(c - d) f (1 + 2m) \sqrt{1 - \sin(e + fx)}}$$

Mathematica [B] time = 6.22, size = 672, normalized size = 2.33

$$6(c + d) \cot \left(\frac{1}{4} (2e + 2fx + \pi) \right) \sin^2 \left(\frac{1}{4} (2e + 2fx + \pi) \right)^{\frac{1}{2}-m} \cos^2 \left(\frac{1}{4} (2e + 2fx - \pi) \right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m \left(\frac{\dots}{\cos^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^(3/2), x]

[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*((B*c - A*d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]/(3*(c + d)*AppellF1[1/2, 1/2 - m, 3/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (6*d*AppellF1[3/2, 1/2 - m, 5/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2 - (B*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, 1/2, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (2*d*AppellF1[3/2, 1/2 - m, 3/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, 1/2, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2))*Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f*(c + d*Sin[e + f*x])^(3/2))

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A) \sqrt{d \sin(fx + e) + c} (a \sin(fx + e) + a)^m}{d^2 \cos(fx + e)^2 - 2cd \sin(fx + e) - c^2 - d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2), x, algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)*(a*sin(f*x + e) + a)^m/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2), x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(a + a \sin(fx + e))^m (A + B \sin(fx + e))}{(c + d \sin(fx + e))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2), x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(a \sin(fx + e) + a)^m}{(d \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m/(d*sin(f*x + e) + c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) (a + a \sin(e + fx))^m}{(c + d \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2),x)

[Out] int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sin(e + fx) + 1))^m (A + B \sin(e + fx))}{(c + d \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))**m*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))**(3/2),x)

[Out] Integral((a*(sin(e + f*x) + 1))**m*(A + B*sin(e + f*x))/(c + d*sin(e + f*x))**(3/2), x)

$$3.346 \quad \int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=270

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx))\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

[Out] (A-B)*AppellF1(1/2+m,-n,1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n*2^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n/(1-sin(f*x+e))^(1/2)+B*AppellF1(3/2+m,-n,1/2,5/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*cos(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)/a/f/(3+2*m)/(((c+d*sin(f*x+e))/(c-d))^n/(1-sin(f*x+e))^(1/2))

Rubi [A] time = 0.43, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2987, 2788, 140, 139, 138}

$$\frac{\sqrt{2}(A-B)\cos(e+fx)(a\sin(e+fx)+a)^m(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};\frac{1}{2},-n;m+\frac{3}{2};\frac{1}{2}(\sin(e+fx))\right)}{f(2m+1)\sqrt{1-\sin(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] (Sqrt[2]*(A - B)*AppellF1[1/2 + m, 1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/c - d)^n) + (Sqrt[2]*B*AppellF1[3/2 + m, 1/2, -n, 5/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Cos[e + f*x]*(a + a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(a*f*(3 + 2*m)*Sqrt[1 - Sin[e + f*x]]*((c + d*Sin[e + f*x])/c - d)^n)

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*

```
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d)
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) +
(f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*(c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx = (A - B) \int (a + a \sin(e + fx))^m (c + d \sin(e + fx))^n dx$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}} dx\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) \operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}+m}(c+dx)^n}{\sqrt{a-ax}} dx\right)}{\sqrt{2} f (a - a \sin(e + fx)) \sqrt{a + a \sin(e + fx)}}$$

$$= \frac{\left(a^2(A - B) \cos(e + fx) \sqrt{\frac{a - a \sin(e + fx)}{a}}\right) (c + d \sin(e + fx))^n}{\sqrt{2}}$$

$$= \frac{\sqrt{2} (A - B) F_1\left(\frac{1}{2} + m; \frac{1}{2}, -n; \frac{3}{2} + m; \frac{1}{2}(1 + \sin(e + fx))\right)}{\sqrt{2}}$$

Mathematica [B] time = 6.08, size = 682, normalized size = 2.53

$$6(c + d) \cot\left(\frac{1}{4}(2e + 2fx + \pi)\right) \sin^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m (c + d \sin(e + fx))^n$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] (6*(c + d)*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*(c + d*Sin[e + f*x])^n*((B*c - A*d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]/(3*(c + d)*AppellF1[1/2, 1/2 - m, -n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*n*AppellF1[3/2, 1/2 - m, 1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2 - (B*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*(c + d*Sin[e + f*x]))/(3*(c + d)*AppellF1[1/2, 1/2 - m, -1 - n, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (-4*d*(1 + n)*AppellF1[3/2, 1/2 - m, -n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, -1 - n, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])*Cos[(2*e + Pi + 2*f*x)/4]^2)*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/(d*f)

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m\left(d \sin(fx + e) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 1.35, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (A + B \sin(e + f x)) (a + a \sin(e + f x))^m (c + d \sin(e + f x))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)

[Out] int((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] Timed out

3.347
$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^{-1-m} dx$$

Optimal. Leaf size=277

$$\frac{\sqrt{2} B \cos(e + fx)(a \sin(e + fx) + a)^{m+1}(c + d \sin(e + fx))^{-m} \left(\frac{c+d \sin(e+fx)}{c-d}\right)^m F_1\left(m + \frac{3}{2}; \frac{1}{2}, m + 1; m + \frac{5}{2}; \frac{1}{2}(\sin(e + fx) + 1)\right)}{af(2m + 3)(c - d)\sqrt{1 - \sin(e + fx)}}$$

[Out] $-2^{(1/2+m)} * a * (A-B) * \cos(f*x+e) * \text{hypergeom}([1/2, 1/2-m], [3/2], 1/2*(c-d)*(1-\sin(f*x+e))/(c+d*\sin(f*x+e))) * (a+a*\sin(f*x+e))^{(-1+m)} * ((c+d)*(1+\sin(f*x+e))/(c+d*\sin(f*x+e)))^{(1/2-m)} / (c+d) / f / ((c+d*\sin(f*x+e))^m) + B * \text{AppellF1}(3/2+m, 1+m, 1/2, 5/2+m, -d*(1+\sin(f*x+e))/(c-d), 1/2+1/2*\sin(f*x+e)) * \cos(f*x+e) * (a+a*\sin(f*x+e))^{(1+m)} * ((c+d*\sin(f*x+e))/(c-d))^m * 2^{(1/2)} / a / (c-d) / f / (3+2*m) / ((c+d*\sin(f*x+e))^m) / (1-\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 0.48, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2987, 2788, 132, 140, 139, 138}

$$\frac{\sqrt{2} B \cos(e + fx)(a \sin(e + fx) + a)^{m+1}(c + d \sin(e + fx))^{-m} \left(\frac{c+d \sin(e+fx)}{c-d}\right)^m F_1\left(m + \frac{3}{2}; \frac{1}{2}, m + 1; m + \frac{5}{2}; \frac{1}{2}(\sin(e + fx) + 1)\right)}{af(2m + 3)(c - d)\sqrt{1 - \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m * (A + B*\text{Sin}[e + f*x]) * (c + d*\text{Sin}[e + f*x])^{(-1 - m)}, x]$

[Out] $-((2^{(1/2 + m)} * a * (A - B) * \text{Cos}[e + f*x] * \text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, ((c - d) * (1 - \text{Sin}[e + f*x])) / (2 * (c + d * \text{Sin}[e + f*x]))]) * (a + a * \text{Sin}[e + f*x])^{(-1 + m)} * (((c + d) * (1 + \text{Sin}[e + f*x])) / (c + d * \text{Sin}[e + f*x]))^{(1/2 - m)} / ((c + d) * f * (c + d * \text{Sin}[e + f*x])^m) + (\text{Sqrt}[2] * B * \text{AppellF1}[3/2 + m, 1/2, 1 + m, 5/2 + m, (1 + \text{Sin}[e + f*x]) / 2, -((d * (1 + \text{Sin}[e + f*x])) / (c - d))] * \text{Cos}[e + f*x] * (a + a * \text{Sin}[e + f*x])^{(1 + m)} * ((c + d * \text{Sin}[e + f*x]) / (c - d))^m) / (a * (c - d) * f * (3 + 2 * m) * \text{Sqrt}[1 - \text{Sin}[e + f*x]] * (c + d * \text{Sin}[e + f*x])^m)$

Rule 132

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)} * ((c_.) + (d_.)*(x_.))^{(n_.)} * ((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] :> \text{Simp}[(a + b*x)^{(m + 1)} * (c + d*x)^n * (e + f*x)^{(p + 1)} * \text{Hypergeometric2F1}[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x)) / ((b*c - a*d) * (e + f*x)))] / (((b*e - a*f)*(m + 1)) * (((b*e - a*f)*(c + d*x)) / ((b*c - a*d) * (e + f*x)))^n), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \&\& \text{EqQ}[m + n + p + 2, 0] \&\& !\text{IntegerQ}[n]$

Rule 138

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)} * ((c_.) + (d_.)*(x_.))^{(n_.)} * ((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] :> \text{Simp}[(a + b*x)^{(m + 1)} * \text{AppellF1}[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f)))] / (b*(m + 1) * (b / (b*c - a*d))^n * (b / (b*e - a*f))^p), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[b / (b*c - a*d), 0] \&\& \text{GtQ}[b / (b*e - a*f), 0] \&\& !(\text{GtQ}[d / (d*a - c*b), 0] \&\& \text{GtQ}[d / (d*e - c*f), 0]) \&\& \text{SimplerQ}[c + d*x, a + b*x] \&\& !(\text{GtQ}[f / (f*a - e*b), 0] \&\& \text{GtQ}[f / (f*c - e*d), 0]) \&\& \text{SimplerQ}[e + f*x, a + b*x]$

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 140

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/
(b*c - a*d), 0] && !SimplerQ[c + d*x, a + b*x] && !SimplerQ[e + f*x, a +
b*x]
```

Rule 2788

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (
f_)*(x_)])^(n_), x_Symbol] := Dist[(a^2*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a + b*x)^(m - 1/2)*(c + d*x
)^n]/Sqrt[a - b*x], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&
!IntegerQ[m]
```

Rule 2987

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Di
st[(A*b - a*B)/b, Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n, x], x]
+ Dist[B/b, Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n, x], x
] /; FreeQ[{a, b, c, d, e, f, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a
^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A*b + a*B, 0]
```

Rubi steps

$$\int (a + a \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^{-1-m} dx = (A - B) \int (a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-1-m} dx$$

$$= \frac{(a^2(A - B) \cos(e + fx)) \operatorname{Subst}\left(\int \frac{(a+ax)^{-\frac{1}{2}}}{\sqrt{a - a \sin(e + fx)}} dx\right)}{f \sqrt{a - a \sin(e + fx)} \sqrt{a + a \sin(e + fx)}}$$

$$= -\frac{2^{\frac{1}{2}+m} a(A - B) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, -\frac{d \sin(e + fx)}{a + a \sin(e + fx)}\right)}{f}$$

$$= -\frac{2^{\frac{1}{2}+m} a(A - B) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, -\frac{d \sin(e + fx)}{a + a \sin(e + fx)}\right)}{f}$$

$$= -\frac{2^{\frac{1}{2}+m} a(A - B) \cos(e + fx) {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, -\frac{d \sin(e + fx)}{a + a \sin(e + fx)}\right)}{f}$$

Mathematica [B] time = 6.94, size = 573, normalized size = 2.07

$$2 \cot\left(\frac{1}{4}(2e + 2fx + \pi)\right) \sin^2\left(\frac{1}{4}(2e + 2fx + \pi)\right)^{\frac{1}{2}-m} \cos^2\left(\frac{1}{4}(2e + 2fx - \pi)\right)^{m-\frac{1}{2}} (a(\sin(e + fx) + 1))^m (c + d \sin(e + fx))^{-1-m}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^(-1 - m),x]

[Out] (2*(Cos[(2*e - Pi + 2*f*x)/4]^2)^(-1/2 + m)*Cot[(2*e + Pi + 2*f*x)/4]*(a*(1 + Sin[e + f*x]))^m*((-3*B*(c + d)^2*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)])/(d*(3*(c + d)*AppellF1[1/2, 1/2 - m, m, 3/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] - (-4*d*m*AppellF1[3/2, 1/2 - m, 1 + m, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)] + (c + d)*(-1 + 2*m)*AppellF1[3/2, 3/2 - m, m, 5/2, Cos[(2*e + Pi + 2*f*x)/4]^2, (2*d*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d)]*Cos[(2*e + Pi + 2*f*x)/4]^2) - A*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]])*((c + d)*Cos[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]))^(1/2 - m) + (B*c*Hypergeometric2F1[1/2, 1/2 - m, 3/2, ((c - d)*Sin[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]])*((c + d)*Cos[(2*e - Pi + 2*f*x)/4]^2)/(c + d*Sin[e + f*x]))^(1/2 - m))/d*(Sin[(2*e + Pi + 2*f*x)/4]^2)^(1/2 - m))/((c + d)*f*(c + d*Sin[e + f*x])^m)

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(B \sin(fx + e) + A\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e) + c\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(1 - m - 1), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(1 - m - 1), x)

maple [F] time = 3.47, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^{-1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x)

[Out] `int((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin (f x + e) + A)(a \sin (f x + e) + a)^m (d \sin (f x + e) + c)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x, algorithm="maxima")`

[Out] `integrate((B*sin(f*x + e) + A)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin (e + f x)) (a + a \sin (e + f x))^m}{(c + d \sin (e + f x))^{m+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m + 1),x)`

[Out] `int(((A + B*sin(e + f*x))*(a + a*sin(e + f*x))^m)/(c + d*sin(e + f*x))^(m + 1), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1-m),x)`

[Out] Timed out

$$3.348 \quad \int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=132

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)}{f(2m+1)}$$

[Out] 2*AppellF1(1/2+m,-n,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*(c+d*sin(f*x+e))^n*2^(1/2)*(1-sin(f*x+e))^(1/2)/f/(1+2*m)/(((c+d*sin(f*x+e))/(c-d))^n)

Rubi [A] time = 0.20, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^n\left(\frac{c+d\sin(e+fx)}{c-d}\right)^{-n}F_1\left(m+\frac{1}{2};-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)}{f(2m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^n,x]

[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, -n, 3/2 + m, (1 + Sin[e + f*x])/2, -((d*(1 + Sin[e + f*x]))/(c - d))]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*(c + d*Sin[e + f*x])^n)/(f*(1 + 2*m)*((c + d*Sin[e + f*x])/(c - d))^n)

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplifierQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplifierQ[e + f*x, a + b*x]

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplifierQ[c + d*x, a + b*x] && !SimplifierQ[e + f*x, a + b*x]

Rule 3008

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])^(p_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^n dx = \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)})}{(\sqrt{2} \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)})}$$

$$= \frac{(\sqrt{2} \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)})}{2\sqrt{2} F_1\left(\frac{1}{2} + m; -\frac{1}{2}, -n; \frac{3}{2} + m; \frac{1}{2}(1 + \sin(e + fx))\right)}$$

Mathematica [F] time = 10.11, size = 0, normalized size = 0.00

$$\int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
n, x]
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a \sin\left(fx + e\right) - a\right)\left(a \sin\left(fx + e\right) + a\right)^m\left(d \sin\left(fx + e\right) + c\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorit
hm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a \sin\left(fx + e\right) - a\right)\left(a \sin\left(fx + e\right) + a\right)^m\left(d \sin\left(fx + e\right) + c\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

maple [F] time = 1.43, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))(a + a \sin(fx + e))^m (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)

[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int (a \sin(fx + e) - a)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + a \sin(e + fx))^m (a - a \sin(e + fx)) (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n,x)

[Out] int((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x))*(c + d*sin(e + f*x))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^n,x)

[Out] Timed out

$$3.349 \quad \int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-1-m} dx$$

Optimal. Leaf size=139

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{1}{2};-\frac{1}{2},m+\frac{1}{2}\right)}{f(2m+1)(c-d)}$$

[Out] 2*AppellF1(1/2+m,1+m,-1/2,3/2+m,-d*(1+sin(f*x+e))/(c-d),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)*((c+d*sin(f*x+e))/(c-d))^m*2^(1/2)*(1-sin(f*x+e))^(1/2)/(c-d)/f/(1+2*m)/((c+d*sin(f*x+e))^m)

Rubi [A] time = 0.23, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3008, 140, 139, 138}

$$\frac{2\sqrt{2}\sqrt{1-\sin(e+fx)}\sec(e+fx)(a\sin(e+fx)+a)^{m+1}(c+d\sin(e+fx))^{-m}\left(\frac{c+d\sin(e+fx)}{c-d}\right)^m F_1\left(m+\frac{1}{2};-\frac{1}{2},m+\frac{1}{2}\right)}{f(2m+1)(c-d)}$$

Antiderivative was successfully verified.

[In] Int[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(-1 - m), x]

[Out] (2*Sqrt[2]*AppellF1[1/2 + m, -1/2, 1 + m, 3/2 + m, (1 + Sin[e + f*x])/2, -(d*(1 + Sin[e + f*x]))/(c - d)]*Sec[e + f*x]*Sqrt[1 - Sin[e + f*x]]*(a + a*Sin[e + f*x])^(1 + m)*((c + d*Sin[e + f*x])/(c - d))^m)/((c - d)*f*(1 + 2*m)*(c + d*Sin[e + f*x])^m)

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplifierQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 140

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && !GtQ[b/(b*c - a*d), 0] && !SimplifierQ[c + d*x, a + b*x] && !SimplifierQ[e + f*x, a + b*x]

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-1-m} dx = \frac{(\sec(e + fx)\sqrt{a - a \sin(e + fx)}\sqrt{a + a \sin(e + fx)})}{(\sqrt{2} \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)})}$$

$$= \frac{(\sqrt{2} a \sec(e + fx)(a - a \sin(e + fx))\sqrt{a + a \sin(e + fx)})}{2\sqrt{2} F_1\left(\frac{1}{2} + m; -\frac{1}{2}, 1 + m; \frac{3}{2} + m; \frac{1}{2}(1 + \dots)\right)}$$

Mathematica [F] time = 4.87, size = 0, normalized size = 0.00

$$\int (a - a \sin(e + fx))(a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-1-m} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

```
[Out] Integrate[(a - a*Sin[e + f*x])*(a + a*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^
(-1 - m), x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a \sin(fx + e) - a\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e) + c\right)^{-m-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m), x, al
gorithm="fricas")
```

```
[Out] integral(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^
(-m - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\left(a \sin(fx + e) - a\right)\left(a \sin(fx + e) + a\right)^m \left(d \sin(fx + e) + c\right)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m),x, algorithm="giac")

[Out] integrate(-(a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(1-m - 1), x)

maple [F] time = 2.75, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))(a + a \sin(fx + e))^m (c + d \sin(fx + e))^{-1-m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m),x)

[Out] int((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int (a \sin(fx + e) - a)(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^{-m-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m),x, algorithm="maxima")

[Out] -integrate((a*sin(f*x + e) - a)*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(1-m - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a \sin(e + fx))^m (a - a \sin(e + fx))}{(c + d \sin(e + fx))^{m+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m + 1),x)

[Out] int(((a + a*sin(e + f*x))^m*(a - a*sin(e + f*x)))/(c + d*sin(e + f*x))^(m + 1), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))*(a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(1-m),x)

[Out] Timed out

$$3.350 \quad \int (a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-2-m} (d - (c - d)m + (c + (c - d)m) \sin(e + fx)) dx$$

Optimal. Leaf size=39

$$\frac{\cos(e + fx)(a \sin(e + fx) + a)^m (c + d \sin(e + fx))^{-m-1}}{f}$$

[Out] $-\cos(f*x+e)*(a+a*\sin(f*x+e))^m*(c+d*\sin(f*x+e))^{(-1-m)}/f$

Rubi [A] time = 0.17, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2974}

$$\frac{\cos(e + fx)(a \sin(e + fx) + a)^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-2 - m)*(d - (c - d)*m + (c + (c - d)*m)*\text{Sin}[e + f*x])}, x]$

[Out] $-\text{((Cos}[e + f*x]*\text{(a + a*Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-1 - m)}/f)$

Rule 2974

$\text{Int}[\text{((a_)} + (\text{b_})*\text{sin}[\text{(e_)} + (\text{f_})*(\text{x_})])^{\text{(m_)}*((\text{A_}) + (\text{B_})*\text{sin}[\text{(e_)} + (\text{f_})*(\text{x_})])*(\text{(c_)} + (\text{d_})*\text{sin}[\text{(e_)} + (\text{f_})*(\text{x_})])^{\text{(n_)}}, \text{x_Symbol}] :> \text{Simp}[\text{((B*c - A*d)*Cos}[e + f*x]*\text{(a + b*Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(n + 1)})/(\text{f*(n + 1)*(c^2 - d^2)}), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[m + n + 2, 0] \&\& \text{EqQ}[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]$

Rubi steps

$$\int (a + a \sin(e + fx))^m (c + d \sin(e + fx))^{-2-m} (d - (c - d)m + (c + (c - d)m) \sin(e + fx)) dx = -\frac{\cos(e + fx)(a \sin(e + fx) + a)^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Mathematica [A] time = 0.68, size = 39, normalized size = 1.00

$$\frac{\cos(e + fx)(a(\sin(e + fx) + 1))^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a + a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-2 - m)*(d - (c - d)*m + (c + (c - d)*m)*\text{Sin}[e + f*x])}, x]$

[Out] $-\text{((Cos}[e + f*x]*\text{(a*(1 + Sin}[e + f*x]))^m*(c + d*\text{Sin}[e + f*x])^{(-1 - m)}/f)$

fricas [A] time = 0.54, size = 56, normalized size = 1.44

$$\frac{(d \cos(fx + e) \sin(fx + e) + c \cos(fx + e))(a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x, algorithm="fricas")

[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 2)/f

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [F] time = 6.47, size = 0, normalized size = 0.00

$$\int (a + a \sin(fx + e))^m (c + d \sin(fx + e))^{-2-m} (d - (c - d)m + (c + (c - d)m) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x)

[Out] int((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 14.77, size = 98, normalized size = 2.51

$$\frac{(a(\sin(e + fx) + 1))^m \left(d \sin(2e + 2fx) - 2c \left(2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right) \right)}{f(c + d \sin(e + fx))^m \left(d^2 \left(2 \sin(e + fx)^2 - 1 \right) + 2c^2 + d^2 + 4cd \sin(e + fx) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + a*sin(e + f*x))^m*(d - m*(c - d) + sin(e + f*x)*(c + m*(c - d))))/(c + d*sin(e + f*x))^(m + 2),x)

[Out] -((a*(sin(e + f*x) + 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d^2 + 4*c*d*sin(e + f*x)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d-(c-d)*m+(c+(c-d)*m)*sin(f*x+e)),x)

[Out] Timed out

$$3.351 \quad \int (a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-2-m} (d + (c + d)m + (c + (c + d)m) \sin(e + fx)) dx$$

Optimal. Leaf size=40

$$-\frac{\cos(e + fx)(a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-m-1}}{f}$$

[Out] $-\cos(f*x+e)*(a-a*\sin(f*x+e))^m*(c+d*\sin(f*x+e))^{(-1-m)}/f$

Rubi [A] time = 0.17, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$, Rules used = {2974}

$$-\frac{\cos(e + fx)(a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a - a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-2 - m)}*(d + (c + d)*m + (c + (c + d)*m)*\text{Sin}[e + f*x]), x]$

[Out] $-\left(\text{Cos}[e + f*x]*(a - a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-1 - m)}\right)/f$

Rule 2974

$\text{Int}[(a + b*\sin[e + f*x])^m*(c + d*\sin[e + f*x])^{(-2 - m)}*(d + (c + d)*m + (c + (c + d)*m)*\sin[e + f*x]), x]$:> $\text{Simp}[(B*c - A*d)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(n + 1)}]/(f*(n + 1)*(c^2 - d^2)), x]$ /; $\text{FreeQ}\{a, b, c, d, e, f, A, B, m, n\}, x$ && $\text{NeQ}[b*c - a*d, 0]$ && $\text{EqQ}[a^2 - b^2, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$ && $\text{EqQ}[m + n + 2, 0]$ && $\text{EqQ}[A*(a*d*m + b*c*(n + 1)) - B*(a*c*m + b*d*(n + 1)), 0]$

Rubi steps

$$\int (a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-2-m} (d + (c + d)m + (c + (c + d)m) \sin(e + fx)) dx = -\frac{\cos(e + fx)(a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Mathematica [A] time = 0.74, size = 40, normalized size = 1.00

$$-\frac{\cos(e + fx)(a - a \sin(e + fx))^m (c + d \sin(e + fx))^{-m-1}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(a - a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-2 - m)}*(d + (c + d)*m + (c + (c + d)*m)*\text{Sin}[e + f*x]), x]$

[Out] $-\left(\text{Cos}[e + f*x]*(a - a*\text{Sin}[e + f*x])^m*(c + d*\text{Sin}[e + f*x])^{(-1 - m)}\right)/f$

fricas [A] time = 0.55, size = 57, normalized size = 1.42

$$-\frac{(d \cos(fx + e) \sin(fx + e) + c \cos(fx + e))(-a \sin(fx + e) + a)^m (d \sin(fx + e) + c)^{-m-2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x, algorithm="fricas")

[Out] -(d*cos(f*x + e)*sin(f*x + e) + c*cos(f*x + e))*(-a*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^(-m - 2)/f

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x, algorithm="giac")

[Out] Timed out

maple [F] time = 6.56, size = 0, normalized size = 0.00

$$\int (a - a \sin(fx + e))^m (c + d \sin(fx + e))^{-2-m} (d + (c + d)m + (c + (c + d)m) \sin(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x)

[Out] int((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x, algorithm="maxima")

[Out] Timed out

mupad [B] time = 14.62, size = 99, normalized size = 2.48

$$\frac{(-a (\sin(e + fx) - 1))^m \left(d \sin(2e + 2fx) - 2c \left(2 \sin\left(\frac{e}{2} + \frac{fx}{2}\right)^2 - 1 \right) \right)}{f (c + d \sin(e + fx))^m \left(d^2 \left(2 \sin(e + fx)^2 - 1 \right) + 2c^2 + d^2 + 4cd \sin(e + fx) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a - a*sin(e + f*x))^m*(d + sin(e + f*x)*(c + m*(c + d)) + m*(c + d)))/(c + d*sin(e + f*x))^(m + 2),x)

[Out] -((-a*(sin(e + f*x) - 1))^m*(d*sin(2*e + 2*f*x) - 2*c*(2*sin(e/2 + (f*x)/2)^2 - 1)))/(f*(c + d*sin(e + f*x))^m*(d^2*(2*sin(e + f*x)^2 - 1) + 2*c^2 + d^2 + 4*c*d*sin(e + f*x)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-a*sin(f*x+e))^m*(c+d*sin(f*x+e))^(2-m)*(d+(c+d)*m+(c+(c+d)*m)*sin(f*x+e)),x)

[Out] Timed out

$$3.352 \quad \int \frac{(a+b \sin(e+fx))^2(A+B \sin(e+fx))}{(c+d \sin(e+fx))^2} dx$$

Optimal. Leaf size=199

$$\frac{(bc-ad)^2(Bc-Ad) \cos(e+fx)}{d^2 f (c^2-d^2) (c+d \sin(e+fx))} \frac{2(bc-ad) (ad^2(Ac-Bd) - b(-Ac^2d + 2Ad^3 + 2Bc^3 - 3Bcd^2)) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e+fx) \right) + d}{\sqrt{c^2-d^2}} \right)}{d^3 f (c^2-d^2)^{3/2}}$$

[Out] $-b*(-A*b*d-2*B*a*d+2*B*b*c)*x/d^3-2*(-a*d+b*c)*(a*d^2*(A*c-B*d)-b*(-A*c^2*d+2*A*d^3+2*B*c^3-3*B*c*d^2))*\arctan((d+c*\tan(1/2*f*x+1/2*e))/(c^2-d^2)^{(1/2)})/d^3/(c^2-d^2)^{(3/2)}/f-b^2*B*\cos(f*x+e)/d^2/f-(-a*d+b*c)^2*(-A*d+B*c)*\cos(f*x+e)/d^2/(c^2-d^2)/f/(c+d*\sin(f*x+e))$

Rubi [A] time = 0.58, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {2988, 3023, 2735, 2660, 618, 204}

$$\frac{2(bc-ad) (ad^2(Ac-Bd) - b(-Ac^2d + 2Ad^3 + 2Bc^3 - 3Bcd^2)) \tan^{-1} \left(\frac{c \tan \left(\frac{1}{2}(e+fx) \right) + d}{\sqrt{c^2-d^2}} \right)}{d^3 f (c^2-d^2)^{3/2}} \frac{(bc-ad)^2(Bc-Ad)}{d^2 f (c^2-d^2) (c+d \sin(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])^2,x]

[Out] $-((b*(2*b*B*c - A*b*d - 2*a*B*d)*x)/d^3 - (2*(b*c - a*d)*(a*d^2*(A*c - B*d) - b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*\text{ArcTan}[(d + c*\text{Tan}[(e + f*x)/2])/ \text{Sqrt}[c^2 - d^2]])/(d^3*(c^2 - d^2)^{(3/2)*f} - (b^2*B*\text{Cos}[e + f*x])/(d^2*f) - ((b*c - a*d)^2*(B*c - A*d)*\text{Cos}[e + f*x])/(d^2*(c^2 - d^2)*f*(c + d*\text{Sin}[e + f*x])))$

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 2988

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^2*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] := Simp[
((B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*(c + d*Sin[e + f*x])^(n + 1))/(f*d^
2*(n + 1)*(c^2 - d^2)), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1)))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a + b \sin(e + fx))^2 (A + B \sin(e + fx))}{(c + d \sin(e + fx))^2} dx = -\frac{(bc - ad)^2 (Bc - Ad) \cos(e + fx)}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))} + \frac{\int \frac{-d(B(bc-ad)^2 - Ad(a^2c + b^2c - 2abd))}{(c + d \sin(e + fx))^2} dx}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))}$$

$$= -\frac{b^2 B \cos(e + fx)}{d^2 f} - \frac{(bc - ad)^2 (Bc - Ad) \cos(e + fx)}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))} + \frac{\int \frac{-d^2 (B(bc-ad)^2 - Ad(a^2c + b^2c - 2abd))}{(c + d \sin(e + fx))^2} dx}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))}$$

$$= -\frac{b(2bBc - Abd - 2aBd)x}{d^3} - \frac{b^2 B \cos(e + fx)}{d^2 f} - \frac{(bc - ad)^2 (Bc - Ad) \cos(e + fx)}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))} + \frac{\int \frac{-d^2 (B(bc-ad)^2 - Ad(a^2c + b^2c - 2abd))}{(c + d \sin(e + fx))^2} dx}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))}$$

$$= -\frac{b(2bBc - Abd - 2aBd)x}{d^3} - \frac{b^2 B \cos(e + fx)}{d^2 f} - \frac{(bc - ad)^2 (Bc - Ad) \cos(e + fx)}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))} + \frac{\int \frac{-d^2 (B(bc-ad)^2 - Ad(a^2c + b^2c - 2abd))}{(c + d \sin(e + fx))^2} dx}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))}$$

$$= -\frac{b(2bBc - Abd - 2aBd)x}{d^3} - \frac{b^2 B \cos(e + fx)}{d^2 f} - \frac{(bc - ad)^2 (Bc - Ad) \cos(e + fx)}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))} + \frac{\int \frac{-d^2 (B(bc-ad)^2 - Ad(a^2c + b^2c - 2abd))}{(c + d \sin(e + fx))^2} dx}{d^2 (c^2 - d^2) f (c + d \sin(e + fx))}$$

$$= -\frac{b(2bBc - Abd - 2aBd)x}{d^3} - \frac{2(bc - ad)(ad^2(Ac - Bd) - b(2Bc^3 - Ad^2))}{d^3 f}$$

Mathematica [A] time = 1.60, size = 188, normalized size = 0.94

$$\frac{2(bc-ad)(ad^2(Bd-Ac)+b(-Ac^2d+2Ad^3+2Bc^3-3Bcd^2)) \tan^{-1}\left(\frac{c \tan\left(\frac{1}{2}(e+fx)\right)+d}{\sqrt{c^2-d^2}}\right)}{(c^2-d^2)^{3/2}} + \frac{b(e+fx)(2aBd + Abd - 2bBc) + \frac{d(bc-ad)^2(Ad-Bc) \cos(e+fx)}{(c-d)(c+d)(c+d \sin(e+fx))}}{d^3 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sin[e + f*x])^2*(A + B*Sin[e + f*x]))/(c + d*Sin[e + f*x])
^2,x]
```

```
[Out] (b*(-2*b*B*c + A*b*d + 2*a*B*d)*(e + f*x) + (2*(b*c - a*d)*(a*d^2*(-(A*c) +
B*d) + b*(2*B*c^3 - A*c^2*d - 3*B*c*d^2 + 2*A*d^3))*ArcTan[(d + c*Tan[(e +
f*x)/2])/Sqrt[c^2 - d^2]])/(c^2 - d^2)^(3/2) - b^2*B*d*Cos[e + f*x] + (d*(
b*c - a*d)^2*(-(B*c) + A*d)*Cos[e + f*x])/((c - d)*(c + d)*(c + d*Sin[e + f
*x]))/(d^3*f)
```

fricas [B] time = 0.61, size = 1308, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="fricas")
```

```
[Out] [-1/2*(2*(2*B*b^2*c^6 - 4*B*b^2*c^4*d^2 + 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^
2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 - (2*B*a*b + A*b^2)*c*d^5)*f*x + (2*
B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b + A*b^2)*c^4*d + (A*a^2 + 4*B*a*b +
2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d
^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2 + 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2
+ 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(f*x
+ e)^2 - 2*c*d*sin(f*x + e) - c^2 - d^2 + 2*(c*cos(f*x + e)*sin(f*x + e) +
d*cos(f*x + e))*sqrt(-c^2 + d^2))/(d^2*cos(f*x + e)^2 - 2*c*d*sin(f*x + e)
- c^2 - d^2)) + 2*(2*B*b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 +
(B*a^2 + 2*A*a*b - 3*B*b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (
B*a^2 + 2*A*a*b - B*b^2)*c*d^5)*cos(f*x + e) + 2*((2*B*b^2*c^5*d - 4*B*b^2*
c^3*d^3 + 2*B*b^2*c*d^5 - (2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c
^2*d^4 - (2*B*a*b + A*b^2)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*
b^2*d^6)*cos(f*x + e))*sin(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x
+ e) + (c^5*d^3 - 2*c^3*d^5 + c*d^7)*f), -((2*B*b^2*c^6 - 4*B*b^2*c^4*d^2
+ 2*B*b^2*c^2*d^4 - (2*B*a*b + A*b^2)*c^5*d + 2*(2*B*a*b + A*b^2)*c^3*d^3 -
(2*B*a*b + A*b^2)*c*d^5)*f*x + (2*B*b^2*c^5 - 3*B*b^2*c^3*d^2 - (2*B*a*b +
A*b^2)*c^4*d + (A*a^2 + 4*B*a*b + 2*A*b^2)*c^2*d^3 - (B*a^2 + 2*A*a*b)*c*d
^4 + (2*B*b^2*c^4*d - 3*B*b^2*c^2*d^3 - (2*B*a*b + A*b^2)*c^3*d^2 + (A*a^2
+ 4*B*a*b + 2*A*b^2)*c*d^4 - (B*a^2 + 2*A*a*b)*d^5)*sin(f*x + e))*sqrt(c^2
- d^2)*arctan(-(c*sin(f*x + e) + d)/(sqrt(c^2 - d^2)*cos(f*x + e))) + (2*B*
b^2*c^5*d + A*a^2*d^6 - (2*B*a*b + A*b^2)*c^4*d^2 + (B*a^2 + 2*A*a*b - 3*B*
b^2)*c^3*d^3 - (A*a^2 - 2*B*a*b - A*b^2)*c^2*d^4 - (B*a^2 + 2*A*a*b - B*b^2
)*c*d^5)*cos(f*x + e) + ((2*B*b^2*c^5*d - 4*B*b^2*c^3*d^3 + 2*B*b^2*c*d^5 -
(2*B*a*b + A*b^2)*c^4*d^2 + 2*(2*B*a*b + A*b^2)*c^2*d^4 - (2*B*a*b + A*b^2
)*d^6)*f*x + (B*b^2*c^4*d^2 - 2*B*b^2*c^2*d^4 + B*b^2*d^6)*cos(f*x + e))*si
n(f*x + e))/((c^4*d^4 - 2*c^2*d^6 + d^8)*f*sin(f*x + e) + (c^5*d^3 - 2*c^3*
d^5 + c*d^7)*f)]
```

giac [B] time = 6.89, size = 776, normalized size = 3.90

$$\frac{2(2Bb^2c^4 - 2Babc^3d - Ab^2c^3d - 3Bb^2c^2d^2 + Aa^2cd^3 + 4Babcd^3 + 2Ab^2cd^3 - Ba^2d^4 - 2Aabd^4) \left(\pi \left[\frac{fx+e}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(c) + \arctan \left(\frac{c \tan \left(\frac{1}{2} \frac{fx+e}{2\pi} + \frac{1}{2} \right) + d}{\sqrt{c^2 - d^2}} \right) \right)}{(c^2d^3 - d^5) \sqrt{c^2 - d^2}} - 2 \left(Bb^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorit
hm="giac")
```

```
[Out] (2*(2*B*b^2*c^4 - 2*B*a*b*c^3*d - A*b^2*c^3*d - 3*B*b^2*c^2*d^2 + A*a^2*c*d
^3 + 4*B*a*b*c*d^3 + 2*A*b^2*c*d^3 - B*a^2*d^4 - 2*A*a*b*d^4)*(pi*floor(1/2
*(f*x + e)/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*f*x + 1/2*e) + d)/sqrt(c^2
- d^2)))/((c^2*d^3 - d^5)*sqrt(c^2 - d^2)) - 2*(B*b^2*c^3*d*tan(1/2*f*x + 1
```

$$\begin{aligned} & /2*e)^3 - 2*B*a*b*c^2*d^2*\tan(1/2*f*x + 1/2*e)^3 - A*b^2*c^2*d^2*\tan(1/2*f*x \\ & + 1/2*e)^3 + B*a^2*c*d^3*\tan(1/2*f*x + 1/2*e)^3 + 2*A*a*b*c*d^3*\tan(1/2*f*x \\ & *x + 1/2*e)^3 - A*a^2*d^4*\tan(1/2*f*x + 1/2*e)^3 + 2*B*b^2*c^4*\tan(1/2*f*x \\ & + 1/2*e)^2 - 2*B*a*b*c^3*d*\tan(1/2*f*x + 1/2*e)^2 - A*b^2*c^3*d*\tan(1/2*f*x \\ & + 1/2*e)^2 + B*a^2*c^2*d^2*\tan(1/2*f*x + 1/2*e)^2 + 2*A*a*b*c^2*d^2*\tan(1/ \\ & 2*f*x + 1/2*e)^2 - B*b^2*c^2*d^2*\tan(1/2*f*x + 1/2*e)^2 - A*a^2*c*d^3*\tan(1 \\ & /2*f*x + 1/2*e)^2 + 3*B*b^2*c^3*d*\tan(1/2*f*x + 1/2*e) - 2*B*a*b*c^2*d^2*\tan \\ & (1/2*f*x + 1/2*e) - A*b^2*c^2*d^2*\tan(1/2*f*x + 1/2*e) + B*a^2*c*d^3*\tan(1 \\ & /2*f*x + 1/2*e) + 2*A*a*b*c*d^3*\tan(1/2*f*x + 1/2*e) - 2*B*b^2*c*d^3*\tan(1/ \\ & 2*f*x + 1/2*e) - A*a^2*d^4*\tan(1/2*f*x + 1/2*e) + 2*B*b^2*c^4 - 2*B*a*b*c^3 \\ & *d - A*b^2*c^3*d + B*a^2*c^2*d^2 + 2*A*a*b*c^2*d^2 - B*b^2*c^2*d^2 - A*a^2*c \\ & *d^3)/((c^3*d^2 - c*d^4)*(c*\tan(1/2*f*x + 1/2*e)^4 + 2*d*\tan(1/2*f*x + 1/2 \\ & *e)^3 + 2*c*\tan(1/2*f*x + 1/2*e)^2 + 2*d*\tan(1/2*f*x + 1/2*e) + c)) - (2*B* \\ & b^2*c - 2*B*a*b*d - A*b^2*d)*(f*x + e)/d^3)/f \end{aligned}$$

maple [B] time = 0.46, size = 1246, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x)

[Out]
$$\begin{aligned} & -4/f*d/(c^2-d^2)^{(3/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1 \\ & /2)})*A*a*b-2/f/d^2/(c^2-d^2)^{(3/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/ \\ & (c^2-d^2)^{(1/2)})*A*b^2*c^3+4/f/d^3/(c^2-d^2)^{(3/2)}*\arctan(1/2*(2*c*\tan(1/2* \\ & f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*b^2*c^4-6/f/d/(c^2-d^2)^{(3/2)}*\arctan(1/2 \\ & *(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*b^2*c^2-2/f*d/(\tan(1/2*f*x \\ & +1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*\tan(1/2*f*x+1/2*e)*B*a^2+2/ \\ & f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*A*b^2*c^2-2 \\ & /f/d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c^3*B*b^ \\ & 2+2/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*\tan(1/2 \\ & *f*x+1/2*e)*A*b^2-4/f/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^ \\ & 2-d^2)*A*a*b*c+8/f/(c^2-d^2)^{(3/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/ \\ & (c^2-d^2)^{(1/2)})*B*a*b*c-2/f/d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e) \\ & *d+c)/(c^2-d^2)*c^2*\tan(1/2*f*x+1/2*e)*B*b^2+4/f/d/(\tan(1/2*f*x+1/2*e)^2*c+ \\ & 2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a*b*c^2-4/f/d^2/(c^2-d^2)^{(3/2)}*\arctan \\ & (1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*a*b*c^3+4/f/(\tan(1/2* \\ & f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*c*\tan(1/2*f*x+1/2*e)*B*a \\ & *b+2/f*d^2/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)/c*\tan \\ & (1/2*f*x+1/2*e)*a^2*A-4/f*d/(\tan(1/2*f*x+1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d \\ & +c)/(c^2-d^2)*\tan(1/2*f*x+1/2*e)*A*a*b+4/f/(c^2-d^2)^{(3/2)}*\arctan(1/2*(2*c* \\ & \tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*b^2*c-2/f/(\tan(1/2*f*x+1/2*e)^2* \\ & c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*B*a^2*c+2/f/(c^2-d^2)^{(3/2)}*\arctan(1/ \\ & 2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*A*a^2*c+2/f*d/(\tan(1/2*f*x+ \\ & 1/2*e)^2*c+2*\tan(1/2*f*x+1/2*e)*d+c)/(c^2-d^2)*a^2*A-2/f/d^2*b^2*B/(1+\tan(1 \\ & /2*f*x+1/2*e)^2)+2/f/d^2*b^2*A*\arctan(\tan(1/2*f*x+1/2*e))-2/f*d/(c^2-d^2)^{(\\ & 3/2)}*\arctan(1/2*(2*c*\tan(1/2*f*x+1/2*e)+2*d)/(c^2-d^2)^{(1/2)})*B*a^2+4/f/d^2 \\ & *b*B*\arctan(\tan(1/2*f*x+1/2*e))*a-4/f/d^3*b^2*B*\arctan(\tan(1/2*f*x+1/2*e))* \\ & c \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` for more details)Is 4*d^2-4*c^2 positive or negative?

mupad [B] time = 27.62, size = 16312, normalized size = 81.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^2)/(c + d*sin(e + f*x))^2,x)

[Out]
$$\begin{aligned} & ((2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*\tan(e/2 + (f*x)/2)^2*(A*a^2*d^3 - 2*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(d^2*(c^2 - d^2)) + (2*\tan(e/2 + (f*x)/2)^3*(A*a^2*d^3 - B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(c*d*(c^2 - d^2)) + (2*\tan(e/2 + (f*x)/2)*(A*a^2*d^3 - 3*B*b^2*c^3 + A*b^2*c^2*d - B*a^2*c*d^2 + 2*B*b^2*c*d^2 - 2*A*a*b*c*d^2 + 2*B*a*b*c^2*d))/(c*d*(c^2 - d^2)))/(f*(c + 2*d*\tan(e/2 + (f*x)/2) + 2*c*\tan(e/2 + (f*x)/2)^2 + c*\tan(e/2 + (f*x)/2)^4 + 2*d*\tan(e/2 + (f*x)/2)^3)) + (\operatorname{atan}(((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4)))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*(((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*\tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*\tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6)))/d^3 - (32*\tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))*1i)/d^3 + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*\tan$$

$$\begin{aligned}
& n(e/2 + (f*x)/2)*(3*c*d^{14} - 8*c^3*d^{12} + 7*c^5*d^{10} - 2*c^7*d^8)/(d^{10} - \\
& 2*c^2*d^8 + c^4*d^6)*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 - (32*\tan(e/ \\
& 2 + (f*x)/2)*(2*A*a^2*c^2*d^{11} - 2*B*a^2*c*d^{12} - 2*A*a^2*c^4*d^9 + 4*A*b^2 \\
& *c^2*d^{11} - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^{10} - 6*B*b^2* \\
& c^3*d^{10} + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^{12} + 4*A*a*b*c^ \\
& 3*d^{10} + 8*B*a*b*c^2*d^{11} - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^{10} - 2* \\
& c^2*d^8 + c^4*d^6))/d^3 - (32*\tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2* \\
& b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 2 \\
& 9*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^ \\
& 10 + B^2*a^4*c*d^{10} + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^ \\
& 2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B* \\
& a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - \\
& 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2* \\
& b^2*c*d^{10} - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4* \\
& d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^{10} - \\
& 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^{10} + 4*A*B*a^3* \\
& b*c*d^{10} + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^ \\
& 4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4* \\
& A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^{10} - 2*c^2*d^8 + c^4*d^6)) \\
& *1i)/d^3)/((64*(A^3*b^6*c^5*d^3 - 2*A^3*b^6*c^3*d^5 - 4*B^3*b^6*c^8 + 6*B^3 \\
& *b^6*c^6*d^2 - 3*A^3*a^2*b^4*c^3*d^5 + A^3*a^2*b^4*c^5*d^3 + 4*A^3*a^3*b^3* \\
& c^2*d^6 - A^3*a^4*b^2*c^3*d^5 + 44*B^3*a^2*b^4*c^4*d^4 - 24*B^3*a^2*b^4*c^6 \\
& *d^2 - 36*B^3*a^3*b^3*c^3*d^5 + 16*B^3*a^3*b^3*c^5*d^3 + 14*B^3*a^4*b^2*c^2 \\
& *d^6 - 4*B^3*a^4*b^2*c^4*d^4 + 8*A*B^2*b^6*c^7*d + 16*B^3*a*b^5*c^7*d - 2*B \\
& ^3*a^5*b*c*d^7 - 13*A*B^2*b^6*c^5*d^3 + 9*A^2*B*b^6*c^4*d^4 - 5*A^2*B*b^6*c \\
& ^6*d^2 + 6*A^3*a*b^5*c^2*d^6 - 2*A^3*a*b^5*c^4*d^4 - 4*A^3*a^2*b^4*c*d^7 - \\
& 26*B^3*a*b^5*c^5*d^3 - 74*A*B^2*a^2*b^4*c^3*d^5 + 24*A*B^2*a^2*b^4*c^5*d^3 \\
& + 44*A*B^2*a^3*b^3*c^2*d^6 + 8*A*B^2*a^3*b^3*c^4*d^4 - 8*A*B^2*a^3*b^3*c^6* \\
& d^2 - 16*A*B^2*a^4*b^2*c^3*d^5 + 4*A*B^2*a^4*b^2*c^5*d^3 + 35*A^2*B*a^2*b^4 \\
& *c^2*d^6 + A^2*B*a^2*b^4*c^4*d^4 - 4*A^2*B*a^2*b^4*c^6*d^2 - 20*A^2*B*a^3*b \\
& ^3*c^3*d^5 + 4*A^2*B*a^3*b^3*c^5*d^3 + 10*A^2*B*a^4*b^2*c^2*d^6 + 2*A^2*B*a \\
& ^4*b^2*c^4*d^4 + 52*A*B^2*a*b^5*c^4*d^4 - 28*A*B^2*a*b^5*c^6*d^2 + 4*A*B^2* \\
& a^2*b^4*c^7*d - 9*A*B^2*a^4*b^2*c*d^7 + 4*A*B^2*a^5*b*c^2*d^6 - 32*A^2*B*a* \\
& b^5*c^3*d^5 + 14*A^2*B*a*b^5*c^5*d^3 - 12*A^2*B*a^3*b^3*c*d^7 - 2*A^2*B*a^5 \\
& *b*c^3*d^5))/d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b^2*c \\
& *2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^ \\
& 4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - \\
& 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B* \\
& b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^ \\
& 5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B \\
& *a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*1i - B*b \\
& ^2*c*2i)*(((32*(c^2*d^{12} - 2*c^4*d^{10} + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d \\
& ^5) + (32*\tan(e/2 + (f*x)/2)*(3*c*d^{14} - 8*c^3*d^{12} + 7*c^5*d^{10} - 2*c^7*d^ \\
& 8))/(d^{10} - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*1i - B*b^2*c*2i))/d^3 \\
& - (32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^{11} + A*b^2*c^3*d^9 + B*a^2 \\
& *c^2*d^{10} - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^{10} - 3*B*b^2*c^4*d^8 + B*b^2*c^6* \\
& d^6 - 2*B*a*b*c*d^{11} + 2*A*a*b*c^2*d^{10} - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9 \\
&))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*\tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^{11} - \\
& 2*B*a^2*c*d^{12} - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^{11} - 6*A*b^2*c^4*d^9 + 2* \\
& A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^{10} - 6*B*b^2*c^3*d^{10} + 10*B*b^2*c^5*d^8 - 4* \\
& B*b^2*c^7*d^6 - 4*A*a*b*c*d^{12} + 4*A*a*b*c^3*d^{10} + 8*B*a*b*c^2*d^{11} - 12*B \\
& *a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^{10} - 2*c^2*d^8 + c^4*d^6))/d^3 - (32*t \\
& \tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 \\
& + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c \\
& ^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^{10} + B^2*a^4*c*d^{10} + 4*A^2*a^2* \\
& b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b \\
& ^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 \\
& - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3 \\
& *c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^{10} - 4*A^2*a^3*b*c^2*d^9
\end{aligned}$$

$$\begin{aligned}
& + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 \\
& - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5)/(d^10 - 2*c^2*d^8 + c^4*d^6))/d^3 - ((b*d*(A*b + 2*B*a)*i - B*b^2*c*2i)*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) + ((b*d*(A*b + 2*B*a)*i - B*b^2*c*2i)*((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(b*d*(A*b + 2*B*a)*i - B*b^2*c*2i))/d^3 - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6))/d^3 - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5))/(d^10 - 2*c^2*d^8 + c^4*d^6))/d^3 + (64*tan(e/2 + (f*x)/2)*(4*A^3*b^6*c^2*d^7 - 16*B^3*b^6*c^9 - 6*A^3*b^6*c^4*d^5 + 2*A^3*b^6*c^6*d^3 - 24*B^3*b^6*c^5*d^4 + 40*B^3*b^6*c^7*d^2 + 2*A^3*a^2*b^4*c^2*d^7 - 2*A^3*a^2*b^4*c^4*d^5 - 96*B^3*a^2*b^4*c^3*d^6 + 144*B^3*a^2*b^4*c^5*d^4 - 48*B^3*a^2*b^4*c^7*d^2 + 48*B^3*a^3*b^3*c^2*d^7 - 64*B^3*a^3*b^3*c^4*d^5 + 16*B^3*a^3*b^3*c^6*d^3 + 8*B^3*a^4*b^2*c^3*d^6 + 24*A*B^2*b^6*c^8*d - 4*A^3*a*b^5*c*d^8 + 48*B^3*a*b^5*c^8*d + 40*A*B^2*b^6*c^4*d^5 - 64*A*B^2*b^6*c^6*d^3 - 22*A^2*B*b^6*c^3*d^6 + 34*A^2*B*b^6*c^5*d^4 - 12*A^2*B*b^6*c^7*d^2 + 4*A^3*a*b^5*c^3*d^6 + 80*B^3*a*b^5*c^4*d^5 - 128*B^3*a*b^5*c^6*d^3 - 8*B^3*a^4*b^2*c*d^8 + 88*A*B^2*a^2*b^4*c^2*d^7 - 104*A*B^2*a^2*b^4*c^4*d^5 + 16*A*B^2*a^2*b^4*c^6*d^3 + 8*A*B^2*a^3*b^3*c^3*d^6 + 16*A*B^2*a^3*b^3*c^5*d^4 + 8*A*B^2*a^4*b^2*c^2*d^7 - 8*A*B^2*a^4*b^2*c^4*d^5 + 10*A^2*B*a^2*b^4*c^3*d^6 + 8*A^2*B*a^2*b^4*c^5*d^4 + 8*A^2*B*a^3*b^3*c^2*d^7 - 8*A^2*B*a^3*b^3*c^4*d^5 - 104*A*B^2*a*b^5*c^3*d^6 + 152*A*B^2*a*b^5*c^5*d^4 - 48*A*B^2*a*b^5*c^7*d^2 - 24*A*B^2*a^3*b^3*c*d^8 + 40*A^2*B*a*b^5*c^2*d^7 - 52*A^2*B*a*b^5*c^4*d^5 + 12*A^2*B*a*b^5*c^6*d^3 - 18*A^2*B*a^2*b^4*c*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*((b*d*(A*b + 2*B*a)*i - B*b^2*c*2i)*2i)/(d^3*f) + (atan(((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2))*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^
\end{aligned}$$

$$\begin{aligned}
& 2*b^4*c^5*d^6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 \\
& - 28*B^2*b^4*c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 \\
& + 4*A^2*a^2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 \\
& - 36*B^2*a^2*b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A* \\
& B*b^4*c^2*d^9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 \\
& - 8*A^2*a*b^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2 \\
& *a^3*b*c^2*d^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3 \\
& *c^6*d^5 - 16*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^ \\
& 9 + 4*B^2*a^3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B* \\
& a*b^3*c^3*d^8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^ \\
& 3*d^8 - 4*A*B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^ \\
& 7 + 4*A*B*a^2*b^2*c^6*d^5)/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(\\
& c + d)^3*(c - d)^3)^(1/2))*((32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - 2*B*a \\
& ^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2*A*b^2* \\
& c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4*B*b^2* \\
& c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B*a*b*c \\
& ^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) - (32*(A*a^2*c^5*d^ \\
& 7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2*c^2*d^10 - B*a^2*c \\
& ^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 - 2*B*a*b*c*d^1 \\
& 1 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9))/(d^9 - 2*c^2*d^7 \\
& + c^4*d^5) + (((32*(c^2*d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c \\
& ^4*d^5) + (32*tan(e/2 + (f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^ \\
& 7*d^8))/(d^10 - 2*c^2*d^8 + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1 \\
& /2)*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2) \\
&))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 \\
& - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6 \\
& *d^3))*(2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d \\
& ^2)*1i)/(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) + ((a*d - b*c)*(-(c + d)^3* \\
& (c - d)^3)^(1/2))*((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A^2*b^4*c^6*d^ \\
& 4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 + 4*B^2*a^2*b \\
& ^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4*A*B*b^4*c^3* \\
& d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3*d^7 + 16*B^2* \\
& a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8*A*B*a*b^3*c^4 \\
& *d^6 + 4*A*B*a*b^3*c^6*d^4))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f \\
& *x)/2)*(A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^6 + 2*A^2*b^4 \\
& *c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4*c^7*d^4 + 8* \\
& B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^10 + B^2*a^4*c*d^10 + 4*A^2*a^2*b^2*c^3*d^8 \\
& - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2*b^2*c^5*d^6 \\
& + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^9 - 32*A*B*b^ \\
& 4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b^3*c^2*d^9 + \\
& 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^10 - 4*A^2*a^3*b*c^2*d^9 + 16*B^2*a \\
& *b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 16*B^2*a*b^3*c \\
& ^8*d^3 - 8*B^2*a^2*b^2*c*d^10 - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^3*b*c^4*d^7 - \\
& 8*A*B*a*b^3*c*d^10 + 4*A*B*a^3*b*c*d^10 + 48*A*B*a*b^3*c^3*d^8 - 40*A*B*a* \\
& b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A*B*a^3*b*c^5*d \\
& ^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^2*b^2*c^6*d^5 \\
&))/(d^10 - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2) \\
& *((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^11 + A*b^2*c^3*d^9 + B*a^2 \\
& *c^2*d^10 - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^10 - 3*B*b^2*c^4*d^8 + B*b^2*c^6* \\
& d^6 - 2*B*a*b*c*d^11 + 2*A*a*b*c^2*d^10 - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9 \\
&))/(d^9 - 2*c^2*d^7 + c^4*d^5) - (32*tan(e/2 + (f*x)/2)*(2*A*a^2*c^2*d^11 - \\
& 2*B*a^2*c*d^12 - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^11 - 6*A*b^2*c^4*d^9 + 2* \\
& A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^10 - 6*B*b^2*c^3*d^10 + 10*B*b^2*c^5*d^8 - 4* \\
& B*b^2*c^7*d^6 - 4*A*a*b*c*d^12 + 4*A*a*b*c^3*d^10 + 8*B*a*b*c^2*d^11 - 12*B \\
& *a*b*c^4*d^9 + 4*B*a*b*c^6*d^7))/(d^10 - 2*c^2*d^8 + c^4*d^6) + (((32*(c^2* \\
& d^12 - 2*c^4*d^10 + c^6*d^8))/(d^9 - 2*c^2*d^7 + c^4*d^5) + (32*tan(e/2 + (\\
& f*x)/2)*(3*c*d^14 - 8*c^3*d^12 + 7*c^5*d^10 - 2*c^7*d^8))/(d^10 - 2*c^2*d^8 \\
& + c^4*d^6))*(a*d - b*c)*(-(c + d)^3*(c - d)^3)^(1/2)*(2*A*b*d^3 + B*a*d^3 \\
& + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2))/(d^9 - 3*c^2*d^7 + 3*c^
\end{aligned}$$

$$\begin{aligned}
& (4d^5 - c^6d^3)) * (2A^3b^3d^3 + B^3a^3d^3 + 2B^3b^3c^3 - A^3a^3c^3d^2 - A^3b^3c^2d^2 - 3B^3b^3c^2d^2)) / (d^9 - 3c^2d^7 + 3c^4d^5 - c^6d^3)) * (2A^3b^3d^3 + B^3a^3d^3 + 2B^3b^3c^3 - A^3a^3c^3d^2 - A^3b^3c^2d^2 - 3B^3b^3c^2d^2) * i) / (d^9 - 3c^2d^7 + 3c^4d^5 - c^6d^3)) / ((64*(A^3b^6c^5d^3 - 2A^3b^6c^3d^5 - 4B^3b^6c^8 + 6B^3b^6c^6d^2 - 3A^3a^2b^4c^3d^5 + A^3a^2b^4c^5d^3 + 4A^3a^3b^3c^2d^6 - A^3a^4b^2c^3d^5 + 44B^3a^2b^4c^4d^4 - 24B^3a^2b^4c^6d^2 - 36B^3a^3b^3c^3d^5 + 16B^3a^3b^3c^5d^3 + 14B^3a^4b^2c^2d^6 - 4B^3a^4b^2c^4d^4 + 8A^3B^2b^6c^7d + 16B^3a^5b^5c^7d - 2B^3a^5b^5c^6d^2 + 6A^3a^5b^5c^2d^6 - 2A^3a^5b^5c^4d^4 - 4A^3a^2b^4c^3d^7 - 26B^3a^5b^5c^5d^3 - 74A^3B^2a^2b^4c^3d^5 + 24A^3B^2a^2b^4c^5d^3 + 44A^3B^2a^3b^3c^2d^6 + 8A^3B^2a^3b^3c^4d^4 - 8A^3B^2a^3b^3c^6d^2 - 16A^3B^2a^4b^2c^3d^5 + 4A^3B^2a^4b^2c^5d^3 + 35A^2B^3a^2b^4c^2d^6 + A^2B^3a^2b^4c^4d^4 - 4A^2B^3a^2b^4c^6d^2 - 20A^2B^3a^3b^3c^3d^5 + 4A^2B^3a^3b^3c^5d^3 + 10A^2B^3a^4b^2c^2d^6 + 2A^2B^3a^4b^2c^4d^4 + 52A^2B^2a^5b^5c^4d^4 - 28A^2B^2a^5b^5c^6d^2 + 4A^2B^2a^2b^4c^7d - 9A^2B^2a^4b^2c^7d + 4A^2B^2a^5b^5c^2d^6 - 32A^2B^2a^5b^5c^3d^5 + 14A^2B^2a^5b^5c^5d^3 - 12A^2B^2a^3b^3c^7d - 2A^2B^2a^5b^5c^3d^5)) / (d^9 - 2c^2d^7 + c^4d^5) + (64 * tan(e/2 + (f*x)/2) * (4A^3b^6c^2d^7 - 16B^3b^6c^9 - 6A^3b^6c^4d^5 + 2A^3b^6c^6d^3 - 24B^3b^6c^5d^4 + 40B^3b^6c^7d^2 + 2A^3a^2b^4c^2d^7 - 2A^3a^2b^4c^4d^5 - 96B^3a^2b^4c^3d^6 + 144B^3a^2b^4c^5d^4 - 48B^3a^2b^4c^7d^2 + 48B^3a^3b^3c^2d^7 - 64B^3a^3b^3c^4d^5 + 16B^3a^3b^3c^6d^3 + 8B^3a^4b^2c^3d^6 + 24A^3B^2b^6c^8d - 4A^3a^5b^5c^8d + 48B^3a^5b^5c^8d + 40A^3B^2b^6c^4d^5 - 64A^3B^2b^6c^6d^3 - 22A^2B^3b^6c^3d^6 + 34A^2B^3b^6c^5d^4 - 12A^2B^3b^6c^7d^2 + 4A^3a^5b^5c^3d^6 + 80B^3a^5b^5c^4d^5 - 128B^3a^5b^5c^6d^3 - 8B^3a^4b^2c^8d + 88A^3B^2a^2b^4c^2d^7 - 104A^3B^2a^2b^4c^4d^5 + 16A^3B^2a^2b^4c^6d^3 + 8A^3B^2a^3b^3c^3d^6 + 16A^3B^2a^3b^3c^5d^4 + 8A^3B^2a^4b^2c^2d^7 - 8A^3B^2a^4b^2c^4d^5 + 10A^2B^3a^2b^4c^3d^6 + 8A^2B^3a^2b^4c^5d^4 + 8A^2B^3a^3b^3c^2d^7 - 8A^2B^3a^3b^3c^4d^5 - 104A^3B^2a^5b^5c^3d^6 + 152A^3B^2a^5b^5c^5d^4 - 48A^3B^2a^5b^5c^7d^2 - 24A^3B^2a^3b^3c^8d + 40A^2B^3a^5b^5c^2d^7 - 52A^2B^3a^5b^5c^4d^5 + 12A^2B^3a^5b^5c^6d^3 - 18A^2B^3a^2b^4c^8d)) / (d^10 - 2c^2d^8 + c^4d^6) + ((a*d - b*c) * (-(c + d)^3 * (c - d)^3)^(1/2) * ((32*(A^2b^4c^2d^8 - 2A^2b^4c^4d^6 + A^2b^4c^6d^4 + 4B^2b^4c^4d^6 - 8B^2b^4c^6d^4 + 4B^2b^4c^8d^2 + 4B^2a^2b^2c^2d^8 - 8B^2a^2b^2c^4d^6 + 4B^2a^2b^2c^6d^4 - 4A^3B^2b^4c^3d^7 + 8A^3B^2b^4c^5d^5 - 4A^3B^2b^4c^7d^3 - 8B^2a^3b^3c^3d^7 + 16B^2a^3b^3c^5d^5 - 8B^2a^3b^3c^7d^3 + 4A^3B^2a^3b^3c^2d^8 - 8A^3B^2a^3b^3c^4d^6 + 4A^3B^2a^3b^3c^6d^4)) / (d^9 - 2c^2d^7 + c^4d^5) - (32 * tan(e/2 + (f*x)/2) * (A^2a^4c^3d^8 + 9A^2b^4c^3d^8 - 8A^2b^4c^5d^6 + 2A^2b^4c^7d^4 - 8B^2b^4c^3d^8 + 29B^2b^4c^5d^6 - 28B^2b^4c^7d^4 + 8B^2b^4c^9d^2 - 2A^2b^4c^3d^10 + B^2a^4c^3d^10 + 4A^2a^2b^2c^3d^8 - 2A^2a^2b^2c^5d^6 + 42B^2a^2b^2c^3d^8 - 36B^2a^2b^2c^5d^6 + 8B^2a^2b^2c^7d^4 - 2A^3B^2a^4c^2d^9 + 8A^3B^2b^4c^2d^9 - 32A^3B^2b^4c^4d^7 + 30A^3B^2b^4c^6d^5 - 8A^3B^2b^4c^8d^3 - 8A^2a^5b^5c^2d^9 + 4A^2a^5b^5c^4d^7 + 4A^2a^2b^2c^3d^10 - 4A^2a^3b^3c^2d^9 + 16B^2a^5b^5c^2d^9 - 64B^2a^5b^5c^4d^7 + 60B^2a^5b^5c^6d^5 - 16B^2a^5b^5c^8d^3 - 8B^2a^2b^2c^3d^10 - 8B^2a^3b^3c^2d^9 + 4B^2a^3b^3c^4d^7 - 8A^3B^2a^5b^5c^3d^10 + 4A^3B^2a^3b^3c^5d^10 + 48A^3B^2a^5b^5c^3d^8 - 40A^3B^2a^5b^5c^5d^6 + 8A^3B^2a^5b^5c^7d^4 + 8A^3B^2a^3b^3c^3d^8 - 4A^3B^2a^3b^3c^5d^6 - 20A^3B^2a^2b^2c^2d^9 + 4A^3B^2a^2b^2c^4d^7 + 4A^3B^2a^2b^2c^6d^5)) / (d^10 - 2c^2d^8 + c^4d^6) + ((a*d - b*c) * (-(c + d)^3 * (c - d)^3)^(1/2) * ((32 * tan(e/2 + (f*x)/2) * (2A^3a^2c^2d^11 - 2B^3a^2c^2d^12 - 2A^3a^2c^4d^9 + 4A^3b^2c^2d^11 - 6A^3b^2c^4d^9 + 2A^3b^2c^6d^7 + 2B^3a^2c^3d^10 - 6B^3b^2c^3d^10 + 10B^3b^2c^5d^8 - 4B^3b^2c^7d^6 - 4A^3a^5b^5c^3d^12 + 4A^3a^5b^5c^5d^10 + 8B^3a^5b^5c^7d^11 - 12B^3a^5b^5c^9d^9 + 4B^3a^5b^5c^6d^7)) / (d^10 - 2c^2d^8 + c^4d^6) - (32*(A^3a^2c^5d^7 - A^3a^2c^3d^9 - A^3b^2c^3d^11 + A^3b^2c^3d^9 + B^3a^2c^
\end{aligned}$$

$$\begin{aligned}
& 2*d^{10} - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^{10} - 3*B*b^2*c^4*d^8 + B*b^2*c^6*d^6 \\
& - 2*B*a*b*c*d^{11} + 2*A*a*b*c^2*d^{10} - 2*A*a*b*c^4*d^8 + 2*B*a*b*c^3*d^9) / \\
& (d^9 - 2*c^2*d^7 + c^4*d^5) + (((32*(c^2*d^{12} - 2*c^4*d^{10} + c^6*d^8)) / (d^9 \\
& - 2*c^2*d^7 + c^4*d^5) + (32*\tan(e/2 + (f*x)/2)*(3*c*d^{14} - 8*c^3*d^{12} + 7 \\
& *c^5*d^{10} - 2*c^7*d^8)) / (d^{10} - 2*c^2*d^8 + c^4*d^6)) * (a*d - b*c) * (-(c + d) \\
& ^3*(c - d)^3)^{(1/2)} * (2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d \\
& - 3*B*b*c*d^2)) / (d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)) * (2*A*b*d^3 + B*a \\
& *d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)) / (d^9 - 3*c^2*d^7 + \\
& 3*c^4*d^5 - c^6*d^3)) * (2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c \\
& ^2*d - 3*B*b*c*d^2)) / (d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3) - ((a*d - b*c) \\
& * (-(c + d)^3*(c - d)^3)^{(1/2)} * ((32*(A^2*b^4*c^2*d^8 - 2*A^2*b^4*c^4*d^6 + A \\
& ^2*b^4*c^6*d^4 + 4*B^2*b^4*c^4*d^6 - 8*B^2*b^4*c^6*d^4 + 4*B^2*b^4*c^8*d^2 \\
& + 4*B^2*a^2*b^2*c^2*d^8 - 8*B^2*a^2*b^2*c^4*d^6 + 4*B^2*a^2*b^2*c^6*d^4 - 4 \\
& *A*B*b^4*c^3*d^7 + 8*A*B*b^4*c^5*d^5 - 4*A*B*b^4*c^7*d^3 - 8*B^2*a*b^3*c^3 \\
& d^7 + 16*B^2*a*b^3*c^5*d^5 - 8*B^2*a*b^3*c^7*d^3 + 4*A*B*a*b^3*c^2*d^8 - 8* \\
& A*B*a*b^3*c^4*d^6 + 4*A*B*a*b^3*c^6*d^4)) / (d^9 - 2*c^2*d^7 + c^4*d^5) - (32 \\
& * \tan(e/2 + (f*x)/2) * (A^2*a^4*c^3*d^8 + 9*A^2*b^4*c^3*d^8 - 8*A^2*b^4*c^5*d^ \\
& 6 + 2*A^2*b^4*c^7*d^4 - 8*B^2*b^4*c^3*d^8 + 29*B^2*b^4*c^5*d^6 - 28*B^2*b^4 \\
& *c^7*d^4 + 8*B^2*b^4*c^9*d^2 - 2*A^2*b^4*c*d^{10} + B^2*a^4*c*d^{10} + 4*A^2*a^ \\
& 2*b^2*c^3*d^8 - 2*A^2*a^2*b^2*c^5*d^6 + 42*B^2*a^2*b^2*c^3*d^8 - 36*B^2*a^2 \\
& *b^2*c^5*d^6 + 8*B^2*a^2*b^2*c^7*d^4 - 2*A*B*a^4*c^2*d^9 + 8*A*B*b^4*c^2*d^ \\
& 9 - 32*A*B*b^4*c^4*d^7 + 30*A*B*b^4*c^6*d^5 - 8*A*B*b^4*c^8*d^3 - 8*A^2*a*b \\
& ^3*c^2*d^9 + 4*A^2*a*b^3*c^4*d^7 + 4*A^2*a^2*b^2*c*d^{10} - 4*A^2*a^3*b*c^2*d \\
& ^9 + 16*B^2*a*b^3*c^2*d^9 - 64*B^2*a*b^3*c^4*d^7 + 60*B^2*a*b^3*c^6*d^5 - 1 \\
& 6*B^2*a*b^3*c^8*d^3 - 8*B^2*a^2*b^2*c*d^{10} - 8*B^2*a^3*b*c^2*d^9 + 4*B^2*a^ \\
& 3*b*c^4*d^7 - 8*A*B*a*b^3*c*d^{10} + 4*A*B*a^3*b*c*d^{10} + 48*A*B*a*b^3*c^3*d^ \\
& 8 - 40*A*B*a*b^3*c^5*d^6 + 8*A*B*a*b^3*c^7*d^4 + 8*A*B*a^3*b*c^3*d^8 - 4*A* \\
& B*a^3*b*c^5*d^6 - 20*A*B*a^2*b^2*c^2*d^9 + 4*A*B*a^2*b^2*c^4*d^7 + 4*A*B*a^ \\
& 2*b^2*c^6*d^5)) / (d^{10} - 2*c^2*d^8 + c^4*d^6) + ((a*d - b*c) * (-(c + d)^3*(c \\
& - d)^3)^{(1/2)} * ((32*(A*a^2*c^5*d^7 - A*a^2*c^3*d^9 - A*b^2*c*d^{11} + A*b^2*c^ \\
& 3*d^9 + B*a^2*c^2*d^{10} - B*a^2*c^4*d^8 + 2*B*b^2*c^2*d^{10} - 3*B*b^2*c^4*d^8 \\
& + B*b^2*c^6*d^6 - 2*B*a*b*c*d^{11} + 2*A*a*b*c^2*d^{10} - 2*A*a*b*c^4*d^8 + 2* \\
& B*a*b*c^3*d^9)) / (d^9 - 2*c^2*d^7 + c^4*d^5) - (32*\tan(e/2 + (f*x)/2) * (2*A*a \\
& ^2*c^2*d^{11} - 2*B*a^2*c*d^{12} - 2*A*a^2*c^4*d^9 + 4*A*b^2*c^2*d^{11} - 6*A*b^2 \\
& *c^4*d^9 + 2*A*b^2*c^6*d^7 + 2*B*a^2*c^3*d^{10} - 6*B*b^2*c^3*d^{10} + 10*B*b^2 \\
& *c^5*d^8 - 4*B*b^2*c^7*d^6 - 4*A*a*b*c*d^{12} + 4*A*a*b*c^3*d^{10} + 8*B*a*b*c^ \\
& 2*d^{11} - 12*B*a*b*c^4*d^9 + 4*B*a*b*c^6*d^7)) / (d^{10} - 2*c^2*d^8 + c^4*d^6) \\
& + (((32*(c^2*d^{12} - 2*c^4*d^{10} + c^6*d^8)) / (d^9 - 2*c^2*d^7 + c^4*d^5) + (3 \\
& 2*\tan(e/2 + (f*x)/2) * (3*c*d^{14} - 8*c^3*d^{12} + 7*c^5*d^{10} - 2*c^7*d^8)) / (d^{1 \\
& 0} - 2*c^2*d^8 + c^4*d^6)) * (a*d - b*c) * (-(c + d)^3*(c - d)^3)^{(1/2)} * (2*A*b*d \\
& ^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)) / (d^9 - 3*c \\
& ^2*d^7 + 3*c^4*d^5 - c^6*d^3)) * (2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 \\
& - A*b*c^2*d - 3*B*b*c*d^2)) / (d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)) * (2*A* \\
& b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2)) / (d^9 - \\
& 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3)) * (a*d - b*c) * (-(c + d)^3*(c - d)^3)^{(1/2)} \\
& * (2*A*b*d^3 + B*a*d^3 + 2*B*b*c^3 - A*a*c*d^2 - A*b*c^2*d - 3*B*b*c*d^2) * 2i \\
&) / (f*(d^9 - 3*c^2*d^7 + 3*c^4*d^5 - c^6*d^3))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))*2*(A+B*sin(f*x+e))/(c+d*sin(f*x+e))*2,x)

[Out] Timed out

$$3.353 \quad \int \frac{(A+B \sin(e+fx))(c+d \sin(e+fx))^{3/2}}{(a+b \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=840

$$\frac{2(Ab - aB)(bc - ad)\sqrt{c + d \sin(e + fx)} \cos(e + fx)}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}} - \frac{(2Ab(bc - ad) - B(-3da^2 + 2bca + b^2d)) \sqrt{c + d \sin(e + fx)}}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}}$$

[Out] $(c-d)*(-2*A*a*b*d+2*A*b^2*c+3*B*a^2*d-2*B*a*b*c-B*b^2*d)*\text{EllipticE}((a+b)^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sin(f*x+e))^{(1/2)},((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)})*\sec(f*x+e)*(a+b*\sin(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sin(f*x+e))/(c+d)/(a+b*\sin(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sin(f*x+e))/(c-d)/(a+b*\sin(f*x+e)))^{(1/2)}/(a-b)/b^2/(-a*d+b*c)/f/(a+b)^{(1/2)}+(2*A*b*d-3*B*a*d+3*B*b*c)*\text{EllipticPi}((a+b)^{(1/2)}*(c+d*\sin(f*x+e))^{(1/2)}/(c+d)^{(1/2)}/(a+b*\sin(f*x+e))^{(1/2)},b*(c+d)/(a+b)/d,((a-b)*(c+d)/(a+b)/(c-d))^{(1/2)})*\sec(f*x+e)*(a+b*\sin(f*x+e))*(c+d)^{(1/2)}*(-(-a*d+b*c)*(1-\sin(f*x+e))/(c+d)/(a+b*\sin(f*x+e)))^{(1/2)}*((-a*d+b*c)*(1+\sin(f*x+e))/(c-d)/(a+b*\sin(f*x+e)))^{(1/2)}/b^3/f/(a+b)^{(1/2)}+(2*A*b*(b*(c-2*d)+a*d)-B*(3*a^2*d-6*a*b*d+b^2*(2*c+d)))*\text{EllipticF}((c+d)^{(1/2)}*(a+b*\sin(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*\sec(f*x+e)*(c+d*\sin(f*x+e))*(a+b)^{(1/2)}*((-a*d+b*c)*(1-\sin(f*x+e))/(a+b)/(c+d*\sin(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sin(f*x+e))/(a-b)/(c+d*\sin(f*x+e)))^{(1/2)}/(a-b)/b^3/f/(c+d)^{(1/2)}+2*(A*b-B*a)*(-a*d+b*c)*\cos(f*x+e)*(c+d*\sin(f*x+e))^{(1/2)}/b/(a^2-b^2)/f/(a+b*\sin(f*x+e))^{(1/2)}-(2*A*b*(-a*d+b*c)-B*(-3*a^2*d+2*a*b*c+b^2*d))*\cos(f*x+e)*(c+d*\sin(f*x+e))^{(1/2)}/b/(a^2-b^2)/f/(a+b*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 3.16, antiderivative size = 840, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {2989, 3061, 3053, 2811, 2998, 2818, 2996}

$$\frac{2(Ab - aB)(bc - ad)\sqrt{c + d \sin(e + fx)} \cos(e + fx)}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}} - \frac{(2Ab(bc - ad) - B(-3da^2 + 2bca + b^2d)) \sqrt{c + d \sin(e + fx)}}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}(((A + B*\text{Sin}[e + f*x])*(c + d*\text{Sin}[e + f*x])^{(3/2)})/(a + b*\text{Sin}[e + f*x])^{(3/2)}, x)$

[Out] $((c - d)*\text{Sqrt}[c + d]*(2*A*b^2*c - 2*a*b*B*c - 2*a*A*b*d + 3*a^2*B*d - b^2*B*d)*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])]/(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*\text{Sec}[e + f*x]*\text{Sqrt}[-(((b*c - a*d)*(1 - \text{Sin}[e + f*x]))/((c + d)*(a + b*\text{Sin}[e + f*x])))]* \text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))]*(a + b*\text{Sin}[e + f*x])/((a - b)*b^2*\text{Sqrt}[a + b]*(b*c - a*d)*f) + (\text{Sqrt}[c + d]*(3*b*B*c + 2*A*b*d - 3*a*B*d)*\text{EllipticPi}[(b*(c + d))/((a + b)*d), \text{ArcSin}[(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])]/(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]*\text{Sec}[e + f*x]*\text{Sqrt}[-(((b*c - a*d)*(1 - \text{Sin}[e + f*x]))/((c + d)*(a + b*\text{Sin}[e + f*x])))]* \text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))]*(a + b*\text{Sin}[e + f*x])/((b^3*\text{Sqrt}[a + b]*f) + (2*(A*b - a*B)*(b*c - a*d)*\text{Cos}[e + f*x]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(b*(a^2 - b^2)*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]) - ((2*A*b*(b*c - a*d) - B*(2*a*b*c - 3*a^2*d + b^2*d))*\text{Cos}[e + f*x]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(b*(a^2 - b^2)*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]) + (\text{Sqrt}[a + b]*(2*A*b*(b*(c - 2*d) + a*d) - B*(3*a^2*d - 6*a*b*d + b^2*(2*c + d)))*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])]/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sec}[e + f*x]*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]*\text{Sqrt}[-(((b*c - a*d)*(1 + \text{Sin}[e + f*x]))/((a + b)*(c + d*\text{Sin}[e + f*x])))]* \text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]$

$-(b)(c + d\sin[e + fx])))(c + d\sin[e + fx])/((a - b)b^3\sqrt{c + d}f)$

Rule 2811

$\text{Int}[\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]}/\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}], x_Symbol] \rightarrow \text{Simp}[(2(a + b\sin[e + fx])\sqrt{(b^2c - a^2)(1 + \sin[e + fx])})/((c - d)(a + b\sin[e + fx]))\sqrt{-((b^2c - a^2)(1 - \sin[e + fx])})}/((c + d)(a + b\sin[e + fx]))]\text{EllipticPi}[(b(c + d))/(d(a + b)), \text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}/\sqrt{a + b\sin[e + fx]}], ((a - b)(c + d))/((a + b)(c - d))]/(d f \text{Rt}[(a + b)/(c + d), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b^2c - a^2, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rule 2818

$\text{Int}[1/(\sqrt{(a_.) + (b_.)\sin[(e_.) + (f_.)x]})\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}], x_Symbol] \rightarrow \text{Simp}[(2(c + d\sin[e + fx])\sqrt{(b^2c - a^2)(1 - \sin[e + fx])})/((a + b)(c + d\sin[e + fx]))\sqrt{-((b^2c - a^2)(1 + \sin[e + fx])})}/((a - b)(c + d\sin[e + fx]))]\text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2](\sqrt{a + b\sin[e + fx]}/\sqrt{c + d\sin[e + fx]})], ((a + b)(c - d))/((a - b)(c + d))]/(f(b^2c - a^2)\text{Rt}[(c + d)/(a + b), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{NeQ}[b^2c - a^2, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/(a + b)]$

Rule 2989

$\text{Int}[(a_.) + (b_.)\sin[(e_.) + (f_.)x])^m((A_.) + (B_.)\sin[(e_.) + (f_.)x])^n, x_Symbol] \rightarrow -\text{Simp}[(b^2c - a^2)(B^2c - A^2)\cos[e + fx](a + b\sin[e + fx])^{m-1}(c + d\sin[e + fx])^{n+1})/(d f (n + 1)(c^2 - d^2)), x] + \text{Dist}[1/(d(n + 1)(c^2 - d^2)), \text{Int}[(a + b\sin[e + fx])^{m-2}(c + d\sin[e + fx])^{n+1})\text{Simp}[b(b^2c - a^2)(B^2c - A^2)(m - 1) + a^2d(aA^2c + bB^2c - (A^2b + a^2B^2)d)(n + 1) + (b(b^2d(B^2c - A^2) + a(A^2cd + B^2(c^2 - 2d^2)))(n + 1) - a(b^2c - a^2)(B^2c - A^2)(n + 2))\sin[e + fx] + b(d(A^2b^2c + a^2B^2c - a^2A^2d)(m + n + 1) - bB^2(c^2m + d^2(n + 1)))\sin[e + fx]^2, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b^2c - a^2, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 1] \&\& \text{LtQ}[n, -1]$

Rule 2996

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x])/((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{3/2}\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}], x_Symbol] \rightarrow \text{Simp}[(-2A^2(c - d)(a + b\sin[e + fx])\sqrt{(b^2c - a^2)(1 + \sin[e + fx])})/((c - d)(a + b\sin[e + fx]))\sqrt{-((b^2c - a^2)(1 - \sin[e + fx])})}/((c + d)(a + b\sin[e + fx]))]\text{EllipticE}[\text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]\sqrt{c + d\sin[e + fx]}/\sqrt{a + b\sin[e + fx]}], ((a - b)(c + d))/((a + b)(c - d))]/(f(b^2c - a^2)^2\text{Rt}[(a + b)/(c + d), 2]\cos[e + fx]), x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b^2c - a^2, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rule 2998

$\text{Int}[(A_.) + (B_.)\sin[(e_.) + (f_.)x])/((a_.) + (b_.)\sin[(e_.) + (f_.)x])^{3/2}\sqrt{(c_.) + (d_.)\sin[(e_.) + (f_.)x]}], x_Symbol] \rightarrow \text{Dist}[(A - B)/(a - b), \text{Int}[1/(\sqrt{a + b\sin[e + fx]})\sqrt{c + d\sin[e + fx]}], x], x] - \text{Dist}[(A^2b - a^2B)/(a - b), \text{Int}[(1 + \sin[e + fx])^{3/2}\sqrt{c + d\sin[e + fx]}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, A, B\}, x\} \&\& \text{NeQ}[b^2c - a^2, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{NeQ}[A, B]$

Rule 3053

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[C/b^2, Int[Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]], x], x] + Dist[1/b^2, Int[(A*b^2 - a^2*C + b*(b*B - 2*a*C)*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3061

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := -Simp[(C*Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*f*Sqrt[a + b*Sin[e + f*x]]), x] + Dist[1/(2*d), Int[(1*Simp[2*a*A*d - C*(b*c - a*d) - 2*(a*c*C - d*(A*b + a*B))*Sin[e + f*x] + (2*b*B*d - C*(b*c + a*d))*Sin[e + f*x]^2, x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^{3/2}}{(a + b \sin(e + fx))^{3/2}} dx = \frac{2(Ab - aB)(bc - ad) \cos(e + fx) \sqrt{c + d \sin(e + fx)}}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}} - \frac{2 \int \frac{1}{2} \dots}{\dots}$$

$$= \frac{2(Ab - aB)(bc - ad) \cos(e + fx) \sqrt{c + d \sin(e + fx)}}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}} - \frac{(2Ab \dots)}{\dots}$$

$$= \frac{2(Ab - aB)(bc - ad) \cos(e + fx) \sqrt{c + d \sin(e + fx)}}{b(a^2 - b^2) f \sqrt{a + b \sin(e + fx)}} - \frac{(2Ab \dots)}{\dots}$$

$$= \frac{\sqrt{c + d} (3bBc + 2Abd - 3aBd) \Pi\left(\frac{b(c+d)}{(a+b)d}; \sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d} \sin(e+fx)}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right)}{\dots}$$

$$= \frac{(c - d) \sqrt{c + d} (2Ab(bc - ad) - B(2abc - 3a^2d + b^2d)) E\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d} \sin(e+fx)}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right)}{\dots}$$

Mathematica [B] time = 6.75, size = 2042, normalized size = 2.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*SIN[e + f*x])*(c + d*SIN[e + f*x])^(3/2))/(a + b*SIN[e + f*x])^(3/2), x]
```

```
[Out] (-2*(A*b^2*c*cos[e + f*x] - a*b*B*c*cos[e + f*x] - a*A*b*d*cos[e + f*x] + a^2*B*d*cos[e + f*x])*Sqrt[c + d*SIN[e + f*x]])/(b*(-a^2 + b^2)*f*Sqrt[a + b*SIN[e + f*x]]) + ((-4*(-b*c) + a*d)*(2*a*A*b*c^2 - 2*b^2*B*c^2 - 2*A*b^2*c*d + 2*a*b*B*c*d + a^2*B*d^2 - b^2*B*d^2)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f
```

```

*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]
]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a
+ b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-
e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*
Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*
(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*
d)*(2*A*b^2*c^2 - 2*a*b*B*c^2 + 4*a^2*B*c*d - 4*b^2*B*c*d - 2*A*b^2*d^2 + 2
*a*b*B*d^2)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF
[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(
b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*
Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*
Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c
+ d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*
x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/
(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[
(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(
-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*
Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*
d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c)
+ a*d)]/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) +
2*(-2*A*b^2*c*d + 2*a*b*B*c*d + 2*a*A*b*d^2 - 3*a^2*B*d^2 + b^2*B*d^2)*((Co
s[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a
- b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a
- b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]
, (2*(-(b*c) + a*d))/((a - b)*(c + d))] *Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt
[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x]])*Sqrt[a + b*Sin[
e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f
*x]))/((c + d)*(a + b*Sin[e + f*x])))) - (2*(-(b*c) + a*d)*(((a + b)*c + a
*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e +
Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*
x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e
+ f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c
+ d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2
)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)]/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
)/(b*d)))/(2*(a - b)*b*(a + b)*f)

```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: Curve
not irreducible after change of variable 0 -> infinity
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^{\frac{3}{2}}}{(b \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="giac")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
a)^(3/2), x)
```

maple [B] time = 100.98, size = 6776580, normalized size = 8067.36

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A)(d \sin(fx + e) + c)^{\frac{3}{2}}}{(b \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(3/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="maxima")
```

```
[Out] integrate((B*sin(f*x + e) + A)*(d*sin(f*x + e) + c)^(3/2)/(b*sin(f*x + e) +
a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^{3/2}}{(a + b \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^(
3/2),x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(3/2))/(a + b*sin(e + f*x))^(
3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx))(c + d \sin(e + fx))^{3/2}}{(a + b \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(3/2)/(a+b*sin(f*x+e))**(3/2),
x)
```

```
[Out] Integral((A + B*sin(e + f*x))*(c + d*sin(e + f*x))**(3/2)/(a + b*sin(e + f*
x))**(3/2), x)
```

$$3.354 \quad \int \frac{(A+B \sin(e+fx)) \sqrt{c+d \sin(e+fx)}}{(a+b \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=630

$$\frac{2\sqrt{a+b}(c-d)(Ab-aB) \sec(e+fx)(c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d \sin(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b}}{\sqrt{a+b}}\right)\right)}{bf(a-b)\sqrt{c+d}(bc-ad)}$$

[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2), ((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/b/(-a*d+b*c)/f/(a+b)^(1/2)+2*(A*b-B*a)*(c-d)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/b/(-a*d+b*c)/f/(c+d)^(1/2)+2*B*EllipticPi((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2), (a+b)*d/b/(c+d), ((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/b^2/f/(c+d)^(1/2)

Rubi [A] time = 0.89, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {2992, 2811, 2795, 2818, 2996}

$$\frac{2\sqrt{a+b}(c-d)(Ab-aB) \sec(e+fx)(c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d \sin(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b}}{\sqrt{a+b}}\right)\right)}{bf(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3/2), x]

[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d]*Sin[e + f*x])/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])]], ((a - b)*(c + d))/((a + b)*(c - d))]*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])/((a - b)*b*Sqrt[a + b]*(b*c - a*d)*f) + (2*Sqrt[a + b]*(A*b - a*B)*(c - d)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x])]], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])/((a - b)*b*Sqrt[c + d]*(b*c - a*d)*f) + (2*Sqrt[a + b]*B*EllipticPi[((a + b)*d)/(b*(c + d)), ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x])]], ((a + b)*(c - d))/((a - b)*(c + d))]*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])/((b^2*Sqrt[c + d]*f)

Rule 2795

Int[Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]/((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2), x_Symbol] := Dist[(c - d)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(b*c - a*d)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b

$\sqrt{c^2 - d^2}, 0]$ && $\text{NeQ}[c^2 - d^2, 0]$

Rule 2811

$\text{Int}[\text{Sqrt}[(a_) + (b_)\sin[(e_) + (f_)(x_)]/\text{Sqrt}[(c_) + (d_)\sin[(e_) + (f_)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*(a + b*\text{Sin}[e + f*x])*\text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((c + d)*(a + b*\text{Sin}[e + f*x])))]*\text{EllipticPi}[(b*(c + d))/(d*(a + b)), \text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(d*f*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rule 2818

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)\sin[(e_) + (f_)(x_)]*\text{Sqrt}[(c_) + (d_)\sin[(e_) + (f_)(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*(c + d*\text{Sin}[e + f*x])*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((a - b)*(c + d*\text{Sin}[e + f*x])))]*\text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2]*(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*\text{Rt}[(c + d)/(a + b), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/(a + b)]$

Rule 2992

$\text{Int}[(((A_) + (B_)\sin[(e_) + (f_)(x_)]*\text{Sqrt}[(c_) + (d_)\sin[(e_) + (f_)(x_)]])^2)/((a_) + (b_)\sin[(e_) + (f_)(x_)]^3, x_Symbol] \rightarrow \text{Dist}[B/b, \text{Int}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]], x], x] + \text{Dist}[(A*b - a*B)/b, \text{Int}[\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/(a + b*\text{Sin}[e + f*x])^3, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0]$

Rule 2996

$\text{Int}[((A_) + (B_)\sin[(e_) + (f_)(x_)])/(((a_) + (b_)\sin[(e_) + (f_)(x_)]^3)*\text{Sqrt}[(c_) + (d_)\sin[(e_) + (f_)(x_)]], x_Symbol] \rightarrow \text{Simp}[(-2*A*(c - d)*(a + b*\text{Sin}[e + f*x])*\text{Sqrt}[(b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((c - d)*(a + b*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((c + d)*(a + b*\text{Sin}[e + f*x])))]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[(a + b)/(c + d), 2]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]/\text{Sqrt}[a + b*\text{Sin}[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*\text{Rt}[(a + b)/(c + d), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}[\{a, b, c, d, e, f, A, B\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{EqQ}[A, B] \&\& \text{PosQ}[(a + b)/(c + d)]$

Rubi steps

$$\int \frac{(A + B \sin(e + fx))\sqrt{c + d \sin(e + fx)}}{(a + b \sin(e + fx))^{3/2}} dx = \frac{B \int \frac{\sqrt{c+d \sin(e+fx)}}{\sqrt{a+b \sin(e+fx)}} dx}{b} + \frac{(Ab - aB) \int \frac{\sqrt{c+d \sin(e+fx)}}{(a+b \sin(e+fx))^{3/2}} dx}{b}$$

$$= \frac{2\sqrt{a+b} B \Pi\left(\frac{(a+b)d}{b(c+d)}; \sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}\right) \middle| \frac{(a+b)(c-d)}{(a-b)(c+d)}\right) \sec(e + \dots)}{b^2 \sqrt{c}}$$

$$= \frac{2(Ab - aB)(c - d)\sqrt{c + d} E\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right) \middle| \frac{(a-b)(c+d)}{(a+b)(c-d)}\right)}{(a - b)b\sqrt{c}}$$

Mathematica [B] time = 10.19, size = 1901, normalized size = 3.02

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/(a + b*Sin[e + f*x])^(3/2), x]
```

```
[Out] (-2*(-(A*b*Cos[e + f*x]) + a*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(a*A*c - b*B*c)*(-(b*c) + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(A*b*c - a*B*c + a*A*d - b*B*d)*((Sqrt[(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]) + (Sqrt[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b)]]], (2*(-(b*c) + a*d))/((a - b)*(c + d))]*Sqrt[c + d*Sin[e + f*x]])/(b*d*Sqrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x]])*Sqrt[a + b*Sin[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x]))]) - (2*(-(b*c) + a*d)*(((a + b)*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[(c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a -
```

$b) * \text{Csc} [(-e + \text{Pi}/2 - f*x)/2]^2 * (c + d * \text{Sin}[e + f*x]) / (-b*c + a*d) / \text{Sqrt}[2]$
 $], (2 * (-b*c + a*d)) / ((a + b) * (-c + d)) * \text{Sec}[e + f*x] * \text{Sin}[(-e + \text{Pi}/2 - f*x)$
 $/2]^4 * \text{Sqrt} [((c + d) * \text{Csc} [(-e + \text{Pi}/2 - f*x)/2]^2 * (a + b * \text{Sin}[e + f*x])) / (-b*c$
 $+ a*d)] * \text{Sqrt} [((-a - b) * \text{Csc} [(-e + \text{Pi}/2 - f*x)/2]^2 * (c + d * \text{Sin}[e + f*x])) /$
 $(-b*c + a*d)] / ((a + b) * d * \text{Sqrt}[a + b * \text{Sin}[e + f*x]] * \text{Sqrt}[c + d * \text{Sin}[e + f*x$
 $]]) / (b*d)) / ((a - b) * (a + b) * f)$

fricas [F] time = 2.23, size = 0, normalized size = 0.00

$$\text{integral} \left(-\frac{(B \sin(fx + e) + A) \sqrt{b \sin(fx + e) + a} \sqrt{d \sin(fx + e) + c}}{b^2 \cos(fx + e)^2 - 2ab \sin(fx + e) - a^2 - b^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="fricas")

[Out] integral(-(B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e)
+ c)/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{d \sin(fx + e) + c}}{(b \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)
^(3/2), x)

maple [B] time = 159.53, size = 3150101, normalized size = 5000.16

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(B \sin(fx + e) + A) \sqrt{d \sin(fx + e) + c}}{(b \sin(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))^(1/2)/(a+b*sin(f*x+e))^(3/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*sqrt(d*sin(f*x + e) + c)/(b*sin(f*x + e) + a)
^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c + d \sin(e + fx)}}{(a + b \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^(3/2), x)
```

```
[Out] int(((A + B*sin(e + f*x))*(c + d*sin(e + f*x))^(1/2))/(a + b*sin(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sin(e + fx)) \sqrt{c + d \sin(e + fx)}}{(a + b \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))*(c+d*sin(f*x+e))**(1/2)/(a+b*sin(f*x+e))**(3/2), x)
```

```
[Out] Integral((A + B*sin(e + f*x))*sqrt(c + d*sin(e + f*x))/(a + b*sin(e + f*x))**(3/2), x)
```

$$3.355 \quad \int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2} \sqrt{c+d \sin(e+fx)}} dx$$

Optimal. Leaf size=417

$$\frac{2\sqrt{a+b}(A-B) \sec(e+fx)(c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d \sin(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}\right)\right)}{f(a-b)\sqrt{c+d}(bc-ad)}$$

[Out] 2*(A*b-B*a)*(c-d)*EllipticE((a+b)^(1/2)*(c+d*sin(f*x+e))^(1/2)/(c+d)^(1/2)/(a+b*sin(f*x+e))^(1/2),((a-b)*(c+d)/(a+b)/(c-d))^(1/2))*sec(f*x+e)*(a+b*sin(f*x+e))*(c+d)^(1/2)*(-(-a*d+b*c)*(1-sin(f*x+e))/(c+d)/(a+b*sin(f*x+e)))^(1/2)*((-a*d+b*c)*(1+sin(f*x+e))/(c-d)/(a+b*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)^2/f/(a+b)^(1/2)+2*(A-B)*EllipticF((c+d)^(1/2)*(a+b*sin(f*x+e))^(1/2)/(a+b)^(1/2)/(c+d*sin(f*x+e))^(1/2),((a+b)*(c-d)/(a-b)/(c+d))^(1/2))*sec(f*x+e)*(c+d*sin(f*x+e))*(a+b)^(1/2)*((-a*d+b*c)*(1-sin(f*x+e))/(a+b)/(c+d*sin(f*x+e)))^(1/2)*(-(-a*d+b*c)*(1+sin(f*x+e))/(a-b)/(c+d*sin(f*x+e)))^(1/2)/(a-b)/(-a*d+b*c)/f/(c+d)^(1/2)

Rubi [A] time = 0.54, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2998, 2818, 2996}

$$\frac{2\sqrt{a+b}(A-B) \sec(e+fx)(c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}} \sqrt{\frac{(bc-ad)(\sin(e+fx)+1)}{(a-b)(c+d \sin(e+fx))}} F\left(\sin^{-1}\left(\frac{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}{\sqrt{a+b} \sqrt{c+d \sin(e+fx)}}\right)\right)}{f(a-b)\sqrt{c+d}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]),x]

[Out] (2*(A*b - a*B)*(c - d)*Sqrt[c + d]*EllipticE[ArcSin[(Sqrt[a + b]*Sqrt[c + d]*Sin[e + f*x])]/(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))*Sec[e + f*x]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/((c - d)*(a + b*Sin[e + f*x]))]*(a + b*Sin[e + f*x])/((a - b)*Sqrt[a + b]*(b*c - a*d)^2*f) + (2*Sqrt[a + b]*(A - B)*EllipticF[ArcSin[(Sqrt[c + d]*Sqrt[a + b*Sin[e + f*x])]/(Sqrt[a + b]*Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))*Sec[e + f*x]*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x])))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*(c + d*Sin[e + f*x])/((a - b)*Sqrt[c + d]*(b*c - a*d)*f)

Rule 2818

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x])/Sqrt[c + d*Sin[e + f*x]]], (a + b)*(c - d)/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Simp[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/

```
((c - d)*(a + b*Sin[e + f*x]))*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(3/2)*Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{3/2} \sqrt{c + d \sin(e + fx)}} dx = \frac{(A - B) \int \frac{1}{\sqrt{a+b \sin(e+fx)} \sqrt{c+d \sin(e+fx)}} dx}{a - b} - \frac{(Ab - aB) \int \frac{1}{(a+b \sin(e+fx))^{3/2} \sqrt{c+d \sin(e+fx)}} dx}{a - b}$$

$$= \frac{2(Ab - aB)(c - d)\sqrt{c + d} E\left(\sin^{-1}\left(\frac{\sqrt{a+b} \sqrt{c+d} \sin(e+fx)}{\sqrt{c+d} \sqrt{a+b \sin(e+fx)}}\right)\right)}{(a - b)\sqrt{c + d}}$$

Mathematica [B] time = 6.56, size = 1949, normalized size = 4.67

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x]
```

```
[Out] (-2*(A*b^2*Cos[e + f*x] - a*b*B*Cos[e + f*x])*Sqrt[c + d*Sin[e + f*x]])/((a^2 - b^2)*(-(b*c) + a*d)*f*Sqrt[a + b*Sin[e + f*x]]) + ((-4*(-(b*c) + a*d)*(-(a*A*b*c) + b^2*B*c + a^2*A*d - A*b^2*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)))*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])]/(-c + d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)))/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-(A*b^2*c) + a*b*B*c - a*A*b*d + a^2*B*d)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)))*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])]/(-c + d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)))/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d)))*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x])]/(-c + d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x])]/(-c + d)))/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])
```


$$2]^2*(c + d*\sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*d*\sqrt{a + b*\sin[e + f*x]}\sqrt{c + d*\sin[e + f*x]}) + 2*(A*b^2*d - a*b*B*d)*((\cos[e + f*x]\sqrt{c + d*\sin[e + f*x]})/(d*\sqrt{a + b*\sin[e + f*x]}) + (\sqrt{(a - b)/(a + b)}*(a + b)*\cos[(-e + \pi/2 - f*x)/2]*\text{EllipticE}[\text{ArcSin}[(\sqrt{(a - b)/(a + b)}*\sin[(-e + \pi/2 - f*x)/2])]/\sqrt{(a + b*\sin[e + f*x])/(a + b)}], (2*(-(b*c) + a*d))/((a - b)*(c + d)))*\sqrt{c + d*\sin[e + f*x]})/(b*d*\sqrt{((a + b)*\cos[(-e + \pi/2 - f*x)/2]^2)/(a + b*\sin[e + f*x])}*\sqrt{a + b*\sin[e + f*x]}\sqrt{(a + b*\sin[e + f*x])/(a + b)}*\sqrt{((a + b)*(c + d*\sin[e + f*x]))/((c + d)*(a + b*\sin[e + f*x]))}) - (2*(-(b*c) + a*d)*(((a + b)*c + a*d)*\sqrt{((c + d)*\cot[(-e + \pi/2 - f*x)/2]^2)/(-c + d)}*\text{EllipticF}[\text{ArcSin}[\sqrt{((-a - b)*\csc[(-e + \pi/2 - f*x)/2]^2*(c + d*\sin[e + f*x]))/(-(b*c) + a*d)}]/\sqrt{2}], (2*(-(b*c) + a*d))/((a + b)*(-c + d))*\sec[e + f*x]*\sin[(-e + \pi/2 - f*x)/2]^4*\sqrt{((c + d)*\csc[(-e + \pi/2 - f*x)/2]^2*(a + b*\sin[e + f*x]))/(-(b*c) + a*d)}*\sqrt{((-a - b)*\csc[(-e + \pi/2 - f*x)/2]^2*(c + d*\sin[e + f*x]))/(-(b*c) + a*d)}))/((a + b)*(c + d)*\sqrt{a + b*\sin[e + f*x]}\sqrt{c + d*\sin[e + f*x]}) - ((b*c + a*d)*\sqrt{((c + d)*\cot[(-e + \pi/2 - f*x)/2]^2)/(-c + d)}*\text{EllipticPi}[(-(b*c) + a*d)/((a + b)*d), \text{ArcSin}[\sqrt{((-a - b)*\csc[(-e + \pi/2 - f*x)/2]^2*(c + d*\sin[e + f*x]))/(-(b*c) + a*d)}]/\sqrt{2}], (2*(-(b*c) + a*d))/((a + b)*(-c + d))*\sec[e + f*x]*\sin[(-e + \pi/2 - f*x)/2]^4*\sqrt{((c + d)*\csc[(-e + \pi/2 - f*x)/2]^2*(a + b*\sin[e + f*x]))/(-(b*c) + a*d)}*\sqrt{((-a - b)*\csc[(-e + \pi/2 - f*x)/2]^2*(c + d*\sin[e + f*x]))/(-(b*c) + a*d)}))/((a + b)*d*\sqrt{a + b*\sin[e + f*x]}\sqrt{c + d*\sin[e + f*x]})))/(b*d)))/((a - b)*(a + b)*(-(b*c) + a*d)*f)$$

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A) \sqrt{b \sin(fx + e) + a} \sqrt{d \sin(fx + e) + c}}{2abd - (b^2c + 2abd) \cos(fx + e)^2 + (a^2 + b^2)c - (b^2d \cos(fx + e)^2 - 2abc - (a^2 + b^2)d) \sin(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(2*a*b*d - (b^2*c + 2*a*b*d)*cos(f*x + e)^2 + (a^2 + b^2)*c - (b^2*d*cos(f*x + e)^2 - 2*a*b*c - (a^2 + b^2)*d)*sin(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{d \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)

maple [B] time = 1.85, size = 99082, normalized size = 237.61

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}} \sqrt{d \sin(fx + e) + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(1/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*sqrt(d*sin(f*x + e) + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{\frac{3}{2}} \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{\frac{3}{2}} \sqrt{c + d \sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(1/2),
x)

[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*sqrt(c + d*sin(e + f*x))), x)

$$3.356 \quad \int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2}(c+d \sin(e+fx))^{3/2}} dx$$

Optimal. Leaf size=544

$$\frac{2 \sec(e+fx) \left(A(a^2 d^2 + b^2(c^2 - 2d^2)) - B(a^2 cd + ab(c^2 - d^2) - b^2 cd) \right) (c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}}}{f \sqrt{a+b} (c-d) \sqrt{c+d} (bc-ad)^3}$$

[Out] $-2*(A*(a^2*d^2+b^2*(c^2-2*d^2))-B*(a^2*c*d-b^2*c*d+a*b*(c^2-d^2)))*\text{EllipticE}((c+d)^{(1/2)}*(a+b*\sin(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*\sec(f*x+e)*(c+d*\sin(f*x+e))*((-a*d+b*c)*(1-\sin(f*x+e))/(a+b)/(c+d*\sin(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sin(f*x+e))/(a-b)/(c+d*\sin(f*x+e)))^{(1/2)}/(c-d)/(-a*d+b*c)^3/f/(a+b)^{(1/2)}/(c+d)^{(1/2)}+2*(-A*a*d+A*b*c-2*A*b*d+B*a*d+B*b*c)*\text{EllipticF}((c+d)^{(1/2)}*(a+b*\sin(f*x+e))^{(1/2)}/(a+b)^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)},((a+b)*(c-d)/(a-b)/(c+d))^{(1/2)})*\sec(f*x+e)*(c+d*\sin(f*x+e))*((-a*d+b*c)*(1-\sin(f*x+e))/(a+b)/(c+d*\sin(f*x+e)))^{(1/2)}*(-(-a*d+b*c)*(1+\sin(f*x+e))/(a-b)/(c+d*\sin(f*x+e)))^{(1/2)}/(c-d)/(-a*d+b*c)^2/f/(a+b)^{(1/2)}/(c+d)^{(1/2)}+2*b*(A*b-B*a)*\cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(a+b*\sin(f*x+e))^{(1/2)}/(c+d*\sin(f*x+e))^{(1/2)}$

Rubi [A] time = 1.38, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3000, 2998, 2818, 2996}

$$\frac{2 \sec(e+fx) \left(a^2(-A)d^2 + a^2 Bcd + abB(c^2 - d^2) - Ab^2(c^2 - 2d^2) - b^2 Bcd \right) (c+d \sin(e+fx)) \sqrt{\frac{(bc-ad)(1-\sin(e+fx))}{(a+b)(c+d \sin(e+fx))}}}{f \sqrt{a+b} (c-d) \sqrt{c+d} (bc-ad)^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + b*\text{Sin}[e + f*x])^{(3/2)}*(c + d*\text{Sin}[e + f*x])^{(3/2)})], x]$

[Out] $(2*b*(A*b - a*B)*\text{Cos}[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) + (2*(a^2*B*c*d - b^2*B*c*d - a^2*A*d^2 - A*b^2*(c^2 - 2*d^2) + a*b*B*(c^2 - d^2))*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sec}[e + f*x]*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 + \text{Sin}[e + f*x]))/((a - b)*(c + d*\text{Sin}[e + f*x]))])*(c + d*\text{Sin}[e + f*x])/(\text{Sqrt}[a + b]*(c - d)*\text{Sqrt}[c + d]*(b*c - a*d)^3*f) + (2*(A*b*c + b*B*c - a*A*d - 2*A*b*d + a*B*d)*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sec}[e + f*x]*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 + \text{Sin}[e + f*x]))/((a - b)*(c + d*\text{Sin}[e + f*x]))])*(c + d*\text{Sin}[e + f*x])/(\text{Sqrt}[a + b]*(c - d)*\text{Sqrt}[c + d]*(b*c - a*d)^2*f)$

Rule 2818

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*\sin[(e_) + (f_)*(x_)])*\text{Sqrt}[(c_) + (d_)*\sin[(e_) + (f_)*(x_)])], x_Symbol] \rightarrow \text{Simp}[(2*(c + d*\text{Sin}[e + f*x])*\text{Sqrt}[(b*c - a*d)*(1 - \text{Sin}[e + f*x])]/((a + b)*(c + d*\text{Sin}[e + f*x]))]*\text{Sqrt}[-((b*c - a*d)*(1 + \text{Sin}[e + f*x])]/((a - b)*(c + d*\text{Sin}[e + f*x])))]*\text{EllipticF}[\text{ArcSin}[\text{Rt}[(c + d)/(a + b), 2]*(\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*\text{Rt}[(c + d)/(a + b), 2]*\text{Cos}[e + f*x]), x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{PosQ}[(c + d)/(a + b)]$

Rule 2996

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*
(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> Sim
p[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 + Sin[e + f*x]))/
((c - d)*(a + b*Sin[e + f*x]))]*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/(c
+ d)*(a + b*Sin[e + f*x]))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt
[c + d*Sin[e + f*x]])/Sqrt[a + b*Sin[e + f*x]]], ((a - b)*(c + d))/(a + b
*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; Fr
eeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]
```

Rule 2998

```
Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*
(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)]]), x_Symbol] :> D
ist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x
]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[
e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e,
f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& NeQ[A, B]
```

Rule 3000

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) +
(f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> -S
imp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)
*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e +
f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2)
+ (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m
+ n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n},
x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Ration
alQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(Inte
gerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{3/2} (c + d \sin(e + fx))^{3/2}} dx = \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} - \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} + \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} \sqrt{c + d \sin(e + fx)}} + \dots$$

Mathematica [B] time = 7.43, size = 2266, normalized size = 4.17

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f
*x])^(3/2)), x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((2*(A*b^3*Cos[e + f*x]
- a*b^2*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^2*(a + b*Sin[e + f*x])
) - (2*(B*c*d^2*Cos[e + f*x] - A*d^3*Cos[e + f*x]))/((b*c - a*d)^2*(c^2 - d
^2)*(c + d*Sin[e + f*x])))/f + ((-4*(-(b*c) + a*d)*(a*A*b^2*c^3 - b^3*B*c^
3 - 2*a^2*A*b*c^2*d + 2*A*b^3*c^2*d + a^3*A*c*d^2 - 2*a*A*b^2*c*d^2 + b^3*B
*c*d^2 + 2*a^2*A*b*d^3 - 2*A*b^3*d^3 - a^3*B*d^3 + a*b^2*B*d^3)*Sqrt[((c +
d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Cs
c[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2
*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^
4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) +
a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*
c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*
x]]) - 4*(-(b*c) + a*d)*(A*b^3*c^3 - a*b^2*B*c^3 + a*A*b^2*c^2*d - 2*a^2*b*
B*c^2*d + b^3*B*c^2*d + a^2*A*b*c*d^2 - 2*A*b^3*c*d^2 - a^3*B*c*d^2 + 2*a*b
^2*B*c*d^2 + a^3*A*d^3 - 2*a*A*b^2*d^3 + a^2*b*B*d^3)*((Sqrt[((c + d)*Cot[(-
e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e +
Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c)
+ a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[
(c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sq
rt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d
)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - (
Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*
d)/((a + b)*d), ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin
[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))
]*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*
x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2
- f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*d*Sqrt[a + b*S
in[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])) + 2*(-(A*b^3*c^2*d) + a*b^2*B*c^2*d
+ a^2*b*B*c*d^2 - b^3*B*c*d^2 - a^2*A*b*d^3 + 2*A*b^3*d^3 - a*b^2*B*d^3)*(
(Cos[e + f*x]*Sqrt[c + d*Sin[e + f*x]])/(d*Sqrt[a + b*Sin[e + f*x]]) + (Sqr
t[(a - b)/(a + b)]*(a + b)*Cos[(-e + Pi/2 - f*x)/2]*EllipticE[ArcSin[(Sqrt[
(a - b)/(a + b)]*Sin[(-e + Pi/2 - f*x)/2])/Sqrt[(a + b*Sin[e + f*x])/(a + b
)]]], (2*(-(b*c) + a*d))/((a - b)*(c + d))] *Sqrt[c + d*Sin[e + f*x]]/(b*d*S
qrt[((a + b)*Cos[(-e + Pi/2 - f*x)/2]^2)/(a + b*Sin[e + f*x]))*Sqrt[a + b*S
in[e + f*x]]*Sqrt[(a + b*Sin[e + f*x])/(a + b)]*Sqrt[((a + b)*(c + d*Sin[e
+ f*x]))/(c + d)*(a + b*Sin[e + f*x]))] - (2*(-(b*c) + a*d)*(((a + b)*c
+ a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin
[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) +
a*d)]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e
+ Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e +
f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Si
n[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqr
t[c + d*Sin[e + f*x]]) - ((b*c + a*d)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2
]^2)/(-c + d)]*EllipticPi[(-(b*c) + a*d)/((a + b)*d), ArcSin[Sqrt[((-a - b)
*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/Sqrt[2]],
(2*(-(b*c) + a*d))/((a + b)*(-c + d))] *Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/
2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c)
+ a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-
(b*c) + a*d)]/((a + b)*d*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]
)))/(b*d))/((a - b)*(a + b)*(c - d)*(c + d)*(-(b*c) + a*d)^2*f)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{(B \sin(fx + e) + A) \sqrt{b \sin(fx + e) + a}}{b^2 d^2 \cos(fx + e)^4 + 4abcd + (a^2 + b^2)c^2 + (a^2 + b^2)d^2 - (b^2c^2 + 4abcd + (a^2 + 2b^2)d^2) \cos(fx + e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2), x,

algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(b^2*d^2*cos(f*x + e)^4 + 4*a*b*c*d + (a^2 + b^2)*c^2 + (a^2 + b^2)*d^2 - (b^2*c^2 + 4*a*b*c*d + (a^2 + 2*b^2)*d^2)*cos(f*x + e)^2 + 2*(a*b*c^2 + a*b*d^2 + (a^2 + b^2)*c*d - (b^2*c*d + a*b*d^2)*cos(f*x + e)^2)*sin(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}} (d \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)

maple [B] time = 3.29, size = 198381, normalized size = 364.67

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}} (d \sin(fx + e) + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(3/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{3/2} (c + d \sin(e + fx))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{\frac{3}{2}} (c + d \sin(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(3/2),  
x)
```

```
[Out] Integral((A + B*sin(e + f*x))/((a + b*sin(e + f*x))**(3/2)*(c + d*sin(e + f  
*x))**(3/2)), x)
```

$$3.357 \quad \int \frac{A+B \sin(e+fx)}{(a+b \sin(e+fx))^{3/2}(c+d \sin(e+fx))^{5/2}} dx$$

Optimal. Leaf size=858

$$\frac{2d \left(A \left((3c^2 - 4d^2) b^2 + a^2 d^2 \right) - B \left(cda^2 + 3b(c^2 - d^2)a - b^2 cd \right) \right) \sqrt{a + b \sin(e + fx)} \cos(e + fx)}{3(a^2 - b^2)(bc - ad)^2(c^2 - d^2)f(c + d \sin(e + fx))^{3/2}} + \frac{1}{(a^2 - b^2)(bc - ad)}$$

[Out] $2*b*(A*b-B*a)*\cos(f*x+e)/(a^2-b^2)/(-a*d+b*c)/f/(c+d*\sin(f*x+e))^{3/2}/(a+b*\sin(f*x+e))^{1/2}+2/3*d*(A*(a^2*d^2+b^2*(3*c^2-4*d^2))-B*(a^2*c*d-b^2*c*d+3*a*b*(c^2-d^2)))*\cos(f*x+e)*(a+b*\sin(f*x+e))^{1/2}/(a^2-b^2)/(-a*d+b*c)^2/(c^2-d^2)/f/(c+d*\sin(f*x+e))^{3/2}+2/3*(B*(2*a^2*b*c*d*(3*c^2-d^2)-2*b^3*c*d*(3*c^2-d^2)-a^3*d^2*(c^2+3*d^2)+a*b^2*(3*c^4-5*c^2*d^2+6*d^4))+A*(4*a^3*c*d^3-4*a*b^2*c*d^3-a^2*b*d^2*(9*c^2-5*d^2)-b^3*(3*c^4-15*c^2*d^2+8*d^4)))*\text{EllipticE}((c+d)^{1/2}*(a+b*\sin(f*x+e))^{1/2}/(a+b)^{1/2}/(c+d*\sin(f*x+e))^{1/2}),((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*\sec(f*x+e)*(c+d*\sin(f*x+e))*((-a*d+b*c)*(1-\sin(f*x+e))/(a+b)/(c+d*\sin(f*x+e)))^{1/2}*(-(-a*d+b*c)*(1+\sin(f*x+e))/(a-b)/(c+d*\sin(f*x+e)))^{1/2}/(c-d)^2/(c+d)^{3/2}/(-a*d+b*c)^4/f/(a+b)^{1/2}-2/3*(B*(a^2*d^2*(c+3*d)-b^2*c*(3*c^2+3*c*d-2*d^2)-6*a*b*d*(c^2-d^2))-A*(a^2*d^2*(3*c+d)-6*a*b*d*(c^2-d^2)+b^2*(3*c^3-9*c^2*d-6*c*d^2+8*d^3)))*\text{EllipticF}((c+d)^{1/2}*(a+b*\sin(f*x+e))^{1/2}/(a+b)^{1/2}/(c+d*\sin(f*x+e))^{1/2}),((a+b)*(c-d)/(a-b)/(c+d))^{1/2})*\sec(f*x+e)*(c+d*\sin(f*x+e))*((-a*d+b*c)*(1-\sin(f*x+e))/(a+b)/(c+d*\sin(f*x+e)))^{1/2}*(-(-a*d+b*c)*(1+\sin(f*x+e))/(a-b)/(c+d*\sin(f*x+e)))^{1/2}/(c-d)^2/(c+d)^{3/2}/(-a*d+b*c)^3/f/(a+b)^{1/2}$

Rubi [A] time = 2.62, antiderivative size = 858, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {3000, 3055, 2998, 2818, 2996}

$$\frac{2d \left(A \left((3c^2 - 4d^2) b^2 + a^2 d^2 \right) - B \left(cda^2 + 3b(c^2 - d^2)a - b^2 cd \right) \right) \sqrt{a + b \sin(e + fx)} \cos(e + fx)}{3(a^2 - b^2)(bc - ad)^2(c^2 - d^2)f(c + d \sin(e + fx))^{3/2}} + \frac{1}{(a^2 - b^2)(bc - ad)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Sin}[e + f*x])/((a + b*\text{Sin}[e + f*x])^{3/2}*(c + d*\text{Sin}[e + f*x])^{5/2}), x]$

[Out] $(2*b*(A*b - a*B)*\text{Cos}[e + f*x])/((a^2 - b^2)*(b*c - a*d)*f*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*(c + d*\text{Sin}[e + f*x])^{3/2}) + (2*d*(A*(a^2*d^2 + b^2*(3*c^2 - 4*d^2)) - B*(a^2*c*d - b^2*c*d + 3*a*b*(c^2 - d^2)))*\text{Cos}[e + f*x]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]/(3*(a^2 - b^2)*(b*c - a*d)^2*(c^2 - d^2)*f*(c + d*\text{Sin}[e + f*x])^{3/2}) + (2*(B*(2*a^2*b*c*d*(3*c^2 - d^2) - 2*b^3*c*d*(3*c^2 - d^2) - a^3*d^2*(c^2 + 3*d^2) + a*b^2*(3*c^4 - 5*c^2*d^2 + 6*d^4)) + A*(4*a^3*c*d^3 - 4*a*b^2*c*d^3 - a^2*b*d^2*(9*c^2 - 5*d^2) - b^3*(3*c^4 - 15*c^2*d^2 + 8*d^4)))*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sec}[e + f*x]*\text{Sqrt}[\frac{(b*c - a*d)*(1 - \text{Sin}[e + f*x])}{(a + b)*(c + d*\text{Sin}[e + f*x])}]]*\text{Sqrt}[\frac{-((b*c - a*d)*(1 + \text{Sin}[e + f*x]))}{(a - b)*(c + d*\text{Sin}[e + f*x])}]]*(c + d*\text{Sin}[e + f*x])/((3*\text{Sqrt}[a + b]*(c - d)^2*(c + d)^{3/2}*(b*c - a*d)^4*f) - (2*(B*(a^2*d^2*(c + 3*d) - b^2*c*(3*c^2 + 3*c*d - 2*d^2) - 6*a*b*d*(c^2 - d^2)) - A*(a^2*d^2*(3*c + d) - 6*a*b*d*(c^2 - d^2) + b^2*(3*c^3 - 9*c^2*d - 6*c*d^2 + 8*d^3)))*\text{EllipticF}[\text{ArcSin}[(\text{Sqrt}[c + d]*\text{Sqrt}[a + b*\text{Sin}[e + f*x]])/(\text{Sqrt}[a + b]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]*\text{Sec}[e + f*x]*\text{Sqrt}[\frac{(b*c - a*d)*(1 - \text{Sin}[e + f*x])}{(a + b)*(c + d*\text{Sin}[e + f*x])}]]*\text{Sqrt}[\frac{-((b*c - a*d)*(1 + \text{Sin}[e + f*x]))}{(a - b)*(c + d*\text{Sin}[e + f*x])}]]*(c + d*\text{Sin}[e + f*x])/((3*\text{Sqrt}[a + b]*(c - d)^2*(c + d)^{3/2}*(b*c - a*d)^3*f)$

Rule 2818

Int[1/(Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(2*(c + d*Sin[e + f*x])*Sqrt[((b*c - a*d)*(1 - Sin[e + f*x]))/((a + b)*(c + d*Sin[e + f*x]))])*Sqrt[-(((b*c - a*d)*(1 + Sin[e + f*x]))/((a - b)*(c + d*Sin[e + f*x])))]*EllipticF[ArcSin[Rt[(c + d)/(a + b), 2]*(Sqrt[a + b*Sin[e + f*x]]/Sqrt[c + d*Sin[e + f*x]])], ((a + b)*(c - d))/((a - b)*(c + d))]/(f*(b*c - a*d)*Rt[(c + d)/(a + b), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && PosQ[(c + d)/(a + b)]

Rule 2996

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Simp[(-2*A*(c - d)*(a + b*Sin[e + f*x])*Sqrt[(b*c - a*d)*(1 + Sin[e + f*x])]/((c - d)*(a + b*Sin[e + f*x])))*Sqrt[-(((b*c - a*d)*(1 - Sin[e + f*x]))/((c + d)*(a + b*Sin[e + f*x])))]*EllipticE[ArcSin[(Rt[(a + b)/(c + d), 2]*Sqrt[c + d*Sin[e + f*x]]/Sqrt[a + b*Sin[e + f*x]])], ((a - b)*(c + d))/((a + b)*(c - d))]/(f*(b*c - a*d)^2*Rt[(a + b)/(c + d), 2]*Cos[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && EqQ[A, B] && PosQ[(a + b)/(c + d)]

Rule 2998

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])/(((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(3/2)*Sqrt[(c_) + (d_)*sin[(e_) + (f_)*(x_)])], x_Symbol] := Dist[(A - B)/(a - b), Int[1/(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]), x], x] - Dist[(A*b - a*B)/(a - b), Int[(1 + Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*Sqrt[c + d*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && NeQ[A, B]

Rule 3000

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := -Simp[((A*b^2 - a*b*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(1 + n))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && RationalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

Rule 3055

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n])

) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || E
 qQ[a, 0]))

Rubi steps

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{3/2} (c + d \sin(e + fx))^{5/2}} dx = \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))^{3/2}}$$

$$= \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))^{3/2}}$$

$$= \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))^{3/2}}$$

$$= \frac{2b(Ab - aB) \cos(e + fx)}{(a^2 - b^2) (bc - ad) f \sqrt{a + b \sin(e + fx)} (c + d \sin(e + fx))^{3/2}}$$

Mathematica [B] time = 8.77, size = 2837, normalized size = 3.31

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(A + B*Sin[e + f*x])/((a + b*Sin[e + f*x])^(3/2)*(c + d*Sin[e + f*x])^(5/2)), x]

[Out] (Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]*((-2*(A*b^4*Cos[e + f*x] - a*b^3*B*Cos[e + f*x]))/((a^2 - b^2)*(-(b*c) + a*d)^3*(a + b*Sin[e + f*x])) + (2*(-(B*c*d^2*Cos[e + f*x]) + A*d^3*Cos[e + f*x]))/(3*(b*c - a*d)^2*(c^2 - d^2)*(c + d*Sin[e + f*x])^2) - (2*(6*b*B*c^3*d^2*Cos[e + f*x] - 9*A*b*c^2*d^3*Cos[e + f*x] - a*B*c^2*d^3*Cos[e + f*x] + 4*a*A*c*d^4*Cos[e + f*x] - 2*b*B*c*d^4*Cos[e + f*x] + 5*A*b*d^5*Cos[e + f*x] - 3*a*B*d^5*Cos[e + f*x]))/(3*(b*c - a*d)^3*(c^2 - d^2)^2*(c + d*Sin[e + f*x])))/f + ((-4*(-(b*c) + a*d)*(-3*a*A*b^3*c^5 + 3*b^4*B*c^5 + 9*a^2*A*b^2*c^4*d - 9*A*b^4*c^4*d - 9*a^3*A*b*c^3*d^2 + 15*a*A*b^3*c^3*d^2 - a^2*b^2*B*c^3*d^2 - 5*b^4*B*c^3*d^2 + 3*a^4*A*c^2*d^3 - 20*a^2*A*b^2*c^2*d^3 + 17*A*b^4*c^2*d^3 + 10*a^3*b*B*c^2*d^3 - 10*a*b^3*B*c^2*d^3 + 5*a^3*A*b*c*d^4 - 8*a*A*b^3*c*d^4 - 4*a^4*B*c*d^4 + 5*a^2*b^2*B*c*d^4 + 2*b^4*B*c*d^4 + a^4*A*d^5 + 7*a^2*A*b^2*d^5 - 8*A*b^4*d^5 - 6*a^3*b*B*d^5 + 6*a*b^3*B*d^5)*Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Csc[(-e + Pi/2 - f*x)/2]^2*(a + b*Sin[e + f*x]))/(-(b*c) + a*d)]*Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]/((a + b)*(c + d)*Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]]) - 4*(-(b*c) + a*d)*(-3*A*b^4*c^5 + 3*a*b^3*B*c^5 - 3*a*A*b^3*c^4*d + 9*a^2*b^2*B*c^4*d - 6*b^4*B*c^4*d - 9*a^2*A*b^2*c^3*d^2 + 15*A*b^4*c^3*d^2 + 5*a^3*b*B*c^3*d^2 - 11*a*b^3*B*c^3*d^2 - 5*a^3*A*b*c^2*d^3 + 11*a*A*b^3*c^2*d^3 - a^4*B*c^2*d^3 - 7*a^2*b^2*B*c^2*d^3 + 2*b^4*B*c^2*d^3 + 4*a^4*A*c*d^4 + a^2*A*b^2*c*d^4 - 8*A*b^4*c*d^4 - 5*a^3*b*B*c*d^4 + 8*a*b^3*B*c*d^4 + 5*a^3*A*b*d^5 - 8*a*A*b^3*d^5 - 3*a^4*B*d^5 + 6*a^2*b^2*B*d^5)*((Sqrt[((c + d)*Cot[(-e + Pi/2 - f*x)/2]^2)/(-c + d)]*EllipticF[ArcSin[Sqrt[((-a - b)*Csc[(-e + Pi/2 - f*x)/2]^2*(c + d*Sin[e + f*x]))/(-(b*c) + a*d)]]/Sqrt[2]], (2*(-(b*c) + a*d))/((a + b)*(-c + d))*Sec[e + f*x]*Sin[(-e + Pi/2 - f*x)/2]^4*Sqrt[((c + d)*Cs

$c[(-e + \text{Pi}/2 - f*x)/2]^2*(a + b*\text{Sin}[e + f*x])/(-b*c) + a*d]]*\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/((a + b)*(c + d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) - (\text{Sqrt}[((c + d)*\text{Cot}[(-e + \text{Pi}/2 - f*x)/2]^2)/(-c + d)]*\text{EllipticPi}[(-b*c) + a*d]/((a + b)*d), \text{ArcSin}[\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/\text{Sqrt}[2]], (2*(-b*c) + a*d)/((a + b)*(-c + d))] * \text{Sec}[e + f*x]*\text{Sin}[(-e + \text{Pi}/2 - f*x)/2]^4*\text{Sqrt}[((c + d)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(a + b*\text{Sin}[e + f*x])/(-b*c) + a*d)]*\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/((a + b)*d*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) + 2*(3*A*b^4*c^4*d - 3*a*b^3*B*c^4*d - 6*a^2*b^2*B*c^3*d^2 + 6*b^4*B*c^3*d^2 + 9*a^2*A*b^2*c^2*d^3 - 15*A*b^4*c^2*d^3 + a^3*b*B*c^2*d^3 + 5*a*b^3*B*c^2*d^3 - 4*a^3*A*b*c*d^4 + 4*a*A*b^3*c*d^4 + 2*a^2*b^2*B*c*d^4 - 2*b^4*B*c*d^4 - 5*a^2*A*b^2*d^5 + 8*A*b^4*d^5 + 3*a^3*b*B*d^5 - 6*a*b^3*B*d^5)*((\text{Cos}[e + f*x]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(d*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]) + (\text{Sqrt}[(a - b)/(a + b)]*(a + b)*\text{Cos}[(-e + \text{Pi}/2 - f*x)/2]*\text{EllipticE}[\text{ArcSin}[(\text{Sqrt}[(a - b)/(a + b)]*\text{Sin}[(-e + \text{Pi}/2 - f*x)/2]]/\text{Sqrt}[(a + b*\text{Sin}[e + f*x])/(a + b)]], (2*(-b*c) + a*d)/((a - b)*(c + d))] * \text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(b*d*\text{Sqrt}[(a + b)*\text{Cos}[(-e + \text{Pi}/2 - f*x)/2]^2)/(a + b*\text{Sin}[e + f*x]))*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[(a + b*\text{Sin}[e + f*x])/(a + b)]*\text{Sqrt}[(a + b)*(c + d*\text{Sin}[e + f*x])/((c + d)*(a + b*\text{Sin}[e + f*x]))] - (2*(-b*c) + a*d)*(((a + b)*c + a*d)*\text{Sqrt}[(c + d)*\text{Cot}[(-e + \text{Pi}/2 - f*x)/2]^2)/(-c + d)]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/\text{Sqrt}[2]], (2*(-b*c) + a*d)/((a + b)*(-c + d))] * \text{Sec}[e + f*x]*\text{Sin}[(-e + \text{Pi}/2 - f*x)/2]^4*\text{Sqrt}[((c + d)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(a + b*\text{Sin}[e + f*x])/(-b*c) + a*d)]*\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/((a + b)*(c + d)*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]]) - ((b*c + a*d)*\text{Sqrt}[(c + d)*\text{Cot}[(-e + \text{Pi}/2 - f*x)/2]^2)/(-c + d)]*\text{EllipticPi}[(-b*c) + a*d]/((a + b)*d), \text{ArcSin}[\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/\text{Sqrt}[2]], (2*(-b*c) + a*d)/((a + b)*(-c + d))] * \text{Sec}[e + f*x]*\text{Sin}[(-e + \text{Pi}/2 - f*x)/2]^4*\text{Sqrt}[((c + d)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(a + b*\text{Sin}[e + f*x])/(-b*c) + a*d)]*\text{Sqrt}[((-a - b)*\text{Csc}[(-e + \text{Pi}/2 - f*x)/2]^2*(c + d*\text{Sin}[e + f*x])/(-b*c) + a*d)]/((a + b)*d*\text{Sqrt}[a + b*\text{Sin}[e + f*x]]*\text{Sqrt}[c + d*\text{Sin}[e + f*x]])/(b*d))/((3*(a - b)*(a + b)*(c - d)^2*(c + d)^2*(-b*c) + a*d)^3*f)$

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral} \left(\frac{6abc^2d + 2abd^3 + (3b^2cd^2 + 2abd^3) \cos^4(fx + e) + (a^2 + b^2)c^3 + 3(a^2 + b^2)cd^2 - (b^2c^3 + 6abc^2d)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2), x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*sqrt(b*sin(f*x + e) + a)*sqrt(d*sin(f*x + e) + c)/(6*a*b*c^2*d + 2*a*b*d^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*cos(f*x + e)^4 + (a^2 + b^2)*c^3 + 3*(a^2 + b^2)*c*d^2 - (b^2*c^3 + 6*a*b*c^2*d + 4*a*b*d^3 + 3*(a^2 + 2*b^2)*c*d^2)*cos(f*x + e)^2 + (b^2*d^3*cos(f*x + e)^4 + 2*a*b*c^3 + 6*a*b*c*d^2 + 3*(a^2 + b^2)*c^2*d + (a^2 + b^2)*d^3 - (3*b^2*c^2*d + 6*a*b*c*d^2 + (a^2 + 2*b^2)*d^3)*cos(f*x + e)^2)*sin(f*x + e)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}}(d \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)

maple [B] time = 19.79, size = 827062, normalized size = 963.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{B \sin(fx + e) + A}{(b \sin(fx + e) + a)^{\frac{3}{2}} (d \sin(fx + e) + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))^(3/2)/(c+d*sin(f*x+e))^(5/2),x,
algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)/((b*sin(f*x + e) + a)^(3/2)*(d*sin(f*x + e) + c)^(5/2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + B \sin(e + fx)}{(a + b \sin(e + fx))^{\frac{3}{2}} (c + d \sin(e + fx))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)),x)

[Out] int((A + B*sin(e + f*x))/((a + b*sin(e + f*x))^(3/2)*(c + d*sin(e + f*x))^(5/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sin(f*x+e))/(a+b*sin(f*x+e))**(3/2)/(c+d*sin(f*x+e))**(5/2),
x)

[Out] Timed out

$$3.358 \quad \int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Optimal. Leaf size=38

$$\text{Int}((A + B \sin(e + fx))(a + b \sin(e + fx))^m (c + d \sin(e + fx))^n, x)$$

[Out] Unintegrable((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] Defer[Int][(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

Rubi steps

$$\int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx = \int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Mathematica [A] time = 17.69, size = 0, normalized size = 0.00

$$\int (a + b \sin(e + fx))^m (A + B \sin(e + fx))(c + d \sin(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n,x]

[Out] Integrate[(a + b*Sin[e + f*x])^m*(A + B*Sin[e + f*x])*(c + d*Sin[e + f*x])^n, x]

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}((B \sin(fx + e) + A)(b \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="fricas")

[Out] integral((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(b \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="giac")

[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

maple [A] time = 1.28, size = 0, normalized size = 0.00

$$\int (a + b \sin(fx + e))^m (A + B \sin(fx + e)) (c + d \sin(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] int((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (B \sin(fx + e) + A)(b \sin(fx + e) + a)^m (d \sin(fx + e) + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x, algorithm="maxima")

[Out] integrate((B*sin(f*x + e) + A)*(b*sin(f*x + e) + a)^m*(d*sin(f*x + e) + c)^n, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int (A + B \sin(e + fx)) (a + b \sin(e + fx))^m (c + d \sin(e + fx))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n,x)

[Out] int((A + B*sin(e + f*x))*(a + b*sin(e + f*x))^m*(c + d*sin(e + f*x))^n, x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(f*x+e))^m*(A+B*sin(f*x+e))*(c+d*sin(f*x+e))^n,x)

[Out] Exception raised: HeuristicGCDFailed

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]],
            If[SpecialFunctionQ[Head[expn]],
              Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
            If[HypergeometricFunctionQ[Head[expn]],
              Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
            If[AppellFunctionQ[Head[expn]],
              Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
            If[Head[expn]===RootSum,
              Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
            If[Head[expn]===Integrate || Head[expn]===Int,
              Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
            9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```



```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```



```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```